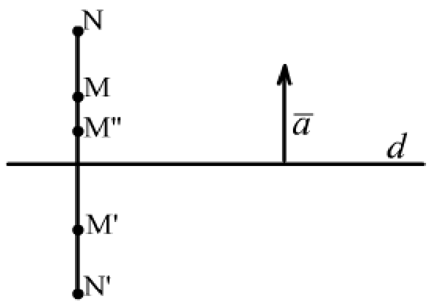
# 11-mavzu: Darsda yechiladigan misollar



54-chizma

**1-misol.** Agar $E$ - ayniy almashtirish bo'lsa, u holda $f\,E = E\,f = f$, $E(M) = M$, va $f(M)=M'$, u holda $M \xrightarrow{f\cdot E} M'$ va $M \xrightarrow{E\cdot f} M'$

**2-misol.** Agar $f_2 = f_1^{-1}$ bo'lsa, u holda har bir M nuqta uchun $f_1 f_1^{-1}$ kompozitsiya ayniy almashtirish bo'ladi.

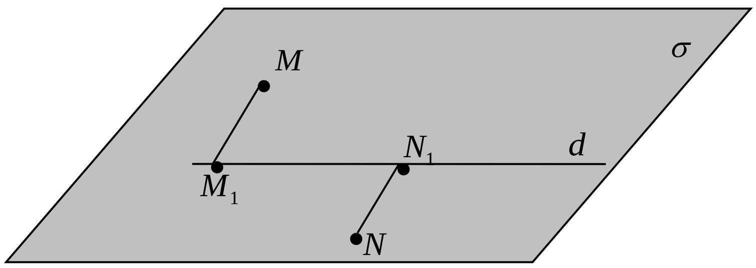
**3-misol.** $f_1$- $d$ to'g'ri chiziqqa nisbatan simmetrik almashtirish $f_2$ - $d$ to'g'ri chiziqqa perpendikulyar $\vec{a}$ vektor qadar parallel ko'chirish (54-chizma) bo'lsin. $f_2 \cdot f_1 \neq f_1 \cdot f_2$ bo'lishini isbotlang.

**4-misol**. Shaxmat donalari $X$ to'plam va shaxmat taxtasidagi kataklari $Y$ to'plam berilgan bo'lsin. Shaxmat donalarini taxtaga terish $f_1$ bilan $X$ to'plamni $Y$ to'plamga akslantirish o'rnatiladi,
ya'ni $f_1: X \rightarrow Y$.

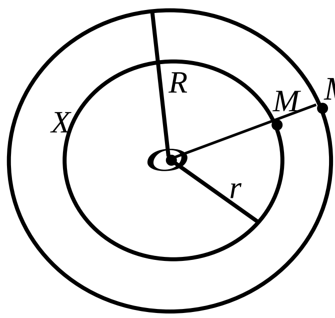
$f: X \rightarrow Y$ akslantirishning muhim xususiy hollari bilan tanishamiz.

1. Agar ixtiyori $x_1, x_2 \in X$ elementlar uchun $x_1 \neq x_2 \rightarrow f(x_1) \neq f(x_2)$ bo'lsa, u holda $X$ to'plamni $Y$ to'plam ichiga akslantirish yoki in'ektsiya deyiladi.

**5-misol.** Yarim aylanani $X$ to'plam deb, yarim aylana diametri orqali o'tuvchi to'g'ri chiziqni $Y$ to'plam deb olaylik (48-chizma). $f_2$-qoida deb $X$ to'plam nuqtalarini $Y$ to'plam nuqtalariga ortogonal proektsiyalarini olsak $X$ to'plam $Y$



49-chizma



50-chizma

to'plam ichiga bir qiymatli akslanadi.

2. Agar $f$ akslantirishda obrazlar to'plami $Y$ to'plamdan iborat bo'lsa, ya'ni $f(X)=Y$ bo'lsa, u holda $f: X \rightarrow Y$ akslantirish $X$ to'plamni $Y$ to'plam ustiga akslantirish yoki *syur'ektsiya* deyiladi.

Ya'ni $f$ akslantirishda $Y$ to'plamning har bir $y$ elementi $X$ to'plamning biror $x$ elementining aksi (obrazi) bo'lsa $f$ akslantirishni $X$ to'plamni $Y$ to'plam ustiga akslantirish yoki *syur'ektsiya* deyiladi.

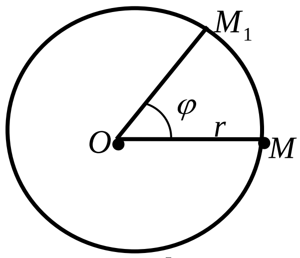
**6-misol.** $\sigma$- tekislikda *d* to'g'ri chiziq berilgan. Tekislikning har bir *M* nuqtasiga uning *d* to'g'ri chiziqdagi ortogonal proektsiyasi $M_1$ nuqtani mos qo'yamiz. Natijada $f_3:\sigma\rightarrow d$ akslantirishga ega bo'lamiz. $f_3$ akslantirish *syurektsiya* bo'ladi, chunki *d* to'g'ri chiziqning har bir nuqtasi proobrazga (asliga) ega (49-chizma).

3. Agar $f:X \to Y$ akslantirish bir vaqtda ham *inektiv* ham *syurektiv* bo'lsa, u holda *f* akslantirishni *o'zaro bir qiymatli* akslantirish yoki *biektiv* akslantirish deyiladi.

**7-misol.** Tekislikda *O* markazli, *r* va *R* radiusli ikkita konsentrik aylanalar berilgan bo'lsin. (50-chizma). *r* radiusli aylananing nuqtalar to'plamini *X*, *R* radiusli aylananing nuqtalar to'plami *Y* bo'lsin.

$f_1$ qoida sifatida *O* nuqtadan chiquvchi nurlarni olaylik. *X* to'plamning har bir *M* nuqtasi *Y* to'plamning *OM* nurida yotuvchi $M_1$ nuqtasiga mos keladi. Natijada $f:X \to Y$ akslantirishga ega bo'lamiz. Bu akslantirish o'zaro bir qiymatli akslantirish bo'ladi.

$f:X \rightarrow Y$ biektiv akslantirish bo'lsin.



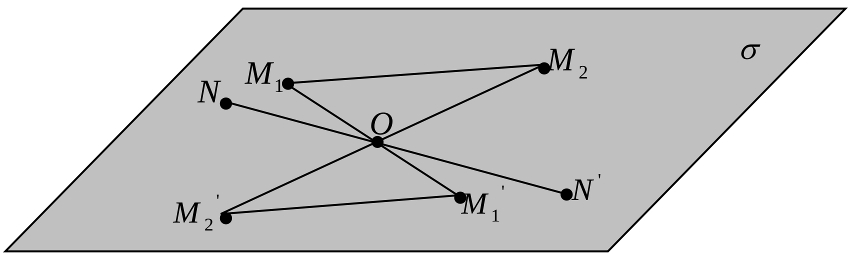

51-chizma

**8-misol**. Yo'nalishli $\sigma$ tekislikda *S(0,r)* aylana berilgan bo'lsin. $\varphi$ - yo'nalishli burchak $-\pi<\varphi<\pi$, $f:S\rightarrow S$ aylanani o'z-o'ziga akslantirishni olaylik.

*f* akslantirish *O* nuqta atrofida $\varphi$ burchakka burishdan iborat, bunda har bir *M* nuqtani *O* nuqta atrofida $\angle MOM_1 = \varphi$ burchakka burib $M_1$ nuqtaga mos qo'yiladi.

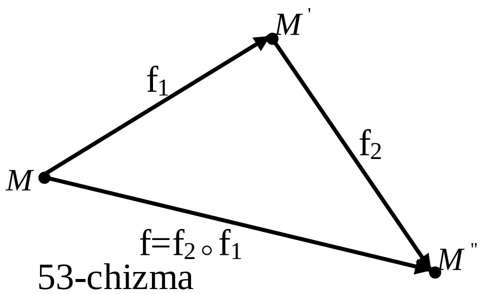
**1-masala.** $\sigma$ tekislik nuqtalarini shu tekislik nuqtalariga almashtiraylik.

Tekislikda *O* nuqta berilgan bo'lsin. Tekislikning har bir *M* nuqtasini *O* nuqtaga nisbatan simmetrik $M^1$ nuqta topiladi. Shunday qilib $f:\sigma\rightarrow\sigma$ almashtirishga ega bo'lamiz. (52-chizma).




52-chizma

3. Tekislikdagi barcha almashtirishlar to'plamini $G$ bilan belgilaylik. Bu to'plamga qarashli ixtiyoriy ikkita $f_1, f_2 \in G$ almashtirishlarni olaylik. Bunda $f_1$ almashtirish M nuqtani $f_1(M)=M'$ nuqtaga, $f_2$ almashtirish $M'$ nuqtani $f_2(M')=M''$ nuqtaga o'tkazsa (53-chizma), u holda $f_1$ va $f_2$ almashtirishlar M ni M'' o'tkazuvchi yangi bir $f(M)=M''$ almashtirishni hosil qiladi.



53-chizma

**9-misol**. Yo'nalishli $\sigma$ tekislikda $S(0,r)$ aylana berilgan bo'lsin. $\varphi$ - yo'nalishli burchak $-\pi<\varphi<\pi$, $f: S \to S$ aylanani o'z-o'ziga akslantirishni olaylik.

$f$ akslantirish $O$ nuqta atrofida $\varphi$ burchakka burishdan iborat, bunda har bir $M$ nuqtani $O$ nuqta atrofida $\angle MOM_1 = \varphi$ burchakka burib $M_1$ nuqtaga mos qo'yiladi.

**Examples 2.5**

i) The function $f:\mathbb{R}\to\mathbb{R}$, $f(x)=ax+b$ is one-to-one for all $a\neq 0$ (in fact, $f(x_1)=f(x_2) \Rightarrow ax_1=ax_2 \Rightarrow x_1=x_2$). Its inverse is $x=f^{-1}(y)=\frac{y-b}{a}$, or $y=f^{-1}(x)=\frac{x-b}{a}$.

ii) The map $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x^2$ is not one-to-one because $f(x)=f(-x)$ for any real $x$. Yet if we consider only values $\geq 0$ for the independent variable, i.e., if we **restrict** $f$ to the interval $[0,+\infty)$, then the function becomes 1-1 (in fact, $f(x_1)=f(x_2) \Rightarrow x_1^2-x_2^2=(x_1-x_2)(x_1+x_2)=0 \Rightarrow x_1=x_2$). The inverse function $x=f^{-1}(y)=\sqrt{y}$ is also defined on $[0,+\infty)$. Conventionally one says that the 'squaring' map $y=x^2$ has the function 'square root' $y=\sqrt{x}$ for inverse (on $[0,+\infty)$). Notice that the restriction of $f$ to the interval $(-\infty,0]$ is 1-1, too; the inverse in this case is $y=-\sqrt{x}$.

iii) The map $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x^3$ is one-to-one. In fact $f(x_1)=f(x_2) \Rightarrow x_1^3-x_2^3=(x_1-x_2)(x_1^2+x_1x_2+x_2^2)=0 \Rightarrow x_1=x_2$ since $x_1^2+x_1x_2+x_2^2=\frac{1}{2}[x_1^2+x_2^2+(x_1+x_2)^2]>0$ for any $x_1\neq x_2$. The inverse function is the 'cubic root' $y=\sqrt[3]{x}$, defined on all $\mathbb{R}$. □