3-misol. To'g'ri burchakli koordinatalar tenglamalar sistemasiga nisbatan

$$5x^2+2y^2+5z^2-4xy-2xz-4yz+10x-4y-2z+4=0$$

tenglama bilan berilgan sirtning ko'rinishi va joylashishi aniqlansin.

Yechish.

$$I_3=\begin{vmatrix}5&-2&-1\\-2&2&-2\\-1&-2&5\end{vmatrix}=0,$$

$$K_4=\begin{vmatrix}5&-2&-1&5\\-2&2&-2&-2\\-1&-2&5&-1\\5&-2&-1&4\end{vmatrix}=0,$$

$$I_2=\begin{vmatrix}5&-2\\-2&2\end{vmatrix}+\begin{vmatrix}5&-1\\-1&5\end{vmatrix}+\begin{vmatrix}2&-2\\-2&5\end{vmatrix}=36,$$

$$K_3=\begin{vmatrix}5&-2&5\\-2&2&-2\\5&-2&4\end{vmatrix}+\begin{vmatrix}5&-1&5\\-1&5&-1\\5&-1&4\end{vmatrix}+\begin{vmatrix}2&-2&-2\\-2&5&-1\\-2&-1&4\end{vmatrix}=-36,$$

$$I_1=5+2+5=12.$$

$$I_3=K_4=0, I_2>0, I_1K_3<0$$

bo'lgani uchun berilgan tenglama elliptik silindrni aniqlaydi.

Xarakteristik $\lambda^3-12\lambda^2+36\lambda=0$ tenglama ildizlari: $\lambda_1=\lambda_2=6, \lambda_3=0$

Sodda tenglamasi

$$6x^2+6y^2-\frac{36}{36}=0$$

yoki

$$x^2+y^2=\frac{1}{6}$$

ko'rinishga ega.

Bu tenglama radiusi $\frac{1}{\sqrt{6}}$ ga teng aylanma silindrni aniqlaydi.

Silindrning o'qi ushbu

$$5x-2y-z+5=0,$$
$$-2x+2y-2z-2=0,$$
$$-x-2y+5z-1=0$$

tenglamalar sistemasidan topiladi, ammo bu sistemadagi ikkita tenglamani olish kifoya.

4-misol. Koordinatalarning to'g'ri burchakli sistemasida

$$x^2+y^2+4z^2+2xy+4xz+4yz-6z+1=0$$

tenglama bilan berilgan sirtning ko'rinishi va joylashishi aniqlansin.

Yechish.

$$I_3=\begin{vmatrix}1&1&2\\1&1&2\\2&2&4\end{vmatrix}=0$$

$$K_4=\begin{vmatrix}1&1&2&0\\1&1&2&0\\2&2&4&-3\\0&0&-3&1\end{vmatrix}=0$$

$$I_2=\begin{vmatrix}1&1\\1&1\end{vmatrix}+\begin{vmatrix}1&2\\2&4\end{vmatrix}+\begin{vmatrix}1&2\\2&4\end{vmatrix}=0$$

$$K_3=\begin{vmatrix}1&1&0\\1&1&0\\0&0&1\end{vmatrix}+\begin{vmatrix}1&2&0\\2&4&-3\\0&-3&1\end{vmatrix}+\begin{vmatrix}1&2&0\\2&4&-3\\0&-3&1\end{vmatrix}=-18$$

$$I_1=1+1+4=6$$

Berilgan tenglama parabolik silindrni aniqlaydi.

Sodda tenglamasi:

$$6x^2-2\sqrt{+\frac{18}{6}}y=0$$

yoki

$$x^2=\frac{y}{\sqrt{3}}.$$

Silindr yasovchilariga perpendikular kesimining parametri

$$P=\frac{1}{2\sqrt{3}}.$$

Silindrning vaziyatini aniqlash uchun uning tenglamasini

$$(x+y+2z+m)^2-[2mx+2my+2(2m+3)z+1=0]$$

ko'rinishda yozib olamiz.

m sonini ikkita tekislikning perpendikularlik shartidan foydalanib topamiz:

$$\begin{aligned} &x+y+2z+m=0,\\ &2mx+2my+2(2m+3)z+1=0,\\ &1m+1m+2(2m+3)=0. \end{aligned}$$

bu yerdan

$$m=-1$$

Shunday qilib silindr yasovchilariga parallel bo'lgan simmetriya tekisligining tenglamasi:

$$x+y+2z-1=0$$

bu tekislikka perpendikular urinma tekislik tenglamasi:

$$-2x-2y+2z+1=0$$

Bu yerdan esa, shu urinma tekislikka perpendikular va silindrning botiqlik tomoniga yo'nalgan

$$\{-2,-2,2\}\downarrow\downarrow\{-1,-1,1\}$$

vektorni topamiz.

5-misol. To'g'ri burchakli koordinatalar sistemasida ushbu

$$y^2+2xy+4xz+2yz-4x-2y=0$$

tenglama bilan berilgan sirtning ko'rinishi va joylashishi aniqlansin.

Quyidagi sirtlarning kanonik tenglamasi va joylashishini aniqlansin.

1754. $x^2+5y^2+z^2+2xy+6xz+2yz-2x+6y+2z=0$.

1755. $2x^2+y^2+2z^2-2xy+2yz+4x-2y=0$.

1756. $x^2+y^2+4z^2+2xy+4xz+4yz-6z+1=0$.

1757. $4x^2+9y^2+z^2-12xy-6yz+4zx+4x-6y+2z-5=0$.

1758. $7x^2+6y^2+5z^2-4xy-4yz-6x-24y+18z+30=0$.

1759. $2x^2+2y^2-5z^2+2xy-2x-4y-4z+2=0$.

1760. $x^2-2y^2+z^2+4xy-8xz-4yz-14x-4y+14z+16=0$.

1761. $2x^2+2y^2+3z^2+4xy+2xz+2yz-4x+6y-2z+3=0$.

1762. $2x^2+5y^2+2z^2-2xy+2yz-4xz+2x-10y-2z-1=0$.

1763. *1)* $x^2+5y^2+z^2+2xy+6xz+2yz-2x+6y+2z=0$;

2) $5x^2+2y^2+5z^2-4xy-2xy-4yz+10x-4y-2z+4=0$;

3) $x^2-2y^2+z^2+4xy-10xz+4yz+2x+4y-10z-1=0$.

1764. $5x^2-y^2+z^2+4xy+6xz+2x+4y+6z-8=0$.

1765. $2x^2+10y^2-2z^2+12xy+8yz+12x+4y+8z-1=0$.

M i s o l. $3x^2+2xy+3y+6x-2y-5=0$ chiziqning o'qlarini toping.

Avvalo berilgan chiziq markazli yoki markazsiz ekanini tekshiramiz. Buning uchun

$$\delta=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

ni tuzamiz. Berilgan chiziq tenglamasidan $a_{11}=3$, $a_{12}=1$, $a_{22}=3$, $a_{20}=-1$, $a_{00}=-5$ bo'lib, $\delta=\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}=9-1=8\neq 0.$

Demak, chiziq markazli, u holda uning o'qi (63.13) ga ko'ra

$$(3x+y+3)+k(x+3y-1)=0$$

tenglamadan aniqlanadi. Tenglamadagi k ushbu k_1, $k_2=-1$ ni qo'shish bilan berilgan chiziq o'qlarining $2x+2y+1=0$, $x-y+2$ tenglamalari hosil bo'ldi.