22 – мавзу: Tekislikning berilish usullari. Tekislikning umumiy tenglamasi. Ax+By+C va Ax+By+Cz+D ko`phadlar ishorasining geometrik ma'nosi.

1-misol. $M_0(2,0,3)$ nuqtadan o'tib, $\vec{a}(1,0,1)$, $\vec{b}(2,1,3)$ vektorlarga parallel tekislikning parametrik va umumiy tenglamasini tuzing.

Yechish. Berilgan $x_0=2$, $y_0=0$, $z_0=3$. $a_1=1$, $a_2=1$, $a_3=1$, $b_1=2$, $b_1=1$, $b_3=3$ qiymatlarni (11.3) parametrik tenglamaga qo'yib topamiz.

$$x=2+u+2\vartheta,$$
$$y=\vartheta,$$
$$z=3+u+3\vartheta.$$

Yuqoridagi ko'rsatilgan M_0 va $\vec{a}$, $\vec{b}$ vektor koordinatalarini (11.1) tenglamaga qo'yib topamiz.

$$\begin{vmatrix} x-1 & y_0 & z-3 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix}=0$$

Bundan tekislikning umumiy tenglamasi

$x+y-z+1=0$.

2-misol. $M_0(1,2,-3)$ nuqtadan o'tib (xoy) tekislikka parallel tekislik tenglamasini tuzing.

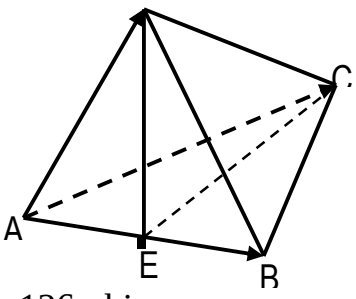
Yechish. Izlangan tekislikning ixtiyoriy nuqtasi $N(x,y,z)$ bo'lsin.

$\overline{M}_0\vec{N}(x-1,y-2,z+3)$, $\vec{e}_1(1,0,0)$, $\vec{e}_2(0,1,0)$ vektorlar komplanar bo'ladi, ya'ni

$$\begin{vmatrix} x-1 & y-2 & z+3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}=0,$$

bundan $z+3=0$-izlangan tekislik tenglamasi kelib chiqadi.

3-misol. *ABCD* tetraedr tasviri berilgan. *A* uchni koordinatalar boshi hamda $\vec{e}_1=\overline{A}\vec{B}$, $\vec{e}_2=\overline{A}\vec{C}$, $\vec{e}_3=\overline{A}\vec{D}$ deb olib, *BDC* va *EDC* tekisliklar tenglamalarini tuzing (bunda *E* nuqta *AB* tomonining o'rta nuqtasi (126-chizma)).



126-chizma

Yechish. Berilishiga ko'ra affin koordinatalar sistemasi $(A,\vec{e}_1,\vec{e}_2,\vec{e}_3)$ dan iborat. Bu koordinatalar sistemasiga nisbatan

$A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$, $D(0,0,1)$, $E\left(\frac{1}{2},0,0\right)$

koordinatalarga ega. U holda BDC tekislik tenglamasini (11.9) tenglamadan foydalanib yozamiz.

$$\begin{vmatrix} x-1 & y & z \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix} = 0,\ \text{bundan}\ x+y+z-1=0.$$

Shunga o'xshash EDC tekislik tenglamasi $2x+y+z-1=0$.

4-misol. $4x-2y+3z-7=0$ tekislik uchlari $A(4,-3,2)$, $B(3,-1,7)$, $C(-2,1,5)$ uchburchak tomonlarining qaysi birini kesadi?

Yechish. Ushbuni hisoblaymiz.

$\delta_A = 4\cdot 4-2\cdot(-3)+3\cdot 2-7=28>0$,

$\delta_B = 4\cdot 3-2\cdot(-1)+3\cdot 7-7=35>0$,

$\delta_C = 4\cdot(-2)-2\cdot 1+3\cdot(-5)-7=-32<0$.

A va B nuqtalar T tekislikning bir tomonida yotadi. C nuqta T tekislikning boshqa tomonida yotadi.

Demak, T tekislik uchburchakning AC va BC tomonlarini kesadi, AB tomonni kesmaydi.

Misollar:

1. Berilgan ikkita (x_1,y_1,z_1) va (x_2,y_2,z_2) nuqtalar biror tekislikka nisbatan simmetrik bo'lsa, bu tekislik tenglamasini tuzing.

2. $ax+by+cz+d_1=0$ va $ax+by+cz+d_2=0$ $(d_1\neq d_2)$ tekisliklarning parallel ekanini ko'rsating.

3. $(ax+by+cz+d)^2-(\alpha x+\beta y+\gamma z+\delta)^2=0$ tenglamani qanoatlantiruvchi nuqtalar to'plamining geometrik o'rni nimadan iborat.

4. $x^2+y^2+z^2+ax+by+cz+d=0$ va $x^2+y^2+z^2+\alpha x+\beta y+\gamma z+\delta=0$

Ikkita sferaning kesishishidan hosil bo'lgan aylanadan o'tadigan tekislik tenglamasini tuzing.[1]

1 топшириқ

M_0 нуқтадан M_1,M_2,M_3 нуқталардан ўтувчи текисликкача бўлган масофани ҳисобланг

[1] Csaba Vincze and Laszlo Kozma 'College Geometry' March 27, 2014 pp 215-225

1. $M_1(0,7,-4)$,	$M_2(4,8,-1)$,	$M_3(-2,1,3)$,	$M_0(-9,10,2)$.
2. $M_1(5,8,3)$,	$M_2(10,5,6)$,	$M_3(8,7,4)$,	$M_0(7,0,1)$.
3. $M_1(1,3,5)$,	$M_2(-5,5,2)$,	$M_3(7,-1,8)$,	$M_0(-3,4,3)$.
4. $M_1(0,-2,-1)$,	$M_2(-3,-1,2)$,	$M_3(1,0,-2)$,	$M_0(-3,3,1)$.
5. $M_1(2,3,1)$,	$M_2(2,0,3)$,	$M_3(1,2,0)$,	$M_0(3,0,5)$.

2 топшириқ

M_0 нуқтадан ўтиб, M_1M_2 векторга перпендикуляр бўлган текислик тенгламасини тузинг.

1. $M_0(3,2,0)$,	$M_1(4,1,5)$,	$M_2(2,-1,4)$.
2. $M_0(-5,-1,0)$,	$M_1(-5,1,-4)$,	$M_2(-2,2,-3)$.
3. $M_0(2,-4,-2)$,	$M_1(-1,-3,-7)$,	$M_2(-4,-1,-5)$.
4. $M_0(-5,3,10)$,	$M_1(0,5,7)$,	$M_2(2,7,8)$.
5. $M_0(2,-10,-4)$,	$M_1(0,-6,-8)$,	$M_2(-2,-5,-9)$.

3 топшириқ

Қуйидаги тенгламалар билан берилган текисликлар орасидаги бурчакни ҳисобланг

1. $3x - y + 3 = 0$,	$x - 2y + 5z - 10 = 0$
2. $x - y + 3z - 5 = 0$,	$x + z - 2 = 0$
3. $5x - 4y + 3z - 3 = 0$,	$4x - y - z + 2 = 0$
4. $6x + 2y - 4z + 17 = 0$,	$3x + 3y - 3z - 8 = 0$
5. $6x + 2y - 4z + 17 = 0$,	$9x + 3y - 6z - 4 = 0$