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MATEMATIK STATISTIKA
(*parametrlarni baholash va gipotezalarni tekshirish misollarda*)

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Qo‘llanma matematik statistika fani chuqur o‘rganiladigan universitetlarning talabalari uchun mo‘ljallangan bo‘lib, u bakalavriatning "Matematika", "Amaliy matematika va informatika", "Informatika va axborot texnologiyalari", "Axborot havfsizligi" va "Mexanika" yo‘nalishlari Davlat Ta’lim Standartlariga mos keladi. Ushbu qo‘llanmadan "Ehtimollar nazariyasi va matematik statistika" mutaxassisligi magistrantlari, katta ilmiy xodim-izlanuvchilar ham foydalanishlari mumkin. Qo‘llanmada matematik statistika bo‘yicha asosiy tushunchalar qisqacha keltirilgan bo‘lib, 300 dan ziyod misol va masalalar berilgan, hamda ulardan 20 dan ortig‘i batafsil yechimi bilan ko‘rsatilgan.

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SO‘Z BOSHI

Matematik statistika fani matematikaning tasodifiy tajribalar natijalarini qayta ishlashning matematik usullariga bag‘ishlangan bo‘limidir. Biz tasodifiy tajriba deganda natijasini oldindan aytib bo‘lmaydigan, ya’ni tasodifiyatga bog‘liq bo‘lgan harakatlar majmuasini tushunamiz. Ma’lumki, ehtimollar nazariyasi tasodifiy jarayonlarni matematik modelini o‘rganadi. Agar bu model to‘g‘ri tanlangan bo‘lsa, biz u yoki bu tasodifiy hodisa ehtimolini hisoblay olamiz va ehtimollarning turg‘unlik xossasiga asoslangan holda bu hodisalarning ro‘y berish sanog‘ini taqriban ayta olamiz. Matematik statistika alohida soha sifatida o‘rganilsada, uning asosiy usullari ehtimollar nazariyasi doirasida qolaveradi. Buning sababi shundaki, matematik statistika masalalari o‘ziga xos hususiyatlarga ega bo‘lib, ma’lum ma’noda ehtimollar nazariyasi masalalariga teskaridir. Agar biz ehtimollar nazariyasida tasodifiy jarayonlar modeli ma’lum deb hisoblangan holda shu jarayonlar haqida ma’lum xulosalar qilsak, matematik statistikada esa aksincha qandaydir tasodifiy hodisalarning sonli xarakterga ega bo‘lgan statistik ma’lumotlariga asoslangan holda tegishli ehtimollik modelini tanlaymiz. Demak, matematik statistika tasodifiy tajribalarni o‘rganish nuqtai nazaridan qaralganda ehtimollar nazariyasini teskari masalalari bilan shug‘ullanar ekan.

Ushbu qo‘llanmada matematik statistikani o‘rganish uchun kerakli barcha tushunchalar keltirilgan bo‘lib, unda mustaqil yechish uchun misollar keltirilgan. Ushbu qo‘llanma universitetlarning bakalavriatura yo‘nalishi talabalari bilan bir qatorda "Ehtimollar nazariyasi va matematik statistika" mutaxassisligi magistrantlari, katta ilmiy xodim-izlanuvchilar uchun ham foydalidir. Bu qo‘llanmadan mustaqil ta’lim uchun ham foydalansa bo‘ladi. Mualliflar qo‘llanmani nashrga tayyorlashdagi yordami uchun N.F.Usmonovaga o‘z minnatdorchiliklarini bildiradilar.

1-§. TASODIFIY MIQDORLAR VA ULARNING SONLI XARAKTERISTIKALARI

Agar Ω elementar hodisalar fazosida aniqlangan ξ sonli funksiya har bir ω elementar hodisaga $\xi(\omega)$ sonni mos qo'ysa, ya'ni $\xi = \xi(\omega)$, $\omega \in \Omega$ bo'lsa, u tasodifiy miqdor deyiladi. Demak, tajriba natijasida u yoki bu qiymatni qabul qilishi oldindan ma'lum bo'lmagan miqdor tasodifiy miqdor deb atalar ekan. Agar t.m. ko'pi bilan sanoqli qiymatlar qabul qilsa, bunday t.m. diskret tipdagi t.m. deyiladi. Agar t.m. qabul qiladigan qiymatlari biror oraliqdan iborat bo'lsa, u holda uzluksiz tipdagi t.m. deyiladi.

Demak, diskret t.m. bir-biridan farqli alohida qiymatlarni, uzluksiz t.m. esa biror oraliqdagi ixtiyoriy qiymatlarni qabul qilar ekan.

$F(x)$ funksiya ξ t.m.ning taqsimot funksiyasi $\forall x \in R$ son uchun quyidagicha aniqlanadi: $F(x) = P\{\xi < x\} = P\{\omega : \ \xi(\omega) < x\}$.

Taqsimot funksiyasi quyidagi xossalarga ega:

1. $F(x)$ chegaralangan: $0 \leq F(x) \leq 1$.

2. $F(x)$ kamaymaydigan funksiya: agar $x_1 < x_2$ bo'lsa, u holda $F(x_1) \leq F(x_2)$.

3. $F(-\infty) = \lim\limits_{x \to -\infty} F(x) = 0, \quad F(+\infty) = \lim\limits_{x \to +\infty} F(x) = 1.$

4. $F(x)$ funksiya chapdan uzluksiz: $\lim\limits_{x \to x_0 - 0} F(x) = F(x_0)$.

5. ξ diskret t.m. t.f. $F(x)$ ning sakrash nuqtalari to'plami ko'pi bilan sanoqlidir.

ξ t.m. uzluksiz deyiladi, agar uning t.f. ixtiyoriy nuqtada uzluksiz bo'lsa. Uzluksiz t.m. *zichlik funksiyasi* yordamida beriladi.

Uzluksiz t.m. *zichlik funksiyasi* deb, shu t.m. taqsimot funksiyasidan olingan birinchi tartibli hosilaga aytiladi. Zichlik funksiyasini $f(x)$ orqali belgilaymiz:

$$f(x) = F'(x). \tag{1}$$

Zichlik funksiyasi quyidagi xossalarga ega:

1. $f(x)$ funksiya manfiy emas: $f(x) \geq 0$.

2. ξ uzluksiz t.m.ning $[a, b]$ oraliqqa tegishli qiymatni qabul qilishi ehtimolligi zichlik funksiyaning a dan b gacha olingan aniq integraliga

teng:

$$P\{a \leq \xi \leq b\} = \int_a^b f(x)dx. \tag{2}$$

3. Uzluksiz t.m. taqsimot funksiyasi zichlik funksiya orqali quyidagicha ifodalanadi:

$$F(x) = \int_{-\infty}^{x} f(t)dt. \tag{3}$$

4. Zichlik funksiyasidan $-\infty$ dan $+\infty$ gacha olingan integrali birga tengdir:

$$\int_{-\infty}^{+\infty} f(x)dx = 1.$$

ξ diskret t.m.ning taqsimot qonuni berilgan bo‘lsin: $\{p_i = P\{\xi = x_i\},\ i = 1, 2, ..., n, ...\}$.

ξ diskret t.m.ning *matematik kutilmasi* $\sum\limits_{i=1}^{\infty} x_i p_i$ qator orqali hisoblanadi va $M\xi$ orqali belgilanadi:

$$M\xi = \sum_{i=1}^{\infty} x_i p_i. \tag{4}$$

ξ uzluksiz t.m.ning matematik kutilmasi

$$M\xi = \int_{-\infty}^{+\infty} xf(x)dx \tag{5}$$

integral orqali hisoblanadi.

ξ t.m.ning dispersiyasi $M(\xi - M\xi)^2$ ifoda bilan aniqlanadi va $D\xi$ orqali belgilanadi. T.m. dispersiyasini hisoblash uchun quyidagi formula qulaydir: $D\xi = M\xi^2 - (M\xi)^2$.

Ikki o‘lchovlik tasodifiy miqdorlar. Faraz qilaylik, $(\Omega, \mathscr{A}, P)$ ehtimollik fazosida aniqlangan ξ, η t.m.lar berilgan bo‘lsin. (ξ, η) vektorga tasodifiy vektor yoki *2-o‘lchovli t.m.* deyiladi. Ikki o‘lchovli t.m.

har bir elementar hodisa ω ga 2 ta ξ va η t.m.larning qabul qiladigan qiymatlarini mos qo'yadi. (ξ,η) ikki o'lchovli diskret t.m. taqsimot qonunini

$$p_{ij} = P\{\xi = x_i, \eta = y_j\}; \quad i = \overline{1,n},\ j = \overline{1,m} \tag{6}$$

sistema yordamida yoki quyidagi jadval ko'rinishda berish mumkin:

	η				
ξ	y_1	y_2	$\dots$	y_m	$P\{\xi = x_i\}\ i = \overline{1,n}$
x_1	p_{11}	p_{12}	$\dots$	p_{1m}	$\sum\limits_{j=1}^{m} p_{1j}$
x_2	p_{21}	p_{22}	$\dots$	p_{2m}	$\sum\limits_{j=1}^{m} p_{2j}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_n	p_{n1}	p_{n2}	$\dots$	p_{nm}	$\sum\limits_{j=1}^{m} p_{nj}$
$P\{\eta = y_j\}$, $j = \overline{1,m}$	$\sum\limits_{i=1}^{n} p_{i1}$	$\sum\limits_{i=1}^{n} p_{i2}$	$\dots$	$\sum\limits_{i=1}^{n} p_{im}$	1

bu yerda barcha p_{ij} ehtimolliklar yig'indisi birga teng (chunki birgalikda bo'lmagan $\{\xi = x_i, \eta = y_j\}\, i = \overline{1,n},\, j = \overline{1,m}$ hodisalar to'la gruppani tashkil etadi: $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m} p_{ij} = 1$).

Agar (ξ,η) ikki o'lchovli diskret t.m.ning birgalikdagi taqsimot qonuni berilgan bo'lsa, har bir komponentaning alohida (marginal) taqsimot qonunlarini topish mumkin. Har bir $i = \overline{1,n}$ uchun $\{\xi = x_i, \eta = y_1\}$, $\{\xi = x_i, \eta = y_2\}$, ..., $\{\xi = x_i, \eta = y_m\}$ hodisalar birgalikda bo'lmagani sababli: $p_{x_i} = P\{\xi = x_i\} = p_{i1} + p_{i2} + \dots + p_{im}$. Demak, (ξ,η) tasodifiy vektorning marginal taqsimotlari $p_{x_i} = P\{\xi = x_i\} = \sum\limits_{j=1}^{m} p_{ij}$, $\ i = \overline{1,n}$, $q_{y_j} = P\{\eta = y_j\} = \sum\limits_{i=1}^{n} p_{ij}\ j = \overline{1,n}$.

$F(x,y) = P\{\xi < x, \eta < y\}$ ikki o'lchovli funksiya (ξ,η) tasodifiy vektorning t.f. yoki ξ,η t.m.larning *birgalikdagi taqsimot funksiyasi* deyiladi.

Ikki oʻlchovli $F(x,y)$ t.f.ning asosiy xossalarini keltiramiz:

1. $\forall x,y: \quad 0 \leq F(x,y) \leq 1$, ya'ni taqsimot funksiya chegaralangan.

2. $F(x,y)$ funksiya har qaysi argumenti boʻyicha kamayuvchi emas va chapdan uzluksiz.

3. $F(x,y)$ funksiyaning kamida bir argumenti $-\infty$ ga teng boʻlsa(limit ma'nosida), u holda $F(x,y)$ funksiya nolga teng: $F(x,-\infty) = F(-\infty,y) = F(-\infty,-\infty) = 0$.

4. Agar $F(x,y)$ funksiyaning bitta argumenti $+\infty$ boʻlsa(limit ma'nosida), u holda

$$F(x,+\infty) = F_\xi(x);\ F(+\infty,y) = F_\eta(y). \tag{7}$$

5. Agar ikkala argumenti $+\infty$ boʻlsa (limit ma'nosida), u holda $F(+\infty,+\infty) = 1$.

Agar ikki oʻlchovlik (ξ,η) t.m. t.f. $F(x,y)$:

1. uzluksiz;

2. har bir argumenti boʻyicha differensiallanuvchi;

3. $F''_{xy}(x,y)$ ikkinchi tartibli aralash hosila mavjud boʻlsa, u holda (ξ,η) uzluksiz t.m. deyiladi.

Ikki oʻlchovlik (ξ,η) *t.m.ning zichlik funksiyasi*

$$f(x,y) = \frac{\partial^2 F(x,y)}{\partial x \partial y} = F''_{xy}(x,y) \tag{8}$$

-ikkinchi tartibli xususiy va aralash hosila orqali aniqlanadi.

$f(x,y)$ zichlik funkiyasi quyidagi xossalarga ega:

1. $f(x,y) \geq 0$;

2. $P\{(\xi,\eta) \in D\} = \int\int\limits_D f(x,y)dxdy;$ (9)

3. $F(x,y) = \int\limits_{-\infty}^{x}\int\limits_{-\infty}^{y} f(u,v)dudv;$ (10)

4. $\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} f(x,y)dxdy = 1;$

5. ξ va η t.m.larning marginal zichlik funksiyalarini quyidagi formulalar yordamida topish mumkin:

$$\int_{-\infty}^{+\infty} f(x,y)dy = f_\xi(x); \quad \int_{-\infty}^{+\infty} f(x,y)dx = f_\eta(y). \tag{11}$$

Agar $\forall x, y \in R$ uchun $\{\xi < x\}$ va $\{\eta < y\}$ hodisalar bog'liqsiz bo'lsa, ξ va η t.m.lar *bog'liqsiz* deyiladi. Endi t.m.lar bog'liqsizligining zarur va yetarli shartini keltiramiz.

Teorema. ξ va η t.m.lar bog'liqsiz bo'lishi uchun barcha x, y lar uchun

$$F(x,y) = F_\xi(x)F_\eta(y) \tag{12}$$

tenglik bajarilishi zarur va yetarlidir.

ξ va η uzluksiz t.m.lar bog'liqsiz bo'lishi uchun $\forall x, y$ lar uchun $f(x,y) = f_\xi(x)\ f_\eta(y)$ tenglikning bajarilishi zarur va yetarlidir;

ξ va η diskret t.m.lar bog'liqsiz bo'lishi uchun esa, ixtiyoriy $i = 1, 2, ...n,\ j = 1, 2, ...m$ larda $P\{\xi = x_i, \eta = y_j\} = P\{\xi = x_i\}\ P\{\eta = y_j\}$ tenglikning bajarilishi zarur va yetarlidir$(n, m \leq \infty)$.

Diskret ξ va η t.m.larning *matematik kutilmalari* birgalikdagi taqsimoti orqali

$$M\xi = m_x = \sum_{i=1}^{n}\sum_{j=1}^{m} x_i p_{ij}, \quad M\eta = m_y = \sum_{i=1}^{n}\sum_{j=1}^{m} y_i p_{ij} \tag{13}$$

formulalar, uzluksiz t.m.larning *matematik kutilmalari* esa

$$M\xi = m_x = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x\ f(x,y)dxdy, \quad M\eta = m_y = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} y\ f(x,y)dxdy \tag{14}$$

integrallar orqali hisoblanadi.

ξ va η t.m.larning *kovariatsiyasi*

$$K_{\xi\eta} = cov(\xi, \eta) = M\left((\xi - m_x)(\eta - m_y)\right) \tag{15}$$

matematik kutilma bilan aniqlanadi. Agar (ξ, η) t.m. diskret bo'lsa, ularning kovariatsiyasi $K_{\xi\eta} = \sum_{i=1}^{n}\sum_{j=1}^{m}(x_i - m_x)(y_j - m_y)p_{ij}$, agar ular uzluksiz bo'lsa, $K_{\xi\eta} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x - m_x)(y - m_y)f(x, y)dxdy$ integral orqali hisoblanadi.

Kovariatsiyani quyidagicha hisoblash ham mumkin:

$$K_{\xi\eta} = cov(\xi, \eta) = M\xi\eta - M\xi M\eta. \qquad (16)$$

ξ va η t.m.larning *korrelatsiya koeffitsienti*

$$r_{\xi\eta} = \frac{cov(\xi, \eta)}{\sqrt{D\xi}\sqrt{D\eta}} \qquad (17)$$

formula bilan aniqlanadi.

Korrelyatsiya koeffisiyentining xossalari:

1. $|r_{\xi\eta}| \leq 1$, ya'ni $-1 \leq r_{\xi\eta} \leq 1$;

2. Agar $\xi \perp \eta$ bo'lsa, u holda $r_{\xi\eta} = 0$;

3. Agar $|r_{\xi\eta}| = 1$ bo'lsa, u holda ξ va η t.m.lar chiziqli funksional bog'liq va aksincha.

Bir argumentning funksiyalari. Agar ξ t.m.ning har bir qiymatiga biror qoida bo'yicha mos ravishda η t.m.ning bitta qiymati mos qo'yilsa, u holda η ni ξ tasodifiy argumentning funksiyasi deyiladi va $\eta = \varphi(\xi)$ kabi yoziladi.

ξ diskret t.m. $x_1, x_2, ..., x_n$ qiymatlarni mos $p_1, p_2, ..., p_n$ ehtimolliklar bilan qabul qilsin: $p_i = P\{\xi = x_i\}$, $i = 1, 2, ..., n$. Ravshanki, $\eta = \varphi(\xi)$ t.m. ham diskret t.m. bo'ladi va uning qabul qiladigan qiymatlari $y_1 = \varphi(x_1)$, $y_2 = \varphi(x_2)$, ..., $y_n = \varphi(x_n)$, mos ehtimolliklari esa $p_1, p_2, ..., p_n$ bo'ladi. Demak, $p_i = P\{\eta = y_i\} = P\{\eta = \varphi(x_i)\}$, $i = 1, 2, ..., n$. Shuni ta'kidlash lozimki, ξ t.m.ning har xil qiymatlariga mos η t.m.ning bir xil qiymatlari mos kelishi mumkin. Bunday hollarda qaytarilayotgan qiymatlarning ehtimolliklarini qo'shish kerak bo'ladi.

$\eta = \varphi(\xi)$ t.m.ning matematik kutilmasi va dispersiyasi quyidagicha aniqlanadi:

$$M\eta = \sum_{i=1}^{n} \varphi(x_i)p_i, \quad D\eta = \sum_{i=1}^{n} (\varphi(x_i) - M\eta)^2 p_i.$$

Zichlik funksiyasi $f(x)$ bo'lgan ξ uzluksiz t.m. va η t.m. ξ t.m.ning funksiyasi $\eta = \varphi(\xi)$ bo'lsin. η t.m.ning taqsimotini aniqlaymiz. $\eta = \varphi(\xi)$ funksiya ξ t.m.ning barcha qiymatlarida uzluksiz, (a, b) intervalda qat'iy o'suvchi va differensiallanuvchi bo'lsin, u holda $y = \varphi(x)$ funksiyaga teskari $x = \psi(y)$ funksiya mavjud. η t.m.ning taqsimot funksiyasi $G(y) = P\{\eta < y\}$ formula orqali aniqlanadi. $\{\eta < y\}$ hodisa $\{\xi < \psi(y)\}$ hodisaga ekvivalent. Yuqoridagilarni e'tiborga olsak,

$$G(y) = P(\eta < y) = P(\xi > \psi(y)) = 1 - P(\xi < \psi(y)) =$$

$$= 1 - F_\xi(\psi(y)) = 1 - \int_a^{\psi(y)} f(x)dx. \qquad (18)$$

(18) ni y bo'yicha differensiallaymiz va η t.m.ning zichlik funksiyasini topamiz: $g(y) = \frac{dG(y)}{dy} = f(\psi(y)) \, \frac{d}{dy}(\psi(y)) = f(\psi(y))\psi'(y)$.

Demak, $g(y) = f(\psi(y))\psi'(y)$. Agar $y = \varphi(x)$ funksiya (a,b) intervalda qat'iy kamayuvchi bo'lsa, u holda $\{\eta < y\}$ hodisa $\{\xi < \psi(y)\}$ hodisaga ekvivalent. Shuning uchun,

$$G(y) = \int_{\psi(y)}^{b} f(x)dx = -\int_{b}^{\psi(y)} f(x)dx.$$

Bu yerdan, $g(y) = -f(\psi(y))\psi'(y)$. Zichlik funksiya manfiy bo'lmasligini hisobga olib, bu formulalarni umumlashtirish mumkin: $g(y) = f(\psi(y)) \left|\psi'(y)\right|$.

Agar ξ zichlik funksiyasi $f(x)$ bo'lgan uzluksiz t.m. bo'lsa, u holda $\eta = \varphi(\xi)$ t.m.ning sonli xarakteristikalarini hisoblash uchun

η t.m.ning taqsimotini qo'llash shart emas:

$$M\eta = M(\varphi(\xi)) = \int_{-\infty}^{+\infty} \varphi(x)f(x)dx,$$

$$D\eta = D(\varphi(\xi)) = \int_{-\infty}^{+\infty} (\varphi(x) - M\eta)^2 f(x)dx. \qquad (19)$$

Ikki argumentning funksiyalari. Agar ξ va η t.m.lar qabul qiladigan qiymatlarining har bir juftligiga biror qoidaga ko'ra ζ t.m. mos qo'yilsa, u holda ζ t.m. ikki ξ va η tasodifiy argumentning funksiyasi deyiladi va $\zeta = \varphi(\xi, \eta)$ kabi belgilanadi. Agar ξ va η t.m.lar bog'liqsiz bo'lsa, u holda $\zeta = \xi + \eta$ t.m. zichlik funksiyasi

$$f_\zeta(z) = f_{\xi+\eta}(z) = \int_{-\infty}^{+\infty} f_\xi(x)f_\eta(z-x)dx,$$

yoki

$$f_\zeta(z) = f_{\xi+\eta}(z) = \int_{-\infty}^{+\infty} f_\xi(z-y)f_\eta(y)dy \qquad (20)$$

formulalar bilan hisoblanadi. Bog'liqsiz t.m.lar yig'indisining taqsimoti shu t.m.lar taqsimotlarining *kompozitsiyasi* deyiladi. ζ t.m.ning zichlik funksiyasi $f_{\xi+\eta} = f_\xi * f_\eta$ ko'rinishda yoziladi, bu yerda * - kompozitsiya belgisi. Umumiy holda, $\xi \perp \eta$ bo'lsa, kompozitsiyani t.f.lar orqali yozish mumkin:

$$F_\zeta(z) = F_{\xi+\eta}(z) = \int_{-\infty}^{+\infty} F_\eta(z-x)dF_\xi(x) = \int_{-\infty}^{+\infty} F_\xi(z-y)dF_\eta(y).$$

1-misol. ξ t.m.ning $f(x)$ zichlik funksiyasi berilgan bo'lsin: $f(x) = \begin{cases} 0, & x < 1, \\ C/x^4, & x \geq 1. \end{cases}$ Quyidagilarni hisoblang: a) C; b)$F(x)$; c) $M\xi$; d) $P(\xi > M\xi)$;

▷ *a*) C sonini zichlikning $\int\limits_{-\infty}^{\infty} f(x)dx = 1$ xossasidan foydalanib hisoblaymiz: $1 = \int\limits_{-\infty}^{\infty} f(x)dx = C\int\limits_{1}^{\infty} x^{-4}dx = \frac{C}{3}; \quad C = 3.$

b) Endi t.f. ni hisoblaymiz: $x \in [1, +\infty)$: $F(x) = \int\limits_{1}^{x} \frac{3}{t^4}dt = -t^{-3}|_1^x = 1 - \frac{1}{x^3}$, $x \in (-\infty, 1)$: $\quad F(x) = 0.$

Demak, $F(x) = \begin{cases} 1 - \frac{1}{x^3}, & x \geq 1, \\ 0, & x < 1. \end{cases}$

c) $M\xi = \int\limits_{-\infty}^{+\infty} x\ f(x)dx = \int\limits_{1}^{+\infty} x\frac{3}{x^4}dx = -3\frac{x^{-2}}{2}\Big|_1^{+\infty} = \frac{3}{2};$

d) $P(\xi > M\xi) = P\left(\xi > \frac{3}{2}\right) = P\left(\frac{3}{2} < \xi < +\infty\right) = F(+\infty) - F\left(\frac{3}{2}\right) = 1 - \left(1 - \frac{1}{\left(\frac{3}{2}\right)^3}\right) = \left(\frac{2}{3}\right)^3.$ ◁

I. ξ t.m.ning $f(x)$ zichlik funksiyasi berilgan bo‘lsin:

1. $f(x) = \begin{cases} Cx, & x \in [0, 1], \\ C, & x \in (1, 2], \\ 0, & \text{aks holda.} \end{cases}$

2. $f(x) = \begin{cases} C(1 - x/3), & x \in [0, 3], \\ 0, & x \notin [0, 3]. \end{cases}$

3. $f(x) = \begin{cases} 0, & x \leq 0, \\ C/(x+1)^4, & x > 0. \end{cases}$

4. $f(x) = \begin{cases} 0, & x < 1, \\ Ce^{1-x}, & x \geq 1. \end{cases}$

5. $f(x) = \begin{cases} C/\sqrt{1 - x^2}, & x \in [-1, 1], \\ 0, & x \notin [-1, 1]. \end{cases}$

6. $f(x) = \begin{cases} C\sqrt{1 - x}, & x \in [0, 1], \\ 0, & x \notin [0, 1]. \end{cases}$

7. $f(x) = \begin{cases} C/(1 + x^2), & x \in [0, \sqrt{3}], \\ 0, & x \notin [0, \sqrt{3}]. \end{cases}$

8. $f(x) = \begin{cases} 0, & x \leq 0, \\ Cxe^{-0.5x}, & x > 0. \end{cases}$

9. $f(x) = \begin{cases} C/x, & x \in [1/e, e], \\ 0, & x \notin [1/e, e]. \end{cases}$

10. $f(x) = \begin{cases} 0, & x < 0, \\ C/(x+1)^5, & x \geq 0. \end{cases}$

11. $f(x) = \begin{cases} C(1 - 0.5\,|x|), & x \in [-2, 2], \\ 0, & x \notin [-2, 2]. \end{cases}$

12. $f\,(x) = \begin{cases} 0, & x \notin (0, \pi/2) \\ C\sin x, & x \in (0, \pi/2) \end{cases}$

13. $f(x) = \begin{cases} C\cos x, & x \in \left[0, \frac{\pi}{2}\right], \\ 0, & x \notin \left[0, \frac{\pi}{2}\right]. \end{cases}$

14. $f(x) = \begin{cases} C\left(|x| + \frac{1}{4}\right), & x \in [-1, 1], \\ 0, & x \notin [-1, 1]. \end{cases}$

15. $f(x) = \begin{cases} 0, & x \leq 0, \\ Cxe^{-x}, & x > 0. \end{cases}$

16. $f(x) = \begin{cases} C\ln x, & x \in [1, e], \\ 0, & x \notin [1, e]. \end{cases}$

17. $f(x) = \begin{cases} 2x/3, & x \in [0, 1], \\ C(3 - x), & x \in (1, 3], \\ 0, \text{aks holda.} \end{cases}$

18. $f(x) = \begin{cases} C(1 - |x|), & x \in [-1, 1], \\ 0, & x \notin [-1, 1]. \end{cases}$

19. $f(x) = \begin{cases} C\sqrt[3]{1 - x}, & x \in [0, 1], \\ 0, & x \notin [0, 1]. \end{cases}$

20. $f(x) = \begin{cases} (x+1)/2, & x \in [-1,0], \\ (C-x)/2C, x \in (0,C], \\ 0, x \notin [-1,C]. \end{cases}$

Quyidagilarni hisoblang: a) C; b) $F(x)$; c) $M\xi$; d) $D\xi$; e) $P(\xi > M\xi)$.

II. Bir argumentning funksiyalari

2-misol. ξ t.m. α parametrli ko‘rsatkichli taqsimotga ega: $P(\xi < x) = 1 - e^{-\alpha x}$, $x \geq 0$. $\eta_1 = \sqrt{\xi}$ t.m.ning zichlik funksiyasini toping.

$\triangleright$ Avval η_1 ning taqsimot funksiyasini topamiz: agar $x \geq 0$ bo‘lsa, $F_{\eta_1}(x) = P(\sqrt{\xi} < x) = P(\xi < x^2) = F_\xi(x^2) = 1 - e^{-\alpha x^2}$; $x < 0$ uchun $F_{\eta_1}(x) = 0$.

Demak, $F_{\eta_1}(x) = \begin{cases} 0, \ x < 0, \\ 1 - e^{-\alpha x^2}, \ x \geq 0, \end{cases}$ t.f.ning zichlik funksiyasi esa $f_{\eta_1}(x) = (F_{\eta_1}(x))' = \begin{cases} 0, \ x < 0, \\ 2\alpha x e^{-\alpha x^2}, \ x \geq 0, \end{cases}$ ekan. $\triangleleft$

3-misol. ξ t.m. $[0,1]$ oraliqda tekis taqsimlangan. Quyidagi t.m.ning zichlik funksiyasini toping: $\eta_2 = -\ln(1-\xi)$.

$\triangleright$ Avval η_2 ning taqsimot funksiyasini topamiz:agar $x > 0$ bo‘lsa, $F_{\eta_2}(x) = P(\eta_2 < x) = P(-\ln(1-\xi) < x) = P(0 \leq \xi \leq 1 - e^{-x}) = F_\xi(1 - e^{-x})$. Bundan $f_{\eta_2}(x)$ ni hisoblaymiz. $x > 0$ da: $f_{\eta_2}(x) = F'_{\eta_2}(-e^{-x}+1)e^{-x} = e^{-x} f_\xi(1 - e^{-x}) = e^{-x}$ va $x \leq 0$ da: $f_{\eta_2}(x) = 0$. $\triangleleft$

Misollar

1. $\xi \sim R(-4,4)$ bo‘lsin, u holda $\eta_1 = \frac{\xi+4}{8}$ va $\eta_2 = \frac{1-\xi}{2}$ t.m.larning zichlik funksiyalarini toping.

2. $\xi \sim E(\alpha)$ bo‘lsin, u holda $\eta_1 = \ln(\xi+1)$ va $\eta_2 = 1/(\xi+1)$ t.m.larning zichlik funksiyalarini toping.

3. $\xi \sim N(a, \sigma^2)$ bo‘lsin, u holda $\eta_1 = 2\xi + 1$ va $\eta_2 = 1 - \xi$ t.m.larning zichlik funksiyalarini toping.

4. ξ t.m.ning t.f. $F_\xi(x) = \begin{cases} 0, \ x \leq 0, \\ x, \ 0 < x < 1, \\ 1, \ x \geq 1 \end{cases}$ bo'lsin. $\eta_1 = \xi^2 + 1$ va $\eta_2 = 1/(1+\xi)$ t.m.larning zichlik funksiyalarini toping.

5. $\xi \sim R(-1,1)$ bo'lsin, u holda $\eta_1 = |\xi|$ va $\eta_2 = \frac{|1-\xi|}{2}$ t.m.larning zichlik funksiyalarini toping.

6. $\xi \sim N(a, \sigma^2)$ bo'lsin, u holda $\eta_1 = 5\xi - 1$ va $\eta_2 = -\xi + 1$ t.m.larning zichlik funksiyalarini toping.

7. $\xi \sim E(\alpha)$ bo'lsin, u holda $\eta_1 = \xi^2$ va $\eta_2 = e^{-\xi}$ t.m.larning zichlik funksiyalarini toping.

8. ξ t.m.ning zichlik funksiyasi $f_\xi(x) = \frac{1}{\pi}\frac{1}{1+x^2}$ bo'lsin. $\eta_1 = \xi^2$ va $\eta_2 = 1/(1+\xi^2)$ t.m.larning zichlik funksiyalarini toping.

9. $\xi \sim N(0,1)$ bo'lsin, u holda $\eta_1 = \xi^2$ va $\eta_2 = \frac{\xi+1}{2}$ t.m.larning zichlik funksiyalarini toping.

10. $\xi \sim R(0,1)$ bo'lsin, u holda $\eta_1 = -\ln(1-\xi)$ va $\eta_2 = tg(\pi(\xi - \frac{1}{2}))$ t.m.larning zichlik funksiyalarini toping.

11. $\xi \sim E(\alpha)$ bo'lsin, u holda $\eta_1 = \xi^2 + 1$ va $\eta_2 = \xi/(\xi+1)$ t.m.larning zichlik funksiyalarini toping.

12. ξ t.m.ning zichlik funksiyasi $f_\xi(x) = \frac{1}{\pi}\frac{1}{1+x^2}$ bo'lsin. $\eta_1 = \frac{2\xi}{1-\xi^2}$ va $\eta_2 = \frac{1}{\xi}$ t.m.larning zichlik funksiyalarini toping.

13. $\xi \sim N(5, \sigma^2)$ bo'lsin, u holda $\eta_1 = 3\xi - 1$ va $\eta_2 = -\xi + 2a$ t.m.larning zichlik funksiyalarini toping.

14. ξ t.m.ning t.f. $F_\xi(x) = \begin{cases} 0, \ x \leq 0, \\ x^3, \ 0 < x < 1, \\ 1, \ x \geq 1 \end{cases}$ bo'lsin. $\eta_1 = \xi^2 + 1$ va $\eta_2 = 1/\xi$ t.m.larning zichlik funksiyalarini toping.

15. $\xi \sim R(-2,2)$ bo'lsin, u holda $\eta_1 = \frac{|\xi|+4}{8}$ va $\eta_2 = \frac{2-\xi}{2}$ t.m.larning zichlik funksiyalarini toping.

16. $\xi \sim E(\alpha)$ bo'lsin, u holda $\eta_1 = \sqrt{\xi + 1}$ va $\eta_2 = \frac{1}{\alpha}\ln\xi$ t.m.larning zichlik funksiyalarini toping.

17. ξ t.m.ning zichlik funksiyasi $f_\xi(x) = \frac{1}{\pi}\frac{1}{1+x^2}$ bo'lsin. $\eta_1 = arctg\xi$ va $\eta_2 = 1/\xi$ t.m.larning zichlik funksiyalarini toping.

18. $\xi \sim N(0,1)$bo'lsin, u holda $\eta_1 = \xi^3$ va $\eta_2 = |\xi|$ t.m.larning zichlik funksiyalarini toping.

19. ξ t.m.ning t.f. $F_{\xi}(x) = \begin{cases} 0, \ x \leq -1, \\ a(x+1)^2, \ -1 < x \leq 2, \\ 1, \ x > 2 \end{cases}$ bo'lsin. $\eta_1 = \xi^2 + 1$ va $\eta_2 = 1/\xi$ t.m.larning zichlik funksiyalarini toping.

20. ξ t.m.ning zichlik funksiyasi $f(x) = \begin{cases} \frac{3}{2}x^2, & |x| \leq h, \\ 0, & |x| > h. \end{cases}$ bo'lsin. $\eta_1 = \xi^2 + 1$ va $\eta_2 = 1/(1+\xi)$ t.m.larning zichlik funksiyalarini toping.

III. Ikki argumentning funksiyalari

4-misol. Agar ξ va η t.m.lar bog'liqsiz bo'lib, $\eta \sim N(0,1)$, $\xi \sim N(0,1)$ bo'lsa, $\zeta = \xi + \eta$ ning taqsimotini toping.

$\triangleright$ (20) formulaga asosan:

$$f_{\zeta}(z) = \int\limits_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-x)^2}{2}} dx = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} e^{-\frac{2x^2 - 2zx + z^2}{2}} dx =$$

$$= \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} e^{-\frac{2\left((x-\frac{z}{2})^2 + \frac{z^2}{4}\right)}{2}} dx = \frac{1}{2\pi} e^{-\frac{z^2}{4}} \int\limits_{-\infty}^{+\infty} e^{-\left(x-\frac{z}{2}\right)^2} d\left(x - \frac{z}{2}\right) =$$

$$= \frac{1}{2\pi} e^{-\frac{z^2}{4}} \sqrt{\pi} = \frac{1}{\sqrt{2}\sqrt{2\pi}} e^{-\frac{z^2}{2(\sqrt{2})^2}},$$

ya'ni $f_{\xi+\eta}(z) = \frac{1}{\sqrt{2}\sqrt{2\pi}} e^{-\frac{z^2}{2(\sqrt{2})^2}}$. Demak, $\zeta \sim N(0,2)$. $\triangleleft$

Misollar

1. $\xi_1, \ldots, \xi_n$ - t.m.lar bog'liqsiz va $\xi_i \sim R(1,b)$ bo'lsa, $\eta = \min\{\xi_1, \ldots, \xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

2. $\xi \sim N(a_1;1)$ $\eta \sim N(a_2;1)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

3. $\xi \sim E(1)$, $\eta \sim E(1/2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \min\{\xi, 2\eta\}$ t.m.ning taqsimotini toping.

4. $\xi_1, \ldots, \xi_n$ - t.m.lar bog'liqsiz va $\xi_i \sim N(a_i, \sigma_i)$, $i = 1, ..., n$ bo'lsa, $\eta = \sum_{i=1}^{n} c\xi_i$ t.m.ning taqsimotini toping.

5. $\xi \sim N(0;1)$, $\eta \sim N(2;4)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

6. $\xi \sim E(1)$, $\eta \sim E(1/2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta - 1$ t.m.ning taqsimotini toping.

7. ξ_1, ξ_2 - t.m.lar bog'liqsiz va $\xi_i \sim E(\lambda_i)$. $\eta = \max\{\xi_1, \xi_2\}$ t.m.ning zichlik va t.f.ni toping.

8. $\xi \sim R(0;2)$, $\eta \sim R(0;2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

9. $\xi \sim N(a_1;1)$, $\eta \sim N(a_2;4)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta - 5$ t.m.ning taqsimotini toping.

10. $\xi_1, \ldots, \xi_n$ - t.m.lar bog'liqsiz va $\xi_i \sim R(-1,1)$ bo'lsa, $\eta = \min\{2\xi_1, \ldots, 2\xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

11. $\xi \sim R[0;1]$, $\eta \sim E(1)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

12. ξ_1, ξ_2, ξ_3 - t.m.lar bog'liqsiz va $\xi_i \sim R(0,1)$ bo'lsa, $\eta = \xi_1 + \xi_2 + \xi_3$ t.m.ning taqsimot funksiyasini toping.

13. $\xi \sim \pi(\lambda_1)$, $\eta \sim \pi(\lambda_2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

14. $\xi \sim (\lambda, \alpha)$, $\eta \sim (\lambda, \alpha)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

15. ξ_1, ξ_2, ξ_3 t.m.lar bog'liqsiz va $\xi_i \sim E(\alpha)$ bo'lsa, $\eta = \xi_1 + \xi_2 + \xi_3$ t.m.ning zichlik funksiyasini toping.

16. $\xi_1, \ldots, \xi_n$ - t.m.lar bog'liqsiz va $\xi_i \sim R(a,1)$ bo'lsa, $\eta = \max\{\xi_1, \ldots, \xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

17. $\xi \sim Bi(n_1, p)$, $\eta \sim Bi(n_2, p)$ va ξ , η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

18. $\xi \sim N(a_1;1)$, $\eta \sim N(a_2;1)$ va ξ , η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi - \eta$ t.m.ning taqsimotini toping.

19. $\xi_1, \ldots, \xi_n$ - t.m.lar bog'liqsiz va $\xi_i \sim E(\lambda_i)$ bo'lsa, $\eta = \min\{\xi_1, \ldots, \xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

20. $\xi \sim R(0;1)$, $\eta \sim R(0;2)$ va ξ , η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \eta - \xi$ t.m.ning taqsimotini toping.

IV. Ikki o'lchovlik diskret tasodifiy miqdorlar

5-misol. (ξ, η) tasodifiy vektorning birgalikdagi taqsimoti:

$\eta \setminus \xi$	0	1
0	1/6	2/6
1	2/6	1/6

Marginal taqsimotlarni, birgalikdagi t.f.ni toping.

▷ Marginal taqsimotlarini topamiz: $P\{\xi = 0\} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$, $P\{\xi = 1\} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$; $P\{\eta = 0\} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$, $P\{\eta = 1\} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$.

Demak, ξ va η t.m.larning alohida taqsimot qonunlari quyidagi ko'rinishga ega bo'ladi: $\begin{cases} \xi: & 0, & 1 \\ p: & \frac{1}{2}, & \frac{1}{2} \end{cases}$ va $\begin{cases} \eta: & 0, & 1 \\ p: & \frac{1}{2}, & \frac{1}{2}. \end{cases}$

(ξ, η) ikki o'lchovlik t.m.ning $F(x, y)$ taqsimot funksiyasini $F(x, y) = \sum\limits_{x_i < x} \sum\limits_{y_j < y} p_{ij}$ formulaga ko'ra hisoblaymiz:

$F(x, y)$	$x \le 0$	$0 < x \le 1$	$x > 1$
$y \le 0$	0	0	0
$0 < y \le 1$	0	1/6	1/2(=1/6+2/6)
$y > 1$	0	1/2(=1/6+2/6)	1(=1/6+2/6+2/6+1/6)

◁

Misollar

(ξ, η) tasodifiy vektorning birgalikdagi taqsimoti berilgan:

1.

$\xi \setminus \eta$	1	2	3
0.1	0.12	0.08	0.40
0.2	0.16	0.10	0.14

2.

$\xi \setminus \eta$	-1	0	1
0	0.1	0.2	0.1
1	0.2	0.3	0.1

3.

$\xi \setminus \eta$	-1	0	1
1	2/9	1/9	0
2	1/9	0	1/9
3	2/9	1/9	1/9

4.

$\xi \setminus \eta$	-1	0	1	2
-1	0.07	0.04	0.1	0.12
0	0.08	0.1	0.07	0.08
1	0.09	0.13	0.1	0.02

5.

$\xi \setminus \eta$	1	2	3	4
1	0.07	0.04	0.11	0.11
2	0.08	0.11	0.06	0.08
3	0.09	0.13	0.1	0.02

6.

$\xi \setminus \eta$	0	1
0	1/8	0
1	1/4	1/8
2	1/8	3/8

7.

$\xi \setminus \eta$	0	1	2
-1	1/4	1/8	1/8
1	1/8	1/8	1/4

8.

$\xi \setminus \eta$	1	2	3
0	C	0.15	0.15
1	0.2	0.3	C

9.

$\xi \setminus \eta$	-1	0	1	2
-1	0.05	0.3	0.15	0.05
1	0.1	0.05	0.25	0.05

10.

$\xi \setminus \eta$	1	2	3
1	2/9	1/9	0
2	1/9	0	1/9
3	2/9	1/9	1/9

11.

$\xi \setminus \eta$	-1	1
0	1/8	0
1	1/4	1/8
2	1/8	3/8

12.

$\xi \setminus \eta$	2	4	6	8
-1	0.1	0.3	0.1	0.1
1	0.1	0	0.25	0.25

13.

$\xi \setminus \eta$	-1	0	1
0	0	0.1	0.4
1	0.2	0.2	0.1

14.

$\xi \setminus \eta$	-1	0	1
0	0.15	0.3	0.3
1	0.1	0.05	0.1

15.

$\xi \setminus \eta$	0	1	2	3
-1	0.05	0.12	0.08	0.04
1	0.09	0.3	0.11	0.21

16.

$\xi \setminus \eta$	4	5
3	0.17	0.1
10	0.13	0.3
12	0.25	0.05

17.

$\xi \setminus \eta$	2	10	12	20
0	0.05	0.11	0.08	0.05
1	0.1	0.3	0.11	0.20

18.

$\xi \setminus \eta$	10	20	30
-10	0.1	0.25	0.15
10	0.15	0.1	0.25

19.

$\xi \setminus \eta$	1	2	3	4
1	0.05	0.04	0.1	0.11
2	0.1	0.1	0.1	0.1
3	0.11	0.1	0.04	0.05

20.

$\xi \setminus \eta$	0	1	2
0	1/4	1/3	1/8
1	1/8	1/16	0
2	C	0	0

Marginal taqsimotlarni, birgalikdagi t.f. $F(x,y)$ ni, $\eta/\xi = x_i$ shartli taqsimotlarni toping va sonli xarakteristikalarini hisoblang.

V. Ikki o'lchovlik uzluksiz tasodifiy miqdorlar

6-misol.

(ξ,η) ikki o'lchovli t.m.ning birgalidagi zichlik funksiyasi berilgan
$f(x,y) = \begin{cases} Ce^{-x-y}, \text{ agar } x \geq 0, y \geq 0, \\ 0, \text{ aks holda.} \end{cases}$

Quyidagilarni toping: 1) C; 2) $F(x,y)$; 3) F_ξ va F_η; 4) f_ξ va f_η; 5) ξ va η t.m.lar bog'liqsizmi?

$\triangleright$ 1) $\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} f(x,y)dxdy = 1$ tenglikdan

$$C\int\limits_0^{+\infty}\int\limits_0^{+\infty} e^{-x-y}dxdy = C\int\limits_0^{+\infty} e^{-x}dx\int\limits_0^{+\infty} e^{-y}dy = C = 1.$$

$$2)\ F(x,y) = \begin{cases} \int\limits_0^x\int\limits_0^y e^{-u-v}dudv = \int\limits_0^x e^{-u}du\int\limits_0^y e^{-v}dv = \\ \qquad = (1-e^{-x})(1-e^{-y}),\ x \geq 0, y \geq 0, \\ 0,\ \text{aks holda.} \end{cases}$$

Demak, $F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}), & x \geq 0, y \geq 0, \\ 0, \text{ aks holda.} \end{cases}$

3) $x \geq 0$ da:

$F_\xi(x) = F(x,+\infty) = \int\limits_0^x \left(\int\limits_0^{+\infty} e^{-u}e^{-v}dv\right)du = \int\limits_0^x e^{-u}du = 1 - e^{-x},$

demak,

$$F_\xi(x) = \begin{cases} 1-e^{-x}, & x \geq 0, \\ 0, x < 0. \end{cases}$$

Aynan shunday,

$$F_\eta(x) = \begin{cases} 1 - e^{-y}, & y \geq 0, \\ 0, y < 0. \end{cases}$$

4) $f_\xi(x) = F'_\xi(x) = \begin{cases} (1 - e^{-x})'_x, & x \geq 0, \\ 0, x < 0, \end{cases} = \begin{cases} e^{-x}, & x \geq 0, \\ 0, x < 0, \end{cases}$

va shu kabi $f_\eta(y) = \begin{cases} e^{-y}, & y \geq 0, \\ 0, y < 0. \end{cases}$

5) $f(x,y) = \begin{cases} e^{-x-y}, x \geq 0, y \geq 0, \\ 0, \text{ aks holda}, \end{cases}$ $f_\xi(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, x < 0, \end{cases}$

$f_\eta(y) = \begin{cases} e^{-y}, & y \geq 0, \\ 0, y < 0, \end{cases}$ $f(x,y) = f_\xi(x) f_\eta(y)$ tenglik o‘rinli, demak, ξ va η t.m.lar bog‘liqsiz. ◁

Misollar

a) (ξ, η) tasodifiy vektorning birgalikdagi zichlik funksiyalari berilgan:

1. $f(x,y) = \begin{cases} C(x^2 + y^2), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

2. $f(x,y) = \begin{cases} C(x^3 + y^3), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

3. $f(x,y) = \begin{cases} C(x^4 + y^4), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

4. $f(x,y) = \begin{cases} C(x^2 + y), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

5. $f(x,y) = \begin{cases} C(2x^2 + y^2), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

6. $f(x,y) = \begin{cases} Cxy, 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

7. $f(x,y) = \begin{cases} Cx^2y^2, 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

8. $f(x,y) = \begin{cases} Cx^3y^3, 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$

9. $f(x,y)=\begin{cases} C(x+y)(1-x)(1-y), 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

10. $f(x,y)=\begin{cases} Cxy, 0\le x\le 1, 0\le y\le 1-x,\\ 0, \text{ aks holda.}\end{cases}$

11. $f(x,y)=\begin{cases} C(1+xy), 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

12. $f(x,y)=\begin{cases} C(1-xy), 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

13. $f(x,y)=\begin{cases} C(1+x^2y^2), 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

14. $f(x,y)=\begin{cases} C(1-x^2y^2), 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

15. $f(x,y)=\begin{cases} C(x-y)^2, 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

16. $f(x,y)=\begin{cases} Ce^{-(x+2y)}, x\ge 0, y\ge 0\\ 0, \text{ aks holda.}\end{cases}$

17. $f(x,y)=\begin{cases} Ce^{-2(x+y)}, x\ge 0, y\ge 0\\ 0, \text{ aks holda.}\end{cases}$

18. $f(x,y)=\begin{cases} Ce^{-(2x+y)}, x\ge 0, y\ge 0\\ 0, \text{ aks holda.}\end{cases}$

19. $f(x,y)=\begin{cases} Cxy(x+y), 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

20. $f(x,y)=\begin{cases} Cxy(x-2y), 0\le x\le 1, 0\le y\le 1,\\ 0, \text{ aks holda.}\end{cases}$

Quyidagilarni hisoblang: C, $f(x)$, $f(y)$, $F(x,y)$, $M\xi$, $M\eta$, $D\xi$, $D\eta$, $\rho(\xi,\eta)$, va ξ, η t.m.larni bog‘liqsizlikga tekshiring.

7-misol. (ξ,η) tasodifiy vektor D sohada tekis taqsimlangan bo‘lsin: D: $x^2+y^2\le 1, y\ge 0$ doiraning yarmi. Birgalikdagi zichlik funksiyasi $f(x,y)$ ni tuzing va $\rho(\xi,\eta)$ ni hisoblang.

$\triangleright$ (ξ,η) tasodifiy vektor D sohada tekis taqsimlangan bo'lsa, uning zuchlik funksiyasi $f(x,y)=\begin{cases}\frac{1}{S(D)}, & (x,y)\in D,\\ 0, & (x,y)\notin D\end{cases}$ bo'ladi. Bu yerda $S(D)$ - D sohaning yuzi. Demak, $S(D)=\frac{1}{2}\pi R^2$, $R=1$ va $f(x,y)=\begin{cases}\frac{2}{\pi}, & (x,y)\in D,\\ 0, & (x,y)\notin D.\end{cases}$ Endi (ξ,η) t.m.ning momentlarini hisoblaymiz:

$$M\xi=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}xf(x,y)\,dxdy=\int\limits_{-1}^{1}dx\int\limits_{0}^{\sqrt{1-x^2}}x\frac{2}{\pi}dy=$$

$$-\frac{1}{\pi}\int\limits_{0}^{1}\sqrt{1-x^2}d\left(1-x^2\right)=-\frac{2}{3}\frac{1}{\pi}(1-x)^{3/2}\Big|_0^1=\frac{2}{3\pi}.$$

$$M\eta=\int\limits_{-1}^{1}\int\limits_{0}^{\sqrt{1-x^2}}y\frac{2}{\pi}dxdy=\frac{1}{\pi}\int\limits_{-1}^{1}\left[y^2\right]_0^{\sqrt{1-x^2}}dx=\frac{1}{\pi}\int\limits_{-1}^{1}\left(1-x^2\right)dx=$$

$$=\frac{2}{\pi}\int\limits_{0}^{1}\left(1-x^2\right)dx=\frac{2}{\pi}\left[x-\frac{x^3}{3}\right]_0^1=\frac{2}{\pi}\left(1-\frac{1}{3}\right)=\frac{4}{3\pi}.$$

$$D\xi=M(\xi-M\xi)^2=\int\limits_{-1}^{1}\int\limits_{0}^{\sqrt{1-x^2}}(x-M\xi)^2\frac{2}{\pi}dxdy=\frac{2}{\pi}\int\limits_{-1}^{1}\int\limits_{0}^{\sqrt{1-x^2}}x^2dxdy=$$

$$=\frac{2}{\pi}\int\limits_{-1}^{1}x^2\left(\int\limits_{0}^{\sqrt{1-x^2}}dy\right)dx=\frac{2}{\pi}\int\limits_{-1}^{1}x^2\left[y\right]_0^{\sqrt{1-x^2}}dx=\frac{2}{\pi}\int\limits_{-1}^{1}x^2\sqrt{1-x^2}dx=$$

$$=\begin{bmatrix}x=\sin t\\ dx=\cos tdt\\ \pm1\to\pm\frac{\pi}{2}\end{bmatrix}=\frac{2}{\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2t\cos t\cos tdt=\frac{1}{2\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}4\sin^2t\cos^2tdt=$$

$$= \frac{1}{2\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 2t dt = \frac{1}{4\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1-\cos 4t)dt = \frac{1}{4\pi}\left[t - \frac{1}{4}\sin 4t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$= \frac{1}{4\pi}\left[\left(\frac{\pi}{2} - 0\right) - \left(-\frac{\pi}{2} - 0\right)\right] = \frac{1}{4\pi} = \frac{1}{4}.$$

$$D\eta = M(\eta - M\eta)^2 = M\eta^2 - (M\eta)^2 = \int\limits_{-1}^{1}\int\limits_{0}^{\sqrt{1-x^2}} y^2\frac{2}{\pi}dxdy - \left(\frac{4}{3\pi}\right)^2 =$$

$$= \frac{2}{\pi}\int\limits_{-1}^{1}\frac{y^3}{3}\Big|_0^{\sqrt{1-x^2}} dx - \frac{4^2}{(3\pi)^2}\frac{2}{3\pi}\int\limits_{-1}^{1}(1-x^2)\sqrt{1-x^2}dx - \frac{16}{3^2\pi^2} =$$

$$= \begin{bmatrix} x = \sin t \\ dx = \cos t dt \\ \pm 1 \to \pm\frac{\pi}{2} \end{bmatrix} = \frac{2}{3\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\sin^2 t)\sqrt{1-\sin^2 t}\cos t dt - \frac{16}{9\pi^2} =$$

$$= \frac{2}{3\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 t\cos t\cos t dt - \frac{16}{9\pi^2} = \frac{2}{3\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\cos^2 t\right)^2 dt - \frac{16}{9\pi^2} =$$

$$= \frac{1}{6\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+\cos 2t)^2 dt - \frac{16}{9\pi^2} = \frac{1}{6\pi}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(1 + 2\cos 2t + \cos^2 2t\right)^2 dt -$$

$$-\frac{16}{9\pi^2} = \frac{1}{6\pi}\, t\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{6\pi}\sin 2\, t\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{6\pi}\frac{1}{2}\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+\cos 4t)\,dt - \frac{16}{9\pi^2} =$$

$$= \frac{1}{6\pi}\pi + \frac{1}{12\pi}\pi + \frac{1}{12\pi}\frac{1}{4}\sin 4\, t\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{16}{9\pi^2} = \frac{1}{6} + \frac{1}{12} - \frac{16}{9\pi^2} =$$

$$= \frac{1}{4} - \frac{16}{9\pi^2} = \frac{9\pi^2 - 64}{36\pi^2}.$$

$$M\xi\eta = \int\limits_{-1}^{1}\int\limits_{0}^{\sqrt{1-x^2}} xy\frac{2}{\pi}dxdy = \frac{2}{\pi}\int\limits_{-1}^{1} x\left(\int\limits_{0}^{\sqrt{1-x^2}} ydy\right)dx =$$

$$= \frac{2}{\pi}\int\limits_{-1}^{1} x\left[\frac{y^2}{2}\right]_0^{\sqrt{1-x^2}} dx = \frac{1}{\pi}\int\limits_{-1}^{1} x\left[y^2\right]_0^{\sqrt{1-x^2}} dx =$$

$$= \frac{1}{\pi}\int\limits_{-1}^{1} x(1-x^2)dx = \frac{1}{2\pi}\int\limits_{-1}^{1}(1-x^2)dx^2 =$$

$$= -\frac{1}{2\pi}\int\limits_{-1}^{1}(1-x^2)d(1-x^2) = -\frac{1}{2\pi}\left[\frac{\left(1-x^2\right)^2}{2}\right]_{-1}^{1} =$$

$$= -\frac{1}{2\pi}(0-0) = 0.$$

$$cov(\xi;\eta) = 0. \qquad \rho(\xi;\eta) = \frac{cov(\xi;\eta)}{\sqrt{D\xi}\sqrt{D\eta}} = \frac{0}{\sqrt{\frac{1}{4}}\sqrt{\frac{9\pi^2-64}{36\pi^2}}} = 0.$$

Misollar

b) (ξ,η) tasodifiy vektor D sohada tekis taqsimlangan bo‘lsin:

1. D: $x^2+y^2 \le 1, x \ge 0, y \ge 0$-doiraning to‘rtdan biri.
2. D: $\frac{x^2}{9}+\frac{y^2}{4} \le 1, x \ge 0, y \ge 0$-ellipsning to‘rtdan biri.
3. D: $\frac{x^2}{4}+y^2 \le 1, x \ge 0, y \ge 0$-ellipsning to‘rtdan biri.
4. D: $\frac{x^2}{a^2}+\frac{y^2}{b^2} \le 1, x \ge 0, y \ge 0$-ellipsning to‘rtdan biri.
5. D: $x^2+(y-1)^2 \le 1, y \ge 1$- doiraning yarmi.
6. D: $x^2+y^2 \le 1, x \ge 0$-doiraning yarmi.
7. D: $x^2+y^2 \le 1$-doira.
8. D: uchlari O(0,0), A(1,0), B(0,1) nuqtalarda bo‘lgan uchburchak.
9. D: uchlari A(1,0), B(1,1), C(0,1) nuqtalarda bo‘lgan uchburchak.
10. D: uchlari A(-1,0), O(0,0), B(0,-1) nuqtalarda bo‘lgan uchburchak.

11. D: uchlari A(1,1), B(-1,1), C(1,-1), D(-1,-1) nuqtalarda bo‘lgan to‘rtburchak.

12. D: uchlari O(0,0), A(1,0), B(1,1), C(0,1) nuqtalarda bo‘lgan to‘rtburchak.

13. D: uchlari O(0,0), A(0,1), B(-1,1), C(-1,0) nuqtalarda bo‘lgan to‘rtburchak.

14. D: uchlari O(0,0), A(1,1), B(0,1), C(1,0) nuqtalarda bo‘lgan to‘rtburchak.

15. D: $y = x^2, y = 0, x = 1$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

16. D: $y = x^2, y = 1, x = 0$chiziqlar bilan chegaralangan egri chiziqli uchburchak.

17. D: $y = \sqrt{x}, y = 0, x = 1$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

18. D: $y = \sqrt{1-x}, y = 0, x = 0$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

19. D: $y = x^3, y = 0, x = 1$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

20. D: $y = x^3, y = 1, x = 0$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

Birgalikdagi zichlik funksiyasi $f(x, y)$ ni tuzing va $\rho(\xi, \eta)$ ni hisoblang.

VI. Katta sonlar qonuni

Agar $\forall\varepsilon > 0$ son uchun

$$\lim_{n\to\infty} P\left\{|X_n - A| < \varepsilon\right\} = 1$$

munosabat o‘rinli bo‘lsa, u holda $X_1, X_2, ..., X_n, ...$ t.m.lar o‘zgarmas son A ga *ehtimollik bo‘yicha yaqinlashadi* deyiladi. Ehtimollik bo‘yicha yaqinlashish $X_n \xrightarrow{P} A$ orqali belgilanadi.

$X_1, X_2, ..., X_n, ...$ t.m.lar ketma-ketligi mos ravishda $MX_1, MX_2, ..., MX_n, ...$ matematik kutilmalarga ega bo‘lib, $\forall\varepsilon > 0$ son uchun $n \to \infty$ da

$$\lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{i=1}^{n} X_i - \frac{1}{n}\sum_{i=1}^{n} MX_i\right| < \varepsilon\right\} = 1$$

munosabat bajarilsa, u holda $X_1, X_2, ..., X_n, ...$ t.m.lar ketma-ketligi *katta sonlar qoniniga bo'ysunadi* deyiladi.

Teorema(Chebishev). Agar bog'liqsiz $X_1, X_2, ..., X_n, ...$ t.m.lar ketma-ketligi uchun shunday $\exists C > 0$ bo'lib, $DX_i \leq C, \ i = 1, 2, ...$ tengsizliklar o'rinli bo'lsa, u holda $\forall \varepsilon > 0$ son uchun

$$\lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{i=1}^{n} X_i - \frac{1}{n}\sum_{i=1}^{n} MX_i\right| < \varepsilon\right\} = 1 \tag{21}$$

munosabat o'rinli bo'ladi.

Bu teoremani isbotlashda quyidagi Chebishev tengsizligidan foydalaniladi: $\forall \varepsilon > 0$ uchun quyidagi tengsizlik o'rinli:

$$P\{|X - MX| \geq \varepsilon\} \leq \frac{DX}{\varepsilon^2}. \tag{22}$$

Agar $X_1, X_2, ..., X_n, ...$ bog'liqsiz va bir xil taqsimlangan t.m.lar, hamda $MX_i = a, \ DX_i = \sigma^2$ bo'lsa, u holda $\forall \varepsilon > 0$ son uchun quyidagi munosabat o'rinli

$$\lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{i=1}^{n} X_i - a\right| < \varepsilon\right\} = 1. \tag{23}$$

Bernulli teoremasi KSQ ning sodda shakli hisoblanadi. U nisbiy chastotaning turg'unligini asoslaydi.

Teorema(Y. Bernulli). Agar A hodisaning bitta tajribada ro'y berishi ehtimolligi p bo'lib, n ta bog'liqsiz tajribada bu hodisa n_A marta ro'y bersa, u holda $\forall \varepsilon > 0$ son uchun

$$\lim_{n\to\infty} P\left\{\left|\frac{n_A}{n} - p\right| < \varepsilon\right\} = 1 \tag{24}$$

munosabat o'rinlidir.

8-misol. $\{\xi_k\}-$ bog'liqsiz t.m.lar ketma-ketligi bo'lib, uning taqsimoti $P(\xi_k = a) = \frac{k}{2k+1}, \ P(\xi_k = -a) = \frac{k+1}{2k+1}$ bo'lsa, $\{\xi_k\}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

▷ KSQ o'rinli ekanligini Chebishev teoremasidan foydalanib tekshiramiz. Buning uchun$D\xi_k$ ni hisoblaymiz:

$$M\xi_k = a\left(\frac{k}{2k+1} - \frac{k+1}{2k+1}\right) = \frac{-a}{2k+1};$$

$$M\xi_k^2 = a^2\left(\frac{k}{2k+1}+\frac{k+1}{2k+1}\right) = a^2;$$

$$D\xi_k = M\xi_k^2-(M\xi_k)^2 = a^2-a^2\frac{1}{(2k+1)^2} = \frac{a^2\left(4k^2+4k\right)}{(2k+1)^2} = \frac{4a^2k\,(k+1)}{(2k+1)^2}.$$

$$D\xi_k = \frac{4a^2k^2\left(1+\frac{1}{k}\right)}{k^2\left(2+\frac{1}{k}\right)} \le 2a^2, \qquad k\to\infty.$$

Demak, $\{\xi_k\}$ t.m.lar ketma-ketligi uchun katta sonlar qonuni o‘rinli. $\triangleleft$

Misollar

1. Agar $P(\xi_k = \pm k) = \frac{1}{2k^2}$, $P(\xi_k = 0) = 1-\frac{1}{k^2}$ bo‘lsa, u holda$\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQga bo‘ysunadimi?

2. $\{\xi_k\}_{k=1}^{\infty}$ t.f. $F_{\xi_k}(x)$ bo‘lgan bog‘liqsiz t.m.lar ketma-ketligi bo‘lsin: $F_{\xi_k}(x) = \begin{cases} 0, x\le 1; \\ 1-x^{-k}, x\ge 1. \end{cases}$ $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQga bo‘ysunadimi?

3. $\xi_1,\xi_2,...$ bir xil taqsimlangan bog‘liqsiz t.m.lar ketma-ketligi va $M\xi_i = 0, D\xi_i = 1$ bo‘lsin. $\eta_k = \xi_k+\xi_{k+1}, k = 1,2,...$ t.m.lar ketma-ketligi uchun KSQ o‘rinlimi?

4. $\{\xi_k\}$ bog‘liqsiz t.m.lar va $P\left(\xi_k = \pm\sqrt{k}\right) = \frac{1}{2}$ bo‘lsa, bu ketma-ketlik uchun KSQ o‘rinlimi?

5. $\{\xi_k\}$ bog‘liqsiz t.m.lar ketma-ketligi bo‘lib, uning taqsimoti $P(\xi_k = a) = \frac{1}{k+1}$, $P(\xi_k = -a) = \frac{k}{k+1}$ bo‘lsin. Chebishev tengsizligi yordamida $\{\xi_k\}$ bog‘liqsiz t.m.lar ketma-ketligi KSQ ga bo‘ysunishini ko‘rsating.

6. $\{\xi_k\}, k = 1,2...$- bog‘liqsiz t.m.lar ketma-ketligi va $\xi_k \sim R(0,2n^{\alpha})$ bo‘lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o‘rinli bo‘ladi?

7. Agar $P(\xi_k = \pm\sqrt{k}) = \frac{1}{2k}$, $P(\xi_k = 0) = 1-\frac{1}{k}$ bo‘lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo‘ysunadimi?

8. Agar $P(\xi_k = \pm 2^k) = 2^{-(2k+1)}$, $P(\xi_k = 0) = 1-2^{-2k}$ bo‘lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo‘ysunadimi?

9. $\xi_1, \xi_2, ...$ bir xil taqsimlangan bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = a, D\xi_i = 1$. α ning qanday qiymatlarida $\eta_k = \alpha(\xi_k + \xi_{k+1}), k = 1, 2, ...$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

10. Agar $P(\xi_k = \pm 2^k) = \frac{1-2^{-k}}{2}$, $P(\xi_k = \pm 1) = 2^{-k-1}$ bo'lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

11. $\{\xi_k\}, k = 1, 2...$- bog'liqsiz t.m.lar ketma-ketligi va $\xi_k \sim R(1, n^{\alpha})$ bo'lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

12. Agar $P(\xi_k = \pm k) = 2^{-k}$, $P(\xi_k = 0) = 1 - 2^{-k+1}$ bo'lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

13. $\xi_1, \xi_2, ...$ bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = 0, D\xi_i = Ck^{\alpha}$, $\alpha \geq 0$, $i = 1, 2, ...$ α ning qanday qiymatlarida$\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

14. $\{\xi_k\}, k = 1, 2...$ - bog'liqsiz t.m.lar ketma-ketligi va $\xi_k \sim R(0, \frac{n^{\alpha}}{2})$ bo'lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

15. Agar $P(\xi_k = \pm 2^{\alpha k}) = 2^{-(2k+1)}$, $P(\xi_k = 0) = 1 - 2^{-2k}$ bo'lsa ,α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

16. $\xi_1, \xi_2, ...$ bir xil taqsimlangan bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = 0, D\xi_i = 1$ bo'lsin. $\eta_k = \xi_k - \xi_{k+1}, k = 1, 2, ...$ ketma-ketlik uchun KSQ o'rinli bo'ladimi?

17. $\{\xi_k\}_{k=1}^{\infty}$ - bog'liqsiz t.m.lar ketma-ketligi bo'lib, t.f. bo'lsin: $F_{\xi_k}(x) = \begin{cases} 0, x \leq 1; \\ 1 - x^{-k}, x \geq 1. \end{cases}$ $\{a\xi_k\}_{k=1}^{\infty}, a > 0$, ketma-ketlik uchun KSQ o'rinli bo'ladimi?

18. Agar $\{\xi_k\}$ ketma-ketlik taqsimoti $P(\xi_k = \alpha) = \frac{k}{2k+1}$, $P(\xi_k = -\alpha) = \frac{k+1}{2k+1}$ bo'lsa, α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

19. $\xi_1, \xi_2, ...$ bir xil taqsimlangan bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = a, D\xi_i = \sigma^2, \sigma < \infty$ bo'lsin. $\eta_k = 2\xi_k - \xi_{k+1}, k = 1, 2, ...$ ketma-ketlik uchun KSQ o'rinlimi?

20. $\{\xi_k\}, k = 1, 2...$- bog'liqsiz t.m.lar ketma-ketligi va $\xi_k \sim E(\alpha/2)$ bo'lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

2-§. STATISTISTIK TANLANMA. TANLANMA XARAKTERISTIKALARI

Statistik izlanish asosida kuzatilmalar, ya'ni statistik tajriba natijalari, sonli ma'lumotlar yotadi. Faraz qilaylik, biror tasodifiy tajribani kuzatish natijasida $x_1,\ldots,x_n$ sonlar olingan bo'lib, ularni bog'liq bo'lmagan va bir xil taqsimlangan $X_1,...,X_n$ t.m.larning qiymatlari deb qaraylik. U holda $x^{(n)} = (x_1,\ldots,x_n)$ vektor $X^{(n)} = (X_1,\ldots,X_n)$ tasodifiy vektorning qiymati bo'ladi. Matematik statistikada $X_1,...,X_n$ tasodifiy miqdorlar n hajmga ega bo'lgan *takroriy tanlanma* yoki shunchaki *tanlanma* deb ataladi. Bunda $X^{(n)}$ vektor tanlanma yoki kuzatilayotgan tasodifiy vektor deb ham ataladi. o'z navbatida esa X_i t.m.ni biror abstrakt ξ t.m.ning har bir tajribadagi amaliy qiymati deb ham qaraymiz. $X^{(n)}$ tanlanma taqsimot qonuni $\mathscr{P}$- taqsimotlar oilasiga tegishli bo'lgan bosh to'plamdan olingan deb hisoblanadi. $\mathscr{P}$ oila parametrik (masalan, $N\left(\theta_1,\theta_2^2\right)$, $R\left(\theta_1,\theta_2\right)$,...) yoki noparametrik tipda bo'lishi mumkin.

Statistika deb, tanlanmadan olingan ixtiyoriy o'lchovli funksiyaga aytiladi: $T(X^{(n)}) = T(X_1,...,X_n)$.

Quyidagi statistikalar, ya'ni tanlanma momentlari muhim ahamiyat kasb etadi:

Tanlanma o'rta qiymati - $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$;

k - tartibli boshlang'ich tanlanma momenti - $\overline{x^k} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i^k$;

k- tartibli markaziy tanlanma moment - $\mu_{kn} = \frac{1}{n}\sum\limits_{i=1}^{n} (X_i-\overline{x})^k$;

Tanlanma dispersiyasi - $S^2 = \frac{1}{n}\sum\limits_{i=1}^{n} (X_i-\overline{x})^2$ (siljimagan tanlanma dispersiyasi - $\overline{S^2} = \frac{1}{n-1}\sum\limits_{i=1}^{n} (X_i-\overline{x})^2$);

Bu statistikalardan tashqari amaliyotda variatsion qator hadlaridan ham keng foydalaniladi: $X_1,...,X_n$ tanlanmaning elementlarini kamaymaslik tartibida joylashtiramiz:

$$X_{(1)} \le X_{(2)} \le ... \le X_{(n)}, \tag{25}$$

hosil bo'lgan qator variatsion qator, elementlari esa tartiblangan statistikalar deyiladi.

$$X_{(1)} = min\{X_1, ..., X_n\},$$

$$X_{(n)} = max\{X_1, ..., X_n\}.$$

9-misol. $X_1, ..., X_n$ tanlanma $Bi\,(n;\theta)$ taqsimotdan olingan bo'lsa, $\overline{x} = \frac{1}{n}\sum X_i$ statistika uchun $M\overline{x},\ D\overline{x}$ ni hisoblang.

$\triangleright$ Har bir $X_1, ..., X_n$ ning zichlik funksiyasi $f\,(x;\theta) = C_n^x\theta^x(1-\theta)^{n-x},\ x = 0, 1, ..., n,\ \theta \in (0, 1)$, hamda $MX_1 = n\theta;\ DX_1 = n\theta\,(1-\theta)$ bo'ladi.

$$M\overline{x} = M\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} MX_i = \frac{1}{n}\sum_{i=1}^{n} n\theta = n\theta.$$

$$D\overline{x} = D\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} DX_i =$$

$$= \frac{1}{n^2}\sum_{i=1}^{n} n\theta\,(1-\theta) = \frac{n\theta\,(1-\theta)}{n} = \theta\,(1-\theta)$$

Demak, $M\overline{x} = MX_1 = n\theta,\quad D\overline{x} = \frac{DX_1}{n} = \theta\,(1-\theta)$ bo'lar ekan. $\triangleleft$

Misollar

1. $X_1, ..., X_n$ tanlanma $R[\theta_1;\theta_2]$ taqsimotdan olingan bo'lsa, $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika uchun $M\overline{x},\ D\overline{x}$ larni hisoblang.

2. $X_1, ..., X_n$ tanlanma $N\left(\theta_1, \theta_2^2\right)$ taqsimotdan olingan bo'lsa, $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika uchun $M\overline{x},\ D\overline{x}$ larni hisoblang.

3. $X_1, ..., X_n$ tanlanma $R[0;\theta]$ taqsimotdan olingan bo'lsa, $X_{(1)} = \min\{X_1, ..., X_n\}$ statistika uchun $MX_{(1)},\ DX_{(1)}$ larni hisoblang.

4. $X_1, ..., X_n$ tanlanma $R[0;\theta]$ taqsimotdan olingan bo'lsa, $X_{(n)} = \max\{X_1, ..., X_n\}$ statistika uchun $MX_{(n)}$, $DX_{(n)}$ larni hisoblang.

5. $X_1, ..., X_n$ tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x;\theta) = \begin{cases} e^{\theta - x}, & x \geq \theta, \\ 0, & x < \theta. \end{cases}$ $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika uchun $M\overline{x}$, $D\overline{x}$ larni hisoblang.

6. $X_1, ..., X_n$ tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x;\theta) = \begin{cases} e^{\theta - x}, & x \geq \theta, \\ 0, & x < \theta. \end{cases}$ $X_{(1)} = \min\{X_1, ..., X_n\}$ statistika uchun $MX_{(1)}$, $DX_{(1)}$ larni hisoblang.

7. $X_1, ..., X_n$ tanlanma $N\left(0, \theta^2\right)$ taqsimotdan olingan bo'lsa, $\overline{x^2} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i^2$ statistika uchun $M\overline{x^2}$, $D\overline{x^2}$ larni hisoblang.

8. $X_1, ..., X_n$ tanlanma $N\left(\theta_1, \theta_2^2\right)$ taqsimotdan olingan bo'lsa, $S^2 = \frac{1}{n}\sum\limits_{i=1}^{n}\left(X_i - \overline{x}\right)^2$, $\overline{S^2} = \frac{1}{n-1}\sum\limits_{i=1}^{n}\left(X_i - \overline{x}\right)^2$ statistikalar uchun MS^2, $M\overline{S^2}$ larni hisoblang.

9. $X_1, ..., X_n$ tanlanma t.f. $F(x)$ bo'lgan taqsimotdan olingan bo'lsa, $F_n(x) = \frac{1}{n}\sum\limits_{i=1}^{n} I\left(X_i < x\right)$, $x \in R^1$ statistika uchun $MF_n(x)$, $DF_n(x)$ larni hisoblang.

10. $X_1, ..., X_n$ tanlanma $R[\theta_1;\theta_2]$ taqsimotdan olingan bo'lsa, $X_{(n)} = \max\{X_1, ..., X_n\}$ statistika uchun $MX_{(n)}$, $DX_{(n)}$ larni hisoblang.

11. $X_1, ..., X_n$ tanlanma $R[\theta_1;\theta_2]$ taqsimotdan olingan bo'lsa, $X_{(1)} = \min\{X_1, ..., X_n\}$ statistika uchun $MX_{(1)}$, $DX_{(1)}$ larni hisoblang.

12. $X_1, ..., X_n$ tanlanma $E\left(\theta\right)$ taqsimotdan olingan bo'lsa, $X_{(n)} = \max\{X_1, ..., X_n\}$ statistika uchun $MX_{(n)}$, $DX_{(n)}$ larni hisoblang.

13. $X_1, ..., X_n$ tanlanma $E\left(\theta\right)$ taqsimotdan olingan bo'lsa, $X_{(1)} = \min\{X_1, ..., X_n\}$ statistika uchun $MX_{(1)}$, $DX_{(1)}$ larni hisoblang.

14. $X_1, ..., X_n$ tanlanma $E\left(\theta\right)$ taqsimotdan olingan bo'lsa,

$\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika uchun $M\overline{x},\ D\overline{x}$ larni hisoblang.

15. $X_1, ..., X_n$ tanlanma $E(\theta)$ taqsimotdan olingan bo'lsa, $S^2 = \frac{1}{n}\sum\limits_{i=1}^{n} (X_i - \overline{x})^2$ statistika uchun $MS^2,\ DS^2$ larni hisoblang.

16. $X_1, ..., X_n$ tanlanma $Bi(n;\theta)$ taqsimotdan olingan bo'lsa, $T_n\left(x^{(n)}\right) = X_1(n - X_1)/(n(n-1))$ statistika uchun $MT_n\left(x^{(n)}\right),\ DT_n\left(x^{(n)}\right)$ larni hisoblang.

17. $X_1, ..., X_n$ tanlanma $\Gamma(1;\theta)$ taqsimotdan olingan bo'lsa, $T_n\left(x^{(n)}\right) = X_1 + X_n$ statistika uchun $MT_n\left(x^{(n)}\right),\ DT_n\left(x^{(n)}\right)$ larni hisoblang.

18. $X_1, ..., X_n$ tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x;\theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0;\theta], \theta > 0, \\ 0, & x \overline{\in} [0;\theta]. \end{cases}$
$T_n\left(x^{(n)}\right) = \frac{3}{2}\overline{x}$ statistika uchun $MT_n\left(x^{(n)}\right),\ DT_n\left(x^{(n)}\right)$ larni hisoblang.

19. $X_1, ..., X_n$ tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin:$f(x;\theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0;\theta], \theta > 0, \\ 0, & x \overline{\in} [0;\theta]. \end{cases}$
$X_{(n)} = \max\{X_1, ..., X_n\}$ statistika uchun $MX_{(n)},\ DX_{(n)}$ larni hisoblang.

20. $X_1, ..., X_n$ tanlanma $\log N\left(\theta_1, \theta_2^2\right)$ taqsimotdan olingan bo'lsa, $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika uchun $M\overline{x},\ D\overline{x}$ larni hisoblang.

3-§. EKSPONENSIAL MODEL

$X_1, ..., X_n$ bog'liqsiz t.m. lar ξ t.m. ning har bir tajribadagi amaliy qiymatlari deb qaraladi va demak, ular bog'liqsiz bir hil taqsimlangan t.m.lardir. ξ t.m. hosil qilgan ehtimollik fazosi $(\mathscr{X}, \mathscr{B}, P)$ bo'lsin, ya'ni $\mathscr{X}$ - ξ t.m.ning qiymatlari to'plami, $\mathscr{B}$ to'plam $\mathscr{X}$ dan olingan to'plamlarning Borel $\sigma-$ algebrasi va P esa $(\mathscr{X}, \mathscr{B})$ o'lchovli fazoda aniqlangan ehtimollik o'lchovidir. $X^{(n)}$ tanlanma hosil qilgan ehtimollik fazosi $(\mathscr{X}^{(n)}, \mathscr{B}^{(n)}, P^{(n)})$ tanlanma fazo (yoki statistik model) deb ataladi. Bu yerda $\mathscr{X}^{(n)} = \mathscr{X} \times ... \times \mathscr{X}$, $\mathscr{B}^{(n)}$ - to'plam $\mathscr{X}^{(n)}$ dagi to'plamlarning σ - algebrasi, $P^{(n)}$ esa $X^{(n)}$

ning $B = B_1 \times ... \times B_n$ da aniqlangan taqsimoti $B_i \in \mathscr{B}$. Qulaylik uchun $P^{(n)}$ ni P kabi yozib u $\mathscr{P}$ oilaga tegishli deb ($\mathscr{P} = \{P\}$), statistik modelni $(\mathscr{X}^{(n)}, \mathscr{B}^{(n)}, \mathscr{P})$ kabi belgilaymiz.

$(\mathscr{X}^{(n)}, \mathscr{B}^{(n)}, \{P_\theta, \theta \in \Theta\})$ parametrik statistik model eksponensial statistik model deyiladi, agar P_θ ning $f(x;\theta)$ zichlik funksiyasi quyidagi ko'rinishda bo'lsa:

$$f(x;\theta) = h(x)\exp\left\{\sum_{i=1}^{n} A_i(\theta)t_i(x) + B(\theta)\right\}.$$

Bu holda $X^{(n)}$ tanlanmaning zichlik funksiyasi $f_n(x^{(n)}, \theta) = \prod\limits_{i=1}^{n} f(x_i, \theta)$ quyidagicha ko'rinishda bo'ladi:

$$f_n(x^{(n)}, \theta) = h_n(x^{(n)})\exp\left\{\sum_{i=1}^{k} A_i(\theta)T_i(x^{(n)}) + nB(\theta)\right\},$$

bu yerda $h_n(x^{(n)}) = \prod\limits_{i=1}^{n} h(x_i)$; $T_i(x^{(n)}) = \sum\limits_{j=1}^{n} t_i(x_j)$.

10-misol. $\Gamma(\theta_1, \theta_2)$ taqsimotlar oilasi eksponensial oilaga tegishlimi?

$\triangleright$ $\Gamma(\theta_1, \theta_2)$ taqsimot zichlik funksiyasini yozib olamiz:

$$f(x,\theta) = \begin{cases} \frac{\theta_1^{-\theta_2}}{\Gamma(\theta_2)} x^{\theta_2 - 1} e^{x/\theta_1}, & x \geq 0, \quad \theta_1, \theta_2 > 0, \\ 0, & x < 0. \end{cases}$$

Bu yerdan $f(x,\theta) = \exp\{x/\theta_1 + (\theta_2 - 1)\ln x + (-\theta_2 \ln\theta_1 - \ln\Gamma(\theta_2))\}$ - eksponensial oilaga tegishli ekanligini ko'rish mumkin: $h(x) \equiv 1$, $A_1(\theta) = 1/\theta_1$, $A_2(\theta) = \theta_2 - 1$, $t_1(x) = x$, $t_2(x) = \ln x$, $B(\theta) = -\theta_2 \ln\theta_1 - \ln\Gamma(\theta_2)$. $\triangleleft$

Misollar

Quyidagi taqsimotlar oilasi eksponensial oilaga tegishlimi?

1. $R\left[-\theta,\theta\right].$ 2. $N\left(\theta_1,\theta_2^2\right).$

3. $W\left(\theta_1,\theta_2\right).$ 4. $K\left(\theta,1\right).$

5. $R\left[\theta_1,\theta_2\right].$ 6. $P\left(\theta_1,\theta_2\right).$

7. $N\left(\theta,2\theta\right).$ 8. $f\left(x,\theta\right)=\begin{cases}\frac{2x}{\theta^2}, & x\in\left[0,\theta\right],\ \theta>0,\\ 0\ , & x\overline{\in}\left[0,\theta\right].\end{cases}$

9. $G\left(\frac{\theta}{2}\right).$ 10. $f(x;\theta)=\begin{cases}e^{\theta-x}, & x\geq\theta,\\ 0, & x<\theta\end{cases}$

11. $\overline{Bi}\left(r,\theta\right).$ 12. $f\left(x,\theta\right)=\begin{cases}3x^2\theta^{-3}e^{-(x/\theta)^3}, x\geq 0,\theta>0,\\ 0\ , \quad x<0.\end{cases}$

13. $R\left[\theta-1,\theta+1\right].$ 14. $f\left(x,\theta\right)=e^{-\frac{x-\theta}{2}},\ \ x,\ \theta\in R.$

15. $\log N\left(\theta_1,\theta_2^2\right).$ 16. $F\left(x,\theta\right)=\begin{cases}1-e^{-\frac{x-\theta_1}{\theta_2}},\ x\geq\theta_1,\ \theta_2>0,\\ 0\ , \quad x<\theta_1.\end{cases}$

17. $Bi\left(n;\frac{\theta}{2}\right).$ 18. $f\left(x,\theta\right)=\begin{cases}\theta\left(\ln^{\theta-1}x\right)/x, & x\in\left[1,e\right],\theta>0,\\ 0\ , & x\overline{\in}\left[1,e\right].\end{cases}$

19. $E\left(2\theta\right).$ 20. $f\left(x,\theta\right)=e^{-x+\theta}\left(1+e^{-x+\theta}\right)^{-2},\ \ x,\theta\in R$

4-§. YETARLI STATISTIKALAR

$\left(\mathscr{X}^{(n)},\mathscr{B}^{(n)},\left\{P_\theta^{(n)},\theta\in\Theta\right\}\right)$- parametrik statistik model va $\mathscr{X}^{(n)}$ dagi P_θ taqsimot yordamida tuzilgan $P_\theta\{X^{(n)}\in B/T(X^{(n)})\}$ shartli taqsimot bo‘lsin, $B\in\mathscr{B}^{(n)}$.

$T(X^{(n)})$ statistika $\{P_\theta,\theta\in\Theta\}$ oila (yoki θ parametr) uchun *yetarli statistika* deyiladi, agar $P_\theta(X^{(n)}\in B/T)$ shartli taqsimotning θ ga bo‘g‘liq bo‘lmagan varianti mavjud bo‘lsa.

Yetarli statistikasining mavjudligini Neyman- Fisherning *faktorlashtirish teoremasi* yordamida osongina tekshirish mumkin. $\{P_\theta,\theta\in\Theta\}\ll\mu$ bo‘lsin ($\mu^{(n)}=\mu\times...\times\mu$).

Teorema(Neyman - Fisher). T statistika θ parametr uchun yetarli statistika bo‘lishi uchun $f_n(x^{(n)};\theta)$ zichlik funksiasi $\mu^{(n)}$

o‘lchovga nisbatan deyarli hamma yerda quyidagicha ifodalanishi zarur va yetarlidir:

$$f_n(x^{(n)};\theta) = \Psi_n(T(x^{(n)});\theta)h_n(x^{(n)}), \tag{26}$$

bu yerda Ψ_n-o‘lchovli funksiya yetarli statistika va θ ga bog‘liq, h_n esa θ ga bog‘liq emas va ψ_n, $h_n \geq 0$.

11-misol. $\pi(\theta)$ taqsimot uchun etarli statistikani ko‘rsating.

$$f(x,\theta) = \frac{e^{-\theta}\theta^x}{x!}, \; x = 0,1,..., \;\; \theta \in (0,1).$$

$\triangleright$ Tanlanma zichlik funksiyasini topamiz:

$$f_n\left(x^{(n)},\theta\right) = \prod_{i=1}^{n} f(x_i,\theta) = \frac{e^{-n\theta}\theta^{\sum x_i}}{\prod\limits_{i=1}^{n} x_i!} = e^{-n\theta}\theta^{\sum x_i}\left(\prod_{i=1}^{n} x_i!\right)^{-1}.$$

Neyman-Fisher faktorlashtirish kriteriysiga ko‘ra:

$$f_n\left(x^{(n)},\theta\right) = e^{-n\theta}\theta^{\sum x_i}\left(\prod_{i=1}^{n} x_i!\right)^{-1} = \Psi_\theta\left(T\left(x^{(n)}\right);\theta\right)h_n\left(x^{(n)}\right);$$

$$\Psi_\theta\left(T\left(x^{(n)}\right);\theta\right) = e^{-n\theta}\theta^{\sum x_i}; \;\; h_n\left(x^{(n)}\right) = \left(\prod_{i=1}^{n} x_i!\right)^{-1}.$$

Demak, $\pi(\theta)$ taqsimot uchun $T\left(x^{(n)}\right) = \sum\limits_{i=1}^{n} x_i$ - etarli statistika bo‘lar ekan. $\triangleleft$

Misollar

Quyidagi taqsimotlar uchun yetarli statistikalarni ko'rsating.

1. $R(\theta_1, \theta_2)$. 2. $N(\theta_1, \theta_2^2)$.

3. $P(\theta_1, \theta_2)$. 4. $\Gamma(\theta_1, \theta_2)$.

5. $W(\theta_1, \theta_2)$. 6. $Ge(\theta)$.

7. $Bi(n; \theta)$. 8. $\Gamma(1; 2\theta_2)$.

9. $\overline{Bi}(r, \theta)$. 10. $E(\theta)$.

11. $K(\theta)$. 12. $\log N(\theta_1, \theta_2^2)$.

13.$P(1; \theta_2)$. 14. $N(\theta, 2\theta)$.

15. $R[-\theta, \theta]$.

16. $f(x,\theta) = \begin{cases} e^{\theta - x}, x \geq \theta, \\ 0\ , \quad x < \theta. \end{cases}$

17. $F(x,\theta) = \begin{cases} 1 - e^{-\frac{x-\theta_1}{\theta_2}}, x \geq \theta_1,\ \theta_2 > 0, \\ 0\ , \quad x < \theta_1. \end{cases}$

18. $f(x,\theta) = \begin{cases} \theta x^{\theta-1}, \quad x \in (0;1),\ \theta > 0, \\ 0\ , \quad x \overline{\in} (0;1). \end{cases}$

19. $f(x,\theta) = e^{-x+\theta}\left(1 + e^{-x+\theta}\right)^2,\ x, \theta \in R.$

20. $f(x,\theta) = \begin{cases} \frac{2x}{\theta^2},\ x \in [0,\theta],\ \theta > 0, \\ 0\ , \quad x \overline{\in} [0,\theta]. \end{cases}$

5-§. TO'LA STATISTIKA

$(\mathcal{Y}, \mathcal{U})$ o'lchovli fazoda aniqlangan $\{Q_\theta\ ,\ \theta \in \Theta\}$ taqsimotlar oilasi to'la deb ataladi, agar ixtiyoriy $\mathcal{U}$ o'lchovli $\varphi(y)$ funksiya uchun

$$\int \phi(y) Q_\theta(dy) = 0\ ,\ \theta \in \Theta \qquad (27)$$

tenglikdan $\{Q_\theta\}$ ga nisbatan deyarli hamma yerda $\phi(y) = 0$ ekanligi kelib chiqsa.

$\left(\mathcal{X}^{(n)}, \mathcal{B}^{(n)}, \left\{P_\theta^{(n)}, \theta \in \Theta\right\}\right)$ statistik modelda aniqlangan.

$T : (\mathcal{X}^{(n)}, \mathcal{B}^{(n)}) \to (\mathcal{Y}, \mathcal{U})$ *statistika to'la* deb ataladi, agar u yaratgan

$$Q_\theta(A) = P_\theta(x^{(n)} : T(x^{(n)}) \in A), \ A \in \mathcal{U}$$

ehtimollar taqsimotlari oilasi to'la bo'lsa.

12-misol. $Bi(n;\theta)$ taqsimot uchun $\sum_{i=1}^{n} X_i$ statistika to'la statistika ekanini ko'rsating.

$\triangleright$ $\theta \in \Theta = (0,1)$ parametrlik binomial taqsimot uchun $T = \sum_{i=1}^{n} X_i$ statistikaning taqsimoti

$$Q_\theta(t) = P_\theta(T = t) = \begin{cases} C_n^t \theta^t (1-\theta)^{n-t}, t \in Y = \{0, 1, ..., n\}, \\ 0, \ \text{aks holda.} \end{cases}$$

bo'ladi. $\varphi = \varphi(t)$ haqiqiy funksiya uchun

$$\sum_{t=0}^{n} \varphi(t) Q_\theta(t) = (1-\theta)^n \sum_{t=0}^{n} C_n^t \varphi(t) \left(\frac{\theta}{1-\theta}\right)^t = 0, \ \theta \in \Theta,$$

bo'lsin. Bu tenglamani chap tomonida darajasi n dan oshmagan $\frac{\theta}{1-\theta}$ ning polinomi turibdi. Bu polinom ko'pi bilan n ta turli nuqtalarda no'lga teng bo'ladi. Ammo bu tenglik barcha $\theta \in \Theta$ lar uchun o'rinli bo'lmoqda. Bu esa, o'z navbatida barcha $t = 0, 1, ..., n$ lar uchun $\varphi(t) = 0$ ekanini ko'rsatadi.

Demak, $\{Q_\theta(t)\ ,\ \theta \in \Theta\}$ oila va $T = \sum_{i=1}^{n} x_i$ statistika to'la ekan. $\triangleleft$

Misollar

1. $R[\theta; \theta+1]$, $\theta \in R$ taqsimot uchun $(X_{(1)}, X_{(n)})$ yetarli statistika, ammo to'la statistika bo'lmasligini ko'rsating.

2. $R[\theta_1; \theta_2]$, $\theta_1 < \theta_2$ taqsimot uchun $(X_{(1)}, X_{(n)})$ vektor to'la statistika bo'lishini ko'rsating.

3. $E(\theta)$ taqsimot uchun $\sum_{i=1}^{n} X_i$ to'la statistika bo'lishini ko'rsating.

4. $N(0;\theta^2)$ taqsimot uchun $\overline{x^2}=\frac{1}{n}\sum\limits_{i=1}^{n}X_i^2$ statistika to'la statistika bo'ladimi?

5. Zichlik funksiyasi $f(x,\theta)=\begin{cases} e^{\theta-x}, x\geq\theta, \\ 0\,, \quad x<\theta \end{cases}$ bo'lgan taqsimot uchun $X_{(1)}$ statistika to'la statistika bo'ladimi?

6. Zichlik funksiyasi $f(x;\theta)=\begin{cases} \theta^{-1}e^{-(x-\beta)/\theta}, & x\geq\beta, \\ 0\,, & x<\beta \end{cases}$ bo'lgan taqsimot uchun $\overline{x}=\frac{1}{n}\sum\limits_{i=1}^{n}X_i$ statistika to'la statistika bo'ladimi?

7. Zichlik funksiyasi $f(x;\theta)=\begin{cases} \alpha^{-1}e^{-(x-\theta)/\alpha}, & x\geq\theta, \\ 0\,, & x<\theta \end{cases}$ bo'lgan taqsimot uchun $X_{(1)}$ statistika to'la statistika bo'ladimi?

8. $R[0;\theta]$ taqsimot uchun $X_{(n)}$ statistika to'la statistika bo'ladimi?

9. $R[\theta;2\theta]$ taqsimot uchun $(X_{(1)},X_{(n)})$ statistika to'la statistika bo'ladimi?

10. $E\left(\frac{1}{\theta}\right)$ taqsimot uchun $\overline{x}=\frac{1}{n}\sum\limits_{i=1}^{n}X_i$ statistika to'la statistika bo'ladimi?

11. Zichlik funksiyasi $f(x,\theta)=\begin{cases} \theta x^{\theta-1}, & x\in(0;1),\ \theta>0, \\ 0\,, & x\overline{\in}(0;1) \end{cases}$ bo'lgan taqsimot uchun $\overline{\ln x}=\frac{1}{n}\sum\limits_{i=1}^{n}\ln X_i$ statistika to'la statistika bo'ladimi?

12. $\pi(\theta)$ taqsimot uchun $\overline{x}=\frac{1}{n}\sum\limits_{i=1}^{n}X_i$ statistika to'la statistika bo'ladimi?

13. $Ge(\theta)$ taqsimot uchun $\overline{x}=\frac{1}{n}\sum\limits_{i=1}^{n}X_i$ statistika to'la statistika bo'ladimi?

14. $W(\lambda;\theta)$ taqsimot uchun $\overline{x^\lambda}=\frac{1}{n}\sum\limits_{i=1}^{n}X_i^\lambda$ statistika to'la statistika bo'ladimi?

15. Zichlik funksiyasi $f(x;\theta)=\begin{cases} \frac{2x}{\theta}e^{-x^2/\theta}, & x\geq 0,\ \theta>0, \\ 0\,, & x<0 \end{cases}$ bo'lgan taqsimot uchun $\overline{x^2}=\frac{1}{n}\sum\limits_{i=1}^{n}X_i^2$ statistika to'la statistika bo'ladimi?

16. $N(\theta;\sigma^2)$ taqsimot uchun $\sum\limits_{i=1}^{n} X_i$ statistika to'la statistika bo'ladimi?

17. Zichlik funksiyasi $f(x,\theta) = \begin{cases} e^{\theta - x}, x \geq \theta, \\ 0\,, \quad x < \theta \end{cases}$ bo'lgan taqsimot uchun $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika to'la statistika bo'ladimi?

18. $\log N(\theta;\sigma^2)$ taqsimot uchun $e^{\overline{x}}$ statistika to'la statistika bo'ladimi?

19. Zichlik funksiyasi $f(x,\theta) = \begin{cases} \frac{2x}{\theta^2}, \ x \in [0,\theta]\,, \ \theta > 0, \\ 0\,, \quad x \overline{\in} [0,\theta] \end{cases}$ bo'lgan taqsimot uchun $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika to'la statistika bo'ladimi?

20. $N(a;\theta^2)$ taqsimot uchun $S_n^2 = \frac{1}{n}\sum\limits_{i=1}^{n} (X_i - a)^2$ statistika to'la statistika bo'ladimi?

6-§. BAHOLAR VA ULARNING XOSSALARI

Agar $T_n(X^{(n)}) = T(X_1, ..., X_n) \in \Theta$ bo'lsa, u holda $T_n(X^{(n)})$ statistika noma'lum θ parametr uchun *baho* deb ataladi.

Agar statistik bahoning matematik kutilmasi noma'lum parametrga teng, ya'ni $MT_n(X^{(n)}) = MT(X_1, \ldots, X_n) = \theta$ bo'lsa, statistik baho θ uchun *siljimagan baho* deyiladi.

Agar $T(X_1, \ldots, X_n)$ statistika noma'lum parametr θ ga ehtimollik bo'yicha yaqinlashsa, ya'ni $\forall \varepsilon > 0$ son uchun:

$$\lim_{n\to\infty} \{P|T(X_1, \ldots, X_n) - \theta| < \varepsilon\} = 1 \tag{28}$$

munosabat o'rinli bo'lsa, u holda $T(X_1, \ldots, X_n)$ statistik baho *asosli baho* deyiladi.

$T_1(X_1, \ldots, X_n)$ va $T_2(X_1, \ldots, X_n)$ lar noma'lum θ parametr uchun baholar bo'lib,

$$M(T_1 - \theta)^2 < M(T_2 - \theta)^2 \tag{29}$$

munosabat bajarilsa, u holda $T_1(X_1, \ldots, X_n)$ baho $T_2(X_1, \ldots, X_n)$ bahoga nisbatan *effektiv* deyiladi. Agar $T_1(X_1, \ldots, X_n)$ va

$T_2(X_1, \ldots, X_n)$ lar noma'lum θ parametr uchun siljimagan baholar bo'lsa, u holda (29) tenglikni $DT_1 < DT_2$ ko'rinishda yozish mumkin.

13-misol. $X_1, ..., X_n$ tanlanma $R\,[0, \theta]$ taqsimotdan olingan bo'lsin, noma'lum parametr θ uchun $2\overline{x}$ bahoning siljimaganligi va asosliligini ko'rsating.

$\triangleright$ *Siljimaganlik:*

$$M\,(2\overline{x}) = 2M\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = 2\frac{1}{n}\sum_{i=1}^{n} MX_i = \frac{2}{n}\sum_{i=1}^{n}\frac{\theta}{2} = \frac{2}{n}\frac{\theta}{2}n = \theta.$$

Asoslilik:

$$D\,(2\overline{x}) = 4D\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{4}{n^2}\sum DX_i = \frac{4}{n^2}\frac{\theta^2}{12}n = \frac{\theta^2}{3n}\xrightarrow[n\to\infty]{} 0,$$

Chebishev tengsizligiga ko'ra, $\forall\varepsilon > 0$ uchun:

$$P\left\{|2\overline{x} - \theta| < \varepsilon\right\} \geq 1 - \frac{D\,(2\overline{x})}{\varepsilon^2} = 1 - \frac{\theta^2}{3\varepsilon n}\xrightarrow[n\to\infty]{} 1.$$

$\triangleleft$

Misollar

1. $X_1, ..., X_n$ tanlanma $R[0; \theta]$ taqsimotdan olingan bo'lsin, noma'lum parametr θ uchun $\frac{n+1}{n}X_{(n)}$ bahoni siljimaganlik va asoslilikka tekshiring.

2. $X_1, ..., X_n$ tanlanma $N\left(\theta_1, \theta_2^2\right)$ taqsimotdan olingan bo'lsin, $\overline{x}$ va S^2 larni mos ravishda θ_1 va θ_2^2 lar uchun siljimaganlik va asoslilikka tekshiring.

3. $X_1, ..., X_n$ tanlanma zichligi $f\,(x, \theta) = \begin{cases} e^{\theta - x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimotdan olingan bo'lsin, noma'lum θ parametr uchun quyidagi baholarni siljimaganlik va asoslilikka tekshiring: a) $X_{(1)}$; b) $\overline{x} - 1$.

4. $X_1, ..., X_n$ tanlanma $N\left(\theta, \sigma^2\right)$ bo'lgan taqsimotdan olingan bo'lsin(bu yerda σ-ma'lum), $T\left(x^{(n)}\right) = \overline{x^2} - \frac{\sigma^2}{n}$ statistika $g(\theta) = \theta^2$ uchun siljimagan baho bo'lishini ko'rsating.

5. $X_1, ..., X_n$ tanlanma $R(\theta_1, \theta_2)$ taqsimotdan olingan bo'lsa, $T\left(X^{(n)}\right) = \frac{1}{2}\left(X_{(n)} + X_{(1)}\right)$ statistika $g(\theta) = \frac{1}{2}(\theta_1+\theta_2)$ uchun siljimagan baho bo'lishini ko'rsating.

6. $X_1, ..., X_n$ tanlanma $R(\theta_1, \theta_2)$ taqsimotdan olingan bo'lsa, $T\left(X^{(n)}\right) = \frac{n+1}{n-1}\left(X_{(n)} + X_{(1)}\right)$ statistika $g(\theta) = \theta_2 - \theta_1$ uchun siljimagan baho bo'lishini ko'rsating.

7. $X_1, ..., X_n$ tanlanma $MX_1 = a$ ma'lum va MX_1^2 chekli bo'lgan taqsimotdan olingan bo'lsin. $T\left(X^{(n)}\right) = \frac{1}{n-1}\sum\limits_{i=1}^{n}\left(X_i - \overline{x}\right)^2$ statistika noma'lum dispersiya uchun siljimagan va asosli baho bo'ladimi?

8. $X_1, ..., X_n$ tanlanma $MX_1 = a$ ma'lum va MX_1^2 chekli bo'lgan taqsimotdan olingan bo'lsin. $T\left(X^{(n)}\right) = \overline{x}^2 - a^2$ statistika noma'lum dispersiya uchun siljimagan va asosli baho bo'ladimi?

9. $X_1, ..., X_n$ tanlanma $MX_1 = a$ ma'lum va MX_1^2 chekli bo'lgan taqsimotdan olingan bo'lsin. $T\left(X^{(n)}\right) = \frac{1}{n}\sum\limits_{i=1}^{n}\left(X_i - a\right)^2$ statistika noma'lum dispersiya uchun siljimagan va asosli baho bo'ladimi?

10. $X_1, ..., X_n$ tanlanma $P(\theta, \beta)$ taqsimotdan olingan bo'lsin(bu yerda β-ma'lum). Noma'lum parametr θ uchun $T\left(X^{(n)}\right) = X_{(1)}$ statistika siljimagan va asosli baho bo'ladimi?

11. $X_1, ..., X_n$ tanlanma zichlik funksiyasi $f(x;\theta) = e^{-x+\theta}\left(1 + e^{-x+\theta}\right)^{-2}$, $-\infty < x < +\infty$, $\theta \in (-\infty, +\infty)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T\left(X^{(n)}\right) = \overline{x}$ statistika siljimagan baho bo'lishini ko'rsating.

12. $X_1, ..., X_n$ tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T\left(X^{(n)}\right) = \frac{1}{\overline{x}}$ statistika siljimagan va asosli baho bo'ladimi?

13. $X_1, ..., X_n$ tanlanma $E(\frac{1}{\sqrt{\theta}})$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T\left(X^{(n)}\right) = (\overline{x})^2$ statistika siljimagan va asosli baho bo'ladimi?

14. $X_1, ..., X_n$ tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T\left(X^{(n)}\right) = \sqrt[k]{k!/\overline{x}^k}$, $k = 1, 2, ...$ statistika siljimagan va asosli baho bo'ladimi?

15. $X_1, ..., X_n$ tanlanma $Bi(1; \sqrt{\theta})$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T\left(X^{(n)}\right) = (\overline{x})^2$ statistika siljimagan va

asosli baho bo‘ladimi?

16. $X_1,...,X_n$ tanlanma $P(\alpha;\theta)$ taqsimotdan olingan bo‘lsin(bu yerda α-ma'lum). Noma'lum parametr θ uchun $T\left(X^{(n)}\right)=\frac{1}{\overline{\ln x}-\ln x_{(1)}}$ statistika siljimagan va asosli baho bo‘ladimi?

17. $X_1,...,X_n$ tanlanma $W(\alpha;\theta)$ taqsimotdan olingan bo‘lsin(bu yerda α-ma'lum). Noma'lum parametr θ uchun $T\left(X^{(n)}\right)=1/(\overline{x^{\alpha}})$ statistika siljimagan va asosli baho bo‘ladimi?

18. $X_1,...,X_n$ tanlanma $Bi(n;\theta)$ taqsimotdan olingan bo‘lsin. $T(X^{(n)})=\frac{n\overline{x}+\alpha}{n+\beta}$, $\alpha\geq 0$; $\beta\geq 0$ baholar oilasini ko‘raylik. $MT(X^{(n)})$ va $\sqrt{DT(X^{(n)})}$ larni hisoblang.

19. $X_1,...,X_n$ tanlanma zichlik funksiyasi quyidagicha bo‘lgan taqsimotdan olingan bo‘lsin: $f(x;\theta)=\begin{cases}\alpha^{-1}e^{-(x-\theta)/\alpha}, & x\geq\theta,\\ 0\,, & x<\theta\end{cases}$ (bu yerda α-ma'lum). Noma'lum parametr θ uchun $T\left(X^{(n)}\right)=X_{(1)}$ statistika siljimagan va asosli baho bo‘ladimi?

20. $X_1,...,X_n$ tanlanma zichlik funksiyasi quyidagicha bo‘lgan taqsimotdan olingan bo‘lsin: $f(x;\theta)=\begin{cases}\theta^{-1}e^{-(x-\beta)/\theta}, & x\geq\beta,\\ 0\,, & x<\beta\end{cases}$ (bu yerda β-ma'lum). Noma'lum parametr θ uchun $T\left(X^{(n)}\right)=\overline{x}+\beta$ statistika siljimagan va asosli baho bo‘ladimi?

7-§. RAO-KRAMER TENGSIZLIGI. EFFEKTIV BAHOLASH

$\left(\mathscr{X}^{(n)},\mathscr{B}^{(n)},\left\{P_\theta^{(n)},\theta\in\Theta\right\}\right)$, $\Theta\subseteq R$ - parametrik statistik modelni qaraylik. Har bir X_i kuzatilmaning $f(x,\theta)$ umumlashgan zichlik funksiyasi uchun *Rao-Kramer regulyarlik shartlarini* kiritamiz:

(I) $N_f=\{x: f(x,\theta)>0\}$ - to‘plam θ ga bog‘liq emas;

(II) $\Theta=R$ yoki Θ - to‘plam R dagi biror interval;

(III) $\frac{\partial}{\partial\theta}f\left(x,\theta\right)$- xususiy hosila mavjud va $\{P_\theta,\theta\in\Theta\}$ga nisbatan deyarli hamma yerda $\forall\theta\in\Theta$ uchun chekli;

(IV) $\forall\theta\in\Theta$ va $i=1,2$ uchun $\int\left|\frac{\partial^i}{\partial\theta^i}f\left(x,\theta\right)\right|\mu\left(dx\right)<\infty$;

(V) $\forall\theta\in\Theta$: $0<I\left(\theta\right)=M_\theta\left[\frac{\partial}{\partial\theta}\ln f\left(\xi,\theta\right)\right]^2<\infty$.

Biz $x^{(n)}$ tanlanmaning $f_n\left(X^{(n)},\theta\right)=\prod\limits_{i=1}^{n}f\left(x_i,\theta\right)$ - zichlik funksiyasini qarayotganimizda (I) - (V) shartlarni f o‘rnida f_n ni ishlatamiz va integrallar $\mathscr{X}^{(n)}$ to‘plam bo‘yicha tushuniladi. $I(\theta)$ funksiya ξ tasodifiy miqdordagi θ parametr haqidagi *Fisher informatsiyasi* deyiladi. $X^{(n)}$ tanlanmaga mos Fisher informatsiyasini $I_n(\theta)$ orqali belgilaymiz.

Ushbu

$$l_n\left(x^{(n)},\theta\right)=\frac{\partial}{\partial\theta}\ln f_n\left(x^{(n)},\theta\right)=\sum_{i=1}^{n}l\left(x_i,\theta\right),$$

$$l\left(x_i,\theta\right)=\frac{\partial}{\partial\theta}\ln f\left(x_i,\theta\right),\ i=1,...,n,$$

- funksiyalar *informantlar* deb ataladi.

$\{f_n(x^{(n)},\theta),\theta\in\Theta\}$ uchun (I) - (V) shartlar bajarilsin. U holda

$$I_n(\theta)=nI(\theta),\quad \theta\in\Theta. \tag{30}$$

Teorema (Rao-Kramer). $\{f_n(x^{(n)},\theta),\theta\in\Theta\}$ oila uchun (I) - (V) shartlar bajarilsin va differensiallanuvchi $g(\theta)$ funksiyaga siljimagan $\widehat{g}_n(X^{(n)})$ baho uchun barcha $\forall\theta\in\Theta$larda

(VI) $\int\left|\hat{g}_n\left(x^{(n)}\right)\frac{\partial}{\partial\theta}f_n\left(x^{(n)},\theta\right)\right|\mu^{(n)}\left(dx^{(n)}\right)<\infty$ va $D_\theta\hat{g}_n<\infty$ bo‘lsin. U holda $\forall\theta\in\Theta$ uchun

$$D_\theta\hat{g}_n\left(X^{(n)}\right)\geq\frac{\left[g‘\left(\theta\right)\right]^2}{nI\left(\theta\right)}. \tag{31}$$

(31) da tenglik bajarilishi uchun $f_n\left(x^{(n)};\theta\right)$ - zichlik funksiyasi quyidagi maxsus eksponensial oilaga tegishli bo‘lishi zarur va yetarlidir:

$$f_n\left(x^{(n)};\theta\right)=\exp\left\{\hat{g}_n\left(x^{(n)}\right)\Psi_1\left(\theta\right)+\Psi_2\left(\theta\right)+k_n\left(x^{(n)}\right)\right\}, \tag{32}$$

bu yerda $\Psi_1\left(\theta\right)\neq 0$. Agar (31) da tenglik o‘rinli bo‘lsa, $\widehat{g}_n$ baho $g(\theta)$ uchun Rao-Kramer ma‘nosida effektiv (yoki shunchaki effektiv) baho deb ataladi.

14-misol. $X_1,...,X_n\sim N(\theta,\sigma^2)$ bo‘lsin. $\hat{\theta}=\overline{x}$ effektiv baho bo‘ladimi?

▷ $f(x,\theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\theta)^2}{2\sigma^2}\right\}$ funksiya uchun regulyarlik shartlari o'rinli. Fisher informatsiyasini hisoblaymiz:

$$\ln f(X_1;\theta) = -\ln(2\pi\sigma^2)^{1/2} - \frac{(X_1-\theta)^2}{2\sigma^2},$$

$$\frac{\partial \ln f(X_1,\theta)}{\partial\theta} = \frac{X_1-\theta}{\sigma^2},$$

$$I(\theta) = M_\theta\left(\frac{\partial \ln f(X_1,\theta)}{\partial\theta}\right)^2 = M_\theta\left(\frac{X_1-\theta}{\sigma^2}\right)^2 = \frac{1}{\sigma^4}M_\theta(X_1-\theta)^2 =$$

$$= \frac{1}{\sigma^4}M_\theta(X_1 - MX_1)^2 = \frac{1}{\sigma^4}D_\theta X_1 = \frac{1}{\sigma^2}.$$

Demak, $I(\theta) = \frac{1}{\sigma^2}$. Endi $\overline{x}$ ning dispersiyasini hisoblaymiz:

$$D_\theta\overline{x} = D_\theta\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} D_\theta X_i = \frac{1}{n^2}n\sigma^2 = \frac{\sigma^2}{n}.$$

Demak, $D_\theta\overline{x} = \frac{1}{nI(\theta)}$, ya'ni $\overline{x}$ baho θ uchun effektiv ekan. Bu yerda $f_n(x^{(n)};\theta)$ uchun (32) formula o'rinlidir:

$$f_n(x^{(n)};\theta) = \exp\left\{\overline{x}\;\frac{n\theta}{\sigma^2} + \left(-\theta^2\frac{n}{2\sigma^2}\right) + \left(-\frac{\overline{x^2}n}{2\sigma^2} - \frac{n}{2}\ln 2\pi - n\ln\sigma\right)\right\}.$$

◁

Misollar

Quyidagi modellarda keltirilgan baholarni effektivlikka tekshiring.

Model	**Baholar**
1. $N(\theta;\sigma^2)$	$\widehat{\theta}=X_1$
2. $N(a;\theta^2)$	$\widehat{\theta}^2=S^2$
3. $\pi(\theta)$	$\widehat{\theta}=\frac{1}{2}(X_1+X_2)$
4. $Bi(n;\theta)$	$\widehat{\theta}=\overline{x}/n$
5. $\pi(\theta)$	$\widehat{\theta}=\overline{x}$
6. $E(1/\theta)$	$\widehat{\theta}=\overline{x}$
7. $\pi(\theta)$	$\widehat{\theta}=X_1$
8. $Ge(\theta)$	$\widehat{\theta}=1/(1+\overline{x})$
9. $\Gamma(\theta,\lambda)$	$\widehat{\theta}=\overline{x}/\lambda$
10. $f(x,\theta)=e^{-x+\theta}\left(1+e^{-x+\theta}\right)^{-2};\ x,\theta\in R$	$\widehat{\theta}=\overline{x}$
11. $f(x;\theta)=\begin{cases}\frac{2x}{\theta}e^{-x^2/\theta}, & x\geq 0,\ \theta>0,\\ 0\ , & x<0\end{cases}$	$\widehat{\theta}=\overline{x^2}$
12. $\overline{Bi}(r,\theta)$	$\widehat{\theta}=\overline{x}/(r+\overline{x})$
13. $\Gamma(\theta,\lambda)$	$\widehat{\theta}=\frac{1}{\overline{x}}(1-1/n)$
14. $E(\theta)$	$\hat{\theta}=\frac{1}{\overline{x}}$
15. $Bi(n;\theta)$	$\hat{\theta}=X_1$
16. $N(a;\theta^2)$	$\hat{\theta}=\overline{x^2}-a^2$
17. $f(x,\theta)=\begin{cases}\theta\left(\ln^{\theta-1}x\right)/x, & x\in[1,e],\theta>0,\\ 0\ , & x\overline{\in}[1,e]\end{cases}$	$\hat{\theta}=\overline{x}$
18. $E(\theta)$	$\hat{\theta}=\frac{n-1}{n\overline{x}}$
19. $Ge(\theta)$	$\hat{\theta}=\frac{1}{\overline{x}}$
20. $N(\theta;\sigma^2)$	$\hat{\theta}^2=(\overline{x})^2$

8-§. MOMENTLAR USULI

X t.f. $F(x,\theta)$ noma'lum parametr $\theta = (\theta_1,...,\theta_r)$ ga bog'liq bo'lgan t.m. bo'lsin. Faraz qilaylik, X t.m.ning birinchi r ta $\alpha_k = MX^k$, $k = 1,...,r$ momentlari mavjud bo'lsin. Tabiiyki, ular noma'lum θ parametrning $\alpha_k = \alpha_k(\theta)$ funksiyalari bo'ladi. $A_{nk} = \sum\limits_{i=1}^{n} X_i^k$, $k = 1,...,r$ tanlanma momentlarini KSQ ga asoslangan holda mos ravishda α_k, $k = 1,...,r$ larga tenglashtirib, yetarlicha katta n larda r ta tenglamalar sistemasini tuzib olamiz:

$$\begin{cases} \alpha_1(\theta) = A_{n1}, \\ \alpha_2(\theta) = A_{n2}, \\ \vdots \\ \alpha_r(\theta) = A_{nr}. \end{cases} \tag{33}$$

Mana shu tenglamalar sistemasini $\theta_1,...,\theta_r$ larga nisbatan yechib, $\widetilde{\theta}_k = \widetilde{\theta}_k(X_1,\ldots,X_n)$, $k = 1,...,r$ yechimlarga ega bo'lamiz. Shunday topilgan $\widetilde{\theta}_k$, $k = 1,...,r$ statistikalar momentlar usuli bilan noma'lum θ_k, $k = 1,...,r$ paramertlar uchun tuzilgan statistik baholar deb ataladi.

15-misol. $f(x,\theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0\,, & x < \theta \end{cases}$ taqsimot noma'lum parametrini momentlar usulida baholang.

▷ Buning uchun matematik kutilmani hisoblaymiz:

$$M_\theta X = \int\limits_{\theta}^{+\infty} xe^{\theta-x}dx = \left| \begin{matrix} x = u & dx = du \\ e^{\theta-x}dx = d\nu & \nu = -e^{\theta-x} \end{matrix} \right| = -xe^{\theta-x}\Big|_{\theta}^{+\infty} +$$

$$+ \int\limits_{\theta}^{+\infty} e^{\theta-x}dx = \theta - e^{\theta-x}\Big|_{\theta}^{+\infty} = \theta + 1.$$

Nazariy momentni mos tanlanma momentga tenglashtirib MUBni topamiz: $\tilde{\theta} = \overline{x} - 1$ ◁

Misollar

Quyidagi taqsimotlar noma'lum parametrlarini momentlar usulida bahosini tuzing.

1. $\Gamma(\theta_1, \theta_2)$.
2. $\overline{Bi}(r; \theta)$.
3. $N(\theta_1, \theta_2^2)$.
4. $R[\theta_1, \theta_2]$.
5. $P(\theta_1, \theta_2)$.
6. $\Gamma(2\theta, 1)$.
7. $F(x,\theta) = \begin{cases} 1 - e^{-\frac{x-\theta_1}{\theta_2}}, \ x \geq \theta_1, \ \theta_2 > 0, \\ 0, \quad x < \theta_1. \end{cases}$
8. $P(2\theta, 1)$.
9. $f(x,\theta) = \begin{cases} \theta x^{\theta-1}, \ x \in [0,1], \\ 0, \quad x \overline{\in} [0,1]. \end{cases}$
10. $W(\theta_1, \theta_2)$.
11. $f(x,\theta) = e^{-\frac{x-\theta}{2}}, \ x > \theta, \ \theta > 0.$
12. $Ge(\theta)$.
13. $f(x,\theta) = e^{-x+\theta}\left(1 + e^{-x+\theta}\right)^{-2}, x, \theta \in R.$
14. $R[-2\theta, \ 2\theta]$.
15. $f(x,\theta) = \begin{cases} \theta\left(\ln^{\theta-1} x\right)/x, \quad x \in [1,e], \theta > 0, \\ 0, \quad x \overline{\in} [1,e]. \end{cases}$
16. $N(\theta, 2\theta)$.
17. $f(x,\theta) = \begin{cases} \frac{2x}{\theta^2}, \ x \in [0,\theta], \ \theta > 0, \\ 0, \quad x \overline{\in} [0,\theta]. \end{cases}$
18. $\log N(\theta_1, \theta_2^2)$.
19. $f(x,\theta) = \begin{cases} 3x^2\theta^{-3}e^{-(x/\theta)^3}, x \geq 0, \ \theta > 0, \\ 0, \quad x < 0. \end{cases}$
20. $E\left(\frac{2}{\theta}\right)$.

9-§. HAQIQATGA MAKSIMAL O'XSHASHLIK USULI

Kuzatilmalari $X_1, ..., X_n$ lardan va umumlashgan zichlik funksiyasi $f(x,\theta)$ bo'lgan X t.m.ni olaylik:

$$f(x,\theta) = \begin{cases} P_\theta\{X = x\} \quad \text{agar } X \text{ diskret t.m. bo'lsa}, \\ \frac{\partial F(x,\theta)}{\partial x}, \ \text{agar } X \text{ uzluksiz t.m. bo'lsa}. \end{cases}$$

Quyidagi funksiyaga

$$L(x_1, ..., x_n, \theta) = f_n\left(x^{(n)}, \theta\right) = f(x_1,\theta) \cdot ... \cdot f(x_n,\theta)$$

haqiqatga maksimal o'xshashlik funksiyasi deyiladi. Faraz qilaylik, $L(x_1,...,x_n,\theta)$ funksiya $\theta \in \Theta$ yopiq sohada biror θ^* nuqtada eng katta qiymatga erishsin:

$$L(x_1,...,x_n,\theta^*) = \max_{\theta\in\Theta} L(x_1,...,x_n,\theta).$$

Haqiqatga maksimal o'xshashlik funksiyasi eng katta qiymatga erishadigan θ^* qiymat noma'lum θ parametr uchun haqiqatga maksimal o'xshashlik usuli bilan tuzilgan statistik baho deb ataladi. Ularni ma'lum shartlarda quyidagi tenglamalar sistemasidan ham topish mumkin:

$$\left.\frac{\partial L(x_1,...,x_n,\theta)}{\partial\theta_k}\right|_{\theta=\theta^*} = 0, \quad k = 1,...,r. \tag{34}$$

(34) sistema haqiqatga maksimal o'xshashlik tenglamalari sistemasi deyiladi. Ko'p hollarda (34) tenglamalar sistemasi o'rniga quyidagi unga ekvivalent tenglamalar sistemasini yechish qulay bo'ladi:

$$\left.\frac{\partial \ln L(x_1,...,x_n,\theta)}{\partial\theta_k}\right|_{\theta=\theta^*} = 0, \quad k = 1,...,r. \tag{35}$$

16-misol. $R\,[-\theta,\theta]$ taqsimot noma'lum parametrini haqiqatga maksimal o'xshashlik usuli bilan baholang.

$\triangleright$ $R\,[-\theta,\theta]$ taqsimot zichlik funksiyasini yozib olamiz:

$$f\,(x,\theta) = \begin{cases} \frac{1}{2\theta}\,, & x \in [-\theta,\theta] \\ 0\,, & x \overline{\in}\,[-\theta,\theta] \end{cases} = \frac{1}{2\theta} I\,(-\theta \le x \le \theta)\,.$$

Haqiqatga o'xshashlik funksiyasini tuzamiz:

$$f_n\left(x^{(n)},\theta\right) = \frac{1}{(2\theta)^n}\prod_{i=1}^{n} I\,(-\theta \le X_i \le \theta) =$$

$$= \frac{1}{(2\theta)^n} I\left(-\theta \le X_{(1)} \le X_{(n)} \le \theta\right).$$

Demak, $\begin{cases} \hat{\theta} = -X_{(1)} - \text{quyi chegara uchun} \\ \hat{\theta} = X_{(n)} - \text{yuqori chegara uchun} \end{cases}$ – HMO'UB bo'ladi, ya'ni yagona θ uchun ikkita baho $-X_{(1)}$ va $X_{(n)}$ lar HMO'UB bo'lar ekan. ◁

17-misol. $\pi(\theta)$ taqsimot noma'lum parametrini haqiqatga maksimal o'xshashlik usuli bilan baholang.

▷ Puasson taqsimoti $p_k = P(\xi = k) = \frac{e^{-\theta}\theta^k}{k!}, \quad k = 0, 1, ...; \theta > 0$ dan haqiqatga o'xshashlik funksiyasini tuzamiz:

$$f_n\left(x^{(n)}, \theta\right) = \frac{e^{-\theta n}\theta^{\sum\limits_{i=1}^{n} x_i}}{\prod\limits_{i=1}^{n} x_i!} = \frac{e^{-\theta n}\theta^{n\overline{x}}}{\prod\limits_{i=1}^{n} x_i!}.$$

Bu funksiya θ bo'yicha uzluksiz differensiallanuvchi ekanligidan, θ bo'yicha ekstremumini topish mumkin. Buning uchun bu funksiyaning logarifmidan foydalanamiz:

$$\ln f_n\left(x^{(n)}, \theta\right) = \ln\left(\frac{e^{-\theta n}\theta^{\sum\limits_{i=1}^{n} x_i}}{\prod\limits_{i=1}^{n} x_i!}\right) = n\overline{x}\ln\theta - \ln\prod_{i=1}^{n} x_i! - \theta n.$$

U holda $\frac{\partial \ln f_n\left(x^{(n)},\theta\right)}{\partial\theta} = \frac{n\overline{x}}{\theta} - n = 0$ tenglamadan θ uchun $\overline{x}$ HMO'UB bo'lar ekan. Demak, $\hat{\theta} = \overline{x}$. ◁

Misollar

Quyidagi taqsimotlar noma'lum parametrlarini haqiqatga maksimal o'xshashlik usuli bilan baholang.

1. $\Gamma(\theta_1, \theta_2)$. 2. $\overline{Bi}(r, \theta)$.

3. $N(\theta_1, \theta_2^2)$. 4. $f(x, \theta) = \frac{\theta}{2} e^{-\theta x},\ \theta > 0$.

5. $E(2\theta)$. 6. $R(-\theta, \theta)$.

7. $N(\theta, 2\theta)$. 8. $R[\theta - 1, \theta + 1]$.

9. $R[\theta_1, \theta_2]$. 10. $f(x, \theta) = \begin{cases} e^{\theta - x}, & x \geq \theta, \\ 0, & x < \theta. \end{cases}$

11. $P(\theta_1, \theta_2)$. 12. $F(x, \theta) = \begin{cases} 1 - e^{-\frac{x - \theta_1}{\theta_2}}, & x \geq \theta_1, \quad \theta_1, \theta_2 > 0, \\ 0, & x < \theta_1. \end{cases}$

13. $\log N(\theta_1, \theta_2^2)$. 14. $f(x, \theta) = \begin{cases} 3x^2 \theta^{-3} e^{-(x/\theta)^3}, & x \geq 0, \theta > 0, \\ 0, & x < 0. \end{cases}$

15. $W(2, \theta)$. 16. $f(x, \theta) = \begin{cases} \theta \left(\ln^{\theta - 1} x\right)/x, & x \in [1, e], \theta > 0, \\ 0, & x \overline{\in} [1, e]. \end{cases}$

17. $G\left(\frac{\theta}{2}\right)$. 18. $f(x, \theta) = e^{-\frac{(x - \theta)}{2}},\ x,\ \theta \in R$.

19. $Bi\left(n; \frac{\theta}{2}\right)$. 20. $f(x, \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \ \theta > 0, \\ 0, & x \overline{\in} [0, \theta]. \end{cases}$

10-§. BAYES BAHOLASH USULI

Bayescha baholashning mohiyati shundan iboratki, unda noma'lum parametr θ tasodifiy miqdor deb qaraladi. θ t.m. ning zichlik funksiyasi $q(\theta)$ *aprior zichlik* deb ataladi. $f_n\left(x^{(n)}, \theta\right)$ tanlanmaning zichlik funksiyasi bo'lsin.

$$f(\theta; x_1, ..., x_n) = f_n\left(x^{(n)}, \theta\right) q(\theta)$$

funksiya $R^{(n)} \times \Theta$ dagi biror taqsimotning zichlik funksiyasi (yoki $x^{(n)}$ va θ ning) birgalikdagi zichlik funksiyasi bo'ladi. Noma'lum parametr θ ning $x_1, ..., x_n$ tanlanma bo'yicha *Bayes bahosi*

$$\theta_n^* = M\left(\theta/x_1, ..., x_n\right) = \int\limits_{\Theta} \theta \; q\left(\theta/x_1, ..., x_n\right) \lambda\left(d\theta\right)$$

ifoda orqali topiladi. Bu yerda $q\left(\theta/x_1, ..., x_n\right)$ noma'lum parametr θ ning *aposterior zichligi* bo‘lib, u quyidagicha hisoblanadi

$$q\left(\theta/x_1, ..., x_n\right) = \frac{f_n\left(x^{(n)}, \theta\right) q\left(\theta\right)}{\int\limits_{\Theta} f_n\left(x^{(n)}, s\right) q\left(s\right) \lambda\left(ds\right)}.$$

18-misol. $X_1, ..., X_n \sim N\left(\theta; 1\right)$ va θ parametr taqsimoti $N\left(0; \sigma^2\right)$ (σ^2 - ma'lum) bo‘lsin. Noma'lum parametr θ ning Bayes bahosini tuzing.

$\triangleright$ Noma'lum parametr taqsimoti $N\left(0; \sigma^2\right)$ bo‘lganligi uchun uning zichlik funksiyasi $q\left(\theta\right) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\theta^2/2\sigma^2}$, $\theta \in R$, $\sigma > 0$ bo‘ladi. $x^{(n)}$ tanlanmaning zichlik funksiyasi esa

$$f_n\left(x^{(n)}, \theta\right) = \frac{1}{\left(2\pi\right)^{n/2}} e^{-\frac{1}{2}\sum\limits_{i=1}^{n}\left(x_i - \theta\right)^2}.$$

$q\left(\theta/x_1, ..., x_n\right)$ aposterior zichlik $q\left(\theta\right) f_n\left(x^{(n)}, \theta\right)$ ga proporsional:

$$\exp\left\{-\frac{\theta^2}{2\sigma^2} - \frac{1}{2}\sum_{i=1}^{n}\left(x_i - \theta\right)^2\right\} = \exp\left\{-\frac{\theta^2\left(1/\sigma^2 + n\right)}{2} + \overline{x}n\theta - \frac{n\overline{x^2}}{2}\right\}.$$

Har ikkala tomon darajalarini tenglashtirsak,

$$\frac{\theta^2}{2}\left(\frac{1}{\sigma^2} + n\right) + \overline{x}\theta n = -\frac{1}{2}\left(\frac{1}{\sigma^2} + n\right)\left(\theta - \frac{\overline{x}n}{1/\sigma^2 + n}\right)^2 + \frac{\left(\overline{x}n\right)}{2\left(1/\sigma^2 + n\right)}$$

hosil bo‘ladi. Bu tenglikdan $q\left(\theta/x_1, ..., x_n\right)$aposterior zichlik $N\left(\frac{\overline{x}n\sigma^2}{1+n\sigma^2}; \frac{\sigma^2}{1+n\sigma^2}\right)$ taqsimotga ega ekanligi kelib chiqadi. Demak, θ uchun Bayes baho

$$\theta_n^* = \int\limits_{\Theta} \theta q\left(\theta / x_1, ..., x_n\right) d\theta = \frac{\overline{x} n \sigma^2}{1 + n\sigma^2}.$$

$\triangleleft$

Misollar

1. $X_1, ..., X_n \sim N(\theta; 1)$ va θ parametr $N\left(a; \sigma^2\right)$ $\left(a, \sigma^2 - \text{ma'lum}\right)$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

2. $X_1, ..., X_n \sim Bi(1; \theta)$ va θ parametr $R[0, 1]$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

3. $X_1, ..., X_n \sim N(\theta; 1)$ va θ parametr $Bi(1; 1/2)$ bo‘lsin. Noma’lum parametr θ ning bayes bahosini toping.

4. $X_1, ..., X_n \sim \pi(\theta)$ va θ parametr $Bi(1; 1/2)$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

5. $X_1, ..., X_n$ tanlanma zichlik funksiyasi $f(x; \theta) = \begin{cases} e^{\theta - x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$ bo‘lgan taqsimotdan olingan va θ parametr taqsimoti $R[0, 1]$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

6. $X_1, ..., X_n \sim R[0, \theta]$ va θ parametr $R[0, 1]$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

7. $X_1, ..., X_n \sim R[0, \theta]$ va θ parametr zichlik funksiyasi $q(\theta) = 1/\theta^2, \ \theta \geq 1$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

8. $X_1, ..., X_n \sim R[0, \theta]$ va θ parametr taqsimoti: $\begin{cases} \theta : & 1 & 2 \\ P_\theta : & \frac{1}{2} & \frac{1}{2} \end{cases}$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

9. $X_1, ..., X_n \sim E(\theta)$ va θ parametr $E(1)$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

10. $X_1, ..., X_n \sim E(\theta)$ va θ parametr $R[0, 1]$ bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

11. $X_1, ..., X_n$ tanlanma zichlik funksiyasi $f(x; \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \\ 0, & x \overline{\in} [0, \theta] \end{cases}$ bo‘lgan taqsimotdan olingan va θ parametr taqsimoti $P(\beta, 1)$ (β - ma’lum) bo‘lsin. Noma’lum parametr θ ning Bayes bahosini toping.

12. $X_1, ..., X_n$ tanlanma zichlik funksiyasi $f(x;\theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0,\theta], \\ 0, & x \overline{\in} [0,\theta] \end{cases}$ va θ parametr zichlik funksiyasi $q(\theta) = 3\theta^2, \ \theta \in [0,1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

13. $X_1, ..., X_n$ tanlanma zichlik funksiyasi $f(x;\theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0,\theta], \\ 0, & x \overline{\in} [0,\theta] \end{cases}$ va θ parametr zichlik funksiyasi $R[0,1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

14. $X_1, ..., X_n \sim Bi(1;\theta)$ va θ parametr taqsimoti $\begin{cases} \theta: & 1/2 & 1/3 \\ P_\theta: & 1/2 & 1/2 \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

15. $X_1, ..., X_n \sim Bi(1;\theta)$ va θ parametr zichlik funksiyasi $q(\theta) = \lambda\theta^{\lambda-1}, \ \theta \in [0,1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

16. $X_1, ..., X_n \sim \pi(\theta)$ va θ parametr taqsimoti $\begin{cases} \theta: & 1 & 2 \\ P_\theta: & 1/3 & 2/3 \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

17. $X_1, ..., X_n \sim G(\theta)$ va θ parametr taqsimoti $\begin{cases} \theta: & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\ P_\theta: & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

18. $X_1, ..., X_n \sim G(\theta)$ va θ parametr taqsimoti $\begin{cases} \theta: & 1 & 2 \\ P_\theta: & \frac{1}{3} & \frac{2}{3} \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

19. $X_1, ..., X_n \sim \pi(\theta)$ va θ parametr $Bi\left(1; \frac{1}{3}\right)$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

20. $X_1, ..., X_n \sim G(\theta)$ va θ parametr $Bi\left(1; \frac{1}{2}\right)$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

11-§. BAHOLARNING ASIMPTOTIK NORMALLIGI

Agar $\sqrt{n}(\theta_n - \theta) \Rightarrow N(0; D(\theta_n))$ (yoki $\frac{\sqrt{n}(\theta_n - \theta)}{D(\theta_n)} \Rightarrow N(0;1)$) shart bajarilsa, θ_n baho noma'lum parametr θ uchun *asimptotik normal* baho deyiladi.

Agar θ_n noma'lum parametr θ uchun a.n. baho bo'lsa, u holda θ_n asosli baho bo'ladi.

19-misol. $X_1, ..., X_n$ tanlanma $R\,[0;\theta]$ taqsimotdan olingan bo'lsin. $2\overline{x}$ statistika noma'lum θ parametr uchun asimptotik normal baho bo'lishini ko'rsating. $\triangleright$ $MX_1 = \theta/2$ va $DX_1 = \theta^2/12$ ekanligini e'tiborga olib, quyidagini yoza olamiz:

$$\sqrt{n}\,(\theta_n - \theta) = \sqrt{n}\,(2\overline{x} - \theta) = \sqrt{n}\left(2\frac{\sum\limits_{i=1}^{n} X_i}{n} - \theta\right) =$$

$$= \frac{\sum\limits_{i=1}^{n} 2X_i - n\theta}{\sqrt{n}} = \frac{\sum\limits_{i=1}^{n} 2X_i - n \cdot M \cdot (2X_1)}{\sqrt{n}}.$$

Markaziy limit teoremaga asosan:

$$\frac{\sum\limits_{i=1}^{n} 2X_i - nM\,(2X_1)}{\sqrt{n}} \Rightarrow N\,(0; D\,(2X_1)) = N\,(0; 4DX_1) = N\left(0; \theta^2/3\right),$$

ya'ni $2\overline{x}$ asimptotik normal baho bo'lar ekan.$\triangleleft$

Misollar

1. $X_1, ..., X_n$ tanlanma chekli dispersiyaga ega taqsimotdan olingan bo'lsin. $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ statistika noma'lum $\theta = MX_1$ parametr uchun asimptotik normal baho bo'lishini ko'rsating.

2. $X_1, ..., X_n$ tanlanma $N\left(0, \theta^2\right)$ taqsimotdan olingan bo'lsin. $\overline{x^2} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i^2$ statistika noma'lum θ^2 parametr uchun asimptotik normal baho bo'lishini ko'rsating.

3. Agar MX_1^4 chekli bo'lsa, $S^2 = \frac{1}{n}\sum\limits_{i=1}^{n} (X_i - \overline{x})^2$ statistika noma'lum $\theta = DX_1$ uchun asimptotik normal baho bo'lishini ko'rsating.

4. $X_1, ..., X_n$ tanlanma $R\,[0, \theta]$ taqsimotdan olingan bo'lsin. $\sqrt[k]{(k+1)\ \overline{x^k}}$, $k \geq 1$ statistika noma'lum parametr θ uchun asimptotik normal baho bo'lishini ko'rsating.

5. $X_1,...,X_n$ tanlanma $R\left[\frac{\theta}{2},\theta\right]$ taqsimotdan olingan bo‘lsin. $\ln(4\overline{x}/3)$ statistika noma’lum parametr ln θ uchun asimptotik normal baho bo‘lishini ko‘rsating.

6. $X_1,...,X_n$ tanlanma $E(\theta)$ taqsimotdan olingan bo‘lsin. $\sqrt[k]{k!/\overline{x^k}}$ statistika noma’lum parametr ln θ uchun asimptotik normal baho bo‘lishini ko‘rsating.

7. $X_1,...,X_n$ tanlanma $E(\theta)$ taqsimotdan olingan bo‘lsin. $\ln\overline{x}$ statistika noma’lum parametr ln θ uchun asimptotik normal baho bo‘lishini ko‘rsating.

8. $X_1,...,X_n$ tanlanma zichligi $f(x,\theta)=\begin{cases} e^{\theta-x}, & x\geq\theta \\ 0\,, & x<\theta \end{cases}$ bo‘lgan taqsimotdan olingan bo‘lsin. Noma’lum parametr θ uhun $\overline{x}-1$ statistika asimptotik normal baho bo‘ladimi?

9. $X_1,...,X_n$ tanlanma zichligi $f(x,\theta)=\begin{cases} e^{\theta-x}, & x\geq\theta \\ 0\,, & x<\theta \end{cases}$ bo‘lgan taqsimotdan olingan bo‘lsin. Noma’lum parametr θ uhun $X_{(1)}$ statistika asimptotik normal baho bo‘ladimi?

10. $X_1,...,X_n$ tanlanma zichlik funksiyasi quyidagicha bo’lgan taqsimotdan olingan bo‘lsin:

$$f(x,\theta)=\begin{cases} \theta^{-1}e^{-(x-b)/\theta}, & x\geq b,\ b\in R,\theta>0 \\ 0\,, & x<b. \end{cases}$$

Noma’lum parametr θ uhun $\overline{x}$ statistika asimptotik normal baho bo‘ladimi?

11. $X_1,...,X_n$ tanlanma zichlik funksiyasi quyidagicha bo’lgan taqsimotdan olingan bo‘lsin:

$$f(x,\theta)=\begin{cases} a^{-1}e^{-(x-\theta)/a}, & x\geq\theta,\theta\in R,a>0 \\ 0\,, & x<\theta. \end{cases}$$

Noma’lum parametr θ uhun $\overline{x}-S$ statistika asimptotik normal baho bo‘ladimi?

12. $X_1,...,X_n$ tanlanma zichlik funksiyasi quyidagicha bo’lgan taqsimotdan olingan bo‘lsin:

$$f(x,\theta)=\begin{cases} \theta^{-1}e^{-(x-b)/\theta}, & x\geq b,\ b\in R,\theta>0 \\ 0\,, & x<b. \end{cases}$$

Noma’lum parametr θ uhun $\overline{X}-X_{(1)}$ statistika asimptotik normal baho bo‘ladimi?

13. $X_1,...,X_n$ tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin:

$$f(x,\theta)=\begin{cases} a^{-1}e^{-(x-\theta)/a}, \ x\geq\theta,\theta\in R, a>0 \\ 0\,, \quad x<\theta. \end{cases}$$

Noma'lum parametr θ uhun $X_{(1)}$ statistika asimptotik normal baho bo'ladimi?

14. $X_1,...,X_n$ tanlanma $P(\theta,\beta)$ (β - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $\frac{1}{\overline{\ln\ x}-\ln X_{(1)}}$ statistika asimptotik normal baho bo'ladimi?

15. $X_1,...,X_n$ tanlanma $P(\alpha,\theta)$ (α - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $X_{(1)}$ statistika asimptotik normal baho bo'ladimi?

16. $X_1,...,X_n$ tanlanma $W(\alpha,\theta)$ (α - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $1/\overline{x^\alpha}$ statistika asimptotik normal baho bo'ladimi?

17. $X_1,...,X_n$ tanlanma $R[0,\theta]$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $\frac{n+1}{n}X_{(n)}$ statistika asimptotik normal baho bo'ladimi?

18. $X_1,...,X_n$ tanlanma $\pi(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $\overline{x}$ statistika asimptotik normal baho bo'ladimi?

19. $X_1,...,X_n$ tanlanma $N\left(a,\theta^2\right)$ (a - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ^2 uchun $\frac{1}{n}\sum\limits_{i=1}^{n}(X_i-a)^2$ statistika asimptotik normal baho bo'ladimi?

20. $X_1,...,X_n$ tanlanma $\Gamma(\theta,1)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $(\overline{x})^{-1}$ statistika asimptotik normal baho bo'ladimi?

21. $X_1,...,X_n$ tanlanma $\Gamma(\theta,1)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $\left(\frac{1}{2n}\sum\limits_{i=1}^{n}X_i^2\right)^{-1/2}$ statistika asimptotik normal baho bo'ladimi?

12-§. BAHOLARNI SOLISHTIRISH

20-misol. $F(x,\theta) = \begin{cases} 1-e^{\theta-x}, x\geq\theta \\ 0\,, \quad x<\theta \end{cases}$, taqsimot uchun $T_1\left(X^{(n)}\right)=\overline{x}-1$ va $T_2\left(X^{(n)}\right)=X_{(1)}$ baholarni o'rtacha kvadratik ma'noda effektivlikka tekshiring.

▷ Buning uchun avval quyidagi momentlarni hisoblaymiz:

$$MX_1=\int\limits_{\theta}^{+\infty} xe^{\theta-x}dx=\begin{vmatrix} x=u & dx=du \\ e^{\theta-x}dx=d\nu & \nu=-e^{\theta-x} \end{vmatrix}=$$

$$-xe^{\theta-x}\,|_{\theta}^{+\infty}+\int\limits_{\theta}^{+\infty} e^{\theta-x}dx=\theta-e^{\theta-x}\,|_{\theta}^{+\infty}=\theta+1;$$

$$MX_1^2=\int\limits_{\theta}^{+\infty} x^2e^{\theta-x}dx=\begin{vmatrix} x^2=u & 2xdx=du \\ e^{\theta-x}dx=dv & v=-e^{\theta-x} \end{vmatrix}=$$

$$-x^2e^{\theta-x}\,|_{\theta}^{+\infty}+2\int\limits_{\theta}^{+\infty} xe^{\theta-x}dx=\theta^2+2\left(\theta+1\right);$$

$MT_1\left(X^{(n)}\right)=M\left(\overline{x}-1\right)=MX_1-1=\theta$, ya'ni $T_1(X^{(n)})$ siljimagan baho bo'lganligi uchun $M\big(T_1(X^{(n)})-\theta\big)^2=DT_1(X^{(n)})$ bo'ladi. Demak, $DT_1(X^{(n)})=D\left(\overline{x}-1\right)=D\overline{x}=\frac{1}{n}DX_1=\frac{1}{n}\left(\theta^2+2\left(\theta+1\right)-\left(\theta+1\right)^2\right)=\frac{1}{n}$.

Endi $T_2\left(X^{(n)}\right)=X_{(1)}$ bahoning momentlarini hisoblaymiz:

$$MX_{(1)}=\int\limits_{\theta}^{+\infty} x\,n\,e^{(\theta-x)(n-1)}e^{\theta-x}dx=n\int\limits_{\theta}^{+\infty} x\,e^{n(\theta-x)}dx=$$

$$=\begin{vmatrix} x=u & dx=du \\ e^{n(\theta-x)}dx=dv & v=-\frac{e^{n(\theta-x)}}{n} \end{vmatrix}=$$

$$= n\left[-x\,\frac{e^{n(\theta-x)}}{n}\,|_{\theta}^{+\infty} + \frac{1}{n}\int_{\theta}^{+\infty} e^{n(\theta-x)}dx\right] =$$

$$= \theta + \int_{\theta}^{+\infty} e^{n(\theta-x)}dx = \theta - \frac{e^{n(\theta-x)}}{n}\,|_{\theta}^{+\infty} = \theta + \frac{1}{n};$$

$$MX_{(1)}^2 = \int_{\theta}^{+\infty} x^2\, n\, e^{n(\theta-x)}dx = \left|\begin{array}{cc} x^2 = u & 2xdx = du \\ e^{n(\theta-x)}dx = dv & v = -\frac{e^{n(\theta-x)}}{n} \end{array}\right| =$$

$$= n\left[-x^2\,\frac{e^{n(\theta-x)}}{n}\,|_{\theta}^{+\infty} + \frac{2}{n}\int_{\theta}^{+\infty} x\, e^{n(\theta-x)}dx\right] = \theta^2 + \frac{2}{n}\left(\theta + \frac{1}{n}\right).$$

$$M(X_{(1)}-\theta)^2 = MX_{(1)}^2 - 2\theta MX_{(1)} + \theta^2 = \theta^2 + \frac{2\theta}{n} + \frac{2}{n^2} - 2\theta^2 - \frac{2\theta}{n} + \theta^2 = \frac{2}{n^2};$$

Demak, $n \geq 3$ uchun $\frac{1}{n} > \frac{2}{n^2}$ bo‘lgani uchun $X_{(1)}$ baho $\overline{x} - 1$ bahoga nisbatan effektiv baho ekan. $\triangleleft$

Misollar

Quyidagi taqsimotlar uchun keltirilgan baholarni o‘rtacha kvadratik ma’noda effektivlikka tekshiring:

1. $N\left(a, \theta^2\right)$: $S^2 = \frac{1}{n-1}\sum\left(X_i - \overline{x}\right)^2$;
   $S_2^2 = \frac{1}{n}\sum\left(X_i - a\right)^2$;
2. $R\,[0;\theta]$: $T_1\left(X^{(n)}\right) = 2\overline{x}$;
   $T_2\left(X^{(n)}\right) = \frac{n+1}{n}X_{(n)}$;
3. $R\,[0,\theta]$: $T_1\left(X^{(n)}\right) = X_{(n)}$;
   $T_2\left(X^{(n)}\right) = X_{(1)} + X_{(n)}$;
4. $R\,[\theta, \theta+1]$: $T_1\left(X^{(n)}\right) = \overline{x} - \frac{1}{2}$;
   $T_2\left(X^{(n)}\right) = X_{(n)} - 1$;

5. $R[\theta,\theta+1]$: $\quad T_1\left(X^{(n)}\right)=\overline{x}-\frac{1}{2}$; $T_2\left(X^{(n)}\right)=X_{(1)}$;

6. $f(x,\theta)=\begin{cases}\frac{2x}{\theta^2}, & x\in[0,\theta],\ \theta>0,\\ 0, & x\overline{\in}[0,\theta];\end{cases}$ $\quad T_1\left(X^{(n)}\right)=\frac{3}{2}\overline{x}$; $T_2\left(X^{(n)}\right)=X_{(n)}$;

7. $F(x,\theta)=\begin{cases}1-e^{\theta-x}, & x\geq\theta,\\ 0, & x<\theta;\end{cases}$ $\quad T_1\left(X^{(n)}\right)=\overline{x}-1$; $T_2\left(X^{(n)}\right)=X_{(1)}-\frac{1}{n}$

8. $f(x,\theta)=\begin{cases}\theta^{-1}, e^{-\frac{(x-b)}{\theta}}, & x\geq b,\ \theta>0,\\ 0, & x<b;\end{cases}$ $\quad T_1\left(X^{(n)}\right)=S$; $T_2\left(X^{(n)}\right)=\overline{x}-X_{(1)}$,

9. $N\left(a;\theta^2\right)$: $\quad T_1\left(X^{(n)}\right)=\overline{x}^2-a^2$; $T_2\left(X^{(n)}\right)=\frac{1}{n-1}\sum\limits_{i=1}^{n}\left(X_i-a\right)^2$;

10. $P(\theta;1)$: $\quad T_1\left(X^{(4)}\right)=X_{(1)}$; $T_2\left(X^{(4)}\right)=\overline{x}$.

13-§. TANLANMA XARAKTERISTIKALARI TAQSIMOTI

21-misol. $X_1,...,X_n \sim R[0;1]$ va $R=X_{(n)}-X_{(1)}$ tanlanma qulochi bo‘lsin. $2n(1-R)$ ning limit taqsimotini toping.

$\triangleright$ $X_{(1)}$ va $X_{(n)}$ larning birgalikdagi zichlik funksiyasi([9]):

$$f(x;y)=\begin{cases}n(n-1)(y-x)^{n-2}, & 0<x<y<1,\\ 0, \text{aks holda}.\end{cases}$$

U holda R va $X_{(n)}$ larning birgalikdagi zichlik funksiyasi

$$g(x)=\begin{cases}n(n-1)x^{n-2}, & 0<x<1,\\ 0, \text{aks holda},\end{cases}$$

bo‘ladi. Bu zichlik yordamida R ning zichlik funksiyasini hisoblaymiz: agar $0<x<1$ bo‘lsa,

$$\varphi_R(x)=\int g(x)dt=\int\limits_x^1 n(n-1)x^{n-2}dt=n(n-1)x^{n-2}(1-x),$$

aks holda $\varphi_R(x) = 0$. R ning zichlik funksiyasi yordamida $2n(1-R)$ t.m.ning taqsimotini topamiz:

$$H_{2n(1-R)}(x) = P(2n(1-R) < x) = P\left(R > 1 - \frac{x}{2n}\right) =$$

$$= 1 - P\left(R \le 1 - \frac{x}{2n}\right) = 1 - G_R\left(1 - \frac{x}{2n}\right).$$

$$h_n(x) = \left(H_{2n(1-R)}(x)\right)' = g_R\left(1 - \frac{x}{2n}\right)\frac{1}{2n} =$$

$$= n(n-1)\Big(1 - \frac{x}{2n}\Big)^{n-2}\left(1 - \left(1 - \frac{x}{2n}\right)\right)\frac{1}{2n} = \frac{n-1}{4n}x\Big(1 - \frac{x}{2n}\Big)^{n-2},$$

$0 < x < 2n$. $2n(1-R)$ ning zichlik funksiyasi

$$h_n(x) = \begin{cases} \frac{n-1}{4n}x\big(1 - \frac{x}{2n}\big)^{n-2}, & 0 < x < 2n, \\ 0, \ \text{aks holda} \end{cases}$$

va $\lim\limits_{n\to\infty}\big(1 - \frac{x}{2n}\big)^{n-2} = e^{-x/2}$ ekanligini hisobga olsak,

$$\lim_{n\to\infty} h_n(x) = 4^{-1}xe^{x/2}I(0 < x < \infty)$$

ni hosil qilamiz.

Demak, $2n(1-R)$ ning limit taqsimoti

$$f(x) = \begin{cases} 4^{-1}xe^{x/2}, & 0 < x < \infty, \\ 0, \text{aks holda}, \end{cases}$$

ya'ni χ^2_4 taqsimot ekan (ozodlik darajasi 4 bo'lgan xi-kvadrat taqsimot). $\triangleleft$

Misollar

1. Agar $X_1, ..., X_n \sim N(a, \sigma^2)$ bo‘lsa, $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ ning taqsimotini toping.

2. Agar $X_1, ..., X_2 \sim N(0, 1)$ bo‘lsa, $Z = X_1^2 + X_2^2$ ning taqsimotini toping.

3. $N(a; \theta_2)$ (a - ma’lum), modelda $\sqrt{n}\frac{\overline{x}-a}{S}$ statistikaning taqsimotini toping.

4. $N(a; \sigma^2)$ modelda $n\frac{(\overline{x}-a)^2}{\sigma^2}$ statistikaning taqsimotini toping.

5. $N(a, \sigma^2)$ modelda $S^2 = \frac{1}{n}\sum\limits_{i=1}^{n}(X_i - \overline{x})^2$ statistikaning taqsimotini toping.

6. $R[a, b]$ modelda $X_{(1)}$ va $X_{(n)}$ statistikalarning taqsimotlarini toping.

7. $E(\alpha)$ modelda $X_{(1)}$ va $X_{(n)}$ statistikalarning taqsimotlarini toping.

8. $N(a, \sigma^2)$ modelda $\frac{nS^2}{\sigma^2}$ statistikaning taqsimotini toping.

9. Agar $X_1, ..., X_n \sim \pi(\lambda)$, $Y_1, ..., Y_m \sim \pi(\lambda)$ va $X_i \perp Y_i$ bo‘lsa, $\overline{x} + \overline{y}$ ning taqsimotini toping.

10. Agar $X_1, ..., X_n \sim N(a_x; \sigma)$, $Y_1, ..., Y_m \sim N(a_y; \sigma)$ va $X_i \perp Y_i$ bo‘lsa, $\overline{x} - \overline{y} - (a_x - a_y)$ ning taqsimotini toping.

11. Agar $X_1, ..., X_n \sim \Gamma(\alpha; \lambda)$ bo‘lsa, $\sum\limits_{i=1}^{n} X_i$ ning taqsimotini toping.

12. Agar $X_1, ..., X_n \sim N(a; \sigma^2)$ bo‘lsa, $\sqrt{n}\frac{\overline{x}-a}{\sigma}$ ning taqsimotini toping.

13. Agar $X_1, ..., X_n \sim N(a; \sigma^2)$ bo‘lsa, $\sum\limits_{i=1}^{n}\left(\frac{X_i - a}{\sigma}\right)^2$ ning taqsimotini toping.

14. Agar $X_1, ..., X_n \sim N(a; \sigma^2)$ bo‘lsa, $\sum\limits_{i=1}^{n}\left(\frac{X_i - \overline{x}}{\sigma}\right)^2$ ning taqsimotini toping.

15. Agar $X_1, ..., X_n \sim \pi(\lambda)$ bo‘lsa, $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ ning taqsimotini toping.

16. Agar $X_1, ..., X_n \sim B_i(1;p)$ bo'lsa, $\sum\limits_{i=1}^{n} X_i$ ning taqsimotini toping.

17. Agar $X_1, ..., X_n \sim E(\alpha)$ bo'lsa, $\overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ ning taqsimotini toping.

18. Agar $X_1, ..., X_n \sim R\,[0;b]$ bo'lsa, $X_{(2)}$ ning taqsimotini toping.

19. Agar $X_1, ..., X_n \sim R\,[0;\theta]$ bo'lsa, $n \to \infty$ da $nX_{(1)}/\theta$ ning limit taqsimotini toping.

20. Agar $X_1, ..., X_n \sim R\,[0;\theta]$ bo'lsa, $n \to \infty$ da $n(\theta - X_{(n)})/\theta$ ning limit taqsimotini toping.

14-§. PARAMETRLARNI INTERVAL BAHOLASH

ξ t.m.ning t.f. $F(x;\theta)$, $\theta \in \Theta$ va $X^{(n)} = (X_1, ..., X_n)$ tanlanma shu taqsimotdan olingan bo'lsin. $T_1(x^{(n)})$ va $T_2(x^{(n)})$, $T_1 < T_2$ statistikalar uchun berilgan $\gamma \in (0;1)$ da barcha $\theta \in \Theta$ uchun

$$P\,(T_1 \le \theta \le T_2) \ge \gamma$$

tengsizlik o'rinli bo'lsa, $\left[T_1(x^{(n)}), T_2(x^{(n)})\right]$ oraliq γ *ishonchlilik oralig'i(intervali)* deyiladi. γ esa ishonchlilik darajasi deyiladi. Odatda γ ni 1 ga yaqin qilib tanlanadi($\gamma = 0,95; 0,99; ...$).

$T_1(x^{(n)})$ va $T_2(x^{(n)})$, $T_1 < T_2$ statistikalar uchun berilgan $\gamma \in (0;1)$ da barcha $\theta \in \Theta$ uchun

$$\liminf_{n\to\infty} P\,(T_1 \le \theta \le T_2) \ge \gamma$$

tengsizlik o'rinli bo'lsa, $\left[T_1(x^{(n)}), T_2(x^{(n)})\right]$ oraliq *asimptotik ishonchlilik oralig'i* deyiladi.

Umumiy holda ishonchlilik intervalini tuzishni keltiramiz:

$X^{(n)} = (X_1, ..., X_n)$ tanlanma va noma'lum parametr θ ga bog'liq, hamda quyidagi shartlarni qanoatlantiruvchi $G(x^{(n)};\theta)$ funksiya berilgan bo'lsin:

1) $G(x^{(n)};\theta)$ ning taqsimoti noma'lum parametr θ ga bog'liq emas;

2) $G(x^{(n)};\theta)$ uzluksiz va θ bo'yicha monoton.

Bunday $G(x^{(n)};\theta)$ t.m. *markaziy statistika* deyiladi. $f_G(t)$ funksiya

$G(x^{(n)};\theta)$ ning zichlik funksiyasi bo‘lsin, u holda ihtiyoriy $\gamma \in (0;1)$ uchun $g_1 < g_2$ sonlarni shunday tanlash mumkinki,

$$P\left(g_1 \leq G(x^{(n)};\theta) \leq g_2\right) = \int_{g_1}^{g_2} f_G(t)dt = \gamma$$

o‘rinli bo‘lsin. g_1, g_2 lar $G(x^{(n)};\theta)$ning taqsimoti kvantillaridir. Odatda g_1, g_2 lar

$$P\left(G(x^{(n)};\theta) \leq g_1\right) = P\left(G(x^{(n)};\theta) \geq g_2\right) = \gamma$$

shartlardan tanlanadi. $g_1 \leq G(x^{(n)};\theta) \leq g_2$ tengsizlikni θ ga nisbatan yechib, $T_1 \leq \theta \leq T_2$ ishonchlilik oralig‘ini hosil qilamiz.

22-misol. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $R[\theta, 2\theta]$, $\theta > 0$ taqsimotdan olingan bo‘lsin. Noma’lum parametr θ uchun ishonchlilik intervalini tuzing.

$\triangleright$ Agar $X_i \sim R\,[\theta, 2\theta]$ bo‘lsa, u holda $Y_i = \frac{X_i}{\theta} - 1 \sim [0,1]$ bo‘ladi. Bu taqsimot noma’lum parametrining HMo‘UB $X_{(n)}$bo‘lgani uchun, markaziy statistikani quyidagi ko‘rinishda olamiz:

$$Y_{(n)} = \max\{Y_1, ..., Y_n\} = \frac{\max\{X_1, ..., X_n\}}{\theta} - 1 = \frac{X_{(n)}}{\theta} - 1 = G(x^{(n)};\theta).$$

Uning taqsimoti $F_{Y_{(n)}}(y) = P_\theta\,(\eta < y) = \begin{cases} 0, \ y < 0, \\ y^n, \ y \in [0,1], \\ 1, \ y > 0. \end{cases}$

Ixtiyoriy musbat g_1 va g_2 lar uchun:

$$P_\theta\,(g_1 \leq G\,(X_i\theta) \leq g_2) = P_\theta\left(g_1 \leq \frac{X_{(n)}}{\theta} - 1 \leq g_2\right) =$$

$$= P_\theta\left(\frac{X_{(n)}}{g_2+1} \leq \theta \leq \frac{X_{(n)}}{g_1+1}\right).$$

Intervalning uzunligi $\frac{X_{(n)}(g_1-g_2)}{(g_1+1)(g_2+1)}$ miqdor g_1,g_2 lar kattalashsa, hamda bir-biriga yaqinlashsa kichrayadi. $Y_{(n)}$ t.m.ning zichlik funksiyasi $[0,1]$

oraliqda ny^{n-1} ga teng va u monoton o'suvchi. Shuning uchun bir-biriga yaqin g_1 va g_2 kvantillar eng katta qiymatlariga $g_2 = 1$ va g_1:

$$\gamma = P_\theta\left(g_1 \leq Y_{(n)} \leq 1\right) = F_{Y_{(n)}}(1) - F_{Y_{(n)}}(g_1) = 1 - g_1^n = \gamma,$$

bo'lsa erishadi. Bu yerdan $g_1 = \sqrt[n]{1-\gamma}$ bo'ladi. Demak,

$$\gamma = P_\theta\left(\sqrt[n]{1-\gamma} \leq Y_{(n)} \leq 1\right) = P_\theta\left(\frac{X_{(n)}}{2} \leq \theta \leq \frac{X_{(n)}}{1+\sqrt[n]{1-\gamma}}\right).$$

$\lhd$

23-misol. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $\pi(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

$\rhd$ MLT ga ko'ra:

$$\frac{\sum\limits_{i=1}^{n} X_i - nMX_1}{\sqrt{nDX_1}} = \sqrt{n}\frac{\overline{x}-\theta}{\sqrt{\theta}} \Rightarrow \eta \sim N(0;1).$$

$n \to \infty$ da $P\left(-c < \sqrt{n}\frac{\overline{x}-\theta}{\sqrt{\theta}} < c\right) \to P(-c < \eta < c) = \gamma$, $c = t_{\frac{1+\gamma}{2}}$-normal taqsimot kvantili. Sust yaqinlashish xossasiga ko'ra, agar $\xi_n \xrightarrow{p} 1$ va $\eta_n \Rightarrow \eta$ bo'lsa, u holda $\xi_n\eta_n \Rightarrow \eta$. $\theta_n = \overline{x}$ asosli baho ekanligidan $\frac{\theta}{\overline{x}} \xrightarrow{p} 1$. Bundan

$$\sqrt{\frac{\theta}{\overline{x}}}\sqrt{n}\frac{\overline{x}-\theta}{\sqrt{\theta}} = \sqrt{n}\frac{\overline{x}-\theta}{\sqrt{\overline{x}}} \Rightarrow \eta.$$

Shuning uchun,

$$P\left(-t_{\frac{1+\gamma}{2}} < \sqrt{n}\frac{\overline{x}-\theta}{\sqrt{\overline{x}}} < t_{\frac{1+\gamma}{2}}\right) \to P\left(-t_{\frac{1+\gamma}{2}} < \eta < t_{\frac{1+\gamma}{2}}\right) = \gamma.$$

Tengsizlikni noma'lum parametr θ ga nisbatan yechsak, $P\left(\overline{x} - \frac{t_{\frac{1+\gamma}{2}}\sqrt{\overline{x}}}{\sqrt{n}} < \theta < \overline{x} + \frac{t_{\frac{1+\gamma}{2}}\sqrt{\overline{x}}}{\sqrt{n}}\right) \to \gamma$, $n \to \infty$, asimptotik ishonchlilik intervalini hosil qilamiz.

Demak, noma'lum parametr θ uchun asimptotik ishonchlilik intervali $\left(\overline{x} - \frac{t_{\frac{1+\gamma}{2}}\sqrt{\overline{x}}}{\sqrt{n}}; \overline{x} + \frac{t_{\frac{1+\gamma}{2}}\sqrt{\overline{x}}}{\sqrt{n}}\right)$ bo'lar ekan. $\triangleleft$

Misollar

1. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $N(\theta, \sigma^2)$ (σ^2 - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

2. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $N(a, \theta^2)$ (a - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ^2 uchun ishonchlilik intervalini tuzing.

3. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ_1 uchun ishonchlilik intervalini tuzing.

4. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ_2^2 uchun ishonchlilik intervalini tuzing.

5. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $R[0, \theta]$, $\theta > 0$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

6. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $\Gamma(\theta; \lambda)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

7. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

8. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $Bi(n; \theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

9. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $f(x, \theta) = \begin{cases} e^{\theta - x}, x \geq \theta \\ 0, \quad x < \theta \end{cases}$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

10. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $Ge(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

11. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $R[0;\theta]$ taqsimotdan olingan bo‘lsin. Noma’lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

12. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $R[\theta, \theta+1]$ taqsimotdan olingan bo‘lsin. Noma’lum parametr θ uchun ishonchlilik intervalini tuzing.

13. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $\Gamma(\theta; 1)$ taqsimotdan olingan bo‘lsin. Noma’lum parametr θ uchun ishonchlilik intervalini tuzing.

15-§. STATISTIK GIPOTEZALARNI TEKSHIRISH

Kuzatilayotgan t.m. haqida aytilgan ixtiyoriy fikrga *statistik gipoteza* deyiladi. Tekshirilishi kerak bo‘lgan gipoteza *asosiy gipoteza* deyiladi va uni H_0 bilan belgilaymiz. Asosiy gipotezadan mantiqan qarama-qarshi bo‘lgan ixtiyoriy gipotezaga *raqobatlashuvchi* yoki *alternativ gipoteza* deb ataladi. Statistik gipoteza sodda deyiladi, agar u taqsimotni bir qiymatli aniqlasa: $H_0 : F = F_0$, aks holda u murakkab deyiladi: $H_0 : F \in \mathscr{F}$. Matematik statistikada statistik gipotezalarni tekshirishda ikki xil xatolikka yo‘l qo‘yilishi mumkin. Statistik yechim asosida asosiy faraz u to‘g‘ri bo‘lgan holda ham rad etilishi mumkin. Bunday xatolik *I tur xatolik* deyiladi. I tur xatolikning ehtimolligini α bilan belgilaymiz. U qiymatdorlik daragasi ham deyiladi. Statistik yechim asosida alternativ gipoteza to‘g‘ri bo‘lsa ham rad etilishi mumkin. Bunday xatolik *II tur xatolik* deyiladi. II tur xatolikning ehtimolligini β bilan belgilaymiz.

Statistik gipotezalarni tekshirish statistik ma’lumotlarga asoslanadi. Faraz qilaylik, $X_1, X_2, ..., X_n$ lar X t.m.ning n ta bog‘liqsiz tajribalardagi kuzatilmalari va X t.m.ning biron - bir xarakteristikasi haqidagi asosiy H_0 gipoteza ko‘rilayotgan bo‘lsin. Statistik ma’lumotlar asosida asosiy gipoteza H_0 ni qabul qilish yoki rad etish qoidasini tuzish kerak. Asosiy gipoteza H_0 ni qabul qilish yoki rad etish qoidasi - H_0 gipotezani tekshirishning statistik kriteriysi deyiladi. Statistik gipotezalarni tekshirish - statistik ma’lumotlar asosida asosiy gipotezani tasdiqlash yoki uni rad etishdan iborat bo‘ladi.

Endi statistik alomatlarni tuzish qoidalari bilan tanishamiz. Odatda statistik kriteriyni qurish empirik ma’lumotlarni asosiy H_0

gipoteza bo‘yicha tavsiflovchi statistika $T = T(X_1, \dots, X_n)$ ni tanlashdan boshlanadi. Bunday tanlashda ikki xossa bajarilishi talab etiladi: a) statistika manfiy qiymatlar qabul qilmaydi; b) asosiy gipoteza to‘g‘ri bo‘lganda statistikaning aniq, H_0 - gipotezaviy taqsimoti ma’lum bo‘lishi kerak.

Faraz qilaylik, bunday stastistika topilgan bo‘lib, $S = \{t : t = T(X_1, \dots, X_n), (X_1, \dots, X_n) -$ tanlanma fazosiga tegishli$\}$ - statistikaning qiymatlar to‘plami bo‘lsin. Amaliyotda har ikki xatoliklarni bir vaqtda minimallashtirish murakkabdir. Shu boisdan, dastlab I tur xatolik α ni fiksirlab, β ni minimallashtiriladi(yoki kriteriy quvvati 1-β maksimallashtiriladi). Oldindan $0 < \alpha < 1$ - sonini tayinlaylik. Endi S sohani shunday kesishmaydigan S_α va $S \backslash S_\alpha$ sohalarga ajratamizki, bunda asosiy gipoteza H_0 to‘g‘ri bo‘lganida T$(X_1, \dots, X_n) \in S_\alpha$ tasodifiy hodisaning ro‘y berish ehtimoli α dan oshmasin:

$$P\left\{T\left(X_1, \dots, X_n\right) \in S_\alpha / H_0\right\} \leq \alpha.$$

Asosiy gipoteza H_0 ni tekshirish qoidasi quyidagicha bo‘ladi: $x = (x_1, ..., x_n)$ t.m. X ning biror tanlanmasi qiymati bo‘lsin. Agar $t = T(x)$ miqdor S_α sohaga tegishli bo‘lsa: $T(x) \in S_\alpha$, u holda asosiy gipoteza H_0 to‘g‘ri bo‘lganida rad etiladi. Aks holda, ya’ni $T(x) \notin S_\alpha$ bo‘lsa, asosiy gipoteza H_0 qabul qilinadi, chunki bu holda statistik ma’lumotlar asosida qilingan xulosalar asosiy gipotezani rad etmaydi. Shuni ta’kidlash lozimki, $t \in S \backslash S_\alpha$ bo‘lishi asosiy gipoteza H_0 ni albatta to‘g‘ri bo‘lishini tasdiqlamaydi, balki bu holat statistik ma’lumotlar va nazariy gipotezaning yetarli darajada muvofiqligini ko‘rsatadi xalos. Yuqorida keltirilgan qoidada $T = T(X_1, \dots, X_n)$ statistikani kriteriy statistikasi, S_α - soha esa kritik soha deyiladi. Odatda α ning qiymatlari uchun 0.1; 0.05; 0.01 sonlari qabul qilinadi. Amaliyotda kritik sohalar $\{t \geq t_\alpha\}$ yoki $\{|t| \geq t_\alpha\}$ ko‘rinishida bo‘ladi.

Faraz qilaylik, S_α kritik soha bo‘lsin. U holda H gipoteza to‘g‘ri bo‘lganida statistikaning qiymati kritik sohaga tegishli bo‘lishi ehtimolligi

$$W\left(H\right) = P\left\{T\left(X_1, ..., X_n\right) \in S_\alpha / H\right\},$$

alomatning quvvat funksiyasi deyiladi. Alomat quvvati $H =$

H_1 bo‘lganida, ya’ni $W(H_1)$ ehtimollik asosiy gipoteza noto‘g‘ri bo‘lganida to‘g‘ri yechimni qabul qilishi ehtimolligini anglatadi. Alomatning siljimaganlik xossasi muhim o‘rin tutadi va bu xossa

$$P\left\{T(X_1,...,X_n) \in S_\alpha / H_1\right\} \le \alpha$$

tengsizlik bilan aniqlanadi.

Mavjud statistik gipotezalarni o‘rganilayotgan taqsimotlar oilasiga qarab ikki guruhga ajratish mumkin: parametrik va noparametrik gipotezalar. T.m.larning taqsimot funksiyasi paramerli taqsimotlar oilasiga tegishli bo‘lsin. Ammo, taqsimotning parametrlari $\theta = (\theta_1, ..., \theta_n)$ noma’lumdir. Masalan, t.m. normal qonunlar oilasiga tegishli bo‘lsa, uning taqsimot funksiyasi ikkita: o‘rta qiymat va dispersiya orqali to‘liq aniqlanadi va H_0 gipoteza bu holda matematik kutilma hamda dispersiya qiymatlari haqida bo‘ladi. Demak, asosiy H_0 gipoteza noma’lum parametr qiymatlari haqida bo‘lar ekan. Bunday statistik gipotezaga *parametrik gipoteza* deb ataladi.

Agarda t.m.ning taqsimot funksiyasi umuman noma’lum bo‘lsa, noparametrik gipoteza qaraladi. Noparametrik gipoteza taqsimot funksiyasining ma’lum xossalarga ega ekanligi haqida bo‘lishi mumkin.

Endi parametrik statistik alomatlarini qaraylik. X t.m.ning asl taqsimot funksiyasi quyidagi taqsimotlar oilasiga tegishli bo‘lsin:

$$\mathscr{F} = \left\{F\left(\cdot\,, \theta\right), \theta \in \Theta\right\}.$$

Bu yerda $\theta = (\theta_1, ..., \theta_r) - r$ - o‘lchovli vektor, $\Theta \subseteq R^r$ parametrlar qiymati to‘plami bo‘lsin. U holda asosiy gipoteza H_0 ga asosan $\theta \in \Theta_0$, alternativ gipotezaga asosan esa $\theta \in \Theta_1 = \Theta \backslash \Theta_0$. Asosiy gipoteza H_0 ni tekshirish uchun S_α va S_α^* ikkita kritik to‘plamlar bo‘lib, ular har birining qiymatdorlik darajasi α bo‘lsin. Faraz qilaylik,

$$W\left(S_\alpha^*, \theta\right) \le W\left(S_\alpha, \theta\right), \quad \theta \in \Theta_0, \tag{36}$$

va

$$W\left(S_\alpha^*, \theta\right) \ge W\left(S_\alpha, \theta\right), \quad \theta \in \Theta_1, \tag{37}$$

bo'lsin. Aytaylik, (37) tengsizlikda hech bo'lmaganda θ ning bitta qiymati uchun qat'iy tengsizlik o'rinli bo'lsin. U holda S_α^* ga asoslangan statistik alomat S_α nikiga nisbatan tekis quvvatliroq deyiladi. Tabiiyki, bu holda S_α^* ga asoslangan statistik alomatni S_α nikiga nisbatan afzal ko'rmoq maqsadga muvofiq bo'ladi.

Agarda (36) va (37) munosabatlar ihtiyoriy S_α uchun o'rinli bo'lsa, S_α^* ga mos alomat *tekis eng quvvatli kriteriy* deyiladi.

Asosiy va alternativ gipotezalar sodda bo'lgan holda t.e.q. kriteriy qurishni ko'ramiz. ξ t.m. va shu t.m. yordamida $X^{(n)} = (X_1, ..., X_n)$ tanlanma olingan bo'lsin. Faraz qilaylik, ξ t.m. uzluksiz va H_0 gipotezaga asosan $f_0(x)$, H_1 gipotezaga asosan esa $f_1(x)$ zichlik funksiyalarga ega bo'lsin.

Demak, $P_{H_0}(x^{(n)}) = \prod\limits_{i=1}^{n} f_0(X_i)$ va $P_{H_1}(x^{(n)}) = \prod\limits_{i=1}^{n} f_1(X_i)$.

Quyidagi nisbatni ko'raylik

$$l(x^{(n)}) = \frac{P_{H_1}(x^{(n)})}{P_{H_0}(x^{(n)})} = \frac{\prod\limits_{i=1}^{n} f_1(x_i)}{\prod\limits_{i=1}^{n} f_0(x_i)}. \tag{38}$$

(38) tenglikka *haqiqatga o'xshashlik nisbati statistikasi* deyiladi.

Teorema(Neyman-Pirson). Sodda gipoteza H_0 ni unga alternativ bo'lgan sodda gipoteza H_1 bo'lgan holda tekshirish uchun t.e.q. kriteriy mavjud va uning kritik sohasi quyidagicha aniqlanadi:

$$S_\alpha = \left\{ x^{(n)} : l(x^{(n)}) = \frac{\prod\limits_{i=1}^{n} f_1(x_i)}{\prod\limits_{i=1}^{n} f_0(x_i)} \geq c \right\}, \tag{39}$$

bu yerda c kritik nuqta $P_{H_0}(l(x^{(n)}) \geq c) = \alpha$ shartdan aniqlanadi.

24-misol. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $N(\theta; \sigma^2)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz:

$$H_0 : \theta = \theta_0 \ \text{ va } \ H_1 : \theta = \theta_1, \ (\theta_1 > \theta_0).$$

I tur xatoligi α bo‘lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

▷ (39) kriteriyning kritik sohasini quyidagidan aniqlaymiz:

$$l\left(x^{(n)}\right)=\frac{\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \exp\left\{-\sum\limits_{j=1}^{n}\frac{(X_j-\theta_1)^2}{2\sigma^2}\right\}}{\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \exp\left\{-\sum\limits_{j=1}^{n}\frac{(X_j-\theta_0)^2}{2\sigma^2}\right\}}\geq C$$

yoki

$$l(x^{(n)})=\exp\left\{\frac{n\overline{x}\left(\theta_1-\theta_0\right)}{\sigma^2}-\frac{n}{2\sigma^2}\left(\theta_1^2-\theta_0^2\right)\right\}\geq C.$$

Tengsizlikni ikki tomonini logarifmlab, $\theta_1>\theta_0$ ekanligini hisobga olsak, quyidagi ekvivalent tengsizlik hosil bo‘ladi:

$$\overline{x}\geq C_1,$$

bu yerda $C_1=\frac{1}{2}(\theta_0+\theta_1)+\frac{\sigma^2\ln C}{(\theta_1-\theta_0)n}$.

Bu yerda C_1 - kritik nuqta va u I-tur hatolik α orqali aniqlanadi. Endi C_1 va β ni berilgan α orqali topamiz. Ma’lumki, $\overline{x}\sim N\left(\theta_0,\frac{\sigma^2}{n}\right)$.

Demak,

$$\alpha=P_{H_0}(\overline{x}>C_1)=P_{H_0}\left(\frac{\overline{x}-\theta_0}{\sigma/\sqrt{n}}\geq\frac{C_1-\theta_0}{\sigma/\sqrt{n}}\right)=1-\Phi\left(\frac{(C_1-\theta_0)\sqrt{n}}{\sigma}\right).$$

Agar U_α orqali α - kvantilni belgilasak, $\Phi(U_\alpha)=\alpha,$ u holda

$$U_{1-\alpha}=\frac{C_1-\theta_0}{\sigma}\sqrt{n}.$$

Bulardan foydalanib, C_1 - kritik chegarani topamiz:

$$C_1=\theta_0+\frac{U_{1-\alpha}\sigma}{\sqrt{n}}.$$

Demak, C_1 soni θ_0 ga bog‘liq, lekin θ_1 ga bog‘liq emas. H_1 gipoteza o‘rinligida $\overline{x}\sim N(\theta_1,\frac{\sigma^2}{\sqrt{n}})$ ekanligidan

$$\beta=P_{H_1}(\overline{x}<C_1)=P_{H_1}\left(\frac{\overline{x}-\theta_1}{\sigma/\sqrt{n}}<\frac{C_1-\theta_1}{\sigma/\sqrt{n}}\right)=$$

$$= \Phi\left(\frac{C_1-\theta_1}{\sigma}\sqrt{n}\right) = \Phi\left(\sqrt{n}\frac{\theta_0-\theta_1}{\sigma} + U_{1-\alpha}\right)$$

bo‘ladi. ◁

Misollar

1. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $N(a;\theta^2)$ taqsimotdan olingan bo‘lsin. Noma’lum parametr θ to‘g‘risida quyidagi ikki sodda gipotezani ko‘ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$. I tur xatoligi α bo‘lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

2. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $Bi(n;\theta)$ taqsimotdan olingan bo‘lsin. Noma’lum parametr θ to‘g‘risida quyidagi ikki sodda gipotezani ko‘ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1 (\theta_1 > \theta_0)$. I tur xatoligi α bo‘lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

3. $X^{(n)} = (X_1, ..., X_n)$ tanlanma $\pi(\theta)$ taqsimotdan olingan bo‘lsin. Noma’lum parametr θ to‘g‘risida quyidagi ikki sodda gipotezani ko‘ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$. I tur xatoligi α bo‘lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

4. ξ t.m.ning taqsimoti $F(x)$ to‘g‘risida quyidagi ikki sodda gipotezani ko‘ramiz: $H_0 : F(x) = R[-a;a]$ va $H_1 : F(x) = N(0;\sigma^2)$. I tur xatoligi α bo‘lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

5. ξ t.m.ning taqsimoti $F(x)$ to‘g‘risida quyidagi ikki sodda gipotezani ko‘ramiz: $H_0 : f(x) = \frac{1}{2}$, $|x| \leq 1$ va $H_1 : F(x) = N(0;1)$. I tur xatoligi α bo‘lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

6. $X^{(n)} = (X_1, ..., X_n)$ tanlanma zichlik funksiyasi $f(x;\theta) = e^{-(x-\theta)}$, $x > \theta$ bo‘lgan taqsimotdan olingan bo‘lsin. Noma’lum parametr θ to‘g‘risida quyidagi ikki sodda gipotezani ko‘ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$. I tur xatoligi α bo‘lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

7. $X^{(n)} = (X_1, ..., X_n)$ tanlanma zichlik funksiyasi $f(x;\theta) = \theta x^{-2}$, $x > \theta$ bo‘lgan taqsimotdan olingan bo‘lsin. Noma’lum

parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ $(\theta_1 \neq \theta_0)$. I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

8. $X^{(n)} = (X_1, ..., X_n)$ tanlanma zichlik funksiyasi $f(x;\theta) = 2\theta^{-2}(\theta - x)$, $x \in (0;\theta)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$. I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

9. $X^{(n)} = (X_1, ..., X_n)$ tanlanma zichlik funksiyasi $f(x;\theta) = 2\left(\theta x + (1-\theta)(1-x)\right)$, $x \in (0;1)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ $(0 \leq \theta_1 < \theta_0 \leq 1)$. I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

10. $X^{(n)} = (X_1, ..., X_n)$ tanlanma zichlik funksiyasi $f(x;\theta) = 2x/\theta^2$, $x \in (0;\theta)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$. I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

16-§. MUVOFIQLIK KRITERIYLARI

I. Kolmogorov kriteriysi

$X^{(n)} = (X_1, ..., X_n)$ tanlanma F taqsimotdan olingan bo'lsin. $H_0 : F = F_0$ va $H_1 : F \neq F_0$ gipotezalarni ko'raylik. Agar F_0 uzluksiz bo'lsa, Kolmogorov kriteriysidan foydalanish mumkin. Quyidagi Kolmogorov statistikasini ko'ramiz:

$$D_n = \sqrt{n} \sup_x \left| \widehat{F}_n(x) - F_0(x) \right|. \tag{40}$$

Agar H_0 gipoteza to'g'ri bo'lsa, u holda X_i larning taqsimoti F_0 bo'ladi. Kolmogorov teoremasiga ko'ra $D_n \Rightarrow \eta$, bu yerda η t.m.ning taqsimoti Kolmogorov taqsimotidir;

η t.m. taqsimot funksiyasi $K(x) = \sum\limits_{j=-\infty}^{\infty} (-1)^j e^{-2j^2x^2}, x > 0$ bo'lsa, u Kolmogorov taqsimotiga ega deyiladi. Bu taqsimot jadvallashtirilgan, ya'ni berilgan α da $K(x_\alpha) = 1-\alpha$ dan x_α ning qiymatini topish mumkin. Quyidagi jadvalda Kolmogorov taqsimotining x_α kritik qiymatlari jadvali keltirilgan:

α	0,4	0,3	0,2	0,1	0,05	0,025	0,01	0,005	0,001
x_α	0,89	0,97	1,07	1,22	1,36	1,48	1,63	1,73	1,95

Kolmogorov kriteriysi bo'yicha gipotezani tekshirish quyidagicha amalga oshiriladi:

a) $\hat{F}_n(x)$ empirik va faraz qilinayotgan $F_0(x)$ nazariy taqsimot funksiyalar quriladi;

b) D_n statistikaning qiymati hisoblanadi;

c) Agar $D_n \leq x_\alpha$ bo'lsa, u holda H_0 gipoteza tajriba natijalariga zid emas, ya'ni H_o gipotezani qabul qilish mumkin. Aks holda, ya'ni $D_n > x_\alpha$ bo'lsa, H_0 gipoteza rad etiladi.

25-misol. Quyida berilgan interval variatsion qator uchun $\alpha = 0.05$ deb hisoblab, Kolmogorov kriteriysi yordamida $H_0 : X \sim N(119,2; 87,48)$ gipotezani tekshiring.

$a_{i-1} - a_i$	94-100	100-106	106-112	112-118	118-124	124-130
n_i	3	7	11	20	28	19

$a_{i-1} - a_i$	130-136	136-142
n_i	10	2

$\triangleright$ Berilgan tanlanma bo'yicha empirik va nazariy taqsimotlarni hisoblaymiz: Nazariy taqsimotni hisoblashda $F(x) = \frac{1}{2} + \Phi\left(\frac{x-a}{\sigma}\right) = \frac{1}{2} + \Phi\left(\frac{x-119,2}{9,35}\right)$ formuladan foydalanamiz. Bu yerda $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_o^x e^{-\frac{t^2}{2}} dt$- Laplas funksiyasi. Bu funksiyaning qiymatlari jadvali ilovada keltirilgan (1-jadval). Masalan,

$$F(100) = \frac{1}{2} + \Phi\left(\frac{100-119,2}{9,35}\right) = \frac{1}{2} + \Phi(-2,05) =$$

$$= 0,5 - 0,47982 = 0,02018 \approx 0,02.$$

Huddi shunday qolgan qiymatlarini ham hisoblaymiz va quyidagi jadvalni hosil qilamiz:

x	100	106	112	118	124	130	136	142
$\hat{F}_n(x)$	0,03	0,1	0,21	0,41	0,69	0,88	0,98	1
$F(x)$	0,02	0,08	0,221	0,449	0,625	0,878	0,964	0,99

Ushbu jadvaldan ko‘rinadiki,

$$D_{100} = \sqrt{100}\sup\left|\widehat{F}_n(118) - F(118)\right| = 10 \cdot 0,039 = 0,39.$$

$\alpha = 0,05$ dagi Kolmogorov kriteriysining $x_{0,05}$ kritik qiymati $1,36$ ga teng. Demak, $D_{100} = 0,39 < 1,36 = x_{0,05}$ va H_o gipoteza tajriba natijalariga zid emas. $\triangleleft$

II. Pirsonning xi-kvadrat kriteriysi

χ^2 - kriteriy guruhlangan ma’lumotlarga asoslanadi. ξ t.m.ning taqsimoti F bo‘lsin. $H_0 : F = F_0$ va $H_1 : F \neq F_1$ gipotezalarni ko‘raylik. ξ ning qiymatlari to‘plami X ni l ta kesishmaydigan $\Delta_1, \Delta_2, ..., \Delta_l$ intervallarga ajratamiz. Δ_i intervalga to‘g‘ri kelgan x_i lar soni (chastotani) n_i deb, F_0 taqsimot bo‘yicha $np_i = n \cdot P(X_j \in \Delta_i)$- nazariy chastotani hisoblaymiz va quyidagi statistikani ko‘ramiz:

$$\chi_n^2 = \sum_{k=1}^{l} \frac{(n_k - np_k)^2}{np_k}. \tag{41}$$

(41) statistika Pirsonning xi-kvadrat statistikasi deyiladi. $n \to \infty$ da (41) statistika ozodlik darajasi $\nu = l-1$ bo‘lgan xi-kvadrat taqsimotga egadir:

$$\chi_n^2 = \sum_{k=1}^{l} \frac{(n_k - np_k)^2}{np_k} \Rightarrow \chi_{l-1}^2.$$

Xi-kvadrat kriteriysi bo‘yicha gipotezani tekshirish quyidagicha amalga oshiriladi:

a) Berilgan tanlanma yordamida interval variatsion qator tuzib olinadi;

b) χ_n^2 statistikaning qiymati hisoblanadi;

c) Berilgan α ga asosan xi-kvadrat taqsimot jadvalidan $\chi_{\nu,\alpha}^2$ kvantil topiladi. Agar $\chi_n^2 < \chi_{\nu,\alpha}^2$ bo‘lsa, u holda H_0 gipoteza tajriba natijalariga zid emas, ya'ni H_o gipotezani qabul qilish mumkin. Aks holda, ya'ni $\chi_n^2 > \chi_{\nu,\alpha}^2$ bo'lsa, H_o gipoteza rad etiladi.

26-misol. Yuqorida ko'rilgan misolda $H_0 : X \sim N\,(119.2; 87,48)$ gipotezani xi-kvadrat kriteriysi yordamida tekshiring.

$\triangleright$ Buning uchun avval p_i: ξ ni $[a_{i-1}, a_i]$ ga tushish ehtimolliklarini hisoblab olamiz. Normal taqsimot xossasiga ko'ra:

$$p_i = P\,(a_{i-1} \leq \xi \leq a_i) = \Phi\left(\frac{a_i - a}{\sigma}\right) - \Phi\left(\frac{a_{i-1} - a}{\sigma}\right) =$$

$$= \Phi\left(\frac{a_i - 119,2}{9,35}\right) - \Phi\left(\frac{a_{i-1} - 119,2}{9,35}\right), \quad i = 1, 2, ..., 8.$$

Masalan,

$$p_1 = P\,(94 \leq \xi \leq 100) = \Phi\left(\frac{100 - 119,2}{9,35}\right) - \Phi\left(\frac{94 - 119,2}{9,35}\right) =$$

$$= \Phi\,(-2,05) - \Phi\,(-2,69) = 0,0166.$$

Qolgan $p_i, i = \overline{2,8}$ ehtimolliklar ham xuddi shunday hisoblanadi. χ_n^2 statistikani hisoblash uchun quyidagi jadvalni tuzib olamiz:

$[x_i, x_{i+1}]$	n_i	p_i	np_i	$(np_i - n)^2$	$\frac{(np_i-n)^2}{np_i}$
$94 - 100$	3 }	0.017	1,7 }	5.76	0.758
$100 - 106$	7 }	0.059	5,9 }		
$106 - 112$	11	0.141	14.1	9.61	0.682
$112 - 118$	20	0.228	22.8	7.84	0.344
$118 - 124$	28	0.247	24.7	10.89	0.441
$124 - 130$	19	0.182	18.2	0.64	0.035
$130 - 136$	10 }	0.087	8,7 }	0.16	0.014
$136 - 142$	2 }	0.029	2,9 }		
Σ	100	0,99	99		$\chi_n^2 = 2,27$

n_1 va n_8 chastotalar 5 dan kichik bo'lganligi sababli qo'shni intervallar bilan birlashtirildi.

Demak, $\chi_n^2 = 2,27$. Birlashtirish hisobiga intervallar soni $l = 6$ ekanligini inobatga olib, $\chi^2_{5;0,05} = 11,07$ ni aniqlaymiz.

$\chi_n^2 = 2,27 < 11.07 = \chi^2_{5;0,05}$ ekanligidan $H_0 : X \sim N(119,2; 87,48)$ gipotezani qabul qilamiz. $\triangleleft$

Misollar

Quyidagi misollarga berilgan tanlanma bo'yicha muvofiqlik kriteriylari yordamida H_0 gipotezani tekshiring.

1. $n = 100$ ta detal uzunligini o'lchatib, quyidagi jadval tuzildi (mm da):

Uzunligi	98	98,5	99	99,5	100	100,5	101	101,5	102	102,5
Chastotasi	2	5	9	16	18	20	14	10	4	2

$H_0 : X \sim N(100,25; 1)$.

2. Tasodifiy sonlar jadvalidan $n = 150$ ta son tanlanadi. $[10i, 10i + 9]$ $(i = 0, 1, ..., 9)$ oraliqqa tushgan sonlar chastotalari n_i : $16, 15, 19, 13, 14, 19, 14, 11, 13, 16$ lardan iborat.
$H_0 : X \sim R[0, 100]$.

3. Telefon stansiyasida har minutda noto'g'ri ulanishlar soni ξ ustida kuzatishlar olib borilib 1 soat davomida quyidagi ma'lumotlar olindi: 3; 1; 3; 1; 4; 2; 2; 4; 0; 3; 0; 2; 2; 0; 2; 1; 4; 3; 3; 1; 4; 2; 2; 1; 1; 2; 1; 0; 3; 4; 1; 3; 2; 7; 2; 0; 0; 1; 3; 3; 1; 2; 4; 2; 0; 2; 3; 1; 2; 5; 1; 1; 0; 1; 1; 2; 2; 1; 1; 5. Muhimlik me'yorini $\alpha = 0,05$ deb tanlab, bu tanlanmaning nazariy taqsimoti Puasson taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

4. $n = 200$ laboratoriyadagi jonivorlarning og'irligi o'lchanib quyidagi jadval tiuzildi(og'irlik grammda):

Intervallar	[30;35]	[35;40]	[40;45]	[45;50]	[50;55]	[55;60]
Chastotalar	6	12	15	22	47	42

Intervallar	[60;65]	[65;70]	[75;80]
Chastotalar	28	17	11

$H_0 : X \sim N(55, 95)$.

5. Quyidagi jadvalda $[15^{00}; 15^{20}]$ vaqt oralig'ida 100 kun davomida telefon chaqiriqlari soni keltirilan.

Chaqiriqlar soni	0	1	2	3	4	5	6	7	8	9	10
Kunlar soni	2	14	23	24	18	9	6	2	1	0	1

$H_0 : X \sim \pi\,(32)$.

6. Ma'lum mahsulotning narxi ξ ning dispersiyasi $\sigma^2 = 2,25$ ga teng va quyidagi statistik tanlanmaga ega:

Narxlar intervali	[3,0;3,6]	[3,6;4,2]	[4,2;4,8]	[4,8;5,4]	[5,4;6,0]	[6,0;6,6]	[6,6;7,2]
Chastotasi	2		35	43	22	15	5

$H_0 : X \sim N\,(5; 2,25)$.

7. 10 soat davomida avtomashinalarning AYOQT da benzin kolonkasi oldiga kelishi kuzatiladi:

Vaqt oralig'i (soatda):	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18
Avtomashinalar soni:	12	40	22	16	28	6	11	33	18	14

$H_0 : X \sim R[8, 18]$.

8. Quyida berilgan interval variatsion qator uchun $\alpha = 0.05$ deb hisoblab, muvofiqlik kriteriylari yordamida tanlanmaning nazariy taqsimoti Puasson taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

$a_{i-1} - a_i$:	2,3-2,5	2,5-2,7	2,7-2,9	2,9-3,1	3,1-3,3	3,3-3,5
n_i :	3	6	9	8	5	2

9. Quyida talabalarning imtihon topshirganliklari to'g'risidagi ma'lumot berilgan:

Topshirilgan imtihonlar soni:	0	1	2	3	4
Talabalar soni:	1	1	1	3	35

$\alpha = 0.05$ deb hisoblab, muvofiqlik kriteriylari yordamida tanlanmaning nazariy taqsimoti Binomial taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

10. Quyida berilgan guruhlangan variatsion qator uchun $\alpha = 0.05$ deb hisoblab, muvofiqlik kriteriylari yordamida tanlanmaning nazariy taqsimoti Puasson taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

x_i	0	1	2	3	4	5	6
n_i	405	366	175	40	8	4	2

TESTLAR

(New York Universiteti 2-kurs bakalavrlariga 1990 yilda berilgan test savollari)

1. R, S va T bog'liqsiz hodisalar va ular bir xil $\frac{1}{3}$ ehtimollikka ega. $P(R \cup S \cup T)$ ni toping?
A. $\frac{1}{27}$; B. $\frac{2}{3}$; C. $\frac{19}{27}$; D. $\frac{26}{27}$; E. 1.

2. X va Y t.m. larning birgalikdagi zichlik funksiyasi berilgan bo'lsin: $f(x,y) = \begin{cases} 25, & 0 \le x \le 2\,,\ x-2 \le y \le x \\ 0, & \text{aks holda.} \end{cases}$ MX^3Y ni toping.
A. $\frac{6}{5}$; B. $\frac{4}{3}$; C. 2; D. 4; E. $\frac{24}{5}$.

3. $X_1, X_2, ..., X_{36}$ va $Y_1, Y_2, ..., Y_{49}$ bog'liqsiz t.m. lar mos matematik kutilmasi $MX = 30.4$ va $MY = 32.1$, o'rtacha kvadratik tarqoqligi $\sigma_X = 12$ va $\sigma_Y = 14$ bo'lgan taqsimotga ega bo'lsin. $P(\overline{x} > \overline{y})$ ni hisoblang.
A. 27; B. 34; C. 50; D. 66; E. 73.

4. X va Y diskret t.m. larning birgalikdagi taqsimot qonuni berilgan:

$Y \setminus X$	-1	0	1
0	1	1	2
1	1	3	2

$Cov(X,Y)$ ni hisoblang. A. -02; B. 0; C. 02; D. 10; E. 12.

5. Nomerlangan kub toki 2 raqami tushmagunga qadar tashlanadi. X t.m. tashlashlar soni bo'lsa, $P(X \le x) \ge \frac{1}{2}$ ni qanoatlantiruvchi eng kichik x ning qiymatini toping.
A. 2; B. 3; C. 4; D. 5; E. 6.

6. X va Y t.m. lar matematik kutilmasi μ dispersiyasi $\sigma^2 > 0$ bo'lgan normal taqsimotga ega va ρ bu t.m. lar korrelyatsiya koeffitsienti. Quyidagi mulohazalarning qaysilari to'g'ri?
I. agar $\rho = 0$ bo'lsa, uholda X va Y t.m. lar bog'liqsiz bo'ladi;
II. $Y - X$ normal taqsimlangan bo'ladi, faqat va faqat $\rho > 0$ bo'lsa;
III. $D(X+Y) < 2\sigma^2$ faqat va faqat $\rho < 0$ bo'lsa;
A. faqat I va II; B. faqat I va III; C. faqat II va III; D. I va III; E. to'g'ri javob yo'q.

7. Zichlik funksiyasi $f(x) = \begin{cases} 0, \text{ agar } x \leq 0 \\ 2\lambda e^{-2\lambda x}, \text{ agar } x > 0 \end{cases}$ bo'lgan ξ tasodifiy miqdor uchun $M\xi$ ni toping.
A.1; B $\frac{1}{\lambda}$; C $\frac{1}{2}$; D λ; E. $\frac{1}{2\lambda}$.

8. $X_1, ..., X_4 \sim N(3; \sigma^2)$, σ^2 - noma'lum bo'lsin. Agar tanlanmaning tajribadagi qiymatlari 4,8,5 va 3 bo'lsa, σ^2 uchun HMo'UB ni qiymatini toping.
A. $\frac{7}{2}$; B. $\frac{9}{2}$; C. $\frac{14}{3}$; D. 5; E. $\frac{15}{2}$.

9. X va Y t.m. larning birgalikdagi zichlik funksiyasi berilgan: $f(x,y) = \begin{cases} \frac{1}{6}, \ x = 1,2,3, \ y = 2,3, \\ 0, \quad \text{aks holda.} \end{cases}$ $U = X + Y$ bo'lsin, u holda U t.m. ning zichlik funksiyasini toping.

A. $g(u) = \begin{cases} \frac{1}{4}, \ u = 3,4,5,6, \\ 0 \ , \ \text{aks holda;} \end{cases}$ B. $g(u) = \begin{cases} \frac{2}{3}, \ u = 3, \\ \frac{1}{5}, \ u = 4,5,6, \\ \\ 0, \ \text{aksholda;} \end{cases}$

C. $g(u) = \begin{cases} \frac{1}{6}, \ u = 3,6, \\ \frac{1}{3}, \ u = 4,5; \\ \\ 0, \ \text{aksholda;} \end{cases}$ D. $g(u) = \begin{cases} \frac{1}{5}, \ u = 2,3,4,5,6, \\ 0 \ , \ \text{aks holda;} \end{cases}$

E. $g(u) = \begin{cases} \frac{1}{4}, \ u = 2,3,4,5,6, \\ 0 \ , \ \text{aks holda.} \end{cases}$

10. $X_1, X_2 \sim R[0,1]$ va $Y = \min(X_1, X_2)$ bo'lsa Y t.m. ning zichlik funksiyasini toping.
A. 1, agar $0 \leq y \leq 1$; B. $2y(1-y)$, agar $0 \leq y \leq 1$;
C. $2y$, agar $0 \leq y \leq 1$; D. $(1-y)$, agar $0 \leq y \leq 1$;
E. $2(1-y)$, agar $0 \leq y \leq 1$

11. $X_1, ..., X_n$ matematik kutilmasi MX va dispersiyasi $DX > 0$ va $Y_1, ..., Y_n$ esa matematik kutilma MY va dispersiya $DY > 0$ taqsimotlardan olingan bo'lsin. Asosiy $H_0 : MX = MY$ gipotezani va alternativ $H_1 : MX \neq MY$ gipotezani ko'raylik. $Z_i = X_i - Y_i$ ga asoslangan St'yudent kriteriysi bo'yicha H_0 gipotezani tekshirishda quyida keltirilgan qaysi mulohazadan foydalaniladi?
I. Z_i normal taqsimotga ega;
II. $\sigma_X^2 = \sigma_Y^2$;
III. $Cov(X_i, Y_i) = 0, \ \ i = 1, ..., n$

A. I,II va III; B. I; C. II; D. III; E. to‘g‘ri javob yo‘q.

12. X_1, X_2 va X_3 diskret t.m. lar bo‘lib, quyidagi zichlik funksiyaga ega bo‘lsin: $g(x) = \begin{cases} \frac{x}{10}, & \text{agar } x = 1,2,3, \\ 0, & \text{aks holda.} \end{cases}$

$P(X_1 < X_2 < X_3)$ ni hisoblang.

A. 0.030; B. 0.050; C. 0.167; D. 0.250; E. 0.350.

13. X uzluksiz t.m. zichlik funksiyasi berilgan:

$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{agar } x > 0, \\ 0, & \text{aks holda.} \end{cases}$ Agar bu taqsimotning medianasi $\frac{1}{3}$ bo‘lsa, λ ni toping.

A. $\frac{1}{3}\ln\frac{1}{2}$; B. $\frac{1}{3}\ln 2$; C. $2\ln\frac{3}{2}$; D. $3\ln 2$; E. 3.

14. $M_X(t)$ funksiya X t.m.ning hosil qiluvchi funksiyasi bo‘lsin. Quyida keltirilgan mulohazalardan qaysilari to‘g‘ri?

I. $M_X(0) = 1$; II. $\frac{d^2 M_X(t)}{dt^2}|_{t=0} = Var[X]$; III. $M_X(t)$ X t.m.ning taqsimotini o‘zaro bir qiymatli aniqlaydi.

A. I va II ; B. I va III; C. II va III; D. I,II va III; E. to‘g‘ri javob yo‘q.

15. X va Y uzluksiz t.m. larning birgalikdagi zichlik funksiyasi berilgan: $f(x,y) = \begin{cases} \frac{12}{25}(x + y^2), & \text{agar } 1 < x < y < 2 \\ 0, & \text{aks holda.} \end{cases}$ Y t.m. ning $f(y)$ marginal zichlik funksiyasini toping.

A. $\frac{6}{25}(2y^3 - y^2 - 1), \quad \text{agar } 1 < y < 2;$

B. $\frac{6}{25}(3 + 2y^2), \quad \text{agar } 1 < y < 2;$

C. $\frac{6}{25}\left[y^2(1 + 2y)\right], \quad \text{agar } 1 < y < 2;$

D. $\frac{3(x+y^2)}{8+6x-3x^2-x^3}, \quad \text{agar } 1 < x < y < 2;$

E. $\frac{3(x+y^2)}{7+3x}, \quad \text{agar } 1 < x, y < 2.$

16. Noma’lum θ parametr uchun uchta bog‘liqsiz ishonchlik intervallari tuzilgan. Agar har bir interval ishonchliligi 0.98 bo‘lsa, bu intervallardan birortasi ham θ ni o‘z ichiga olmasligi ehtimolligini toping.

A. 0.0192; B. 0.0297; C. 0.0588; D. 0.9412; E. 0.9703.

17. $X_1, ..., X_8 \sim N(MX, 9)$ va $Y_1, ..., Y_8 \sim N(MY, 9)$ bo‘lsin. Asosiy gipoteza $H_0 : MX = MY$ ni $H_1 : MX > MY$ alternativ gipotezaga qarshi tekshirishda $W = \overline{x} - \overline{y}$ statistikadan foyalaniladi. Agar $W > 3$ da H_0 rad etilsa, $MX - MY = 4.5$ bo‘lganda kriteriy

quvvatini toping.
A. 05; B. 07; C. 16; D. 84; E. 93.

18. $X_1, X_2 \sim \pi(\theta)$ bo'lsin. Asosiy gipoteza $H_0 : \theta = 5$ ni $H_1 : \theta \neq 5$ alternativ gipotezaga qarshi tekshirishda $\overline{x} = \frac{X_1+X_2}{2}$ statistikadan foyalaniladi. Agar $|\overline{x} - 5| \geq 4$ kritik soha bo'lsa, 1-tur xatolik ehtimolligini toping.
A. $1 - \sum\limits_{y=2}^{8} \frac{e^{-5}5^y}{y!}$; B. $1 - \sum\limits_{y=1}^{9} \frac{e^{-5}5^y}{y!}$; C. $1 - \sum\limits_{y=2}^{8} \frac{e^{-10}10^y}{y!}$; D. $1 - \sum\limits_{y=0}^{17} \frac{e^{-10}10^y}{y!}$;
E. $1 - \sum\limits_{y=3}^{17} \frac{e^{-10}10^y}{y!}$.

19. X uzluksiz t.m. ning birgalikdagi ziclik funksiyasi berilgan:
$f(x) = \begin{cases} \frac{1}{9}x(4-x) \text{ agar } 0 < x < 3, \\ 0, \quad \text{aks holda.} \end{cases}$ X ning modasining hisoblang:
A. $\frac{4}{9}$; B. 1; C. $\frac{3}{2}$; D. $\frac{7}{4}$; E. 2.

20. X va Y t.m. larning birgalikdagi taqsimot qonuni berilgan:

$Y \setminus X$	1	5
2	$\theta_1 + \theta_2$	$\theta_1 + 2\theta_2$
4	$\theta_1 + 2\theta_2$	$\theta_1 + \theta_2$

θ_1 va θ_2 parametrlar birgalikdagi taqsimot xossalarini va $-0.25 \leq \theta_1 \leq 0.25$, $0 \leq \theta_2 \leq 0.35$ shartlarni qanoatlantiradi. (θ_1, θ_2) ning qanday qiymatida X va Y t.m.lar bog'liqsiz bo'ladi?
A. $\left(0, \frac{1}{6}\right)$; B. $\left(\frac{1}{4}, 0\right)$; C. $\left(-\frac{1}{4}, \frac{1}{3}\right)$; D. $\left(-\frac{1}{8}, \frac{1}{4}\right)$; E. $\left(\frac{1}{16}, \frac{1}{8}\right)$.

21. X, Y va Z bog'liqsiz normal taqsimlangan tasodifiy miqdorlar va $MX = 2$, $MY = 1$, $MZ = 2$ va $DX > 0$ bo'lsin. Agar $W = c\left[\frac{4(X-2)^2}{(Y-1)^2+(Z-2)^2}\right]$ bo'lsa, c ning qanday qiymatida W t.m. $F_{1,2}$ Fisher taqsimotiga ega bo'ladi.
A. 0.25; B. 0.50; C. 1; D. 2; E. 4.

22. $X_1, ..., X_{15} \sim N\left(\mu, \sigma^2\right)$ va $\overline{x} = \sum\limits_{i=1}^{15} \frac{X_i}{15}$ va $T = \sum\limits_{i=1}^{15} (X_i - \overline{x})^2$ bo'lsin. Asosiy gipoteza $H_0 : \sigma^2 \leq 10$ ga qarshi alternativ $H_1 : \sigma^2 > 10$ gipotezani ko'raylik. $1 - \alpha = 0.05$ bo'lganda kritik sohani toping.
A. H_0 rad etiladi, agar $T \geq 23.69$; B. H_0 rad etiladi, agar $T \geq 25.00$; C. H_0 rad etiladi, agar $T \geq 236.90$; D. H_0 rad etiladi, agar $T \geq 250.00$;

E. H_0 rad etiladi, agar $T \geq 261.20$.

23. Quyida keltirilgan hodisalarning qaysilari $(B \cap C) \cup (A' \cap B \cap C')$ hodisaga teng kuchli:
I. $B \cap (A' \cup C)$; II. $(A' \cap B) \cup (B \cap C)$; III. $(A' \cap C') \cup (B \cap C)$.
A. I va II ; B. I va III; C. II va III; D. I, II va III; E. to‘g‘ri javob yo‘q;

24. Yashikda 100 shardan r tasi qizil, qolganlari esa oq rangda. Tavakkaliga 20 ta shar olinadi. Agar olingan sharlardan hech bo‘lmasa 10 tasi qizil bo‘lsa, asosiy gipoteza $H_0 : r = 30$ rad etilib, $H_1 : r > 30$ alternativ gipoteza qabul qilinadi. $r = 40$ bo‘lsa, 2-tur xatolik ehtimolligini hisoblang.

A. $\dfrac{\binom{40}{10}}{\binom{100}{20}}$; B. $\sum\limits_{k=10}^{20} \dfrac{\binom{30}{k}\binom{70}{20-k}}{\binom{100}{20}}$;

C. $\sum\limits_{k=0}^{9} \dfrac{\binom{30}{k}\binom{70}{20-k}}{\binom{100}{20}}$; D. $\sum\limits_{k=10}^{20} \dfrac{\binom{40}{k}\binom{60}{20-k}}{\binom{100}{20}}$;

E. $\sum\limits_{k=0}^{9} \dfrac{\binom{40}{k}\binom{60}{20-k}}{\binom{100}{20}}$.

25. X va Y uzluksiz t.m. lar birgalikdagi zichlik funksiyasi berilgan:
$f(x,y) = \begin{cases} 2(x+y) \text{ agar } 0 < x < y < 1 \\ 0, \quad \text{aks holda.} \end{cases}$. MY ni hisoblang.
A. $\frac{5}{12}$; B. $\frac{1}{2}$; C. $\frac{3}{4}$; D. 1; E. $\frac{2}{6}$.

26. $X_1, X_2, ..., X_n$ tanlanma zichlik funksiyasi
$f(x) = \begin{cases} \frac{1}{\theta_2-\theta_1}, & \text{agar } \theta_1 \leq x \leq \theta_2 \\ 0, & \text{aks holda} \end{cases}$, $\theta_2 > 0$, $\theta_1 < 0$, bo‘lgan taqsimotdan olingan. Asosiy $H_0 : \theta_1 = -\theta_2$ gipoteza va unga alternativ $H_1 : \theta_1 \neq -\theta_2$ gipotezalarni ko‘raylik. H_0 gipotezani tekshirish uchun qurilgan haqiqatga o‘xshashlik nisbati statistikasining kritik sohasini toping.

A. $\frac{\max(X_i)-\min(X_i)}{\max(X_i)} \leq k$; B. $\frac{\max|X_i|-\min|X_i|}{\max(X_i)} \leq k$; C. $\frac{\max(X_i)-\min(X_i)}{\max|X_i|} \leq k$;
D. $\frac{\max(X_i)}{\max(X_i)-\min(X_i)} \leq k$; E. $\frac{\max|X_i|}{\max(X_i)} \leq k$.

27. X_1, X_2, X_3 lar bog‘liqsiz va mos ravishda $\theta, 2\theta, 3\theta$ parametrli Puasson taqsimoti bo‘yicha taqsimlangan bo‘lsin. Noma’lum parametr θ uchun HMo‘UB toping.
A. $\frac{1}{2}\overline{x}$; B. $\overline{x}$; C. $\frac{X_1+2X_3+3X_3}{6}$; D. $\frac{3X_1+2X_2+X_3}{6}$; E. $\frac{6X_1+3X_2+2X_3}{11}$.

28. A va B hodisalarning ehtimolliklari mos ravishda 0.3 va 0.1 ga teng. o‘tkazilgan bog‘liqsiz 5 ta tajribada A 3 marta, 10 ta tajribada B hodisa 6 marta ro‘y berishi ehtimolligini toping.
A. $\binom{5}{3}(0.3)^3(0.7)^2 + \binom{10}{6}(0.1)^6(0.9)^4$;
B. $\binom{5}{3}(0.3)^3(0.7)^2 + \binom{10}{6}(0.1)^6(0.9)^4$;
C. $\binom{15}{9}(0.4)^9(0.6)^6$; D. $\frac{\binom{5}{3}\binom{10}{6}}{\binom{15}{9}}$; E. $(0.6)(0.9)$.

29. X va Y t.m.lar markazi (0,0) nuqtada radiusi 1 bo‘lgan doirada tekis taqsimlangan bo‘lsin. Y t.m.ning $X = x$ dagi shartli zichlik funksiyasini toping.
A. $\frac{1}{2\pi\sqrt{1-x^2}}$, agar $-1 < y < 1$;
B. $\frac{1}{2\sqrt{1-x^2}}$, agar $-1 < y < 1$;
C. $\frac{1}{\pi}$, agar $-\sqrt{1-x^2} < y < \sqrt{1-x^2}$;
D. $\frac{1}{2\pi\sqrt{1-x^2}}$, agar $-\sqrt{1-x^2} < y < \sqrt{1-x^2}$;
E. $\frac{1}{2\sqrt{1-x^2}}$, agar $-\sqrt{1-x^2} < y < \sqrt{1-x^2}$.

30. Agar $M\xi = a$, $D\xi = \sigma^2$ bo‘lib, $\eta = \frac{\xi-a}{\sigma}$ bo‘lsa, $M\xi$ va $D\eta$ ni toping.
A. a;1; B. 0;1; C. 0;σ^2; D. $a;\frac{1}{\sigma}$; E.0;2.

31. $X_1, ..., X_n \sim N(\mu, 50)$ bo‘lsin. $\overline{x} \geq 13.75$ kritik soha yordamida asosiy gipoteza $H_0 : \mu = 10$ alternativ gipoteza $H_1 : \mu = 15$ qarshi tekshiriladi. 2 - tur xatolik 0.31 dan kichik yoki teng bo‘lishi uchun eng kichik tanlanma hajmini toping.

A. 2; B. 4; C. 5; D. 8; E. 20.

32. $X_1, ...X_n$ tanlanma zichlik funksiyasi

$$f(x) = \begin{cases} \theta(a-x)^{\theta-1} \,\text{agar}\, a-1 \le x \le a \\ 0, \quad \text{aks holda.} \end{cases} \quad a > 0, \;\; \theta > 0,$$

bo‘lgan taqsimotdan olingan bo‘lsin. Asosiy gipoteza $H_0 : \theta = \theta_0$ va unga alternativ gipoteza $H_1 : \theta > \theta_0$ bo‘lsin. Uning uchun eng quvvatli kritik sohani toping.

A. $\sum\limits_{i=1}^{n} \ln(a - X_i) \ge k$; B. $\sum\limits_{i=1}^{n} \ln(a - X_i) \le k$; C. $\sum\limits_{i=1}^{n} X_i \ge k$;

D. $\sum\limits_{i=1}^{n} X_i \le k$; E. $\prod\limits_{i=1}^{n} \ln(X_i - a) \le k$.

33. X t.m. zichlik funksiyasi:

$f(x;\theta) = \begin{cases} \left(\frac{1}{\theta}\right) x^{(1-\theta)/\theta} \text{agar}\, 0 < x < 1 \\ 0, \quad \text{aks holda.} \end{cases}$ $\theta > 0$ bo‘lsin. Noma’lum parametr θ uchun MUB ni toping.

A. $\frac{1-\overline{x}}{\overline{x}}$; B. $\frac{\overline{x}-1}{\overline{x}}$; C. $\frac{\overline{x}}{1-\overline{x}}$; D. $\frac{\overline{x}}{\overline{x}-1}$; E. $\frac{1}{1+\overline{x}}$

34. X va Y diskret t.m. lar birgalikdagi zichlik funksiyasi berilgan:$f(x,y) = \frac{x^2+y^2}{56}$ agar $x = 1,2,3,$ va $y = 1,...,x$. $P(Y = 3/Y \ge 2)$ ni hisoblang.

A. $\frac{9}{28}$; B. $\frac{1}{3}$; C. $\frac{6}{13}$; D. $\frac{41}{54}$; E. $\frac{6}{7}$.

35. Quyida keltirilgan funksiyalarning qaysilari taqsimot funksiya bo‘ladi:

I. $F(x) = \begin{cases} 0 \,\text{agar}\, x \le 1 \\ x^2 - 2x + 1 \,\text{agar}\, 1 < x \le 2 \\ 1 \,\text{agar}\, x > 2 \end{cases}$;

II. $F(x) = \begin{cases} 0 \,\text{agar}\, x \le 0 \\ x^2 - 2x + 1 \,\text{agar}\, 0 < x \le 1 + \sqrt{2}\, x^2 - 2x \le 0 \\ 1 \,\text{agar}\, x > 1 + \sqrt{2} \end{cases}$;

III. $F(x) = \begin{cases} 0 \,\text{agar}\, x < 2 \\ \frac{1}{2} \,\text{agar} x = 2 \\ x^2 - 2x + \frac{1}{2} \,\text{agar}\, 2 < x \le 1 + \frac{\sqrt{6}}{2} \\ 1 \;\text{agar}\; x > 1 + \frac{\sqrt{6}}{2} \end{cases}$

A. I va II; B. I va III; C. II va III; DI,II va III; E to‘g‘ri javob yo‘q.

36. $X_1, X_2 \sim N(0,1)$ va $M(c|X_1 - X_2|) = 1$bo'lsa, c ni toping.
A. $\sqrt{\pi}$; B. $\frac{1}{\sqrt{\pi}}$; C. $\frac{\sqrt{2\pi}}{4}$; D. $\frac{2}{\sqrt{\pi}}$; E. $\frac{\sqrt{\pi}}{2}$.

37. $X_1, ..., X_9 \sim N(0,4)$ va $Y_1, ..., Y_8 \sim N(0,9)$ bo'lsa, quyida keltirilganlarning qaysilari $P\left[\frac{8.2\sum\limits_{i=1}^{9} X_i^2}{\sum\limits_{j=1}^{8} Y_j^2} < 1\right]$ ehtimollikka yaqin?
A. 0.010; B. 0.025; C. 0.050; D. 0.950; E. 0.975.

38. X va Y uzluksiz t.m. lar birgalikdagi zichlik funksiyasi $f(x,y)$, marginal zichlik funksiyalari $f_X(x), f_Y(y)$ lar (0,1) da 0 dan farqli bo'lsin. Quyida keltirilgan tengliklarning qaysi biri to'g'ri?
A. $M(X^2Y^3) = \left(\int_0^1 x^2dx\right)\left(\int_0^1 y^3dy\right)$; B. $M(X^2) = \int_0^1 x^2 f(x,y)dx$;
C. $M(X^2Y^3) = \left(\int_0^1 x^2 f(x,y)dx\right)\left(\int_0^1 y^3 f(x,y)dy\right)$;
D. $M(X^2) = \int_0^1 x^2 f_X(x)dx$; E. $M(Y^3) = \int_0^1 y^3 f_X(x)dx$.

39. $Y \sim R(0,1)$ va $Z = -a\ln(1-Y)$, $a > 0$ bo'lsa, Z ning taqsimoti?
A. Koshi; B. Lognormal; C.Normal; D.Ko'rsatkichli; E. Tekis.

40. Agar ξ t.m. t.f. $F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-cx}, & x \geq 0 \end{cases}$ bo'lib, $F\left(\frac{1}{3}\right) = \frac{1}{2}$ bo'lsa, c ni toping.
A. $3\ln 2$; B. $2\ln\frac{3}{2}$; C. $\frac{1}{3}\ln 2$; D. 3; E.1.

41. A, B va$C$ hodisalar uchun $P(A/C) = 0.05$, $P(B/C) = 0.05$ o'rinli bo'lsa, quyida keltirilganlarning qaysi biri to'g'ri?
A. $P[A \cap B \backslash C] = (0.05)^2$; B. $P[A' \cap B' \backslash C] \geq 0.90$;
C. $P[A \cup B \backslash C] \leq 0.05$; D. $P[A \cup B \backslash C'] \geq 1 - (0.05)^2$;
E. $P[A \cup B \backslash C'] \geq 0.10$.

42. X, Y va Z lar bog'liqsiz va Puasson taqsimotiga ega bo'lsin. $MX = 3, MY = 1$ va $MZ = 4$ bo'lsa, $P(X + Y + Z \leq 1)$?
A. $13e^{-12}$; B. $9e^{-8}$; C. $\frac{13}{12}e^{-1/12}$; D. $9e^{-1/8}$; E. $\frac{9}{8}e^{-1/8}$.

43. Agar ξ t.m. $N(2;1)$ normal qonun bo'yicha taqsimlangan bo'lsa, $P(\xi < M\xi)$ni toping. A.1/2; B.1/3; C.1/4; D.1/5 ; E.1/6.

44. Agar $X_1, ..., X_n$ t.m.lar birgalikdagi zichlik funksiyasi

$$f(x) = \begin{cases} \theta(\theta+1)x^{\theta-1}(1-x) \text{ agar } 0 < x < 1 \\ 0, \quad \text{aks holda}, \end{cases}$$

$\theta > 0$ bo'lsa, θ uchun yetarli statistikani toping.
A. $\sum\limits_{i=1}^{n} X_i$; B. $\sum\limits_{i=1}^{n} X_i^2$; C. $\prod\limits_{i=1}^{n} X_i$; D. $\frac{\prod\limits_{i=1}^{n} X_i}{\prod\limits_{i=1}^{n}(1-X_i)}$; E. $\left(\prod\limits_{i=1}^{n} X_i\right)\prod\limits_{i=1}^{n}(1-X_i)$.

45. Agar $X_1, ..., X_{10}$ t.m.lar dispersiyasi $\sigma^2 > 0$ bo'lgan normal taqsimotga ega bo'lsa, 95% aniqlikda σ^2 ishonchlilik intervalini tuzing.
A. $(10.162, \infty)$; B. $(8.589, \infty)$; C. $(2.00, \infty)$; D. $(1.848, \infty)$;
E. $(1.720, \infty)$.

46. Idishda 4 ta qizil va 6 ta oq shar bor. Tavakkaliga 3 ta shar olinadi. Tanlangan sharlardan hech bo'lmasa 2 tasi oq rangda bo'lsa, 1 ta qizil va 2 ta oq shar olinishi ehtimolini toping.
A. $\frac{1}{2}$; B. $\frac{2}{3}$; C. $\frac{3}{4}$; D. $\frac{9}{11}$; E. $\frac{54}{55}$.

47. Anketa tarqatilganda 50% aholi tezda javob qaytaradi, 40 % aholi esa 2-marta yuborilganda javob qaytaradi. Agar anketa 4 ta kishiga yuborilgan bo'lsa va 1 ta anketa takroran javob bermagan ihtiyoriy 4 kishidan 1 tasiga yuborilsa, kamida 3 ta kishi umuman javob bermasligi ehtimolligini toping.
A. $(0.3^4) + 4(0.3^3)(0.7)$; B. $4(0.3^3)(0.7)$; C. $0.1^4 + 4(0.1^3)(0.9)$;
D. $0.4(0.3)(0.7^3) + 0.7^4$; E. $0.9^4 + 4(0.9^3)(0.1)$

48. Mijozlar tasodifiy va bog'liqsiz ravishda xizmat ko'rsatish oynasiga borib murojaat qilishadi. Murojaat qilish orasidagi vaqt o'rta qiymati 12 minut bo'lgan ko'rsatkichli taqsimotga ega. X t.m. 1 soat ichida murojaat qilganlar soni bo'lsa, $P(X = 10)$ ni hisoblang.
A. $\frac{10e^{-12}}{10!}$; B. $\frac{10^{12}e^{-10}}{10!}$; C. $\frac{12^{10}e^{-10}}{10!}$; D. $\frac{12^{10}e^{-12}}{10!}$; E. $\frac{5^{10}e^{-5}}{10!}$.

49. X va Y uzluksiz t.m. lar birgalikdagi zichlik funksiyasi berilgan:

$$f(x,y) = \begin{cases} xy \text{ agar } 0 \le x \le 2 \,\text{va}\, 0 \le y \le 1 \\ 0, \quad \text{aks holda.} \end{cases}$$

$P\left[\frac{X}{2} \le Y \le X\right]$ ni toping. A. $\frac{3}{32}$; B. $\frac{1}{8}$; C. $\frac{1}{4}$; D. $\frac{3}{8}$; E. $\frac{3}{4}$.

50. Agar ξ va η bog'liqsiz t.m.lar bo'lib, $M\xi = 1$, $M\eta = 2$, $D\xi = 1$, $D\eta = 4$ bo'lsa, $M(\xi + \eta + 1)^2$ ni toping.
A. 21; B. 10; C. 6; D. 11; E.5.

ILOVA

I. Statistikadagi muhim taqsimotlar

1. χ^2 taqsimot

ξ_i t.m.lar bog'liqsiz va $\xi_i \sim N(0,1)$, $i = 1,...,k$ bo'lsa, $\eta = \xi_1^2 + ... + \xi_k^2$ t.m. ozodlik darajasi k bo'lgan xi-kvadrat taqsimlangan deyiladi va bu taqsimotni $\chi^2(k)$ kabi belgilanadi. χ^2 - taqsimot zichlik funksiyasi:

$$f_{x^2}(x) = \begin{cases} \frac{1}{2^{k/2}\,\Gamma(k/2)} \cdot x^{\frac{k-2}{2}} \cdot e^{-\frac{x}{2}}, & x > 0,\ k > 0, \\ 0, & x \le 0, \end{cases}$$

bu yerda $\Gamma(k)$ gamma funksiya. η t.m. momentlari $M\eta = k,\ D\eta = 2k$.

Xi - kvadrat taqsimotining boshqa taqsimotlar bilan bog'liqligi

∘ $k = 2$ da xi-kvadrat taqsimot parametri $1/2$ bo'lgan eksponensial qonun bilan ustma-ust tushadi: $\chi^2(2) = E(1/2)$.

∘ Xi-kvadrat taqsimot parametrlari $(k/2; 1/2)$ bo'lgan gamma taqsimotning xususiy holidir: $\chi^2(k) = \Gamma(k/2; 1/2)$.

∘ Agar $\eta_1 \sim \chi^2(k_1)$, $\eta_2 \sim \chi^2(k_2)$ va $\eta_1 \perp \eta_2$ bo'lsa, $F = \frac{\eta_1/k_1}{\eta_2/k_2}$ bu t.m. ozodlik darajasi (k_1, k_2) bo'lgan Fisher taqsimoti bo'yicha taqsimlangan bo'ladi (Fisher taqsimoti quyida keltiriladi).

2. St'yudent taqsimoti

Bog'liqsiz va $\xi_0, \xi_1, ..., \xi_n \sim N(0,1)$ bo'lsa, $t = \frac{\xi_0}{\sum\limits_{i=1}^{n} \xi_i}$ t.m. St'yudent taqsimotiga ega deyiladi va $t(n)$ kabi belgilanadi. St'yudent taqsimoti zichlik funksiyasi:

$$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}},\ x \in (-\infty, +\infty),\ n > 0$$

bu yerda $\Gamma(n)$ gamma funksiya. Bu t.m. momentlari $Mt = 0$, agar $n > 2$ bo'lsa, $Dt = \frac{n}{n-2}$.

St'yudent taqsimotining boshqa taqsimotlar bilan bog'liqligi:

∘ Koshi taqsimoti St'yudent taqsimotining xususiy holidir: $t(1) = K(0,1)$.

∘ Agar $\xi \sim t(n)$ bo'lsa, u holda $\xi^2 \sim F(1,n)$ Fisher taqsimoti bo'yicha taqsimlangan bo'ladi.

3. Fisher taqsimoti

Agar $\eta_1 \sim \chi^2(k_1)$, $\eta_2 \sim \chi^2(k_2)$ va $\eta_1 \perp \eta_2$ bo'lsa, $F = \frac{\eta_1/k_1}{\eta_2/k_2}$ bu t.m. ozodlik darajasi (k_1, k_2) bo'lgan Fisher taqsimoti bo'yicha taqsimlangan deyiladi va $F(k_1, k_2)$ kabi belgilanadi. Fisher taqsimoti zichlik funksiyasi:

$$f(x) = \begin{cases} 0\,, \quad x \le 0 \\ \frac{\Gamma\left(\frac{k_1+k_2}{2}\right)}{\Gamma\left(\frac{k_1}{2}\right)\cdot\Gamma\left(\frac{k_2}{2}\right)} \left(\frac{k_1}{k_2}\right)^{\frac{k_1}{2}} \frac{x^{\frac{k_1}{2}-1}}{\left(1+\frac{k_1}{k_2}\right)^{\frac{k_1+k_2}{2}}} \quad , \quad x > 0 \end{cases}$$

Bu t.m. momentlari: agar $k_2 > 2$ bo'lsa, $MF = \frac{k_2}{k_2-2}$, va $k_2 > 4$ bo'lsa, $DF = \frac{2k_2^2(k_1+k_2-2)}{k_1(k_2-2)^2(k_2-4)}$.

II. Statistik jadvallar

1-jadval

Normal taqsimot qiymatlari $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_0^x e^{-\frac{t^2}{2}} dt$

x	0	1	2	3	4	5	6	7	8	9
0.0	0.000	0.039	0.079	0.119	0.159	0.199	0.239	0.279	0.319	0.359
0.1	03983	04380	04776	05172	05567	05962	06356	06749	07142	07535
0.2	07926	08317	08706	09095	09483	09871	10257	10642	11026	11409
0.3	11791	12172	12552	12930	13307	13683	14058	14431	14803	15173
0.4	15542	15910	16276	16640	17003	17364	17724	18082	18439	18793
0.5	19146	19497	19847	20194	20540	20884	21226	21566	21904	22240
0.6	22575	22907	23237	23565	23891	24215	24537	24857	25175	25490
0.7	25804	26115	26424	26730	27035	27337	27637	27935	28230	28524
0.8	28814	29103	29389	29673	29955	30234	30511	30785	31057	31327
0.9	31594	31859	32121	32381	32639	32894	33147	33398	33646	33891
1.0	34134	34375	34614	34849	35083	35314	35543	35769	35993	36214
1.1	36433	36650	36864	37076	37286	37493	37698	37900	38100	38298
1.2	38493	38686	38877	39065	39251	39435	39617	39796	39973	40147
1.3	40320	40490	40658	40824	40988	41149	41308	41466	41621	41774
1.4	41924	42073	42220	42364	42507	42647	42785	42922	43056	43189
1.5	43319	43448	43574	43699	43822	43943	44062	44179	44295	44408
1.6	44520	44630	44738	44845	44950	45053	45154	45254	45352	45449
1.7	45543	45637	45728	45818	45907	45994	46080	46164	46246	46327
1.8	46407	46485	46562	46638	46712	46784	46856	46926	46995	47062
1.9	47128	47193	47257	47320	47381	47441	47500	47558	47615	47670
2.0	47725	47778	47831	47882	47932	47982	48030	48077	48124	48169
2.1	48214	48257	48300	48341	48382	48422	48461	48500	48537	48574
2.2	48610	48645	48679	48713	48745	48778	48809	48840	48870	48899
2.3	48928	48956	48983	49010	49036	49061	49086	49111	49134	49158
2.4	49180	49202	49224	49245	49266	49286	49305	49324	49343	49361

2.5	49379	49396	49413	49430	49446	49461	49477	49492	49506	49520
2.6	49534	49547	49560	49573	49585	49598	49609	49621	49632	49643
2.7	49653	49664	49674	49683	49693	49702	49711	49720	49728	49736
2.8	49744	49752	49760	49767	49774	49781	49788	49795	49801	49807
2.9	49813	49819	49825	49831	49836	49841	49846	49851	49856	49861

x	3.0	3.5	4.0	5.0
$\Phi_0(x)$	0.49865	0.49977	0.499968	0.49999997

2-jadval

$N(0,1)$ - normal taqsimot kvantillari

p	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
U_p	1.282	1.645	1.960	2.326	2.576	3.090	3.291

3-jadval

$t_p(k)$- St'yudent taqsimoti kvantili
k- ozodlik darajasi; p - kvantil tartibi

$$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{\pi n}}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}, \; x \in (-\infty, +\infty), \; n > 0$$

$k \setminus p$	0.750	0.900	0.950	0.975	0.990	0.995	0.999
1	1.000	3.078	6.314	12.706	31.821	63.657	318
2	0.816	1.886	2.920	4.303	6.965	9.925	22.3
3	0.765	1.638	2.353	3.182	4.541	5.841	10.2
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785

8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

4-jadval

$\chi^2(k)$ - taqsimoti kvantili
k- ozodlik darajasi; p - kvantil tartibi

$k \setminus p$	0.90	0.95	0.99	19	27.20	30.14	36.19
1	2.71	3.84	6.63	20	28.41	31.14	37.57
2	4.61	5.99	9.21	21	29.62	32.67	38.93
3	6.25	7.81	11.34	22	30.81	33.92	40.29
4	7.78	9.49	13.28	23	32.01	35.17	41.64
5	9.24	11.07	15.09	24	33.20	36.42	42.98
6	10.64	12.59	16.81	25	34.38	37.65	44.31
7	12.02	14.07	18.48	26	35.56	38.89	45.64
8	13.36	15.51	20.09	27	36.74	40.11	46.96
9	14.68	16.92	21.67	28	37.92	41.34	48.28
10	15.99	18.31	23.21	29	39.09	42.56	49.59
11	17.28	19.68	24.72	30	40.26	43.77	50.89
12	18.55	21.03	26.22	40	51.80	55.76	63.69
13	19.81	22.36	27.69	50	63.17	67.50	76.15
14	21.06	23.68	29.14	60	74.40	79.08	88.38
15	22.31	25.00	30.58	70	85.53	90.53	100.42
16	23.54	26.30	32.00	80	96.58	101.88	112.33
17	24.77	27.59	33.41	90	107.56	113.14	124.12
18	25.99	28.87	34.81	100	118.50	124.34	135.81

Xi-kvadrat taqsimoti zichlik funksiyasi $\chi^2(k)$:

$$f_{\chi^2}(x) = \begin{cases} 0, & x \leq 0; \\ 2^{-k/2}\Gamma^{-1}(k/2)x^{(k-2)/2}e^{-x/2}, & x > 0. \end{cases}$$

5-jadval

$F_p(k_1, k_2)$ - *Fisher taqsimoti kvantili* k_1, k_2- *ozodlik darajalari; p - kvantil tartibi* p=0.95

$k_2 \setminus k_1$	4	6	12	24	30	40	60	120	∞
1	224	234	245	249	250	251	252	253	254
2	19.2	19.3	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	9.1	8.9	8.7	8.6	8.6	8.6	8.6	8.5	8.5
4	6.4	6.2	5.9	5.8	5.7	5.7	5.7	5.7	5.6
5	5.2	5.0	4.7	4.5	4.5	4.5	4.4	4.4	4.4
6	4.5	4.3	4.0	3.8	3.8	3.8	3.7	3.7	3.7
7	4.1	3.9	3.6	3.4	3.4	3.3	3.3	3.3	3.2
8	3.8	3.6	3.3	3.1	3.1	3.0	3.0	3.0	2.9
9	3.6	3.4	3.1	2.9	2.9	2.8	2.8	2.7	2.7
10	3.5	3.2	2.9	2.7	2.7	2.7	2.6	2.6	2.5
11	3.4	3.1	2.8	2.6	2.6	2.5	2.5	2.4	2.4
12	3.3	3.0	2.7	2.5	2.5	2.4	2.4	2.3	2.3
13	3.2	2.9	2.6	2.4	2.4	2.3	2.3	2.3	2.2
14	3.1	2.8	2.5	2.3	2.3	2.3	2.2	2.2	2.1
15	3.1	2.8	2.5	2.3	2.2	2.2	2.2	2.1	2.1
16	3.0	2.7	2.4	2.2	2.2	2.2	2.1	2.1	2.0
17	3.0	2.7	2.4	2.2	2.1	2.1	2.1	2.0	2.0
18	2.9	2.7	2.3	2.1	2.1	2.1	2.1	2.0	1.9
19	2.9	2.6	2.3	2.1	2.1	2.0	2.0	1.9	1.9
20	2.9	2.6	2.3	2.1	2.0	2.0	1.9	1.9	1.8
22	2.8	2.5	2.2	2.0	2.0	1.9	1.9	1.8	1.8
24	2.8	2.5	2.2	2.0	1.9	1.9	1.8	1.8	1.7
26	2.7	2.5	2.1	1.9	1.9	1.9	1.8	1.7	1.7
28	2.7	2.4	2.1	1.9	1.9	1.8	1.8	1.7	1.7
30	2.7	2.4	2.1	1.9	1.8	1.8	1.7	1.7	1.6
40	2.6	2.3	2.0	1.8	1.7	1.7	1.6	1.6	1.5
60	2.5	2.3	1.9	1.7	1.6	1.6	1.5	1.5	1.4
120	2.4	2.2	1.8	1.6	1.6	1.5	1.4	1.4	1.3
∞	2.4	2.1	1.8	1.5	1.5	1.4	1.3	1.2	1.0

6-jadval Tekis taqsimlangan tasodifiy sonlar

98	52	01	77	67	14	90	56	86	07	22	10	94	05	58	60	97	09	34	33	50	50	07	39	98	11	80
50	54	31	39	80	82	77	32	50	72	56	82	48	29	40	52	42	01	52	77	56	78	51	83	45	29	96
34	06	28	89	80	83	13	74	67	00	78	18	47	54	06	10	68	71	17	78	17	88	68	54	02	00	86
50	75	84	01	36	76	66	79	51	90	36	47	64	93	29	60	91	10	62	99	59	46	73	48	87	51	76
49	69	91	82	60	89	28	93	78	56	13	68	23	47	83	41	13	65	48	11	76	74	17	46	85	09	50
58	04	77	69	74	73	03	95	71	86	40	21	81	65	44	80	12	43	56	35	17	72	70	80	15	45	31
82	23	74	21	11	57	82	53	14	38	55	37	63	74	35	09	98	17	77	40	27	72	14	43	23	60	02
10	45	52	16	42	37	96	28	60	26	55	69	91	62	68	03	66	25	22	91	48	36	93	68	72	03	76
62	11	39	90	94	40	05	64	18	09	89	32	05	05	14	22	56	85	14	46	42	75	67	88	96	29	77
88	22	54	38	21	45	98	91	49	91	45	23	68	47	92	76	86	46	16	28	35	54	94	75	08	99	23
37	08	92	00	48	80	33	69	45	98	26	94	03	68	58	70	29	73	41	35	53	14	03	33	40	42	05
08	23	41	44	10	48	19	49	85	15	74	79	54	32	97	92	65	75	57	60	04	08	81	22	22	20	64
13	12	55	07	37	42	11	10	00	20	40	12	86	07	46	97	96	64	48	94	39	28	70	72	53	15	63
60	64	93	29	16	50	53	44	84	40	21	95	25	63	43	65	17	70	82	07	20	73	17	90	61	19	69
04	46	26	45	74	77	74	51	92	43	37	29	65	39	45	95	93	42	58	26	05	27	15	47	44	52	66
95	27	07	99	53	59	36	78	38	48	82	39	61	01	18	33	21	15	94	66	94	55	72	85	73	67	89
75	43	87	54	62	24	44	31	91	19	04	25	92	92	92	74	59	73	42	48	11	62	13	97	34	40	87
21	16	86	84	87	67	03	07	11	20	59	25	70	14	66	70	23	52	37	83	17	73	20	88	98	37	68
93	59	14	16	26	25	22	96	63	05	52	28	25	62	04	49	35	24	94	75	24	63	38	24	45	86	25
10	25	61	96	27	93	35	65	33	71	24	72	00	54	99	76	54	64	05	18	81	59	96	11	96	38	96
54	69	28	23	91	23	28	72	95	29	35	96	31	53	07	26	89	80	93	54	33	35	13	54	62	77	97
45	00	24	90	10	33	93	33	59	80	80	83	91	45	42	72	68	42	83	60	94	97	00	13	02	12	48
92	78	56	52	01	06	46	05	88	52	36	01	39	09	22	86	77	28	14	40	77	93	91	08	36	47	70
61	74	29	41	32	17	90	05	97	87	37	92	52	41	05	56	70	70	07	86	74	31	71	57	85	39	41
18	38	69	23	46	14	06	20	11	74	52	04	15	95	66	00	00	18	74	39	24	23	97	11	89	63	35
19	56	54	14	30	01	75	87	53	79	40	41	92	15	85	66	67	43	68	06	84	96	28	52	07	45	15
51	49	38	19	47	60	72	46	43	66	79	45	43	59	04	79	00	33	20	82	66	95	41	94	86	43	19
94	36	16	81	08	51	34	88	88	15	53	01	54	03	54	56	05	01	45	11	76	98	08	62	48	26	45
24	02	84	04	44	99	90	88	96	39	09	47	34	07	35	44	13	18	80	33	18	51	62	32	41	94	15
09	49	89	43	54	85	81	88	69	54	19	94	37	54	87	30	43	80	95	10	04	06	96	38	27	07	74
20	15	12	33	87	25	01	62	52	98	94	62	46	11	71	79	75	24	91	40	71	96	12	82	96	69	86
10	25	91	74	85	22	05	39	00	38	75	95	79	18	63	33	25	37	98	14	50	65	71	31	01	02	46
74	05	45	56	14	27	77	93	89	19	36	74	02	94	39	02	77	55	73	22	70	97	79	01	71	19	52
52	75	80	21	80	81	45	17	48	54	17	84	58	11	80	99	33	71	43	05	33	51	29	69	56	12	71

JAVOBLAR

1 – §

I. 1. C=2/3; $F(x) = \begin{cases} 0, & x<1 \\ x^2/3, & x\in[0,1], \\ 2x/3-1/3, & x\in(1,2], \\ 1, & x>2; \end{cases}$ $M\xi$=11/9; $D\xi$=37/162; $P(\xi > M\xi)$=14/27. 3. C=3; $F(x) = \begin{cases} 0, & x\le 0 \\ 1-\frac{1}{(x+1)^3}, & x>0; \end{cases}$ $M\xi$ =1/2; $D\xi$=3/4; $P(\xi > M\xi)$=8/27. 5. $C = \frac{1}{\pi}$; $M\xi$=0; $D\xi$=1/2. 7. $C = \frac{3}{\pi}$; $F(x) = \begin{cases} 0, & x<0 \\ \frac{3}{\pi}arctgx, & x\in[0,\sqrt{3}], \\ 1, & x>\sqrt{3}; \end{cases}$ $M\xi = \frac{3}{\pi}\ln 2$; $D\xi$=1,092. 9. C=1/2;

$F(x) = \begin{cases} 0, & x<1/e, \\ \frac{1}{2}\ln x+\frac{1}{2}, & x\in[1/e,], \\ 1, & x>e; \end{cases}$ $M\xi = sh1$, $D\xi = 0,5(1-e^{-2})$.

11. C=1/2; $F(x) = \begin{cases} 0, & x<-2 \\ \frac{1}{2}x-\frac{1}{8}\text{sign}(x)x^2, & x\in[-2,2], \\ 1, & x>2; \end{cases}$

$M\xi$=0; $D\xi$=2/3; $P(\xi > M\xi)$=1/2. 13. C=1; $F(x) = \begin{cases} 0, & x<0 \\ \sin x, & x\in[0,\pi/2], \\ 1, & x>\pi/2; \end{cases}$ $M\xi = \frac{\pi-2}{2}$; $D\xi = \pi-3$. 15. C=1; $M\xi$=2; $D\xi$=2. 17. C=1/3; $M\xi$=4/3; $D\xi$=7/18. 19. C=4/3; $M\xi$=3/7; $D\xi$=0,0735.

II. 1. $\eta_1 \sim R(0,1)$. 2. $f_{\eta_2}(x) = \begin{cases} \frac{\alpha}{x^2}e^{-\alpha\left(\frac{1}{x}-1\right)}, & x\in(0,1), \\ 0, & x\notin(0,1). \end{cases}$

3. $\eta_1 \sim N(2a+1;4\sigma^2)$. 4. $f_{\eta_2}(x) = \begin{cases} \frac{1}{x^2}, & x\in(0,1), \\ 0, & x\notin(0,1). \end{cases}$

5. $\eta_1 \sim R(0,1)$. 6. $\eta_2 \sim N(1-a;\sigma^2)$.

7. $f_{\eta_1}(x) = \begin{cases} \frac{\alpha}{2\sqrt{x}}e^{-\alpha\sqrt{x}}, & x>0, \\ 0, & x\le 0. \end{cases}$ 8. $f_{\eta_2}(x) = \frac{1}{\pi\sqrt{x(1-x)}}, \quad x\in(0,1)$.

9. $\eta_1 \sim \Gamma(1/2,1/2)$. 10. $f_{\eta_2}(x) = \frac{1}{\pi}\frac{1}{1+x^2}, x\in R$.

11. $f_{\eta_1}(x)=\begin{cases}\frac{\alpha}{2\sqrt{x-1}}e^{-\alpha\sqrt{x-1}}, & x>1,\\ 0, & x\le 1.\end{cases}$ 12. $f_{\eta_2}(x)=\frac{1}{\pi}\frac{1}{1+x^2}, x\in R.$

13. $\eta_1\sim N(14;9\sigma^2)$. 14. $f_{\eta_2}(x)=\begin{cases}0, & x<1,\\ 3/x^4, & x\ge 1.\end{cases}$

15. $\eta_1\sim R(1/4,3/4)$. 16. $f_{\eta_2}(x)=\alpha^2\exp\{-\alpha(e^{\alpha x}-x)\}, x\in R.$

17. $\eta_1\sim R\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. 18. $f_{\eta_2}(x)=\begin{cases}\frac{1}{\sqrt{2\pi x}}e^{-x/2}, & x>0,\\ 0, & x\le 0.\end{cases}$

19. $f_{\eta_1}(x)=\begin{cases}\frac{2a}{\sqrt{x-1}}, & x\in(2,5),\\ 0, & x\notin(2,5).\end{cases}$

20. $f_{\eta_2}(x)=\begin{cases}\frac{3(1-x)^2}{2x^4}, & x>1/2,\\ 0, & x\le 1/2.\end{cases}$

III. 1. $f(x)=n\left(\frac{b-x}{b-1}\right)^{n-1}\frac{1}{b-1},\ x\in(1,b)$. 3. $\varsigma\sim E(5/4)$.
5. $\varsigma\sim N(2,5)$. 7. $F(x)=1-e^{-\lambda_1 x}-e^{-\lambda_2 x}+e^{-(\lambda_1+\lambda_2)x},\ x>0$.
9. $\varsigma\sim N(a_1+a_2-5,5)$. 11. $f(x)=\begin{cases}0, \quad x\le 0,\\ 1-e^{-x},\ 0<x\le 1,\\ (e-1)e^{-x}, x>1.\end{cases}$

13. $\varsigma\sim\pi(\lambda_1+\lambda_2)$. 17. $\varsigma\sim Bi(n_1+n_2,p)$. 19. $\varsigma\sim E\left(\sum_{i=1}^{n}\lambda_i\right)$.

V. a) 2. -9/13. 4. $\sqrt{15}/2\sqrt{7}$. 6. 0. 8. 0. 10. -2/3. 12. -2/13. 14. -45/411. 16. 0. 18. 0. 20. -5/139.
b) 1. $\frac{2(9\pi-32)}{9\pi^2-64}\approx-0,3$. 3. $\frac{2(9\pi-32)}{9\pi^2-64}\approx-0,3$. 7. 0. 9. -0,5. 11. 0. 13. 0. 15. $\frac{\sqrt{35}}{\sqrt{111}}\approx 0,562$. 17. $\frac{\sqrt{35}}{2\sqrt{57}}\approx 0,392$. 19. $\frac{9\sqrt{5}}{8\sqrt{19}}\approx 0,577$.
VI. 2. bo‘ysunadi. 4.yo‘q. 6. $\alpha<0$. 8. bo‘ysunadi. 10. yo‘q. 12. bo‘ysunadi. 16. bo‘ysunadi. 18. bo‘ysunadi. 20. $\alpha>0$-ixtiyoriy.

2 – §

1. $M\overline{x}=\frac{\theta_1+\theta_2}{2},\ D\overline{x}=\frac{(\theta_2-\theta_1)^2}{12n}$. 3. $MX_{(1)}=\frac{\theta}{n+1},\ DX_{(1)}=\frac{n\theta^2}{(n+1)^2(n+2)}$. 5. $M\overline{x}=\theta+1,\ D\overline{x}=\frac{1}{n}$. 7. $M\overline{x^2}=\theta^2,\ D\overline{x^2}=\frac{2(n-1)}{n^2}\theta^4$.
9. $MF_n(x)=F(x),\ DF_n(x)=\frac{1}{n}F(x)(1-F(x))$.

3 – §

2. tegishli. 4. yo‘q. 6. yo‘q. 8. tegishli. 10. yo‘q. 12. tegishli. 14. tegishli. 16.yo‘q. 18. tegishli.

4 – §

1. $\left(X_{(1)}, X_{(n)}\right)$; 3. $\left(X_{(1)}, \sum\limits_{i=1}^{n} X_i\right)$; 7. $\sum\limits_{i=1}^{n} X_i$; 9. $\sum\limits_{i=1}^{n} X_i$; 11. $X^{(n)} = (X_1, \ldots, X_n)$; 13. $\sum\limits_{i=1}^{n} X_i$; 15. $X_{(1)}$; 17. $\left(X_{(1)}, \sum\limits_{i=1}^{n} X_i\right)$; 19. $X^{(n)} = (X_1, \ldots, X_n)$;

6 – §

1. siljimagan, asosli baho; 3. a) siljigan, asosli baho; b) siljimagan, asosli baho; 7. siljimagan, asosli baho; 9. siljimagan, asosli baho; 13. siljigan, asosli baho; 15. siljigan, asosli baho; 17. siljigan, asosli baho; 19. siljigan, asosli baho.

8 – §

1. $\left(\frac{S^2}{\overline{x}}, \ \frac{\overline{x}}{S}\right)$; 3. $\left(\overline{x}, \ S^2\right)$; 5. $\left(e^{\overline{x}-1/S}, \ S\right)$; 9. $\overline{x}/(1-\overline{x})$; 11. $\overline{x}/2-2$; 13. $\frac{1}{2}e^{\overline{x}-1}$; 17. $\frac{3\overline{x}}{2}$; 19. $\sqrt[k]{\overline{x^k}/\Gamma\left(1+\frac{k}{3}\right)}$.

9 – §

3. $\left(\overline{x}, \ S^2\right)$; 6. $\left(-X_{(1)}, X_{(n)}\right)$; 8. $\left(X_{(1)}+1, X_{(n)}-1\right)$; 10. $X_{(1)}$; 11. $\left(1/\left(\overline{\ln x} - \ln X_{(1)}\right), X_{(1)}\right)$; 12. $\left(X_{(1)}, \overline{x} - X_{(1)}\right)$; 14. $\overline{x^2}$; 16. $-\frac{1}{\overline{\ln\ln x}}$; 17. $2\overline{x}$; 20. $X_{(n)}$.

10 – §

1. $\frac{\overline{x}n\sigma^2+a}{1+n\sigma^2}$; 3. $\left(1+e^{n/2-n\overline{x}}\right)^{-1}$; 5. $ae^{na}/(e^{na}-1)-1/n$, bu yerda $a = \min(1, X_{(1)})$; 7. $\frac{n+1}{n}\max(X_{(n)}, 1)$; 9. $\frac{n+1}{1+\overline{x}n}$; 11. $\max(X_{(n)}, 1)\frac{\beta+2n}{\beta+2n-1}$; 13. $\frac{2n-1}{2n-2}\frac{X_{(n)}^{2n-1}-X_{(n)}}{X_{(n)}^{2n-1}-1}$; 15. $\frac{\overline{x}n+\lambda}{n+1+\lambda}$; 17. $\frac{3^{n\overline{x}}+4\cdot 2^{n\overline{x}}+9}{4(3^{n\overline{x}}+2\cdot 2^{n\overline{x}}+3)}$; 19. $\frac{\overline{x}n+1}{n+1}$.

11 – §

8. a.n. baho. 10. a.n. baho. 12. a.n. baho. 14. a.n. baho. 16. a.n. baho. 18. a.n. baho. 20. a.n. baho.

12 – §

1. $D_\theta S^2 = \frac{2\theta^4}{n-1}$; $D_\theta S_2^2 = \frac{2\theta^4}{n}$; 3. $M_\theta(T_1-\theta)^2 = M_\theta(T_2-\theta)^2 = \frac{2\theta^2}{(n+1)(n+2)}$ 5. $M_\theta\big(X_{(1)}-\theta\big)^2 < M_\theta(\overline{x}-1/2-\theta)^2$; 7. $M_\theta\big(X_{(1)}-\frac{1}{n}-\theta\big)^2 = \frac{1}{n^2}$, $M_\theta(\overline{x}-1-\theta)^2 = \frac{1}{n}$.

13 – §

2. χ_2^2 - xi-kvadrat; 4. χ_n^2 - xi-kvadrat; 10. $N\,(0,2\sigma/n)$; 12. $N\,(0,1)$; 14. χ_{n-1}^2 - xi-kvadrat; 16. $Bi(n,p)$; 18. $f(x)=\frac{n(n-1)x(b-x)^{n-2}}{b^n}$, $x\in[0,b]$; 20. $E(1)$.

14 – §

1. $\left(\overline{x}-\frac{\sigma}{\sqrt{n}}t_{\frac{1+\gamma}{2}};\overline{x}+\frac{\sigma}{\sqrt{n}}t_{\frac{1+\gamma}{2}}\right)$;
3. $\left(\overline{x}-\frac{\overline{S}^2}{\sqrt{n}}t_{n-1,\,\frac{1+\gamma}{2}};\overline{x}+\frac{\overline{S}^2}{\sqrt{n}}t_{n-1,\,\frac{1+\gamma}{2}}\right)$; 5. $\left(X_{(n)};X_{(n)}/\sqrt[n]{1-\gamma}\right)$;
7. $\left((\overline{x})^{-1}\left(1-\frac{t_{\frac{1+\gamma}{2}}}{\sqrt{n}}\right);(\overline{x})^{-1}\left(1+\frac{t_{\frac{1+\gamma}{2}}}{\sqrt{n}}\right)\right)$;
9. $\left(X_{(1)}+\ln(1-\gamma)/n;X_{(1)}\right)$; 11. $\left(X_{(1)}-1+\sqrt[n]{1-\gamma};X_{(1)}\right)$;
13. $(0;-(\ln(1-\gamma))/X_1)$.

Testlarning javoblari

1. C.	6. B.	11. B.	16. C.	21. B.	26. C.	31.D.	36. E.
2. A.	7. E.	12. B.	17. D.	22. C.	27. A.	37. B.	32. A.
3. A.	8. E.	13. D.	18. E.	23. A.	28. B.	33. A.	38. D.
4. A.	9. C.	14. B.	19. E.	24. E.	29. E.	34. C.	39. D.
5. C.	10. E.	15. A.	20.B.	25. C.	30. B.	35. B.	40. D.

41. B.	46. C.
42. B.	47. A.
43. A.	48. E.
44. C.	49. D.
45. C.	50. A.

BELGILASHLAR VA QISQARTMALAR RO‘YXATI

t.m.	tasodifiy miqdor
t.f.	taqsimot funksiyasi
t.e.q.	tekis eng quvvatli
a.n.	asimptotik normal
KSQ	katta sonlar qonuni
MUB	momentlar usuli bahosi
HMO‘UB	haqiqatga maksimal o‘xshashlik usuli bahosi
$P(A)$	A hodisaning ehtimolligi
$M\xi$	ξ t.m. matematik kutilmasi
$D\xi$	ξ t.m. dispersiyasi
$I(A)$	A hodisa indikatori
$X^{(n)} = (X_1, ..., X_n)$	statistik tanlanma
$X_{(k)}$	$k-$ tartiblangan statistika
$\xrightarrow{P}$	ehtimollik bo‘yicha yaqinlashish
$\Rightarrow$	taqsimot bo‘yicha yaqinlashish
$\xi \sim N(a;\sigma^2)$	ξ t.m. $(a;\sigma^2)$ paramertli normal taqsimotga ega
$\triangleright$	yechimning boshlanishi
$\triangleleft$	yechimning tugashi
$\perp$	bog‘liqsizlik
$f(x;\theta)$	absolyut uzluksiz t.m. uchun zichlik funksiyasi, diskret t.m. uchun ehtimollik
$F(x;\theta)$	t.f.
N_f	taqsimot bardori (nositel)
θ	noma’lum parametr
Θ	parametrik fazo

BA'ZI MUHIM TAQSIMOTLAR OILASI

1	Normal taqsimot	$f(x,\theta)=\frac{1}{\sqrt{2\pi}\theta_2}e^{-\frac{(x-\theta_1)^2}{2\theta_2^2}}$
	$N\left(\theta_1,\theta_2^2\right)$	$N_f=\{x:-\infty<x<+\infty\}$
		$\theta=(\theta_1,\theta_2)\in\Theta=(-\infty;+\infty)\times(0;+\infty)$
		$M\xi=\theta_1,\ D\xi=\theta_2^2$
2	Tekis taqsimot	$f(x,\theta)=\begin{cases}\frac{1}{\theta_2-\theta_1}, & \theta_1<x<\theta_2\\ 0, & \text{aks holda}\end{cases}$
	$R(\theta_1,\theta_2)$	$N_f=\{x:\theta_1<x<\theta_2\}$
		$\theta=(\theta_1,\theta_2)\in\Theta;\ -\infty<\theta_1<\theta_2<+\infty$
		$M\xi=\frac{\theta_1+\theta_2}{2},\ D\xi=\frac{(\theta_2-\theta_1)^2}{12}$
3	Ko'rsatkichli taqsimot	$f(x,\theta)=\begin{cases}\theta e^{-\theta x}, & x>0\\ 0, & x\le 0\end{cases}$
	$E(\theta)$	$N_f=\{x:x>0\}$
		$\theta\in\Theta=(0;+\infty)$
		$M\xi=1/\theta,\ D\xi=1/\theta^2$
4	Gamma taqsimot	$f(x,\theta)=\begin{cases}\frac{x^{\theta_2-1}e^{-x/\theta_1}}{\Gamma(\theta_2)\theta_1^{\theta_2}}, & x\ge 0\\ 0, & x<0\end{cases}$
	$\Gamma(\theta_1,\theta_2)$	$N_f=\{x:x>0\}$
		$\theta=(\theta_1,\theta_2)\in\Theta=(0;+\infty)\times(0;+\infty)$
		$M\xi=\theta_1\theta_2,\ D\xi=\theta_1^2\theta_2$
5	Lognormal taqsimot	$f(x,\theta)=\begin{cases}\frac{1}{\sqrt{2\pi}\theta_2 x}e^{-\frac{(\ln x-\theta_1)^2}{2\theta_2^2}}, & x>0\\ 0, & x\le 0\end{cases}$
	$LogN\left(\theta_1,\theta_2^2\right)$	$N_f=\{x:x>0\}$

		$\theta = (\theta_1, \theta_2) \in \Theta = (-\infty; +\infty) \times (0; +\infty)$
		$M\xi = e^{\theta_1 + \theta_2^2/2}, \ M\xi^2 = e^{2\theta_1 + 2\theta_2^2}$
6	Veybull taqsimoti	$f(x,\theta) = \begin{cases} \frac{\theta_1}{\theta_2}\left(\frac{x}{\theta_2}\right)^{\theta_1 - 1} e^{-\left(\frac{x}{\theta_2}\right)^{\theta_1}}, & x > 0, \\ 0, & x \le 0 \end{cases}$
	$W(\theta_1, \theta_2)$	$N_f = \{x : x > 0\}$
		$\theta = (\theta_1, \theta_2) \in \Theta = (0; +\infty) \times (0; +\infty)$
		$M\xi = \theta_2 \Gamma\left(1 + \frac{1}{\theta_1}\right), \ M\xi^2 = \theta_2^2 \Gamma\left(1 + \frac{1}{\theta_1}\right)$
7	Pareto taqsimoti	$f(x,\theta) = \begin{cases} \theta_2 e^{-\theta_2(x - \ln\theta_1)}, & x > \ln\theta_1 \\ 0 \quad , & x \le \ln\theta_1 \end{cases}$
	$P(\theta_1, \theta_2)$	$N_f = \{x : x > \ln\theta_1\}$
		$\theta = (\theta_1, \theta_2) \in \Theta = (0; +\infty) \times (0; +\infty)$
		$M\xi = \ln\theta_1 + 1/\theta_2, \ D\xi = 1/\theta_2^2$
8	Koshi taqsimoti	$f(x,\theta) = \frac{1}{\pi\theta_2} \frac{1}{1 + \left(\frac{x - \theta_1}{\theta_2}\right)^2}$
	$K(\theta_1, \theta_2)$	$N_f = \{x : -\infty < x < +\infty\}$
		$\theta = (\theta_1, \theta_2) \in \Theta = (-\infty; +\infty) \times (0; +\infty)$
		$M\xi$ mavjud emas
9	Puasson taqsimoti	$f(x,\theta) = \frac{e^{-\theta} \cdot \theta^x}{x!}$
	$\pi(\theta)$	$N_f = \{x : x = 0, 1, 2, ...\}$
		$\theta \in \Theta = (0; +\infty)$
		$M\xi = D\xi = \theta$

10	Geometrik taqsimot $Ge\,(\theta)$	$f\,(x,\theta)=\theta(1-\theta)^{x-1}$ $N_f=\{x:x=1,2,...\}$ $\theta\in\Theta=(0;1)$ $M\xi=1/\theta,\ \ D\xi=\theta/(1-\theta)$
11	Binomial taqsimot $Bi\,(n;\theta)$	$f\,(x,\theta)=C_n^x\theta^x(1-\theta)^{n-x}$ $N_f=\{x:x=0,1,2,...,n\}$ $\theta\in\Theta=(0;1)$ $M\xi=n\theta,\ \ D\xi=n\theta(1-\theta)$
12	Teskari binomial taqsimot $\overline{Bi}\,(r;\theta)$	$f\,(x,\theta)=C_{r+x-1}^x\theta^x(1-\theta)^r$ $N_f=\{x:x=0,1,2,...\}$ $\theta\in\Theta=(0;1)$ $M\xi=r\theta/(1-\theta),$ $D\xi=r\theta/(1-\theta)^2$

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MUNDARIJA

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MATEMATIK STATISTIKA

(PARAMETRLARNI BAHOLASH VA GIPOTEZALARNI TEKSHIRISH MISOLLARDA)

(O'QUV QO'LLANMA)

Muharrir: I.Mamatova

Bosishga ruxsat etildi: 11.04.2013 y. Bichimi $60 \times 84\,{}^{1}/_{16}$
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O'zMU bosmaxonasida bosildi.