28 – мавзу: Giperbaloid va uning xossalari. Parabaloid va uning xossalari. Ikkinchi tartibli sirtning to`g`ri chiziqli yasovchilari

Masala. To'g'ri burchakli dekart koordinatalar sistemasida $A(0,0,3)$ va $B(0,0,-3)$ nuqtalar berilgan. Fazoda shunday nuqtalarning geometrik o'rnini topingki, bu nuqtalar har biridan A va B nuqtalargacha bo'lgan masofalar ayirmasining absolyut qiymati 4 ga teng bo'lsin.

Yechish. Faraz qilaylik, aytilgan geometrik o'ringa tegishli nuqta $N(x, y, z)$ bo'lsin. U holda masala shartiga ko'ra

$$|\rho(A, N) - \rho(B, N)| = 4$$

buni koordinatalar yozsak

$$\left|\sqrt{x^2 + y^2 + (z-3)^2} - \sqrt{x^2 + y^2 + (z+3)^2}\right| = 4$$

yoki

$$\sqrt{x^2 + y^2 + (z-3)^2} = \sqrt{x^2 + y^2 + (z+3)^2} \pm 4$$

bundan

$$x^2 + y^2 + z^2 - 6z + 9 = x^2 + y^2 + z^2 + 6z + 9 \pm 8\sqrt{x^2 + y^2 + (z+3)^2} + 16$$

yoki

$$\mp\sqrt{x^2 + y^2 + (z+3)^2} = 16 + 2z$$

Yana bir marta kvadratga ko'tarib, soddalashtirsak, ikki pallali giperboloid tenglamasiga ega bo'lamiz: $-\frac{x^2}{5} - \frac{y^2}{5} + \frac{z^2}{4} = 1.$

Masala. $M_0(6, 2, 8)$ nuqtadan o'tuvchi

$$\frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{16} = 1$$

bir pallali giperboloidning to'g'ri chiziqli yasovchilarini toping.

Yechish. Bir pallali giperboloid uchun (36.3) tenglamalar sistemasini yozaylik.

$$\begin{cases} \lambda\left(\frac{x}{3} + \frac{z}{4}\right) = \mu\left(1 + \frac{y}{2}\right) \\ \mu\left(\frac{x}{3} - \frac{z}{4}\right) = \lambda\left(1 - \frac{y}{2}\right) \end{cases}$$

Bu tenglamalarga $x = 6$, $y = 2$, $z = 8$ qiymatlarni qo'yib topamiz: $2\lambda = \mu$. Bu tenglikni $\lambda = 1$, $\mu = 2$ sonlari qanoatlantiradi. Bu qiymatlarni sistemaga qo'yib topamiz:

$$\begin{cases} 4x - 12y + 3z - 24 = 0 \\ 4x + 3y - 3z - 6 = 0 \end{cases}$$

Bu tenglamalar sistemasi M_0 nuqtadan o'tuvchi to'g'ri chiziqli yasovchilarining bitta oilasini aniqlaydi. Shunga o'xshash ishlarni (36.4) tenglamalar sistemasi uchun bajarib, sirtning M_0 nuqtadan o'tuvchi to'g'ri chiziqli yasovchilar oilasiga ega bo'lamiz.

$$\begin{cases} 4x - 3z = 0 \\ y - 2 = 0 \end{cases}$$

Giperbolik paraboloidning kanonik

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z \qquad (36.5)$$

tenglamasini quyidagicha yozib olamiz:

$$\left(\frac{x}{a} + \frac{y}{b}\right)\left(\frac{x}{a} - \frac{y}{b}\right) = 2z .$$