19-mavzu:

1 – m i s o l. Chiziqning ushbu $x^2+6xy+y^2+6x+2y-1=0$ tenglamasini kanonik ko'rinishga keltirib, chizmasini yasang.

Ye ch i sh. Bu yerda: $a_{11}=1$, $a_{12}=3$, $a_{22}=1$, $a_{10}=3$, $a_{20}=1$, $a_{00}=-1$. $a_{12}=3\neq 0$; berilgan tenglamani kanonik holda yozish uchun quyidagi ishlarni bajaramiz:

1) xarakteristik tenglamani tuzamiz: $\lambda_2-2\lambda-8=0$, $\lambda_{1,2}=1\pm\sqrt{1+8}=1\pm 3\Rightarrow \lambda_1=4, \lambda_2=-2$;

2) $\operatorname{tg}\alpha_1=\dfrac{\lambda_1-a_{11}}{a_{12}}=\dfrac{4-1}{3}=1\Rightarrow \alpha=45^\circ$,

$\sin\alpha_1=\dfrac{tg\alpha_1}{\sqrt{1+tg^2\alpha_1}}=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$, $\cos\alpha_1=\dfrac{1}{\sqrt{1+tg^2\alpha_1}}=\dfrac{\sqrt{2}}{2}$.

3) $(O,\vec{i},\vec{j})$ reperni $\alpha_1=45^0$ burchakka burishdan $(O,\vec{i}\,`,\vec{j}\,`)$ reper hosil bo'ladi, uning koordinata vektorlari:

$$\vec{i}\,`=\frac{\sqrt{2}}{2}\vec{i}+\frac{\sqrt{2}}{2}\vec{j},\quad \vec{j}\,`=-\frac{\sqrt{2}}{2}\vec{i}+\frac{\sqrt{2}}{2}\vec{j}.$$

1) $a`_{10}=a_{10}\cos\alpha_1+a_{20}\sin\alpha_1$, $a`_{20}=-a_{10}\sin\alpha_1+a_{20}\cos\alpha_1$ formulalar bo'yicha $a`_{10}$, $a`_{20}$ koeffitsiyentlarni topamiz:

$$a`_{10}=\frac{3\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=2\sqrt{2},\quad a`_{20}=-\frac{3\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}.$$

B` reperda chiziqning tenglamasi:

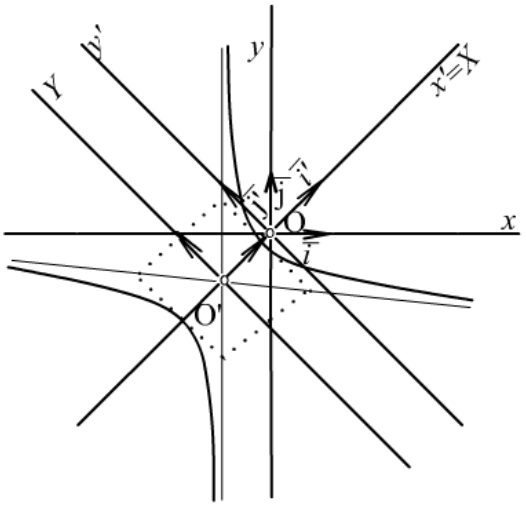
$$4x`^2-2y`^2+4\sqrt{2}x`-2\sqrt{2}y`-1=0$$

2) Bu tenglamani koordinatalar boshi O ni ko'chirish bilan soddalashtiramiz. Buning uchun tenglamaning chap tomonidagi hadlardan $x`$, $y`$ ga nisbatan to'la kvadratlar ajratamiz;

$$\left(4x`^2+\frac{4\sqrt{2}}{4}x`+\left(\frac{\sqrt{2}}{2}\right)^2-\left(\frac{\sqrt{2}}{2}\right)^2\right)-2\left(y`^2+\frac{2\sqrt{2}}{2}y`+\left(\frac{\sqrt{2}}{2}\right)^2-\left(\frac{\sqrt{2}}{2}\right)^2\right)-1=0,$$

$$4\left(x`^2+\sqrt{2}x`+\left(\frac{\sqrt{2}}{2}\right)^2\right)-2\left(y`^2+\sqrt{2}y`+\left(\frac{\sqrt{2}}{2}\right)^2\right)-2+1\cdot$$

$$\begin{cases}X=x`+\dfrac{\sqrt{2}}{2}\\ Y=y`+\dfrac{\sqrt{2}}{2}\end{cases}\Rightarrow\begin{cases}x`=X+(-\dfrac{\sqrt{2}}{2}),\\ y`=Y+(-\dfrac{\sqrt{2}}{2}).\end{cases}$$



105-chizma

Chiziqning tenglamasi kanonik ko’rinishga keladi:

$4X^2-2Y^2=2$ yoki $\frac{4X^2}{2}-\frac{2Y^2}{2}=1\Rightarrow\frac{X^2}{\frac{1}{2}}-\frac{Y^2}{1}=1.$

Bu yerda $a=\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2}$, $b=1;$ giperbolaning kanonik tenglamasi hosil qilinadi. 105 – chizmada bu giperbola yasalgan.

2 – m i s o l. $4x^2-4xy+y^2-2x-14y+7=0.$

Ye ch i sh. Bu yerda: $a_{11}=4$, $a_{12}=-2$, $a_{22}=1$, $a_{10}=-1$,

$a_{20}=-7$, $a_{00}=7$.

1) xarakteristik tenglama $\lambda^2-5\lambda=0$, ildizlari:

$\lambda_1=0$, $\lambda_2=5$;

2) $\operatorname{tg}\alpha_1=\frac{\lambda_1-a_{11}}{a_{12}}=\frac{-4}{-2}=2\Rightarrow\sin\alpha_1=\frac{2}{\sqrt{5}},\cos\alpha_1=\frac{1}{\sqrt{5}};$

3) $(O,\vec{i},\vec{j})$ reperni $\operatorname{tg}\alpha_1=2$ dan aniqlanadigan α_1 burchakka burishdan hosil bo’ladigan $(O,\vec{i}`,\vec{j}`)$ reperning koordinata vektorlari:

$$\vec{i}`=\frac{\sqrt{5}}{5}\vec{i}+\frac{2\sqrt{5}}{5}\vec{j},\quad \vec{j}`=-\frac{2\sqrt{5}}{5}\vec{i}+\frac{\sqrt{5}}{5}\vec{j};$$

4) $a`_{10}=-3\sqrt{5}$, $a_{20}=-\sqrt{5}$. B` reperda chiziqning tenglamasi:

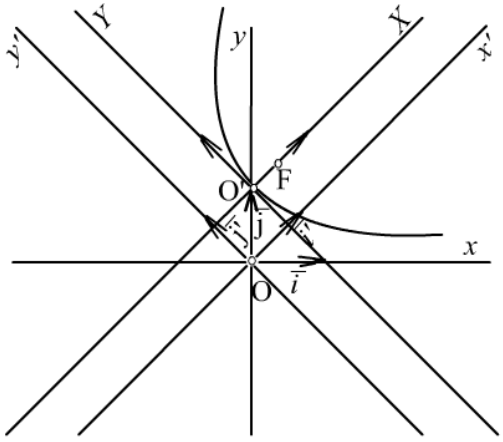
$5y`^2-6\sqrt{5}x`-2\sqrt{5}y`+7=0;$

5) endi koordinatalar boshini ko'chiramiz. Bu tenglamaning chap tomonidagi

$$5\left(y`-\frac{\sqrt{5}}{5}\right)^2-6\sqrt{5}\left(x`-\frac{\sqrt{2}}{2}\right)=0$$

hadlardan $y`$ ga nisbatan to'la kvadrat ajratamiz: $\begin{cases} X=x`-\frac{\sqrt{5}}{5} \\ Y=y`-\frac{\sqrt{5}}{5} \end{cases} \Rightarrow \begin{cases} x`=X+\frac{\sqrt{5}}{5}, \\ y`=Y+\frac{\sqrt{5}}{5}. \end{cases}$

Chiziqning O ni $O'(\frac{\sqrt{5}}{5},\frac{\sqrt{5}}{5})$ nuqtaga ko'chirishdan hosil bo'lgan $(O`,\vec{i}`,\vec{j}`)$ reperdagi tenglamasi: $5Y^2-6\sqrt{5}X=0$ yoki $Y^2=\frac{6\sqrt{5}}{5}X.$



Bu tenglama 106-chizmada tasvirlangan parabolani ifodalaydi.

106-chizma

3 – m i s o l. *$9x^2+16y^2-24xy+30x-40y-25=0$*

Ye ch i sh. Bu yerda: $a_{11}=9$, $a_{12}=-12$, $a_{22}=16$, $a_{10}=15$, $a_{20}=-20$, $a_{00}=-25$

1) chiziqning xarakteristik tenglamasi:
$\lambda^2-25\lambda=0\Rightarrow\lambda_1=25,\ \lambda_2=0;$

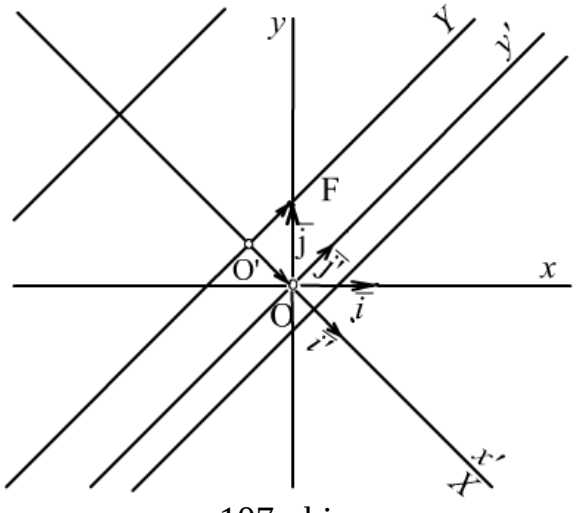
2) $\text{tg}\alpha_1=-\frac{4}{3}\Rightarrow\sin\alpha_1=-\frac{4}{5},\cos\alpha_1=\frac{3}{5};$

3) $(O,\vec{i},\vec{j})$ reperning koordinata vektorlari, $\vec{i}`=\frac{3}{5}\vec{i}-\frac{4}{5}\vec{j},\quad \vec{j}`=\frac{4}{5}\vec{i}+\frac{3}{5}\vec{j};$

4) $a`_{10}=25$, $a`_{10}=0$ chiziqning tenglamasi $x`^2+2x`-1=0$ ko'rinishda bo'ladi. Bundan $(x`+1)^2-2=0$

5) koordinatalar boshi O' ni $\begin{cases} x`=X-1, \\ y`=Y \end{cases}$ formulalar bo'yicha

$O`(-1,0)$ nuqtaga ko'chirsak, chiziq tenglamasi $X^2-2=0$ ko'rinishini oladi. Bu tenglama ordinatalar o'qiga parallel ikki



107-chizma

to’g’ri chiziqni aniqlaydi (107 –chizma)

Savollar va masalalar.

1. Elliptik paraboloidni ta’riflang.
2. Elliptik paraboloidning xossalarini ayting va shaklini yasang.
3. Giperbolik paraboloidning ta’rifini ayting.
4. Bu sirt qanday xossalarga ega?
5. Qanday sirtlarni to’g’ri chiziqli sirtlar deyiladi?
6. Bir pallali giperboloidning to’g’ri chiziqli yasovchilarini ayting.
7. Ular qanday xossalarga ega?
8. Giperbolik paraboloidning to’g’ri chiziqli yasovchilari haqida nimalarni bilasiz?
9. Giperbolik paraboloid qanday xossalarga ega?
10. 1) Uchi koordinatalar boshida, o’qi $o\,y$ o’q bilan ustma-ust tushgan $A_1(1,-2,1)$ va $A_2(-3,-3,2)$ nuqtalardan o’tuvchi;

    2) Uchi koordinatalar boshida, o’qi $o\,x$ dan iborat. $N_1(1,2,1)$ va $N_2(2,4,0)$ nuqtalardan o’tuvchi paraboloid tenglamasini tuzing.
11. Giperbolik paraboloidda $4x^2-z^2=y$ yotuvchi $M(1,3,-1)$ nuqtadan o’tadigan to’g’ri chiziqli yasovchilarning tenglamasini tuzing.

    Javob: $\frac{x}{1}=\frac{y+1}{4}=\frac{z-1}{-2}$; $\frac{x}{1}=\frac{y+9}{12}=\frac{z+3}{2}$.
12. $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ bir pallali giperboloidning $6x+4y+3z-17=0$ tekislikka parallel to’g’ri chiziqli yasovchilarini tenglamasini yozing.

    Javob: $\frac{x}{1}=\frac{y-3}{0}=\frac{z}{-2}$, $\frac{x-2}{0}=\frac{y}{3}=\frac{z}{-4}$.
13. $\frac{x^2}{72}+\frac{z^2}{4}=z$ paraboloidning $x-y-2z=0$ tekislikka parallel bo’lgan urinma tekisligini toping.

    Javob: $x-y-2z-2=0$
14. $\frac{x^2}{1}-\frac{y^2}{4}=z$ giperbolik paraboloid va uning urinma tekisligidan biri $10x-2y-2z-21=0$ berilgan. Urinma tekislik bilan sirtning kesishishidan hosil bo’lgan har ikkala to’g’ri chiziqning tenglamalarini toping.

    Javob: $\frac{x-5}{1}=\frac{y-4}{2}=\frac{z-21}{6}$ va $\frac{x-5}{1}=\frac{y-4}{-2}=\frac{z-21}{14}$.