5-mavzu: Darsda yechiladigan misollar

1-misol. Ikkita $(0,\ \vec{e}_1,\ \vec{e}_2)$ va $(0,\ \vec{e}_1',\ \vec{e}_2')$ affin reperlar berilgan bo'lib, bunda $o'(1,\ 2),\ \vec{e}_1(-1,\ 1),\ \vec{e}_2(2,\ -1)$ bo'lsin. *N* nuqtaning eski reperga nisbatan koordinatalari *x= 2, y=1* ekanligi ma'lumligini bilgan holda bu nuqtaning yangi reperga nisbatan *x', y'* koordinatalarini toping.

Yechish Berilgan: $c_{11}=-1, c_{21}=1, c_{12}=2, c_{22}=-1, x_0=1, y_0=2.$ Bu qiymatlarni (6.4) ga qo'yib quyidagilarga ega bo'lamiz.

$$x=-x'+2y'+1 \qquad \begin{cases} -x'+2y'=1 \\ x'-y'=-1 \end{cases}$$

$$y=x'-y'+2$$

bu sistemani yechib $x'=2, y'=0.$

Yangi sistemada *N* nuqtaning koordinatalari $x'=2, y'=0.$

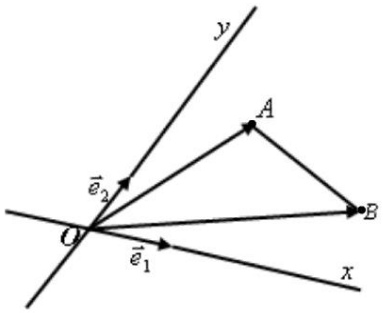
1-masala. *AB* vektorning boshi $A(x_1, y_1)$ va oxiri $B(x_2, y_2)$ koordinatalari bilan berilgan bo'lsa, $\overrightarrow{AB}$ vektor koordinatasini toping.(18-chizma)

Yechish:

$\overrightarrow{OA}=x_1\vec{e}_1+y_1\vec{e}_2$

$\overrightarrow{OB}=x_2\vec{e}_1+y_2\vec{e}_2 \quad \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(x_2-x_1)\vec{e}_1-(y_2-y_1)\vec{e}_2$

bundan $\overrightarrow{AB}(x_2-x_1;\ y_2-y_1)$



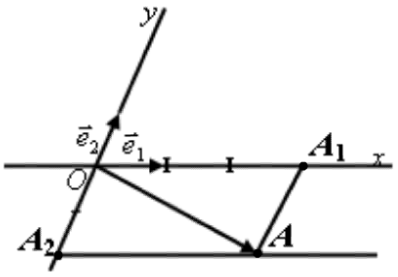
18-chizma

2-**misol.** Affin koordinatalar sistemasi berilgan *A(3, -2), B(0, 3), C(-2, 0)* nuqtalarni yasang.

Yechish. A nuqtani yasash uchun $\overrightarrow{OA}=3\vec{e}_1-2\vec{e}_2$ vektorni yasaymiz.

Buning uchun *0* nuqtadan boshlab $\vec{e}_1$ vektorga kollinear $\overrightarrow{OA_1}=3\vec{e}_1$ vektorni, $\vec{e}_2$ vektorga kollinear $\overrightarrow{OA_2}=-2\vec{e}_2$ vektorlarni yasaymiz.

Bu vektorlarning yig'indisini yasasak $\overrightarrow{OA}$ vektorga ega bo'lamiz va *A* nuqtani topamiz.



19-chizma

3-misol. Uchlari *A(1, 2), B(0, 5), C(-2, 3)* nuqtalarda bo'lgan uchburchak medianalarining kesishgan nuqtasini toping.

Yechish *AD* mediana *D(x, y)* nuqta *BC* tomon o'rta nuqtasi $x_D=-1, y_D=4, D(-1, 4).$

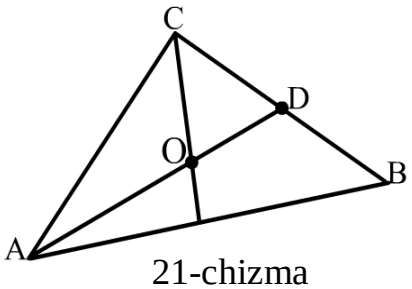
Uchburchak medianalar kesishgan nuqtasi *O(x, y)* bo'lsin, u holda

$$\frac{AO}{OD} = \lambda = 2:1,\ \lambda = 2$$

$$x = \frac{x_1 + \lambda x_2}{1+\lambda} = \frac{1+2(-1)}{3} = -\frac{1}{3}$$

$$y = \frac{y_1 + \lambda y_2}{1+\lambda} = \frac{2+2\cdot 4}{3} = \frac{10}{3}$$

Demak, $O(-\frac{1}{3},\ \frac{10}{3})$.



21-chizma

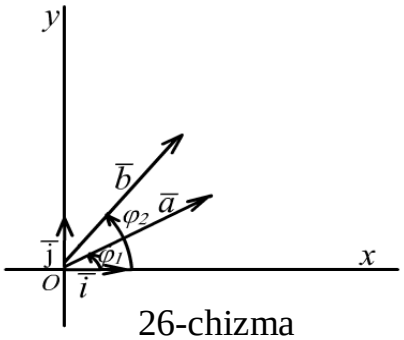
3-masala. Ortogonal $\vec{i}$, $\vec{j}$ bazisga nisbatan $\vec{a}(a_1,\ a_2)$ $\vec{b}(b_1,\ b_2)$ vektorlar koordinatalari bilan berilgan . $(\vec{a}\wedge\vec{b})$ yo'nalishli burchakni toping.

Yechish Bu masalani yechish uchun $cos(\vec{a}\wedge\vec{b})$ va $sin(\vec{a}\wedge\vec{b})$ larni topish yetarlidir. $(\vec{a}\wedge\vec{b}) = \varphi,$ $(\vec{i}\wedge\vec{a}) = \varphi_1$ $(\vec{i}\wedge\vec{b}) = \varphi_2$ (26-chizma)

U holda

$$\cos\varphi = \cos((\vec{a}\wedge\vec{i}) + (\vec{i}\wedge\vec{b})) = \cos((\vec{i}\wedge\vec{b}) - (\vec{i}\wedge\vec{a})) = \cos(\varphi_2 - \varphi_1)$$

$$\sin\varphi = \sin((\vec{a}\wedge\vec{i}) + (\vec{i}\wedge\vec{b})) = \sin((\vec{i}\wedge\vec{b}) - (\vec{i}\wedge\vec{a})) = \sin(\varphi_2 - \varphi_1)$$



26-chizma

Shunday qilib,

$$\cos\varphi = \cos\varphi_2\cos\varphi_1 + \sin\varphi_2\sin\varphi_1$$

$$\sin\varphi = \sin\varphi_2\cos\varphi_1 - \cos\varphi_2\sin\varphi_1$$

$$a_1 = |\vec{a}|\cos\varphi_1 \qquad a_2 = |\vec{a}|\sin\varphi_2$$

$$b_1 = |\vec{b}|\cos\varphi_2, \qquad b_2 = |\vec{b}|\sin\varphi_1$$

Bulardan $\cos\varphi_1$, $\cos\varphi_2$, $\sin\varphi_1$, $\sin\varphi_2$ qiymatlarini (5.6) ga qo'yib quyidagiga ega bo'lamiz.

$$\cos\varphi = \frac{a_1b_1 + a_2b_2}{|\vec{a}||\vec{b}|} \qquad \sin\varphi = \frac{a_1b_2 - b_1a_2}{|\vec{a}||\vec{b}|}$$

Tekshirish uchun savollar va mashqlar

1. Tekislikdagi koordinatalar sistemasi deb nimaga aytiladi?
2. Affin va Dekart koordinatalar sistemasini ta'riflang.
3. Dekart koordinatalar sistemasining affin koordinatalar sistemasidan farqini tushuntiring.
4. Affin koordinatalar sistemasini olib, ushbu nuqtalarni yasang:

   A(2, 1), B($\frac{1}{2}$, -1), C(-1, 4), D($\sqrt{2}$, -2), E(0, 1), F(2, 0), G(-3, -2).

5. *A(2, 5), B(1, -1), C(2, -2), D(1, 7)* nuqtalar berilgan. $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{DB}$, $\overrightarrow{BC}$ va $\overrightarrow{CA}$ vektorlarning koordinatalarini toping.
6. $\vec{a}(3, 4)$ vektorni boshi *A(-2, 3)* nuqtada oxiri *B(x, y)* nuqta koordinatasini toping.

Javob: *(1, 7)*

7. Uchlari *A(2, 3), B(-1, 2)* nuqtalarda bo'lgan *AB* kesmani ushbu nisbatlarda $\lambda_1 = 1,\ \lambda_2 = -2,\ \lambda_3 = \frac{1}{2},\ \lambda_4 = 3$ bo'luvchi nuqtaning koordinatalarini toping.

Javob: $(\frac{1}{2}, \frac{5}{2})$, (-4, 1), $(1, \frac{8}{3})$, $(-\frac{1}{4}, \frac{9}{4})$.

8. Uchburchak tomonlarining o'rtalari *P(3, -2), Q(1, 6), C(-4, 2)* nuqtalarda bo'lsa, uning uchlarining koordinatalarini toping.
9. Uchburchak og'irlik markazining koordinatalari, uning uchining koordinatalari bilan qanday ifodalanadi.

Javob: $x = \frac{x_1 + x_2 + x_3}{3},\quad y = \frac{y_1 + y_2 + y_3}{3}$.