4-mavzu: Darsda yechiladigan misollar

1-misol. $\vec{a}(2,2,3),\vec{b}(2,-2,0),\vec{c}(5,-1,4)$ vektorlarning qaysi jufti perpendikulyar?

Yechish $\vec{a}\cdot\vec{b}$, $\vec{a}\cdot\vec{c}$, $\vec{b}\cdot\vec{c}$ skalyar ko'paytmalarini tekshiramiz:

$\vec{a}\cdot\vec{b}=2\cdot2+2\cdot(-2)+3\cdot0=4-4+0=0;\quad \vec{a}\cdot\vec{c}=10-2+12=20;\vec{b}\cdot\vec{c}=10+2+0=12;$

Bundan $\vec{a}\perp\vec{b}$.

2-misol. $\vec{a}(1,\ -1,\ 0)\quad \vec{b}(1,\ -2,\ 2)$ vektorlar orasidagi burchakni toping.

Yechish (6.3) formuladan foydalanamiz.

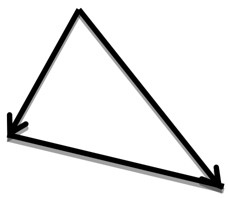
$\cos(\vec{a}\wedge\vec{b})=\dfrac{1+2+0}{\sqrt{1+1+0}\cdot\sqrt{1+4+4}}=\dfrac{\sqrt{2}}{2}$. Bundan $(\vec{a}\wedge\vec{b})=\varphi=45^0$.

3- misol. $|\vec{a}|=3,\ |\vec{b}|=4$ bo'lib, $\varphi=60^0$ bo'lsa, $\vec{a}\cdot\vec{b}$ ni toping.

Echish: $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos\varphi=3\cdot4\cos60^0=12\cdot\dfrac{1}{2}=6$.

1199.[2] ABC uchburchak tomonlarining uzunliklari berilgan: $BC=5,\ CA=6,\ AB=7$. $\overrightarrow{BA},\overrightarrow{BC}$ vektorlarning skalyar ko'paytmasi topilsin.

Echish: $(\overrightarrow{BA},\overrightarrow{BC})=|BA|\cdot|BC|\cos(B)$. Kosinuslar teorimasiga ko'ra



$$\cos(B)=\frac{(|BC|^2+|BA|^2-|CA|^2)}{2|BA|\cdot|BC|}=\frac{19}{35}\Rightarrow$$

$$(\overrightarrow{BA},\overrightarrow{BC})=5\cdot7\cdot\frac{19}{35}=19$$

Javob: 19

1210.[2] Koordinatalari bilan berilgan vektorlarning skalyar ko'paytmasi hisoblansin: $\boldsymbol{a},\boldsymbol{b}$ Koordinatalari bilan berilgan vektorlarning skalyar ko'paytmasi hisoblansin: $\boldsymbol{a}=\{5,\ 2\},\ \boldsymbol{b}=\{-3,\ 6\}$;

Echish: Koordinatalari bilan berilgan $\boldsymbol{a}=\{a_1,\ a_2\}\ va\ \boldsymbol{b}=\{b_1,\ b_2\}$ vektorlarning skalyar ko'paytmasi $(\boldsymbol{a},\boldsymbol{b})=a_1\cdot b_1+a_2\cdot b_2$ formula orqali hisoblanadi:

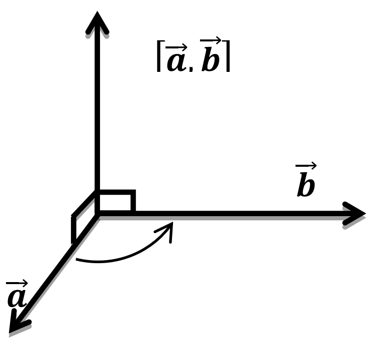
$$(\boldsymbol{a},\boldsymbol{b})=5\cdot(-3)+2\cdot6=-3$$

Javob: -3

1200.[2] Quyidagi hollarning har birida $[\boldsymbol{a}\ \boldsymbol{b}]$ vektor ko'paytma topilsin:

a={2, 3, 1}, b={5, 6, 4};

Echish: Koordinatalari bilan berilgan $\boldsymbol{a}=\{a_1,\ a_2,a_3\}\ va\ \boldsymbol{b}=\{b_1,\ b_2,b_3\}$ vektorlarning vektor ko'paytmasi $[\boldsymbol{a}\ \boldsymbol{b}]=\left\{\begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix},-\begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix},\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}\right\}$ formula orqali hisoblanadi:

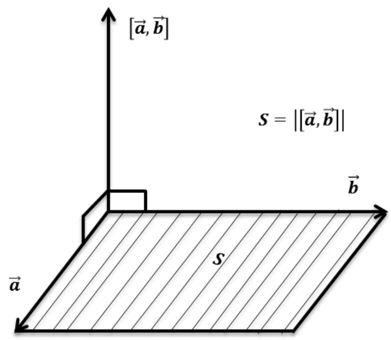


$$[\boldsymbol{a}\ \boldsymbol{b}] = \left\{\begin{vmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{6} & \mathbf{4} \end{vmatrix}, -\begin{vmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{5} & \mathbf{4} \end{vmatrix}, \begin{vmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{5} & \mathbf{6} \end{vmatrix}\right\} = \{\mathbf{6}, -\mathbf{3}, -\mathbf{3}\}$$

Javob: $[\boldsymbol{a}\ \boldsymbol{b}] = \{\mathbf{6}, -\mathbf{3}, -\mathbf{3}\}$

1201.[2] a={8, 4, 1}, b={2, -2, 1} vektorlardan yasalgan parallelogramm yuzi hisoblansin.

Echish: a={8, 4, 1}, b={2, -2, 1} vektorlardan yasalgan parallelogramm yuzi bu vektorlarning vektor ko'paytmasidan hosil bo'lgan vektorning uzunligiga teng, ya'ni



$$S = |[\vec{a}, \vec{b}]| = \sqrt{\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}^2 + \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}^2 + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}^2}$$

$$= \sqrt{\begin{vmatrix} 3 & 1 \\ 6 & 4 \end{vmatrix}^2 + \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}^2 + \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}^2} = \sqrt{36 + 9 + 9}$$

$$= 3\sqrt{5}$$

Javob: $\boldsymbol{S} = \mathbf{3}\sqrt{\mathbf{5}}$

1170.[2] a={3, 1, 2}, b={2, 7, 4} c={1, 2, 1} vektorlar berilgan. 1) abc topilsin.

Echish: Koordinatalari bilan berilgan $\boldsymbol{a} = \{\boldsymbol{a_1},\ \boldsymbol{a_2}, \boldsymbol{a_3}\}, \boldsymbol{b} = \{\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}\}\ \boldsymbol{va}\ \ \boldsymbol{c} = \{\boldsymbol{c_1},\ \boldsymbol{c_2}, \boldsymbol{c_3}\}$ vektorlarning skalyar ko'paytmasi

$$\boldsymbol{abc} = \begin{vmatrix} \boldsymbol{a_1} & \boldsymbol{a_2} & \boldsymbol{a_3} \\ \boldsymbol{b_1} & \boldsymbol{b_2} & \boldsymbol{b_3} \\ \boldsymbol{c_1} & \boldsymbol{c_2} & \boldsymbol{c_3} \end{vmatrix}$$

formula orqali hisoblanadi:

$$\boldsymbol{abc} = \begin{vmatrix} \mathbf{3} & \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{7} & \mathbf{4} \\ \mathbf{1} & \mathbf{2} & \mathbf{1} \end{vmatrix}$$

$$= \mathbf{3 \cdot 7 \cdot 1 + 1 \cdot 4 \cdot 1 + 2 \cdot 2 \cdot 2 - 2 \cdot 7 \cdot 1 - 1 \cdot 2 \cdot 1 - 3 \cdot 4 \cdot 1} =$$
$$= -\mathbf{7}$$

Javob: $\boldsymbol{abc} = -\mathbf{7}$

Misollar

1201.[2] p = a(bc)-b(ac) va c vektorlarning bir-biriga perpendikularligi isbotlansin.

1203.[2] Tomonlari birga teng bo'lgan tengtomonli *ABC* uchburchak berilgan. $\overrightarrow{BC}$ = a, $\overrightarrow{CA}$ = b, $\overrightarrow{AB}$ = c deb ab + bc + ca ifoda hisoblansin.

1210.[2] Koordinatalari bilan berilgan a, b vektorlarning skalyar ko'paytmasi hisoblansin:

a={5, 2}, b={-3, 6};

a={6, -8}, b={12, 9};

a={3, -5}, b={7, 4}.

1211.[2] a={5, 2}, b={7, -3} vektorlar berilgan. Bir vaqtning o'zida ikkita

ax=38, bx=30 tenglamani qanoatlantiradigan x vektor topilsin.
1213.[2] a={3, -2}, b={-5, 1}, c={0, 4} vektorlar berilgan.
1) $3a^2–4ab+5b^2–6bc–2c^2$;
2) $2(ab)c–3b^2a+(ac)b$ topilsin.
1192[2]. *a, b* vektorlarni bilgan holda:
[(a+b)(a–b)];
[a(a+b)];
$\left[\frac{a+b}{2}\left(b-\frac{a}{2}\right)\right]$ topilsin.
1193[2]. $[a\ b]^2+(a\ b)^2=a^2b^2$ ekanligini ko’rsating.
1195[2]. $[a\ (b+\lambda a)]=[(a+\mu b)\ b]=[a\ b]$ ekanligini ko’rsating.
1196[2]. Bir nuqtadan chiquvchi uchta komplanar a, b, c vektorlar berilgan. Ularning oxirlaridan o’tgan tekislikning [a b]+[b c]+[c a] vektorga perpendikularligini ko’rsatilsin.
1200[2]. Quyidagi hollarning har birida [a b] vektor ko’paytma topilsin:
a={2, 3, 1}, b={5, 6, 4};
a={5, -2, 1}, b={4, 0, 6};
a={-2, 6, -4}, b={3, -9, 6}.
1170[2]. a={3, 1, 2}, b={2, 7, 4} c={1, 2, 1} vektorlar berilgan.1) abc; 2) [[a b]c] 3) [a [b c]] topilsin.
1.4.10.[1,p39]. $\vec{\imath},\vec{\jmath},\vec{k}$ ortonormal bazis. $\vec{a}=2\vec{\imath}+\vec{\jmath}-\vec{k}$ *va* $\vec{b}=\vec{\imath}-3\vec{\jmath}+\vec{k}$ vektorlardan yasalgan parallelogramm yuzi hisoblansin.
1.4.12.[1,p39]Uchta nokomplanar a, b, c vektorlar berilgan. ax=α, bx=β, cx=γ tenglamalar sistemasini qanoatlantiradigan x vektor topilsin.
1175[2].a={2, 1, -1}, a_2={-3, 0, 2}, a_3={5, 1, -2} vektorlar uchligiga o’zarolik munosabatida bo’lgan uchlik topilsin (oldingi misolga qarang).

Tekshirish uchun savollar va mashqlar.

1. Skalyar ko’paytma, vektorlar ustidagi chiziqli amaldan farqini tushuntiring?
2. Skalyar ko’paytma ta’rifini ayting.
3. Skalyar ko’paytma xossalarini ayting.
4. Ortogonal bazis haqida nimalarni bilasiz?
5. Koordinatalar bilan berilgan vektorning skalyar ko’paytmasi.
6. Koordinatalari bilan berilgan vektor xossalari.
7. 1-masala. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi $\varphi=\frac{2}{3}\pi$, $|\vec{a}|=3$, $|\vec{b}|=4$ bo’lsa, quyidagilarni xisoblang: 1) $\vec{a}\cdot\vec{b}$; 2) $\vec{a}^2$; 3) $\vec{b}^2$; 4) $(\vec{a}+\vec{b})^2$; 5) $(3\vec{a}-2\vec{b})(\vec{a}+2\vec{b})$.

Javob: 1)- 6; 2) 9; 3) 16; 4) 13; 5) - 16.

8. 2-masala $|\vec{a}|=3,\ |\vec{b}|=5$ berilgan. α ning qanday qiymatida $\vec{a}+\alpha\vec{b}$ va $\vec{a}-\alpha\vec{b}$ vektorlar o'zaro perpendikulyar bo'ladi.

Javob: $\alpha=\pm\frac{3}{5}$.

9. 3-masala. $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlar perpendikulyar bo'lishi uchun $\vec{a}$, $\vec{b}$ vektorlar qanday shartlari qanoatlantirishi kerak.

Javob: $\vec{a} = \vec{b}$.

10. 4- masala $\vec{a}$ *(4;-2;-4)* va $\vec{b}$ *(6;-3;2)* vektorlar berilgan: 1) $\vec{a}$ $\vec{b}$; 2) $\vec{a}$, $\vec{b}$ vektorlar orasidan burchak 3) $\sqrt{b^2}$; 4)(2 $\vec{a}$ -3 $\vec{b}$) ($\vec{a}$ +2 $\vec{b}$).

Javob: 1) 22; 2) 6; 3) 7; 4) - 200