O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi

Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universiteti

Rossiya Federatsiyasining fan va oliy ta’lim vazirligi
Sibir federal universiteti

T.T.To‘ychiev, J.K.Tishabaev, D.X.Djumaboev, A.M.Kitmanov

Kompleks o‘zgaruvchili funksiyalar nazariyasi fanidan
MUSTAQIL ISHLAR

O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi tomonidan o‘quv qo‘llanma sifatida chop etishga ruxsat berildi (2018 yil 14 iyundagi 531-sonli buyruq)

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Qo‘llanma kompleks o‘zgaruvchili funksiyalar nazariyasi fanidan mustaqil ishlarni bajarish uchun mo‘ljallangan bo‘lib, shu fanning o‘quv dasturi asosida tuzilgan va o‘quv adabiyoti Davlat ta’lim standartining bakalavr mutaxassisligi 51301100 – matematika , 5140300 – mexanika va 5140200 – fizika yo‘nalishlariga mos keladi.

Qo‘llanma kompleks sonlar va kompleks argumentli funksiyalar, elementar funksiyalar va ular yordamida bajariladigan konform akslantirishlar, kompleks argumentli funksiyaning integrali va chegirmalar nazariyasi mavzularini o‘z ichiga oladi. Qo‘llanmada 3 ta mustaqil ish, 1092 ta misol va masalalar keltirilgan bo‘lib, ulardan 52 ta misol va masalalar batafsil yechimi bilan keltirilgan. Bu misol va masalalar yechimlari Maple matematik paketi yordamida ham keltirilgan.

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ISBN

MUNDARIJA

Mirzo Ulug‘bek nomidagi
O‘zbekiston Milliy universiteti
100 yilligiga bag‘ishlanadi

So‘z boshi

O‘zbekiston Respublikasining Ta’lim to‘g‘risidagi Qonuni va Kadrlar tayyorlash Milliy dasturi talablarini amalga oshirishda O‘zbekiston Milliy Universiteti matematika fakulteti matematik analiz kafedrasi jamoasi mas’uliyatini his etgan holda ilmiy-tadqiqot ishlari va ilmiy pedagogik kadrlar tayyorlash samaradorligini oshirish maqsadlarini ko‘zlab o‘z oldiga qator vazifalarni belgiladi.

Ilm-fan jadal taraqqiy etayotgan, zamonaviy axborot-kommunikatsiya tizimlari vositalari keng joriy etilayotgan jamiyatda turli fan sohalarida bilimlarning tez yangilanib borishi, ta’lim oluvchilar oldiga ularni jadal egallash bilan bir qatorda, muntazam va mustaqil ravishda bilim izlash vazifasini qo‘ymoqda.

Bu vazifani hal qilish maqsadida o‘quv rejalariga matematik va kompleks analiz fanlaridan mustaqil ta’lim olish kiritildi. O‘z navbatida o‘quv dasturlarida rejaga mos ravishda o‘zgartirishlar amalga oshirildi.

Hozirgi vaqtda matematik va kompleks analizning uslublari fan, texnika va iqtisodiyotning turli-tuman masalalarini hal qilishda keng qo‘llanilmoqda. Xalq xo‘jaligining barcha sohalarida kompyuterlarning va matematik usullarning yalpi qo‘llanilishi munoabati bilan bu usullarning ahamiyati yanada ortdi.

Yuqorida qayd etib belgilangan vazifalar bajarilishining isboti sifatida yuzaga kelgan ushbu qo‘llanma kompleks o‘zgaruvchili funksiyalar nazariyasi fanidan mustaqil ishlarni bajarishga mo‘ljallangan bo‘lib, o‘quv adabiyoti Davlat ta’lim standartining bakalavr mutaxassisligi “Matematika” , “Mexanika” va “Fizika” yo‘nalishlariga mos keladi.

Qo‘llanma uch paragrafdan iborat bo‘lib ularda “Kompleks sonlar va kompleks argumentli funksiyalar”, “Elementar funksiyalar va ular yordamida bajariladigan konform akslantirishlar” va “Kompleks argumentli funksiyaning integrali va chegirmalar nazariyasi’’ mavzulari bo‘yicha 3 ta mustaqil ish tavsiya etilgan. Har bir mustaqil ishni berishdan avval shu mustaqil ishni bajarish uchun lozim bo‘ladigan asosiy tushuncha va teoremalar keltirilgan. So‘ng 21 ta variantdan iborat bo‘lgan vazifa mustaqil yechish uchun tavsiya qilingan. Talabaning mavzularni o‘zlashtirishini hamda ishni bajarishini yengillashtirish maqsadida har bir paragrafning oxirida 1 ta variantdagi (21-variant) barcha misol va masalalar to‘liq yechib ko‘rsatilgan. Bunda aksariyat misollar ikki usulda yechilgan.Avval analitik yo’l bilan yechilgan bo’lsa,undan so’ng misolning mohiyatini chuqurroq ochib berish maqsadida shu misol Maple matematik paket yordamida yechib, chizmalari bilan keltirilgan.

Qo‘llanmani yozishda mualliflar tomonidan mavzularning oddiy va sodda tilda, tushunarli va ravon bayon etilishiga harakat qilindi. Shu munosabat bilan mualliflar qo‘llanma talabalarda bilim olishga intilish hissi, mustaqil fikrlash malakalarining shakllanishiga xizmat qiladi deb umid bildiradilar hamda u talabalarga kompleks o‘zgaruvchili funksiyalar nazariyasi fanining aytib o‘tilgan mavzulari bo‘yicha bilimlarini oshirishda yordam beradi deb ishonadilar.

1-§. 1-MUSTAQIL ISH

KOMPLEKS SONLAR VA KOMPLEKS ARGUMENTLI FUNKSIYALAR

Kompleks sonlar va ular ustida amallar.

Kompleks sonning geometrik tasviri.

Kompleks sonning trigonometrik va ko‘rsatkichli ko‘rinishlari.

Kompleks tekislikda soha va egri chiziq.

Stereografik proyeksiya.

Kompleks argumentli funksiyalar, ularning limiti, uzluksizligi.

Funksiyaning differensiallanuvchiligi. Koshi-Riman shartlari.

Garmonik funksiyalar.

Hosila moduli va argumentining geometrik ma’nosi.

Konform akslantirishlar.

-A-

ASOSIY TUSHUNCHA VA TEOREMALAR

1^0. **Kompleks sonlar va ular ustida amallar**.

Ma’lumki, kompleks son

$$z = x + iy \qquad (1)$$

ko‘rinishda ifodalanadi, bunda x va y lar haqiqiy sonlar i esa $(i^2 = -1)$ mavhum birlikdir.

Odatda x haqiqiy songa z kompleks sonning *haqiqiy qismi*, y haqiqiy songa esa z kompleks sonning *mavhum qismi* deyiladi va

$$x = \operatorname{Re} z, \qquad y = \operatorname{Im} z$$

kabi belgilanadi.

Agar (1) da $y = 0$ bo‘lsa, $z = x + i \cdot 0 = x$ bo‘lib, z haqiqiy x songa teng bo‘ladi. Agar (1) da $x = 0$ bo‘lsa , $z = 0 + i \cdot y = iy$ bo‘lib, z sof mavhum son bo‘ladi. (1) da $x = 0$, $y = 0$ bo‘lsa, z kompleks son 0 ga teng bo‘ladi.

Ikkita $z_1 = x_1 + iy_1$ va $z_2 = x_2 + iy_2$ kompleks sonlar berilgan bo‘lib, $x_1 = x_2$, $y_1 = y_2$ bo‘lsa, unda z_1 va z_2 kompleks sonlar *bir biriga teng* deyiladi. Agar $x_1 = x_2$, $y_1 = -y_2$ bo‘lsa, y holda z_2 kompleks son z_1 ga *qo‘shma kompleks son* deyiladi va $\bar{z}_1$ kabi belgilanadi.

Demak, $z = x + iy$ bo‘lsa, $\bar{z} = \overline{x + iy} = x - iy$ bo‘ladi. Masalan, $z = 2 + \frac{1}{3}i$ kompleks sonning qo‘shmasi $\bar{z} = 2 - \frac{1}{3}i$ bo‘ladi.

Aytaylik, ikkita $z_1 = x_1 + iy_1$ va $z_2 = x_2 + iy_2$ kompleks sonlar berilgan bo‘lsin. Ular ustidagi arifmetik amallar quyidagi qoidalar asosida aniqlanadi.

1) $z_1 + z_2 := (x_1 + x_2) + i(y_1 + y_2)$;

2) $z_1 z_2 := (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2)$;

3) $\dfrac{z_1}{z_2} := \dfrac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\,\dfrac{y_1 x_2 - x_1 y_2}{x_2^2 + y_2^2}$;

4) $z^n = \underbrace{z \cdot z \cdots z}_{n}$.

Izoh. $z_1 \cdot z_2$ ko‘paytma $(x_1 + iy_1)(x_2 + iy_2)$ ifodani hadma-had ko‘paytirishdan hosil bo‘lishini ko‘rish qiyin emas:

$$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1 x_2 + x_1 i y_2 + i y_1 x_2 + i^2 y_1 y_2 =$$

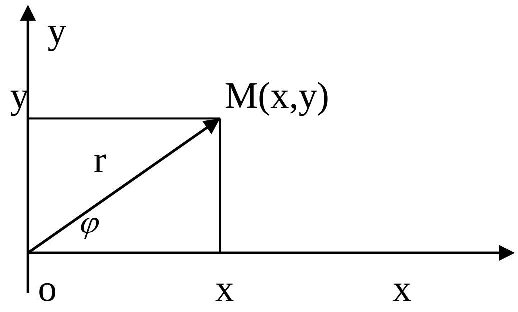
$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + i y_1 x_2).$$

$\dfrac{z_1}{z_2} = \dfrac{x_1 + iy_1}{x_2 + iy_2}$ nisbatni hisoblashda kasrning surat va maxrajini $\bar{z}_2 = x_2 - iy_2$ ga ko‘paytiriladi:

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\,\frac{y_1 x_2 - x_1 y_2}{x_2^2 + y_2^2}.$$

2^0. **Kompleks sonning geometrik tasviri**.

Tekislikda, *Oxy* Dekart koordinatlar sistemasida $z = x + iy$ kompleks son koordinatlari *x* va *y* bo‘lgan M(*x*,*y*) nuqtani ifodalaydi (1-chizma).



1-chizma.

Shu M(*x*,*y*) nuqta $z = x + iy$ kompleks sonning *geometrik tasviri deyiladi.* Demak, har bir kompleks son tekislikda bitta nuqtani ifodalaydi. Aksincha, tekislikdagi har bir nuqta haqiqiy qismi shu nuqtaning abssissasiga, mavhum qismi esa ordinatasiga teng bo‘lgan kompleks sonni ifodalaydi.

Shunday qilib, tekislikning barcha nuqtalari to‘plami bilan barcha kompleks sonlar to‘plami orasida o‘zaro bir qiymatli moslik mavjud. Bunda barcha haqiqiy sonlarning geometrik tasviri abssissalar o‘qini, barcha sof mavhum sonlarning geometrik tasviri ((0,0) nuqtadan farqli) esa ordinatalar o‘qini ifodalaydi. Shuning uchun abssissalar o‘qini *haqiqiy o‘q*, ordinatalar o‘qini esa *mavhum o‘q* deyiladi. Oxy tekislikni esa *kompleks tekislik* deyiladi va C harfi bilan belgilanadi.

1-chizmadagi $\overrightarrow{OM}$ vektorga M(*x*,*y*) nuqtaning *radius vektori* deyilib, bu vektorning uzunligi r ga $z = x + iy$ *kompleks sonning moduli* deyiladi va $|z|$ kabi belgilanadi. $\overrightarrow{OM}$ vektor bilan Ox haqiqiy o‘qning musbat yo‘nalishi orasidagi φ burchak *z kompleks sonning argumenti* deyiladi va $\varphi = \arg z$ kabi belgilanadi.

Agar $z = x + iy$ kompleks son berilgan bo‘lsa uning moduli va argumenti quyidagi tengliklar yordamida hisoblanadi:

$$r = |z| = \sqrt{x^2 + y^2}\,; \qquad (2)$$

$$\varphi = \arg z = \begin{cases} arctg\dfrac{y}{x}, \quad agar\ x \geq 0, y \geq 0\ bo'lsa, \\ arctg\dfrac{y}{x} + \pi,\ agar\ x < 0\ \ bo'lsa \\ arctg\dfrac{y}{x} + 2\pi,\ \ agar\ \ x \geq 0, y < 0\ ,bo'lsa \end{cases} \qquad (3)$$

1-chizmadan

$$\cos\varphi = \frac{x}{r},\ \sin\varphi = \frac{y}{r}$$

ekanligini hosil qilamiz va bundan

$$z = x + iy = r(\cos\varphi + i\sin\varphi) \qquad (4)$$

ifodaga ega bo‘lamiz. Bu ifoda z kompleks *sonning trigonometrik ifodasi (shakli)* deyiladi.

$$e^{i\varphi} = \cos\varphi + i\sin\varphi \qquad (5)$$

tenglik *Eyler formulasi* deyiladi. Bundan

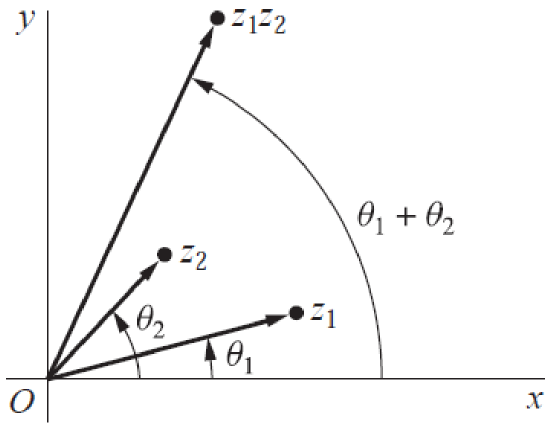
$$z = re^{i\varphi}$$

kompleks sonning ko‘rsatkichli ko‘rinishi kelib chiqadi.

1-Teorema. *Ikkita* z_1 *ва* z_2 *kompleks son ko‘paytmasining moduli shu kompleks sonlar modullarining ko‘paytmasiga teng:*

$$|z_1 z_2| = |z_1| \cdot |z_2|.$$

Ikkita kompleks son ko‘paytmasining argumenti shu kompleks sonlar argumentlarining yig‘indisiga teng.



2-chizma.

2-Teorema. *Ushbu*

$$|z^n| = |z|^n, \arg z^n = n \arg z \ (n \in N)$$

tengliklar o'rinlidir.

3-Teorema. *Ikkita kompleks son nisbati* $\frac{z_1}{z_2}$ *uchun*

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$$

tengliklar o'rinli.

Izoh. Kompleks sonlar argumentlariga doir keltirilgan tengliklarda kompleks son argumenti shu songa mos radius vektorning tekislikdagi holati ma'nosida tushuniladi.

(4)–munosabat va 2-teoremadan z^n uchun

$$z^n = [r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi) \qquad (6)$$

Muavr formulasi kelib chiqadi.

3^0. Kompleks tekislikda soha va egri chiziq.

Aytaylik,

$$x = x(t), \quad y = y(t)$$

funksiyalar $[\alpha,\beta]$ da $([\alpha,\beta]\subset R)$ aniqlangan va uzluksiz bo‘lsin. Unda

$$z = x + iy$$

kompleks son haqiqiy o‘zgaruvchi t ga bog‘liq bo‘lib,

$$z = z(t) = x(t) + iy(t)$$

haqiqiy argumentli kompleks qiymatli funksiyaga ega bo‘lamiz.

Ravshanki, t o‘zgaruvchi $[\alpha,\beta]$ da o‘zgarganda $z(t)$ funksiyaning qiymatlari C da o‘zgarib, biror egri chiziqni tashkil etadi. Shu sababli

$$z = z(t) \qquad (\alpha \le t \le \beta)$$

funksiyaga egri chiziqning ***parametrik tenglamasi*** deyiladi.

Agar $z = z(t)$ da $\forall t_1, t_2 \in [\alpha,\beta]$ uchun $t_1 \ne t_2$ bo‘lishidan $z(t_1) \ne z(t_2)$ bo‘lishi kelib chiqsa, u holda $z = z(t)$ egri chiziq *sodda chiziq* deyiladi.

Agar $z(\alpha) = z(\beta)$ bo‘lsa, $z = z(t)$ egri chiziq *yopiq chiziq* deyiladi.

Kompleks tekislik C da biror z_0 nuqta hamda $\varepsilon > 0$ son olaylik.

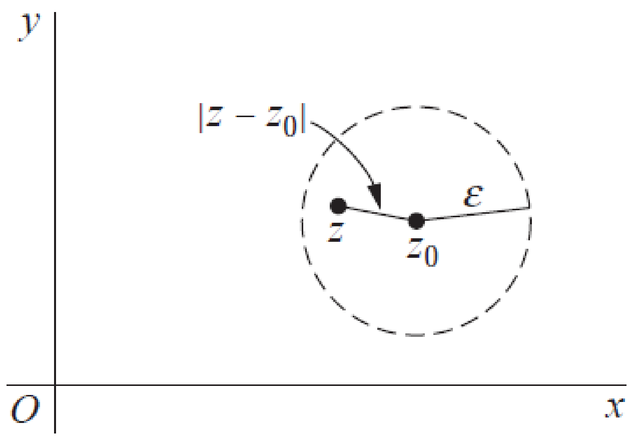
1-Ta’rif. *Ushbu*

$$\{z \in C : |z - z_0| < \varepsilon\}$$

to‘plam z_0 *nuqtaning* ε***-atrofi*** *deyiladi va* $U(z_0,\varepsilon)$ *kabi belgilanadi:*

$$U(z_0,\varepsilon) = \{z \in C : |z - z_0| < \varepsilon\}.$$

Ravshanki, $U(z_0.\varepsilon)$ atrof markazi z_0 nuqtada, radiusi ε bo‘lgan ochiq doira bo‘ladi.



3-chizma.

C da biror D to‘plam berilgan bo‘lsin ($D \subset C$). Agar $z_0 \in D$ nuqtaning $\exists U(z_0, \varepsilon)$ atrofi mavjud bo‘lib, $U(z_0, \varepsilon) \subset D$ bo‘lsa, u holda z_0 nuqta D to‘plamning ***ichki nuqtasi*** deyiladi.

2-Ta’rif. *Agar D to‘plamining har bir nuqtasi uning ichki nuqtasi bo‘lsa, u holda D* ***ochiq to‘plam*** *deyiladi.*

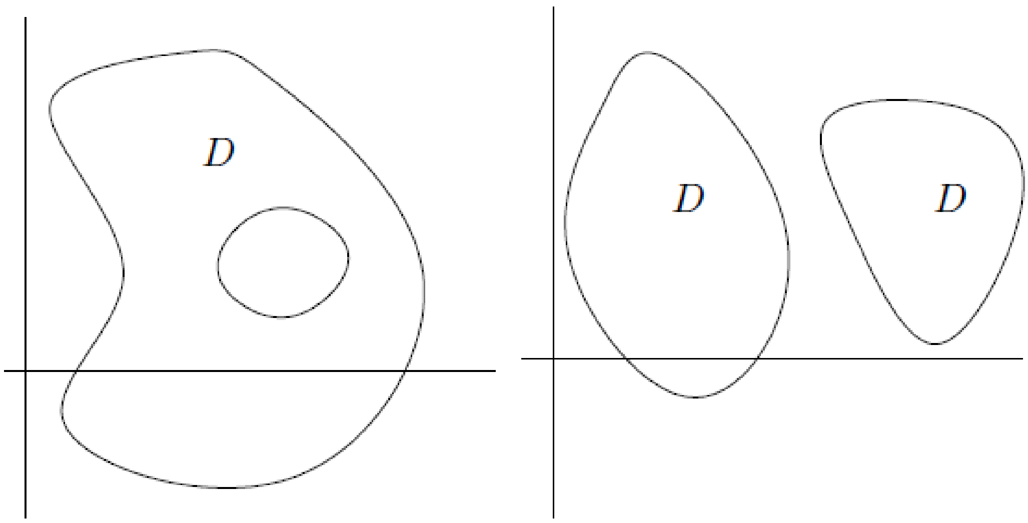
C da biror F to‘plam berilgan bo‘lsin ($F \subset C$)

3-Ta’rif. *Agar $z_0 \in C$ nuqtaning ixtiyoriy $U(z_0, \varepsilon)$ atrofida F to‘plamning z_0 nuqtadan farqli kamida bitta nuqtasi bo‘lsa, z_0 nuqta F to‘plamning* ***limit nuqtasi*** *deyiladi.*

4- Ta’rif. *Agar F to‘plamning barcha limit nuqtalari shu to‘plamga tegishli bo‘lsa, F* ***yopiq to‘plam*** *deyiladi.*

5-Ta’rif. *Agar D to‘plamning ixtiyoriy z_1, z_2 nuqtarlarini D to‘plamda to‘liq yotuvchi birorta uzluksiz γ egri chiziq yordamida birlashtirish mumkin bo‘lsa, u holda D* ***bog‘lamli to‘plam*** *deyiladi.*

6-Ta’rif. *Agar $D(D \subset C)$ to‘plam ochiq hamda bog‘lamli to‘plam bo‘lsa, bunday to‘plam* ***soha*** *deb ataladi.*



a) *b)*

4-chizma. a) soha , b) soha emas

D sohaning o‘ziga tegishli bo‘lmagan limit nuqtalaridan tashkil topgan to‘plam D ***sohaning chegarasi*** deyiladi va ∂D kabi belgilanadi.

Ushbu

$$D \cup \partial D$$

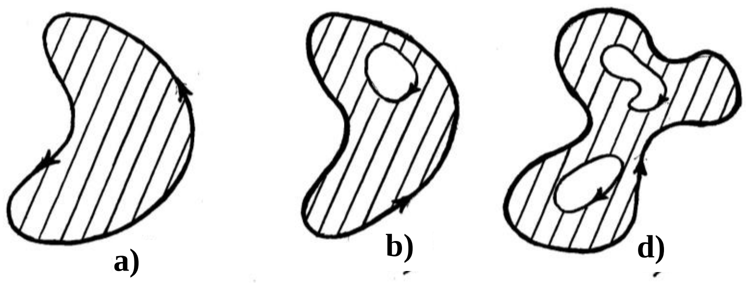
to‘plam $\overline{D}$ kabi belgilanadi. Demak, $\overline{D} := D \cup \partial D$.

Agar D sohaning chegarasi ∂D bog‘lamli to‘plam bo‘lsa, D ***bir bog‘lamli***, aks holda esa ***ko‘p bog‘lamli soha*** deyiladi.

D soha chegarasi ∂D ning bog‘lamli komponentlari soniga qarab D sohani ***bir bog‘lamli, ikki bog‘lamli, n bog‘lamli soha*** deb ataymiz.

Soha chegarasining ***musbat yo‘nalishi*** deb shunday yo‘nalishni qabul qilamizki, kuzatuvchi bu yo‘nalish bo‘ylab harakat qilganda soha unga nisbatan har doim chapda joylashgan bo‘ladi.

Masalan, 5-chizmada a) bir bog‘lamli, b) ikki bog‘lamli, d) uch bog‘lamli sohalar tasvirlangan bo‘lib, soha chegaralarining musbat yo‘nalishlari strelkalar bilan ko‘rsatilgan.



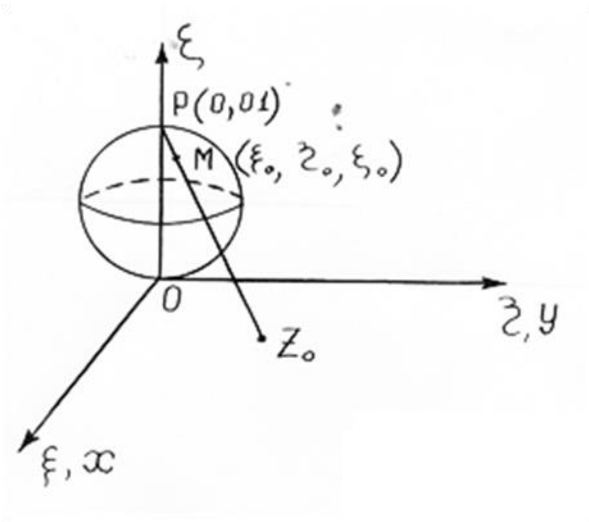
5-chizma

4^0. **Stereografik proeksiya.**

R^3 fazoda (ξ, η, ζ) Dekart koordinatalar sistemasini olaylik. Bu fazoda

$$S = \{(\xi, \eta, \zeta) \in R^3 : \xi^2 + \eta^2 + \zeta^2 = \zeta\}$$

sferani qaraymiz. Faraz qilaylik ξ va η o‘qlar mos ravishda x va y o‘qlari bilan ustma-ust tushsin (6-chizma).



6-chizma

Ravshanki, qaralayotgan S sfera Oxy tekisligiga koordinata boshida urinadi. Kompleks tekislikda $z_0 = x_0 + iy_0$ nuqta olib, bu nuqtani sferaning P nuqtasi bilan to‘g‘ri chiziq kesmasi yordamida birlashtiramiz. Natijada, bu to‘g‘ri chiziq sferani $M(\xi_0, \eta_0, \zeta_0)$ nuqtada kesadi. Demak, kompleks tekislikdagi har bir nuqta S sferadagi biror nuqta bilan ifodalanadi, va aksincha, S sferadagi har bir nuqtaga (P nuqtadan boshqa) kompleks tekislikda yagona nuqta mos keladi.

Shunday qilib, $S \setminus \{P\}$ to‘plam bilan C kompleks tekislik o‘rtasida o‘zaro bir qiymatli moslik o‘rnatildi. Odatda bu moslik ***kompleks tekislikning stereografik proyeksiyasi*** deyiladi.

Agar z_0 nuqta ∞ ga intilsa, bu z_0 nuqtaga S sferada mos keluvchi nuqtaning P ga yaqinlashishini ko‘rish qiyin emas. Bu hol P nuqtaga kompleks tekislikda $z = \infty$ nuqtani mos qo‘yish tabiiyligini ko‘rsatadi. Demak, kompleks tekisligidagi yagona $z = \infty$ nuqta S sferada P nuqta bilan ifodalanadi. Kompleks tekislik cheksiz uzoqlashgan nuqta $z = \infty$ bilan birgalikda kengaytirilgan kompleks tekislik deb ataladi va $\overline{C}$ kabi belgilanadi. S sferadagi $M(\xi, \eta, \zeta)$ va kompleks tekislikdagi $z = x + iy$ nuqta orasidagi moslik quyidagi formulalar yordamida aniqlanadi:

$$\xi = \frac{x}{1+|z|^2}, \eta = \frac{y}{1+|z|^2}, \zeta = \frac{|z|^2}{1+|z|^2}; \qquad (7)$$

$$x = \frac{\xi}{1-\zeta}, \ y = \frac{\eta}{1-\zeta}. \qquad (8)$$

Bu tengliklardan foydalanib, *sferik masofa tushunchasini* kiritamiz. Aytaylik, $z_1, z_2 \in \overline{C}$ nuqtalar berilgan b o‘lsin, z_1 va z_2 nuqtalar orasidagi sferik masofa deganda, ularning Riman sferasi S dagi obrazlari orasidagi masofa tushuniladi va u $\rho(z_1, z_2)$ kabi belgilanadi. (7) va (8) – tengliklar yordamida ushbu formulalarni keltirib chiqarish qiyin emas.

$$\rho(z_1, z_2) = \frac{|z_2 - z_1|}{\sqrt{1+|z_1|^2} \cdot \sqrt{1+|z_2|^2}} \qquad (z_1 \neq \infty; z_2 \neq \infty); \qquad (9)$$

$$\rho(z,\infty) = \frac{1}{\sqrt{1+|z|^2}}. \qquad (10)$$

5^0. Kompleks argumentli funksiyalar.

Kompleks sonlar tekisligi C da biror E to‘plam berilgan bo‘lsin ($E \subset C$).

1-Ta’rif. *Agar E to‘plamdagi har bir z kompleks songa f qoida yoki qonunga ko‘ra bitta w kompleks son mos qo‘yilgan bo‘lsa, E to‘plamda funksiya berilgan (aniqlangan) deb ataladi va u*

$$f : z \to w \qquad yoki \qquad w = f(z)$$

kabi belgilanadi.

Bunda E to‘plam funksiyaning ***aniqlanish to‘plami***, *z* ***erkli o‘zgaruvchi*** *yoki* ***funksiya argumenti***, w esa z o‘zgaruvchining ***funksiyasi*** deyiladi.

Aytaylik $w = f(z)$ funksiya biror E (E$\subset$C) to‘plamda berilgan bo‘lsin. Bu funksiyani

$$w = f(x+iy) = u + iv \qquad (x \in R, y \in R)$$

ko‘rinishda ham yozish mumkin. Bu esa E to‘plamda ikki o‘zgaruvchili ikkita

$$\begin{cases} u = u(x, y), \\ v = v(x, y) \end{cases}$$

funksiyalarning aniqlanishiga olib keladi. Bundan bitta kompleks o‘zgaruvchili $w = f(z)$ funksiyaning berilishi ikkita ikki o‘zgaruvchili haqiqiy funksiyalar

$$\begin{cases} u = u(x, y), \\ v = v(x, y) \end{cases}$$

berilishiga ekvivalent bo‘lishi kelib chiqadi.

$w = f(z)$ funksiya $E \subset C$ to‘plamda berilgan bo‘lib, z o‘zgaruvchi E to‘plamda o‘zgarganda funksiyaning mos qiymatlaridan iborat to‘plam

$$F = \{f(z) : z \in E\}$$

bo‘lsin. Bu to‘plamga funksiyaning ***qiymatlari to‘plami*** deyiladi.

$E \subset C$ to‘plamda $w = f(z)$ funksiyaning berilishi Oxy kompleks tekisligidagi E to‘plamni (to‘plam nuqtalarini) Ouv kompleks tekisligidagi F to‘plamga (to‘plam nuqtalariga) aks ettirishdan iborat. Shu sababli $w = f(z)$ ni E to‘plamni F to‘plamga ***akslantirish*** deyiladi.

Odatda $w = f(z)$ funksiyani geometrik tasvirlash uchun bu akslantrish yordamida aniqlangan E va F to‘plamlar mos ravishda Oxy va Ouv kompleks tekisliklarida chiziladi.

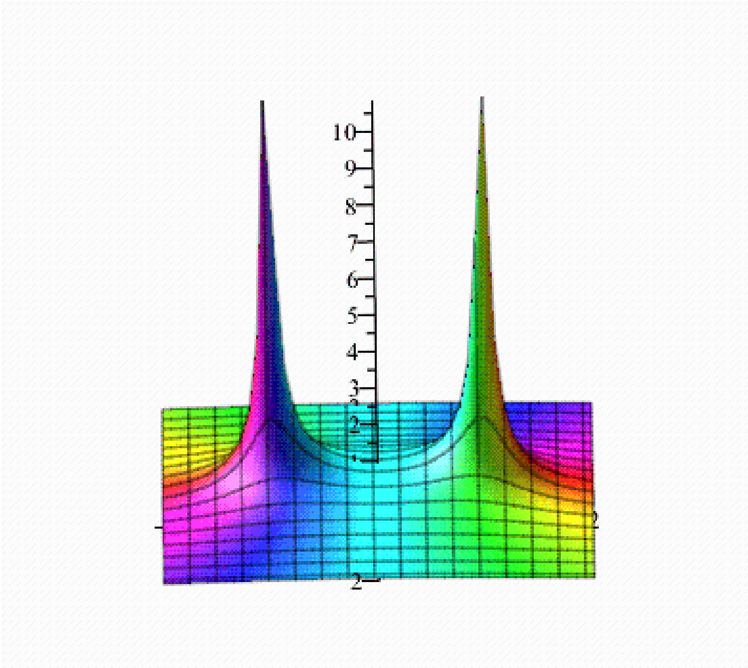
Ba'zida funksiyani geometrik tasvirlash uchun boshqacha usul ham qo‘llaniladi. Uch o‘lchovli (x, y, ρ) fazoda $\rho = |f(z)|$ sirt chiziladi.Bu sirtga $w = f(z)$ funksiyaning **relyefi** deb ataladi.

Misol tariqasida $f(z) = \dfrac{1}{1 + z^2}$ funksiyaning relyefini Maple matematik paketidan foydalanib chizamiz.

$with(plots)$:

> $f := z \to \dfrac{1}{1+z^2};$

> $complexplot3d(f, -2-2I..2+2I, grid=[50, 50])$



7-chizma

$w = f(z)$ funksiya E to‘plamda (E⊂C) berilgan bo‘lib, F esa shu funksiya qiymatlaridan iborat to‘plam bo‘lsin

$$F = \{f(z) : z \in E\}.$$

So‘ngra F to‘plamda o‘z navbatida biror $\zeta = \varphi(w)$ funksiya berilgan bo‘lsin. Natijada E to‘plamdan olingan har bir z ga F to‘plamda bitta $w(f : z \to w)$ son va F to‘plamdan olingan bunday w songa bitta $\zeta(\varphi : w \to \zeta)$ son $(\zeta \in C)$ mos qo‘yiladi. Demak, E to‘plamdan olingan har bir z ga bitta ζ son mos qo‘yilib, $\zeta = \varphi(f(z))$ funksiya hosil bo‘ladi. Bunday funksiya ***murakkab funksiya*** deyiladi.

$w = f(z)$ funksiya E to‘plamda berilgan bo‘lib, F to‘plam esa shu funksiya qiymatlaridan iborat to‘plam bo‘lsin. F to‘plamdan olingan har bir w kompleks songa E to‘plamda faqat bitta z sonni mos qo‘yadigan funksiyaga

$w = f(z)$ funksiyaga nisbatan ***teskari funksiya*** deyiladi va u $z = f^{-1}(w)$ kabi belgilanadi.

2-Ta'rif. *Agar argument z ning E to'plamdan olingan ixtiyoriy z_1 va z_2 qiymatlari uchun $z_1 \neq z_2$ bo'lishidan $f(z_1) \neq f(z_2)$ bo'lishi kelib chiqsa, $f(z)$ funksiya E to'plamda **bir yaproqli** (yoki **bir varaqli**) funksiya deb ataladi.*

Misol. $f(z) = \dfrac{1}{2z-3}$ funksiyani $E = \{z \in C;\ |z| < \dfrac{3}{2}\}$ doirada bir yaproqlilikka tekshiring.

$\triangleleft$ Faraz qilaylik, $z_1, z_2 \in E$ lar uchun $f(z_1) = f(z_2)$, ya'ni

$\dfrac{1}{2z_1 - 3} = \dfrac{1}{2z_2 - 3}$ bo'lsin $. \Rightarrow 2z_1 - 3 = 2z_2 - 3 \Rightarrow z_1 = z_2 . \Rightarrow f(z)$ funksiya E to'plamda bir yaproqli.$\triangleright$

Faraz qilaylik $w = f(z)$ funksiya $E \subset C$ to'plamda berilgan bo'lib, z_0 nuqta shu E to'plamning limit nuqtasi bo'lsin.

3-Ta'rif. *Agar $\forall \varepsilon > 0$ son uchun $\exists \delta = \delta(z_0, \varepsilon) > 0$ son topilsaki, argument z ning $0 < |z - z_0| < \delta$ tengsizlikni qanoatlantiruvchi barcha $z \in E$ qiymatlarida $|f(z) - A| < \varepsilon$ tengsizlik bajarilsa, u holda A kompleks son $f(z)$ funksiyaning $z \to z_0$ dagi **limiti** deb ataladi va*

$$\lim_{z \to z_0} f(z) = A$$

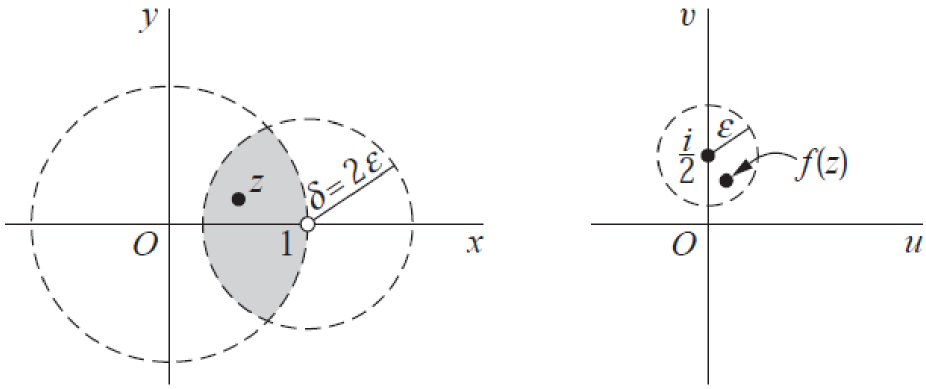
kabi belgilanadi.

Misol. $f(z) = \dfrac{i \cdot \bar{z}}{2}$, $E = \{z \in C : |z| < 1\}$, funksiya uchun

$\lim\limits_{z \to 1} f(z) = \dfrac{i}{2}$ ekanligini ta'rif yordamida ko'rsating.

$\triangleleft$ $\forall z \in E$ uchun $\left| f(z) - \dfrac{i}{2} \right| = \left| \dfrac{i \cdot \bar{z}}{2} - \dfrac{i}{2} \right| = \left| \dfrac{z-1}{2} \right|$. Agar $\forall \varepsilon > 0$ son uchun

$\delta = 2\varepsilon$ son deb olinsa, u holda argument z ning $0 < |z - 1| < \delta$ tengsizlikni qanoatlantiruvchi barcha $z \in E$ qiymatlarida $\left| f(z) - \frac{i}{2} \right| < \varepsilon$ tengsizlik bajarildi (8-chizma) . ▹



8-chizma

$f(z) = u(x, y) + iv(x, y)$ funksiyaning limitini hisoblash $u(x, y)$ va $v(x, y)$ larning limitlarini hisoblashga keltirilishi mumkin.

1-Teorema. *$w = f(z)$ funksiya $z \to z_0 (z_0 = x_0 + iy_0)$ da $A = \alpha + i\beta$ limitga ega ($\lim\limits_{z \to z_0} f(z) = A$) bo'lishi uchun*

$$\lim_{\substack{x \to x_0 \\ y \to y_0}} u(x, y) = \alpha, \quad \lim_{\substack{x \to x_0 \\ y \to y_0}} v(x, y) = \beta$$

bo'lishi zarur va yetarli.

Aytaylik $w = f(z)$ funksiya $E \subset C$ to'plamda berilgan bo'lib, z_0 nuqta shu E to'plamning o'ziga tegishli bo'lgan limit nuqtasi bo'lsin.

4-Ta'rif. *Agar $\forall \varepsilon > 0$ uchun $\exists \delta = \delta(z_0, \varepsilon) > 0$ son topilsaki, argument z ning $|z - z_0| < \delta$ tengsizlikni qanoatlatiruvchi barcha $z \in E$ qiymatlarida*

$$|f(z) - f(z_0)| < \varepsilon$$

tengsizlik bajarilsa u holda $f(z)$ funksiya z_0 nuqtada uzluksiz deb ataladi.(Ravshanki ,bu xolda $\lim\limits_{z \to z_0} f(z) = f(z_0)$ bo'ladi)

Odatda $z-z_0$ ayirma *funksiya* ***argumentining orttirmasi*** deyiladi, uni Δz kabi belgilanadi: $\Delta z = z - z_0$, $f(z)-f(z_0)$ ayirma esa ***funksiya orttirmasi*** deyilib uni Δf kabi belgilanadi:

$$\Delta f = f(z) - f(z_0).$$

Shu tushunchalardan foydalanib, z_0 nuqtadagi funksiya uzluksizligi 4-ta'rifini quyidagicha ham aytish mumkin:

Agar

$$\lim_{\Delta z\to 0} \Delta f = 0$$

bo'lsa, $f(z)$ funksiya z_0 nuqtada ***uzluksiz*** deyiladi.

5-Ta'rif. *Agar $f(z)$ funksiya E to'plamning har bir nuqtasida uzluksiz bo'lsa, u holda $f(z)$ funksiya E to'plamda uzluksiz deyiladi.*

2-Teorema. $f(z) = u(x.y) + iv(x,y)$ *funksiyaning* $z_0 = x_0 + iy_0$ *nuqtada uzluksiz bo'lishi uchun $u = u(x,y)$ hamda $v = v(x,y)$ funksiyalarning (x_0, y_0) nuqtada uzluksiz bo'lishi zarur va yetarli.*

$w = f(z)$ funksiya $E \subset C$ to'plamda berilgan bo'lsin.

6-Ta'rif. *Agar $\forall\varepsilon > 0$ son uchun $\exists\delta = \delta(\varepsilon) > 0$ son topilsaki, E to'plamning $|z' - z''| < \delta$ tengsizlikni qanoatlantiruvchi ixtiyoriy $z', z'' \in E$ nuqtalarida*

$$|f(z') - f(z'')| < \varepsilon$$

tengsizlik bajarilsa, $f(z)$ funksiya E to'plamda ***tekis uzluksiz*** *deyiladi.*

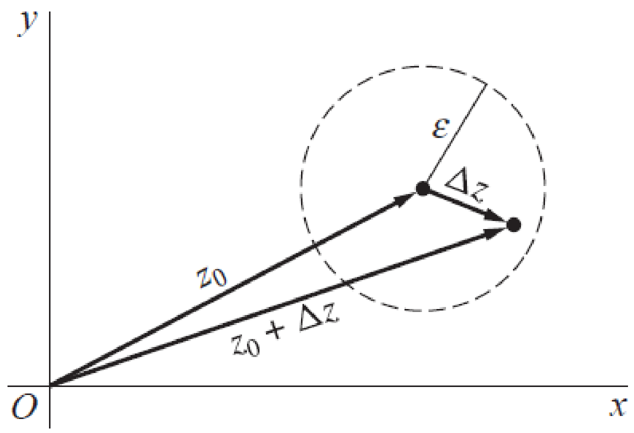
3-Teorema. (Kantor teoremasi). *Agar $f(z)$ funksiya chegaralangan yopiq to'plamda uzluksiz bo'lsa, funksiya shu to'plamda tekis uzluksiz bo'ladi.*

6⁰. Funksiyaning differensiallanuvchiligi. **Koshi-Riman shartlari**.

Biror $E \subset C$ sohada $w = f(z)$ funksiya berilgan bo‘lsin. Ixtiyoriy $z_0 \in E$ nuqta olib, unga shunday Δz orttirma beraylikki, $z_0 + \Delta z \in E$ bo‘lsin (9-chizma). Natijada, $f(z)$ funksiya ham z_0 nuqtada

$$\Delta w = \Delta f(z_0) = f(z_0 + \Delta z) - f(z_0)$$

orttirmasiga ega bo‘ladi.



9-chizma

1-Ta’rif. *Agar* $\Delta z \to 0$ *da* $\frac{\Delta w}{\Delta z}$ *nisbatning limiti*

$$\lim_{\Delta z \to 0} \frac{\Delta w}{\Delta z} = \lim_{\Delta z \to 0} \frac{f(z_0 + \Delta z) - f(z_0)}{\Delta z}$$

mavjud va chekli bo‘lsa, bu limit kompleks o‘zgaruvchili $f(z)$ *funksiyaning* z_0 *nuqtadagi hosilasi deb ataladi va* $f'(z_0)$ *kabi belgilanadi:*

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{f(z_0 + \Delta z) - f(z_0)}{\Delta z} \qquad (11)$$

Faraz qilaylik, $f(z) = u(x.y) + iv(x.y)$ funksiya $z_0 = x_0 + iy_0$ **(**$z_0 \in C$**)** nuqtaning biror atrofida aniqlangan bo‘lsin.

2-Ta’rif. *Agar* $u(x, y)$ *va* $v(x, y)$ *funksiyalar* x, y *o‘zgaruvchilarning funksiyasi sifatida* (x_0, y_0) *nuqtada differensiallanuvchi bo‘lsa,* $f(z)$ *funksiya* z_0 *nuqtada haqiqiy analiz ma’nosida differensiallanuvchi deyiladi.*

Bu holda $du(x_0, y_0) + idv(x_0, y_0)$ ifoda $f(z)$ funksiyaning z_0 nuqtadagi *differensiali* deyiladi:

$$df = du + idv.$$

Teorema. $f(z) = u(x, y) + iv(x, y)$ *funksiyaning* z_0 *nuqtada* $f'(z_0)$ *hosilaga ega bo'lishi uchun bu funksiyaning* $z_0(x_0, y_0)$ *nuqtada haqiqiy analiz ma'nosida differensiallanuvchi bo'lib,*

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases} \qquad (12)$$

shartlarning bajarilishi zarur va yetarli.

Odatda (12) shartlar *Koshi-Riman shartlari* deyiladi.

Kompleks analizda ushbu $dz = dx + idy$, $d\bar{z} = dx - idy$,

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right) \qquad \frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right)$$

belgilashlar yordamida $f(z) = u(x, y) + iv(x, y)$ funksiyaning to'la differensiali $df = du + idv$, $df = \dfrac{\partial f}{\partial z}dz + \dfrac{\partial f}{\partial \bar{z}}d\bar{z}$

ko'rinishda qulay ifodalanadi.

Yuqorida keltirilgan (12) – Koshi-Riman shartlari

$$\frac{\partial f}{\partial \bar{z}} = 0 \qquad (13)$$

tenglikka ekvivalent bo'ladi.

Agar $w = f(z)$ funksiya z_0 nuqtada hosilaga ega bo'lsa, bu nuqtada $\dfrac{\partial f}{\partial \bar{z}} = 0$

bo'lib, f ning hosilasi $f'(z_0) = \dfrac{\partial f}{\partial z}$, differensiali esa

$$df = \frac{\partial f}{\partial z} dz = f'(z_0) dz$$

ko'rinishda bo'ladi. Kompleks analizda hosilaga ega bo'lgan funksiyalar *C – differensiallanuvchi* funksiyalar deyiladi.

Amaliyotda funksiyalarni C-differensiallanuvchilikka tekshirishda Koshi-Riman shartlaridan foydalaniladi.

Qutb koordinatlar sistemasida

$$f(z) = u(x, y) + iv(x, y) = u(\rho\cos\varphi, \rho\sin\varphi) + iv(\rho\cos\varphi, \rho\sin\varphi)$$

funksiya uchun Koshi-Riman shartlari

$$\begin{cases} \dfrac{\partial u}{\partial \rho} = \dfrac{1}{\rho} \cdot \dfrac{\partial v}{\partial \varphi} \\ \\ \dfrac{\partial v}{\partial \rho} = -\dfrac{1}{\rho} \dfrac{\partial u}{\partial \varphi} \end{cases} \qquad (14)$$

ko'rinishda bo'ladi.

Faraz qilaylik, $w = f(z)$ funksiya biror $E \subset C$ sohada berilgan bo'lsin.

3-Ta'rif. *Agar $f(z)$ funksiya $z_0 \in C$ nuqtaning biror $U(z_0, \varepsilon)$ atrofida S-differensiallanuvchi bo'lsa, u holda $f(z)$ funksiya z_0 nuqtada golomorf funksiya deyiladi.*

4-Ta'rif. *Agar $f(z)$ funksiya E sohaning har bir nuqtasida golomorf bo'lsa, funksiya E sohada golomorf deyiladi.*

Odatda E sohada golomorf funksiyalar sinfi $O(E)$ kabi belgilanadi.

5-Ta'rif. *Agar $g(z) = f(\frac{1}{z})$ funksiya $z = 0$ nuqtada golomorf bo'lsa, $f(z)$ funksiya "∞" nuqtada golomorf deyiladi.*

6-Ta'rif. *Agar $\overline{f(z)}$ funksiya $z_0 \in C$ nuqtada golomorf bo'lsa, $f(z)$ funksiya z_0 nuqtada antigolomorf deyiladi.*

7^0. **Garmonik funksiyalar**.

Faraz qilaylik, R^2 fazodagi $E(E \subset R^2)$ sohada $F = F(x, y)$ funksiya berilgan bo‘lib, u shu sohada ikkinchi tartibli $\frac{\partial^2 F(x,y)}{\partial x^2}$, $\frac{\partial^2 F(x,y)}{\partial y^2}$ uzluksiz xususiy hosilalarga ega bo‘lsin.

Ta’rif. *Agar E sohaning har bir nuqtasida*

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = 0 \qquad (15)$$

tenglik bajarilsa, $F = F(x, y)$ funksiya E sohada garmonik funksiya deyiladi.

(15) – tenglamani *Laplas tenglamasi* deyiladi. Bu tenglama ushbu

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Laplas operatori yordamida $\Delta F = 0$ shaklda ham yoziladi. Laplas operatori uchun

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \left(\frac{\partial}{\partial x} - \frac{i\partial}{\partial y}\right) \cdot \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) = 4\frac{\partial^2}{\partial z \partial \bar{z}}$$

bo‘lishini e’tiborga olsak, unda (15) - tenglikni

$$\frac{\partial^2 F}{\partial z \partial \bar{z}} = 0 \qquad (16)$$

shaklda yozish mumkinligini ko‘ramiz.

Teorema. *$E \subset C$ sohada golomorf bo‘lgan har qanday $f(z)$ funksiyaning haqiqiy va mavhum qismlari $u(x, y)$ va $v(x, y)$ funksiyalar shu sohada garmonik bo‘ladilar*.

Eslatma. Ixtiyoriy ikkita $u(x, y)$ va $v(x, y)$ garmonik funksiyalar uchun $f(z) = \mathrm{u(x, y)} + \mathrm{iv(x, y)}$ funksiyaning golomorf bo‘lishi shart emas. f ning golomorf bo‘lishi uchun u va v lar Koshi-Riman shartlari orqali bog‘langan bo‘lishlari lozim. Bunday holda u va v garmonik funksiyalar qo‘*shma garmonik funksiyalar* deyiladi.

Bir bog‘lamli $E \subset C$ sohada $u(z) = u(x, y)$ garmonik funksiya bo‘lib, $z_0 \in E$ tayinlangan nuqta bo‘lsin. U holda

$$v(z) = \int_{z_0}^{z} -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy \qquad (17)$$

ikkinchi tur egri chiziqli integral $u(z)$ funksiyaga qo‘shma garmonik funksiya $v(z)$ ni aniqlaydi.

8⁰. Hosila moduli va argumentining geometrik ma’nosi. Konform akslantirishlar.

Faraz qilaylik, $w = f(z)$ funksiya biror $E \subset C$ sohada berilgan bo‘lsin. Uni (z) tekislikning nuqtalarini (w) tekislik nuqtalariga akslantirish deb qaraymiz. Aytaylik, $w = f(z)$ funksiya $z_0 \in E$ nuqtada $f'(z_0)$ $(f'(z_0) \neq 0)$ hosilaga ega bo‘lsin. Unda $w = f(z)$ akslantirish yordamida $|z - z_0| = r$ aylana, cheksiz kichik miqdor $o(|z - z_0|)$ e’tiborga olinmasa

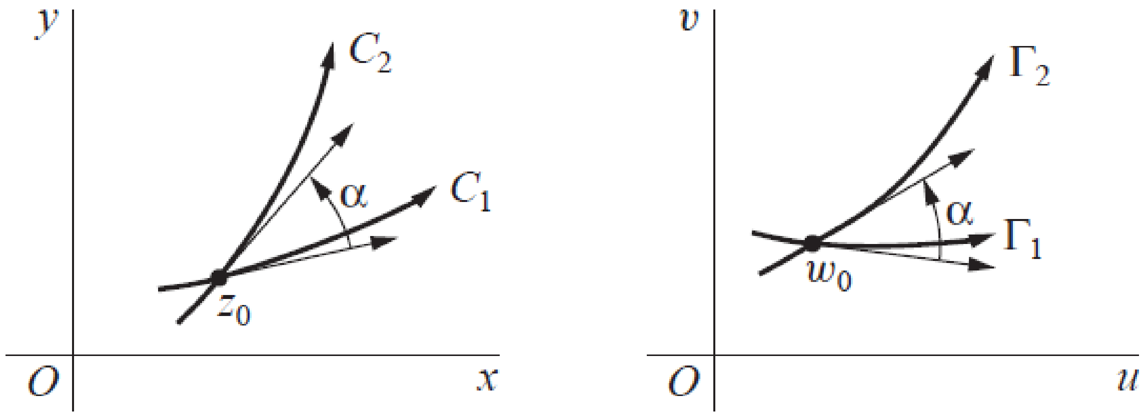
$$|w - w_0| = |f'(z_0)| \cdot r$$

aylanaga akslanadi. Agar $|f'(z_0)| < 1$ bo‘lsa, unda $|z - z_0| = r$ *aylana siqiladi*, $|f'(z_0)| > 1$ bo‘lganda esa *aylana cho‘ziladi*.

Demak, funksiya hosilasining moduli $w = f(z)$ akslantirishda *«cho‘zilish koeffitsientini»* bildirar ekan.

Endi $w = f(z)$ akslantirish z_0 nuqtadan o‘tuvchi γ silliq chiziqni (w) tekislikdagi Γ chiziqqa akslantirsin. Bu holda funksiya hosilasining argumenti $w = f(z)$ akslantirishda γ chiziqni qanday burchakka burishini bildiradi.

$f'(z_0) \neq 0$ bo‘lgan holda (z_0) nuqtadan o‘tuvchi ikki C_1 va C_2 egri chiziqlar orasidagi burchak α bo‘lsa, $w = f(z)$ akslantirishda bu chiziqlarning akslari Γ_1 va Γ_2 lar orasidagi burchak ham α ga teng bo‘ladi(10-chizma).



10-chizma

Aytaylik, $w = f(z)$ funksiya $E \subset C$ sohada berilgan bo‘lib, $z_0 \in E$ bo‘lsin.

1-Ta’rif. *Agar* $w = f(z)$ *akslantirish*

1) markazi z_0 *nuqtada bo‘lgan cheksiz kichik aylanani cheksiz kichik aylanaga o‘tkazish xossasiga,*

2) z_0 *nuqtadan o‘tuvchi har qanday ikkita chiziq orasidagi burchakning miqdorini ham, yo‘nalishini ham saqlash xosssasiga ega bo‘lsa,* $w = f(z)$ *akslantirish* z_0 *nuqtada konform akslantirish deb ataladi.*

Agar bu ta’rifdgi 2-shartda burilish burchagining miqdori o‘zgarmay, yo‘nalishi qarama-qarshisiga o‘zgarsa, bunday akslantirish *II-tur konform akslantirish* deyiladi.

2-Ta’rif. *Agar* $E \subset C$ *sohada aniqlangan* $w = f(z)$ *akslantirish uchun*

1) $w = f(z)$ *funksiya* E *sohada bir yaproqli funksiya,*

2) E *sohaning har bir nuqtasida konform bo‘lsa, berilgan akslantirish* E *sohada konform akslantirish deb ataladi.*

Konform akslantirishlar quyidagi xossalarga ega:

1) Konform akslantirishga teskari bo‘lgan akslantirish ham konform akslantirish bo‘ladi.

2) Chekli sondagi konform akslantirishlarning superpozitsiyasi yana konform akslantirish bo‘ladi.

Teorema. *Agar* $w = f(z)$ *akslantirish* $E \subset C$ *sohada bir yaproqli bo‘lib,* $f'(z) \neq 0$ *bo‘lsa, u holda akslantirish shu sohada konform bo‘ladi.*

Nazorat savollari.

1. Kompleks sonlar ustida amallar.
2. Kompleks sonni geometrik tasvirlash.
3. Kompleks sonning moduli va argumentini hisoblash.
4. Kompleks sonning trigonometrik va ko‘rsatkichli ko‘rinishlari.
5. Muavr formulasi.
6. Kompleks tekislikda egri chiziq tushunchasi.
7. Kompleks tekislikda soha tushunchasi.
8. Bir bog‘lamli va ko‘p bog‘lamli sohalar.
9. Stereografik proeksiya.
10. Sferik masofa tushunchasi.
11. Kompleks argumentli funksiya, murakkab va teskari funksiya tushunchalari.
12. Bir yaproqli funksiya ta’rifi va unga misollar.
13. Kompleks argumentli funksiyaning limiti va uzluksizligi.
14. Haqiqiy analiz ma’nosida differensiallanuvchilik ta’rifi.
15. C–differensiallanuvchilik ta’rifi.
16. Golomorf funksiyalar.
17. Garmonik funksiyalar.
18. Golomorf va garmonik funksiyalar orasidagi bog‘lanish.
19. Qo‘shma garmonik funksiyalar va ularni topish.
20. Hosila modulining geometrik ma’nosi.
21. Hosila argumentining geometrik ma’nosi.
22. Nuqtada va sohada konform akslantirishlar.

- B -

MUSTAQIL YECHISH UCHUN MISOL VA MASALALAR

1-Masala. Quyidagi z_1 va z_2 kompleks sonlarning yig‘indisi, ayirmasi, ko‘paytmasi, nisbati hamda $z_1+\frac{1}{z_2}$ ni toping:

1.1 $z_1=\sqrt{2}+i$, $z_2=\sqrt{2}-i$.

1.2. $z_1=1+\sqrt{2}i$, $z_2=1-i\sqrt{2}$.

1.3. $z_1=2+3i$, $z_2=2-3i$.

1.4 $z_1=2+i\sqrt{3}$, $z_2=2-i\sqrt{3}$.

1.5. $z_1=3+4i$, $z_2=3-4i$.

1.6. $z_1=5+2i$, $z_2=5-2i$.

1.7. $z_1=2+i\sqrt{3}$, $z_2=3+i\sqrt{2}$.

1.8. $z_1=2-i\sqrt{3}$, $z_2=3-i\sqrt{2}$.

1.9. $z_1=\sqrt{2}+i\sqrt{3}$, $z_2=\sqrt{3}+i\sqrt{2}$.

1.10. $z_1=3+4i$, $z_2=4+3i$.

1.11. $z_1=3-4i$, $z_2=4-3i$.

1.12. $z_1=1+\sqrt{5}i$, $z_2=1-\sqrt{5}i$.

1.13. $z_1=2+\sqrt{5}i$, $z_2=2-\sqrt{5}i$.

1.14. $z_1=2+\sqrt{5}i$, $z_2=\sqrt{5}+2i$.

1.15. $z_1=2-\sqrt{5}i$, $z_2=\sqrt{5}+2i$.

1.16. $z_1=3-\sqrt{5}i$, $z_2=3+\sqrt{5}i$.

1.17. $z_1=\sqrt{3}+\sqrt{5}i$, $z_2=\sqrt{3}-\sqrt{5}i$.

1.18. $z_1=\sqrt{5}+\sqrt{3}i$, $z_2=\sqrt{5}-\sqrt{3}i$.

1.19. $z_1 = 3+\sqrt{5}i$, $\qquad z_2 = 3-\sqrt{5}i$.

1.20. $z_1 = \sqrt{5}+3i$, $\qquad z_2 = \sqrt{5}+3i$.

1.21. $z_1 = \sqrt{3}+i\sqrt{2}$, $\qquad z_2 = \sqrt{3}-i\sqrt{2}$.

2-Masala. Amallarni bajaring, hosil bo‘lgan kompleks sonlarning moduli va argumentini topib, ularni kompleks tekislikda tasvirlang.

2.1. $(\sqrt{3}+i\sqrt{3})^6\cdot(1+i)^3$.

2.2. $(\sqrt{3}+i\sqrt{3})^4\cdot(1-i)^4$.

2.3. $(-\sqrt{3}+3i)^6\cdot(3+i\sqrt{3})^4$.

2.4. $(-\sqrt{3}-3i)^3\cdot(3+i\sqrt{3})^6$.

2.5. $(\sqrt{3}+3i)^5\cdot(3+i\sqrt{3})^3$.

2.6. $(\sqrt{3}-3i)^4\cdot(3+i\sqrt{3})^6$.

2.7. $(\sqrt{3}+3i)^3\cdot(1+i)^5$.

2.8. $(\sqrt{3}+3i)^4\cdot(1-i)^5$.

2.9. $(3+i\sqrt{3})^4\cdot(1+i)^5$.

2.10. $(3+i\sqrt{3})^3\cdot(1-i)^5$.

2.11. $(-1+i\dfrac{\sqrt{3}}{3})^6\cdot(1+i)^3$.

2.12. $(-1+i\dfrac{\sqrt{3}}{3})^4\cdot(1-i)^4$.

2.13. $(1-i\dfrac{\sqrt{3}}{3})^6\cdot(1+i)^4$.

2.14. $(1-i\dfrac{\sqrt{3}}{3})^3\cdot(1+i)^6$.

2.15. $(1+i\dfrac{\sqrt{3}}{3})^5\cdot(1+i)^3$.

2.16. $(1+i\dfrac{\sqrt{3}}{3})^4\cdot(1-i)^6$.

2.17. $(-1+i)^3\cdot(1+i\sqrt{3})^5$.

2.18. $(-1+i)^4\cdot(1-i\sqrt{3})^5$.

2.19. $(1+i)^4\cdot(1+i\sqrt{3})^5$.

2.20. $(1-i)^3\cdot(1-i\sqrt{3})^5$.

2.21. $(1-i)^3\cdot(1+i\sqrt{3})^8$.

3-Masala. Quyidagi tengsizliklarni qanoatlantiruvchi barcha nuqtalar to‘plamini kompleks tekislik C da tasvirlang.

3.1. $1<|z+2-3i|\le 3$.

3.2. $1\le|z+1+i|<2$.

3.3. $1<|z-1+i|\le 2$.

3.4. $1\le|z+1-i|<2$.

3.5. $1<|z-3+4i|\le 3$.

3.6. $1\le|z+3-4i|<3$.

3.7. $1<|z-1+2i|\le 2$.

3.8. $1\le|z+1-2i|\le 3$.

3.9. $1<|z-2+i|\le 2$.

3.10. $1\le|z+2-i|\le 3$.

3.11. $1<|z+2+i|\le 2$.

3.12. $1<|z-2-3i|\le 3$.

3.13. $2\le|z-1-3i|<3$.

3.14. $2<|z+1-3i|\le 3$.

3.15. $2\le|z+1+3i|<3$.

3.16. $2<|z-1+3i|\le 3$.

3.17. $2\le|z-3i|<3$.

3.18. $2<|z+3-i|\le 3$.

3.19. $2\le|z+3|<3$.

3.20. $2<|z-3+i|\le 3$.

3.21. $1<|z-2+3i|\le 3$.

4-Masala. Quyidagi tengsizliklar sistemasini qanoatlantiruvchi nuqtalar to‘plamini C tekislikda tasvirlang.

4.1. $\begin{cases}(\operatorname{Im} z)^2<2\operatorname{Re} z,\\ (\operatorname{Re} z)^2\le \operatorname{Im} z.\end{cases}$

4.2. $\begin{cases}(\operatorname{Im} z)^2<\operatorname{Re} z,\\ \operatorname{Re} z+\operatorname{Im} z\le 3.\end{cases}$

4.3. $\begin{cases}(\operatorname{Im} z)^2<\operatorname{Re} z,\\ |z|<2.\end{cases}$

4.4. $\begin{cases}(\operatorname{Im} z)^2<\operatorname{Re} z,\\ \dfrac{\pi}{4}\le \arg z<\dfrac{\pi}{2}\end{cases}$

4.5. $\begin{cases}(\operatorname{Im} z)^2<\operatorname{Re} z,\\ |z|>1.\end{cases}$

4.6. $\begin{cases}(\operatorname{Re} z)^2\le \operatorname{Im} z,\\ \operatorname{Re} z+\operatorname{Im} z\le 4.\end{cases}$

4.7. $\begin{cases}(\operatorname{Re} z)^2\le \operatorname{Im} z,\\ |z-i|<1.\end{cases}$

4.8. $\begin{cases}(\operatorname{Re} z)^2\le \operatorname{Im} z,\\ |z-1|<1.\end{cases}$

4.9. $\begin{cases}(\operatorname{Re} z)^2\le \operatorname{Im} z,\\ \dfrac{\pi}{3}<\arg z<\pi\end{cases}$

4.10. $\begin{cases}|z-1|<1,\\ |z-i|<1.\end{cases}$

4.11. $\begin{cases}|z-1|<1,\\ \dfrac{\pi}{4}\le \arg z<\dfrac{\pi}{2}.\end{cases}$

4.12. $\begin{cases}|z-i|>2,\\ \dfrac{\pi}{4}\le \arg z<\dfrac{\pi}{3}\end{cases}$

4.13. $\begin{cases} \operatorname{Re} z + \operatorname{Im} z > 1, \\ |z| < 2. \end{cases}$

4.14. $\begin{cases} \operatorname{Re} z + \operatorname{Im} z \le 3, \\ \dfrac{\pi}{3} < \arg z < \dfrac{\pi}{2} \end{cases}$

4.15. $\begin{cases} \dfrac{\pi}{4} < \arg z \le \dfrac{\pi}{2} \\ 1 < |z| \le 3. \end{cases}$

4.16. $\begin{cases} 1 < |z| \le 3, \\ (\operatorname{Im} z)^2 < \operatorname{Re} z. \end{cases}$

4.17. $\begin{cases} 1 < |z| \le 3, \\ (\operatorname{Re} z)^2 < \operatorname{Im} z. \end{cases}$

4.18. $\begin{cases} 1 < |z| \le 2, \\ \operatorname{Im} z > 0 \end{cases}$

4.19. $\begin{cases} 1 < |z| \le 2, \\ \operatorname{Re} z > 0. \end{cases}$

4.20. $\begin{cases} 1 < |z| \le 2, \\ \operatorname{Im} z > \operatorname{Re} z. \end{cases}$

4.21. $\begin{cases} \operatorname{Re} z + \operatorname{Im} z > 1, \\ 0 < \arg z < \dfrac{\pi}{4} \end{cases}$

5-Masala. Quyidagi tenglama aylananing tenglamasi ekanligini isbotlang va bu aylana markazining koordinatalari hamda radiusini toping**.**

5.1. $|z|^2 + (1-i)z + (1+i)\overline{z} + 1 = 0$**.**

5.2. $|z|^2 + (1-4i)z + (1+4i)\overline{z} + 6 = 0$**.**

5.3. $|z|^2 + (2-3i)z + (2+3i)\overline{z} + 11 = 0$**.**

5.4. $|z|^2 + (3-2i)z + (3+2i)\overline{z} + 12 = 0$**.**

5.5. $|z|^2 + (4-3i)z + (4+3i)\overline{z} + 20 = 0$**.**

5.6. $|z|^2 + (2-4i)z + (2+4i)\overline{z} + 9 = 0$**.**

5.7. $|z|^2 + (4-5i)z + (4+5i)\overline{z} + 21 = 0$**.**

5.8. $|z|^2 + (4-i)z + (4+i)\overline{z} + 16 = 0$.

5.9. $|z|^2 + (3-i)z + (3+i)\overline{z} + 9 = 0$.

5.10. $|z|^2 + (4-4i)z + (4+4i)\overline{z} + 24 = 0$.

5.11. $|z|^2 + (1-5i)z + (1+5i)\overline{z} + 25 = 0$.

5.12. $|z|^2 + (5-i)z + (5+i)\overline{z} + 25 = 0$.

5.13. $|z|^2 + (2-5i)z + (2+5i)\overline{z} + 28 = 0$.

5.14. $|z|^2 + (5-2i)z + (5+2i)\overline{z} + 28 = 0$.

5.15. $|z|^2 + (3-5i)z + (3+5i)\overline{z} + 25 = 0$.

5.16. $|z|^2 + (5-3i)z + (5+3i)\overline{z} + 25 = 0$.

5.17. $|z|^2 + (5-4i)z + (5+4i)\overline{z} + 25 = 0$.

5.18. $|z|^2 + (2-2i)z + (2+2i)\overline{z} + 7 = 0$.

5.19. $|z|^2 + (3-3i)z + (3+3i)\overline{z} + 16 = 0$.

5.20. $|z|^2 + (4-4i)z + (4+4i)\overline{z} + 28 = 0$.

5.21. $|z|^2 + (4-3i)z + (4+3i)\overline{z} + 21 = 0$.

6-Masala. **C** kompleks tekisligidagi z nuqtaning **S** Riman sferasidagi obrazini toping**.**

6.1. $1+i$.	**6.2.** $1-i$.	**6.3.** $2+i$.
6.4. $2i+1$.	**6.5.** $2-i$.	**6.6.** $-2+i$.
6.7. $2+2i$.	**6.8.** $2-2i$.	**6.9.** $3+i$.
6.10. $3-i$.	**6.11.** $-1+i$.	**6.12.** $1+3i$.
6.13. $1-3i$.	**6.14.** $3+2i$.	**6.15.** $3-2i$.
6.16. $2+3i$.	**6.17.** $2-3i$.	**6.18.** $-2+3i$.

6.19. $-2-3i$. **6.20.** $3-3i$. **6.21.** $\frac{1+i}{\sqrt{2}}$.

7-Masala. **H**isoblang**.**

7.1. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{2^k}\sin(\frac{k\pi}{4})$. **7.2.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{2^k}\sin(\frac{k\pi}{3})$.

7.3. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{2^k}\cos(\frac{k\pi}{3})$. **7.4.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{2^k}{3^k}\cos(\frac{k\pi}{4})$.

7.5. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{2^k}{3^k}\sin(\frac{k\pi}{4})$. **7.6.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{2^k}{3^k}\cos(\frac{k\pi}{3})$.

7.7. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{2^k}{3^k}\sin(\frac{k\pi}{3})$. **7.8.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{2^k}{3^k}\cos(\frac{k\pi}{6})$.

7.9. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{2^k}{3^k}\sin(\frac{k\pi}{6})$. **7.10.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{3^k}\cos(\frac{k\pi}{3})$.

7.11. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{3^k}\sin(\frac{k\pi}{3})$. **7.12.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{3^k}\cos(\frac{k\pi}{4})$.

7.13. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{3^k}\sin(\frac{k\pi}{4})$. **7.14.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{3^k}\cos(\frac{k\pi}{6})$.

7.15. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{3^k}\sin(\frac{k\pi}{6})$. **7.16.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{4^k}\cos(\frac{k\pi}{3})$.

7.17. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{4^k}\sin(\frac{k\pi}{3})$. **7.18.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{4^k}\cos(\frac{k\pi}{4})$.

7.19. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{4^k}\sin(\frac{k\pi}{4})$. **7.20.** $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{3^k}{4^k}\cos(\frac{k\pi}{4})$.

7.21. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{2^k}\cos(\frac{k\pi}{4})$.

8-Masala. Quyidagi funksiyalar aniqlagan egri chiziqlarni toping**.**

8.1. $z = t + it^2 \quad (0 \le t < +\infty)$.

8.2. $z = 2t + it^2 \quad (0 \le t < +\infty)$.

8.3. $z = t + i2t^2 \qquad (0 \le t < +\infty)$.

8.4. $z = t + \frac{i}{t} \qquad (-\infty < t < 0)$.

8.5. $z = t + \frac{i}{t} \qquad (0 < t < +\infty)$.

8.6. $z = 2t + \frac{i}{t} \qquad (-\infty < t < 0)$.

8.7. $z = t + \frac{i}{2t} \qquad (0 < t < +\infty)$.

8.8. $z = 4t^2 + it^4 \qquad (-\infty < t < +\infty)$.

8.9. $z = t^2 + i\frac{t}{16}^4 \qquad (-\infty < t < +\infty)$.

8.10. $z = 9t^2 + it^4 \qquad (-\infty < t < +\infty)$.

8.11. $z = \mathrm{Re} \cdot e^{i2t} \qquad (0 \le t \le \frac{\pi}{4})$.

8.12. $z = \mathrm{Re}\, e^{i3t} \qquad (0 \le t \le \frac{\pi}{6})$.

8.13. $z = \mathrm{Im}\, e^{i2t} \qquad (0 \le t \le \frac{\pi}{4})$.

8.14. $z = \mathrm{Im}\, e^{i3t} \qquad (0 \le t \le \frac{\pi}{6})$.

8.15. $z = 2t \qquad (0 \le t \le 3)$.

8.16. $z = 2 + it \qquad (2 \le t \le 5)$.

8.17. $z = t + 3i \qquad (1 \le t \le 2)$.

8.18. $z = 2 + i + [(3 + 2i) - (2 + i)]t \qquad (0 \le t \le 1)$.

8.19. $z = 3 + 2i + [(5 + 4i) - (3 + 2i)]t \qquad (0 \le t \le 1)$.

8.20. $z = 3 + 2i + [(4 + 4i) - (3 + 2i)]t \qquad (0 \le t \le 1)$.

8.21. $z = 1 + i + [(2 + 3i) - (1 + i)]t \qquad (0 \le t \le 1)$.

9-Masala. Quyidagi $f(z)$ funksiyalarni berilgan sohalarda bir yaproqlikka tekshiring**.**

9.1. $f(z)=z^2;\qquad E=\{\operatorname{Re} z>0\}$

9.2. $f(z)=z^2;\qquad E=\{\operatorname{Im} z>0\}$.

9.3. $f(z)=z^3;\qquad E=\{0<\arg z<\frac{\pi}{2}\}$.

9.4. $f(z)=z^2;\qquad E=\{|z|<1\}$.

9.5. $f(z)=z^2;\qquad E=\{|z|<1, 0<\arg z<\frac{3\pi}{2}\}$.

9.6. $f(z)=z^2;\qquad E=\{|z|>2\}$.

9.7. $f(z)=\frac{1}{2}(z+\frac{1}{z});\qquad E=\{|z|<1\}$.

9.8. $f(z)=\frac{1}{2}(z+\frac{2}{z});\qquad E=\{|z|<2\}$.

9.9. $f(z)=\frac{1}{2}(z+\frac{2}{z});\qquad E=\{\operatorname{Im} z>0\}$.

9.10. $f(z)=\frac{1}{2}(z+\frac{2}{z});\qquad E=\{\operatorname{Re} z>0\}$.

9.11. $f(z)=\frac{1}{2}(z+\frac{2}{z});\qquad E=\{\frac{\pi}{2}<\arg z<\frac{3\pi}{2}\}$.

9.12. $f(z)=\frac{1}{z+3};\qquad E=\{|z|<3\}$.

9.13. $f(z)=\frac{1}{z+3};\qquad E=\{|z|>3\}$.

9.14. $f(z)=\frac{1}{z+4};\qquad E=\{|z|<4\}$.

9.15. $f(z)=\frac{1}{z+3};\qquad E=\{\operatorname{Re} z>3\}$.

9.16. $f(z)=\dfrac{1}{z+i}$; $E=\{\operatorname{Re} z>1\}$.

9.17. $f(z)=e^{2x}(\cos 2y+i\sin 2y)$; $E=\{\operatorname{Im} z>0\}$.

9.18. $f(z)=e^{2x}(\cos 2y+i\sin 2y)$; $E=\{0<\operatorname{Im} z<\pi\}$.

9.19. $f(z)=e^{2x}(\cos 2y+i\sin 2y)$; $E=\{|z|<1\}$.

9.20. $f(z)=e^{2x}(\cos 2y+i\sin 2y)$; $E=\{0<\operatorname{Re} z<\dfrac{1}{2}\}$.

9.21. $f(z)=\dfrac{1}{2}(z+\dfrac{1}{z})$; $E=\{|z|<2\}$.

10-Masala. Berilgan funksiyalarni uzluksizlikka tekshiring**.**

10.1. $f(z)=\dfrac{1}{z^2-1}$.

10.2. $f(z)=\dfrac{1}{(z-1)(z+i)}$.

10.3. $f(z)=\dfrac{1}{(z+1)(z+i)}$.

10.4. $f(z)=\dfrac{1}{(z-2)(z+i)}$.

10.5. $f(z)=\dfrac{1}{z^2+4}$.

10.6. $f(z)=\dfrac{1}{(z+2)(z+i)}$.

10.7. $f(z)=\dfrac{1}{(z-2)(z-1)}$.

10.8. $f(z)=\dfrac{1}{(z+2)(z-1)}$.

10.9. $f(z)=\dfrac{1}{(z-2)(z-i)}$.

10.10. $f(z)=\dfrac{1}{(z+2)(z-i)}$.

10.11. $f(z)=\dfrac{z}{(z+2)(z+i)}$.

10.12. $f(z)=\dfrac{z}{(z-2)(z+i)}$.

10.13. $f(z)=\dfrac{1}{(2z+1)(z+i)}$.

10.14. $f(z)=\dfrac{1z}{(2z+1)(z-i)}$.

10.15. $f(z)=\dfrac{1}{(2z-1)(z+i)}$.

10.16. $f(z)=\dfrac{1}{(2z-1)(z-i)}$.

10.17. $f(z)=\dfrac{z}{(2z-i)(z-1)}$.

10.18. $f(z)=\dfrac{z}{(2z-i)(z+1)}$.

10.19. $f(z)=\dfrac{z}{(2z+i)(z-1)}$. **10.20.** $f(z)=\dfrac{z}{(3z+i)(z-1)}$.

10.21. $f(z)=\dfrac{1}{z^2+1}$

11-Masala. Funksiya hosilasini ta’rif yordamida hisoblang.

11.1. $f(z)=\dfrac{1}{z+i}\quad (z\neq -i)$. **11.2.** $f(z)=\dfrac{1}{z+1}\quad (z\neq -1)$.

11.3. $f(z)=\dfrac{1}{z-1}\quad (z\neq 1)$. **11.4.** $f(z)=\dfrac{1}{z-i}\quad (z\neq i)$.

11.5. $f(z)=\dfrac{1}{2z-1}\quad (z\neq \dfrac{1}{2})$. **11.6.** $f(z)=\dfrac{1}{2z+1}\quad (z\neq -\dfrac{1}{2})$.

11.7. $f(z)=\dfrac{1}{2z-i}\quad (z\neq \dfrac{i}{2})$. **11.8.** $f(z)=\dfrac{1}{2z+i}\quad (z\neq -\dfrac{i}{2})$.

11.9. $f(z)=z^2$. **11.10.** $f(z)=z^3$.

11.11. $f(z)=z^2+2z$. **11.12.** $f(z)=z^3-z+1$.

11.13. $f(z)=1-3z^2$. **11.14.** $f(z)=z+2z^2$.

11.15. $f(z)=3z\ \ -1$. **11.16.** $f(z)=2z\ \ +3$.

11.17. $f(z)=\dfrac{1}{z}\quad (z\neq 0)$. **11.18.** $f(z)=\dfrac{z}{2}+5$.

11.19. $f(z)=\dfrac{2z}{3}$. **11.20.** $f(z)=e^x(\cos y+i\sin y)$.

11.21. $f(z)=\dfrac{1}{z+2}\quad (z\neq -2)$.

12-Masala. Quyidagi funksiyalarni C-differensiallanuvchanlikka tekshiring

12.1. $f(z)=\operatorname{Re} z$. **12.2.** $f(z)=z^2\operatorname{Re} z$.

12.3. $f(z)=(\operatorname{Re} z)^2$. **12.4.** $f(z)=z^2\operatorname{Im} z$.

12.5. $f(z)=\operatorname{Re} z^2$. **12.6.** $f(z)=z\cdot(\operatorname{Re} z)^2$.

12.7. $f(z)=[\operatorname{Re} z]^2\cdot \operatorname{Im} z$. **12.8.** $f(z)=[\operatorname{Im} z]^2\cdot \operatorname{Re} z$.

12.9. $f(z)=z(\operatorname{Re} z+\operatorname{Im} z)$. **12.10.** $f(z)=\operatorname{Im} z^2$.

12.11. $f(z)=|z|^2$. **12.12.** $f(z)=|\bar{z}|^2$.

12.13. $f(z)=z\operatorname{Re} z$. **12.14.** $f(z)=\bar{z}\cdot \operatorname{Im} z$.

12.15. $f(z)=\operatorname{Im} z$. **12.16.** $f(z)=z$.

12.17. $f(z)=\bar{z}$. **12.18.** $f(z)=2xy-i(x^2+y^2)$.

12.19. $f(z)=2xy+i(x^2+y^2)$. **12.20.** $f(z)=2xy+i(x^2-y^2)$.

12.21. $f(z)=z\operatorname{Im} z$.

13-Masala. Berilgan funksiyalarni golomorflikka tekshiring.

13.1. $f(z)=x+y+i(ax+by)$. **13.2.** $f(z)=x^2-y^2+ibxy$.

13.3. $f(z)=\dfrac{x}{x^2+y^2}-i\dfrac{ay}{x^2+y^2}$. **13.4.** $f(z)=x+2y+i(ax-by)$.

13.5. $f(z)=x-y+i(ax-by)$. **13.6.** $f(z)=x+y+i(ax-y)$.

13.7. $f(z)=a(x^2-y^2)+2ixy$. **13.8.** $f(z)=x^2+ay^2+ibxy$.

13.9. $f(z)=x+y+i(x+ay)$. **13.10.** $f(z)=\dfrac{ax}{x^2+y^2}+i\dfrac{y}{x^2+y^2}$.

13.11. $f(z)=x^2+ay^2-ibxy$. **13.12.** $f(z)=x-y+i(ay+bx)$.

1313. $f(z)=x^2-y^2+iaxy$. **13.14.** $f(z)=ax+by+icy$.

13.15. $f(z)=ax+y+i(bx+cy)$. **13.16.** $f(z)=x^2-ay^2+i2xy$.

13.17. $f(z)=\dfrac{ax}{x^2+y^2}+i\dfrac{by}{x^2+y^2}$. **13.18.** $f(z)=x-2y+i(bx+cy)$.

13.19. $f(z)=ax+i(bx+cy)$. **13.20.** $f(z)=ax+y+i(bx+cy)$.

13.21. $f(z)=x+ay+i(bx+cy)$.

14-Masala. $\gamma - z_0$ nuqtadan chiquvchi $\arg(z - z_0) = \varphi$ nur bo‘lsin. Quyidagi misollardagi akslantirishlar uchun z_0 nuqtadagi cho‘zilish koeffitsenti $R(\varphi)$ va burilish burchagi $\alpha(\varphi)$ ni toping.

14.1. $w = z^2$, $z_0 = i$. **14.2.** $w = \overline{z^2}$, $z_0 = 1$.

14.3. $w = \bar{z} + 2z$ $z_0 = 0$. **14.4.** $w = z^2$, $z_0 = \frac{i}{4}$.

14.5. $w = z^2$, $z_0 = 1 - i$. **14.6.** $w = z^2$, $z_0 = -1 + i$.

14.7. $w = z^3$, $z_0 = i$. **14.8.** $w = z^3$, $z_0 = -\frac{i}{4}$.

14.9 $w = z^2 + 2\bar{z}$, $z_0 = 1$. **14.10.** $w = z^2 - 2\bar{z}$, $z_0 = i$.

14.11 . $w = e^{2x}(\cos 2y + \sin 2y)$; $z_0 = 0$.

14.12 . $w = e^{2x}(\cos 2y - i\sin 2y)$; $z_0 = 0$.

14.13 . $w = \frac{z-1}{z+1}$, $z_0 = 1$. **14.14.** $w = \frac{z-(1+i)}{z+1+i}$, $z_0 = 1 + i$.

14.15. $w = \frac{z-2+i}{z+2-i}$, $z_0 = 2 - i$. **14.16.** $w = \frac{z-2i}{z+2i}$, $z_0 = 2i$.

14.17. $w = \frac{z+2}{z-2}$, $z_0 = -2$. **14.18.** $w = \frac{z-2}{z+2}$, $z_0 = 2$.

14.19. $w = \frac{z+2i}{z-2i}$, $z_0 = -2i$. **14.20.** $w = \frac{z+1-i}{z-1+i}$, $z_0 = -1 + i$.

14.21. $w = \frac{z-i}{z+i}$, $z_0 = i$.

15-Masala. Quyida berilgan $u(x, y)$ garmonik funksiyalarga ko‘rsatilgan sohalarda qo‘shma garmonik bo‘lgan $v(x, y)$ funksiyalarni toping va ular yordamida golomorf $f(z) = u(x, y) + iv(x, y)$ funksiyani quring.

15.1. $u(x, y) = 4xy$ $E = C$. **15.2.** $u(x, y) = 2x - 3y + 5$, $E = C$.

15.3. $u(x, y) = \frac{2x+y}{3(x^2+y^2)}$, $E = \{0 < |z| < \infty\}$.

15.4. $u(x,y)=2(x^2-y^2)+4xy \quad E=C$.

15.5. $u(x,y)=x^2-y^2-2xy, \quad E=C$.

15.6. $u(x,y)=\dfrac{x-2y}{2(x^2+y^2)}$, $E=\{0<|z|<\infty\}$.

15.7. $u(x,y)=x^2-y^2+x, \quad E=C$.

15.8. $u(x,y)=3(x^2-y^2)-6xy, \quad E=C$.

15.9. $u(x,y)=x+2y-1, \quad E=C$.

15.10 $u(x,y)=\dfrac{x}{x^2+y^2}$, $E=\{0<|z|<\infty\}$.

15.11. $u(x,y)=\dfrac{x+y}{4(x^2+y^2)}$, $E=\{0<|z|<\infty\}$.

15.12. $u(x,y)=y^2-x^2+2xy, \quad E=C$.

15.13. $u(x,y)=x^2-3xy^2, \quad E=C$.

15.14. $u(x,y)=-x+4y-5, \quad E=C$.

15.15. $u(x,y)=xy+1, \quad E=C$.

15.16. $u(x,y)=\dfrac{x+2y}{x^2+y^2}$, $E=\{0<|z|<\infty\}$.

15.17. $u(x,y)=2x^3-6xy^2, \quad E=C$.

15.18. $u(x,y)=x^2-y^2+xy, \; E=C$.

15.19. $u(x,y)=2x+4y-1, \quad E=C$.

15.20. $u(x,y)=y^2-x^2-4xy, \quad E=C$.

15.21. $u(x,y)=2(x^2-y^2)-1, \quad E=C$.

16-Masala. Quyidagi funksiyalarning konformlik sohalari topilsin.

16.1. $f(z)=z+\dfrac{1}{z}$. **16.2.** $f(z)=\dfrac{2z+1}{z-1}$.

16.3. $f(z)=z^2+1$. **16.4.** $f(z)=z^2-1$.

16.5. $f(z)=2z^2+z-1$. **16.6.** $f(z)=z^2-2z$.

16.7. $f(z)=z^3-1$. **16.8.** $f(z)=z^3+1$.

16.9. $f(z)=z^3+3z$ **16.10.** $f(z)=z^3-3z$.

16.11. $f(z)=\dfrac{z-3}{2z+1}$ **16.12.** $f(z)=\dfrac{z+4}{2z-5}$.

16.13. $f(z)=e^x(\cos y+\sin y)$. **16.14.** $f(z)=e^{2x}(\cos 2y+i\sin 2y)$.

16.15. $f(z)=e^{-x}(\cos y-\sin y)$. **16.16.** $f(z)=e^{-2x}(\cos 2y-i\sin 2y)$.

16.17. $f(z)=\dfrac{1}{2}(z+\dfrac{1}{z})$. **16.18** $f(z)=\dfrac{1}{2}(z-\dfrac{1}{z})$.

16.19. $f(z)=3z^2-6z$ **16.20.** $f(z)=z^3-8$.

16.21. $f(z)=4z^2-8z$.

- C -

NAMUNAVIY VARIANT YECHIMI.

Namunaviy variant sifatida 21-variantni olib, undagi misol va masalalarning yechimlarini namuna sifatida keltiramiz.

1.21-Masala. Quyidagi

$$z_1=\sqrt{3}+i\sqrt{2} \quad va \quad z_2=\sqrt{3}-i\sqrt{2}$$

kompleks sonlarning yig‘indisi, ayirmasi, ko‘paytmasi, nisbati hamda $z_1+\dfrac{1}{z_2}$ ni toping.

$\lhd$ $z_1+z_2=(\sqrt{3}+i\sqrt{2})+(\sqrt{3}-i\sqrt{2})=(\sqrt{3}+\sqrt{3})+i(\sqrt{2}-\sqrt{2})=2\sqrt{3}.$

$z_1-z_2=(\sqrt{3}+i\sqrt{2})-(\sqrt{3}-i\sqrt{2})=(\sqrt{3}-\sqrt{3})+i(\sqrt{2}+\sqrt{2})=2\sqrt{2}i.$

$z_1\cdot z_2=(\sqrt{3}+i\sqrt{2})\cdot(\sqrt{3}-i\sqrt{2})=(\sqrt{3})^2-(i\sqrt{2})^2=3-2i^2=5$.

$$\frac{z_1}{z} = \frac{\sqrt{3}+i\sqrt{2}}{\sqrt{3}-i\sqrt{2}} = \frac{(\sqrt{3}+i\sqrt{2})\cdot(\sqrt{3}+i\sqrt{2})}{(\sqrt{3})^2-(i\sqrt{2})^2} = \frac{3+i2\sqrt{3}\cdot\sqrt{2}+2i^2}{3+2} =$$

$$= \frac{(3-2)+i\cdot 2\sqrt{6}}{5} = \frac{1}{5} + i\,\frac{2\sqrt{6}}{5}.$$

$$z_1 + \frac{1}{z_2} = \sqrt{3}+i\sqrt{2} + \frac{1}{\sqrt{3}-i\sqrt{2}} = \sqrt{3}+i\sqrt{2} + \frac{\sqrt{3}+i\sqrt{2}}{3+2} =$$

$$= \sqrt{3}+i\sqrt{2} + \frac{\sqrt{3}}{5} + i\,\frac{\sqrt{2}}{5} = \frac{6\sqrt{3}}{5} + i\,\frac{6\sqrt{2}}{5}. \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

$> z1 := \sqrt{3} + I\sqrt{2}$

$$z1 := \sqrt{3} + \mathrm{I}\sqrt{2}$$

$> z2 := \sqrt{3} - I\sqrt{2}$

$$z2 := \sqrt{3} - \mathrm{I}\sqrt{2}$$

$> a := z1 + z2$

$$a := 2\sqrt{3}$$

$> b := z1 - z2$

$$b := 2\,\mathrm{I}\sqrt{2}$$

$> c := z1 \cdot z2$

$$c := (\sqrt{3} + \mathrm{I}\sqrt{2})\,(\sqrt{3} - \mathrm{I}\sqrt{2})$$

$> c := evalc(c)$

$$c := 5$$

$> d := \frac{z1}{z2}$

$$d := \frac{\sqrt{3} + \mathrm{I}\sqrt{2}}{\sqrt{3} - \mathrm{I}\sqrt{2}}$$

$> d := evalc(d)$

$$d := \frac{1}{5} + \frac{2}{5} I \sqrt{3} \sqrt{2}$$

$> r := z1 + \frac{1}{z2}$

$$r := \sqrt{3} + I\sqrt{2} + \frac{1}{\sqrt{3} - I\sqrt{2}}$$

$> r := evalc(r)$

$$r := \frac{6}{5}\sqrt{3} + \frac{6}{5} I \sqrt{2}$$

2.21-Masala. Amallarni bajaring, hosil bo‘lgan kompleks sonning moduli va argumentini toping, uni kompleks tekislikda tasvirlang $(1-i)^3 \cdot (1+i\sqrt{3})^8$.

$\triangleleft$ Oldin $z_1 = 1-i$ *ва* $z_2 = 1+i\sqrt{3}$ sonlarning moduli va argumentini (2) va (3) – formulalardan foydalanib topib, so‘ng ularni trigonometrik shaklda yozamiz va Muavr formulasidan foydalanamiz:

$$z_1 = 1-i \Rightarrow |z_1| = \sqrt{1^2 + (-1)^2} = \sqrt{2}.$$

$$\arg z_1 = arctg\frac{x}{y} + 2\pi = -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4} \Rightarrow z_1 = \sqrt{2}.$$

$$\cdot(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}) \Rightarrow z_1^3 = (1-i)^3 = 2\sqrt{2}(\cos\frac{21\pi}{4} + i\sin\frac{21\pi}{4}) =$$

$$2\sqrt{2}(-\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}) = 2\sqrt{2}\cdot(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}) = -2(1+i).$$

$$z_2 = 1+i\sqrt{3} \Rightarrow |z_2| = \sqrt{1^2 + (\sqrt{3})^2} = 2, \quad \arg z_2 = arctg\sqrt{3} = \frac{\pi}{3} \Rightarrow z_2 = 1+i\sqrt{3} =$$

$$\cdot = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) \Rightarrow z_2^8 = (1+i\sqrt{3})^8 = 2^8(\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}) =$$

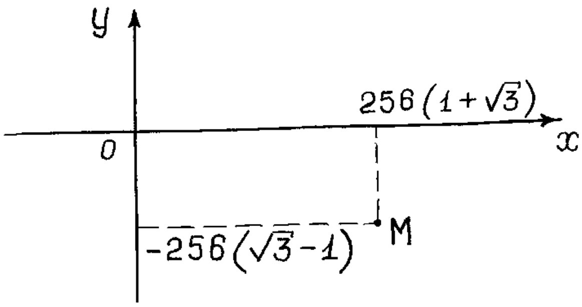
$$= 256(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}) = 256\cdot(-\frac{1}{2} + i\frac{\sqrt{3}}{2}) = -128(1-i\sqrt{3}).$$

Demak,

$$z=(1-i)^3\cdot(1+i\sqrt{3})^8=z_1^3\cdot z_2^8=256(1+i)(1-i\sqrt{3})=$$

$$=256\cdot[(1+\sqrt{3})+i(1-\sqrt{3})]=256\cdot(1+\sqrt{3})-i\cdot 256\cdot(\sqrt{3}-1).$$

Bu kompleks son tekislikda $M(256(1+\sqrt{3}),\,-256(\sqrt{3}-1))$ nuqtani ifodalaydi (11-chizma)▹



11 - chizma

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $z1 := 1 - I$

$$z1 := 1 - \mathrm{I}$$

> $|z1|$

$$\sqrt{2}$$

> argument$(z1)$

$$-\frac{1}{4}\pi$$

> $z2 := 1 + I\sqrt{3}$

$$z2 := 1 + \mathrm{I}\sqrt{3}$$

> $|z2|$

$$2$$

> argument$(z2)$

$$\frac{1}{3}\pi$$

$> a := (z1)^3$

$$a := -2 - 2\,I$$

$> b := (z2)^8$

$$b := \left(1 + I\sqrt{3}\right)^8$$

$> b := evalc\left(\left(1 + I\sqrt{3}\right)^8\right)$

$$b := -128 + 128\,I\sqrt{3}$$

$> c := a \cdot b$

$$c := (-2 - 2\,I)\left(-128 + 128\,I\sqrt{3}\right)$$

$> c := evalc(c)$

$$c := 256 + 256\sqrt{3} + I\left(256 - 256\sqrt{3}\right)$$

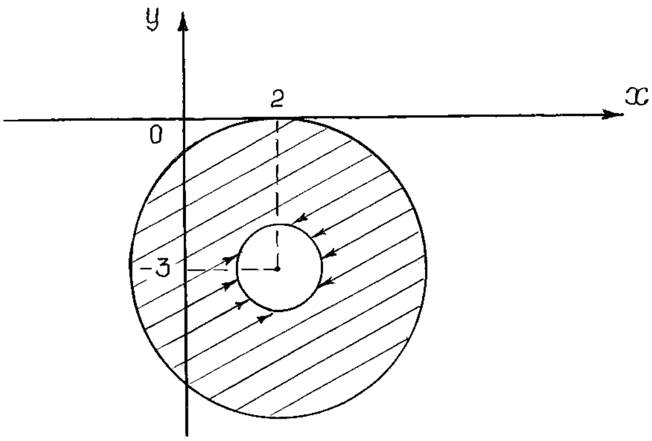
3.21-Masala. Quyidagi $1 < |z - 2 + 3i| \le 3$ tengsizlikni qanoatlantiruvchi barcha nuqtalar to‘plamini C kompleks tekislikda tasvirlang.

$\triangleleft$ $|z-2+3i| = |x+iy-2+3i| = |(x-2)+i(y+3)| = \sqrt{(x-2)^2+(y+3)^2}$

bo‘lgani uchun berilgan $1 < |z-2+3i| \le 3$ to‘plam

$$1 < (x-2)^2 + (y+3)^2 \le 9$$

xalqadan iborat bo‘ladi. Bu markazi (2;-3) nuqtada radiuslari 1 va 3 ga teng bo‘lgan konsentrik aylanalar orasidagi nuqtalar va radiusi 3 ga teng aylana nuqtalarini o‘z ichiga olgan xalqadir (12-chizma)$\triangleright$



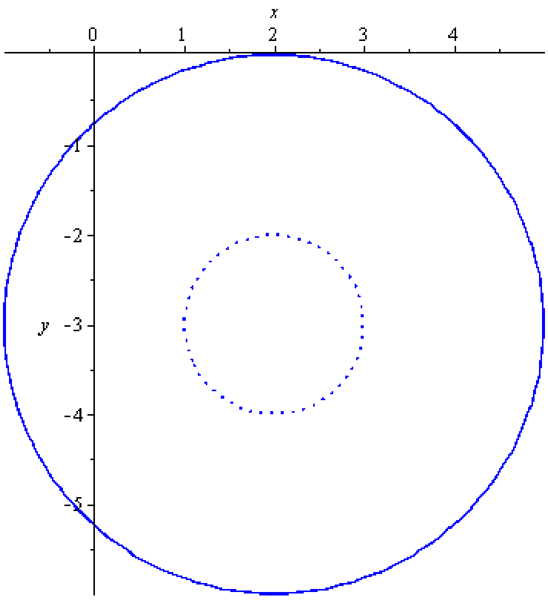
12 - chizma

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots)$:

>

> $implicitplot\left(\left[(x-2)^2+(y+3)^2>1,\ (x-2)^2+(y+3)^2\le 9\right], x=-2..6, y=-7..1, grid=[50, 50], color=[blue, blue]\right);$



> $with(plots)$:

> $r <$ \`|z-z0| ≤R\`; $z0 := 2-3\cdot I$; $r := 1$; $R := 3$;

$$1 < |z-z0| \le R$$

$$z0 := 2 - 3\,I$$

$$r := 1$$

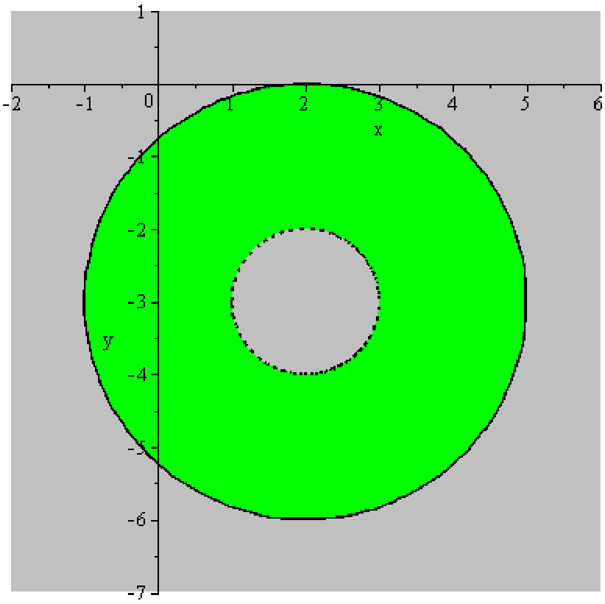
$$R := 3$$

> $p1 := implicitplot(subs(z = x + I*y, \mathrm{abs}(z\text{-}z0) = R), x = (\mathrm{Re}(z0) - \mathrm{R} - 1)\,..(\mathrm{Re}(z0) + \mathrm{R} + 1), y = (\mathrm{Im}(z0) - \mathrm{R} - 1)\,..(\mathrm{Im}(z0) + \mathrm{R} + 1), filled = true, coloring = [\mathrm{green}, \mathrm{grey}], linestyle = 1)$:

> $p2 := inequal(\{(x - \mathrm{Re}(z0))^2 + (y - \mathrm{Im}(z0))^2 < r^2\}, x = (\mathrm{Re}(z0) - r)\,..(\mathrm{Re}(z0) + r), y = (\mathrm{Im}(z0) - r)\,..(\mathrm{Im}(z0) + r), color = grey)$:

> $p3 := plot([\mathrm{Im}(z0) + \mathrm{sqrt}(r^2 - (x - \mathrm{Re}(z0))^2), \mathrm{Im}(z0) - \mathrm{sqrt}(r^2 - (x - \mathrm{Re}(z0))^2)], x = (\mathrm{Re}(z0) - r)\,..(\mathrm{Re}(z0) + r), y = (\mathrm{Im}(z0) - r)\,..(\mathrm{Im}(z0) + r), \mathrm{color} = \mathrm{black}, \mathrm{linestyle} = 2)$:

> $display(p2, p3, p1);$



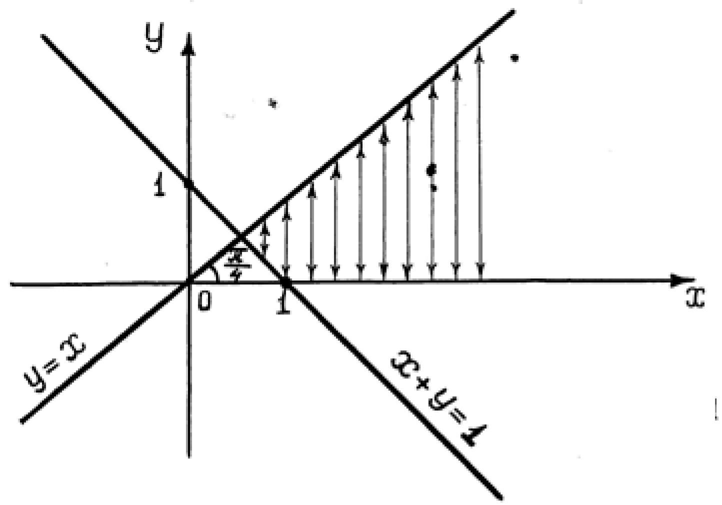
4.21-Masala. Quyidagi

$$\begin{cases} \mathrm{Re}\,z + \mathrm{Im}\,z > 1, \\ 0 < \arg z < \dfrac{\pi}{4} \end{cases}$$

tengsizliklar sistemasini qanoatlantiruvchi nuqtalar to‘plamini C tekislikda tasvirlang.

$$\triangleleft \begin{cases} \operatorname{Re} z = x, \\ \operatorname{Im} z = y \end{cases} \qquad va \quad 0 < \arg z < \frac{\pi}{4} \Rightarrow \begin{cases} x + y > 1 \\ 0 < y < x \end{cases}$$

Bu to‘plam 13-chizmada tasvirlangan $\triangleright$



13-chizma

5.21-Masala. Ushbu

$$|z|^2 + (4-3i)z + (4+3i)\bar{z} + 21 = 0$$

tenglama aylananing tenglamasi ekanligini isbotlang va bu aylana markazining koordinatlari hamda radiusini toping.

$$\triangleleft \quad \left. \begin{array}{l} z = x + iy \\ \bar{z} = x - iy \\ |z| = \sqrt{x^2 + y^2} \end{array} \right\} \Rightarrow 0 = |z|^2 + (4-3i)z + (4+3i)\bar{z} + 21 =$$

$$= x^2 + y^2 + (4-3i)(x+iy) + (4+3i)\cdot(x-iy) + 21 = x^2 + y^2 + 8x + 6y + 21 =$$

$$= (x+4)^2 + (y+3)^2 - 4 \Rightarrow (x+4)^2 + (y+3)^2 = 2^2.$$

Bu markazi (-4,-3) nuqtada va radiusi 2 ga teng bo‘lgan aylananing tenglamasi $\triangleright$

6.21-Masala. C kompleks tekisligidagi $z = \dfrac{1+i}{\sqrt{2}}$ nuqtaning S Riman sferasidagi obrazini toping.

◃ Bu masalani yechishda (7)- formulalardan foydalanamiz.

$z=\frac{1+i}{\sqrt{2}}=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\Rightarrow x=\frac{\sqrt{2}}{2}\ ,\ \ y=\frac{\sqrt{2}}{2}\ ,|z|=\sqrt{\frac{1}{2}+\frac{1}{2}}.=1$ Bu yerdan (7)-

formulaga ko‘ra $\xi=\frac{x}{1+|z|^2}=\frac{\sqrt{2}}{4};\quad \eta=\frac{y}{1+|z|^2}=\frac{\sqrt{2}}{4};\ \zeta=\frac{|z|^2}{1+|z|^2}=\frac{1}{2}$

ekanligini topamiz. Demak, berilgan nuqtaning Riman sferasidagi obrazi

$(\frac{\sqrt{2}}{4};\frac{\sqrt{2}}{4};\frac{1}{2})$ ekan(14-chizma)▹

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

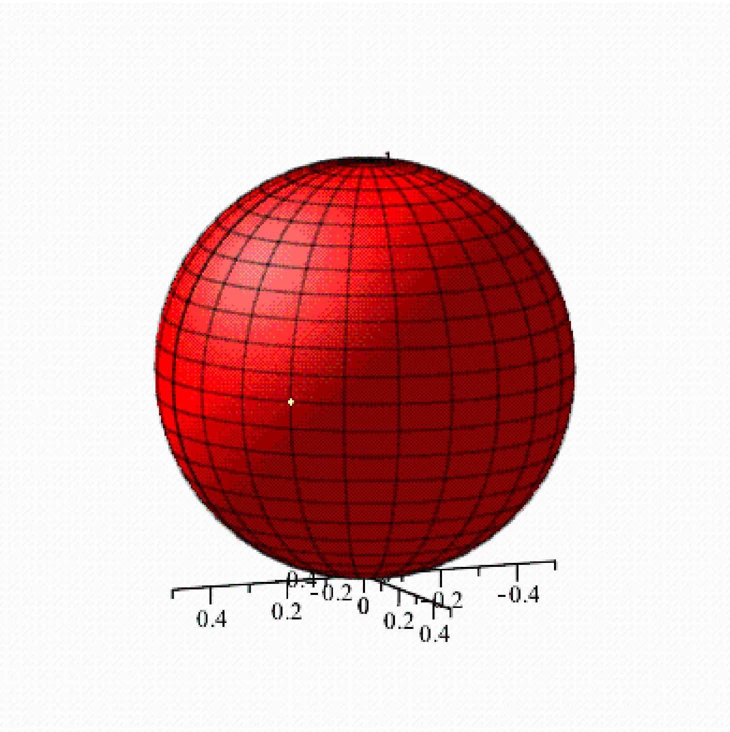
$>$ $with(geom3d)$:

$>$

$>$ $sphere\left(s, \left[point\left(o, 0, 0, \frac{1}{2}\right), \frac{1}{2}\right]\right), point\left(P, \frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{1}{2}\right)$

$$s, P$$

$>$ $draw([s(color=red), P(color=blue)])$



14-chizma

7.21-Masala. Hisoblang. $\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n}\frac{1}{2^k}\cos\frac{k\pi}{4}$.

◁ Berilgan limitni hisoblash uchun oldin

$$z_n=\sum_{k=0}^{n}\frac{1}{2^k}\left(\cos\frac{k\pi}{4}+i\sin\frac{k\pi}{4}\right)=\sum_{k=0}^{n}\frac{e^{i\frac{k\pi}{4}}}{2^k}$$

ketma-ketlikning limitini topamiz:

$$\lim_{n\to\infty}z_n=\lim_{n\to\infty}\sum_{k=0}^{n}\frac{e^{i\frac{k\pi}{4}}}{2^k}=\lim_{n\to\infty}\sum_{k=0}^{n}\left(\frac{e^{i\frac{\pi}{4}}}{2}\right)^k=\lim_{n\to\infty}\frac{1-\dfrac{e^{\frac{in\pi}{4}}}{2^n}}{1-\dfrac{e^{i\frac{\pi}{4}}}{2}}=\left(\left(\left|e^{\frac{in\pi}{4}}\right|=\right.\right.$$

$$=\left|\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right|=1))=\frac{1}{1-\dfrac{e^{i\frac{\pi}{4}}}{2}}.$$

Bundan $\Rightarrow$

$$\lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{2^k}\cos\frac{k\pi}{4}=\lim_{n\to\infty}\operatorname{Re}z_n=\operatorname{Re}\frac{1}{1-\frac{e^{\frac{i\pi}{4}}}{2}}=\operatorname{Re}\frac{2}{2-e^{\frac{i\pi}{4}}}=\operatorname{Re}\frac{2}{2-(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})}=$$

$$=\operatorname{Re}\frac{4}{4-\sqrt{2}-i\sqrt{2}}=\frac{4(4-\sqrt{2})}{(4-\sqrt{2})^2+(\sqrt{2})^2}=\frac{4-\sqrt{2}}{5-2\sqrt{2}}. \quad \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

$$A:=\sum_{i=0}^{n}\left(\frac{1}{2^i}\cdot\cos\left(\frac{i\cdot\pi}{4}\right)\right)$$

$$A:=\frac{\left(-\frac{16}{17}-\frac{3}{17}\sqrt{2}\right)\cos\left(\frac{1}{4}(n+1)\pi\right)}{2^{n+1}}+\frac{\left(\frac{4}{17}+\frac{5}{17}\sqrt{2}\right)\sin\left(\frac{1}{4}(n+1)\pi\right)}{2^{n+1}}+\frac{16}{17}+\frac{3}{17}\sqrt{2}$$

> $\lim_{n\to\infty}A$

$$\frac{16}{17}+\frac{3}{17}\sqrt{2}$$

>

8.21-Masala. Quyidagi

$$z=(1+i)+[(2+3i)-(1+i)\}]t \qquad (0\le t\le 1)$$

funksiya aniqlagan egri chiziqni toping.

$\triangleleft \quad z(t)=x(t)+y(t)=(1+i)+[(2+3i)-(1+i)]t=1+i+(1+2i)t=$

$=1+t+i(1+2t)$

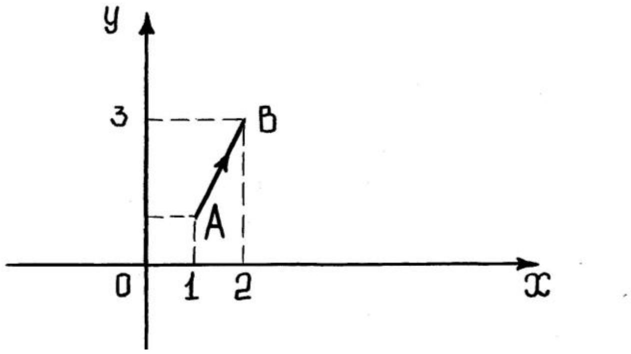
tenglikdan, berilgan chiziqning parametrik tenglamasi.

$$\begin{cases} x(t)=1+t, \\ y(t)=1+2t, \qquad 0\le t\le 1 \end{cases}$$

ekanligini, bu yerdan esa

$$y = 1 + 2t = 2(1+t) - 1 = 2x - 1, \quad 1 \le x \le 2$$

ekanligini topamiz. Demak, berilgan chiziq 7-chizmada tasvirlangan A(1;1) nuqtadan B(2;3) nuqtaga qarab yo‘nalgan AB kesmadan iborat ekan ▹



15 - chizma

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

```
> with(plots) :
> z(t) := (1 + I) + ((2 + 3·I) − (1 + I))·t
```

$$z := t \rightarrow 1 + I + (1 + 2\,I)\,t$$

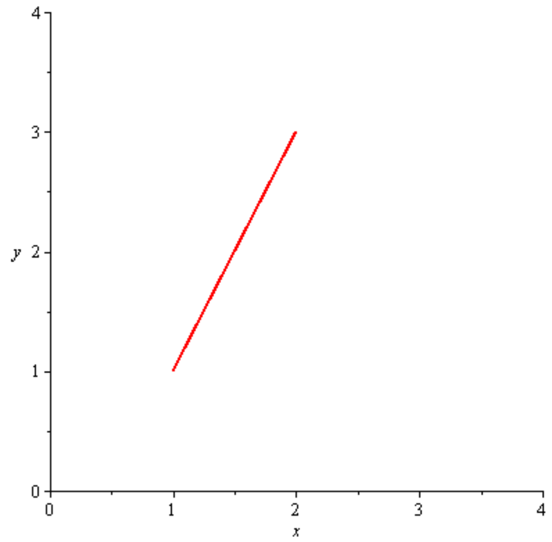
```
> x(t) := 1 + t
```

$$x := t \rightarrow 1 + t$$

```
> y(t) := 1 + 2 t
```

$$y := t \rightarrow 1 + 2\,t$$

```
> plot([x(t), y(t), t = 0..1], x = 0..4, y = 0..4, color
      = red, thickness = 2, grid = [100, 100])
```



>

9.21-Masala. $f(z)=\frac{1}{2}(z+\frac{1}{z})$ funksiyani $E=\{|z|<2\}$ sohada bir yaproqlikka tekshiring.

$\triangleleft$ Bu masalani echishda 5^0 punktdagi 2-ta'rifdan foydalanamiz. Faraz qilaylik $z_1, z_2, \in E$ lar uchun $f(z_1)=f(z_2)$, ya'ni $\frac{1}{2}(z_1+\frac{1}{z_1})=\frac{1}{2}(z_2+\frac{1}{z_2})$ bo‘lsin $\Rightarrow \frac{(z_2-z_1)(1-z_1z_2)}{z_1z_2}=0\Rightarrow$

Berilgan funksiyaning E to‘plamda bir yaproqli bo‘lishi uchun shu to‘plamning

$$z_1z_2=1$$

tenglikni qanoatlantiruvchi z_1, z_2 nuqtalarni o‘zida saqlamasligi zarur va yetarli. Lekin,

$$z_1=i\in E\ ,\qquad z_2=-i\in E\qquad \text{ва}\qquad z_1\cdot z_2=1.\Rightarrow f(z)$$

funksiya E sohada bir yaproqli bo‘lmaydi$\triangleright$

10.21-Masala. $f(z)=\frac{1}{z^2+1}$ funksiyani uzluksizlikka tekshiring.

◁ $z^2+1=0 \Rightarrow z^2=-1 \Rightarrow z=\pm i$ nuqtalar funksiyaning uzilish nuqtalari. Qolgan barcha nuqtalarda funksiyaning uzluksiz ekanligini ko‘rsatamiz. $\forall z \in C \setminus \{-i,\, i\}$ uchun

$$\Delta f(z) = f(z+\Delta z) - f(z) = \frac{1}{(z+\Delta z)^2+1} - \frac{1}{z^2+1} = \frac{-\Delta z\cdot(2z+1)}{[(z+\Delta z)^2+1]\cdot(z^2+1)}$$

bo‘lib, bu tenglikdan $\lim\limits_{\Delta z\to 0}\Delta f(z)=0$ ekanligi kelib chiqadi. Bu esa $f(z)=\dfrac{1}{z^2+1}$ funksiyaning $\forall z \in C \setminus \{-i,\, i\}$ nuqtada uzluksiz ekanligini anglatadi ▷

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots):$

> $f := \dfrac{1}{1+z^2}$

$$f := \frac{1}{z^2+1}$$

> $solve\left(\{1+z^2=0\},\ \{z\}\right)$

$$\{z=\mathrm{I}\},\ \{z=-\mathrm{I}\}$$

> $\lim\limits_{z\to I} f$

$$undefined$$

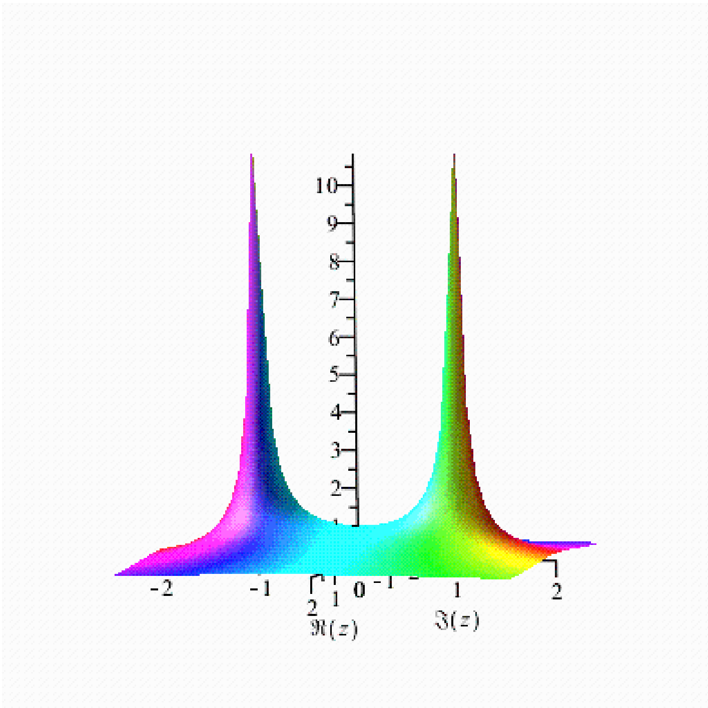
> $\lim\limits_{z\to -I} f$

$$undefined$$

> $\lim\limits_{z\to a} f$

$$\frac{1}{a^2+1}$$

> $complexplot3d(f,\ z=-2-2\,I..2+2\,I,\ grid=[50,\ 50])$



11.21-Masala. $f(z)=\dfrac{1}{z+2}$ $(z\neq -2)$ funksiyaning hosilasi ta'rif yordamida hisoblansin.

◁ $\forall\ z\in C\setminus\{-2\}$ uchun (11)-formuladan foydalanib topamiz:

$$f'(z)\ :=\lim_{\Delta z\to 0}\frac{f(z+\Delta z)-f(z)}{\Delta z}=\lim_{\Delta z\to 0}\frac{\dfrac{1}{z+\Delta z+2}-\dfrac{1}{z+2}}{\Delta z}=$$

$$\lim_{\Delta z\to 0}\frac{-1}{(z+\Delta z+2)(z+2)}=-\frac{1}{(z+2)^2}\ .\quad ▷$$

12.21-Masala. $f(z)=z\cdot\operatorname{Im}z$ funksiyani C–differensiallanuvchanlikka tekshiring.

◁ Bu masalani 6^0 punktda keltirilgan teoremadan foydalanib yechamiz.

$$f(z)=z\cdot\operatorname{Im}z=(x+iy)\cdot y=xy+iy^2\ \Rightarrow u(x,y)=xy,\ v(x,y)=y^2.$$

Bu funksiyalar $\forall(x,y)\in R^2$ nuqtada haqiqiy analiz ma'nosida differensiallanuvchi. Endi bu funksiyalar uchun Koshi-Riman shartlarini tekshiramiz.Ushbu

$$\frac{\partial u}{\partial x} = y\ ;\quad \frac{\partial u}{\partial y} = x\ ;\quad \frac{\partial v}{\partial x} = 0\ ;\quad \frac{\partial v}{\partial y} = 2y \text{ tengliklardan}$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases}$$

Koshi-Riman shartlari faqat (0,0) nuqtadagi bajarilishi kelib chiqadi. Demak, $f(z) = z \cdot \operatorname{Im} z$ funksiya faqat $z = 0$ nuqtada C–differensiallanuvchi ▹

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $f(z) := z \cdot \operatorname{Im}(z)$

$$f := z \to z\,\Im(z)$$

> $u := x \cdot y$

$$u := x\,y$$

> $v := y^2$

$$v := y^2$$

> $solve\left(\left\{\frac{\partial}{\partial x}\,u = \frac{\partial}{\partial y}\,v,\ \frac{\partial}{\partial y}\,u = -\frac{\partial}{\partial x}\,v\right\},\ \{x, y\}\right)$

$$\{x = 0,\ y = 0\}$$

13.21-Masala. $f(z) = x + ay + i(bx + cy)$ funksiyani golomorflikka tekshiring.

◃ $f(z) = x + ay + i(bx + cy) \Rightarrow u(x, y) = x + ay,\ \ v(x, y) = bx + cy$

funksiyalar R^2 da haqiqiy analiz ma’nosida differensiallanuvchi. Bu funksiyalar uchun Koshi-Riman shartlarini tekshiramiz.

$$\frac{\partial u}{\partial x} = 1\ ;\quad \frac{\partial u}{\partial y} = a\ ;\quad \frac{\partial v}{\partial x} = b\ ;\quad \frac{\partial v}{\partial y} = c \qquad \text{va}$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases} \Rightarrow \begin{cases} c = 1 \\ b = -a \end{cases} \Rightarrow b = -a \quad \text{ва} \quad c = 1$$

shartlar bajarilsa, $f(z)$ funksiya C da golomorf bo‘ladi va

$f(z) = x + ay + i(-ax + y) = (1 - ai)z$ tenglik bajariladi ▷

14.21-Masala. Faraz qilaylik γ – i nuqtadan chiquvchi $\arg(z-i) = \varphi$ nur bo‘lsin. $w = \dfrac{z-i}{z+i}$ akslantirish uchun i nuqtadagi cho‘zilish koeffitsienti $R(\varphi)$ va burilish burchagi $\alpha(\varphi)$ ni toping.

◁ $w = \dfrac{z-i}{z+i} \Rightarrow \forall z \in C \setminus \{-i\}$ uchun

$w'(z) = (\dfrac{z-i}{z+i})' = \dfrac{2i}{(z+i)^2} \Rightarrow w'(i) = -\dfrac{i}{2}$. Demak,

$R(\varphi) = |w'(i)| = \left|-\dfrac{i}{2}\right| = \dfrac{1}{2}$ ва $\alpha(\varphi) = \arg w'(i) = \arg(-\dfrac{i}{2}) = \dfrac{3\pi}{2}$ ▷

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

$f(z) := \dfrac{z-I}{z+I}$

$$f := z \to \frac{z - \mathrm{I}}{z + \mathrm{I}}$$

> $\dfrac{d}{dz} f(z)$

$$\frac{1}{z+\mathrm{I}} - \frac{z-\mathrm{I}}{(z+\mathrm{I})^2}$$

> $a(z) := \dfrac{1}{z+\mathrm{I}} - \dfrac{z-\mathrm{I}}{(z+\mathrm{I})^2}$

$$a := z \to \frac{1}{z+\mathrm{I}} - \frac{z-\mathrm{I}}{(z+\mathrm{I})^2}$$

> $a(I)$

$$-\frac{1}{2}\,\mathrm{I}$$

> $k = |a(I)|$

$$k=\frac{1}{2}$$

> $\theta = \mathrm{argument}(a(I))$

$$\theta = -\frac{1}{2}\,\pi$$

15.21-Masala. Berilgan $u(x,y)=2(x^2-y^2)-1$ garmonik funksiyaga E=C sohada qo'shma garmonik bo'lgan $v(x,y)$ funksiyani toping va ular yordamida golomorf $f(z)=u(x,y)+iv(x,y)$ funksiyani quring.

◃ $v(x,y)$ funksiya $u(x,y)$ funksiyaga qo'shma garmonik funksiya bo'lgani uchun ular Koshi-Riman shartlarini bajarishi kerak:

$$\begin{cases}\dfrac{\partial v}{\partial x}=-\dfrac{\partial u}{\partial y}=4y\\[2ex] \dfrac{\partial v}{\partial y}=\dfrac{\partial u}{\partial x}=4x\end{cases}\Rightarrow\begin{cases}v(x,y)=\int 4y\,dx+\varphi(y)=4xy+\varphi(y)\\[2ex] \dfrac{\partial v}{\partial y}=4x+\varphi'(y)=\dfrac{\partial u}{\partial x}=4x\Rightarrow 4x+\varphi'(y)=4x\Rightarrow\varphi'(y)=0\Rightarrow\end{cases}$$

$\Rightarrow \varphi(y)=const=c. \Rightarrow v(x,y)=4xy+c$

qo'shma garmonik funksiya.

$\Rightarrow f(z)=u+iv=2(x^2-y^2)-1+i(4xy+c)=2(x+iy)^2-1+ic=2z^2-1+ic.$ ▹

$u(x,y)$ va $v(x,y)$ funksiyalarning grafigi 16 va 17- chizmalarda keltirilgan.

Bu misolni Maple matematik paketida yechishni ko'rsatamiz.

> $with(plots):$

> $with(VectorCalculus):$

> $u := 2(x^2-y^2)-1$

$$u := 2x^2-2y^2-1$$

> $\frac{\partial}{\partial x}u$

$$4x$$

> $\frac{\partial}{\partial y} u$

$$-4y$$

> $Laplacian(\mathrm{u}, [x, y])$

$$0$$

> $PDE := \frac{\partial}{\partial x} v(x, y) = 4y$

$$PDE := \frac{\partial}{\partial x} v(x, y) = 4y$$

> $ans := pdsolve(PDE)$

$$ans := v(x, y) = 4yx + \_F1(y)$$

> $\frac{\partial}{\partial y} \mathrm{ans}$

$$\frac{\partial}{\partial y} v(x, y) = 4x + \frac{\mathrm{d}}{\mathrm{d}y} \_F1(y)$$

> $4x + \frac{\mathrm{d}}{\mathrm{d}y} \_F1(y) = 4x$

$$4x + \frac{\mathrm{d}}{\mathrm{d}y} \_F1(y) = 4x$$

> **# isolate for diff(_F1(y),y)**
isolate(, diff(_F1(y),y));

$$\frac{\mathrm{d}}{\mathrm{d}y} \_F1(y) = 0$$

> $c := dsolve\left(\frac{\mathrm{d}}{\mathrm{d}y} \_F1(y) = 0\right)$

$$c := \_F1(y) = \_C1$$

> $v := 4xy + c$

$$v := \_F1(y) + 4xy = \_C1 + 4xy$$

>

> $f = u + I \cdot v$

$$f = \left(\mathrm{I}\,(\_F1(y) + 4xy) + 2x^2 - 2y^2 - 1 = \mathrm{I}\,(\_C1 + 4xy) + 2x^2 - 2y^2 - 1\right)$$

> $f = 2\,z^2 - 1 + Ic$

$$f = 2\,(x + \mathrm{I}\,y)^2 + Ic - 1$$

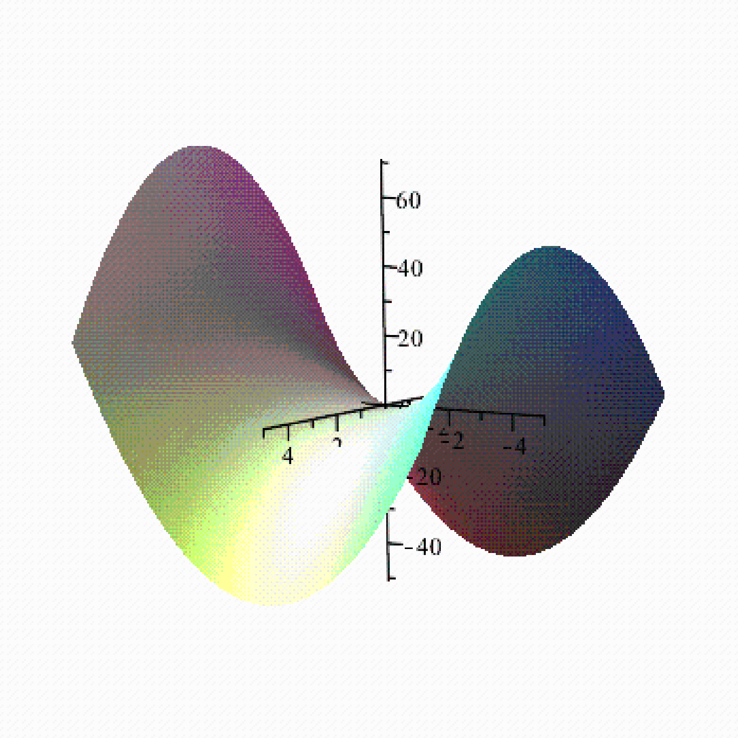
> $v := 4\,xy$

$$v := 4\,xy$$

> $Laplacian(\mathrm{v}, [x, y])$

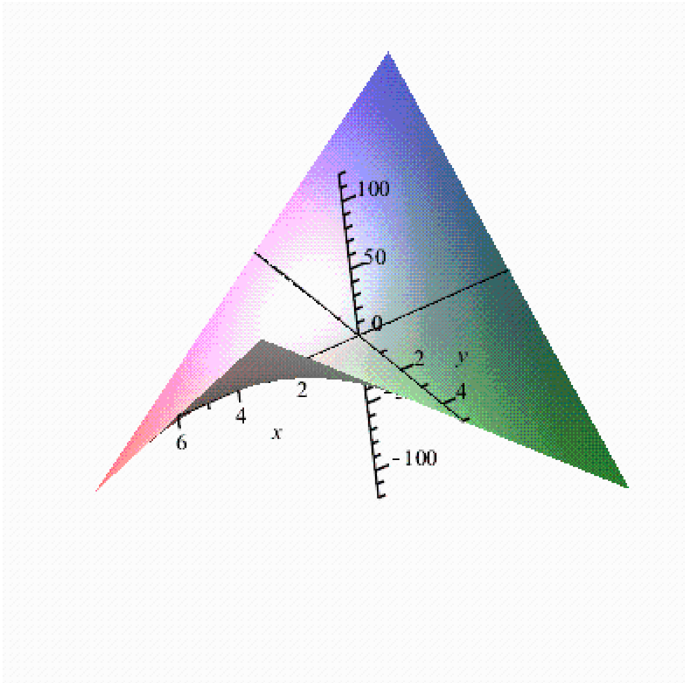
$$0$$

> $plot3d(\mathrm{u}, x = -5\,..6, y = -5\,..5, grid = [30, 30])$



16-chizma

> $plot3d(4 \cdot x \cdot y, x = -5\,..6, y = -5\,..5, grid = [30, 30])$



17-chizma

16.21-Masala. Quyidagi

$$f(z) = 4z^2 - 8z$$

funksiyaning konformlik sohasi topilsin.

▹ Bu masalani 8^0 punktdagi teoremadan foydalanib yechamiz.

$$f'(z) = (4z^2 - 8z)' = 8(z-1) \neq 0 \Rightarrow z \neq 1.$$

Funksiyani bir yaproqlikka tekshiramiz. Faraz qilaylik, $f(z_1) = f(z_2)$ bo‘lsin.

$\Rightarrow 4z_1^2 - 8z_1 = 4z_2^2 - 8z_2 \Rightarrow 4(z_1 - z_2)(z_1 + z_2 - 2) = 0 \Rightarrow$

Berilgan funksiyaning E to‘plamda bir yaproqli bo‘lishi uchun shu to‘plamning

$$z_1 + z_2 = 2 \qquad (18)$$

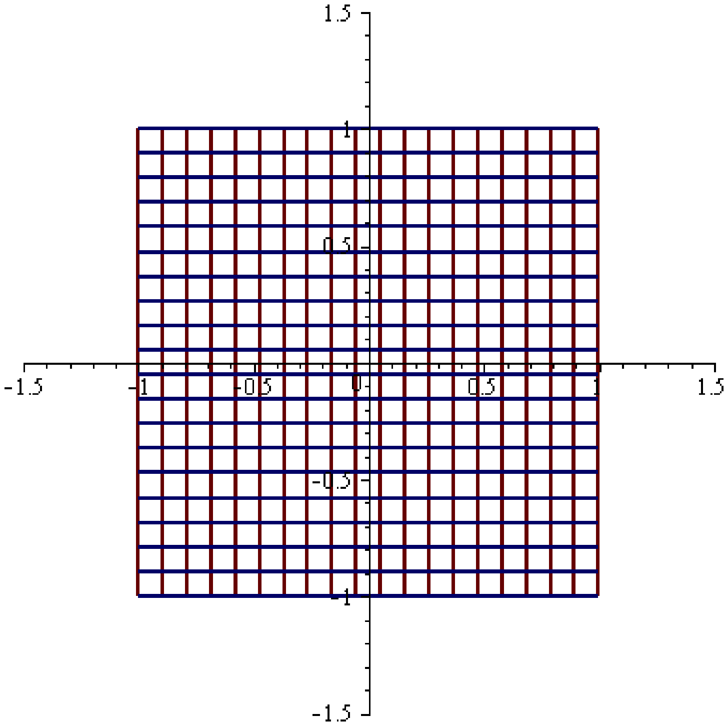
tenglikni qanoatlantiruvchi z_1, z_2 nuqtalarni o‘zida saqlamasligi zarur va yetarli.

Shunday qilib, $f(z) = 4z^2 - 8z$ funksiya $z = 1$ nuqtani va (18)–tenglikni qanoatlantiruvchi nuqtalarni o‘zida saqlamaydigan ixtiyoriy $E \subset C$ sohada konform bo‘lar ekan. Masalan, $E = \{z \in C: -1 \leq x \leq 1, -1 \leq y \leq 1\}$ sohada bu funksiya conform emas (18-chizma). Ammo, $G=\{z\in C:-2\le x\le -1,-1\le y\le \frac{1}{2}\}$ sohada konform bo‘ladi (19-chizma). ▹

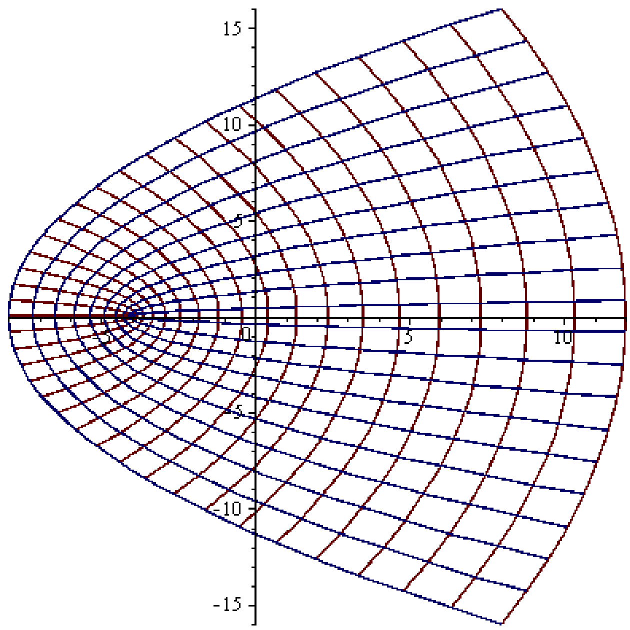
Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots)$:

> $conformal\left(z, z=-1-I..1+I, -1.5-1.5\cdot I..1.5+1.5\cdot I, grid=[20, 20]\right)$

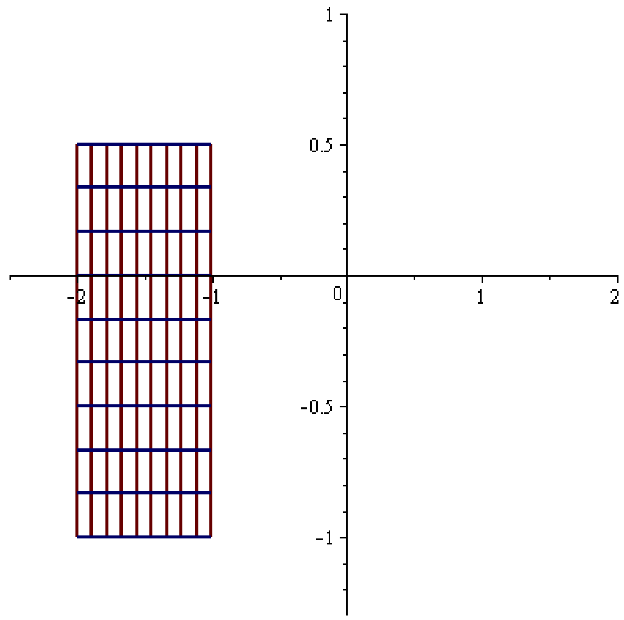


> $conformal\left(4\cdot z^2-8\cdot z, z=-1-I..1+I, grid=[20, 20]\right)$

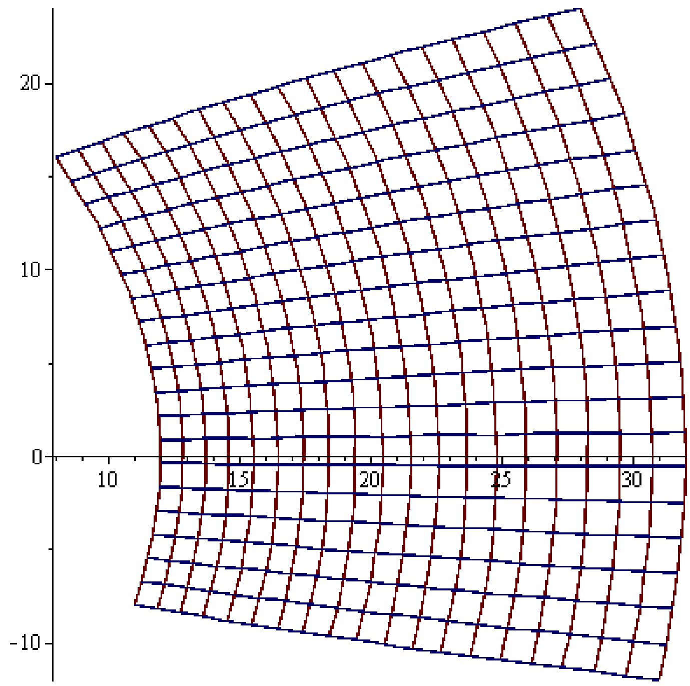


18-chizma

> $conformal\left(z, z = -2 - I..-1 + \frac{1}{2}\,\mathrm{I}, -2.5 - 1.3\cdot\mathrm{I}..2 + I, grid = [10, 10]\right)$



> $conformal\left(4\cdot z^2 - 8\cdot z, z = -2 - I..-1 + \frac{1}{2}\,\mathrm{I}, grid = [20, 20]\right)$



>

19-chizma.

2-§. 2-MUSTAQIL ISH

ELEMENTAR FUNKSIYALAR VA ULAR YORDAMIDA BAJARILADIGAN KONFORM AKSLANTIRISHLAR

Riman teoremasi.

Sohaning saqlanish prinsipi.

Chiziqli funksiya.

Kasr-chiziqli funksiya.

Darajali funksiya.

Jukovskiy funksiyasi.

Ko‘rsatkichli funksiya.

Trigonometrik funksiyalar.

Ko‘p qiymatli funksiyalar.

Simmetriya prinsipi.

- A -

ASOSIY TUSHUNCHA VA TEOREMALAR

Konform akslantirishlar nazariyasida asosan quyidagi ikki masala o‘rganiladi:

1-masala. C kompleks tekislikdagi biror E sohada (E$\subset$C) $w = f(z)$ akslantirish berilgan holda sohaning aksini, ya’ni $w(E)$ ni topish.

2-masala. Ikkita ixtiyoriy $E \subset C_z$ $F \subset C_w$ sohalar berilgan holda E sohani F sohaga aksalantiruvchi konform $w = f(z)$ akslantirishni topish.

Bu masalalarni hal qilishda quyidagi tasdiqlardan foydalaniladi.

1-Teorema. (Riman teoremasi). *Agar E va F lar mos ravishda kengaytirilgan kompleks tekislik* $\overline{C_z}$ *hamda* $\overline{C}_w$ *lardan olingan va chegarasi 2 ta nuqtadan kam bo‘lmagan bir bog‘lamli sohalar bo‘lsa, E sohani F sohaga konform akslantiruvchi* $w = f(z)$ *funksiya mavjud.*

2-Teorema. (sohaning saqlanish prinsipi). *Agar* $f(z)$ *funksiya E sohada golomorf bo‘lib,* $f(z) \neq const$ *bo‘lsa ,* $f(E)$ *ham soha bo‘ladi.*

Amaliyotda ko‘pincha berilgan E sohani o‘zidan soddaroq bo‘lgan sohaga, masalan birlik doira yoki yuqori yarim tekislikka konform akslantirish masalasini yechish talab qilinadi. Bu masalani hal qilishda biz kompleks argumentli elementar funksiyalar sinfini, birinchi navbatda ularning geometrik xossalarini tatbiq qilish uslublarini o‘rganishimiz zarur.

1^0. Chiziqli funksiya

1-Ta’rif. *Ushbu*

$$w = az + b \quad (a, b \in C \ , a \neq 0) \qquad (1)$$

ko‘rinishdagi funksiya chiziqli funksiya (akslantirish) deb ataladi.

Chiziqli funksiya C_z kompleks tekislikni C_w kompleks tekislikka konform akslantiradi.

Chiziqli funksiyaning xususiy hollarini qaraymiz:

1) Aytaylik,

$$w = z + b \quad (b \in C)$$

bo‘lsin. Bu funksiya parallel ko‘chirishni amalga oshiradi.

2) Aytaylik,

$$w = e^{i\alpha} \cdot z \quad (\alpha \in R)$$

bo‘lsin. Bu funksiya C_z tekislikdagi xar bir z nuqtani koordinata boshi atrofida soat strelkasiga teskari yo‘nalishda α burchakka burishni amalga oshiradi.

Masalan,

$$w = iz = (\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})z = e^{i\frac{\pi}{2}} \cdot z$$

funksiya koordinata boshi atrofida 90^0 ga,

$$w = -z$$

esa 180^0 ga burishni amalga oshiradi.

3) Aytaylik,

$$w = kz \quad (k > 0)$$

bo‘lsin. Bu funksiya berilgan sohani unga o‘xshash sohaga cho‘zib ($\kappa > 1$ da) yoki siqib ($\kappa < 1$ da) akslantiradi.

Umuman ,

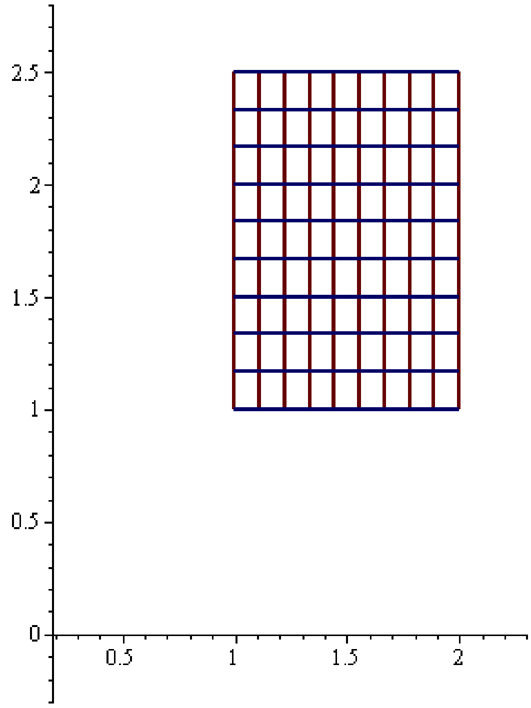
$$w = az + b \quad (a, b \in C)$$

funksiya yordamida bajariladigan akslantirish C_z tekislikdagi sohani «cho‘zish», biror burchakka burish hamda parallel ko‘chirishni amalga oshiradi. Amaliyotda bu funksiyaning shu xossalaridan foydalaniladi.

Misol. $D = \{z \in C : 1 < \operatorname{Re} z < 2, 1 < \operatorname{Im} z < 2.5\}$ sohani $w = (1+i)z + 2 - i$ akslantirish yordamida aksi topilsin.

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

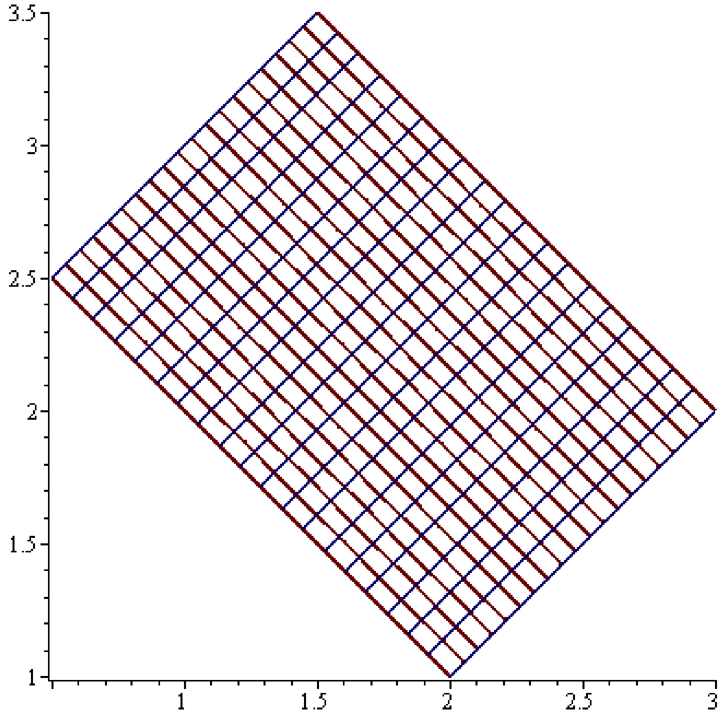
```
> with(plots) :
> conformal(z, z = 1 + I..2 +  2.5·I, 0.2 − 0.3·I..2.3 + 2.8·I, grid = [10, 10])
```



> $w := (1+I)\cdot z + 2 - I$

$$w := (1+\mathrm{I})\,z + 2 - \mathrm{I}$$

> *conformal*(w, z = 1 + I ..2 + 2.5· I, *grid* = [20, 20])



20-chizma.

Faraz qilaylik, $w = f(z)$ funksiya C tekislikdagi biror E sohada berilgan bo‘lsin.

2-Ta’rif. *Agar* $a \in E$ *nuqtada*

$$f(a) = a$$

tenglik bajarilsa, u holda $z = a$ *nuqta* $w = f(z)$ *akslantirishning qo‘zg‘almas nuqtasi deyiladi.*

$w = az + b$ chiziqli akslantirish $a \neq 1$ bo‘lganda ikkita

$$z_1 = \infty, \ z_2 = \frac{b}{1-a}$$

qo‘zgalmas nuqtalarga ega.

Agar $a = 1$ bo‘lsa, $z = \infty$ shu chiziqli akslantirishning karrali qo‘zgalmas nuqtasi bo‘ladi.

2^0. Kasr- chiziqli funksiya.

1-Ta’rif. *Ushbu*

$$w = \frac{az+b}{cz+d} \quad (a, \ b, \ c, \ d \in C) \qquad (2)$$

ko‘rinishdagi funksiya kasr-chiziqli funksiya (kasr-chiziqli akslantirish) deb ataladi.

Bu ta’rifda $ad - bc \neq 0$ deb qaraymiz, aks holda $\frac{a}{c} = \frac{b}{d}$ bo‘lib, w funksiya o‘zgarmasga aylanadi.

Kasr-chiziqli funksiya kengaytirilgan $\overline{C}_z$ kompleks tekislikni kengaytirilgan $\overline{C}_w$ kompleks tekislikka konform akslantiradi.

Kasr-chiziqli akslantirishlar qator xossalarga ega.

1-Xossa. *Kasr-chiziqli akslantirishlarning superpozitsiyasi yana kasr-chiziqli akslantirish bo‘ladi; kasr-chiziqli akslantirishga teskari bo‘lgan akslantirish ham kasr-chiziqli bo‘ladi.*

2-Xossa. *Ixtieriy kasr-chiziqli akslantirish* $\overline{C}_z$ *dagi aylana yoki to'g'ri chiziqni* $\overline{C}_w$ *dagi aylana yoki to'gri chiziqqa akslantiradi.*

Bu xossani kasr-chiziqli akslantirishning *doiraviylik xossasi* deyiladi (to'gri chiziq odatda radiusi cheksizga teng bo'lgan aylana deb qaraladi).

Izoh. Kasr-chiziqli funksiya yordamida aylana aylanaga yoki to'g'ri chiziqqa akslanishini aniqlash uchun funksiyaning maxrajini nolga aylantiruvchi $z=-\dfrac{d}{c}$ nuqtaning qaralayotgan aylanaga tegishli yoki tegishli emasligini aniqlash kifoyadir.

Masalan,

$$w=\frac{1}{z-3}$$

akslantirish $\{z:|\,z\,|=2\}$ aylanani aylanaga, $\{z:|\,z\,|=3\}$ aylanani esa to'g'ri chiziqqa o'tkazadi.

Tekislikdagi γ to'gri chiziqqa nisbatan simmetrik nuqtalar tushunchasi o'quvchiga elementar matematikadan ma'lum. Endi bu tushunchani aylanaga nisbatan keltiraylik.

2-Ta'rif. *Agar* z_1 *va* z_1^* *nuqtalar uchi*

$$\gamma=\{z\in C\,:|z-z_0|=R\}$$

aylana markazida bo'lgan bitta nurda yotib, ulardan aylana markazigacha bo'lgan masofalar ko'paytmasi γ *aylana radiusining kvadratiga teng bo'lsa, ya'ni*

$$\begin{cases}\arg(z_1^*-z_0)=\arg(z_1-z_0),\\ |z_1^*-z_0|\cdot|z_1-z_0|=R^2\end{cases}$$

tengliklar o'rinli bo'lsa, z_1 *va* z_1^* *nuqtalar C kompleks tekislikdagi* γ ***aylanaga nisbatan simmetrik nuqtalar*** *deyiladi.*

Agar z_1 va z_1^* nuqtalar γ aylanaga nisbatan simmetrik nuqtalar bo‘lsa,u xolda

$$z_1^* - z_0 = \frac{R^2}{\overline{z_1 - z_0}} \qquad (3)$$

bo‘ladi.

3-Xossa. *Har qanday kasr-chiziqli akslantirish natijasida* (z) *tekislikdagi* γ *aylana yoki to‘g‘ri chiziqqa nisbatan simmetrik bo‘lgan* z_1 *va* z_1^* *nuqtalarning aksi* (w) *tekislikda* γ *aylananing aksi bo‘lgan* $w(\gamma)$ *aylana yoki to‘g‘ri chiziqqa nisbatan simmetrik bo‘lgan* w_1 *va* w_1^* *nuqtalardan iborat bo‘ladi.*

Bu xossa kasr-chiziqli akslantirishda ***simmetriklikning saqlanish xossasi*** deyiladi.

4-Xossa. (z) *tekislikda berilgan har xil* z_1, z_2, z_3 *nuqtalarni* (w) *tekislikda berilgan har xil* w_1, w_2, w_3 *nuqtalarga akslantiruvchi kasr-chiziqli funksiya mavjud va u yagonadir.*

Bu akslantirish ushbu

$$\frac{w - w_1}{w - w_2} \cdot \frac{w_3 - w_2}{w_3 - w_1} = \frac{z - z_1}{z - z_2} \cdot \frac{z_3 - z_2}{z_3 - z_1} \qquad (4)$$

munosabatdan topiladi. (4)–munosabatga ***angarmonik nisbat*** deb ataladi.

5-Xossa. *Ushbu*

$$w = e^{i\theta} \cdot \frac{z - a}{z - \bar{a}}, \qquad \operatorname{Im} a > 0 \qquad (5)$$

kasr-chiziqli funksiya yuqori yarim tekislik $\{\operatorname{Im} z > 0\}$ *ni birlik doira* $\{|w| < 1\}$ *ga akslantiradi, bunda* θ *-ixtiyoriy haqiqiy son.*

6-Xossa. *Ushbu*

$$w = e^{i\theta} \cdot \frac{z - a}{1 - \bar{a}z}, \qquad |a| < 1 \qquad (6)$$

kasr-chiziqli funksiya (z) *tekislikdagi birlik doira* $\{|z|<1\}$ *ni* (w) *tekislikdagi birlik doira* $\{|w|<1\}$ *ga akslantiradi, bunda* θ *-ixtiyoriy haqiqiy son.*

Misol. $D=\{z:\ \mathrm{Re}\,z>0,\ \mathrm{Im}\,z>0\}$ sohani $w=\dfrac{1}{z}$ akslantirish yordamida aksi topilsin.

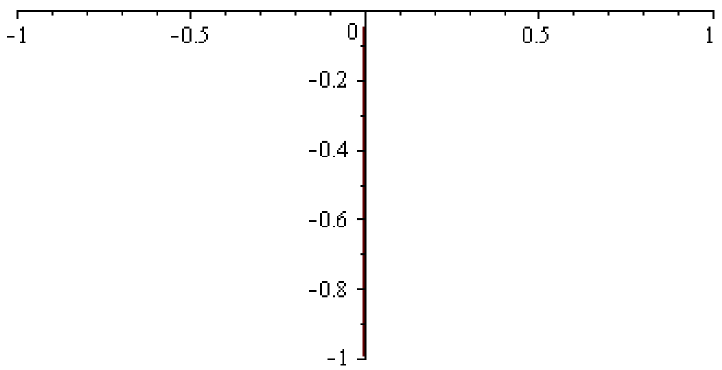
Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots)$:

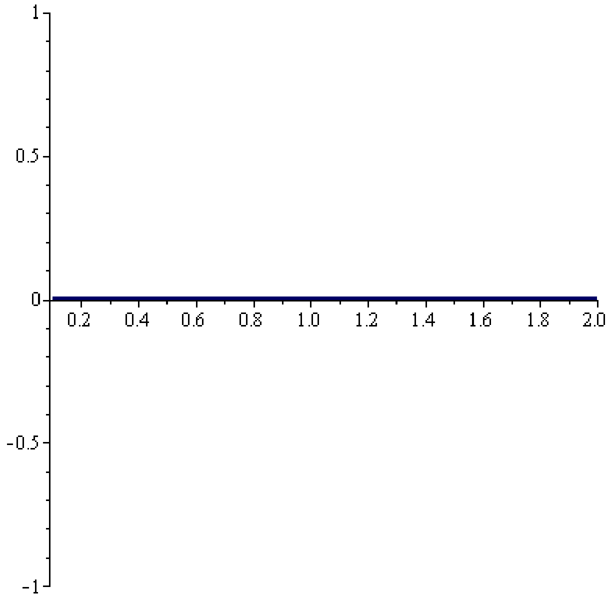
> $w := \dfrac{1}{z}$

$w := \dfrac{1}{z}$

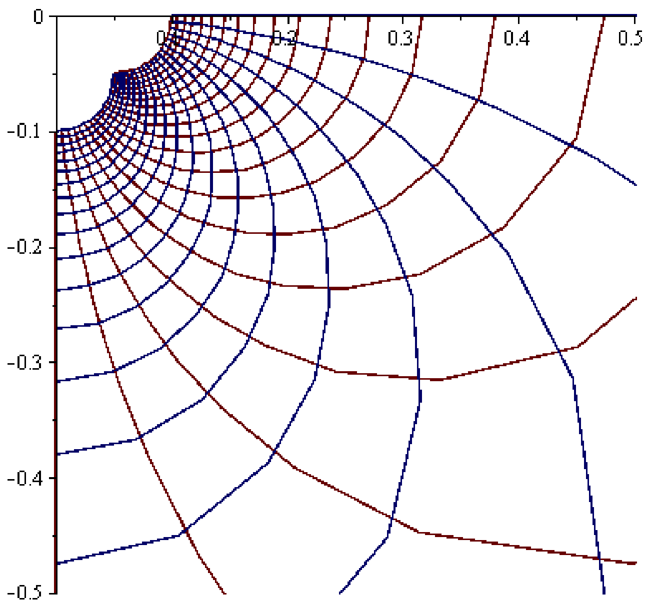
> $conformal$(w, z = 0 + 0· I..0 + 20· I, grid = [10, 10])



> $conformal$(w, z = 0 + 0· I..10 + 0· I, grid = [10, 10])



> *conformal*(w, z = 0 + 0· I..10 + 10· I, 0 − 0.5·I..0.5, *grid* = [20, 20])



21-chizma.

3⁰. Darajali funksiya.

Ta'rif. *Ushbu*

$$w = z^n \quad (n \in N,\ n > 1)` \qquad (7)$$

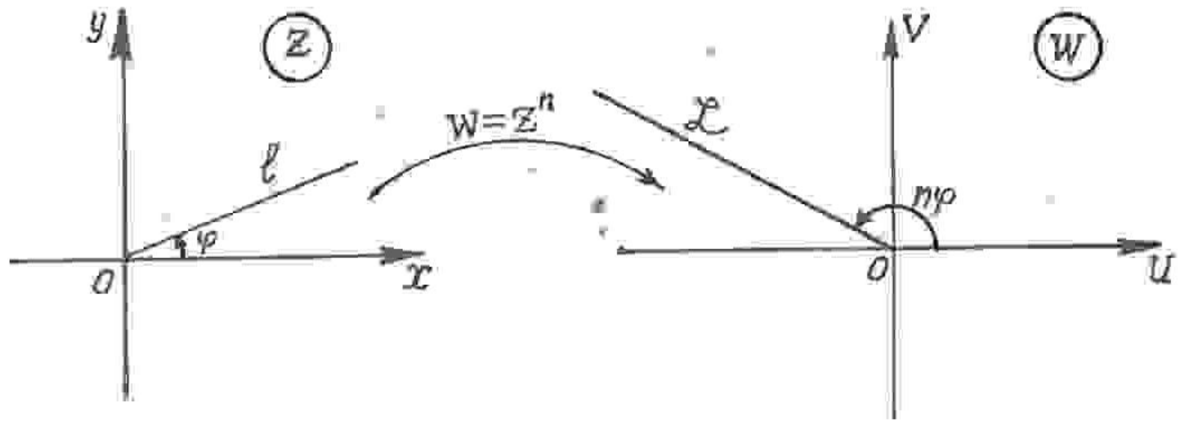
ko'rinishdagi funksiya darajali funksiya deyiladi.

Darajali funksiya C da golomorf va bu funksiya yordamida bajariladigan akslantirish $\forall z \in C \setminus \{0\}$ nuqtada konform bo‘ladi: $w' = nz^{n-1}$ hosila $C \setminus \{0\}$ da noldan farqlidir.

Agar $z = re^{i\varphi}$, $w = \rho e^{i\Psi}$ deyilsa ,

$$\begin{cases} \rho = r^n, \\ \psi = n\varphi \end{cases} \qquad (8)$$

ekanligini ko‘ramiz. Bu tengliklardan $w = z^n$ funksiya argumenti φ ga teng bo‘lgan, 0 nuqtadan chiquvchi ℓ nurni, argumenti $n\varphi$ ga teng bo‘lgan L nurga akslantirishini ko‘ramiz (22-chizma).



22- chizma

Agarda biz (z) tekisligida orasidagi burchagi $\frac{2\pi}{n}$ dan kichik bo‘lgan koordinata boshidan chiquvchi ikkita nur bilan chegaralangan *D* sohani qarasak, $w = z^n$ funksiyaning bu sohada bir yaproqli ekanligini ko‘ramiz.

Masalan, $w = z^n$ funksiya

$$\frac{2k\pi}{n} < \arg z < \frac{2(k+1)\pi}{n}, \quad k = 0,1,\ldots,n-1$$

sohaning har birida bir yaproqli, demak, konform bo‘lib, ularning har birini (w) teksligidagi $C \setminus R_+ = C \setminus [0,+\infty)$ sohaga akslantiradi.

Amaliyotda $w = z^n$ funksiyasidan burchakli sohalarni o'zidan soddaroq sohalarga akslantirishda foydalaniladi.

Misol. $D = \{z: \operatorname{Re} z = 1\}$ *va* $G = \{z: \operatorname{Im} z = 1\}$ chiziqlarni $w = z^2$ akslantirish yordamida aksi topilsin.

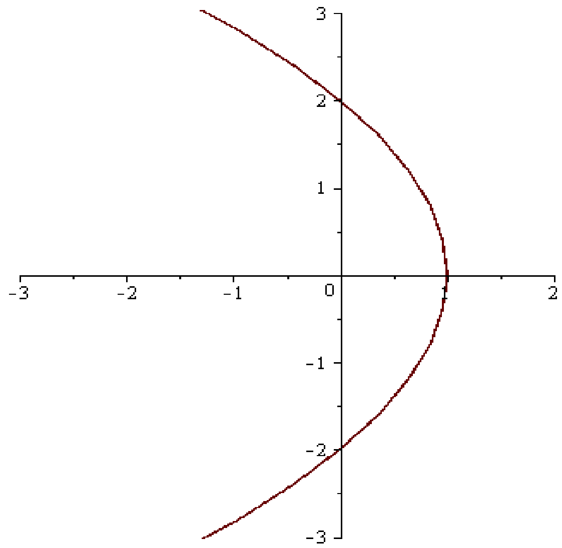
Bu misolni Maple matematik paketida yechishni ko'rsatamiz.

> *with*(*plots*) :

> $w := z^2$

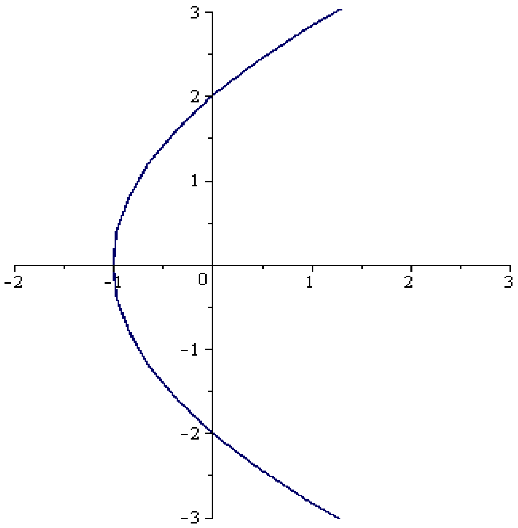
$w := z^2$

> *conformal*(w, z = 1 − 2· I ..1 + 2· I, −3 − 3·I ..2 + 3·I, grid = [20, 20])



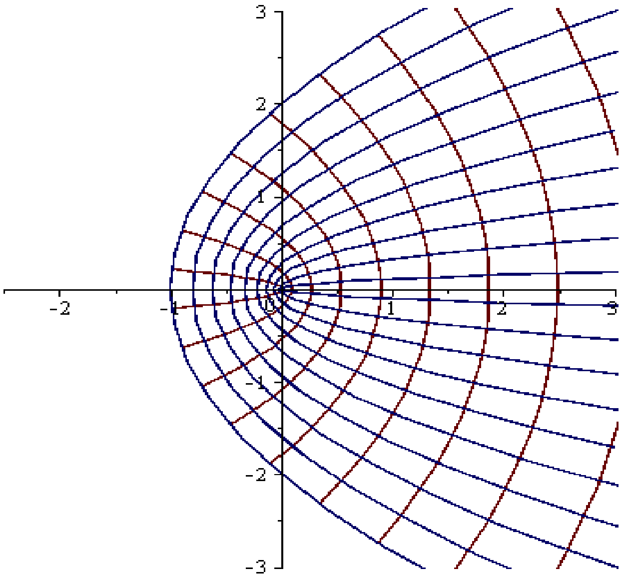
23- chizma

> *conformal*(w, z = −2 + I ..2 + I, −2 − 3·I ..3 + 3·I, grid = [20, 20])



24- chizma

> *conformal*(w, z=−2 − I..2 + I, −2.5 − 3·I..3 + 3·I, grid=[20, 20])



25- chizma

4^0. **Jukovskiy funksiyasi.**

Ta'rif. *Ushbu*

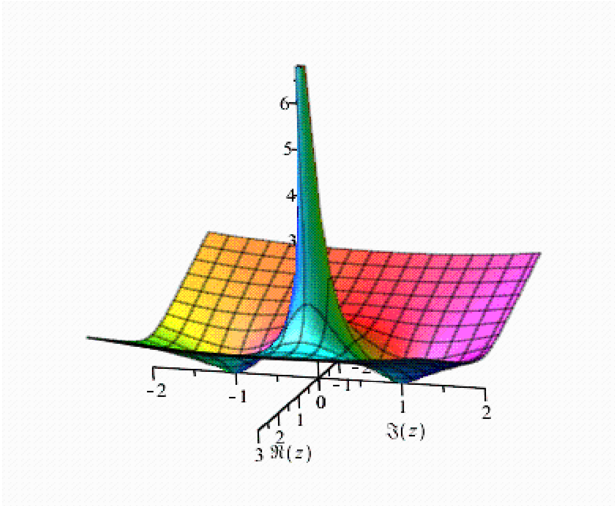
$$w = \frac{1}{2}\left(z + \frac{1}{z}\right) \qquad (9)$$

funksiya Jukovskiy funksiyasi deb ataladi.

Bu funksiyaning relyefi 26- chizmada tasvirlangan.

> *with*(*plots*) :

> $complexplot3d\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=-3-\ 2I..3+2I, grid=[50, 50]\right)$



26- chizma

Bu funksiya $z=0$ va $z=\infty$ nuqtalardan tashqari butun tekislikda *golomorf* funksiyadir.

Jukovskiy funksiyasining hosilasi $w'=\frac{1}{2}(1-\frac{1}{z^2})$ bo‘lib, $\{+1;-1\}$ nuqtalardan tashqarida $w'\neq 0$ dir. $w=\frac{1}{2}(z+\frac{1}{z})$ funksiya yordamidagi akslantirish $\{+1;-1\}$ nuqtalardan tashqarida ($z=0$, $z=\infty$ nuqtalarda ham) *konformdir*.

(9)–funksiya biror $E\subset C$ sohada bir yaproqli bo‘lishi uchun bu soha ushbu

$$z_1\cdot z_2=1 \quad\cdot \qquad (10)$$

munosabatni qanoatlantiruvchi z_1 va z_2 nuqtalarga ega bo‘lmasligi zarur va yetarli.

Bunday soha sifatida $U=\{z\in C:|z|<1\}$ yoki $U^*=\{z\in C:|z|>1\}$ sohalarni olish mumkin. Jukovskiy funksiyasi bu sohalarning har birini [-1; 1] kesmaning tashqarisiga konform akslantiradi.

Agar Jukovskiy funksiyasida

$$z = re^{i\varphi} \ , \quad w = u + iv$$

deyilsa, unda

$$u + iv = \frac{1}{2}(re^{i\varphi} + \frac{1}{r}e^{-i\varphi})$$

bo‘lib,

$$\begin{cases} u = \frac{1}{2}(r + \frac{1}{r})\cos\varphi. \\ \\ v = \frac{1}{2}(r - \frac{1}{r})\sin\varphi \end{cases} \qquad (11)$$

bo‘ladi. (11) dan (9)-akslantirish uchun quyidagilar kelib chiqadi.

1) (z) tekislikdagi $\{z \in C : |z| = r \ , r > 1\}$ aylana (w) tekislikdagi fokuslari (-1; 0) va (1; 0) nuqtalarda, yarim o‘qlari

$$a = \frac{1}{2}\left(r + \frac{1}{r}\right), \quad b = \frac{1}{2}\left(r - \frac{1}{r}\right)$$

bo‘lgan ellipsga akslanadi (27- chizma).

2) (z) tekislikdagi $\{z \in C : |z| = r, \ r < 1\}$ aylana (w) tekislikdagi fokuslari (-1; 0) va (1; 0) nuqtalarda, yarim o‘qlari

$$a = \frac{1}{2}\left(r + \frac{1}{r}\right), \quad b = \frac{1}{2}\left(\frac{1}{r} - r\right)$$

bo‘lgan ellipsga akslanadi (28- chizma).

3) (z) tekislikdagi $\{z \in C : |z| = 1\}$ aylana (w) tekislikdagi (-1; 0) va (1; 0) nuqtalarni tutashtiruvchi kesmaga akslanadi (29- chizma).

4) (z) tekislikdagi $\{z \in C : \arg z = 0\}$ nur (w) tekislikdagi $\{w \in C : \arg w = 0\}$ nurga, $\{z \in C : \arg z = \pi\}$ nur esa $\{w \in C : \arg w = \pi\}$ nurga akslanadi (30 va 31- chizma).

5) (z) tekislikdagi $\{z \in C : \arg z = \frac{\pi}{2}\}$ hamda $\{z \in C : \arg z = \frac{3\pi}{2}\}$ nurlarning har biri (w) tekislikdagi $\{w \in C : \mathrm{Re}\, w = 0\}$ to'g'ri chiziqqa akslanadi(32- chizma).

6) (z) tekislikdagi

$$\{z \in C : \arg z = \varphi; \ \ \varphi \neq 0 \ , \ \varphi \neq \frac{\pi}{2} \ , \ \ \varphi \neq \pi \ \ \varphi = \frac{3\pi}{2}\}$$

nur (w) tekislikdagi ushbu

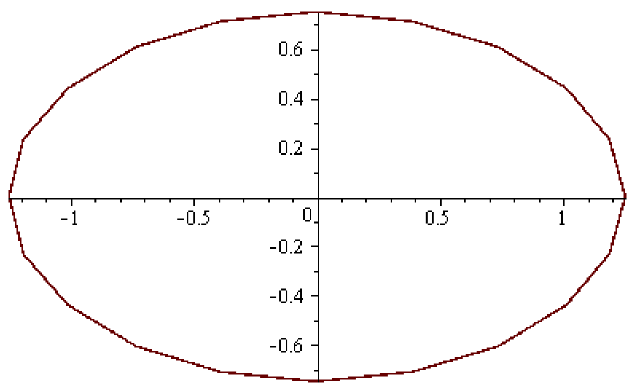
$$\frac{u^2}{\cos^2 \varphi} - \frac{v^2}{\sin^2 \varphi} = 1$$

giperbolaning mos «shoxchasiga» akslanadi(33- chizma).

Endi bu xossalarni Maple matematik paketi yordamida keltiramiz.

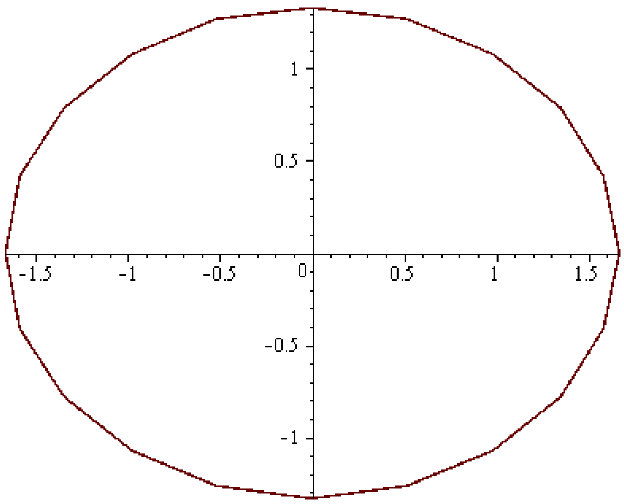
> $with(plots):$

> $conformal\left(\frac{1}{2} \cdot \left(z + \frac{1}{z}\right), z = 2 - \pi \cdot I..2 + \pi \cdot I, grid = [20, 20], coords = \mathrm{polar}\right)$



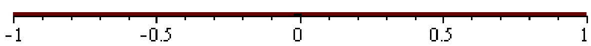
27- chizma

> $conformal\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=\frac{1}{3}-\pi\cdot I..\frac{1}{3}+\pi\cdot I, grid=[20, 20], coords=\text{polar}\right)$



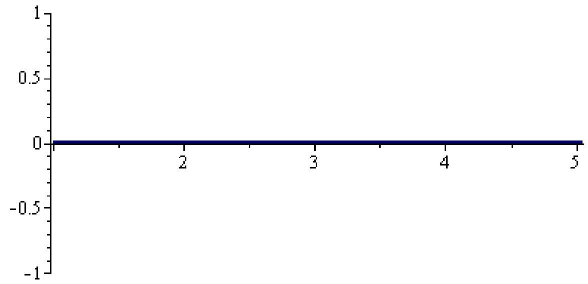
28- chizma

> $conformal\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=1-\pi\cdot I..1+\pi\cdot I, grid=[20, 20], coords=\text{polar}\right)$



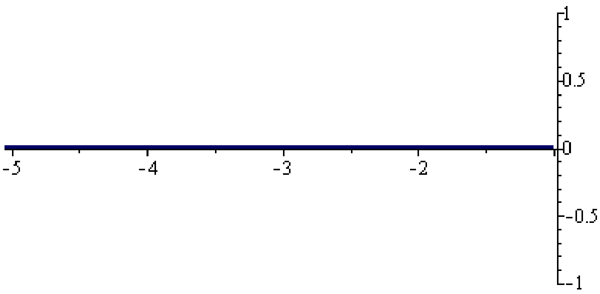
29- chizma

> $conformal\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=0-0\cdot I..10+0\cdot I, grid=[50, 50], coords=\text{polar}\right)$



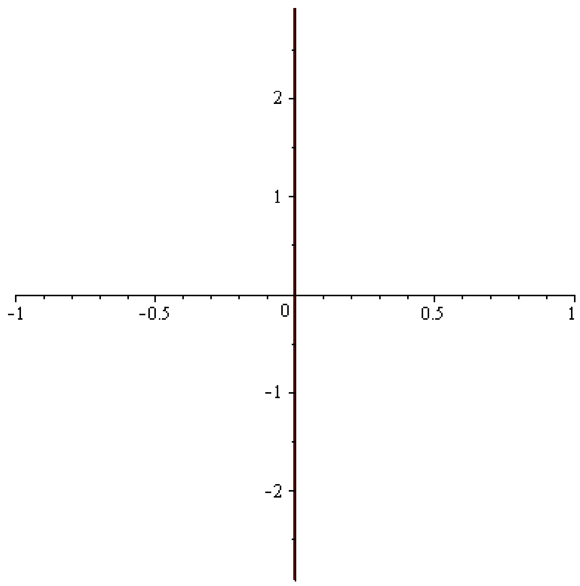
30- chizma

> $conformal\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=-10-0\cdot I..0+0\cdot I, grid=[50,50], coords=\mathrm{polar}\right)$



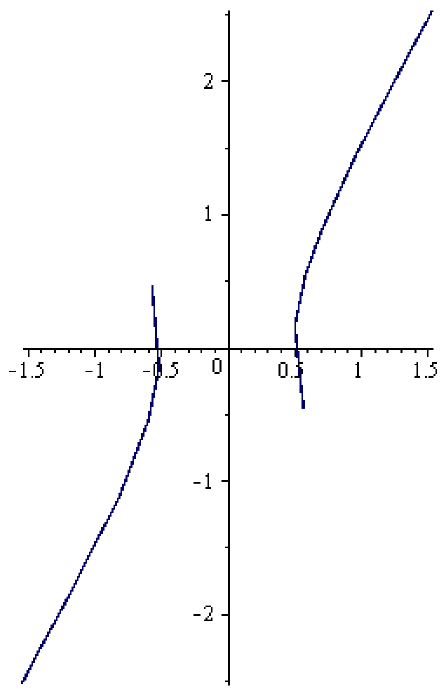
31- chizma

> $conformal\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=0-6\,I..0+6\,I, grid=[20,20]\right)$



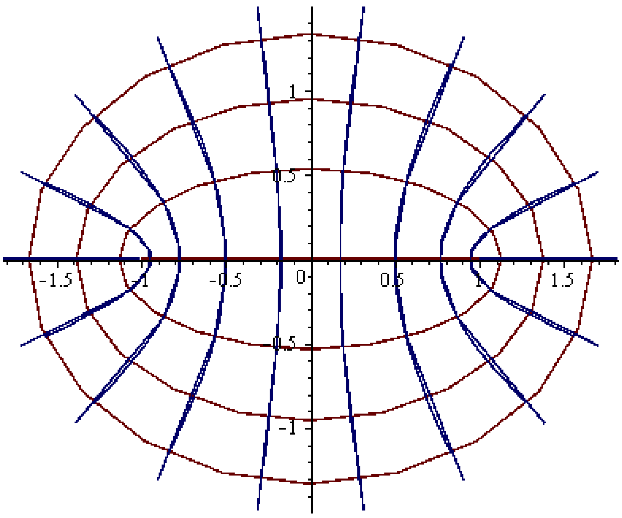
32- chizma

> $conformal\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=-6+\frac{\pi}{3}\cdot I..6+\frac{\pi}{3}\cdot I, grid=[20,20], coords=\mathrm{polar}\right)$



33- chizma

> $conformal\left(\frac{1}{2}\cdot\left(z+\frac{1}{z}\right), z=-3-\pi\cdot I..3+\pi\cdot I, grid=[10,10], coords=\text{polar}\right)$



5⁰. e^z funksiyasi.

Ta'rif. *Ushbu*

$$e^z := \lim_{n\to\infty}(1+\frac{z}{n})^n \qquad (z\in C)$$

funksiya ko'rsatkichli funksiya deyiladi.

Agar $z = x + iy$ desak,

$$e^z = e^x(\cos y + i\sin y) \qquad (12)$$

tenglik o'rinli.

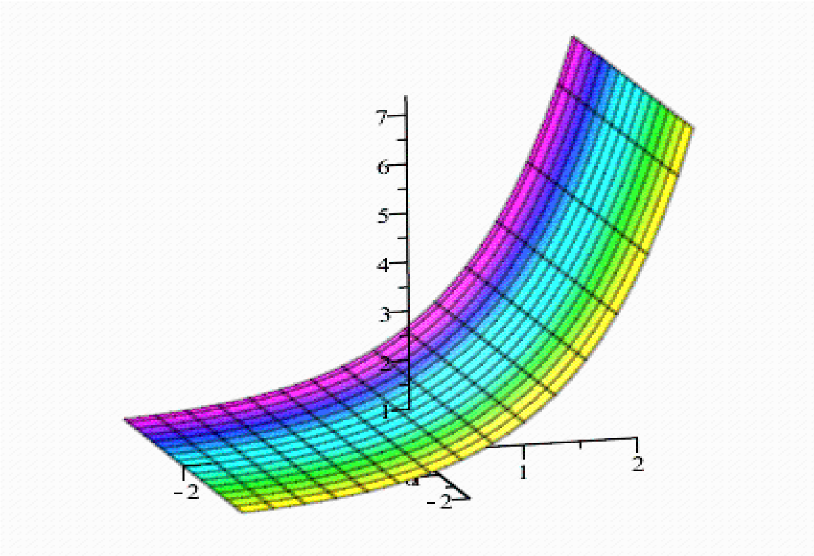
Bu funksiyaning relyefi 34- chizmada tasvirlangan.

> $with(plots):$

> > $w := z \to e^z;$

$$w := z \to e^z$$

> $complexplot3d(w,-2-2I..2+2I, grid=[30,30]);$



34- chizma

Ko‘rsatkichli $w = e^z$ funksiya quyidagi xossalarga ega:

1) e^z funksiya C kompleks tekislikda golomorf va uning hosilasi

$$(e^z)' = e^z$$

bo‘ladi.

2) e^z funksiya uchun

$$e^{z_1+z_2} = e^{z_1} \cdot e^{z_2} \quad (z_1 \in C, \ z_2 \in C)$$

bo‘ladi.

3) e^z funksiya davriy bo‘lib, uning asosiy davri $2\pi i$ bo‘ladi:

$$e^{z+2\pi i} = e^z$$

4) $\forall z \in C$ uchun $(e^z)' \neq 0$ bo‘lib, $w = e^z$ funksiya yordamidagi akslantirish C tekislikning har bir nuqtasida konform akslantirish bo‘ladi.

(12)–tenglikka ko‘ra, $|e^z| = e^x$, $\arg e^z = y$ bo‘lib, $w = e^z$ funksiya (z) tekislikdagi $\{x = x_0\}$ to‘g‘ri chiziqni $\{|w| = e^{x_0}\}$ aylanaga(35- chizma), $\{y = y_0\}$ to‘gri chiziqni esa $\{\arg w = y_0\}$ nurga akslantiradi(36- chizma). $w = e^z$ funksiya $\Pi_k = \{y_0 < \operatorname{Im} z < y_0 + 2\pi\}$, sohada bir yaproqli bo‘ladi (bu yerda $y_0 \in R$ bo‘lgan ixtiyoriy nuqta). Jumladan, $w = e^z$ funksiya ushbu

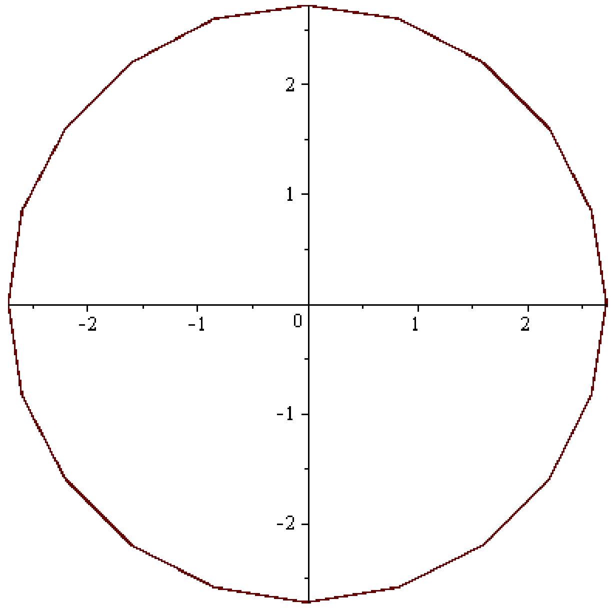
$$\Pi_k = \{z:\ 2k\pi < \mathrm{Im}\, z < 2(k+1)\pi\}, \quad k = 0, \pm 1, \pm 2, ...$$

sohalarning har birini (w) tekislikdagi $C \setminus R_+$ ga konform akslantiradi. Xuddi shunga o'xshash $w = e^z$ funksiya $\{z: 0 < \mathrm{Im}\, z < \pi\}$ yo'lakni yuqori yarim tekislikka konform akslantiradi(37- chizma).

Endi bu xossalarni Maple matematik paketi yordamida keltiramiz

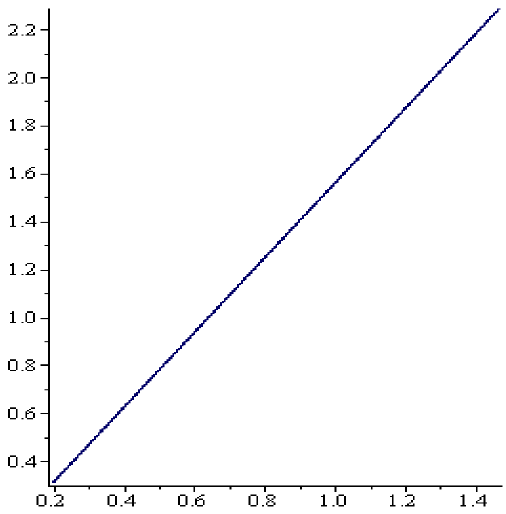
> $with(plots):$

> $conformal(e^z, z = 1 - \pi I..1 + \pi I, grid = [20, 20])$



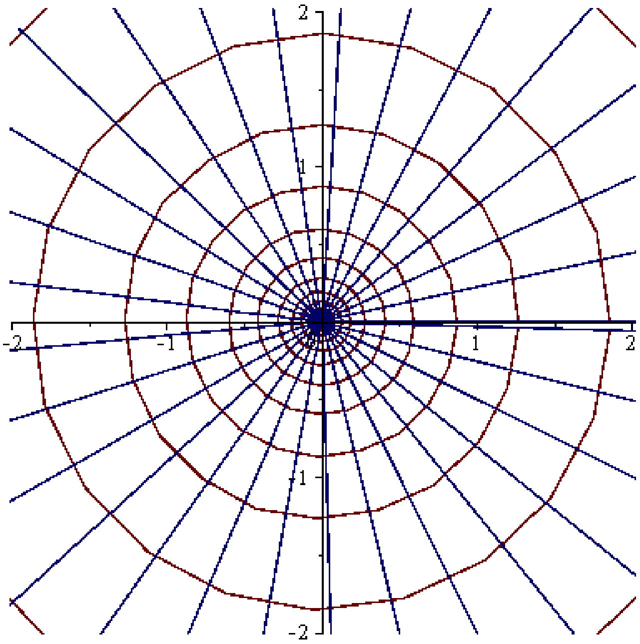
35- chizma

> $conformal(e^z, z = -1 + I..1 + I, grid = [20, 20])$

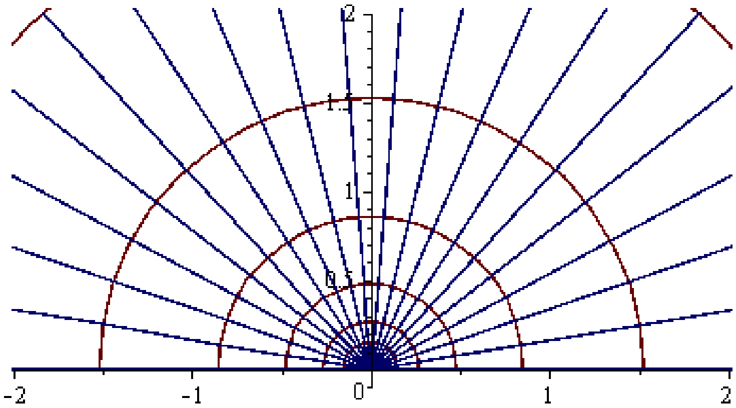


36- chizma

> $conformal\left(e^{z}, z=-10+0\cdot I..1+1.99\cdot \pi I, -2-2\cdot I..2+2\cdot I, grid=[30, 30]\right)$



> $conformal\left(e^{z}, z=-10+0\cdot I..1+\pi I, -2-0.1\,I..2+2\cdot I, grid=[20, 20]\right)$



37- chizma

6⁰. Trigonometrik funksiyalar.

(12)-tenglikda $x = 0$ desak,

$$\begin{cases} e^{iy} = \cos y + i\sin y \\ e^{-iy} = \cos y - i\sin y \end{cases}$$

tengliklarga ega bo‘lib,bundan

$$\cos y = \frac{e^{iy} + e^{-iy}}{2}, \quad \sin y = \frac{e^{iy} - e^{-iy}}{2i} \qquad (13)$$

ifodalarni hosil qilamiz (13)-formulalar ixtiyoriy haqiqiy son uchun o‘rinli bo‘lib, ulardan biz

$$w = \cos z, \quad w = \sin z$$

funksiyalarni aniqlashda foydalanamiz.

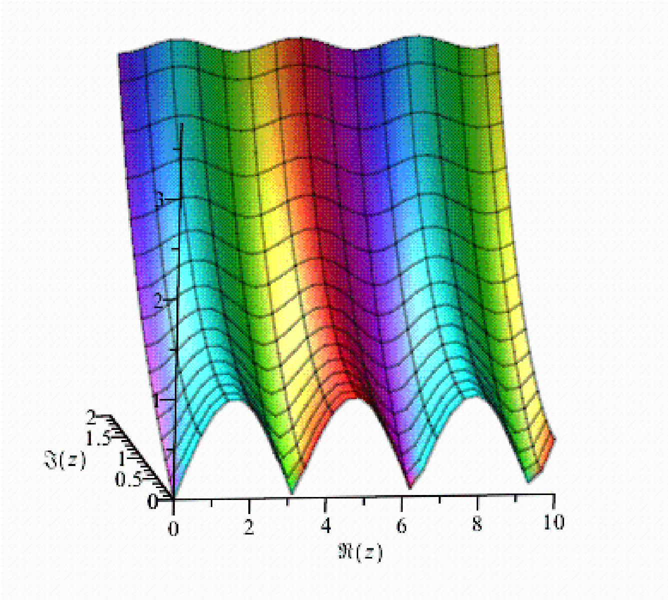
Ta’rif. *Ushbu*

$$\begin{cases} \cos z = \dfrac{e^{iz} + e^{-iz}}{2}, \quad \sin z = \dfrac{e^{iz} - e^{-iz}}{2i}, \\ tgz = \dfrac{\sin z}{\cos z} = \dfrac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}, \\ ctgz = \dfrac{\cos z}{\sin z} = \dfrac{i(e^{iz} + e^{-iz})}{e^{iz} - e^{-iz}} \end{cases} \qquad (14)$$

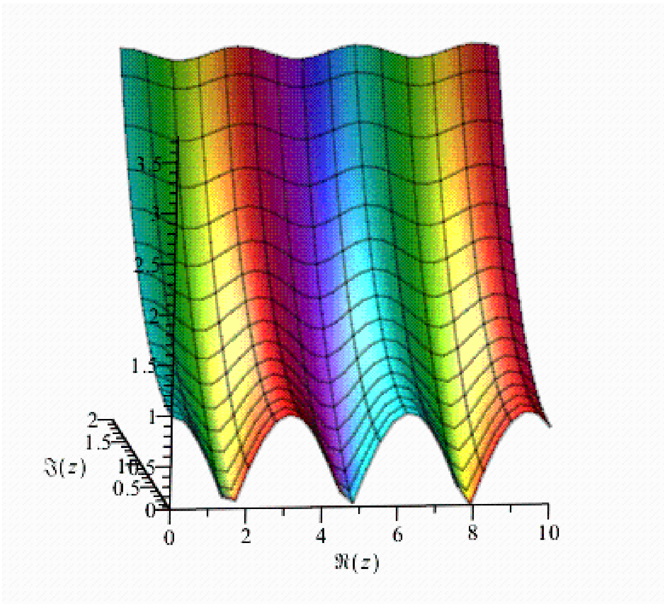
tengliklar yordamida aniqlangan funksiyalarga kompleks argumentli trigonometrik funksiyalar deb ataladi.

Bu funksiyalarning relyeflarini keltiramiz

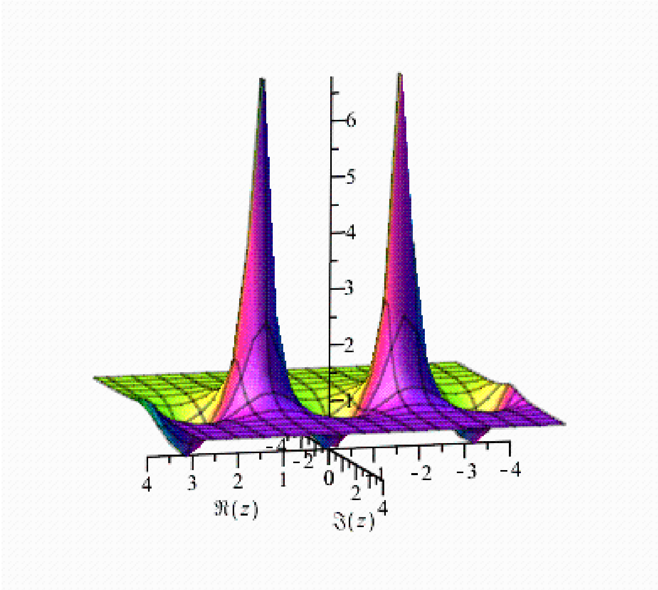
```
> with(plots) :
> complexplot3d(sin(z), z = 10 − 0 I..0 + 2 I, grid = [30, 30]);
```



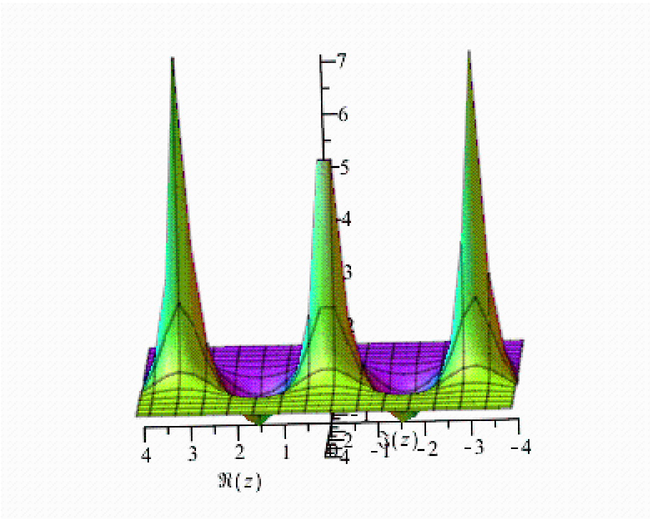
complexplot3d(cos(z), $z = 10 - 0\,I..0 + 2\,I$, *grid* = [30, 30])



> *complexplot3d*(tan(z), $z = -4 - 4\,I..4 + 4\,I$, *grid* = [30, 30])



> $complexplot3d\left(\frac{1}{\tan(z)}, z=-4-4I..4+4I, grid=[30,30]\right)$



Trigonometrik funksiyalarning asosiy xossalarini keltiramiz.

1) $\cos z$ va $\sin z$ funksiyalar C kompleks tekislikda golomorf va ularning hosilalari

$$(\cos z)' = -\sin z,$$

$$(\sin z)' = \cos z$$

bo‘ladi.

2) tgz funksiya

$$\left\{z \in C; \quad z \neq \frac{\pi}{2} + k\pi \;\;, \quad k = 0, \pm 1, \pm 2, \ldots\right\}$$

to‘plamda, ctgz funksiya esa

$$\{z \in C; \;\;, z \neq k\pi, \;\; k = 0, \pm 1, \pm 2, \ldots\}$$

to‘plamda golomorf bo‘ladi.

3) sinz, tgz, ctgz funksiyalar toq, cosz esa juft funksiya bo‘ladi.

4) Trinogometrik funksiyalar davriy bo‘lib, cosz va sinz ning davri 2π ga, tgz va ctgz ning davri π ga tengdir.

5) Haqiqiy o‘zgaruvchili trigonometrik funksiyalar orasidagi munosabatlarni ifodalovchi formulalarning ko‘pchiligi kompleks o‘zgaruvchili bo‘lgan holda ham o‘rinli bo‘ladi.

Izoh. Kompleks argumentli cosz va sinz funksiyalarning haqiqiy argumentli cosz va sinz funksiyalardan farqli tomoni shundaki, ular chegaralangan bo‘lishi shart emas. Masalan w=cosz funksiyaning kompleks tekslik C da chegaranlanmaganligini ko‘rsataylik,

◁ Ma’lumki,

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \quad .$$

Bu tenglikda z=iy deb olamiz. Unda

$$\cos(iy) = \frac{e^{i(iy)} + e^{-i(iy)}}{2} = \frac{e^{-y} + e^{y}}{2}$$

bo‘ladi. Ravshanki,

$$\lim_{y \to +\infty} \frac{e^{-y} + e^{y}}{2} = \infty$$

Bu esa w=cosz funksiyaning C da chegaralanmaganligini bildiradi ▷

6) Ushbu

cos(iz)=chz , i sinz=-shz ,

cosz=ch(iz), sinz=-ish(iz)

munosabatlar o‘rinli, bunda

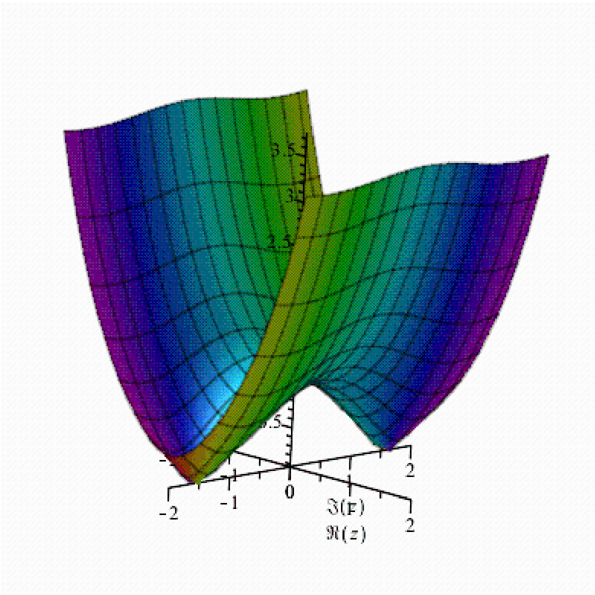
$$ch\,z = \frac{e^{z} + e^{-z}}{2}, \quad sh\,z = \frac{e^{z} - e^{-z}}{2}. \qquad (15)$$

Odatda, (15)-funksiyalar *giperbolik funksiyalar* deyiladi.

Bu funksiyalarning relyeflari 38 va 39 – chizmalarda tasvirlangan.

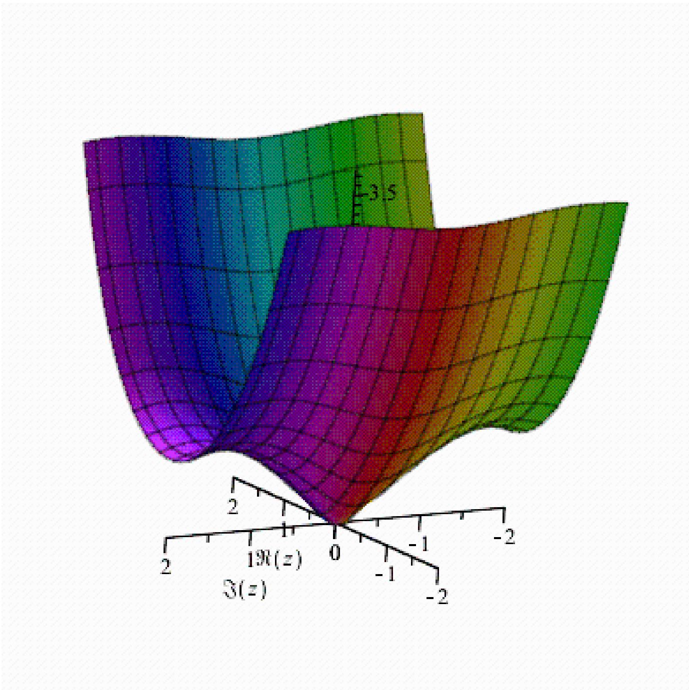
> *with*(*plots*) :

> *complexplot3d*(cosh(*z*), *z* =−2 − 2 *I*..2 + 2 *I*, *grid* = [30, 30]);



38- chizma

> *complexplot3d*(sinh(z), *z* =−2 − 2 *I*..2 + 2 *I*, *grid* = [30, 30])



39- chizma

7) Trigonometrik funksiyalar yordamida bajariladigan akslantirishlar bir nechta bizga ma’lum akslantirishlarning kompozitsiyasi natijasidan iborat bo‘ladi.

Misol. Ushbu

$$w = \sin z$$

funksiya yordamida bajariladigan akslantirish (z) tekisligidagi

$$D = \{z \in C : -\frac{\pi}{2} < \operatorname{Re} z < \frac{\pi}{2}, \operatorname{Im} z > 0\}$$

sohani (yarim yo‘lakni) (w) tekislikdagi qanday sohaga akslantiradi?

◁ Berilgan $w = \sin z$ funksiya yordamida bajariladigan akslantirish bizga ma’lum bo‘lgan

$$w_1 = iz \quad , \quad w_2 = e^{w_1} \quad , \quad w_3 = \frac{w_2}{i}$$

akslantirishlar kompozitsiyasidan iborat bo‘lib,

$$w = \sin z = \frac{1}{2}(w_3 + \frac{1}{w_3})$$

bo‘ladi. Binobarin, bu akslantirishlarni, ketma-ket bajarish natijasida $w = \sin z$ uchun $w(D)$ topiladi:

1) D soha $w_1 = iz$ akslantirish natijada

$$D_1 = \{w_1 \in C : \operatorname{Re} w_1 < 0 \ , \qquad -\frac{\pi}{2} < \operatorname{Im} w_1 < \frac{\pi}{2}\}$$

sohaga o‘tadi.

2) D_1 soha $w_2 = e^{w_1}$ akslantirish natijasida

$$D_2 = \{w_2 \in C : |w_2| < 1 \ , \qquad -\frac{\pi}{2} < \arg w_2 < \frac{\pi}{2}\}$$

yarim doiraga o‘tadi.

3) D_2 soha $w_3 = \frac{w_2}{i}$ akslantirish natijasida

$$D_3 = \{w_3 \in C : |w_3| < 1 , \qquad \pi < \arg w_3 < 2\pi\}$$

sohaga o‘tadi.

4) D_3 soha $w = \sin z = \frac{1}{2}(w_3 + \frac{1}{w_3})$ akslantirish natijasida

$$w(D) = \{w \in C : \operatorname{Im} w > 0\}$$

sohaga o‘tadi.

Demak, $w = \sin z$ akslantirish (z) tekislikdagi

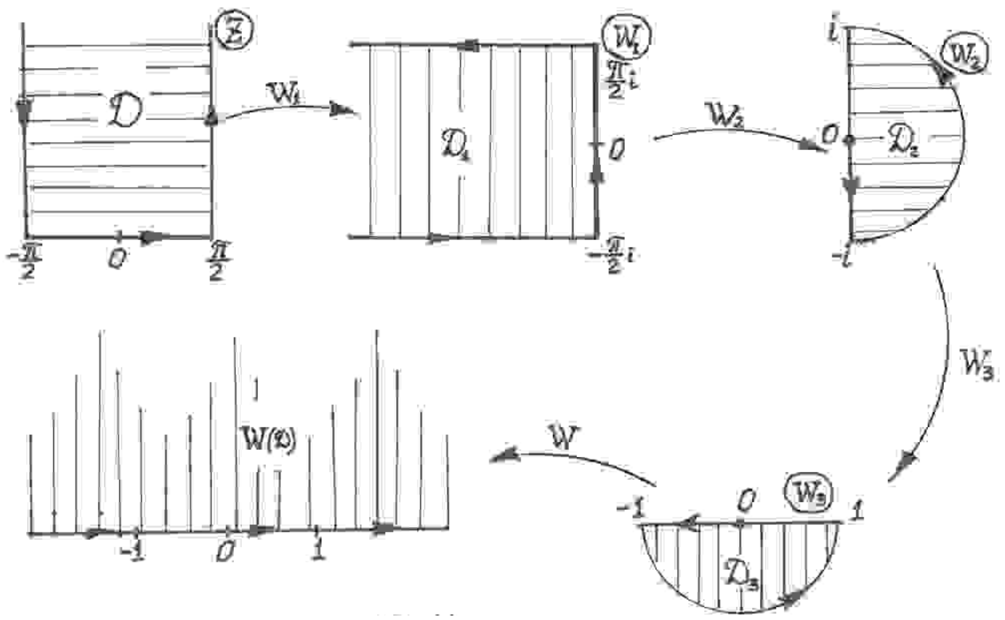
$$D = \{z \in C : -\frac{\pi}{2} < \operatorname{Re} z < \frac{\pi}{2} \ , \operatorname{Im} z > 0\}$$

sohani (w) tekislikdagi

$$w(D) = \{w \in C : \operatorname{Im} w > 0\}$$

yuqori yarim tekislikka akslantirar ekan.

Olingan funksiyalar D sohani qaysi yo‘l bilan $w(D)$ sohaga akslantirishi 40-chizmada ko‘rsatilgan ▻

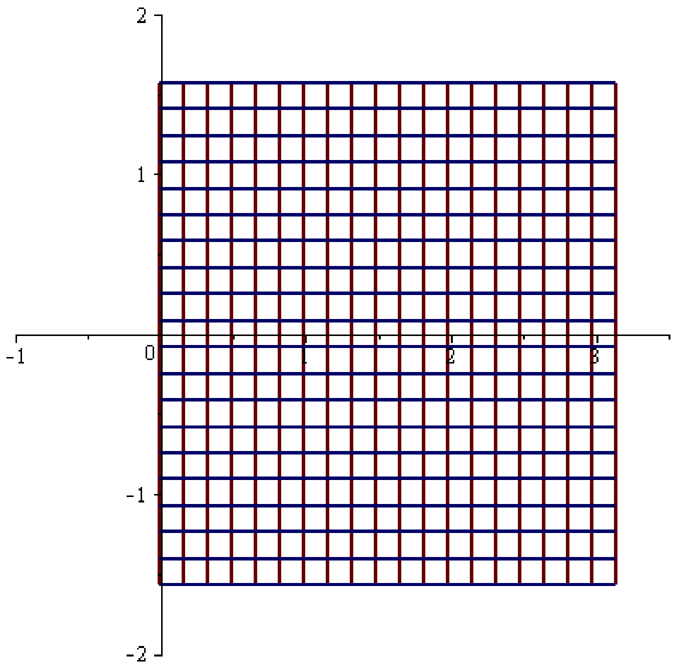


40-chizma

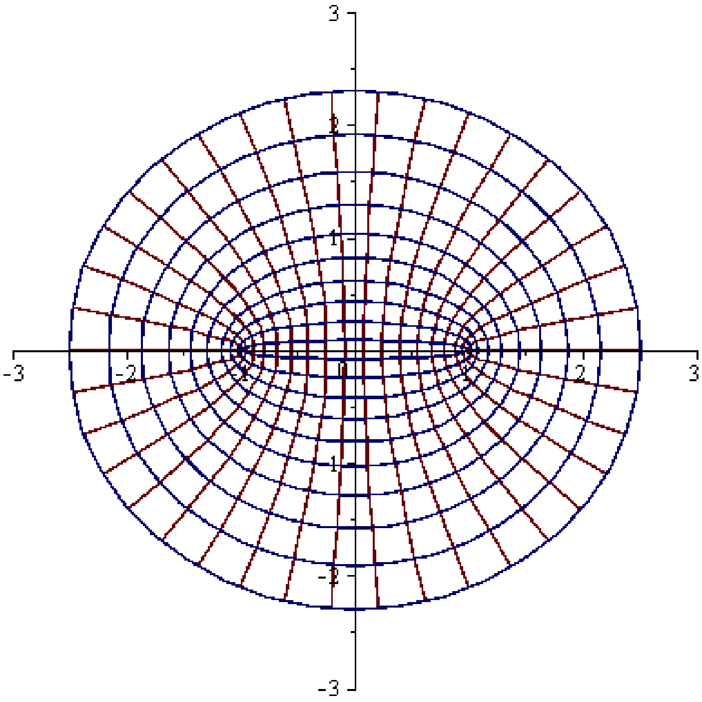
Misol. $D=\{0<\operatorname{Re}z<\pi, -\frac{\pi}{2}<\operatorname{Im}z<\frac{\pi}{2}\}$ sohani $w=cosz$ akslantirishdagi aksini Maple matematik paketi yordamida topilsin.

> $with(plots):$

> $conformal\left(z, z=0-\frac{\pi}{2}\cdot I..\pi+\frac{\pi}{2}\cdot I, -1-2\cdot I..3.5+2\cdot I, grid=[20,20]\right)$



$> conformal\left(\cos(z), z=0-\frac{\pi}{2}\cdot I..\pi+\frac{\pi}{2}\cdot I, -3-3\cdot I..3+3\cdot I, grid=[20,20]\right)$



7^0. Ko‘p qiymatli funksiyalar.

Kompleks argumentli funksiyalar nazariyasida golomorf funksiyaga teskari bo‘lgan funksiyani o‘rganish masalasi ham muhim o‘rinda turadi. Aksariyat hollarda bunday funksiyalar bir qiymatli bo‘lmay, argumentning bitta qiymatiga bir nechta (ba’zi holda cheksiz ko‘p) kompleks son mos qo‘yiladi. Bunday funksiyalarni qat’iy matematik asosda berish yo‘lida kompleks analizga Riman sirtlari termini kiritiladi. Biz bu yerda eng sodda ko‘p qiymatli funksiyalarni qarash bilan kifoyalanamiz.

a) $w=\sqrt[n]{z}$ **(**$n\geq 2-butun\ son$**) funksiyasi.**

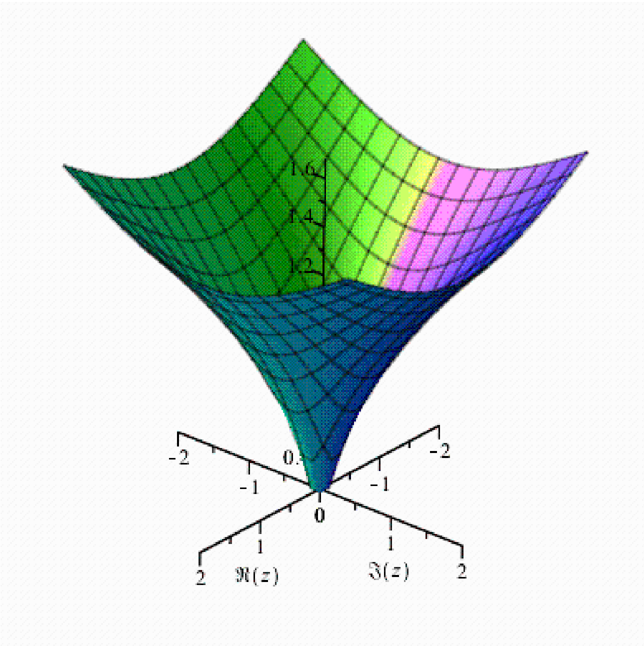
1-Ta’rif. *Ushbu*

$$w^n = z \qquad (16)$$

tenglamaning yechimlariga z *kompleks sonning n-darajali ildizlari deyiladi va* $w=\sqrt[n]{z}$ *kabi belgilanadi.*

$w=\sqrt{z}$ funksiyaning releyfi 41- chizmada tasvirlangan.

$> complexplot3d(\sqrt{z}, z=-2-2I..2+2I, grid=[30,30])$



41- chizma

(16)–tenglamaning yechimlari ildiz chiqarish uchun *Muavr formulasiga* ko‘ra

$$\sqrt[n]{z} = \sqrt[n]{r} \cdot e^{\frac{\varphi+2k\pi}{n}i} = \sqrt[n]{r} \cdot (\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}),\ k \in Z \qquad (17)$$

tenglik yordamida topiladi. Bu yechimlar k ning $0,1,2,.....,(n-1)$ qiymatlarida bir-biridan farq qilib, k-ning boshqa qiymatlarida esa ular takrorlanadi. Shuning uchun ham $\sqrt[n]{z}$ n- ta qiymatli bo‘lib, bu qiymatlar

$$\sqrt[n]{|z|} \cdot e^{\frac{\arg z+2k\pi}{n} \cdot i}, \qquad , \ k = 0,1,2,\cdots,(n-1) \qquad (18)$$

$w = \sqrt[n]{z}$ ning funksional xossalarini o‘rganishda quyidagi sodda, lekin muhim teoremadan foydalaniladi.

Teorema. (Teskari funksiyaning konformligi haqidagi teorema). *Faraz qilaylik,* $w = f(z)$ *funksiya* (z) *tekislikdagi D sohani* (w) *tekislikdagi G sohaga konform akslantiruvchi funksiya bo‘lsin. U holda bu funksiyaga teskari bo‘lgan* $z = f^{-1}(w)$ *funksiya G ni D ga konform akslantiradi.*

O‘quvchiga $z = w^n$ funksiyaning bir yaproqli bo‘ladigan sohalari 3^0-punktdan ma’lum: $z = w^n$ funksiya ushbu har bir

$$D_k=\{\frac{2k\pi}{n}<\arg w<\frac{2(k+1)\pi}{n}\}\ ,\quad k=0,1,2,\ldots,(n-1),$$

sohada bir yaproqli bo‘lib, bu sohani u $G=C\setminus R_+$ sohaga konform akslantiradi. $k=0$ desak $z=w^n$ funksiya

$$D_0=\{0<\arg w<\frac{2\pi}{n}\}$$

sohani G ga konform akslantiradi. Keltirilgan teoremaga ko‘ra bu akslantirishning teskarisi G *ni* D_0 ga konform akslantiradi. Bu teskari funksiya (18) dagi

$$\sqrt[n]{|z|}\cdot e^{i\cdot\frac{\arg z}{n}}$$

ga mos kelib, bu bir qiymatli funksiyaga $\sqrt[n]{z}$ ko‘p qiymatli funksiyaning *0-tarmog‘i* deyiladi va u $(\sqrt[n]{z})_0$ kabi belgilanadi. Xuddi shunday $z=w^n$ funksiya

$$D_1=\{\frac{2\pi}{n}<\arg z<2\cdot\frac{2\pi}{n}\}$$

sohani ham G ga konform akslantiradi. Bu funksiyaning teskarisi G *ni* D_1 ga akslantirib, unga $\sqrt[n]{z}$ ning *1-tarmog‘i* deyiladi va u $(\sqrt[n]{z})_1$ kabi belgilanadi. Bu jarayonni davom ettirib, $\sqrt[n]{z}$ ko‘p qiymatli funksiyadan n ta bir qiymatli tarmoqlar $(\sqrt[n]{z})_0,(\sqrt[n]{z})_1,\ldots(\sqrt[n]{z})_{n-1}$ larni ajrata olamiz. Bu har bir $(\sqrt[n]{z})_k$, $k=0,1,\ldots(n-1),$ tarmoq G da bir qiymatli va uni D_k sohaga konform akslantiradi.

Misol. $D=C\setminus R_+$ sohani birlik doiraga konform akslantiring.

$\lhd$ $(\sqrt{z})_0$ tarmoqning xossasiga ko‘ra $w_1=(\sqrt{z})_0$ funksiya D ni yuqori yarim tekislikka konform akslantiradi. (5)-formulaga ko‘ra $w=\frac{w_1-i}{w_1+i}$ kasr-chiziqli funksiya yuqori yarim tekislikni birlik doiraga akslantiradi. Demak $w=\dfrac{(\sqrt{z})_0-i}{(\sqrt{z})_0+i}$ funksiya $C\setminus R_+$ ni birlik doiraga konform akslantiradi ▹

$w=\sqrt[n]{z}$ ko‘p qiymatli funksiyada $(\sqrt[n]{z})_0, (\sqrt[n]{z})_1, \ldots, (\sqrt[n]{z})_{n-1}$ bir qiymatli funksiyalarning hosil qilinishi ko‘p qiymatli funksiyalardan *tarmoq ajratish* deyilib, bu yerda biz tarmoq ajratishning bitta uslubini berdik. Bu tarmoqlardan odatda $w=(\sqrt[n]{z})_0$ tarmoq ko‘p ishlatiladi. Amaliyotda bu funksiyalardan burchak sohalarni kichraytirish (siqish) uchun foydalaniladi.

Ba’zi bir masalalarni yechishda ko‘p qiymatli $w=\sqrt[n]{z}$ funksiyaning bir qiymatli tarmoqlarini berilgan shartlarga qarab ham ajratishga to‘g‘ri keladi. Masalan, $n=2$ bo‘lganda, ikki qiymatli $w=\sqrt{z}$ funksiyaning ikkita bir qiymatli $(w)_0$ *va* $(w)_1$ tarmoqlarini quyidagicha ham ajratish mumkin:

$$(w)_0=\sqrt{z} \quad , \quad \sqrt{-1}=i \quad (yoki\ \sqrt{1}=1)$$

va

$$(w)_1=\sqrt{z} \quad , \quad \sqrt{-1}=-i \quad (yoki\ \sqrt{1}=-1)$$

$(w)_0$ tarmoq $C\setminus R_+$ ni yuqori yarim tekislikka, $(w)_1$ tarmoq esa $C\setminus R_+$ ni quyi yarim tekislikka konform akslantiradi.

b) w=Lnz funksiyasi.

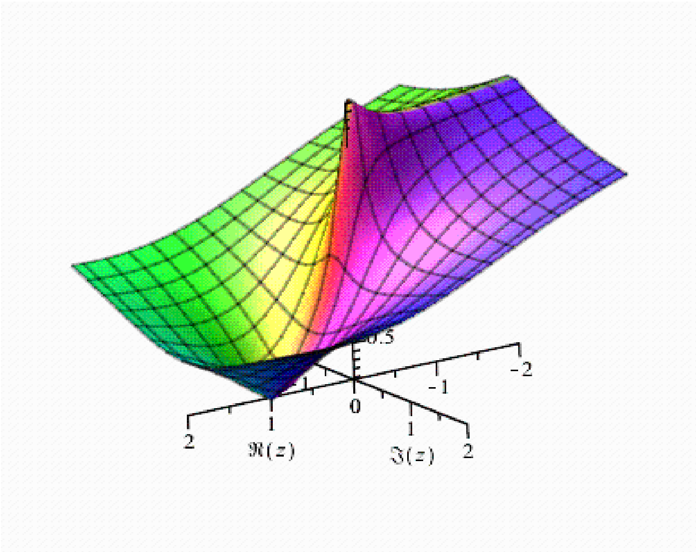
2-Ta’rif. *Ushbu*

$$e^w=z \qquad (19)$$

tenglamaning yechimlari z kompleks sonning logarifmi deyiladi va $w=Lnz$ *kabi belgilanadi.*

Bu funksiyaning releyfi 42- chizmada tasvirlangan.

```
> complexplot3d(ln(z), z=-2 - 2I..2 + 2I, grid = [30, 30])
```



42- chizma

(19)–tenglamani yechish uchun z ni $z = re^{i\varphi}$ ko‘rinishda, w ni esa $w = u + iv$ shaklda ifodalaymiz:

$$e^{u+iv} = re^{i\varphi}.$$

Bunda $e^{u} = r$, $e^{iv} = e^{i\varphi}$ tengliklarga ega bo‘lib, yechim $u = \ln r$, $v = \varphi + 2k\pi$, $k \in Z$ ekanligini ko‘ramiz. Demak,

$$w = Lnz = \ln|z| + i(\arg z + 2k\pi)\ , \quad k \in Z \qquad (20)$$

bo‘lib, Lnz funksiya ko‘p qiymatlidir.

e^{w} funksiya

$$\Pi_k = \{w \in C : 2k\pi < \operatorname{Im} w < 2(k+1)\pi\},\ k \in Z$$

sohalarda bir yaproqli va bu sohalarning har birini $C \setminus R_{+}$ ga konform akslantirishini bilamiz. Teskari funksiyaning konformligi haqidagi teoremadan foydalansak, biz $w = Lnz$ funksiyasidan cheksiz ko‘p tarmoqlar

$$w = (Lnz)_k = \ln|z| + i(\arg z + 2k\pi)\ ,\ k \in Z$$

ni ajratish mumkin ekanligini hosil qilamiz. Bu har bir tarmoq $G = C \setminus R_{+}$ da golomorf bo‘lib, uni Π_k yo‘lakka konform akslantiradi.

Kelishuvga ko‘ra $(Lnz)_0 = \ln z$ deb belgilanadi va bu funksiyaga *Lnz funksiyaning bosh tarmog‘i* deyiladi.

Misol. $z_0 = i$ nuqtani $w_0 = \frac{5\pi i}{2}$ nuqtaga o‘tkazadigan *logarifmning bir qiymatli tarmog‘i* yordamida

$$D = \{z : z \notin (-\infty, 0]\}$$

sohaning aksini toping.

$\triangleleft$ Lnz funksiyaning

$$w = (Lnz)_k = \ln z + 2k\pi i \quad , \quad k = 0,1,2,...$$

tarmoqlaridan qaysi birini tanlashimiz kerakligini

$$w(i) = \frac{5\pi i}{2}$$

shartdan aniqlaymiz:

$$\frac{5\pi i}{2} = \ln i + 2k\pi i = \ln|i| + i\arg i + 2k\pi i = i \cdot \frac{\pi}{2} + 2k\pi i \,.$$

Bu yerdan k=1 ekanligini topamiz. Demak, Lnz ning kerakli tarmog‘i

$$w = (Lnz)_1 = \ln z + 2\pi i$$

ekan. $w_1 = \ln z$ funksiya yordamida D sohaning

$$\{w_1 : -\pi < \operatorname{Im} w_1 < \pi\}$$

yo‘lakka akslanishini tekshirish qiyin emas. $w = w_1 + 2\pi i$ funksiya yordamida esa yo‘lak

$$\{w : \pi < \operatorname{Im} w < 3\pi\}$$

yo‘lakka akslanadi $\triangleright$

v) Kompleks sonni kompleks darajaga ko‘tarish.

$w = Lnz$ funksiyasidan foydalanib, ixtiyoriy $z \neq 0$ va *a* kompleks sonlar uchun ta’rifga ko‘ra

$$z^a = e^{aLnz} = e^{a[\ln|z| + i(\arg z + 2k\pi)]} \qquad (21)$$

deb qabul qilinadi.

Masalan,

$$i^i = e^{iLni} = e^{i[\ln|i| + i(\arg i + 2k\pi)]} = e^{i\cdot i(\frac{\pi}{2} + 2k\pi)} = e^{-\frac{\pi}{2} - 2k\pi}, k \in Z.$$

Demak, i^i ning cheksiz ko‘p qiymatlari mavjud bo‘lib, ularning hammasi haqiqiy sonlardir.

(21)–munosabat yordamida biz ixtiyoriy kompleks son uchun

$$w = z^a$$

funksiyasini o‘rganishimiz mumkin. Amaliyotda a - haqiqiy son bo‘lgan hol ko‘p qo‘llanilib, $w = z^a$ funksiya burchak sohalarni konform akslantirishda foydalidir.

g) Teskari trigonometrik funksiyalar.

Kompleks o‘zgaruvchili funksiyalar nazariyasida teskari funksiya tushunchasi haqiqiy o‘zgaruvchili funksiyalar sinfidagi kabi kiritiladi.

Masalan,

$$w = Arc\cos z$$

funksiya $z = \cos w$ tenglamani qanoatlantiruvchi barcha w larning qiymatlari to‘plamidan iborat, ya’ni $\cos z$ funksiyaga teskari funksiyadir.

$$Arc\sin z, \quad Arctgz, \quad Arcctgz$$

va boshqa funksiyalar ham shunga o‘xshash aniqlanadi.

Ta’rifdan foydalanib

$$Arc\cos z = -iLn(z + \sqrt{z^2 - 1}) \qquad (22)$$

tenglikning o‘rinli ekanligini ko‘rsatish qiyin emas. Bu yerda ildizning barcha qiymatlari olinadi.

(22)-tenglikdan ko‘rinib turibdiki, logarifmik funksiya kabi $Arc\cos z$ funksiya ham bir qiymatli emas. $Arc\cos z$ funksiyaning bosh qiymati $w = \arccos z$ deb olinadi va ushbu

$$w = \arccos z = -i\ln(z + \sqrt{z^2 - 1}) \qquad (23)$$

tenglik yordamida aniqlanadi.

$w = Arc\cos z$ funksiya

$$\{z : \operatorname{Im} z > 0\}$$

yuqori yarim tekislikda cheksiz ko‘p qiymatli bo‘lib, (22)–tenglikdan foydalanib uning bir qiymatli tarmoqlarini ajratish mumkin. Ular

$$(Arc\cos z)_k = -i(Ln(z + \sqrt{z^2 - 1}))_k \qquad k = 0, \pm 1, \pm 2, \ldots$$

tenglik yordamida aniqlanadi. Masalan, $k=0$ bo‘lsa,

$$(Arc\cos z)_0 = \arccos z = -i\ln(z + \sqrt{z^2 - 1})$$

funksiya

$$\{z : \operatorname{Im} z > 0\}$$

sohani

$$\{w : 0 < \operatorname{Re} w < \pi, \quad \operatorname{Im} z < 0\}$$

yarim yo‘lakka konform akslantiradi.

8⁰. Simmetriya prinsipi.

Bir sohani ikkinchi sohaga konform akslantirishda simmetriya prinsipidan keng foydalaniladi.

Faraz qilaylik, $f_1(z)$ funksiya D_1 sohada ($D_1 \subset C$) berilgan hamda shu sohada konform bo‘lsin. Bunda D_1 sohaning chegarasi ∂D_1 ning biror qismi $\gamma(\gamma \subset \partial D_1)$ aylana yoyi yoki to‘g‘ri chiziq kesmasidan iborat. Bu $f_1(z)$ akslantirish D_1 sohani G_1 sohaga, γ chiziqni Γ chiziqqa (Γ - aylana yoyi yoki to‘g‘ri chiziq kesmasi) akslantirsin:

$$G_1 = f_1(D_{1,})$$

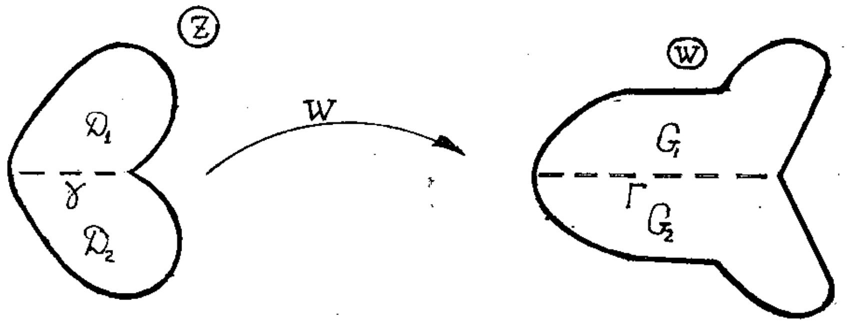
$$\Gamma = f_1(\gamma).$$

D_1 sohaning γ yoyga nisbatan simmetrik bo‘lgan sohasi D_2, G_1 sohaning Γ yoyga nisbati simmetrik bo‘lgan sohasi esa G_2 bo‘lsin. $f_2(z)$ funksiyani D_2

sohada shunday aniqlaymizki, uning qiymatlari $f_1(z)$ funksiyaning G_1 dagi qiymatlariga Γ yoyga nisbatan simmetrik bo‘lgan qiymatlarni qabul qilsin. U holda $f_2(z)$ funksiya D_2 *ni* G_2 *ga*, ushbu

$$w = \begin{cases} f_1(z) \quad , \quad z \in D_1, \\ f_1(z) = f_2(z) \, , \quad z \in \gamma \, , \\ f_2(z) \quad , \quad z \in D_2 \end{cases}$$

funksiya esa $D_1 \cup \gamma \cup D_2$ sohani $G_1 \cup \Gamma \cup G_2$ sohaga konform akslantiradi (43-chizma).



43- chizma

Odatda, yuqoridagi tasdiq *simmetriya prinsipi yo*ki *Riman–Shvars teoremasi* deb ataladi.

Eslatma. Agar γ va Γ lar haqiqiy o‘qdagi kesmalar bo‘lsa, u holda $f_2(z)$ funksiya ushbu

$$f_2(z) = \overline{f_1(\bar{z})}$$

tenglik yordamida aniqlanadi.

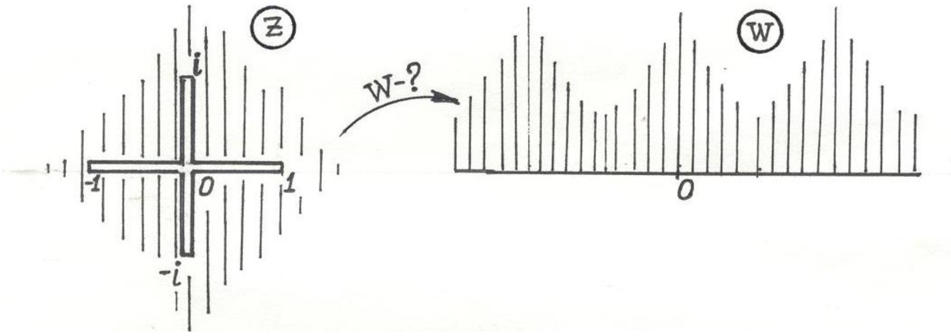
Misol. Ushbu

$$D = \{z \in C : z \notin [-1;1], \ z \notin [-i; i]\}$$

sohani yuqori yarim tekislik

$$\{w \in C : \operatorname{Im} w > 0\}$$

ka konform akslantiruvchi $w = w(z)$ funksiyani toping (44-chizma).



44 - chizma

◁ Quyidagi

$$D_1 = \{z \in C : \operatorname{Im} z > 0 \ , \ z \notin [0,1]\}$$

sohada

$$w_1 = z^2$$

funksiyani qaraymiz. Ravshanki, bu akslantirish D_1 sohada konform bo‘ladi.

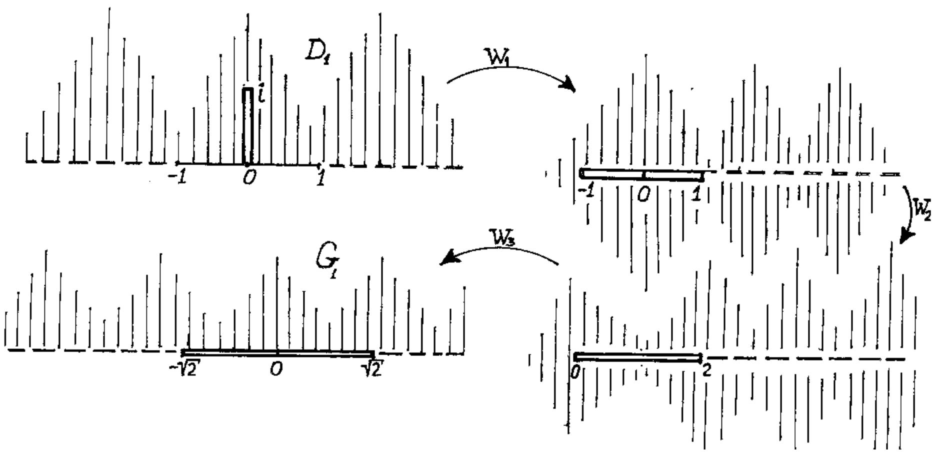
Endi D_1 sohani yuqori yarim tekislikka akslantiramiz. Bu quyidagi

$$w_1 = z^2$$

$$w_2 = w_1 + 1, \qquad (24)$$

$$w_3 = \sqrt{w_2} \ , \quad \sqrt{-1} = i$$

akslantirishlarni ketma-ket bajarish natijasida sodir bo‘ladi. ((24)– akslantirishlarning bajarilish jarayoni 45-chizmada tasvirlangan):



45-chizma

Shunday qilib, D_1 soha ushbu

$$w_3 = \sqrt{w_2} = \sqrt{w_1 + 1} = \sqrt{z^2 + 1}\ ,\ \sqrt{-1} = i$$

funksiya yordamida

$$G_1 = \{w_3 \in C : \operatorname{Im} w_3 > 0\}$$

yuqori yarim tekislikka konform akslanar ekan. Endi simmetriya prinsipidan foydalanib, D sohani

$$w_3 = \sqrt{z^2 + 1}\ ,\ \sqrt{-1} = i$$

funksiya yordamida

$$G = \{w_3 \in C : w_3 \notin [-\sqrt{2}, \sqrt{2}]\}$$

sohaga konform akslantiramiz. Bu sohani yuqori yarim tekislik

$$\{w \in C : \operatorname{Im} w > 0\}$$

ka konform akslantirish quyidagi

$$w_4 = \frac{w_3 + \sqrt{2}}{\sqrt{2} - w_3},$$

$$w = \sqrt{w_4}\quad ,\quad \sqrt{-1} = i$$

akslantirishlarni ketma-ket bajarilishi natijasida amalga oshiriladi.

Demak, $D = \{z \in C : z \notin [-1;1],\ z \notin [-i; i]\}$ sohani yuqori yarim tekislik $\{w \in C : \operatorname{Im} w > 0\}$ ka konform akslantiruvchi funksiya

$$w = \sqrt{w_4} = \sqrt{\frac{w_3 + \sqrt{2}}{\sqrt{2} - w_3}} = \sqrt{\frac{\sqrt{z^2 + 1} + \sqrt{2}}{\sqrt{2} - \sqrt{z^2 + 1}}}\quad ,\quad \sqrt{-1} = i$$

bo‘ladi ▹

9^0. **Asosiy elementar funksiyalar yordamida bajariladigan konform akslantirishlar**.

Biz bu punktda amaliyotda ko‘p uchraydigan asosiy elementar funksiyalar va ular yordamida bajariladigan konform akslantirishlarni bir joyga jamlab chizmalardan foydalangan holda keltiramiz.

I. **Kasr- chiziqli funksiya**.

1) Angarmonik nisbat.

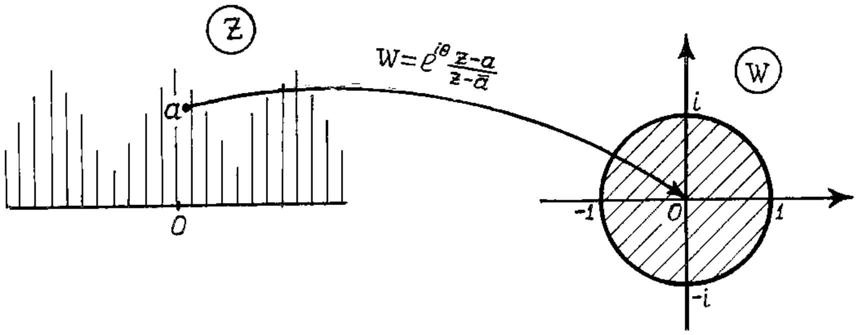
Berilgan $z_1, z_2, z_3 \in C_z$ nuqtalarni mos ravishda $w_1, w_2, w_3 \in C_w$ nuqtalarga akslantiruvchi kasr–chiziqli funksiya ushbu

$$\frac{w-w_1}{w-w_2} \cdot \frac{w_3-w_2}{w_3-w_1} = \frac{z-z_1}{z-z_2} \cdot \frac{z_3-z_2}{z_3-z_1}$$

angarmonik nisbatdan topiladi.

2) $w = e^{i\theta} \cdot \dfrac{z-a}{z-\overline{a}}$, $\quad \operatorname{Im} a > 0$ *va* $D = \{z : \operatorname{Im} z > 0\}$

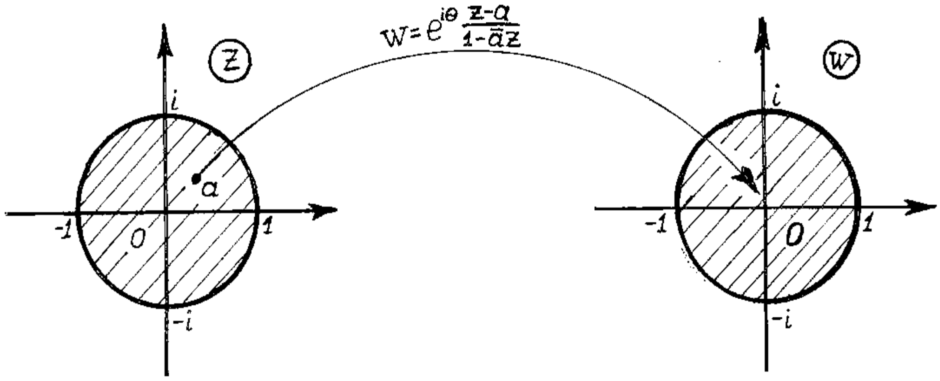
bo‘lsa, $w(D) = \{w : |w| < 1\}$ bo‘ladi (46-chizma).



46 - chizma

3) $w = e^{i\theta} \cdot \dfrac{z-a}{1-\overline{a}z}$, $\quad |a| < 1$ *va* $D = \{z : |z| < 1\}$

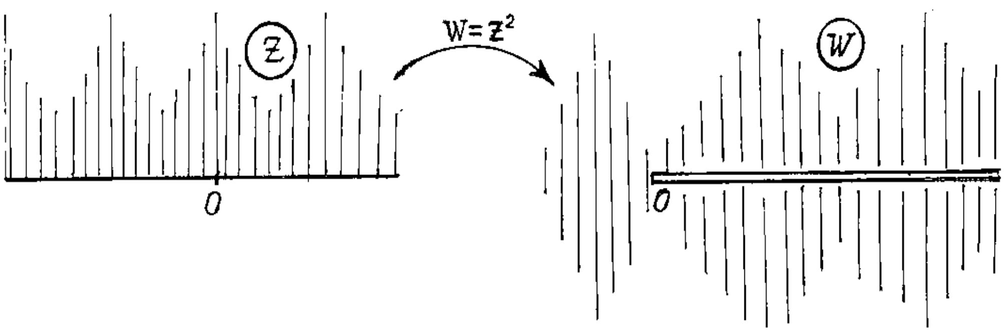
bo‘lsa, $w(D) = \{w : |w| < 1\}$ bo‘ladi (47-Chizma).



47 - chizma

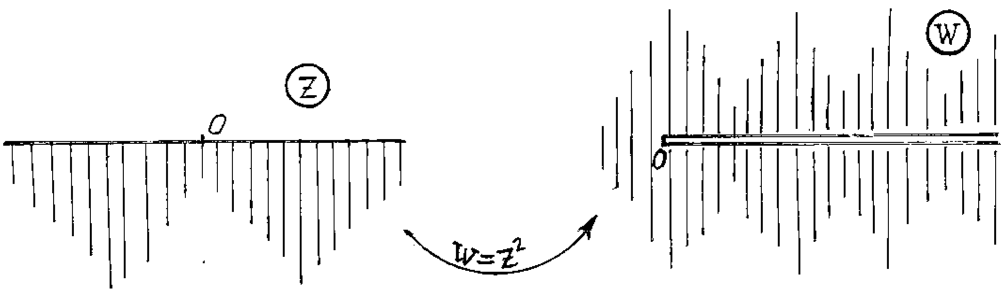
II. **Darajali funksiya va unga teskari bo‘lgan funksiyalar**.

1) $w=z^2$ *va* $D=\{z:\operatorname{Im} z>0\}$ *bo'lsa,* $w(D)=C\setminus R_+$ bo‘ladi (48-chizma).



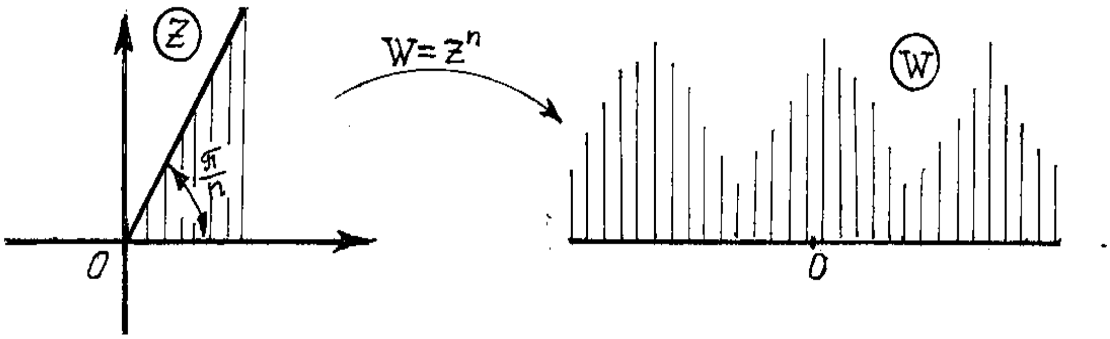
48-chizma

2) $w=z^2$ *va* $D=\{z:\operatorname{Im} z<0\}$ *bo'lsa,* $w(D)=C\setminus R_+$ bo‘ladi (49-chizma).



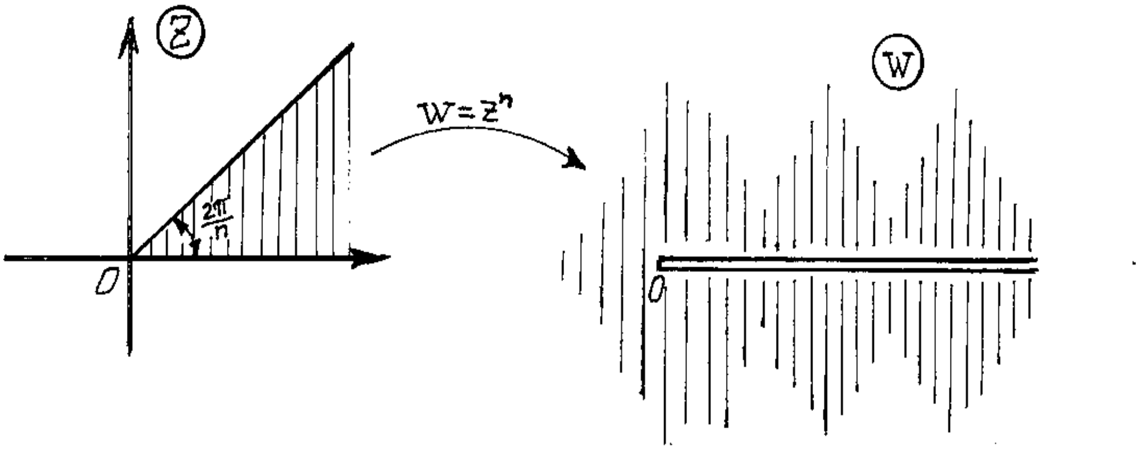
49 - chizma

3) $w=z^n$ *va* $D=\{0:0<\arg z<\frac{\pi}{n}\}$ *bo'lsa,* $w(D)=\{w:\operatorname{Im} w>0\}$ bo‘ladi (50-chizma).



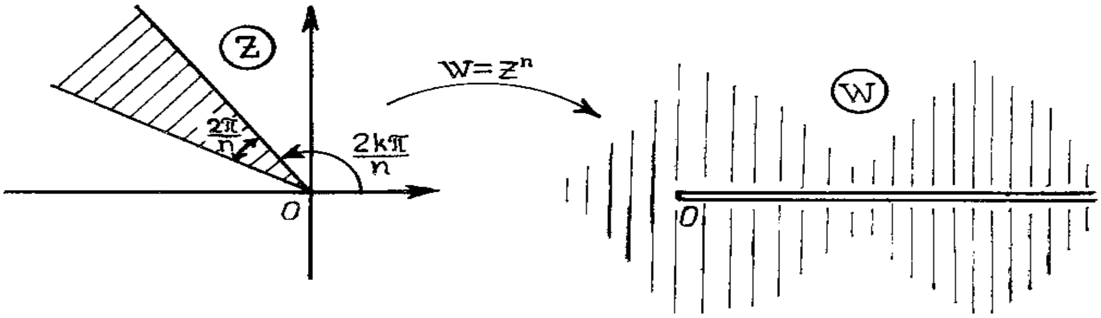
50 - chizma

4) $w = z^n$ *va* $D = \{0 : 0 < \arg z < \frac{2\pi}{n}\}$ *bo'lsa,* $w(D) = C \setminus R_+$ bo‘ladi (51-chizma).



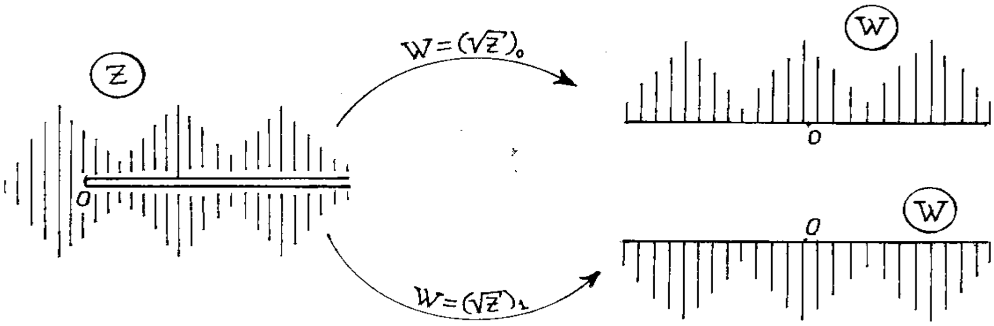
51-chizma

5) $w = z^n$ *va* $D = \left\{ \frac{2k\pi}{n} < \arg z < \frac{2(k+1)\pi}{n} \right\},\ k = 0,1,...n-1,$ bo‘lsa, $w(D) = C \setminus R_+$ bo‘ladi (52-chizma).



52 - chizma

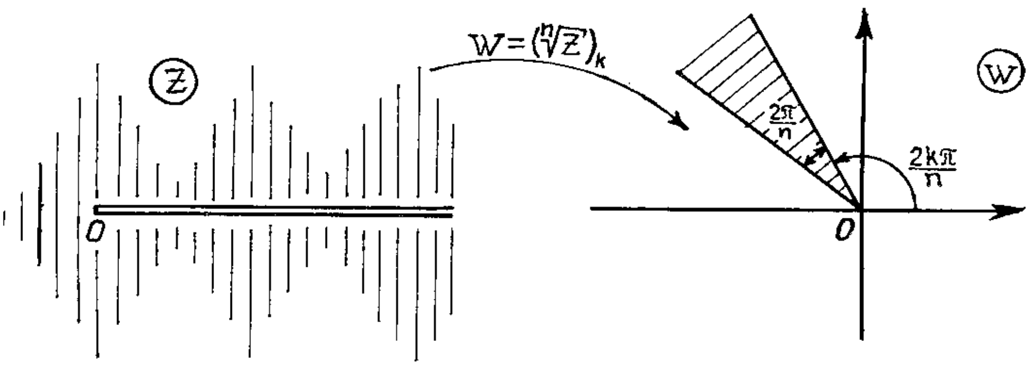
6) $w=(\sqrt{z})_0$ $(yoki\ \ w=\sqrt{z},\ \sqrt{-1}=i)$ $va\ D=C\setminus R_+$ va bo‘lsa, $w(D)=\{w:\operatorname{Im}w>0\}$ va $w=(\sqrt{z})_1$ $(yoki\ w=\sqrt{z},\ \sqrt{-1}=-i)$ bo‘lsa, $w(D)=\{w:\operatorname{Im}w<0\}$ bo‘ladi (53-chizma).



53 - chizma

7) $w=(\sqrt[n]{z})_k$, $k=0,1,...n-1$ $va\ D=C\setminus R_+$ bo‘lsa,

$w(D)=\{w:\dfrac{2k\pi}{n}<\arg w<\dfrac{2(k+1)\pi}{n}\}$ bo‘ladi (54-chizma).

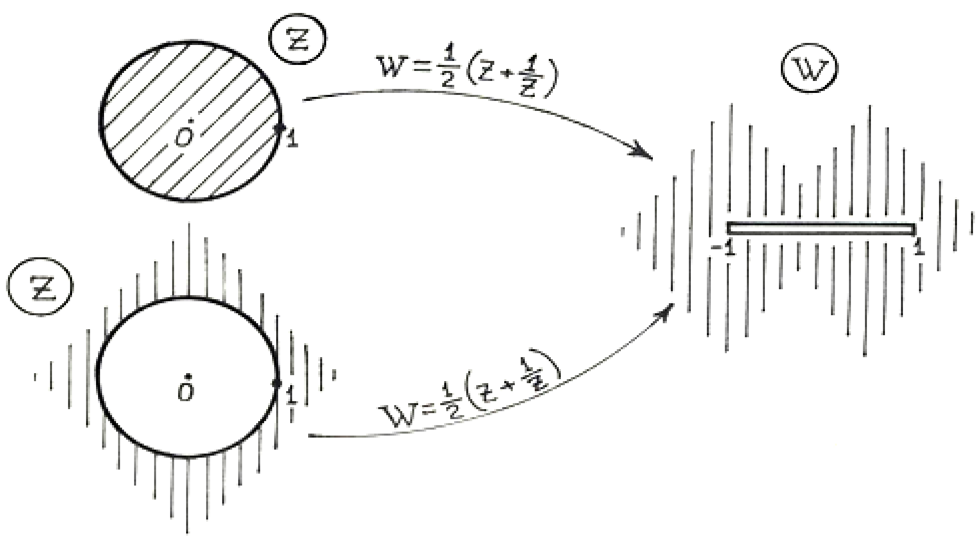


54 - chizma

III. **Jukovskiy funksiyasi va unga teskari funksiya**.

1) $w=\dfrac{1}{2}(z+\dfrac{1}{z})$ $va\ D=\{z:|z|<1\}\ bo'lsa\ w(D)=\{w:w\notin[-1;1]\}$ bo‘ladi (55-chizma).

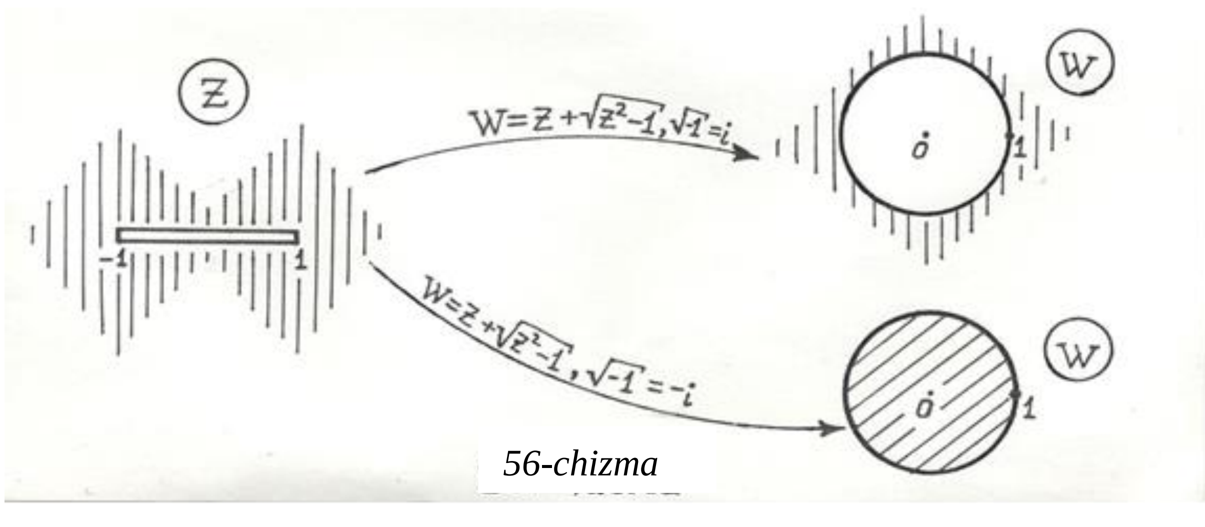
2) $w=\dfrac{1}{2}(z+\dfrac{1}{z})$ $va\ D=\{z:|z|>1\}\ bo'lsa\ w(D)=\{w:w\notin[-1;1]\}$ bo‘ladi (55-chizma).



55-chizma

3) $w = z + \sqrt{z^2 - 1}\,;\ \sqrt{-1} = i\ \ (yoki\ w(\infty) = \infty)\ va\ D = \{z : z \notin [-1;1]\}$ bo‘lsa, $w(D) = \{w : |w| > 1\}$ bo‘ladi (56-chizma).

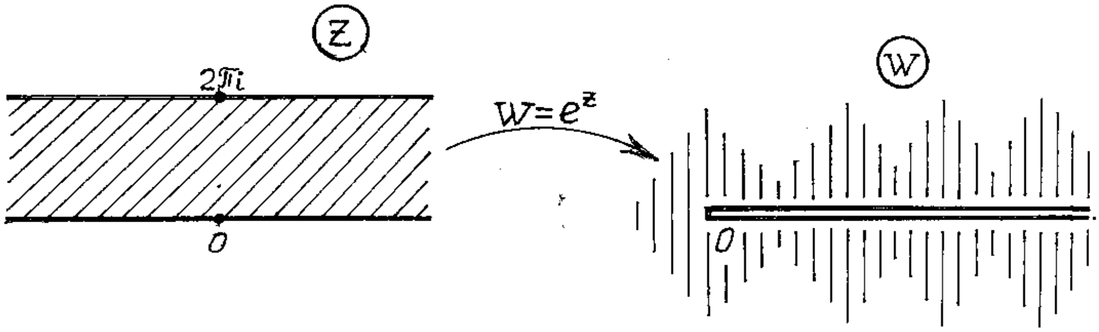
4) $w = z + \sqrt{z^2 - 1}\,;\ \sqrt{-1} = -i\ \ (yoki\ w(\infty) = 0)\ va\ D = \{z : z \notin [-1;1]\}$ bo‘lsa, $w(D) = \{w : |w| < 1\}$ bo‘ladi (56-chizma).



56-chizma

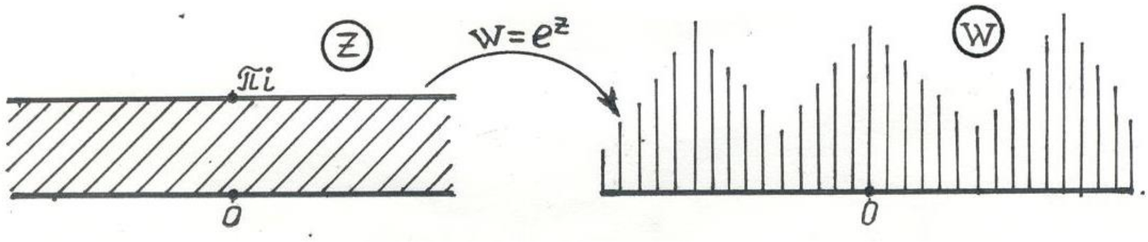
Ko‘rsatgichli va logarifmik funksiyalar.

1) $w = e^z\ \ va\ \ D = \{z : 0 < \operatorname{Im} z < 2\pi\}\ bo'lsa\ \ w(D) = C \setminus R_+$ bo‘ladi (57-chizma).



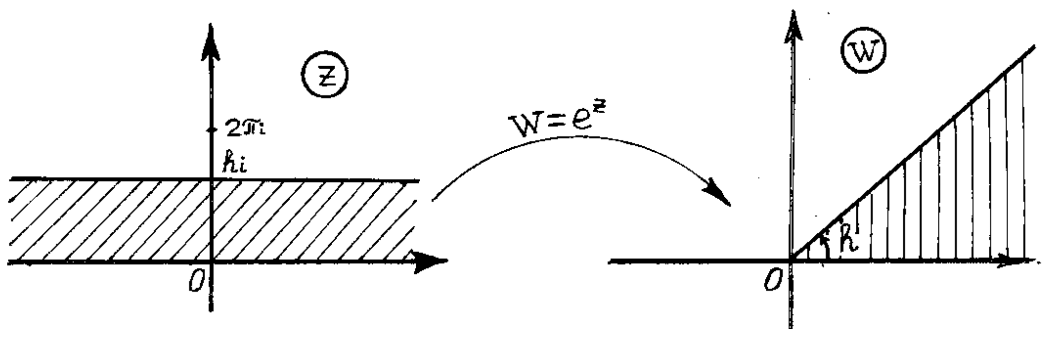
57 - chizma

2) $w=e^z$ *va* $D=\{z:0<\operatorname{Im}z<\pi\}$ *bo'lsa* $w(D)=\{w:\operatorname{Im}w>0\}$ bo‘ladi (58-chizma).



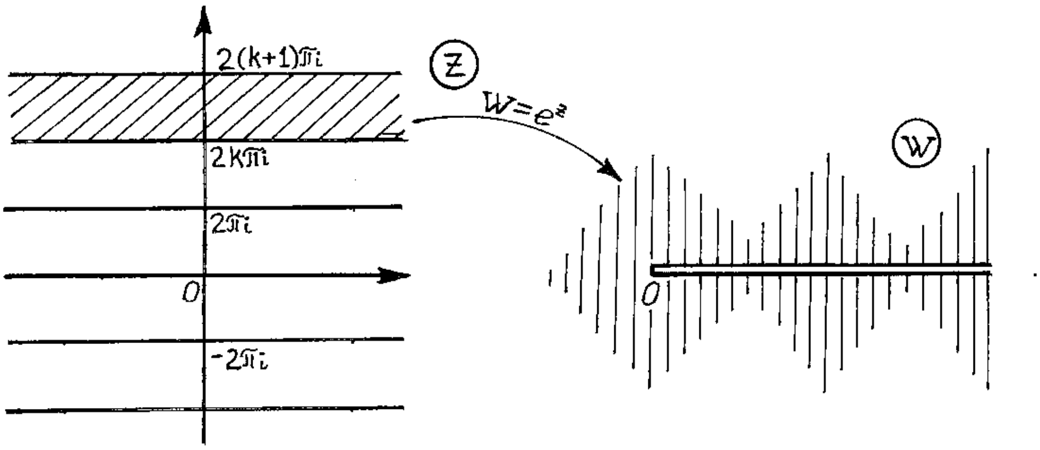
58 - chizma

3) $w=e^z$ *va* $D=\{z:0<\operatorname{Im}z<h,\ h<2\pi\}$ bo‘lsa, $w(D)=\{w:0<\arg w<h\}$ bo‘ladi (59-chizma).



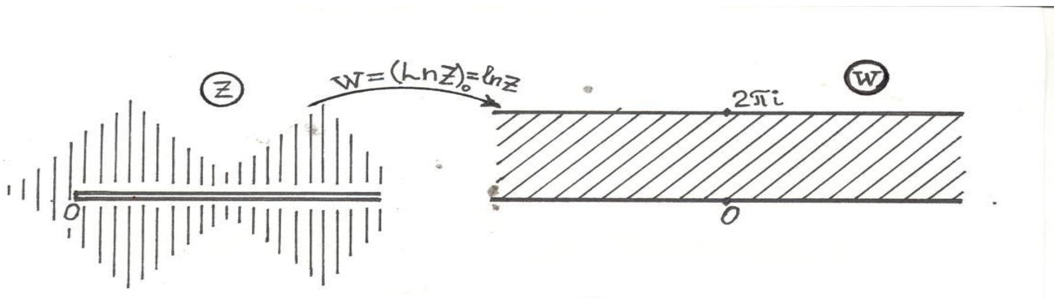
59- chizma

4) $w=e^z$ *va* $D=\{z:2k\pi<\operatorname{Im}z<2(k+1)\pi\}$ $k=0,\pm1,\pm2,...)$ bo‘lsa, $w(D)=C\setminus R_+$ bo‘ladi (60-chizma).



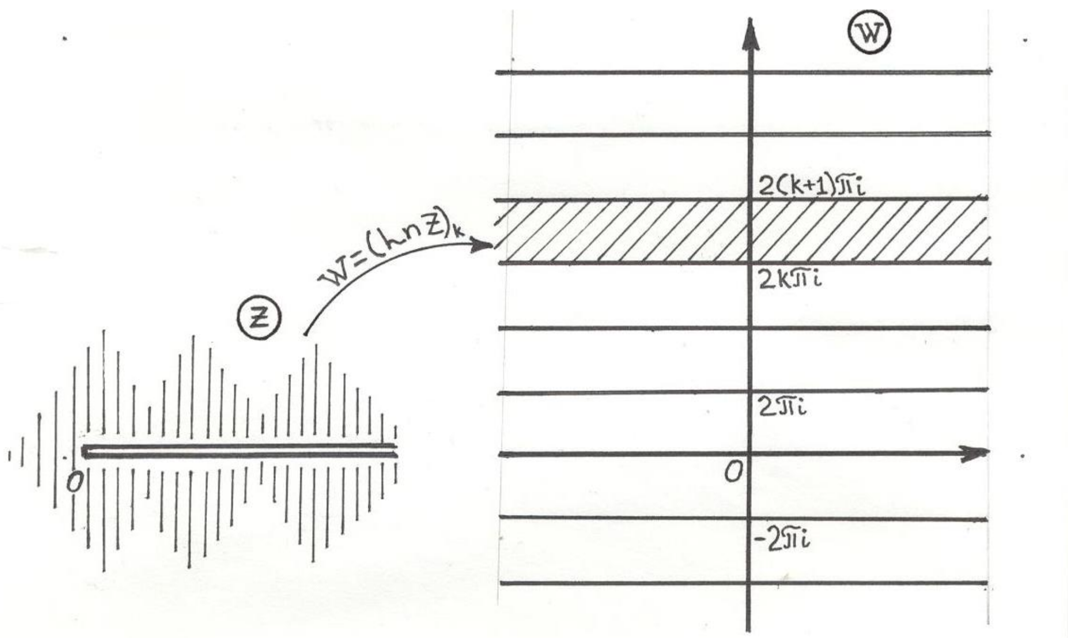
60 - chizma

5) $w=(Lnz)_0=\ln z$ *va* $D=C\setminus R_+$ *bo'lsa* $w(D)=\{w:0<\operatorname{Im}w<2\pi\}$ bo‘ladi (61- chizma).



61 - chizma

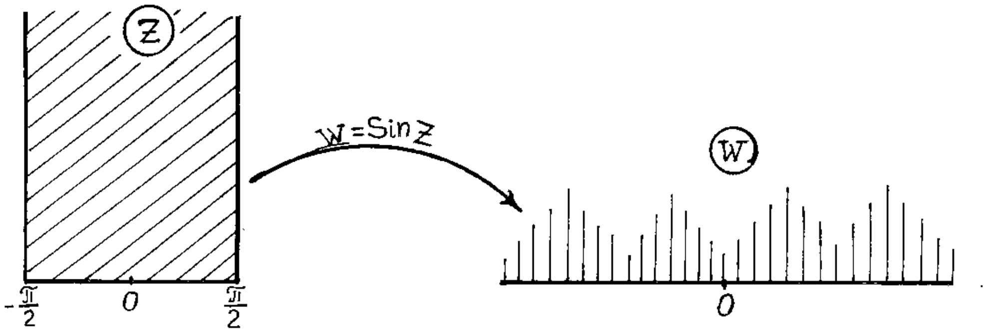
6) $w=(Lnz)_k$ *va* $D=C\setminus R_+$ *bo'lsa,* $w(D)=\{w:2k\pi<\operatorname{Im}w<2(k+1)\pi\}$ $(k=0,\pm1,\pm2,...)$ bo‘ladi (62-chizma).



62-chizma.

V. Trigonometrik va teskari trigonometrik funksiyalar.

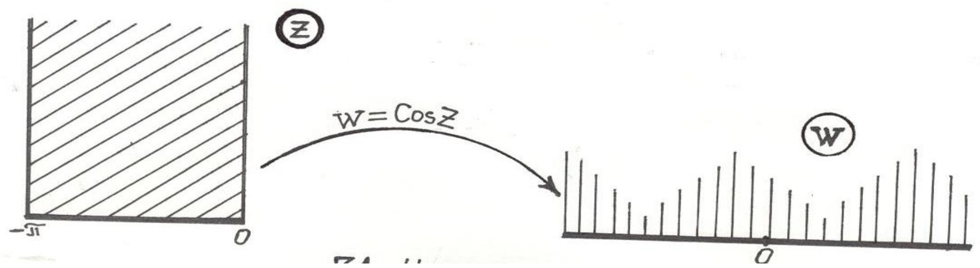
1) $w = \sin z$ *va* $D = \left\{ z : -\frac{\pi}{2} < \operatorname{Re} z < \frac{\pi}{2}, \operatorname{Im} z > 0 \right\}$ bo‘lsa, $w(D) = \{w : \operatorname{Im} w > 0\}$ bo‘ladi (63-chizma).



63- chizma

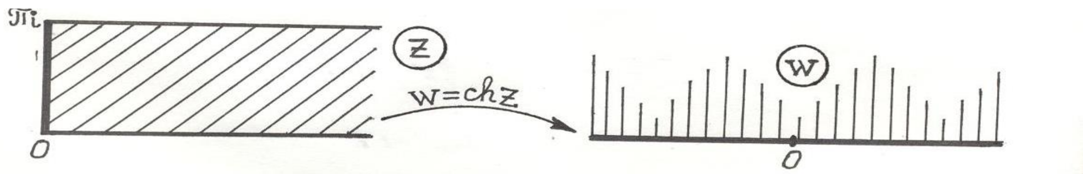
2) $w = \cos z$ *va* $D = \{z : -\pi < \operatorname{Re} z < 0, \operatorname{Im} z > 0\}$ bo‘lsa,

$w(D) = \{w : \operatorname{Im} w > 0\}$ bo‘ladi (64-chizma).



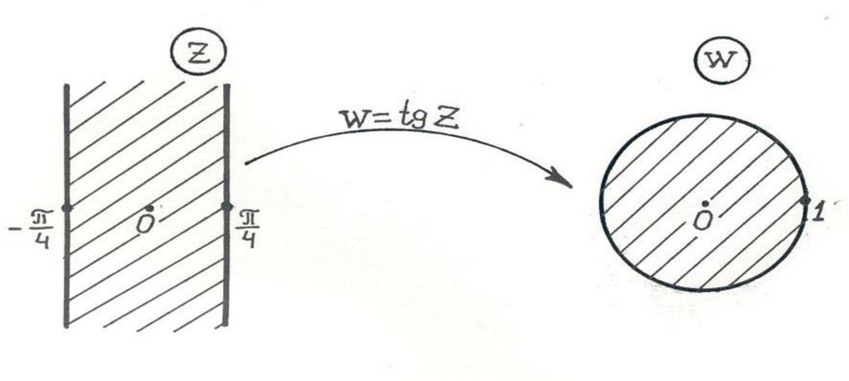
64 - chizma

3) $w = chz$ *va* $D = \{z : 0 < \operatorname{Im} z < \pi, \operatorname{Re} z > 0\}$ bo‘lsa, $w(D) = \{w : \operatorname{Im} w > 0\}$ bo‘ladi (65-chizma).



65 - chizma

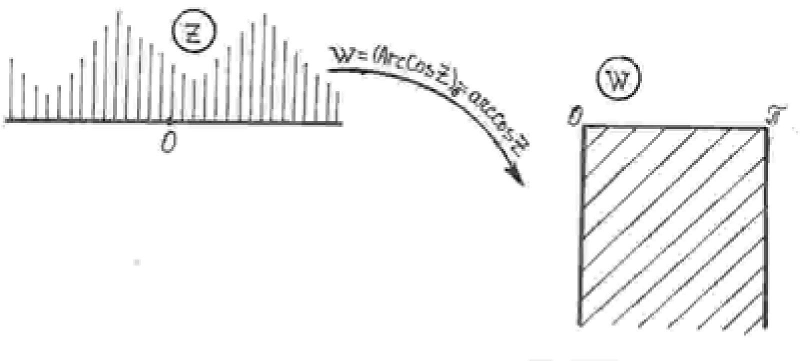
4) $w = tgz$ *va* $D = \{z : -\frac{\pi}{4} < \mathrm{Re}\, z < \frac{\pi}{4}\}$ bo‘lsa, $w(D) = \{w : |w| < 1\}$ bo‘ladi (66- chizma).



66 - chizma

5) $w = (Arc\cos z)_0 = \arccos z$ *ва* $D = \{z : \mathrm{Im}\, z > 0\}$ bo‘lsa,

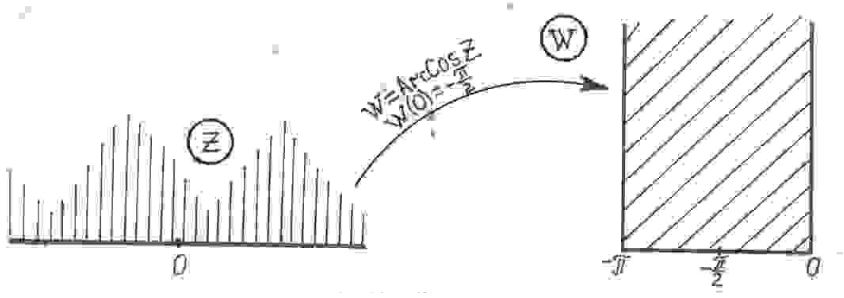
$w(D) = \{w_i : 0 < \mathrm{Re}\, w < \pi,\ \mathrm{Im}\, w < 0\}$ bo‘ladi (67-chizma).



67-chizma

6) $w = Arc\cos z,\ w(0) = -\frac{\pi}{2}$ *va* $D = \{z : \mathrm{Im}\, z > 0\}$ bo‘lsa,

$w(D) = \{w : -\pi < \mathrm{Re}\, w < 0,\ \mathrm{Im}\, z > 0\}$ bo‘ladi (68- chizma).



68 – chizma

Nazorat savollari.

1. Konform akslantirishlar nazariyasining asosiy masalalari.
2. Riman teoremasi.
3. Sohaning saqlanish prinsipi.
4. Chiziqli funksiya va uning xossalari.
5. Kasr-chiziqli akslantirishning doiraviylik xossasi.
6. Kasr-chiziqli akslantirishda simmetriklikning saqlanish xossasi.
7. Angarmonik nisbat.
8. Yuqori yarim tekislikni birlik doiraga akslantiruvchi kasr-chiziqli funksiyaning umumiy ko‘rinishi.
9. Birlik doirani birlik doiraga akslantiruvchi kasr-chiziqli funksiyaning umumiy ko‘rinishi.
10. Darajali funksiya va uning xossalari.
11. Jukovskiy funksiyasi va uning xossalari.
12. Ko‘rsatkichli funksiya va uning xossalari.
13. Trigonometrik funksiyalar va ularning xossalari.
14. Darajali funksiyaga teskari bo‘lgan $w=\sqrt[n]{z}$ $(n \geq 2 - butun\ son)$ funksiyasi.
15. $w = Ln\ z$ funksiyasi.
16. Kompleks sonni kompleks darajaga ko‘tarish.
17. Teskari trigonometrik funksiyalar.

18. Simmetriya prinsipi.

- B -

MUSTAQIL YECHISH UCHUN MISOL VA MASALALAR

1-Masala. Berilgan D sohaning $w = f(z)$ chiziqli funksiya yordamidagi aksini toping.

1.1. $D = \{|z-1| < 2\}, w = 1-2iz$. **1.2.** $D = \{|z-i| < 2\}, w = 1-2iz$.

1.3. $D = \{|z+1| < 2\}, w = 1-2iz$. **1.4.** $D = \{|z+i| < 2\}, w = 1-2iz$.

1.5. $D = \{|z-1| < 2\}, w = 1+2iz$. **1.6.** $D = \{|z-i| < 2\}, w = 1+2iz$.

1.7. $D = \{|z+1| < 2\}, w = 1+2iz$. **1.8.** $D = \{|z+i| < 2\}, w = 1+iz$.

1.9. $D = \{|z-1| < 2\}, w = iz+1+i$. **1.10.** $D = \{|z-i| < 2\}, w = iz+1+i$.

1.11. $D = \{|z+1| < 2\}, w = iz+1+i$. **1.12.** $D = \{|z+i| < 2\}, w = iz+1+i$.

1.13. $D = \{|z-1| < 2\}, w = iz-1+i$. **1.14.** $D = \{|z-i| < 2\}, w = iz-1+i$.

1.15. $D = \{|z+1| < 2\}, w = iz-1+i$. **1.16.** $D = \{|z+i| < 2\}, w = iz-1+i$.

1.17. $D = \{|z-1| < 2\}, w = iz+1-i$. **1.18.** $D = \{|z-i| < 2\}, w = iz+1-i$.

1.19. $D = \{|z+1| < 2\}, w = iz+1-i$. **1.20.** $D = \{|z+i| < 2\}, w = iz+1-i$.

1.21. $D = \{|z-1-i| < 2\}, w = iz+1+i$.

2-Masala. Berilgan z_0 nuqtani qo‘zg‘almas qoldirib, z_1 nuqtani w_1 nuqtaga o‘tkazadigan chiziqli akslantirishni toping.

2.1. $z_0 = 1+i$, $z_1 = i$, $w_1 = -i$.

2.2. $z_0 = 1-i$, $z_1 = i$, $w_1 = -i$.

2.3. $z_0 = 1+i$, $z_1 = 2+i$, $w_1 = i$.

2.4. $z_0 = 1-i$, $z_1 = 1+i$, $w_1 = i$.

2.5. $z_0 = 1+i$, $z_1 = 1-i$, $w_1 = i$.

2.6. $z_0 = 1 - i$, $z_1 = 2 - i$, $w_1 = i$.

2.7. $z_0 = 1 + i$, $z_1 = 2 + i$, $w_1 = 1 - i$.

2.8. $z_0 = 1 + i$, $z_1 = 2 + i$, $w_1 = 1 + i$.

2.9. $z_0 = 1 + i$, $z_1 = 2 - i$, $w_1 = 1 - i$.

2.10. $z_0 = 1 + i$, $z_1 = 2 - i$, $w_1 = 1 + i$.

2.11. $z_0 = 1 + i$, $z_1 = 2 + i$, $w_1 = 2 - i$.

2.12. $z_0 = 1 + i$, $z_1 = 2 - i$, $w_1 = 2 + i$.

2.13. $z_0 = 1 + i$, $z_1 = 1 + 2i$, $w_1 = 2 - i$.

2.14. $z_0 = 1 + i$, $z_1 = 1 - 2i$, $w_1 = 2 - i$.

2.15. $z_0 = 1 + i$, $z_1 = 1 + 2i$, $w_1 = 2 + i$.

2.16. $z_0 = 1 + i$, $z_1 = 1 - 2i$, $w_1 = 2 + i$.

2.17. $z_0 = 1 + i$, $z_1 = 1 + 2i$, $w_1 = i$.

2.18. $z_0 = 1 + i$, $z_1 = 1 - 2i$, $w_1 = i$.

2.19. $z_0 = 1 + i$, $z_1 = 1 + 2i$, $w_1 = -i$.

2.20. $z_0 = 1 + i$, $z_1 = 1 - 2i$, $w_1 = -i$.

2.21. $z_0 = 1 + 2i$, $z_1 = i$, $w_1 = -i$.

3-Masala. Quyidagi akslantirishlar uchun chekli qo‘zg‘almas nuqta z_0 (agar u mavjud bo‘lsa), burilish burchagi φ va cho‘zilish koeffitsienti k-ni toping. Akslantirishni $w - z_0 = \lambda(z - z_0)$ kanonik ko‘rinishiga keltiring.

3.1. $w = z + 1 - 2i$. **3.2.** $w = z + 1 + 2i$. **3.3.** $w = z - 1 - 2i$.

3.4. $w = z - 1 + 2i$. **3.5.** $w = z + 2 - i$. **3.6.** $w = z + 2 + i$.

3.7. $w = z - 2 + i$. **3.8.** $w = z - 2 - i$. **3.9.** $w = z - 2 + 2i$.

3.10. $w = z + 2 - 2i$. **3.11.** $w = 2z + 1 - 2i$. **3.12.** $w = 2z + 1 + 2i$.

3.13. $w = 2z - 1 - 2i$. **3.14.** $w = 2z - 1 + 2i$. **3.15.** $w = 2z + 2 - i$.

3.16. $w = 2z + 2 + i$. **3.17.** $w = 2z - 2 + i$. **3.18.** $w = 2z - 2 - i$.

3.19. $w = 2z - 1 + i$. **3.20.** $w = 2z + 1 - i$. **3.21.** $w = 2z + 1 - 3i$.

4-Masala. Berilgan D doirani G doiraga akslantiruvchi chiziqli funksiyani toping.

4.1. $D=\{|z-1+i|<2\}$, $G=\{|w-i|<4\}$.

4.2. $D=\{|z-1+i|<2\}$, $G=\{|w+i|<4\}$.

4.3. $D=\{|z+1-i|<2\}$, $G=\{|w-i|<4\}$.

4.4. $D=\{|z+1-i|<2\}$, $G=\{|w+i|<4\}$.

4.5. $D=\{|z-1|<2\}$, $G=\{|w+i|<3\}$.

4.6. $D=\{|z-i|<2\}$, $G=\{|w+1|<3\}$.

4.7. $D=\{|z+1|<2\}$, $G=\{|w+i|<3\}$.

4.8. $D=\{|z+i|<2\}$, $G=\{|w+1|<3\}$.

4.9. $D=\{|z-i|<3\}$, $G=\{|w+i|<4\}$.

4.10. $D=\{|z-1|<3\}$, $G=\{|w+i|<4\}$.

4.11. $D=\{|z-1+i|<4\}$, $G=\{|w-i|<3\}$.

4.12. $D=\{|z-1+i|<4\}$, $G=\{|w+i|<2\}$.

4.13. $D=\{|z+1-i|<4\}$, $G=\{|w-i|<2\}$.

4.14. $D=\{|z+1-i|<4\}$, $G=\{|w+i|<5\}$.

4.15. $D=\{|z-1|<4\}$, $G=\{|w+i|<2\}$.

4.16. $D=\{|z-i|<4\}$, $G=\{|w+i|<2\}$.

4.17. $D=\{|z+1|<4\}$, $G=\{|w+i|<2\}$.

4.18. $D=\{|z+i|<4\}$, $G=\{|w+1|<2\}$.

4.19. $D=\{|z-i|<4\}$, $G=\{|w+i|<3\}$.

4.20. $D=\{|z-1|<4\}$, $G=\{|w+1|<2\}$.

4.21. $D=\{|z-i|<2\}$, $G=\{|w-2|<4\}$.

5-Masala. Berilgan D sohaning kasr-chiziqli $w = f(z)$ akslantirish yordamida aksini toping.

5.1. $D = \{|z| > 1\}$, $\quad w = \dfrac{z-1}{z+i}$.

5.2. $D = \{x < 0, y < 0\}$, $\quad w = \dfrac{1}{z}$.

5.3. $D = \{|z| < 1\}$, $\quad w = \dfrac{z+i}{z+1}$.

5.4. $D = \{\operatorname{Im} z < 1\}$, $\quad w = \dfrac{z-i}{z}$.

5.5. $D = \{0 < \operatorname{Re} z < 2\}$, $\quad w = \dfrac{1}{z-2}$.

5.6. $D = \{\dfrac{\pi}{4} < \arg z < \dfrac{\pi}{2}\}$, $\quad w = \dfrac{1}{z}$.

5.7. $D = \{|z| < 1, |z-1| < \sqrt{2}\}$, $\quad w = \dfrac{z-i}{z+i}$.

5.8. $D = \{|z| > 1, |z-1| < \sqrt{2}\}$, $\quad w = \dfrac{z-i}{z+i}$.

5.9. $D = \{|z-1| > 2\}$, $\quad w = \dfrac{2iz}{z+3}$.

5.10. $D = \{|z-1| > 2\}$, $\quad w = \dfrac{z+1}{z-2}$.

5.11. $D = \{|z-1| < 3\}$, $\quad w = \dfrac{z-1}{2z-6}$.

5.12. $D = \{\operatorname{Re} z > 1\}$, $\quad w = \dfrac{z}{z-1+i}$.

5.13. $D = \{\operatorname{Re} z > 1\}$, $\quad w = \dfrac{z}{z-2}$.

5.14. $D = \{\operatorname{Re} z > 1\}$, $\quad w = \dfrac{z-3+i}{z+1+i}$.

5.15. $D=\{|z|<1, \operatorname{Im} z<0\}, \qquad w=\dfrac{1-z}{1+z}$.

5.16. $D=\{|z+i|>1, \operatorname{Im} z>1\} \qquad w=\dfrac{1}{z}$.

5.17. $D=\{1<|z|<2\}, \qquad w=\dfrac{1}{z-2}$.

5.18. $D=\{x>0, y<0\}, \qquad w=\dfrac{z-i}{z+i}$.

5.19. $D=\{|z|<1, \operatorname{Im} z<0\}, \qquad w=\dfrac{2z-i}{2+iz}$.

5.20. $D=\{\dfrac{3\pi}{4}<\arg z<\pi\}, \qquad w=\dfrac{z}{z+1}$.

5.21. $D=\{0<\operatorname{Re} z<1\}, \qquad w=\dfrac{z-1}{z}$.

6-Masala. Quyidagi shartlarni qanoatlantiruvchi kasr-chiziqli $w(z)$ akslantirishni toping.

6.1. $w(1)=1, \qquad w(0)=-1, \qquad w(i)=i$.

6.2. $w(\frac{1}{2})=\frac{1}{2}, \qquad w(2)=2, \qquad w(\frac{5}{4}+\frac{3}{4}i)=\infty$.

6.3. $w(0)=2, \qquad w(1+i)=2+i, \qquad w(2i)=0$.

6.4. $w(4)=0, \qquad w(2+2i)=1+i, \qquad w(0)=2i$.

6.5. $w(0)=0, \qquad w(i)=2, \qquad w(2i)=3$.

6.6. $w(0)=0, \qquad w(2)=i, \qquad w(3)=2i$.

6.7. $w(1)=0, \qquad w(1+i)=\infty, \qquad w(3i)=3i$.

6.8. $w(0)=1, \qquad w(\infty)=1+i, \qquad w(3)=4i$.

6.9. $w(i)=2, \qquad w(\infty)=2i, \qquad w(-i)=0$.

6.10. $w(2)=i, \qquad w(2i)=\infty, \qquad w(0)=3i$.

6.11. $w(i)=-2, \qquad w(\infty)=4i, \qquad w(-i)=2$.

6.12. $w(-2)=i, \quad w(4i)=\infty, \quad w(2)=-i$.

6.13. $w(0)=-1, \quad w(2i)=i, \quad w(1+i)=1-i$.

6.14. $w(i)=-1, \quad w(\infty)=i, \quad w(1)=1+i$.

6.15. $w(i)=-1, \quad w(1)=\infty, \quad w(1+i)=i$.

6.16. $w(\infty)=-1, \quad w(i)=\infty, \quad w(i)=i$.

6.17. $w(0)=-1, \quad w(\infty)=\infty, \quad w(1)=i$.

6.18. $w(1)=1, \quad w(\infty)=-1, \quad w(i)=i$.

6.19. $w(\frac{1}{2})=\frac{1}{2}, \quad w(2)=2, \quad w(\infty)=\frac{5}{4}+\frac{3}{4}i$.

6.20. $w(2)=0, \quad w(2+i)=1+i, \quad w(\infty)=\infty$.

6.21. $w(-1)=i, \quad w(i)=\infty, \quad w(1+i)=1$.

7-Masala. D sohani G sohaga akslantiruvchi va quyidagi shartlarni qanoatlantiruvchi kasr-chiziqli $w(z)$ funksiyani toping.

7.1. $D=\{\operatorname{Im} z>0\}$, $G=\{\operatorname{Im} w<0\}$, $w(i)=-i$, $\arg w'(i)=-\frac{\pi}{2}$.

7.2. $D=\{\operatorname{Im} z>0\}$, $G=\{\operatorname{Im} w<0\}$, $w(2i)=-2i$, $\arg w'(2i)=-\frac{\pi}{2}$.

7.3. $D=\{\operatorname{Im} z<0\}$, $G=\{\operatorname{Im} w>0\}$, $w(-i)=i$, $\arg w'(-i)=\frac{\pi}{2}$.

7.4. $D=\{\operatorname{Im} z<0\}$, $G=\{\operatorname{Im} w>0\}$, $w(-2i)=2i$, $\arg w'(-2i)=\frac{\pi}{2}$.

7.5. $D=\{|z|<1\}$, $G=\{|w|<1\}$, $w\left(\frac{1}{4}\right)=0$, $\arg w'\left(\frac{1}{4}\right)=0$.

7.6. $D=\{|z|<1\}$, $G=\{|w|<1\}$, $w\left(-\frac{1}{2}\right)=0$, $\arg w'\left(-\frac{1}{2}\right)=0$.

7.7. $D=\{|z|<1\}$, $G=\{|w|<1\}$, $w\left(\frac{i}{4}\right)=0$, $\arg w'\left(\frac{i}{4}\right)=\frac{\pi}{2}$.

7.8. $D=\{|z|<1\}$, $G=\{|w|<1\}$, $w\left(-\frac{i}{2}\right)=0$, $\arg w'\left(-\frac{i}{2}\right)=\frac{\pi}{2}$.

7.9. $D=\{|z|<2\}$, $G=\{|w|<4\}$, $w(1)=0$, $\arg w'(1)=\dfrac{\pi}{2}$.

7.10. $D=\{|z|<1\}$, $G=\{|w|<2\}$, $w\left(\dfrac{i}{2}\right)=0$, $\arg w'\left(\dfrac{i}{2}\right)=0$.

7.11. $D=\{|z|<1\}$, $G=\{|w-1|<1\}$, $w(0)=\dfrac{1}{4}$, $\arg w'(0)=0$.

7.12. $D=\{\operatorname{Im} z>0\}$, $G=\{|w|<1\}$, $w(i)=0$, $\arg w'(i)=0$.

7.13. $D=\{\operatorname{Im} z<0\}$, $G=\{|w|<2\}$, $w(-i)=0$, $\arg w'(-i)=0$.

7.14. $D=\{\operatorname{Im} z>0\}$, $G=\{|w|<1\}$, $w(1+i)=0$, $\arg w'(1+i)=\dfrac{\pi}{2}$.

7.15. $D=\{\operatorname{Im} z>0\}$, $G=\{|w|<1\}$, $w(-1+2i)=0$, $\arg w'(-1+2i)=\dfrac{\pi}{2}$.

7.16. $D=\{\operatorname{Im} z>0\}$, $G=\{|w+1|<1\}$, $w(i)=0$, $\arg w'(i)=1$.

7.17. $D=\{|z-2i|<1\}$, $G=\{\operatorname{Im} w>\operatorname{Re} w\}$, $w(2i)=-2$, $w(i)=0$.

7.18. $D=\{\operatorname{Im} z>0\}$, $G=\{\operatorname{Im} w>0\}$, $w(i)=i$, $\arg w'(i)=\dfrac{\pi}{2}$.

7.19. $D=\{\operatorname{Im} z>0\}$, $G=\{\operatorname{Im} w>0\}$, $w(2i)=i$, $\arg w'(2i)=0$.

7.20. $D=\{|z|<3\}$, $G=\{\operatorname{Re} w<0\}$, $w(0)=-1$, $\arg w'(0)=\dfrac{\pi}{2}$.

7.21. $D=\{|z|<2\}$, $G=\{\operatorname{Re} w>0\}$, $w(0)=1$, $\arg w'(0)=\dfrac{\pi}{2}$.

8-Masala. Quyidagi D to‘plamning berilgan akslantirish yordamidagi aksini toping.

8.1. $D=\{\operatorname{Re} z=2\}$ $\qquad w=z^2$.

8.2. $D=\{\operatorname{Im} z=3\}$ $\qquad w=z^2$.

8.3. $D=\{\arg z=\dfrac{\pi}{3}\}$ $\qquad w=z^4$.

8.4. $D=\{|z|=2, \dfrac{\pi}{3}<\arg z<\dfrac{2\pi}{3}\}$ $\qquad w=z^2$.

8.5. $D=\{\operatorname{Im} z>1\}$ $\qquad w=z^2$.

8.6. $D=\{\operatorname{Re} z>1\}$ $\qquad w=z^2$.

8.7. $D=\{|z|<2, \frac{\pi}{2}<\arg z<\pi\}$ $\qquad w=z^2$.

8.8. $D=\{|z|>2, \frac{5\pi}{4}<\arg z<\frac{3\pi}{2}\}$ $\qquad w=z^2$.

8.9. $D=\{\operatorname{Im} z<0\}$ $\qquad w=z^2$.

8.10. $D=\{\operatorname{Re} z<-1\}$ $\qquad w=z^2$.

8.11. $D=\{|z|<4, \frac{\pi}{4}<\arg z<\frac{3\pi}{4}\}$ $\qquad w=z^2$.

8.12. $D=\{|z|>3, \operatorname{Re} z>0\}$ $\qquad w=z^2$.

8.13. $D=\{|z|>2, \arg z=\frac{\pi}{4}\}$ $\qquad w=z^3$.

8.14. $D=\{|\arg z|<\frac{\pi}{4}, z\notin[0,1]\}$ $\qquad w=z^4$.

8.15. $D=\{|z|=4, \frac{\pi}{4}<\arg z<\frac{\pi}{2}\}$ $\qquad w=z^4$.

8.16. $D=\{|z|>1, \pi<\arg z<\frac{3\pi}{2}\}$ $\qquad w=z^2$.

8.17. $D=\{\operatorname{Re} z>0\ ,\ z\notin[1,+\infty)\}$ $\qquad w=z^2$.

8.18. $D=\{\operatorname{Im} z<0,\ z\notin(-\infty,-2]\}$ $\qquad w=z^2$.

8.19. $D=\{|z|>2, \arg z=\frac{\pi}{3}\}$ $\qquad w=z^6$.

8.20. $D=\{|z|<3, \arg z=\frac{\pi}{4}\}$ $\qquad w=z^4$.

8.21. $D=\{|z|=2, \frac{\pi}{6}<\arg z<\frac{\pi}{3}\}$ $\qquad w=z^6$.

9-Masala. Jukovskiy funksiyasidan foydalanib quyidagi to‘plamlarning aksini toping.

9.1. $|z| = \frac{1}{2}$, $\frac{\pi}{4} < \arg z < \frac{3\pi}{4}$.

9.2. $|z| = 2$, $\frac{3\pi}{4} < \arg z < \frac{5\pi}{4}$.

9.3. $|z| > 2$, $z \notin [2,+\infty)$.

9.4. $|z| < \frac{1}{2}$, $z \notin [-\frac{1}{2};0]$.

9.5. $\frac{\pi}{4} < \arg z < \frac{3\pi}{4}$, $z \notin [i,+i\infty)$.

9.6. $\frac{\pi}{4} < \arg z < \frac{3\pi}{4}$, $z \notin [0,4i]$.

9.7. $|z| < 1$, $z \notin [-1;0]$.

9.8. $|z| < 1$, $\operatorname{Im} z > 0$ $z \notin [\frac{i}{2};i]$.

9.9. $|z| < \frac{1}{2}$, $0 < \arg z < \frac{\pi}{2}]$.

9.10. $|z| < \frac{1}{2}$, $\frac{5\pi}{4} < \arg z < \frac{7\pi}{4}$.

9.11. $|z| > 2$, $0 < \arg z < \frac{\pi}{2}$.

9.12. $|z| > 2$, $\frac{5\pi}{4} < \arg z < \frac{7\pi}{4}$.

9.13. $\operatorname{Re} z > 0$, $\operatorname{Im} z > 0$.

9.14. $\operatorname{Re} z < 0$, $\operatorname{Im} z < 0$.

9.15. $|z| < \frac{1}{2}$, $\operatorname{Im} z > 0$.

9.16. $|z| < \frac{1}{2}$, $\operatorname{Im} z < 0$.

9.17. $|z| > 2$, $\operatorname{Im} z > 0$.

9.18. $|z| < 2$, $\operatorname{Im} z < 0$.

9.19. $1 < |z| < 2$, $\operatorname{Im} z > 0$.

9.20. $\frac{1}{2} < |z| < 2$, $\operatorname{Im} z > 0$ $\operatorname{Re} z > 0$.

9.21. $\operatorname{Im} z > 0,$ $z \notin \{|z| = 1,\ 0 < \arg z \le \frac{\pi}{4},\ \frac{3\pi}{4} \le \arg z \le \pi\}$.

10-Masala. Quyidagi to‘plamlarning e^z akslantirish yordamidagi aksini toping.

10.1. $0 < \operatorname{Re} z < \pi$, $\operatorname{Im} z < 0$.

10.2. $-\pi < \operatorname{Re} z < 0$, $\operatorname{Im} z > 0$.

10.3. $\operatorname{Re} z > 0$ $, \frac{\pi}{2} < \operatorname{Im} z < \pi$.

10.4. $\operatorname{Re} z < 0$ $, -\frac{\pi}{2} < \operatorname{Im} z < 0$.

10.5. $1 < \operatorname{Re} z < 2$ $, 0 < \operatorname{Im} z < \pi$.

10.6. $2 < \operatorname{Re} z < 3$ $\frac{\pi}{2} < \operatorname{Im} z < \frac{3\pi}{2}$.

10.7. $\operatorname{Re} z > 0$ $, 0 < \operatorname{Im} z < \frac{\pi}{2}$.

10.8. $\operatorname{Re} z < 0$, $-\frac{\pi}{2} < \operatorname{Im} z < 0$.

10.9. $\frac{\pi}{2} < \operatorname{Im} z < \frac{3\pi}{2}$.

10.10. $0 < \operatorname{Im} z < \pi$, $\operatorname{Re} z > 0$.

10.11. $-\frac{\pi}{4} < \operatorname{Im} z < \frac{\pi}{4}$.

10.12. $-\frac{\pi}{4} < \operatorname{Im} z < \frac{\pi}{4}$, $\operatorname{Re} z > 0$.

10.13. $-\frac{\pi}{4} < \operatorname{Im} z < \frac{\pi}{4}, \quad \operatorname{Re} z < 0$.

10.14. $\operatorname{Im} z = 2 \cdot \operatorname{Re} z + 1$.

10.15. $\operatorname{Re} z < \operatorname{Im} z < \operatorname{Re} z + 2\pi$.

10.16. $\operatorname{Im} z = 2\operatorname{Re} z$.

10.17. $\operatorname{Im} z = \operatorname{Re} z + 1$.

10.18. $\operatorname{Im} z + \operatorname{Re} z = 2$.

10.19. $\operatorname{Im} z - \operatorname{Re} z = 3$.

10.20. $1 < \operatorname{Re} z < 4$, $\quad \frac{\pi}{2} < \operatorname{Im} z < \pi$.

10.21. $0 < \operatorname{Re} z < 2$, $\quad \pi < \operatorname{Im} z < 2\pi$.

11-Masala. Quyidagi D to‘plamning berilgan $w = f(z)$ akslantirish yordamidagi aksini toping.

11.1. $D = \{\operatorname{Re} z = 2\}$, $\quad w = \cos z$.

11.2. $D = \{\operatorname{Im} z = 2\}$, $\quad w = \cos z$.

11.3. $D = \{0 < \operatorname{Re} z < \frac{\pi}{2}, \operatorname{Im} z < 0\}$, $\quad w = \cos z$.

11.4. $D = \{-\pi < \operatorname{Re} z < 0 , \operatorname{Im} z < 0\}$, $\quad w = \cos z$.

11.5. $D = \{-\frac{\pi}{2} < \operatorname{Re} z < \frac{\pi}{2}, \operatorname{Im} z < 0\}$, $\quad w = \cos z$.

11.6. $D = \{-\pi < \operatorname{Re} z < 0\}$, $\quad w = \cos z$.

11.7. $D = \{0 < \operatorname{Re} z < \pi , -\frac{\pi}{2} < \operatorname{Im} z < \frac{\pi}{2}\}$, $\quad w = \cos z$.

11.8. $D = \{\operatorname{Re} z = 2\}$, $\quad w = \sin z$.

11.9. $D = \{\operatorname{Im} z = 2\}$, $\quad w = \sin z$.

11.10. $D = \{0 < \operatorname{Re} z < \frac{\pi}{2}, \operatorname{Im} z < 0\}$, $\quad w = \sin z$.

11.11. $D = \{-\pi < \operatorname{Re} z < 0 , \operatorname{Im} z < 0\}$, $\quad w = \sin z$.

11.12. $D=\{-\frac{\pi}{2}<\operatorname{Re}z<\frac{\pi}{2}, \operatorname{Im}z<0\}, \quad w=\sin z$.

11.13. $D=\{-\pi<\operatorname{Re}z<0\}, \quad w=\sin z$.

11.14. $D=\{0<\operatorname{Re}z<\pi, -\frac{\pi}{2}<\operatorname{Im}z<\frac{\pi}{2}\}, \quad w=\sin z$.

11.15. $D=\{0<\operatorname{Re}z<\frac{\pi}{4}\}, \quad w=tgz$.

11.16. $D=\{-\pi<\operatorname{Re}z<0\}, \quad w=tgz$.

11.17. $D=\{0<\operatorname{Re}z<\frac{\pi}{4}\}, \quad w=ctgz$.

11.18. $D=\{0<\operatorname{Re}z<1, \operatorname{Im}z>0\}, \quad w=tg\pi z$.

11.19. $D=\{0<\operatorname{Re}z<\pi, \operatorname{Im}z>0\}, \quad w=\sin z$.

11.20. $D=\{-\frac{\pi}{4}<\operatorname{Re}z<0\}, \quad w=tgz$.

11.21. $D=\{-\frac{\pi}{2}<\operatorname{Re}z<\frac{\pi}{2}, \operatorname{Im}z>0\}, \quad w=\sin z$.

12-Masala. Berilgan D sohani $G=\{\operatorname{Im}w>0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta $w(z)$ funksiyani toping.

12.1. $D=\{|z|<1 \ , \ \operatorname{Im}z>0\}$.

12.2. $D=\{|z|<1 \ , \ \operatorname{Im}z<0\}$.

12.3. $D=\{|z|<1 \ , \ \operatorname{Re}z>0\}$.

12.4. $D=\{|z|<1 \ , \ \operatorname{Re}z<0\}$.

12.5. $D=\{-\pi<\operatorname{Im}z<\pi \ , \ z\notin[1,+\infty)\}$.

12.6. $D=\{|z+1|>1 \ , \ |z-2|>2\}$.

12.7. $D=\{|z+2|>2 \ , \ |z-1|>1\}$.

12.8. $D=\{|z-1|>1 \ , \ \operatorname{Re}z>0\}$.

12.9. $D=\{|z+1|>1 \ , \ \operatorname{Re}z<0\}$.

12.10. $D=\{|z-i|>1\ \ ,\ |z-2i|<2\}$.

12.11. $D=\{|z+i|>1\ \ ,\ |z+2i|<2\}$.

12.12. $D=\{|z-1|>1\ \ ,\ |z-2|<2\}$.

12.13. $D=\{|z+1|>1\ \ ,\ |z+2|<2\}$.

12.14. $D=\{0<\operatorname{Re} z<1\ \ ,\operatorname{Im} z>0\}$.

12.15. $D=\{-1<\operatorname{Re} z<0\ \ ,\operatorname{Im} z<0\}$.

12.16. $D=\{\operatorname{Re} z>0\ \ ,-\frac{\pi}{2}<\operatorname{Im} z<\frac{\pi}{2}\}$.

12.17. $D=\{\operatorname{Re} z<0\ \ ,-\frac{\pi}{2}<\operatorname{Im} z<\frac{\pi}{2}\}$.

12.18. $D=\{\operatorname{Re} z>0\ \ ,|z-1|>1\ ,|z-2|<2\}$.

12.19. $D=\{\operatorname{Re} z>0\ \ ,|z+1|>1\ ,|z+2|>2\}$.

12.20. $D=\{\operatorname{Re} z<0\ \ ,\operatorname{Im} z>0\ \ ,|z-i|>1\}$.

12.21. $D=\{\operatorname{Re} z>0\ \ ,-\pi<\operatorname{Im} z<\pi\}$.

13-Masala. Quyidagi tenglamalarni yeching.

13.1. $z^5+2=i$. **13.2.** $z^4-1=i$. **13.3.** $z^3+2i=2$.

13.4. $z^2-z+1=i$. **13.5.** $z^2-4i=2$. **13.6.** $z^5+32=0$.

13.7. $z^3+81=0$. **13.7.** $z^5+1=0$. **13.9.** $z^4+z^2+1=0$.

13.10. $z^7+1=0$. **13.11.** $z^2+4i=3$. **13.12.** $z^8=1-i$.

13.13. $z^2=i$. **13.14.** $z^2+i=1$. **13.15.** $z^3-1=0$.

13.16. $z^4+1=0$. **13.17.** $z^3+2=2i$. **13.18.** $z^3-i=0$.

13.19. $z^6+8=0$. **13.20** $z^2-4i=3$. **13.21.** $z^5+4=3i$.

14-Masala. $w=\sqrt{z}$ funksiyaning quyida berilgan shartni qanoatlantiruvchi bir qiymatli tarmog‘i yordamida D sohaning aksini toping.

14.1. $D=\{\operatorname{Re} z>0\}$, $\sqrt{1}=1$.

14.2. $D=\{\operatorname{Re} z<0\}, \quad \sqrt{-1}=i$.

14.3. $D=\{z\notin(-\infty,2]\}, \quad \sqrt{4}=2$.

14.4. $D=\{z\notin[-2,+\infty)\}$ $, \sqrt{4}=2i$.

14.5. $D=\{|z|<1 \text{ , } \operatorname{Im} z>0\}, \ \sqrt{\dfrac{i}{2}}=\dfrac{1+i}{2}$.

14.6. $D=\{|z|<1 \text{ , } \operatorname{Im} z<0\}, \ \sqrt{-\dfrac{i}{2}}=\dfrac{1-i}{2}$.

14.7. $D=\{|z|>1, \ \dfrac{3\pi}{4}<\arg z<\dfrac{5\pi}{4}\}, \ \sqrt{-1}=i$.

14.8. $D=\{|z|>1, \ \dfrac{3\pi}{4}<\arg<\dfrac{5\pi}{4}\}, \quad \sqrt{-1}=-i$.

14.9. $D=\{(\operatorname{Im} z)^2>2\operatorname{Re} z+1\}$ $, \ \sqrt{-1}=-i$.

14.10. $D=\{(\operatorname{Im} z)^2>2\operatorname{Re} z+1\}, \ \sqrt{-1}=i$.

14.11. $D=\{\operatorname{Im} z>0\}, \ \sqrt{i}=-\dfrac{1+i}{\sqrt{2}}$.

14.12. $D=\{\operatorname{Im} z>0\}, \ \sqrt{i}=\dfrac{1+i}{\sqrt{2}}$.

14.13. $D=\{z\notin[2i,+i\infty)\}, \quad \sqrt{1}=1$.

14.14. $D=\{z\notin[-i\infty,-2i)\}, \quad \sqrt{1}=1$.

14.15. $D=\{z\notin[1,+\infty)\}, \quad \sqrt{-1}=i$.

14.16. $D=\{|z|<1, \operatorname{Re} z>0\}, \quad \sqrt{1}=1$.

14.17. $D=\{|z|<1, \operatorname{Re} z>0\}, \quad \sqrt{1}=-1$.

14.18. $D=\{|z|<4, \operatorname{Re} z<0\}, \quad \sqrt{1}=1$.

14.19. $D=\{|z|<4, \operatorname{Re} z<0\}, \quad \sqrt{1}=-1$.

14.20. $D=\{\operatorname{Re} z>0, |z|>1\}, \quad \sqrt{1}=1$.

14.21. $D=\{\operatorname{Im} z>0, (\operatorname{Im} z)^2>4\operatorname{Re} z+4\}, \quad \sqrt{-1}=i$.

15-Masala. Quyidagi sohalarni $\{\operatorname{Im} w > 0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta $w(z)$ funksiyani toping.

15.1. $\operatorname{Im} z > 0$, $z \notin [0,2i]$.

15.2. $\operatorname{Re} z < 0$, $z \notin [-2,0]$.

15.3. $|z| < 2$, $0 < \arg z < \dfrac{3\pi}{2}$.

15.4. $|z| > 2$, $0 < \arg z < \dfrac{3\pi}{2}$.

15.5. $|z| < 2$, $|z - 2i| < 2$.

15.6. $|z| > 2,\ \ |z - 2i| > 2$.

15.7. $z \notin [-2,3]$.

15.8. $z \notin [-2i, 2i]$.

15.9. $z \notin \{(-\infty,-2] \cup [2,+\infty)\}$.

15.10. $z \notin \{(-i\infty,-i] \cup [i,+i\infty)\}$.

15.11. $\operatorname{Im} z > 0, z \notin \{|z| = 1,\ 0 \le \arg z \le \dfrac{\pi}{4}\}$.

15.12. $\operatorname{Im} z > 0, z \notin \{|z| = 1, \dfrac{3\pi}{4} \le \arg z \le \pi\}$.

15.13. $\operatorname{Im} z > 0, (\operatorname{Im} z)^2 > 2\operatorname{Re} z + 1$.

15.14. $|z-1| < 2, |z+1| < 2$.

15.15. $|z-1| > 2, |z+1| > 2$.

15.16. $\operatorname{Im} z > 0,\ |z - i| < 2$.

15.17. $z \notin \{|z| = 1\ ,\ \ 0 \le \arg z \le \pi\}$.

15.18. $\dfrac{\pi}{4} < \arg z < \dfrac{3\pi}{4}$, $z \notin [i, +i\infty)$.

15.19. $\operatorname{Im} z > 0,\ \ z \notin [2i, +\infty i)$.

15.20. $\operatorname{Re} z < 0$, $z \notin (-\infty, -1]$.

15.21. $z \notin \{|z| \le 1,\ 0 \le \arg z \le \pi\}$, $z \notin [-i, 0]$.

16-Masala. Quyidagi sohalarni $\{\operatorname{Im} w > 0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta $w(z)$ funksiyani toping.

16.1. $z \notin [-2i, 2i]$.

16.2. $z \notin \{-\infty; -2] \cup [4, +\infty)\}$.

16.3. $|z| > 1,\ z \notin \{[-2, -1] \cup [1, 2]\}$.

16.4. $|z| > 1$, $z \in (-\infty, -2]$.

16.5. $|z| > 4,\ z \notin \{[-4, -2] \cup [-1, 4]\}$.

16.6. $\operatorname{Im} z > 0, z \notin \{|z| = 1, \frac{\pi}{2} \le \arg z \le \pi\}$.

16.7. $|z| < 1, \operatorname{Im} z > 0, z \notin [0, \frac{i}{2}]$.

16.8. $|z| < 1, 0 < \arg z < \frac{\pi}{2}, z \notin \{\arg z = \frac{\pi}{4}, 0 \le |z| \le \frac{1}{4}\}$.

16.9. $|z| > 1, 0 < \arg z < \frac{\pi}{2}, z \notin \{\arg z = \frac{\pi}{4}, |z| \ge 2\}$.

16.10. $\operatorname{Re} z > 0, \operatorname{Im} z > 0, z \notin \{|z| = 2, \frac{\pi}{4} \le \arg z \le \frac{\pi}{2}\}$.

16.11. $4(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2 \ge 4, z \notin [2i; 3i]$.

16.12. $\frac{\pi}{2} < \arg z < 2\pi, z \notin \{|z| = 1, \frac{\pi}{2} \le \arg z \le \pi\}$.

16.13. $z \notin [0, +\infty), z \notin \{|z| < 1,\ \ \frac{\pi}{2} \le \arg z \le \frac{3\pi}{2}\}$.

16.14. $\operatorname{Im} z > 0,\ z \notin \{|z| \le 1,\ \frac{\pi}{4} \le \arg z \le \frac{3\pi}{4}\}$.

16.15. $-\pi < \operatorname{Im} z < \pi,\ z \notin [0, +\infty)$.

16.16. $-\pi < \operatorname{Im} z < \pi,\ z \notin \{(-\infty, 0] \cup [\pi, +\infty)\}$.

16.17. $-\pi < \operatorname{Im} z < \pi,\ \ z \notin [-\pi i, 0]$.

16.18. $-\pi < \operatorname{Im} z < \pi,\ \ z \notin \{[-\pi i, -\frac{2\pi}{2}] \cup [0, \pi i]\}$.

16.19. $-1 < \operatorname{Re} z < 1, \operatorname{Im} z > 0,\ \ z \notin [0, i]$.

16.20. $-1 < \operatorname{Re} z < 1, \operatorname{Im} z > 0,\ \ z \notin [i, +i\infty)$.

16.21. $|z-2i| > 2,\ \ |z+2i| > 2,\ \ z \notin [-2;2]$.

17-M**asala**. Quyidagi ifodalarning barcha qiymatlarini toping.

17.1. $Ln5$. **17.2.** $Ln(-1)$. **17.3.** Lni.

17.4. $\ln(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2})$. **17.5.** $\ln i$. **17.6.** $Ln(1+i\sqrt{3})$.

17.7. $(\frac{1+i}{\sqrt{2}})^i$. **17.8.** $(-1)^i$. **17.9.** $(-3+4i)^{1+i}$.

17.10. $2^{\overline{i}}$. **17.11.** $(-i)^i$. **17.12.** $(\frac{1-i}{\sqrt{2}})^{1+i}$.

17.13. $Arc\sin 1$. **17.14.** $Arc\cos\frac{1}{2}$. **17.15.** $Arc\cos 2$.

17.16. $Arc\cos i$. **17.17.** $Arcctg1$. **17.18.** $Arcctg(1+2i)$.

17.19. $Arc\sin\frac{4i}{3}$. **17.20.** $Arc\cos\frac{3i}{4}$. **17.21.** $Arc\sin 2$.

18-Masala. Quyidagi sohalarning $w = Lnz$ funksiyaning qo‘yilgan shartni qanoatlantiruvchi bir qiymatli tarmog‘i yordamidagi aksini toping.

18.1. $D = \{\operatorname{Im} z > 0\}, \quad w(i) = \frac{\pi i}{2}$.

18.2. $D = \{\operatorname{Im} z < 0\}, \quad w(-i) = -\frac{\pi i}{2}$.

18.3. $D = \{z \notin (-\infty, 0]\}$, $w(1) = 4\pi i$.

18.4. $D = \{z \notin (-\infty, 0]\}$, $w(-i) = -\frac{\pi i}{2}$.

18.5. $D = \{z \notin [0, +\infty)\}$, $w(1) = 4\pi i$.

18.6. $D=\{z\notin[0,+\infty)\}$, $w(-i)=-\dfrac{\pi i}{2}$.

18.7. $D=\{z\notin[0,+\infty)\}$, $w(i)=\dfrac{5\pi i}{2}$.

18.8. $D=\{z\notin(-\infty,0]\}$, $w(i)=\dfrac{5\pi i}{2}$.

18.9. $D=\{z\notin[0,+\infty)\}$, $w(-i)=\pi i$.

18.10. $D=\{z\notin(-\infty,0]\}$, $w(-1)=\pi i$.

18.11. $D=\{z\notin[0,+\infty)\}$, $w(-i)=-\dfrac{\pi i}{2}$.

18.12. $D=\{z\notin(-\infty,0]\}$, $w(-i)=-\dfrac{\pi i}{2}$.

18.13. $D=\{z\notin[0,+\infty)\}$, $w(i)=\dfrac{\pi i}{2}$.

18.14. $D=\{z\notin(-\infty,0]\}$, $w(i)=\dfrac{\pi i}{2}$.

18.15. $D=\{z\notin[0,+\infty)\}$, $w(\dfrac{-1-\sqrt{3}i}{2})=\dfrac{10\pi i}{3}$.

18.16. $D=\{z\notin(-\infty,0]\}$, $w(\dfrac{-1-\sqrt{3}i}{2})=\dfrac{10\pi i}{3}$.

18.17. $D=\{z\notin[0,+\infty)\}$, $w(-1)=-\pi i$.

18.18. $D=\{z\notin(-\infty,0]\}$, $w(-1)=-\pi i$.

18.19. $D=\{|z|<1, \operatorname{Im} z>0\}$, $w(i-i0)=-\dfrac{3\pi i}{2}$.

18.20. $D=\{|z|<1, z\notin[0,1]\}, w(-1+0)=-\pi i$.

18.21. $D=\{z\notin(-\infty,0], z\notin[1,+\infty)\}, w(i)=\dfrac{\pi i}{2}$.

19-Masala. Simmetriya prinsipidan foydalanib, $D=\{|z|<1\}$ birlik doiraning berilgan funksiya yordamidagi aksini toping.

19.1. $w=\dfrac{z}{\sqrt[20]{(1+z^{20})^2}}$.   **19.2.** $w=\dfrac{z}{\sqrt[19]{(1+z^{19})^2}}$.

19.3. $w=\dfrac{z}{\sqrt[18]{(1+z^{18})^2}}$.   **19.4.** $w=\dfrac{z}{\sqrt[17]{(1+z^{17})^2}}$.

19.5. $w=\dfrac{z}{\sqrt[16]{(1+z^{16})^2}}$.   **19.6.** $w=\dfrac{z}{\sqrt[15]{(1+z^{15})^2}}$.

19.7. $w=\dfrac{z}{\sqrt[14]{(1+z^{14})^2}}$.   **19.8.** $w=\dfrac{z}{\sqrt[13]{(1+z^{13})^2}}$.

19.9. $w=\dfrac{z}{\sqrt[12]{(1+z^{12})^2}}$.   **19.10.** $w=\dfrac{z}{\sqrt[11]{(1+z^{11})^2}}$.

19.11. $w=\dfrac{z}{\sqrt[10]{(1+z^{10})^2}}$.   **19.12.** $w=\dfrac{z}{\sqrt[9]{(1+z^{9})^2}}$.

19.13. $w=\dfrac{z}{\sqrt[8]{(1+z^{8})^2}}$.   **19.14.** $w=\dfrac{z}{\sqrt[7]{(1+z^{7})^2}}$.

19.15. $w=\dfrac{z}{\sqrt[6]{(1+z^{6})^2}}$.   **19.16.** $w=\dfrac{z}{\sqrt[5]{(1+z^{5})^2}}$.

19.17. $w=\dfrac{z}{\sqrt[4]{(1+z^{4})^2}}$.   **19.18.** $w=\dfrac{z}{\sqrt[3]{(1+z^{3})^2}}$.

19.19. $w=\dfrac{z}{\sqrt[21]{(1+z^{21})^2}}$.   **19.20.** $w=\dfrac{z}{\sqrt[22]{(1+z^{22})^2}}$.

19.21. $w=\dfrac{z}{\sqrt[n]{(1+z^{n})^2}}$.

20-Masala. Simmetriya prinsipidan foydalanib, berilgan sohalarni $\{\operatorname{Im} w>0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta funksiyani toping.

20.1. $z\notin[-1,2],\ z\notin[-i,i]$.

20.2. $z\notin[-2;1],\ z\notin[-i,i]$.

20.3. $z \notin [-1,1],\ z \notin [-i,2i]$.

20.4. $z \notin [-1,1],\ z \notin [-2i,i]$.

20.5. $z \notin [-1,1],\ z \notin [-i;0]$.

20.6. $z \notin [-1,1],\ z \notin [0,i]$.

20.7. $z \notin [0,1],\ z \notin [-i,i]$.

20.8. $z \notin [-1,0],\ z \notin [-i,i]$.

20.9. $|z| > 1,\ z \notin [-2;-1],\ z \notin [-2i;-i],\ z \notin [i;2i]$.

20.10. $|z| < 2,\ z \notin \{[-1;2] \cup [-i;i]\}$.

20.11. $|z| > 1,\ z \notin \{[1;2] \cup [-2i;-i] \cup [i;2i]\}$.

20.12. $|z| < 2,\ z \notin \{[-2;1] \cup [-i;i]\}$.

20.13. $|z| > 1,\ z \notin \{[-2;-1] \cup [1;2] \cup [i;2i]\}$.

20.14. $|z| < 2,\ z \notin \{[-1;1] \cup [-i;2i]\}$.

20.15. $|z| > 1,\ z \notin \{[-2;-1] \cup [1,2] \cup [-2i;-i]\}$.

20.16. $|z| < 2,\ z \notin \{[-1,1] \cup [-2i,i]\}$.

20.17. $0 < \operatorname{Re} z < 1,\ z \notin \{\operatorname{Re} z = \frac{1}{2},\ -\infty < \operatorname{Im} z \le -2\}$.

20.18. $z \notin [-2,2],\ z \notin [0,2i]$.

20.19. $-1 < \operatorname{Re} z < 0,\ \ z \notin \{\operatorname{Re} z = -\frac{1}{2},\ 2 < \operatorname{Im} z < \infty\}$.

20.20. $z \notin [0,2],\ \ z \notin [-2i,2i]$.

20.21. $0 < \operatorname{Re} z < 1,\ z \notin \{\operatorname{Re} z = \frac{1}{2},\ 2 \le \operatorname{Im} z < \infty\}$.

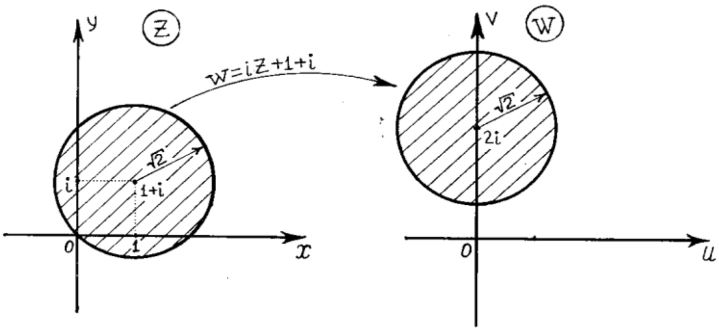
- C -

NAMUNAVIY VARIANT YECHIMI.

1.21-Masala. Berilgan $D=\{|z-1-i|<2\}$ sohaning $w=iz+1+i$ funksiya yordamidagi aksini toping.

$\triangleleft$ $w=iz+1+i$ tenglamani z ga nisbatan yechamiz:

$z=-iw+i-1 \Rightarrow |z-1-i|=|-iw-2|=|-i(w-2i)|=|-i|\cdot|w-2i|=|w-2i| \Rightarrow D$ doiraning aksi $G=w(D)=\{|w-2i|<\sqrt{2}\}$ doira ekan. (69-chizma) $\triangleright$



69- chizma

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots)$:

>

> $w = I\cdot z + 1 + I$

$$w = \mathrm{I}z + 1 + \mathrm{I}$$

> $solve(\{w = I\cdot z + 1 + I\}, z)$

$$\{z = \mathrm{I}(-w + 1 + \mathrm{I})\}$$

> $z := \mathrm{I}(-w + 1 + \mathrm{I})$

$$z := \mathrm{I}(-w + 1 + \mathrm{I})$$

> $a := evalc(z)$

$$a := -1 + \mathrm{I}(-w + 1)$$

> $b := a - 1 - I$

$$b := -2 - \mathrm{I} + \mathrm{I}(-w + 1)$$

> $c := evalc(b)$

$$c := -2 - \mathrm{I}w$$

> $I \cdot c$

$$I(-2 - I w)$$

> $evalc(I \cdot c)$

$$-2I + w$$

> $|-2I + w|$

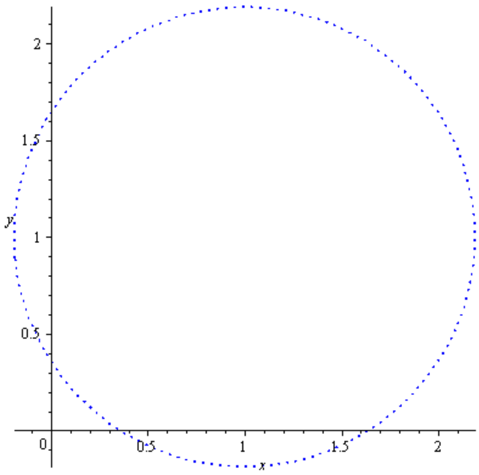
$$|-2I + w|$$

> $|-2I + w| < \sqrt{2}$

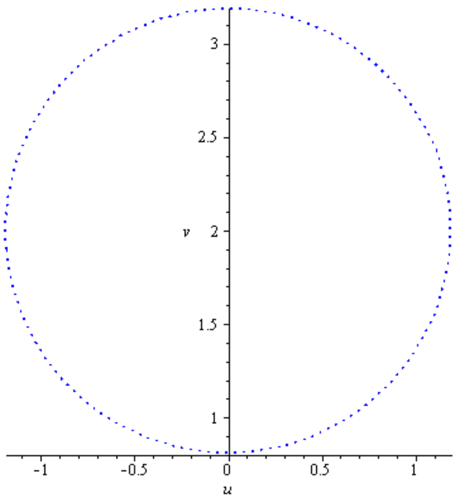
$$|-2I + w| < \sqrt{2}$$

> $with(plots):$

> $implicitplot\left((x-1)^2 + (y-1)^2 < \sqrt{2}, x = -1..3, y = -1..3, grid = [50, 50], color = blue\right)$



> $implicitplot\left(u^2 + (v-2)^2 < \sqrt{2}, u = -2..2, v = -1..5, grid = [50, 50], color = blue\right)$



>

2.21-Masala. Berilgan $z_0 = 1 + 2i$ nuqtani qo‘zg‘almas qoldirib, $z_1 = i$ nuqtani $w_1 = -i$ nuqtaga o‘tkizadigan chiziqli akslantirishni toping.

◁ Ma’lumki, chiziqli akslantirishning umumiy ko‘rinishi $w = az + b$. Bu yerdagi $a, b \in C$ noma’lumlarni masala shartidan foydalanib topamiz:

$$\begin{cases} a(1+2i) + b = 1 + 2i, \\ ai + b = -i. \end{cases} \Rightarrow a = 2 + i \ , \ b = 1 - 3i.$$

Demak, $w = (2+i)z + 1 - 3i$ ▷

3.21-Masala. Quyidagi $w = 2z + 1 - 3i$ uchun chekli qo‘zgalmas nuqta z_0 (agar u mavjud bo‘lsa), burilish burchagi φ va cho‘zilish koeffitsienti k ni toping. Akslantirishni $w - z_0 = \lambda(z - z_0)$ kanonik ko‘rinishga keltiring.

◁ Qo‘zgalmas nuqtani $w(z_0) = z_0$ tenglikdan foydalanib topamiz:

$$2z_0 + 1 - 3i = z_0 \Rightarrow z_0 = -1 + 3i \Rightarrow w - z_0 = 2z + 1 - 3i - z_0 =$$

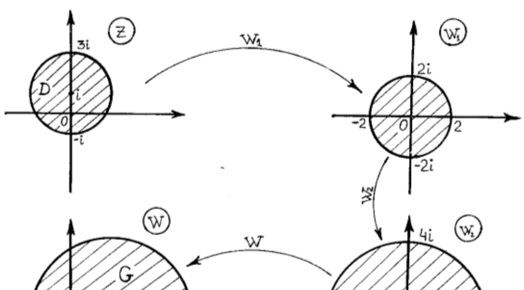
$$= 2z + 1 - 3i + 1 - 3i = 2(z + 1 - 3i).$$

Demak, $w + 1 - 3i = 2(z + 1 - 3i)$. Bu yerdan

$z_0 = -1 + 3i, \ \varphi = 0, \ k = 2; w + 1 - 3i = 2(z + 1 - 3i)$ natijaga kelamiz ▷

4.21-Masala. Berilgan $D = \{|z - i| < 2\}$ doirani $G = \{|w - 2| < 4\}$ doiraga akslantiruvchi chiziqli funksiyani toping.

◁ Ushbu $w_1 = z - i$ funksiyani qaraylik. Bu funksiya berilgan *D* doirani (w_1) tekislikda markazi koordinata boshida bo‘lgan $|w_1| < 2$ doiraga akslantiradi. Endi $w_2 = 2w_1$ *ва* $w = w_2 + 2$ akslantirishlardan ketma-ket foydalansak berilgan doira *G* doiraga akslanadi (70-chizma).



70-chizma

Demak , $w = w_2 + 2 = 2w_1 + 2 = 2(z-i) + 2 = 2z + 2(1-i)$ $\rhd$

5.21-Masala. Berilgan $D = \{0 < \operatorname{Re} z < 1\}$ sohaning $w = \dfrac{z-1}{z}$ akslantirish yordamidagi aksini toping.

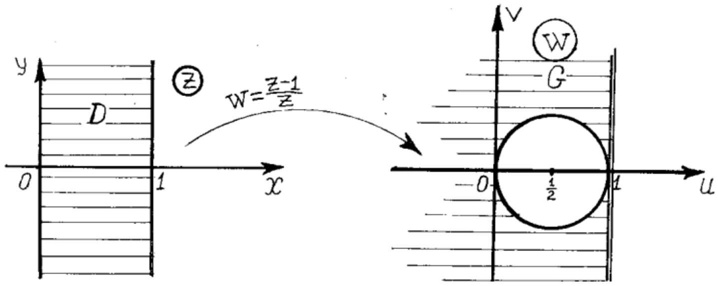
$\lhd$ Bu masalani yechish uchun sohaning saqlanish prinsipi va kasr-chiziqli akslatirishning doiraviylik prinsipidan foydalanamiz. $G = w(D)$ desak, $\partial D = w(\partial D)$ bo‘ladi.

$\partial D = \{\operatorname{Re} z = 0\} \cup \{\operatorname{Re} z = 1\}$. $z \in \{\operatorname{Re} z = 0\}$ va $w(0) = \infty$ bo‘lgani uchun $\{\operatorname{Re} z = 0\}$ to‘g‘ri chiziqning aksi to‘g‘ri chiziq bo‘ladi. Uni topish uchun $z_1 = i$ *ва* $z_2 = -i \in \{\operatorname{Re} z = 0\}$ nuqtalarni olib, ularning obrazlarini topamiz: $w(i) = \dfrac{i-1}{i} = 1 + i,\ \ w(-i) = \dfrac{-i-1}{-i} = 1 - i. \Rightarrow$ Bu nuqtalardan o‘tuvchi to‘g‘ri chiziq $\operatorname{Re} w = 1$. $\operatorname{Re} z = 1$ to‘g‘ri chiziqning aksi esa aylana bo‘ladi, chunki bu chiziqning ustida $w = \dfrac{z-1}{z}$ funksiyani ∞ ga aylantiradigan nuqta yo‘q. Uni topish uchun $w = \dfrac{z-1}{z}$ tenglamadan *z* ni topamiz:

$$z = \frac{-1}{w-1} = \frac{-1}{u+iv-1} = \frac{-1}{u-1+iv} = \frac{-(u-1-iv)}{(u-1)^2+v^2} = \frac{1-u}{(u-1)^2+v^2} + i\frac{v}{(u-1)^2+v^2}$$

Bu yerdan va $\operatorname{Re} z = 1$ dan $\Rightarrow \dfrac{1-u}{(u-1)^2+v^2} = 1 \Rightarrow (u-1)^2 + v^2 = 1-u \Rightarrow (u-\frac{1}{2})^2 + v^2 = \frac{1}{4} \Rightarrow \left|w-\frac{1}{2}\right| = \frac{1}{2}$

Demak, $\partial G = \{\operatorname{Re} w = 1\} \cup \{\left|w-\frac{1}{2}\right| = \frac{1}{2}\} \Rightarrow G = \{\operatorname{Re} w < 1, \left|w-\frac{1}{2}\right| > \frac{1}{2}\}$ (71-chizma). ▹



71- chizma

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

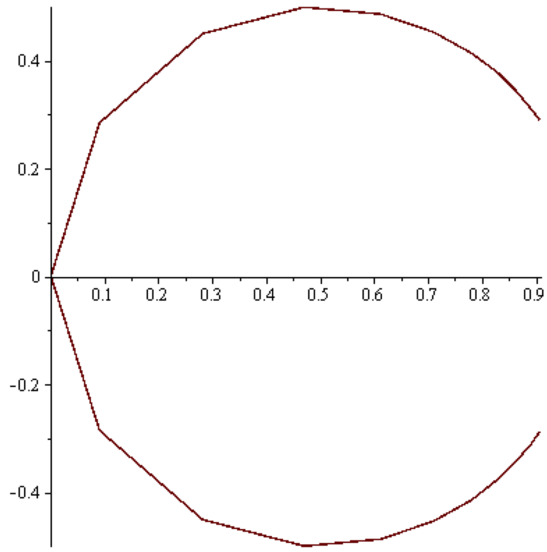
> $with(plots)$:

>

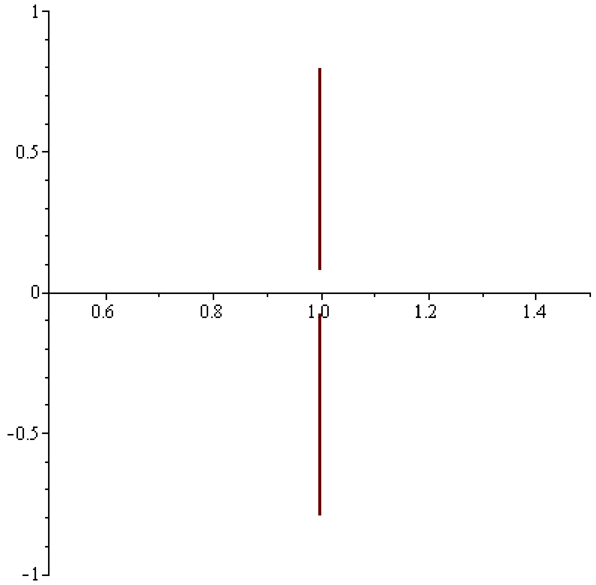
> $w := \dfrac{z-1}{z}$

$$w := \frac{z-1}{z}$$

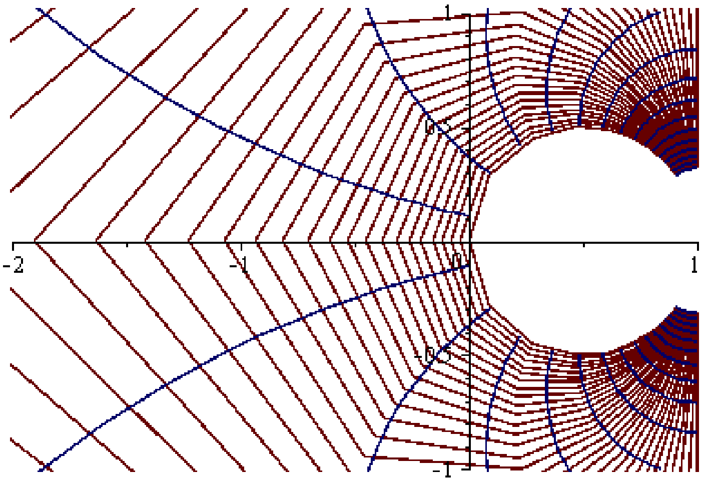
> $conformal(\mathrm{w}, \mathrm{z} = 1 - \pi \cdot \mathrm{I}..1 + \pi \cdot I, grid = [30, 30])$



> $conformal\left(\mathrm{w}, z=0-4\cdot\pi\cdot I..0+4\cdot\pi\cdot I, 1-I..1+I, grid=[30, 30]\right)$



$conformal\left(\mathrm{w}, \mathrm{z}=0-\pi\cdot \mathrm{I}..1+\pi\cdot \mathrm{I}, -2-\mathrm{I}..1+\mathrm{I}, \mathrm{grid}=[30, 30]\right)$



6.21-Masala. Quyidagi $w(-1)=i$, $w(i)=\infty$, $w(1+i)=1$ shartlarni qanoatlantiruvchi kasr-chiziqli $w(z)$ akslantirishni toping.

$\triangleleft$ Bu masalani yechish uchun ushbu

$$\frac{w-w_1}{w-w_2}\cdot\frac{w_3-w_2}{w_3-w_1}=\frac{z-z_1}{z-z_2}\cdot\frac{z_3-z_2}{z_3-z_1}$$

angarmonik nisbatdan foydalanamiz. Bizning holda $z_1=-1$, $z_2=i$, $z_3=1+i$ *ва* $w_1=i$, $w_2=\infty$, $w_3=1$. $w_2=\infty$ bo‘lgani uchun angarmonik nisbat quyidagi

$$\frac{w-w_1}{w_3-w_1}=\frac{z-z_1}{z-z_2}\cdot\frac{z_3-z_2}{z_3-z_1}$$

ko‘rinishga keladi. Bu yerdan

$$\frac{w-i}{1-i}=\frac{z+1}{z-i}\cdot\frac{1+i-i}{1+i+1}$$

va $w=\dfrac{(1+2i)z+6-3i}{5(z-i)}$ ekanligini topamiz ▹

$$> solve\left(\left\{\frac{w-I}{1-I}=\frac{(z+1)}{(z-I)}\cdot\frac{(1+I-I)}{(1+I+1)}\right\},\{w\}\right)$$

$$\left\{w=\frac{1}{5}\,\frac{9+3\,\mathrm{I}-z+3\,\mathrm{I}\,z}{\mathrm{I}\,z+1-\mathrm{I}+z}\right\}$$

7.21-Masala. $D=(|z|<2\}$ sohani $G=\{\operatorname{Re}w>0\}$ sohaga akslantiruvchi va $w(0)=1$, $\arg w^1(0)=\dfrac{\pi}{2}$ shartlarni qanoatlantiruvchi kasr-chiziqli $w(z)$ funksiyani toping.

◃ Avval (5)-formuladan foydalanib G ni D ga konform akslantiruvchi kasr-chiziqli funksiyaning umumiy ko‘rinishini topib olamiz. Buning uchun ushbu $z_1=iw$, $z_2=e^{i\theta}\dfrac{z_1-a}{z_1-\overline{a}}$ *ва* $z=2z_2$ akslantirishlarni ketma-ket bajarish yetarli ekanligini ko‘rish qiyin emas.

Demak,

$$z=2z_2=2e^{i\theta}\frac{z_1-a}{z_1-\overline{a}}=2e^{i\theta}\cdot\frac{iw-a}{iw-\overline{a}}.$$

Bu tenglamani w ga nisbatan yechib, D ni G ga akslantiruvchi funksiyaning umumiy ko‘rinishi

$$w = -i\frac{\bar{a}z - 2ae^{i\theta}}{z - 2e^{i\theta}}$$

ekanligini hosil qilamiz. Bu yerdagi a *ва* θ noma’lumlarni berilgan shartlardan foydalanib topamiz:

$$w(0) = 1 \Rightarrow -i\cdot\frac{-2ae^{i\theta}}{-2e^{i\theta}} = -ai = 1 \Leftarrow a = +i \Rightarrow w = -i\frac{-iz - 2ie^{i\theta}}{z - 2e^{i\theta}} = -\frac{z + 2e^{i\theta}}{z - 2e^{i\theta}}$$

$\arg w'(0) = \frac{\pi}{2}$ shartdan θ ni topamiz:

$$\arg w'(0) = \arg(-\frac{4e^{i\theta}}{(z - 2e^{i\theta})^2})\Big|_{z=0} = \arg\frac{-1}{e^{i\theta}} = \pi - \theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2} \Rightarrow$$

$$\Rightarrow e^{i\theta} = e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i.$$

Demak,

$$w = -\frac{z + 2i}{z - 2i}$$

funksiya masala shartini qanoatlantiruvchi funksiya bo‘lar ekan▹

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $z1 := I\cdot w$

$$z1 := \mathrm{I}\, w$$

> $z2 := e^{I\cdot\theta}\cdot\frac{z1 - a}{z1 - conjugate(a)}$

$$z2 := \frac{e^{\mathrm{I}\,\theta}\,(\mathrm{I}\,w - a)}{\mathrm{I}\,w - \bar{a}}$$

> $z3 := 2\cdot z2$

$$z3 := \frac{2\,e^{\mathrm{I}\,\theta}\,(\mathrm{I}\,w - a)}{\mathrm{I}\,w - \bar{a}}$$

>

> $solve\left(\left\{z = \frac{2\,e^{I\cdot\theta}\,(I\,w - a)}{I\,w - \overline{a}}\right\}, \{w\}\right)$

$$\left\{w = -\frac{I\left(2\,e^{I\,\theta}\,a - z\,\overline{a}\right)}{2\,e^{I\,\theta} - z}\right\}$$

> $w(z) := -\frac{I\left(2\,e^{I\,\theta}\,a - z\,\overline{a}\right)}{2\,e^{I\,\theta} - z}$

$$w := z \rightarrow -\frac{I\left(2\,e^{I\,\theta}\,a - z\,\overline{a}\right)}{2\,e^{I\,\theta} - z}$$

> $w(0)$

$$-I\,a$$

> $solve(\{-I\cdot a = 1\}, \{a\})$

$$\{a = I\}$$

> $w(z) := -\frac{I\left(2\,e^{I\,\theta}\cdot I + z\cdot I\right)}{2\,e^{I\,\theta} - z}$

$$w := z \rightarrow -\frac{I\left(2\,I\,e^{I\,\theta} + I\,z\right)}{2\,e^{I\,\theta} - z}$$

> $\frac{d}{d\,z}\,w(z)$

$$\frac{1}{2\,e^{I\,\theta} - z} - \frac{I\left(2\,I\,e^{I\,\theta} + I\,z\right)}{\left(2\,e^{I\,\theta} - z\right)^2}$$

> **simplify( );**

$$\frac{4\,e^{I\,\theta}}{\left(2\,e^{I\,\theta} - z\right)^2}$$

> $h(z) := \frac{1}{2\,e^{I\,\theta} - z} - \frac{I\left(2\,I\,e^{I\,\theta} + I\,z\right)}{\left(2\,e^{I\,\theta} - z\right)^2}$

$$h := z \to \frac{1}{2\,e^{I\theta} - z} - \frac{I\left(2\,I\,e^{I\theta} + I\,z\right)}{\left(2\,e^{I\theta} - z\right)^2}$$

> $h(0)$

$$\frac{1}{e^{I\theta}}$$

> $\operatorname{argument}\left(\frac{1}{e^{I\theta}}\right)$

$$\operatorname{argument}\left(\frac{1}{e^{I\theta}}\right)$$

> **simplify( );**

$$\operatorname{argument}\left(e^{-I\theta}\right)$$

> $evalc\left(\operatorname{argument}\left(e^{-I\theta}\right)\right)$

$$-\theta$$

> $\theta := -\frac{\pi}{2}$

$$\theta := -\frac{1}{2}\pi$$

> $e^{-I\theta}$

$$I$$

> $w(z) := -\frac{I\,(2\,I \cdot I + z \cdot I)}{2\,I - z}$

$$w := z \to -\frac{I\,(-2 + I\,z)}{2\,I - z}$$

8.21-Masala. Quyidagi $D = \{|z| = 2, \frac{\pi}{6} < \arg z < \frac{\pi}{3}\}$ to'plamning $w = z^6$ funksiya yordamidagi aksini toping.

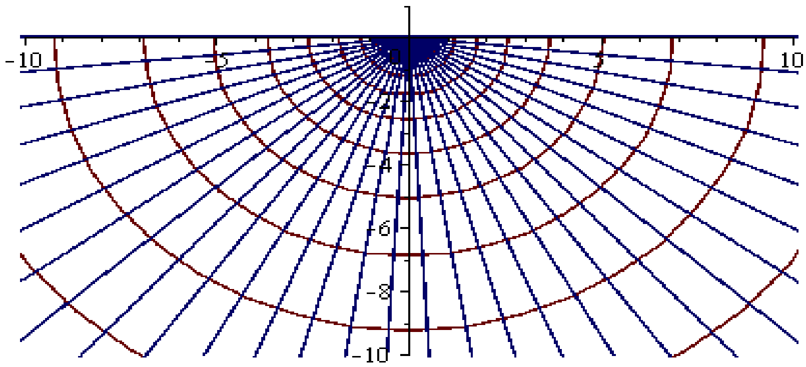
◁ Agar $z = re^{i\varphi}$ *ва* $w = \rho e^{i\varphi}$ desak, unda $w = z^6$ dan $\rho = r^6$ *ва* $\psi = 6\varphi$ ekanligi kelib chiqadi. Unda $G = w(D) = \{|w| = 64, \ \pi < \arg w < 2\pi\}$ bo'ladi (72-chizma) ▷

Bu misolni Maple matematik paketida yechishni ko'rsatamiz.

> > $with(plots)$:

>

> $conformal\left(z^6, z = 0 + \frac{\pi}{6}\cdot I..2 + \frac{\pi}{3}\cdot I, -10 - 10\cdot I..10 + I, grid = [30, 30],\right.$
$\left. coords = \text{polar}\right)$



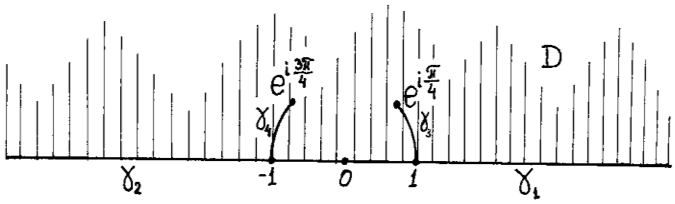
72-chizma

9.21-Masala. Jukovskiy funksiyasidan foydalanib ushbu

$D = \{\operatorname{Im} z > 0,\ \ z \notin \{|z| = 1,\ 0 \le \arg z \le \frac{\pi}{4},\ \frac{3\pi}{4} \le \arg z \le \pi\}$ to'plamning aksini

toping.

◁Bu masalani yechish uchun birinchi navbatda D sohaning chizmasini chizib olamiz (73-chizma) va (11)-formulalardan foydalanamiz.



73-Chizma.

$$\begin{cases} u = \frac{1}{2}(r + \frac{1}{r})\cos\varphi, \\ \\ v = \frac{1}{2}(r - \frac{1}{r})\sin\varphi \end{cases} \qquad (11)$$

Ma’lumki sohaning saqlanish prinsipiga ko‘ra $w(\partial D) = \partial G$ bo‘ladi. Agar

$\gamma_1 = [0,+\infty),\ \gamma_2 = (-\infty,0],\ \gamma_3 = \{r = 1,\ 0 \le \varphi \le \frac{\pi}{4}\}$ *ва* $\gamma_4 = \{r = 1, \frac{3\pi}{4} \le \varphi \le \pi\}$ desak, $\partial D = \gamma_1 \cup \gamma_2 \cup \gamma_3 \cup \gamma_4$ bo‘ladi va (11)–formulalardan foydalansak,

$$w(\gamma_1) = [1,+\infty),\ w(\gamma_2) = (-\infty,-1],\ w(\gamma_3) = [\frac{\sqrt{2}}{2},1] \text{ ва } w(\gamma_4) = [-\frac{\sqrt{2}}{2},-1]$$

ekanligini topamiz.

$$\Rightarrow\ \partial G = w(\gamma_1) \cup w(\gamma_2) \cup w(\gamma_3) \cup w(\gamma_4) = (-\infty,-\frac{\sqrt{2}}{2}] \cup [\frac{\sqrt{2}}{2},+\infty).$$

Demak, $G = w(D) = \{w \notin (-\infty,-\frac{\sqrt{2}}{2}], w \notin [\frac{\sqrt{2}}{2},+\infty)\}$ ▹

10.21-Masala. Quyidagi $D = \{0 < \operatorname{Re} z < 2,\ \pi < \operatorname{Im} z < 2\pi\}$ to‘plamning $w = e^z$ akslantirish yordamidagi aksini toping.

◃ Agar $z = x + iy$ *ва* $w = \rho e^{i\varphi}$ desak ,

$$\begin{cases} \rho = e^x \\ \\ \varphi = y \end{cases} \qquad (*)$$

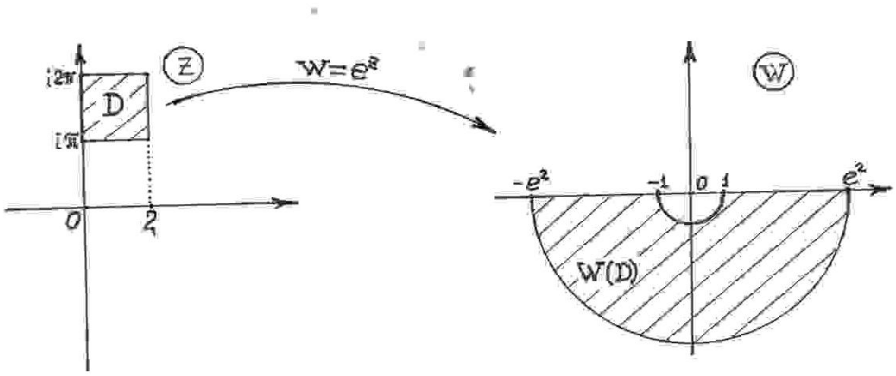
sistemani hosil qilamiz. Unda D sohada

$$e^0 < \rho < e^2 \ ,\quad \pi < \psi < 2\pi$$

bo‘ladi. Shularni e’tiborga olib topamiz:

$$w(D) = \{1 < |w| < e^2,\quad \pi < \arg w < 2\pi\}$$

D hamda $w(D)$ tuplamlar 74-chizmada tasvirlangan▹

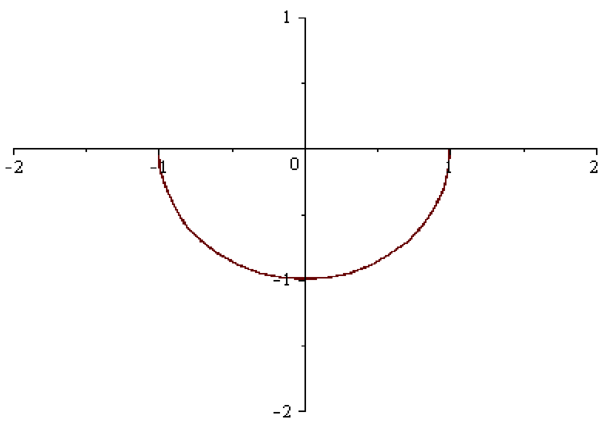


74- chizma

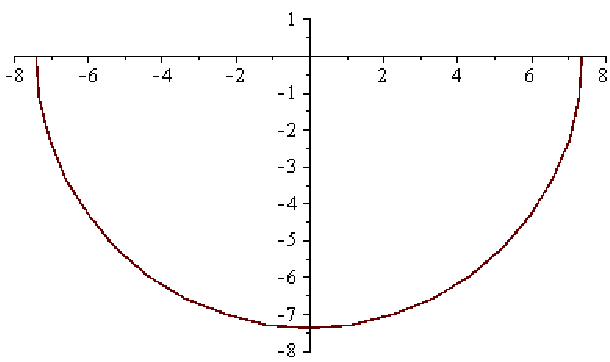
Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots):$

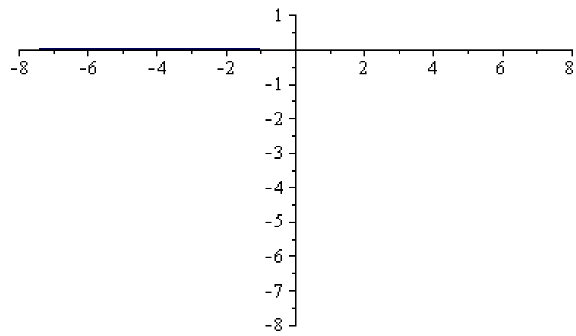
> $conformal\left(e^z, z = 0 + \pi \cdot I..0 + 2\pi \cdot I, -2 - 2I..2 + 1 \cdot I, grid = [20, 20]\right)$



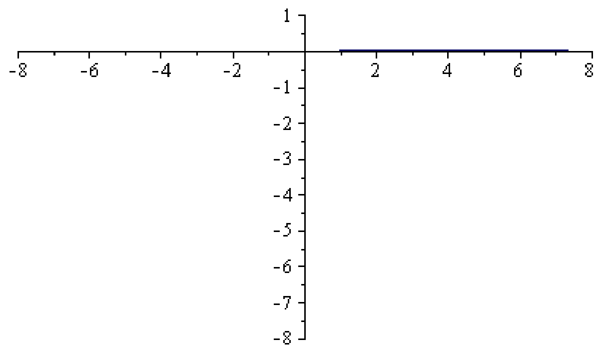
> $conformal\left(e^z, z = 2 + \pi \cdot I..2 + 2\pi \cdot I, -8 - 8I..8 + 1 \cdot I, grid = [20, 20]\right)$



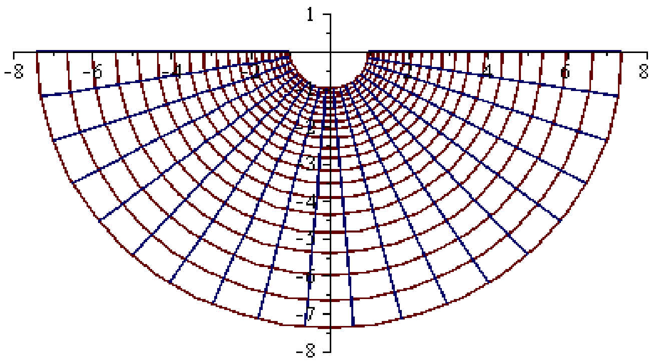
> $conformal\left(e^z, z = 0 + \pi \cdot I..2 + \pi \cdot I, -8 - 8I..8 + 1 \cdot I, grid = [20, 20]\right)$



> $conformal\left(e^{z}, z = 0 + 2\pi \cdot I..2 + 2\pi \cdot I, -8 - 8I..8 + 1 \cdot I, grid = [20, 20]\right)$



> $conformal\left(e^{z}, z = 0 + \pi \cdot I..2 + 2\pi \cdot I, -8 - 8I..8 + 1 \cdot I, grid = [20, 20]\right)$



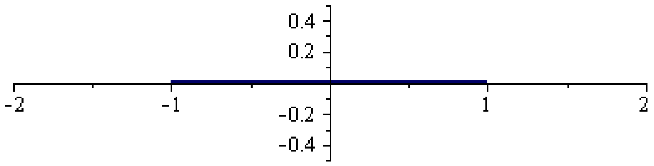
11.21-Masala. Quyidagi $D=\{-\frac{\pi}{2}<\operatorname{Re}z<\frac{\pi}{2},\ \operatorname{Im}z>0\}$ sohaning $w=\sin z$ funksiya yordamidagi aksini toping.

◁ Bu masalaning yechimi 6^0-punktda keltirilgan ▷

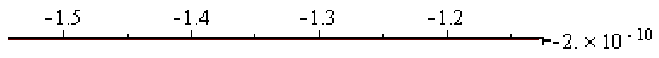
Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots)$:

> $conformal\left(\sin(z), z=-\frac{\pi}{2}..\frac{\pi}{2}, -2-0.5\cdot I..2+0.5\cdot I, grid=[20,20]\right)$

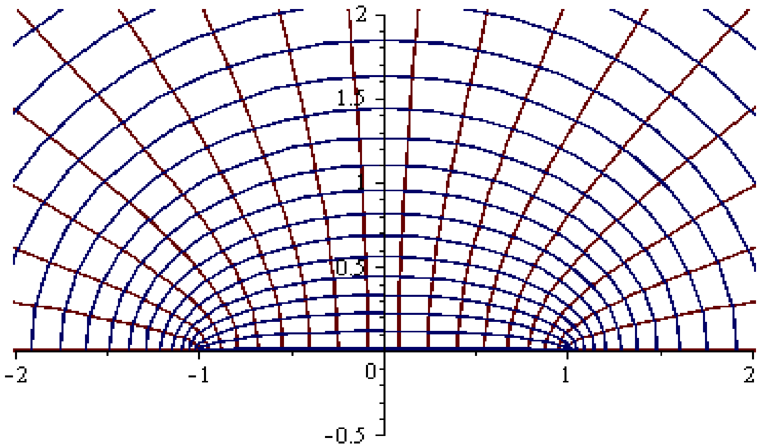


> $conformal\left(\sin(z), z=-\frac{\pi}{2}+\mathrm{I}..-\frac{\pi}{2}+\frac{1}{2}\mathrm{I}, grid=[20,20]\right)$



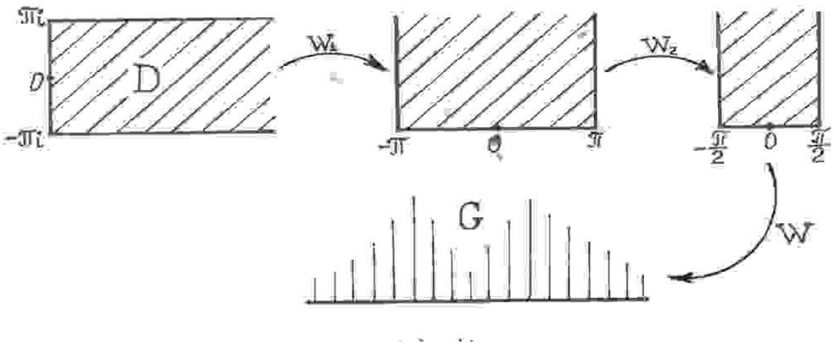
> $conformal\left(\sin(z), z=\frac{\pi}{2}+I..\frac{\pi}{2}+\frac{1}{2}\cdot I, grid=[20,20]\right)$

> $conformal\left(\sin(z),\ z=-\frac{\pi}{2}+0.01\cdot I..\frac{\pi}{2}+2\cdot I, -2-0.5\cdot I..2+2I, grid=[20,20]\right)$



12.21-Masala. Ushbu $D=\{\operatorname{Re} z>0,\ -\pi<\operatorname{Im} z<\pi\}$ sohani $G=\{\operatorname{Im} z>0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta $w(z)$ funksiyani toping.

$\triangleleft$ Bu tipdagi masalalarni 9^0-punktda keltirilgan akslantirishlardan foydalanib yechish maqsadga muvofiqdir. Biz V dagi 1)-akslantirishdan foydalanamiz. Kerakli akslantirishni topish uchun $w_1=iz$, $w_2=\frac{w_1}{2}$, $w=\sin w_2$ akslantirishlarni bajarish kifoya (75-chizma).



75-chizma

Demak, $w = \sin w_2 = \sin\frac{w_1}{2} = \sin\frac{iz}{2} = \frac{e^{i\cdot\frac{iz}{2}} - e^{-i\frac{iz}{2}}}{2i} = i\cdot\frac{e^{\frac{z}{2}} - e^{-\frac{z}{2}}}{2} = ish\frac{z}{2}$ ▷

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

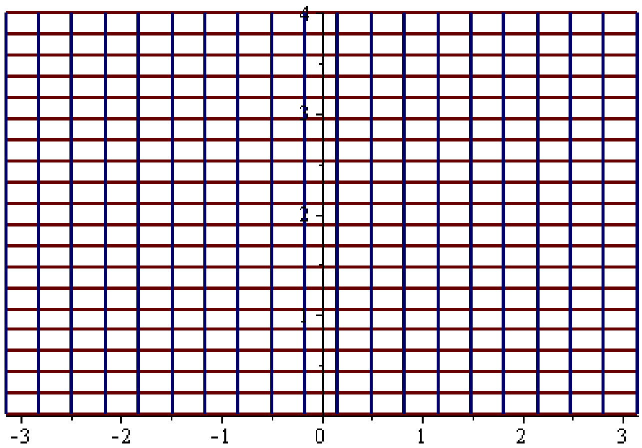
> $with(plots)$:

> $w1 := I\cdot z$

$w1 := \mathrm{I}z$

>

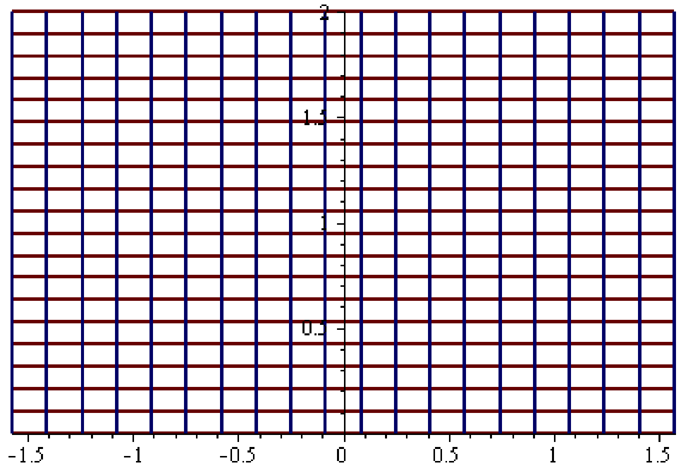
> $conformal(\mathrm{w1}, \mathrm{z} = 0 - \pi\cdot\mathrm{I}..4 + \pi\cdot\mathrm{I}, grid = [20, 20])$



> $w2 := \frac{w1}{2}$

$w2 := \frac{1}{2}\mathrm{I}z$

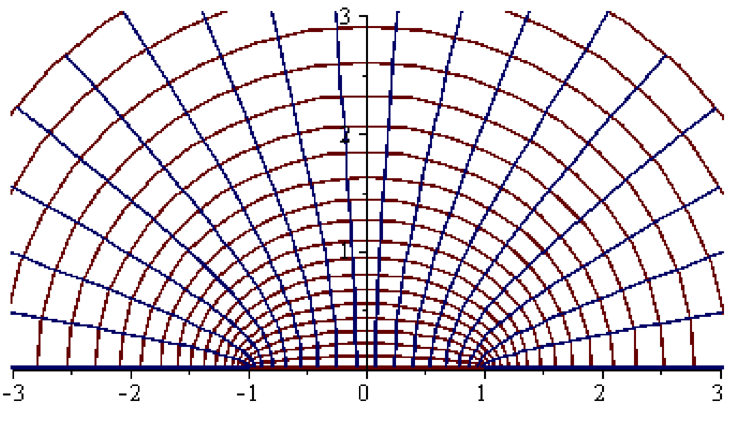
> $conformal(\mathrm{w2}, \mathrm{z} = 0 - \pi \cdot \mathrm{I} .. 4 + \pi \cdot \mathrm{I}, grid = [20, 20])$



> $w := \sin(w2)$

$$w := \mathrm{I} \sinh\left(\frac{1}{2} z\right)$$

> $conformal(\mathrm{w}, \mathrm{z} = 0 - \pi \cdot \mathrm{I} .. 4 + \pi \cdot \mathrm{I}, -3 .. 3 + 3 \cdot \mathrm{I}, \mathrm{grid} = [20, 20])$



13.21-Masala. Quyidagi $z^5 + 4 = 3i$ tenglamani yeching.

$$\triangleleft\ z^5+4=3i \Rightarrow z^5=-4+3i \Rightarrow z=\sqrt[5]{-4+3i}=$$

$$=\sqrt[5]{|-4+3i|}\cdot\left[\cos\frac{\arg(-4+3i)+2k\pi}{5}+i\sin\frac{\arg(-4+3i)+2k\pi}{5}\right]=$$

$$=\sqrt[5]{5}\cdot\left[\cos\frac{(2k+1)\pi-arctg\frac{3}{4}}{5}+i\sin\frac{(2k+1)\pi-arctg\frac{3}{4}}{5}\right],\ \ k=0,1,2,3,4\ \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

$$>\ solve\left(\{z^5+4=3\cdot I\},\ \{z\}\right)$$

$$\{z=(-4+3\,\mathrm{I})^{1/5}\},\ \left\{z=\left(\frac{1}{4}\sqrt{5}-\frac{1}{4}+\frac{1}{4}\mathrm{I}\sqrt{2}\sqrt{5+\sqrt{5}}\right)(-4+3\,\mathrm{I})^{1/5}\right\},\ \left\{z=\left(-\frac{1}{4}\sqrt{5}-\frac{1}{4}+\frac{1}{4}\mathrm{I}\sqrt{2}\sqrt{5-\sqrt{5}}\right)(-4+3\,\mathrm{I})^{1/5}\right\},\ \left\{z=\left(-\frac{1}{4}\sqrt{5}-\frac{1}{4}-\frac{1}{4}\mathrm{I}\sqrt{2}\sqrt{5-\sqrt{5}}\right)(-4+3\,\mathrm{I})^{1/5}\right\},\ \left\{z=\left(\frac{1}{4}\sqrt{5}-\frac{1}{4}-\frac{1}{4}\mathrm{I}\sqrt{2}\sqrt{5+\sqrt{5}}\right)(-4+3\,\mathrm{I})^{1/5}\right\}$$

$$>\ z1 := evalc\left((-4+3\,\mathrm{I})^{1/5}\right)$$

$$z1 := 5^{1/5}\sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right)+\frac{3}{10}\pi\right)+\mathrm{I}\,5^{1/5}\sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right)+\frac{1}{5}\pi\right)$$

>

$$z2 := evalc\left(\left(\frac{1}{4}\sqrt{5} - \frac{1}{4} + \frac{1}{4}\,\mathrm{I}\sqrt{2}\,\sqrt{5+\sqrt{5}}\right)(-4 + 3\,\mathrm{I})^{1/5}\right)$$

$$z2 := \left(\frac{1}{4}\sqrt{5} - \frac{1}{4}\right) 5^{1/5} \sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{3}{10}\pi\right) - \frac{1}{4}\sqrt{2}\,\sqrt{5+\sqrt{5}}\, 5^{1/5} \sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{1}{5}\pi\right) + \mathrm{I}\left(\frac{1}{4}\sqrt{2}\,\sqrt{5+\sqrt{5}}\, 5^{1/5} \sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{3}{10}\pi\right) + \left(\frac{1}{4}\sqrt{5} - \frac{1}{4}\right) 5^{1/5} \sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{1}{5}\pi\right)\right)$$

>

$$z3 := evalc\left(\left(-\frac{1}{4}\sqrt{5} - \frac{1}{4} + \frac{1}{4}\,\mathrm{I}\sqrt{2}\,\sqrt{5-\sqrt{5}}\right)(-4 + 3\,\mathrm{I})^{1/5}\right)$$

$$z3 := \left(-\frac{1}{4}\sqrt{5} - \frac{1}{4}\right) 5^{1/5} \sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{3}{10}\pi\right) - \frac{1}{4}\sqrt{2}\sqrt{5-\sqrt{5}}\, 5^{1/5} \sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{1}{5}\pi\right) + \mathrm{I}\left(\frac{1}{4}\sqrt{2}\sqrt{5-\sqrt{5}}\, 5^{1/5} \sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{3}{10}\pi\right) + \left(-\frac{1}{4}\sqrt{5} - \frac{1}{4}\right) 5^{1/5} \sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{1}{5}\pi\right)\right)$$

> $z4 := evalc\left(\left(-\frac{1}{4}\sqrt{5} - \frac{1}{4} - \frac{1}{4}\mathrm{I}\sqrt{2}\sqrt{5-\sqrt{5}}\right)(-4+3\,\mathrm{I})^{1/5}\right)$

$$z4 := \left(-\frac{1}{4}\sqrt{5} - \frac{1}{4}\right) 5^{1/5} \sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{3}{10}\pi\right) + \frac{1}{4}\sqrt{2}\sqrt{5-\sqrt{5}}\, 5^{1/5} \sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{1}{5}\pi\right) + \mathrm{I}\left(-\frac{1}{4}\sqrt{2}\sqrt{5-\sqrt{5}}\, 5^{1/5} \sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{3}{10}\pi\right) + \left(-\frac{1}{4}\sqrt{5} - \frac{1}{4}\right) 5^{1/5} \sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right) + \frac{1}{5}\pi\right)\right)$$

> $z5 := evalc\left(\left(\frac{1}{4}\sqrt{5}-\frac{1}{4}-\frac{1}{4}\,\mathrm{I}\sqrt{2}\,\sqrt{5+\sqrt{5}}\right)(-4+3\,\mathrm{I})^{1/5}\right)$

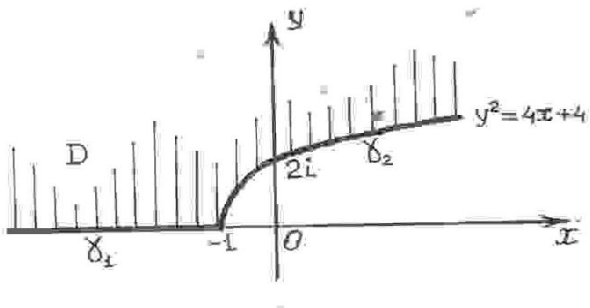
$$z5 := \left(\frac{1}{4}\sqrt{5}-\frac{1}{4}\right)5^{1/5}\sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right)+\frac{3}{10}\pi\right)+\frac{1}{4}\sqrt{2}\,\sqrt{5+\sqrt{5}}\,5^{1/5}\sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right)+\frac{1}{5}\pi\right)+\mathrm{I}\left(-\frac{1}{4}\sqrt{2}\,\sqrt{5+\sqrt{5}}\,5^{1/5}\sin\left(\frac{1}{5}\arctan\left(\frac{3}{4}\right)+\frac{3}{10}\pi\right)+\left(\frac{1}{4}\sqrt{5}-\frac{1}{4}\right)5^{1/5}\sin\left(-\frac{1}{5}\arctan\left(\frac{3}{4}\right)+\frac{1}{5}\pi\right)\right)$$

14.21-Masala. $w=\sqrt{z}$ funksiyaning $\sqrt{-1}=i$ shartni qanoatlantiruvchi bir qiymatli tarmog‘i yordamida

$$D=\{\operatorname{Im} z>0\ ,\quad (\operatorname{Im} z)^2>4\operatorname{Re} z+4\}$$

sohaning aksini toping.

◃ Avval D sohaning chizmasini chizib olamiz (76-chizma).



76-chizma

Keyin $z = re^{i\varphi}$, $w = \rho e^{i\psi}$ deb,

$$w = \sqrt{r}(\cos\frac{\varphi + 2k\pi}{2} + i\cdot\sin\frac{\varphi + 2k\pi}{2}) \quad (k = 0,1)$$

tenglik va $\sqrt{-1} = \sqrt{1}\cdot(\cos\frac{\pi + 2k\pi}{2} + i\cdot\sin\frac{\pi + 2k\pi}{2}) = i$ shartdan k=0 ekanligini topamiz.

Demak,

$$w = \sqrt{z} = \sqrt{r}(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}) = \sqrt{r}\cdot e^{i\frac{\varphi}{2}} \quad \text{ekan} \quad \Rightarrow \begin{cases} \rho = \sqrt{r}, \\ \psi = \frac{\varphi}{2}. \end{cases} \qquad (*)$$

munosabatlardan foydalanib, D sohaning chegarasi ∂D ning obrazi ∂G ni topamiz:

$\gamma_1 = (-\infty,-1] = \{\varphi = \pi,\ 1 \le r < +\infty\}$ va $\gamma_2 = \{y^2 = 4x + 4,\ \ y \ge 0\}$ desak, $\partial D = \gamma_1 \cup \gamma_2$ bo‘ladi. $w(\gamma_1) = \{\psi = \frac{\pi}{2}, 1 \le \rho < +\infty\}$ ekanligini to‘g‘ridan to‘g‘ri (*) munosabatdan kelib chiqadi. Endi $w(\gamma_2)$ topamiz:

$$w = \sqrt{z} \Rightarrow w^2 = z \Rightarrow (u + iv)^2 = x + iy \Rightarrow \begin{cases} x = u^2 - v^2 \\ y = 2uv \end{cases} \qquad (**)$$

(**) va

$$y^2 = 4x + 4 \Rightarrow 4u^2v^2 = 4u^2 - 4v^2 + 4 \Rightarrow u^2v^2 - u^2 + v^2 - 1 = 0 \Rightarrow$$

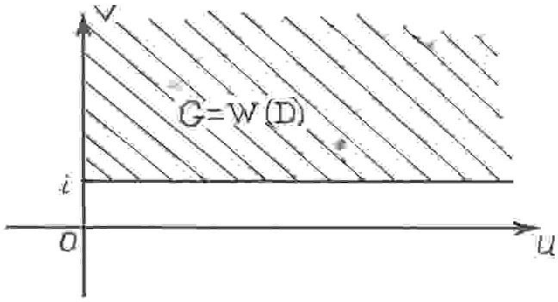
$$(v^2 - 1)(u^2 + 1) = 0 \Rightarrow v = 1, u \ge 0.$$

Demak,

$$w(\gamma_2) = \{v = 1, u \ge 0\} \Rightarrow \partial G = w(\gamma_1) \cup w(\gamma_2) =$$

$$= \{\arg w = \frac{\pi}{2}, 1 \le |w| < +\infty\} \cup \{\operatorname{Im} w = 1,\ \operatorname{Re} w \ge 0\}$$

Bu yerdan va misol shartidan $G = \{\operatorname{Re} w > 0,\ \operatorname{Im} w > 1\}$ ekanligini hosil qilamiz (77-chizma)▹



77 - chizma

15.21-Masala. Quyidagi

$$D=\{z\notin\{|z|\le 1,\ 0\le \arg z\le \pi\},\ z\notin[-i,0]\}$$

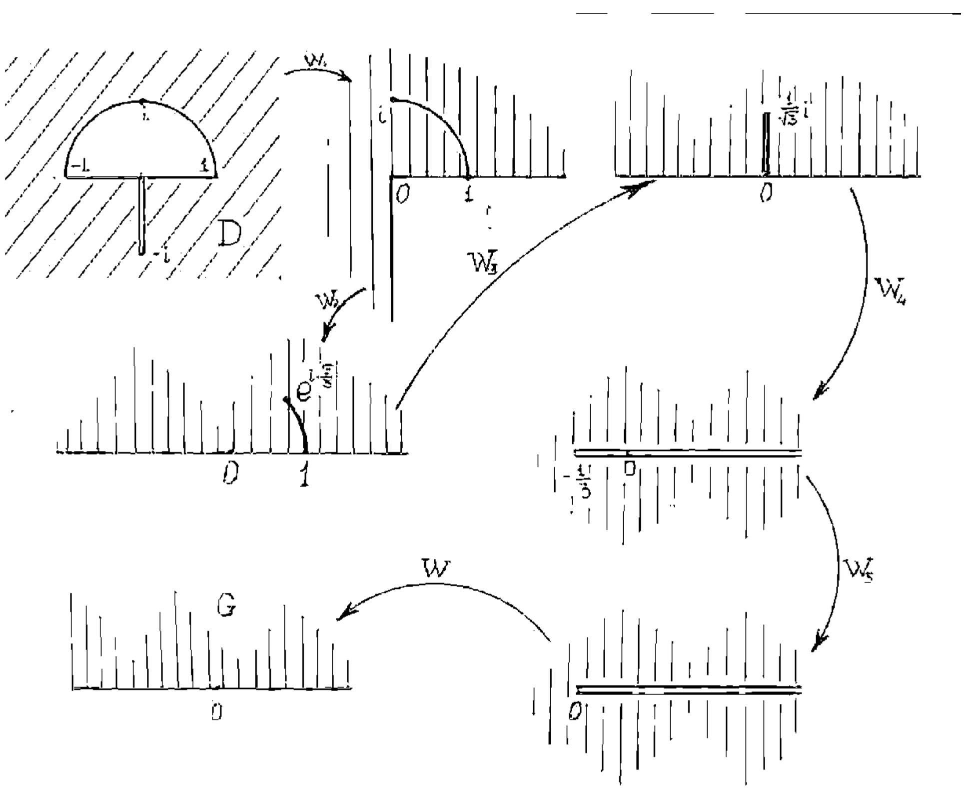
sohani $G=\{\operatorname{Im} w>0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta $w(z)$ funksiyani toping.

◃Masala shartini qanoatlantiruvchi konform akslantirishni quyidagi akslantirishlarni ketma-ket bajarish yordamida topamiz:

$$w_1=\frac{1-z}{1+z}\ ,\quad w_2=w_1^{\frac{2}{3}}\ ,\quad w_3=\frac{w_2-1}{w_2+1}\ ,\ w_4=w_3^2\ ,\quad w_5=w_4+\frac{1}{3}\ ,$$

$$w=\sqrt{w_5}\,,\ \sqrt{-1}=i$$

Olingan funksiyalar D sohani qaysi yo‘l bilan G sohaga akslantirishi 78-chizmada ko‘rsatilgan.



78 - chizma

Demak, masala shartini qanoatlantiruvchi funksiya

$$w=\sqrt{w_5}=\sqrt{w_4+\frac{1}{3}}=\sqrt{w_3^2+\frac{1}{3}}=\sqrt{(\frac{w_2-1}{w_2+1})^2+\frac{1}{3}}=\sqrt{(\frac{w_1^{\frac{2}{3}}-1}{w_1^{\frac{2}{3}}+1})^2+\frac{1}{3}}$$

$$=\sqrt{[\frac{(1-z)^{\frac{2}{3}}-(1+z)^{\frac{2}{3}}}{(1-z)^{\frac{2}{3}}+(1+z)^{\frac{2}{3}}}]^2+\frac{1}{3}},\sqrt{-1}=i$$

ekan ▹

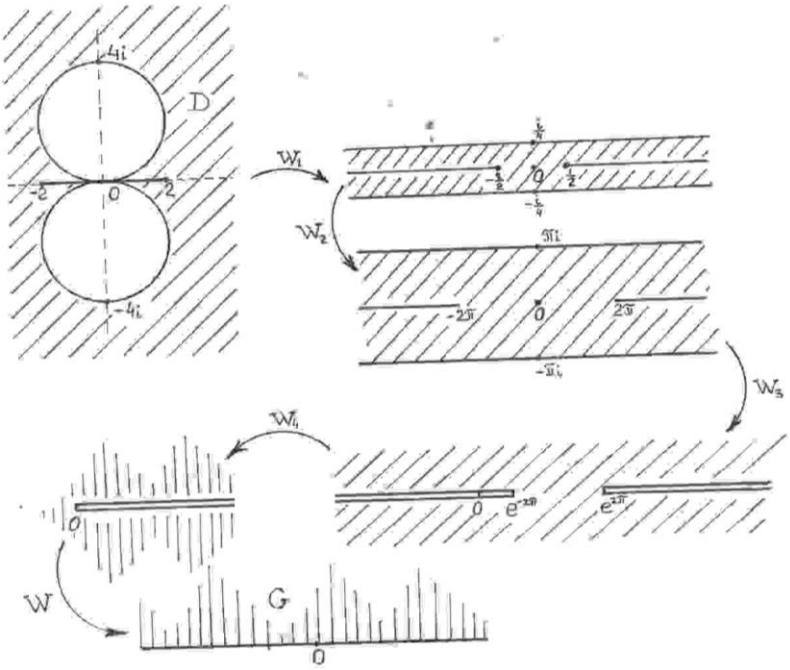
16.21-Masala. Quyidagi

$$D=\{|z-2i|>2,\ |z+2i|>2\ ,\ \ z\notin[-2,2]\}$$

sohani $G=\{\operatorname{Im} w>0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta $w(z)$ funksiyani toping.

◃ Masala shartini qanoatlantiruvchi konform akslantirishni topish uchun quyidagi akslantirishlarni ketma-ket bajarish kifoya (79-chizma):

$$w_1=\frac{1}{z}\ ,\ w_2=4\pi w_1\ ,\ w_3=e^{w_2}\ ,\ w_4=\frac{e^{2\pi}-w_3}{e^{-2\pi}-w_3},\ w=\sqrt{w_4}\ ,\ \sqrt{-1}-i.$$



79 - chizma

Demak, $w=\sqrt{w_4}=\sqrt{\dfrac{e^{2\pi}-w_3}{e^{-2\pi}-w_3}}=\sqrt{\dfrac{e^{2\pi}-e^{\frac{4\pi}{z}}}{e^{-2\pi}-e^{\frac{4\pi}{z}}}}\ ,\ \sqrt{-1}=i.$ ▷

17.21-Masala. Quyidagi $Arc\sin 2$ ifodaning barcha qiymatlarini toping.

◁ Bu tipdagi masalalarni yechishda isbotlash qiyin bo‘lmagan quyidagi tengliklardan foydalaniladi.

1) $Arc\sin z=-iLni(z+\sqrt{z^2-1}).$

2) $Arc\cos z=-iLn(z+\sqrt{z^2-1}).$

3) $Arctgz=\frac{1}{2}Ln\frac{i+z}{i-z}=\frac{1}{2i}Ln\frac{1+iz}{1-iz}.$

4) $Arcctgz=\frac{i}{2}Ln\frac{z-i}{z+i}$

Bu tengliklarda ildizning barcha qiymatlari olingan. Biz 1)-tenglik va (20)-formuladan foydalanamiz:

$$Arc\sin 2 = -iLni(2+\sqrt{2^2-1}) = -iLn(2\pm\sqrt{3})i =$$

$$= -i[\ln(2\pm\sqrt{3}) + i\cdot\frac{\pi}{2} + 2k\pi i] = \frac{\pi}{2} + 2k\pi - i\ln(2\pm\sqrt{3}) =$$

$$= \frac{(4k+1)\pi}{2} - i\cdot\ln(2\pm\sqrt{3}) \;, \quad k\in Z. \qquad \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

```
> solve( {sin(z) = 2}, {z})
```

$$\{z = \arcsin(2)\}$$

```
> evalc(arcsin(2))
```

$$\frac{1}{2}\pi - \mathrm{I}\ln\left(2+\sqrt{3}\right)$$

18.21-Masala. Quyidagi $D=\{z\notin(-\infty,0]\,,\,z\notin[1,+\infty)\}$ sohaning $w=Lnz$ funksiyaning $w(i)=\frac{\pi i}{2}$ shartni qanoatlantiruvchi bir qiymatli tarmog‘i yordamidagi aksini toping.

◁ Lnz funksiyaning

$$w=(Lnz)_k = \ln z + 2k\pi i \qquad , \qquad k=0,\pm1,\pm2,...$$

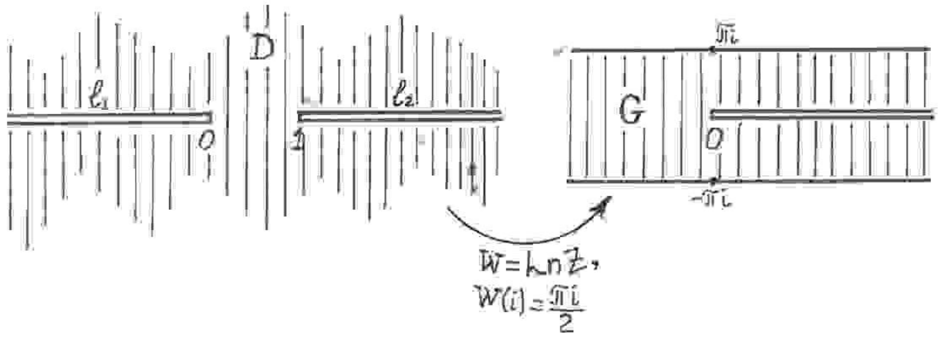
tarmoqlaridan qaysi birini tanlashimiz kerakligini $w(i)=\frac{\pi i}{2}$ shartdan aniqlaymiz:

$$\frac{\pi i}{2} = \ln i + 2k\pi i = \ln|i| + i\arg i + 2k\pi i = i\frac{\pi}{2} + 2k\pi i$$

Bu yerdan $k=0$, ekanligini topamiz. Demak, Lnz ning kerakli tarmog‘i $w=(Lnz)_0=\ln z$ ekan. $w=\ln z$ akslantirish yordamida D sohaning aksini topish uchun $w=u+iv$ *ва* $z=re^{i\varphi}$ desak,

$$\begin{cases} u=\ln r, \\ v=\varphi \end{cases} \quad (*)$$

ekanligini ko‘ramiz. Agar $l_1=(-\infty,0]$ *ва* $l_2=[1,+\infty)$ desak, $\partial D=l_1\cup l_2$ bo‘ladi. (*) tenglikka ko‘ra $w(l_2)=\{v=0,\ 0\le u<+\infty\}$ *ва* l_1 nurning yuqori qirg‘og‘i $\{v=\pi\}$ to‘g‘ri chiziqqa, pastki qirg‘og‘i esa $\{v=-\pi\}$ to‘g‘ri chiziqqa akslanadi. Demak, $G=\{-\pi<\operatorname{Im}w<\pi\ ,\ w\notin[0,+\infty)\}$ ekan. (80-chizma) ▹



80- chizma

19.21-Masala. Simmetriya prinsipidan foydalanib , $D=\{|z|<1\}$ birlik doiraning $w=\dfrac{z}{\sqrt[n]{(1+z^n)^2}}$ funksiya yordamidagi aksini toping.

◃ D birlik doirani uchlari $z=0$ nuqtada va kengligi $\dfrac{2\pi}{n}$ ga teng bo‘lgan $D_0, D_1, D_2,..., D_{n-1}$ n-ta sektorga ajratamiz. Ravshanki,

$$D_0=\{z\in C:-\frac{\pi}{n}<\arg z<\frac{\pi}{n}\ ,\ |z|<1\}$$

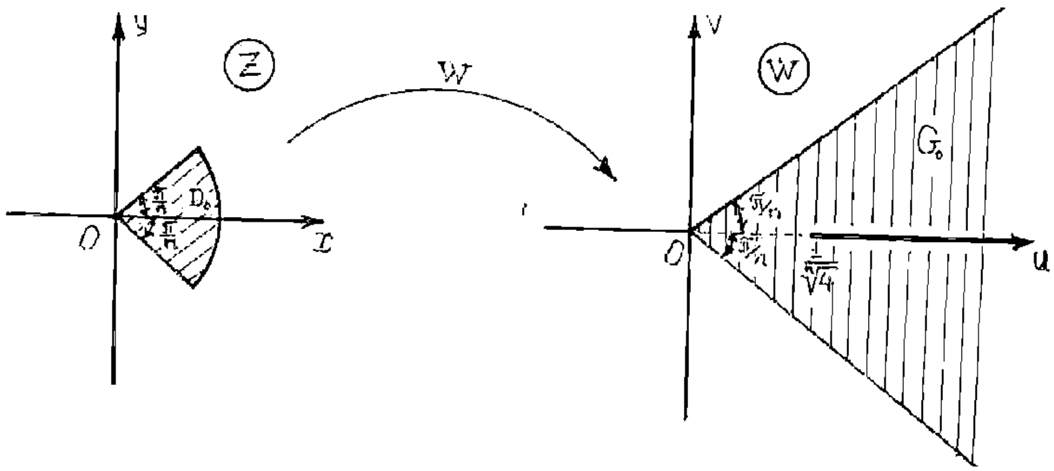
deb olish mumkin. Bunda berilgan w funksiyani quyidagicha yozib olamiz:

$$w=\frac{z}{\sqrt[n]{(z^n+1)^2}}=\sqrt[n]{\frac{z^n}{z^{2n}+2z^n+1}}=\frac{1}{\sqrt{z^n+2+\frac{1}{z^n}}}=\frac{1}{\sqrt{2\cdot[\frac{1}{2}(z^n+\frac{1}{z^n})+1}}.$$

Agar $w_1=z^n$, $w_2=\frac{1}{2}(w_1+\frac{1}{w_1})$, $w_3=w_2+1$ *ва* $w_4=\frac{1}{2w_3}$ deyilsa, unda w funksiya ushbu $w=(\sqrt[n]{w_4})_0$ ko‘rinishga keladi. Bu akslantirishlardan foydalanib, D_0 ning aksi

$$G_0=\{w\in C:-\frac{\pi}{n}<\arg w<\frac{\pi}{n},\quad w\notin[\frac{1}{\sqrt[n]{4}},+\infty)\}$$

bo‘lishini topamiz (81-chizma).



81- chizma

Shu mulohaza asosida, simmetriya prinsipini *n* marta qo‘llash natijasida

$w=\frac{z}{\sqrt[n]{(z^n+1)^2}}$ funksiya birlik doira $D=\{z\in C:|z|<1\}$ ni n-ta

$\{\arg w=\frac{2\pi k}{n}\ ,\ |w|\geq\frac{1}{\sqrt[n]{4}}\}$, $k=\overline{0,n-1}$ nurlar bo‘yiga qirqilgan (w)

tekislikka akslantirishini topamiz ▹

20.21-Masala. Simmetriya prinsipidan foydalanib

$$D=\{0<\operatorname{Re}z<1,\ z\notin\{\operatorname{Re}z=\frac{1}{2},\ 2\leq\operatorname{Im}z<\infty\}\}$$

sohani $\{\operatorname{Im} w > 0\}$ yuqori yarim tekislikka konform akslantiruvchi birorta funksiyani toping.

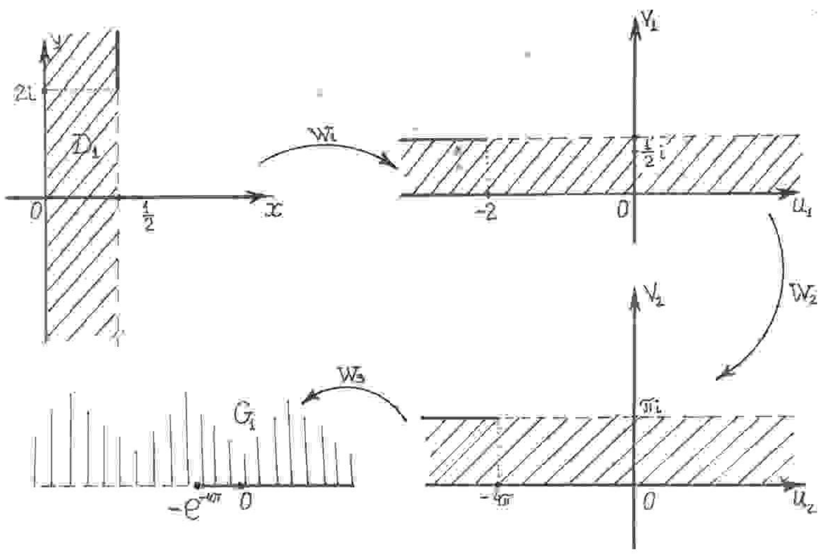
◃ Quyidagi $D_1 = \{0 < \operatorname{Re} z < \frac{1}{2}\}$ sohani qaraymiz. Bu soha

$$w_1 = iz,\ w_2 = 2\pi w_1,\ w_3 = e^{w_2} \qquad (25)$$

akslantirishlarni birin-ketin bajarish natijasida

$$G_1 = (\operatorname{Im} w_3 > 0\}$$

yuqori yarim tekislikka konform akslanadi. (25)-akslantirishlarning bajarilishi jarayoni 82-chizmada tasvirlangan.



82 - chizma

Simmetriya prinsipidan foydalanib, berilgan soha $w_3 = e^{w_2} = e^{2\pi w_1} = e^{2\pi iz}$ funksiya yordamida $G = \{w_3 \notin [-e^{-4\pi}, +\infty)$ sohaga konform akslanishini topamiz. Bu G soha

$$w_4 = w_3 + e^{-4\pi} \ \textit{ва}\ \ w = \sqrt{w_4}\,,\ \sqrt{-1} = i$$

akslantirishlar yordamida $\{\operatorname{Im} w > 0\}$ yuqori yarim tekislikka akslanadi.

Demak, berilgan sohani yuqori yarim tekislikka konform akslantiruvchi funksiya ushbu

$$w = \sqrt{w_4} = \sqrt{e^{2\pi iz} + e^{-4\pi}}\ ,\ \sqrt{-1} = i\,,$$

ko‘rinishda bo‘ladi ▹

3-§. 3-MUSTAQIL ISH
KOMPLEKS ARGUMENTLI FUNKSIYANING INTEGRALI VA CHEGIRMALAR NAZARIYASI

Kompleks argumentli funksiyaning integrali tushunchasi.

Koshining integral teoremasi.

Koshining integral formulasi.

Darajali qatorlar.

Golomorf funksiyalarning xossalari.

Loran qatori.

Funksiyaning yakkalangan maxsus nuqtalari.

Chegirmalar va ularni hisoblash.

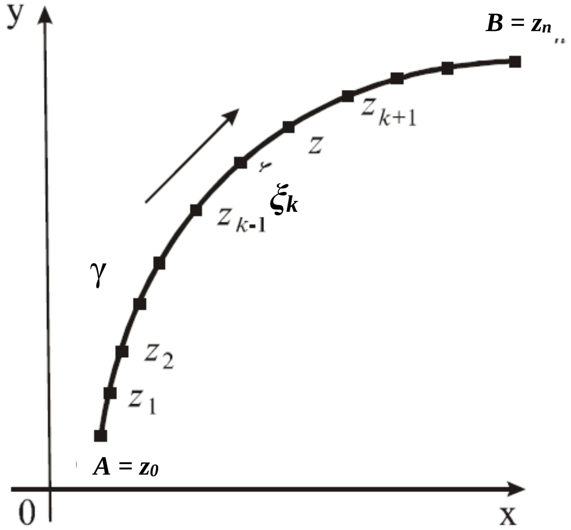
Integralni chegirmalar yordamida hisoblash.

- A -

ASOSIY TUSHUNCHA VA TEOREMALAR.

1^0. Integral tushunchasi.

Kompleks tekislik C da to‘g‘rilanuvchi $\gamma = \overset{\cup}{AB}$ egri chiziq berilgan bo‘lsin. Bu egri chiziqni A dan B ga qarab $z_0, z_1, ..., z_n$ nuqtalar yordamida n ta $\gamma_1, \gamma_2, ..., \gamma_n$ yoylarga ajratamiz (83-chizma).



83 - chizma

γ_k yoylarning $(k=1,2,...,n)$ uzunliklarini l_k *va* $\lambda=\max\limits_{1\le k\le n} l_k$ deb belgilaymiz. Aytaylik, γ egri chiziqda $f(z)$ funksiya berilgan bo‘lsin. $\forall\ \xi_k\in\gamma_k$ nuqta olib, quyidagi

$$\sigma=\sum_{k=1}^{n} f(\xi_k)\cdot(z_k-z_{k-1}) \qquad (1)$$

integral yigindini tuzamiz.

Ta’rif. *Agar* $\lambda\to 0$ *da* $f(z)$ *funksiyaning integral yigindisi* γ *egri chiziqning bo‘linish usuliga hamda* γ_k *dagi* ξ_k *nuqtaning tanlab olinishiga bog‘liq bo‘lmagan holda chekli limitga ega bo‘lsa, bu limit* $f(z)$ *funksiyaning* γ *egri chiziq bo‘yicha integrali deb ataladi va*

$$\int_\gamma f(z)dz \qquad (2)$$

kabi belgilanadi.

Demak,

$$\int_\gamma f(z)dz:=\lim_{\lambda\to 0}\sum_{k=1}^{n} f(\xi_k)\cdot(z_k-z_{k-1}). \qquad (3)$$

Agar $z=x+iy,\quad f(z)=u(x,y)+iv(x,y)=u+iv$ deyilsa, unda ushbu

$$\int_\gamma f(z)dz:=\int_\gamma udx-vdy+i\int_\gamma vdx+udy \qquad (4)$$

tenglik hosil bo‘ladi.

1-Teorema. $f(z)$ *funksiyaning* γ *egri chiziq bo‘yicha integrali*

$$\int_\gamma f(z)dz$$

ning mavjud bo‘lishi uchun quyidagi

$$\int_\gamma udx-vdy \quad \textit{ва} \quad \int_\gamma vdx+udy$$

egri chiziqli integrallarning mavjud bo‘lishi zarur va yetarli.

Xususan, $f(z)$ funksiya uzluksiz bo‘lsa uning integrali mavjud bo‘ladi.

2-Teorema. *Agar $f(z)$ funksiya γ egri chiziqda berilgan va uzluksiz, γ egri chiziq ushbu*

$$z = z(t) \qquad (\alpha \le t \le \beta)$$

tenglama bilan berilgan bo‘lib , $z'(t) \ne 0$ bo‘lsa, u holda

$$\int_{\gamma} f(z)dz = \int_{\alpha}^{\beta} f[|z(t)|] \cdot z'(t)dt \qquad (5)$$

bo‘ladi.

Bu formuladan kompleks argumentli funksiya integralini hisoblashda foydalaniladi.

Misol. Ushbu

$$I_n = \int_{\gamma} (z-a)^n dz \qquad (n - \text{butun son})$$

integralni hisoblang, bunda $\gamma = \{z \in C : |z-a| = \rho,\ \rho > 0\}$ aylanadan iborat (yo‘nalish soat strelkasining yo‘nalishiga qarama-qarshi olingan).

◃ γ aylananing tenglamasini quyidagi

$$z = z(t) = a + \rho \cdot e^{it} \quad (0 \le t \le 2\pi)$$

ko‘rinishida yozib olamiz. Unda

$$dz = d(a + \rho \cdot e^{it}) = i\rho \cdot e^{it} dt$$

bo‘lib, (5)-formulaga ko‘ra

$$I_n = \int_{\gamma} (z-a)^n dz = i\rho^{n+1} \int_{0}^{2\pi} e^{it(n+1)} dt$$

bo‘ladi. Agar $n \ne -1$ bo‘lsa,

$$I_n = i\rho^{n+1} \int_{0}^{2\pi} e^{it(n+1)} dt = i\rho^{n+1} \cdot \frac{e^{it(n+1)}}{i(n+1)} \Big|_{0}^{2\pi} = 0$$

bo‘ladi. Agar $n = -1$ bo‘lsa

$$I_{-1} = i \int_{0}^{2\pi} dt = 2\pi i$$

bo‘ladi. Demak,

$$\int_{\gamma}(z-a)^n dz = \int_{|z-a|=\rho}(z-a)^n dz = \begin{cases} 0, & agar\, n \neq -1\, bo'lsa \\ 2\pi i, & agar\;\; n=-1\, bo'lsa \end{cases} \qquad \triangleright$$

2^0. Koshining integral teoremasi.

Kompleks o‘zgaruvchili funksiyalar nazariyasida fundamental teoremalardan biri Koshining integral teoremasidir.

1-Teorema. (Koshining integral teoremasi) *Faraz qilaylik,* $f(z)$ *funksiya kompleks tekislik* C *dagi bir bog‘lamli D sohada golomorf bo‘lsin. U holda ixtiyoriy to‘g‘rilanuvchi yopiq egri chiziq* $\gamma \subset\subset D$ *uchun*

$$\oint_{\gamma} f(z)dz = 0$$

bo‘ladi.

Yuqorida, 1^0-punktda biz ko‘rdikki $f(z) = \dfrac{1}{z-a}$ funksiyasidan $\gamma : |z-a| = \rho$ aylana bo‘yicha olingan integral $2\pi i$ ga teng. Bu misolda $f(z)$ funksiya $C \setminus \{a\}$ da golomorf bo‘lib,bu soha bir bog‘lamli emas. Shuning uchun ham $\oint_{\gamma} f(z)dz \neq 0$ bo‘ldi. Demak, 1-teoremadagi *D* sohaning bir bog‘lamli bo‘lishi muhim shart ekan.

2-Teorema. $D \subset C$ *soha bir bog‘lamli, chegarasi to‘g‘rilanuvchi chiziqdan iborat bo‘lgan soha bo‘lib,* $f(z)$ *funksiya D da golomorf,* $\overline{D}$ *da uzluksiz* ($f(z) \in O(D) \cap C(\overline{D})$ *bo‘lsin. U holda*

$$\oint_{\partial D} f(z)dz = 0$$

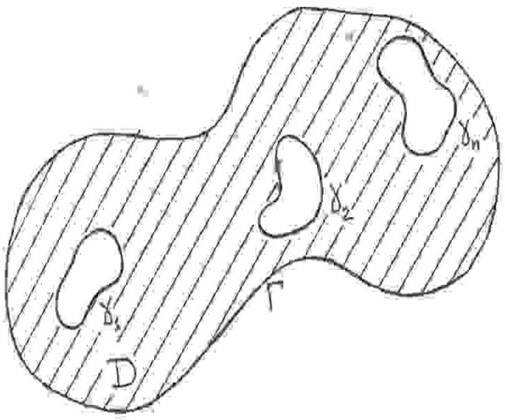
bo‘ladi.

3-Teorema. (Ko‘p bog‘lamli soha uchun Koshi teoremasi) *Faraz qilaylik,* $D \subset C$ *soha chegarasi* $\Gamma, \gamma_1, ...\gamma_n$ *to‘g‘rilanuvchi chiziqlardan tashkil topgan* *ko'p bog'lamli soha bo'lsin (84-chizma). Agar* $f(z)\in O(D)\cap C(\overline{D})$ *bo'lsa , u holda*

$$\int_{\partial D} f(z)dz = \int_{\Gamma\cup\gamma_1^-\cup...\cup\gamma_n^-} f(z)dz = 0$$

tenglik o'rinlidir.

Bu tenglikni quyidagicha ham yozish mumkin $\int_{\Gamma} f(z)dz = \sum_{k=1}^{n}\int_{\gamma_k} f(z)dz$ (6)

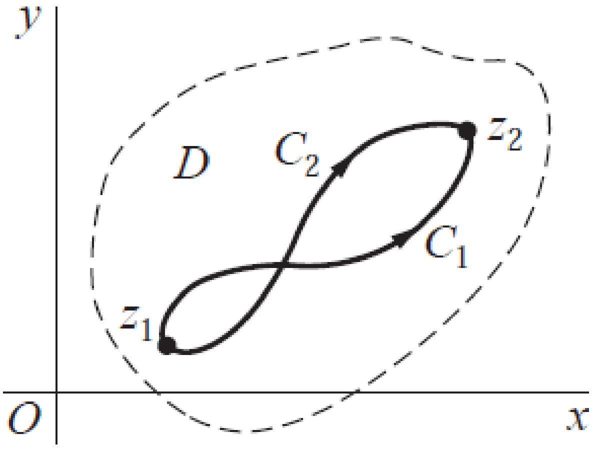


84-chizma.

Natija. *Faraz qilaylik,* $D\subset C$ *bir bog'lamli soha bo'lib,* C_1, C_2 *chiziqlarning har biri (*$C_1\subset D, C_2\subset D$*) boshi* z_1 *va oxiri* z_2 *nuqtada bo'lgan chiziqlar bo'lsin (85-chizma). Agar* $f(z)\in O(D)$ *bo'lsa, u holda*

$$\int_{C_1} f(z)dz = \int_{C_2} f(z)dz \qquad (7)$$

bo'ladi.



85-chizma.

(7)-tenglik, qaralayotgan integralning z_1 va z_2 nuqtalarigagina bog‘liq bo‘lib, integrallash yo‘liga bog‘liq bo‘lmasligini bildiradi. Shuni e’tiborga olib, (7)-integralni

$$\int_{z1}^{z_2} f(z)dz \qquad (8)$$

kabi belgilash ham mumkin.

1-Misol. Ushbu

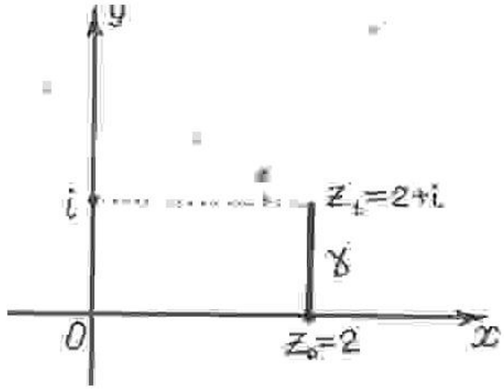
$$\int_{2}^{2+i} z^2 dz$$

integralni hisoblang.

$\triangleleft$ Ravshanki, $f(z) = z^2 \in O(C)$. Binobarin, berilgan integral $z_0 = 2,\ \ z_1 = 2 + i$ nuqtalarni birlashtiruvchi yo‘lga bog‘liq bo‘lmaydi. Shundan foydalanib integrallash chizig‘i γ sifatida

$$\gamma = \{z = x + iy \in C : x = 2, \quad 0 \le y \le 1\}$$

to‘g‘ri chiziq kesmasini olamiz (86-chizma)



86-chizma

Bu γ chiziqda

$$z = 2 + iy, \qquad dz = idy$$

bo‘lishidan foydalanib topamiz:

$$\int_2^{2+i} z^2 dz = \int_\gamma z^2 dz = \int_0^1 (2+iy)^2 \cdot idy = i\int_0^1 (4 + 4iy - y^2)dy =$$

$$= i(4y + 2iy^2 - \frac{y^3}{3})\Big|_0^1 = -2 + \frac{11}{3}i. \qquad \triangleright$$

2-Misol. Ushbu

$$\int_1^2 \frac{dz}{z} \qquad (z \neq 0)$$

integralning qiymati $z_0 = 1$ va $z_1 = 2$ nuqtalarni birlashtiruvchi yo‘lga bog‘liq bo‘ladimi (yo‘l koordinata boshidan o‘tmaydi deb faraz qilinadi)?

$\triangleleft$ Ravshanki,

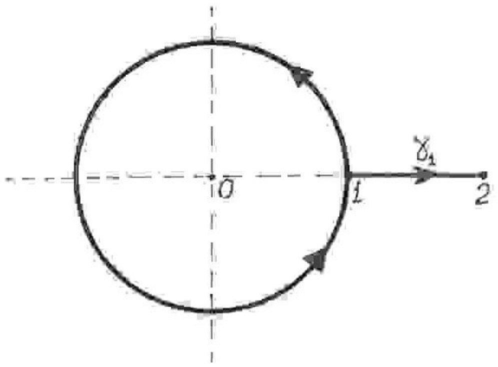
$$f(z) = \frac{1}{z}$$

funksiya $D = C \setminus \{0\}$ sohada golomorf. Ayni paytda bu bir bog‘lamli soha emas. Demak, Koshining integral teoremasidan foydalanib bo‘lmaydi. $z_0 = 1$ va $z_0 = 2$ nuqtalarni birlashtiruvchi ikkita γ_1 hamda γ_2 chiziqlarni

$$\gamma_1 = \{z = x + iy \in C : \ 1 \le x \le 2 \ , \ \ y = 0\},$$

$$\gamma_2 = \{z \in C : |z| = 1\} \cup \gamma_1$$

deb olamiz (87- chizma).



87-chizma.

γ_1 chiziqda

$z = x, \ \ dz = dx$ bo‘lib,

$$\int_1^2 \frac{dz}{z} = \int_{\gamma_1} \frac{dz}{z} = \int_1^2 \frac{dx}{x} = \ln x \Big|_1^2 = \ln 2$$

$|z| = 1$ aylanada $z = e^{i\varphi} (0 \le \varphi \le 2\pi), dz = ie^{i\varphi} d\varphi$ bo‘lib,

$$\int_1^2 \frac{dz}{z} = \int_{\gamma_2} \frac{dz}{z} = \oint_{|z|=1} \frac{dz}{z} + \int_{\gamma_1} \frac{dz}{z} = 2\pi \int_0 \frac{ie^{i\varphi}}{e^{i\varphi}} d\varphi + \int_1^2 \frac{dx}{x} = 2\pi i + \ln 2$$

bo‘ladi. Demak, berilgan integral integrallash yo‘liga bog‘liq ekan ▷

Agar (8)-integralda z_0 nuqtani tayinlab, z_1 ni esa z o‘zgaruvchi sifatida qaralsa, (8)-integral z o‘zgaruvchining funksiyasi bo‘ladi:

$$F(z) = \int_{z_0}^{z} f(z) dz .$$

4-Teorema. *Agar* $f(z)$ *funksiya bir bog‘lamli* $D \subset C$ *sohada golomorf bo‘lsa, u holda* $F(z)$ *funksiya ham* D *sohada golomorf bo‘lib,*

$$F'(z) = f(z) \qquad (z \in D)$$

bo‘ladi.

Bu teoremadan ko‘rinadiki, bir bog‘lamli sohada golomorf funksiya $f(z)$ ning boshlang‘ich funksiyasi mavjuddir.

5-Teorema. *Agar* $\Phi(z)$ *funksiya* $D \subset C$ *sohada* $f(z)$ *ning boshlang'ich funksiyasi bo'lsa, u holda*

$$\int_{z_0}^{z} f(z)dz = \Phi(z) - \Phi(z_0) = \Phi(z)\Big|_{z_0}^{z} \qquad (9)$$

formula (Nyuton-Leybnits formulasi) o'rinli bo'ladi, bunda z_0 *va* z *nuqtalar D sohaga tegishli ixtiyoriy nuqtalar.*

3^0. **Koshining integral formulasi**.

Kompleks tekislik C da chegarasi to'g'rilanuvchi chiziq bo'lgan chegaralangan D sohani qaraylik. Kuzatuvchi bu soha chegarasi ∂D bo'ylab harakat qilganda soha har doim chap tomonda qolsin.

1-Teorema. *Agar* $f(z) \in O(D) \cap C(\overline{D})$ *bo'lsa, u holda*

$$\frac{1}{2\pi i}\int_{\partial D}\frac{f(\xi)}{\xi - z}d\xi = \begin{cases} f(z), \ agar \ \ z \in D \ bo'lsa \\ 0, \ agar \ \ z \notin \overline{D} \ bo'lsa \end{cases} \qquad (10)$$

tenglik o'rinli bo'ladi.

Odatda (10)-formula *Koshining integral formulasi* deyiladi. Bu formula $f(z)$ ning $z \in D$ nuqtadagi qiymatini chegaradagi qiymatlar bilan bog'laydigan formuladir.

1-Misol. Ushbu

$$\oint_{\gamma}\frac{dz}{z^2+4}$$

integralni hisoblang, bunda γ egri chiziq C tekislikning $\pm 2i$ nuqtalaridan o'tmaydigan ixtiyoriy yopiq chiziq.

◃ Faraz qilaylik, γ yopiq chiziq bilan chegaralangan to'plam D bo'lsin.

a) $\pm 2i \notin \overline{D}$ bo'lsin. Bu holda

$$\varphi(z) = \frac{1}{z^2+4} \in O(\overline{D})$$

bo'lib, Koshining integral teoremasiga ko'ra

$$\oint_{\gamma}\varphi(z)dz=\oint_{\gamma}\frac{dz}{z^2+4}=0$$

bo‘ladi.

b) $+2i\notin D; -2i\notin \overline{D}$ bo‘lsin. Bu holda, avvalo integral ostidagi funksiyani

$$\frac{1}{z^2+4}=\frac{1}{(z+2i)(z-2i)}=\frac{\frac{1}{z+2i}}{z-2i}$$

ko‘rinishida yozib olamiz. Unda

$$f(z)=\frac{1}{z+2i}\qquad ,a=2i$$

lar uchun 1-teoremaning shartlari bajarilganligi sababli (10)-formulaga ko‘ra

$$\oint_{\gamma}\frac{dz}{z^2+4}=\oint_{\gamma}\frac{f(z)}{z-2i}dz=2\pi i\cdot f(2i)=\frac{2\pi i}{2i+2i}=\frac{\pi}{2}$$

bo‘ladi.

c) $-2i\in D,\quad 2i\notin \overline{D}$ bo‘lsin. Bunda yuqoridagi b) holdagiga o‘xshash mulohoza yuritish bilan topamiz:

$$\oint_{\gamma}\frac{dz}{z^2+4}=\oint_{\gamma}\frac{\frac{1}{z-2i}}{z+2i}=2\pi i\cdot\frac{1}{z-2i}\Big|_{z=-2i}=-\frac{\pi}{2}.$$

d) $2i\in D,\ -2i\in D$ bo‘lsin. Bu holda, avval integral ostidagi funksiyani sodda kasrlarga ajratamiz:

$$\frac{1}{z^2+4}=\frac{1}{(z+2i)(z-2i)}=\frac{1}{4i}\left(\frac{1}{z-2i}-\frac{1}{z+2i}\right).$$

U holda

$$\oint_{\gamma}\frac{dz}{z^2+4}=\frac{1}{4i}\left[\oint_{\gamma}\frac{dz}{z-2i}-\oint_{\gamma}\frac{dz}{z+2i}\right]=\frac{1}{4i}\cdot 2\pi i(1-1)=0$$

bo‘lishini topamiz $\triangleright$

(10)-formuladagi $\frac{1}{2\pi i}\int_{\partial D}\frac{f(\xi)}{\xi - z}d\xi$ integralga *Koshi integrali* deyiladi. Koshi integralida ∂D kontur soha chegarasi bo‘lib, $f(\xi)$ funksiya D sohada golomorfdir. Endi, faraz qilaylik, C tekislikda ixtiyoriy to‘g‘rilanuvchi Γ kontur va Γ da aniqlangan hamda uzluksiz $f(\xi)$ funksiya berilgan bo‘lsin. U holda ushbu

$$F(z) = \frac{1}{2\pi i}\int_{\Gamma}\frac{f(\xi)}{\xi - z}d\xi$$

integralga *Koshi tipidagi integral* deyiladi.

2-Teorema. *Koshi tipidagi integral* $C\setminus\Gamma$ *sohada* $F(z)$ *funksiyasini aniqlab, bu funksiya ushbu xossalarga egadir:*

a) $F(z)$ *funksiya* $C\setminus\Gamma$ *da golomorf,*

b) $\lim_{z\to\infty} F(z) = 0$,

c) $F(z)$ *funksiyaning istalgan tartibli hosilasi* $F^{(n)}(z)$ *mavjud va*

$$F^{(n)}(z) = \frac{n!}{2\pi i}\int_{\Gamma}\frac{f(\xi)}{(\xi - z)^{n+1}}d\xi$$

tenglik o‘rinli.

Natija. *Golomorf funksiya istalgan tartibli hosilaga egadir*.

Haqiqatan ham, golomorf funksiyani Koshi integrali yordamida ifodalash mumkin. Koshi integralining istalgan tartibli hosilasi mavjudligidan berilgan funksiya ham istalgan tartibli hosilaga ega:

$$f^{(n)}(z) = \frac{n!}{2\pi i}\int_{\partial D}\frac{f(\xi)}{(\xi - z)^{n+1}}d\xi \qquad (11)$$

2-Misol. Ushbu

$$\oint_{\gamma}\frac{e^{2z}}{(z+3)^4}dz$$

integralni hisoblang, bunda γ chiziq C tekislikdagi $z=-3$ nuqtani o‘z ichiga oladigan ixtiyoriy yopiq kontur.

◁ γ kontur bilan chegaralangan sohani D deb belgilaymiz. Ravshanki, $f(z)=e^{2z}$ funksiya va D soha uchun 2-teoremaning shartlari bajariladi. Unda (11)-formuladan foydalanib topamiz:

$$\oint_{\gamma}\frac{e^{2z}}{(z+3)^4}dz=\oint_{\gamma}\frac{f(z)}{(z+3)^4}dz=\frac{2\pi i}{3!}\cdot f'''(-3)=$$

$$=\frac{2\pi i}{6}\cdot 2^3\cdot e^{-6}=\frac{8\pi i}{3e^6}. \qquad ▷$$

4^0**. Darajali qatorlar**.

Ushbu

$$\sum_{n=0}^{\infty}c_n(z-a)^n=c_0+c_1(z-a)+...+c_n(z-a)^n+... \qquad (12)$$

qatorga *darajali qator* deyiladi (bunda $c_0,c_1,...c_n,...$ hamda a-kompleks sonlar).

Agar (12)-qatorda $z-a=\xi$ deyilsa, u holda (12) qator $\sum_{n=0}^{\infty}a_n\xi^n$ ko‘rinishdagi darajali qatorga keladi. Binobarin, shu ko‘rinishdagi qatorlarni o‘rganish biz uchun yetarli bo‘ladi.

1-Teorema. *(Abel teoremasi) Agar*

$$\sum_{n=0}^{\infty}c_nz^n=c_0+c_1z+c_2z^2+...+c_nz^n+... \qquad (13)$$

darajali qator z *ning* $z=z_0\,(z_0\neq 0)$ *qiymatida yaqinlashuvchi bo‘lsa, u holda qator*

$$\{z\in C:|z|<|z_0|\}$$

doirada absolyut yaqinlashuvchi bo‘ladi. Agar (13)-qator z *ning* $z=z_1$ *qiymatida uzoqlashuvchi bo‘lsa, u holda qator*

$$\{z \in C : |z| > |z_1|\}$$

to'plamda uzoqlashuvchi bo'ladi.

Darajali qatorning yaqinlashish sohasi

$$\bigcup = \{z \in C : |z| < r\}$$

doiradan iborat bo'lib, qatorning yaqinlashish radiusi r ushbu

$$r = \frac{1}{\overline{\lim\limits_{n\to\infty}} \sqrt[n]{|c_n|}} \qquad (14)$$

Koshi-Adamar formulasidan topiladi.

(13)-darajali qator o'zining yaqinlashish sohasiga tegishli bo'lgan ixtiyoriy

$$\{z \in C : |z| \le \rho\}, \quad \rho < r$$

yopiq doirada *tekis yaqinlashuvchi* bo'ladi.

Funksiyalarni darajali qatorlarga yoyish qatorlar nazariyasidagi muhim masalalardan hisoblanadi. Bu masala quyidagi teorema yordamida hal etiladi.

2-Teorema. *Agar* $f(z)$ *funksiya* $D \subset C$ *sohada golomorf bo'lsa, u holda* D *sohadagi ixtiyoriy*

$$U = \{z \in C : |z - a| < r\} \qquad (\forall a \in D)$$

doirada $(U \subset D)$ *uni darajali qatorga yoyish mumkin:*

$$f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n . \qquad (15)$$

Bu yerda c_n -koeffitsientlar

$$c_n = \frac{f^{(n)}(a)}{n!} = \frac{1}{2\pi i} \int\limits_{|z-a|=\rho} \frac{f(z)}{(z-a)^{n+1}} dz \;\;, \;\; 0 < \rho < r, \;\; (n = 0,1,2...) \qquad (16)$$

formulalar yordamida hisoblanadi.

Odatda, koeffitsientlari (16)-tengliklar yordamida aniqlanadigan (15)-qatorga *Teylor qatori* deyiladi.

Amaliyotda ko‘pchilik masalalarni hal qilishda elementar funksiyalarning Teylor qatoriga yoyilmalaridan foydalaniladi:

1) $\frac{1}{1-z}=\sum_{n=0}^{\infty} z^n \qquad |z|<1.$

2) $e^z=\sum_{n=0}^{\infty}\frac{z^n}{n!} \qquad |z|\in C.$

3) $\sin z=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{z^{2n-1}}{(2n-1)!}\ , \qquad z\in C.$

4) $\cos z=\sum_{n=0}^{\infty}(-1)^{n}\frac{z^{2n}}{(2n)!}\ , \qquad z\in C.$

5) $shz=\sum_{n=1}^{\infty}\frac{z^{2n-1}}{(2n-1)!}\ , \qquad z\in C.$

6) $chz=\sum_{n=0}^{\infty}\frac{z^{2n}}{(2n)!}\ , \qquad z\in C.$

7) $(1+z)^{\alpha}=1+\sum_{n=1}^{\infty}\frac{\alpha\cdot(\alpha-1)...(\alpha-n+1)}{n!}z^n\ , \qquad |z|<1.$

8) $\ln(1+z)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{z^n}{n}\ , \qquad |z|<1.$

3-Teorema. *Aytaylik,* $U=\{z\in C:|z-a|<r\}$ *doira berilgan bo‘lib,* $f(z)\in O(U)$ *ва* $M=\max\limits_{z\in\partial U}|f(z)|$ *bo‘lsin. U holda* $f(z)$ *funksiyaning a nuqta atrofidagi Teylor qatori*

$$f(z)=\sum_{n=0}^{\infty}c_n\cdot(z-a)^n$$

koeffitsientlari uchun ushbu

$$|c_n|\le\frac{M}{r^n} \qquad (n=0,1,2,...) \qquad (17)$$

Koshi tengsizliklari o‘rinli bo‘ladi.

5^0. **Golomorf funksiyalarning xossalari**.

1-Teorema. *Agar* $f(z)$ *funksiya D sohada golomorf bo'lsa, u holda* $\forall n \in N$ *uchun* $f^{(n)}(z)$ *mavjud va u D sohada golomorf bo'ladi.*

2-Teorema. (Liuvill teoremasi) *Agar* $f(z)$ *funksiya butun tekislik C da golomorf bo'lib, chegaralangan* $(|f(z)| \le M)$ *bo'lsa, u holda* $f(z) \equiv const$ *bo'ladi.*

Faraz qilaylik, $f(z)$ funksiya biror $a \in C$ nuqtaning atrofida golomorf bo'lsin. Agar $f(a)=0$ bo'lsa, a soni $f(z)$ *funksiyaning noli* deyiladi. Agar $f(a)=f'(a)=...=f^{(n-1)}(a)=0$ bo'lib, $f^{(n)}(a)\ne 0$ bo'lsa, a soni $f(z)$ funksiyaning *n-tartibli yo*ki *n karrali noli* deyiladi. Xususan, $n=1$ da *a oddiy nol* deyiladi.

Agar $f(z)$ funksiya $z=\infty$ da golomorf bo'lib, $f(\infty)=0$ bo'lsa, ∞ *nuqta funksiya noli* deyiladi. Funksiyaning bunday nolining tartibi

$$g(z) = f(\frac{1}{z})$$

funksiyaning $z=0$ nuqtadagi noli tartibi bilan aniqlanadi.

3-Teorema. *Agar* $f(z)$ *funksiya* $(f(z)\ne 0)$ $a \in C$ *nuqtaning atrofida golomorf bo'lib, a son funksiyaning n–tartibli noli bo'lsa,*

$$f(z) = (z-a)^n \varphi(z)$$

tenglik o'rinli bo'ladi, bunda $\varphi(z)$ *funksiya a nuqtaning atrofida golomorf va* $\varphi(a)\ne 0.$

4-Teorema. (yagonalik teoremasi) *Aytaylik,* $f(z)$ *va* $g(z)$ *funksiyalar* $D \subset C$ *sohada golomorf bo'lib, kamida bitta limit nuqtaga ega bo'lgan* $E \subset D$ *to'plamda* $f(z)=g(z)$ *bo'lsin. U holda barcha* $z \in D$ *lar uchun* $f(z)\equiv g(z)$ *bo'ladi.*

5-Teorema. (modulning maksimum prinsipi) *Agar* $f(z)$ *funksiya* $D \subset C$ *sohada golomorf bo'lib, uning moduli* $|f|$ *birorta ichki* $z_0 \in D$ *nuqtada (lokal) maksimumga erishsa, u holda* $f(z) \equiv const$ *bo'ladi.*

6^0. **Loran** q**atori**.

Ushbu

$$... + c_{-n} \cdot \frac{1}{(z-a)^n} + c_{-(n-1)} \cdot \frac{1}{(z-a)^{n-1}} + ... + c_{-1} \cdot \frac{1}{z-a} +$$

$$+ c_0 + c_1(z-a) + c_2(z-a)^2 + ... + c_n(z-a)^n + ...$$

ifoda Loran qatori deyiladi va

$$\sum_{n=-\infty}^{+\infty} c_n (z-a)^n$$

kabi belgilanadi. Loran qatori

$$\sum_{n=0}^{\infty} c_n (z-a)^n \qquad (18)$$

va

$$\sum_{n=-1}^{-\infty} c_n (z-a)^n \qquad (19)$$

qatorlar yig'indisi sifatida ifodalanadi. (18)-qatorga *Loran qatorining to'g'ri qismi*, (19) ga esa *bosh qismi* deyiladi.

(18)–darajali qatorning yaqinlashish radiusi

$$R = \frac{1}{\overline{\lim\limits_{n \to \infty}} \sqrt[n]{|c_n|}} \qquad (20)$$

formula yordamida topilib, uning yaqinlashish sohasi

$$\{z \in C : |z-a| < R\}$$

bo'ladi. (19)-qatorning yaqinlashish radiusi

$$r = \overline{\lim_{n \to \infty}} \sqrt[n]{|c_{-n}|} \qquad (21)$$

formula yordamida topiladi va uning yaqinlashish sohasi

$$\{z \in C : |z-a| > r\}$$

bo‘ladi. Berilgan Loran qatorining yaqinlashish sohasi

$$\{z \in C : r < |z-a| < R\}$$

xalqadan iborat bo‘ladi.

Teorema. *Agar* $f(z)$ *funksiya* $U = \{r < |z-a| < R\}$ *xalqada golomorf bo‘lsa, u shu xalqada Loran qatoriga yoyiladi:*

$$\sum_{n=-\infty}^{+\infty} c_n (z-a)^n \qquad (22)$$

Qatorning koeffitsientlari ushbu

$$c_n = \frac{1}{2\pi i} \oint_{|z-a|=\rho} \frac{f(z)}{(z-a)^{n+1}} dz \quad (n = 0, \pm 1, \pm 2, \ldots) \qquad (23)$$

formulalar yordamida topiladi ($r < \rho < R$).

Loran qatorini yaqinlashish sohasida hadlab differensiallash va integrallash mumkin.

7^0. **Funksiyaning yakkalangan maxsus nuqtalari**.

Biror $f(z)$ funksiyani qaraylik. Bu funksiya uchun a nuqtada $(a \in \overline{C})$ golomorflik sharti bajarilmasa a nuqta $f(z)$ funksiyaning *maxsus nuqtasi* deyiladi.

Ta’rif. *Agar a maxsus nuqtaning shunday*

$$\overset{\circ}{U}(a) = \{z \in C : 0 < |z-a| < \varepsilon\}$$

o‘yilgan atrofi topilsaki, $f(z)$ *funksiya* $\overset{\circ}{U}(a)$ *da golomorf bo‘lsa, a nuqta* $f(z)$ *funksiyaning yakkalangan maxsus nuqtasi deyiladi.*

Faraz qilaylik, a nuqta $f(z)$ funksiyaning yakkalangan maxsus nuqtasi bo‘lsin.

1) Agar

$$\lim_{z \to a} f(z) = A$$

(A-chekli son) bo‘lsa, *a* nuqta $f(z)$ funksiyaning *bartaraf qilinadigan maxsus nuqtasi* deyiladi.

2) Agar

$$\lim_{z\to a} f(z) = \infty$$

bo‘lsa, *a* nuqta $f(z)$ funksiyaning *qutb nuqtasi* deyiladi.

3) Agar $z \to a$ da $f(z)$ funksiyaning limiti mavjud bo‘lmasa, *a* nuqta $f(z)$ *funksiyaning o‘ta maxsus nuqtasi* deyiladi.

Eslatma. *A* nuqta $f(z)$ funksiyaning bartaraf qilinadigan maxsus nuqtasi bo‘lsa,

$$f(a) = \lim_{z\to a} f(z)$$

deb olinishi natijasida maxsuslik bartaraf etiladi.

Agar *a* nuqta $f(z)$ funksiyaning qutb nuqtasi bo‘lsa, u holda shu nuqta $\dfrac{1}{f(z)}$ funksiyaning noli bo‘ladi. $\dfrac{1}{f(z)}$ funksiya nolining tartibiga $f(z)$ *funksiya qutbining tartibi* deyiladi.

Endi funksiyaning maxsus nuqtalari bilan uning Loran qatori orasidagi bog‘lanishini ifodalaydigan tasdiqlarni keltiramiz.

1-Teorema. $f(z)$ *funksiyaning yakkalangan maxsus a nuqtasi uning bartaraf qilish mumkin bo‘lgan maxsus nuqtasi bo‘lishi uchun* $f(z)$ *funksiyaning a nuqta atrofida Loran qatoriga yoyilmasida bosh qismining bo‘lmasligi, ya’ni*

$$f(z) = \sum_{n=0}^{\infty} c_n \cdot (z-a)^n$$

bo‘lishi zarur va yetarli.

2-Teorema. $f(z)$ *funksiyaning yakkalangan a nuqtasi uning qutb nuqtasi bo‘lishi uchun* $f(z)$ *funksiyaning a nuqta atrofida Loran qatoriga* *yoyilmasida bosh qism tarkibida chekli sondagi noldan farqli hadlarning bo'lishi, ya'ni*

$$f(z)=\sum_{n=-m}^{\infty}c_n\cdot(z-a)^n \qquad (m>0)$$

bo'lishi zarur va yetarli.

3-Teorema. $f(z)$ *funksiyaning yakkalangan maxsus a nuqtasi uning o'ta maxsus nuqtasi bo'lishi uchun* $f(z)$ *funksiyaning a nuqta atrofida Loran qatoriga yoyilmasida bosh qism tarkibida cheksiz ko'p sondagi noldan farqli hadlarning bo'lishi zarur va yetarli.*

8^0. **Chegirmalar va ularni hisoblash**.

Faraz qilaylik, $f(z)$ funksiya $\{0<|z-a|<\delta\}$ da golomorf bo'lib, a nuqta bu funksiyaning yakkalangan maxsus nuqtasi bo'lsin.

1-Ta'rif. *Ushbu*

$$\frac{1}{2\pi i}\oint_{|z-a|=\rho}f(z)dz \qquad (0<\rho<\delta)$$

integral $f(z)$ *funksiyaning a nuqtadagi chegirmasi deyiladi va* $\underset{z=a}{res}\, f(z)$ *kabi belgilanadi:*

$$\underset{z=a}{res}\, f(z)=\frac{1}{2\pi i}\oint_{|z-a|=\rho}f(z)dz.$$

Ravshanki, $f(z)$ funksiya a nuqtada golomorf bo'lsa, $\underset{z=a}{res}\, f(z)=0$ bo'ladi.

Aytaylik, $f(z)$ funksiya $\{r<|z|<\infty\}$ da golomorf bo'lsin.

2-Ta'rif. *Ushbu*

$$-\frac{1}{2\pi i}\oint_{|z|=\rho}f(z)dz \qquad (\rho>r)$$

integral $f(z)$ *funksiyaning* $z=\infty$ *nuqtadagi chegirmasi deyiladi va* $\underset{z=\infty}{res}\, f(z)$ *kabi belgilanadi:*

$$\underset{z=\infty}{res}\, f(z) = -\frac{1}{2\pi i}\oint_{|z|=\rho} f(z)dz \quad .$$

1-Teorema. *Agar* $f(z)$ *funksiya* $\{0<|z-a|<r\}$ *xalqada Loran qatori*

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-a)^n$$

ga yoyilgan bo'lsa, u holda

$$\underset{z=a}{res}\, f(z) = c_{-1} \qquad (24)$$

bo'ladi. Agar $f(z)$ funksiya $\{r<|z|<\infty\}$ xalqada Loran qatori

$$f(z) = \sum_{n=-\infty}^{\infty} c_n z^n$$

ga yoyilgan bo'lsa, u holda

$$\underset{z=\infty}{res}\, f(z) = -c_{-1} \qquad (25)$$

2-Teorema. (Chegirmalarning yigindisi haqidagi teorema). *Agar* $f(z)$ *funksiya* $C\setminus\{a_1, a_2,..., a_n\}$ *to'plamda golomorf bo'lsa, u holda*

$$\sum_{k=1}^{n} \underset{z=a_k}{res}\, f(z) + \underset{z=\infty}{res}\, f(z) = 0 \qquad (26)$$

bo'ladi.

Endi funksiya chegirmalarini hisoblashda foydalanadigan formulalarni keltiramiz.

1) Agar $z=a$ nuqta $f(z)$ funksiyaning *birinchi tartibli qutb nuqtasi* bo'lsa,

$$\underset{z=a}{res}\, f(z) = \lim_{z\to a}(z-a)\cdot f(z) \qquad (27)$$

bo'ladi.

2) Agar $f(z)=\frac{\varphi(z)}{\psi(z)}$ uchun $\varphi(z)$ *ва* $\psi(z)$ funksiyalar *a* nuqtaga golomorf bo‘lib, $\psi(a)=0,\ \ \psi'(a)\neq 0$ bo‘lsa , u holda

$$\underset{z=a}{res}\, f(z)=\frac{\varphi(a)}{\psi'(a)} \qquad (28)$$

bo‘ladi.

3) Agar $z=a$ nuqta $f(z)$ funksiyaning *n-tartibli qutb nuqtasi* bo‘lsa,

$$\underset{z=a}{res}\, f(z)=\frac{1}{(n-1)!}\cdot\lim_{z\to a}\frac{d^{n-1}[(z-a)^n f(z)]}{dz^{n-1}} \qquad (29)$$

bo‘ladi.

4) Agar $z=\infty$ nuqtada $f(z)$ funksiya golomorf bo‘lsa,

$$\underset{z=\infty}{res}\, f(z)=\lim_{z\to\infty} z[f(\infty)-f(z)] \qquad (30)$$

bo‘ladi.

5) Agar $f(z)=\varphi(\frac{1}{z})$ bo‘lib, $\varphi(z)$ funksiya $z=0$ nuqtada golomorf bo‘lsa,

$$\underset{z=\infty}{res}\, f(z)=-\varphi'(0) \qquad (31)$$

bo‘ladi.

9^0. **Integrallarni chegirmalar yordamida hisoblash**.

Chegirmalar yordamida turli integrallarni hisoblash mumkin. Bunda quyidagi teorema muhim rol o‘ynaydi.

Teorema (Koshi teoremasi). *Faraz qilaylik ,*

1) $f(z)$ *funksiya* $D\setminus\{a_1,a_2,...a_n\}$ *sohada golomorf* $(D\subset C,\ \ a_1,a_2,...,a_n\in D)$,

2) $f(z)$ *funksiya sohaning chegarasigacha aniqlangan va* $\overline{D}\setminus\{a_1,a_2,...a_n\}$ *da uzluksiz,*

3) ∂D *- to‘g‘rilanuvchi yopiq kontur bo‘lsin. U holda*

$$\int_{\partial D} f(z)dz = 2\pi i \sum_{k=1}^{n} \underset{z=a_k}{res} f(z) \qquad (32)$$

formula o'rinlidir.

Izoh. (32)-formula $\infty \in D$ bo'lgan hol uchun ham o'rinlidir. Faqat bu holda $z=\infty$ ni $f(z)$ uchun maxsus nuqta deb hisoblash hamda ∂D chiziq orientatsiyasini soat strelkasi yo'nalishida olish kifoyadir.

Yuqorida keltirilgan Koshi teoremasidan amaliyotda yopiq kontur bo'yicha olingan integrallarni hisoblashda foydalaniladi.

10⁰. Aniq integrallarni chegirmalar yordamida hisoblash.

Aniq integrallarni ham chegirmalar yordamida hisoblash mumkin. Bunda aniq integral kompleks o'zgaruvchili funksiyaning kontur bo'yicha olingan integraliga keltirilib hisoblanadi.

a) $\int_0^{2\pi} R(\cos x, \sin x)dx$ **ko'rinishdagi integrallarni hisoblash.**

Ushbu

$$I = \int_0^{2\pi} R(\cos x, \sin x)dx \qquad (33)$$

integral berilgan bo'lib, uni hisoblash talab etilsin, bunda $R(\cos x, \sin x) - \cos x$ *va* $\sin x$ larning ratsional funksiyasi va u $[0, 2\pi]$ da uzluksiz.

Eyler formulasiga ko'ra

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

bo'lishini e'tiborga olib, so'ng

$$z = e^{ix}$$

deb belgilash kiritsak, unda

$$x \in [0,2\pi] \Rightarrow z \in \{\ z \in C : |z| = 1\},$$

$$\cos x = \frac{1}{2}(z + \frac{1}{z}) \ , \quad \sin x = \frac{1}{2i}(z - \frac{1}{z}), \ dx = \frac{1}{iz}dz$$

bo‘lib, berilgan (33)-integral quyidagicha

$$I = \int_0^{2\pi} R(\cos x, \sin x)dx = \oint_{|z|=1} \tilde{R}(z)dz$$

bo‘ladi, bunda

$$\tilde{R}(z) = \frac{1}{iz} R(\frac{1}{2}(z + \frac{1}{z}), \frac{1}{2i}(z - \frac{1}{z})).$$

Hosil bo‘lgan integral oldingi punktdagi (32)-formula yordamida hisoblanadi.

b) Xosmas integrallarni hisoblash.

Chegirmalar nazariyasidan foydalanib xosmas integrallarni ham hisoblash mumkin. Bu quyidagi teoremaga asoslangan.

Teorema. $f(z)$ *funksiya* $\{z \in C : \operatorname{Im} z > 0\}$ *sohaning chekli sondagi maxsus nuqtalaridan tashqari barcha nuqtalarida golomorf bo‘lib, uning chegarasida uzluksiz bo‘lsin. Agar*

$$\lim_{r \to \infty} \int_{\gamma_r} f(z)dz = 0 \qquad (\gamma_r = \{|z| = r \ , \ \ 0 \le \arg z \le \pi\}) \qquad (34)$$

bo‘lsa, u holda $\int_{-\infty}^{+\infty} f(x)dx$ *yaqinlashuvchi bo‘lib,*

$$\int_{-\infty}^{+\infty} f(x)dx = 2\pi i \sum_{\operatorname{Im} z_k \succ 0} \underset{z=z_k}{res} f(z) \qquad (35)$$

bo‘ladi.

Bu teoremadagi (34)-shartning bajarilishini ko‘rsatishda quyidagi lemmalardan foydaniladi.

1-Lemma (Jordan lemmasi). *Agar*

$$\lim_{r \to \infty} r \max_{z \in \gamma_r} |f(z)| = 0 \qquad (36)$$

bo‘lsa,

$$\lim_{r\to\infty}\int_{\gamma_r} f(z)dz = 0 \qquad (37)$$

bo'ladi.

2-Lemma.(Jordan lemmasi). *Agar*

$$\lim_{r\to\infty}\max_{z\in\gamma_r}|f(z)| = 0 \qquad (38)$$

bo'lsa, u holda $\forall\lambda > 0$ *uchun*

$$\lim_{r\to\infty}\int_{\gamma_r} f(z)e^{i\lambda z}dz = 0 \qquad (39)$$

bo'ladi.

Endi

$$\int_{-\infty}^{+\infty} e^{i\lambda x}R(x)dx$$

ko'rinishdagi xosmas integrallarni qaraylik.

Agar $\lim\limits_{r\to\infty}\max\limits_{z\in\gamma}|R(z)| = 0$ bo'lsa, u holda bu integralga 2-lemmani va yuqoridagi teoremani qo'llash natijasida quyidagi formulalarni hosil qilamiz:

$$\int_{-\infty}^{+\infty} R(x)\cos\lambda x dx = -2\pi\cdot\mathrm{Im}\left\{\sum_{\mathrm{Im}\, z_k \succ 0} \underset{z=z_k}{res}[e^{i\lambda z}\cdot R(z)]\right\}, \qquad (40)$$

$$\int_{-\infty}^{+\infty} R(x)\sin\lambda x dx = 2\pi\cdot\mathrm{Re}\left\{\sum_{\mathrm{Im}\, z_k \succ 0} \underset{z=z_k}{res}[e^{i\lambda z}\cdot R(z)]\right\}, \qquad (41)$$

Misol. Ushbu

$$\int_{-\infty}^{+\infty}\frac{\sin x}{x^2-2x+2}dx$$

integralni hisoblang.

◁ $f(z)$ funksiya deb

$$f(z) = \frac{e^{iz}}{z^2-2z+2} = \frac{e^{iz}}{[z-(1+i)]\cdot[z-(1-i)]}$$

ni olamiz. Bu funksiyaning 2 ta $z_1 = 1+i$ va $z_2 = 1-i$ qutb nuqtalari bo'lib, ulardan $z_1 = 1+i \in \{\mathrm{Im}\, z > 0\}$ bo'ladi.

$R(z) = \dfrac{1}{z^2 - 2z + 2}$ funksiya uchun $z \to \infty$ da $R(z) \sim \dfrac{1}{z^2}$ bo‘lganidan 2-lemma shartining bajarilishi ta’minlanadi. Unda (41)-formulaga ko‘ra

$$\int_{-\infty}^{+\infty} \frac{\sin x}{x^2 - 2x + 2} dx = 2\pi \cdot \mathrm{Re}[\operatorname*{res}_{z=z_1} f(z)]$$

bo‘ladi.

(27)-formuladan foydalanib $\operatorname*{res}_{z=z_1} f(z)$ ni hisoblaymiz:

$$\operatorname*{res}_{z=z_1} f(z) = \lim_{z \to 1+i} \left\{ \frac{e^{iz}}{[z-(1+i)] \cdot [z-(1-i)]} \cdot [z-(1+i)] \right\} =$$

$$= \frac{e^{i(1+i)}}{2i} = \frac{e^{-1}}{2}(\sin 1 - i \cos 1).$$

Demak,

$$\int_{-\infty}^{+\infty} \frac{\sin x}{x^2 - 2x + 2} dx = 2\pi \cdot \mathrm{Re}\left[\frac{e^{-1}}{2}(\sin 1 - i \cos 1) \right] = \pi e^{-1} \sin 1. \qquad \triangleright$$

Nazorat savollari.

1. Kompleks argumentli funksiya integralining ta’rifi.
2. Integral mavjud bo‘lishining zaruriy va yetarli shartlari.
3. Koshining integral teoremasi.
4. Ko‘p bog‘lamli soha uchun Koshi teoremasi.
5. Kompleks argumentli funksiya integralining integrallash yo‘liga bog‘liq bo‘lmasligi.
6. Nyuton–Leybnits formulasi.
7. Koshining integral formulasi.
8. Koshi integrali va Koshi tipidagi integral.
9. Darajali qatorlar va ularning xossalari.
10. Elementar funksiyalarning darajali qatorga yoyilmalari.
11. Liuvill teoremasi.
12. Golomorf funksiyaning nollari.

13. Yagonalik teoremasi.
14. Modulning maksimum prinsipi.
15. Loran qatorlari va ularning xossalari.
16. Funksiyaning yakkalangan maxsus nuqtalari.
17. Yakkalangan maxsus nuqtalar va Loran qatori orasidagi bog‘lanish.
18. Chegirmaning ta’rifi va chegirma bilan Loran qatorining koeffitsientlari orasidagi bog‘lanish.
19. Chegirmalarning yig‘indisi haqidagi teorema.
20. Chegirmalarni hisoblash formulalari.
21. Kompleks argumentli funksiyalardan yopiq kontur bo‘yicha olingan integrallarni chegirmalar yordamida hisoblash.
22. $\int_0^{2\pi} R(\cos x, \sin x)dx$ ko‘rinishidagi integrallarni hisoblash.
23. $\int_{-\infty}^{+\infty} f(x)dx$ ko‘rinishidagi integrallarni hisoblash.
24. Jordan lemmalari.
25. $\int_{-\infty}^{+\infty} R(x)\cos\lambda x dx$ ko‘rinishidagi integrallarni hisoblash.
26. $\int_{-\infty}^{+\infty} R(x)\sin\lambda x dx$ ko‘rinishidagi integrallarni hisoblash.

- B -

MUSTAQIL YECHISH UCHUN MISOL VA MASALALAR

1-Masala. Boshi $a(a \in C)$ oxiri $b(b \in C)$ nuqtada bo‘lgan γ to‘g‘ri chiziq kesmasi bo‘yicha quyidagi integrallarni ta’rif yordamada hisoblang.

1.1. $\int_\gamma (3z+1)dz, \quad a = 1+i, \quad b = 1-i.$

1.2. $\int_\gamma (z-i)dz, \quad a = 1+i, \quad b = 1+2i.$

1.3. $\int_\gamma (z+i)dz, \quad a = 1+i, \quad b = i.$

1.4. $\int_\gamma (3z - i)dz, \quad a = 1 + i, \quad b = -1 - i.$

1.5. $\int_\gamma (3z + i)dz, \quad a = 2i, \quad b = 1 - i.$

1.6. $\int_\gamma (z + 2i)dz, \quad a = 2i, \quad b = 1 + i.$

1.7. $\int_\gamma (z - 2i)dz, \quad a = 2i, \quad b = -1 - i.$

1.8. $\int_\gamma (z - 2)dz, \quad a = 2, \quad b = 1 + i.$

1.9. $\int_\gamma (z + 2)dz, \quad a = 2, \quad b = 1 - i.$

1.10. $\int_\gamma (3z - 1)dz, \quad a = 2, \quad b = -1 + i.$

1.11. $\int_\gamma (3z - 2)dz, \quad a = 2, \quad b = -1 - i.$

1.12. $\int_\gamma (z + 3)dz, \quad a = 1 + i, \quad b = i.$

1.13. $\int_\gamma (z - 3)dz, \quad a = 1 + i, \quad b = -i.$

1.14. $\int_\gamma (z - 3i)dz, \quad a = 1 - i, \quad b = i.$

1.15. $\int_\gamma (z + 3i)dz, \quad a = 1 - i, \quad b = -i.$

1.16. $\int_\gamma (2z - 3)dz, \quad a = -1 + i, \quad b = i.$

1.17. $\int_\gamma (2z + 3)dz, \quad a = -1 + i, \quad b = -i.$

1.18. $\int_\gamma (2z - 3i)dz, \quad a = -1 - i, \quad b = i.$

1.19. $\int_\gamma (2z + 3i)dz, \quad a = -1 - i, \quad b = -i.$

1.20. $\int\limits_{\gamma}(3z-i)dz,\qquad a=2+i,\quad b=2-i.$

1.21. $\int\limits_{\gamma}(2z-1)dz,\qquad a=2+2i,\quad b=i.$

2-Masala. Quyidagi integrallarni berilgan z_0 va z_1 nuqtalarni tutashtiruvchi γ to‘g‘ri chiziq bo‘yicha hisoblang.

2.1. $\int\limits_{\gamma}(x+iy^2)dz,\qquad z_0=1+i,\quad z_1=2+3i.$

2.2. $\int\limits_{\gamma}(x^2+iy^2)dz,\qquad z_0=2+2i,\quad z_1=3+4i.$

2.3. $\int\limits_{\gamma}(x^2+iy)dz,\qquad z_0=1+i,\quad z_1=2+3i.$

2.4. $\int\limits_{\gamma}(x+iy^2)dz,\qquad z_0=2+2i,\quad z_1=3+4i.$

2.5. $\int\limits_{\gamma}(x^2-iy^2)dz,\qquad z_0=1+i,\quad z_1=2+3i.$

2.6. $\int\limits_{\gamma}(x^2+iy^2)dz,\qquad z_0=2+2i,\quad z_1=3+4i.$

2.7. $\int\limits_{\gamma}zdz,\qquad z_0=1+i,\quad z_1=2+3i.$

2.8. $\int\limits_{\gamma}(x^2-iy^2)dz,\qquad z_0=2+2i,\quad z_1=3+4i.$

2.9. $\int\limits_{\gamma}\overline{z}dz,\qquad z_0=1+i,\quad z_1=2+3i.$

2.10. $\int\limits_{\gamma}\overline{z}dz,\qquad z_0=2+2i,\quad z_1=3+4i.$

2.11. $\int\limits_{\gamma}(x^2+iy^2)dz,\qquad z_0=1+i,\quad z_1=3+2i.$

2.12. $\int\limits_{\gamma}(x^2+iy^2)dz,\qquad z_0=2+2i,\quad z_1=4+3i.$

2.13. $\int\limits_{\gamma}(x+iy^2)dz,\qquad z_0=1+i,\quad z_1=3+2i.$

2.14. $\int\limits_{\gamma}(x^2 - iy^2)dz, \quad z_0 = 2+2i, \ z_1 = 4+3i.$

2.15. $\int\limits_{\gamma}(x^2 + iy)dz, \quad z_0 = 1+i, \ z_1 = 3+2i.$

2.16. $\int\limits_{\gamma}(x + iy^2)dz, \quad z_0 = 1+2i, \ z_1 = 3+4i.$

2.17. $\int\limits_{\gamma}(x^2 - iy^2)dz, \quad z_0 = 1+i, \ z_1 = 3+2i.$

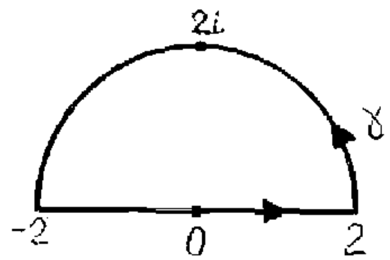
2.18. $\int\limits_{\gamma}(x^2 + iy)dz, \quad z_0 = 1+2i, \ z_1 = 3+4i.$

2.19. $\int\limits_{\gamma} zdz, \quad z_0 = 1+i, \ z_1 = 3+2i.$

2.20. $\int\limits_{\gamma}(x^2 - iy)dz, \quad z_0 = 1+2i, \ z_1 = 3+4i.$

2.21. $\int\limits_{\gamma}(x^2 + iy^2)dz, \quad z_0 = 1+i, \ z_1 = 2+3i.$

3-Masala. 88-chizmada tasvirlangan γ chiziq bo‘yicha olingan quyidagi integrallarni hisoblang.



88-chizma

3.1. $\oint\limits_{\gamma}\frac{\bar{z}}{z}dz.$ **3.2.** $\oint\limits_{\gamma}\frac{2z-\bar{z}}{\bar{z}}dz.$

3.3. $\oint\limits_{\gamma}\frac{2\bar{z}+z}{z}dz.$ **3.4.** $\oint\limits_{\gamma}\frac{3z-\bar{z}}{\bar{z}}dz.$

3.5. $\oint\limits_{\gamma}\frac{3\bar{z}+z}{z}dz.$ **3.6.** $\oint\limits_{\gamma}\frac{3\bar{z}-2z}{z}dz.$

3.7. $\oint\limits_{\gamma} \frac{2z - 3\bar{z}}{\bar{z}} dz.$ **3.8.** $\oint\limits_{\gamma} \frac{3\bar{z} - z}{z} dz.$

3.9. $\oint\limits_{\gamma} \frac{2\bar{z} + 3z}{z} dz.$ **3.10.** $\oint\limits_{\gamma} \frac{5z - 6\bar{z}}{\bar{z}} dz.$

3.11. $\oint\limits_{\gamma} \frac{6\bar{z} - 5z}{z} dz.$ **3.12.** $\oint\limits_{\gamma} \frac{4z - 5\bar{z}}{\bar{z}} dz.$

3.13. $\oint\limits_{\gamma} \frac{4\bar{z} + 5z}{z} dz.$ **3.14.** $\oint\limits_{\gamma} \frac{5z - 4\bar{z}}{\bar{z}} dz.$

3.15. $\oint\limits_{\gamma} \frac{5\bar{z} + 4}{z} dz.$ **3.16.** $\oint\limits_{\gamma} \frac{7z - 8\bar{z}}{\bar{z}} dz.$

3.17. $\oint\limits_{\gamma} \frac{7\bar{z} + 8z}{z} dz.$ **3.18.** $\oint\limits_{\gamma} \frac{8z - 5\bar{z}}{\bar{z}} dz.$

3.19. $\oint\limits_{\gamma} \frac{6\bar{z} - 7z}{z} dz.$ **3.20.** $\oint\limits_{\gamma} \frac{6z + 7\bar{z}}{\bar{z}} dz.$

3.21. $\oint\limits_{\gamma} \frac{7z + 6\bar{z}}{\bar{z}} dz.$

4-Masala. Agar $\gamma : x = a\cos t,\ y = b\sin t,\ 0 < t \le 2\pi,$ ellips bo‘lsa, quyidagi integrallar hisoblansin.

4.1. $\int\limits_{\gamma} y dz, \quad a = 2, \quad b = 3.$

4.2. $\int\limits_{\gamma} z dz, \quad a = 2, \quad b = 3.$

4.3. $\int\limits_{\gamma} \bar{z} dz, \quad a = 2, \quad b = 3.$

4.4. $\int\limits_{\gamma} (2x - iy) dz, \quad a = 2, \quad b = 3.$

4.5. $\int\limits_{\gamma} (x - iy) dz, \quad a = 2, \quad b = 3.$

4.6. $\int_{\gamma} x^2 dz, \quad a=3, \quad b=2.$

4.7. $\int_{\gamma} y^2 dz, \quad a=3, \quad b=2.$

4.8. $\int_{\gamma} (x^2 - iy)dz, \quad a=3, \quad b=2.$

4.9. $\int_{\gamma} (x - iy^2)dz, \quad a=3, \quad b=2.$

4.10. $\int_{\gamma} (x + i\cdot 2y)dz, \; a=3, \; b=2.$

4.11. $\int_{\gamma} (2x + iy)dz, \quad a=3, \quad b=2.$

4.12. $\int_{\gamma} (x - i\cdot 2y)dz, \; a=2, \; b=3.$

4.13. $\int_{\gamma} (3x - iy)dz, \quad a=3, \quad b=2.$

4.14. $\int_{\gamma} (x - i3y)dz, \quad a=3, \; b=2.$

4.15. $\int_{\gamma} (3x + iy)dz, \quad a=2, \quad b=3.$

4.16. $\int_{\gamma} (x + i3y)dz, \quad a=2, \; b=3.$

4.17. $\int_{\gamma} (3x - 2iy)dz, \; a=3, \; b=2.$

4.18. $\int_{\gamma} (2x - 3iy)dz, \quad a=3, \; b=2.$

4.19. $\int_{\gamma} (3x + 2iy)dz, \; a=2, b=3.$

4.20. $\int_{\gamma} (2x + 3iy)dz, \quad a=2, \; b=3.$

4.21. $\int_{\gamma} (4x + 3iy)dz, \; a=3, \; b=2.$

5-Masala. Quyidagi integrallarni hisoblang.

5.1. $\int_{-3}^{-3+i} zdz$. **5.2.** $\int_{i}^{2+i} z^2dz$. **5.3.** $\int_{1}^{1+i} zdz$.

5.4. $\int_{3i}^{1+3i} zdz$. **5.5.** $\int_{3}^{3+i} zdz$. **5.6.** $\int_{-2i}^{1-2i} zdz$.

5.7. $\int_{-2}^{1-2i} z^2dz$. **5.8.** $\int_{2}^{2+i} zdz$. **5.9.** $\int_{2}^{2+i} z^2dz$.

5.10. $\int_{1+2i}^{2+i} zdz$. **5.11.** $\int_{1+i}^{2+i} z^2dz$. **5.12.** $\int_{-2}^{-2+3i} zdz$.

5.13. $\int_{-1+2i}^{2+2i} zdz$. **5.14.** $\int_{-1+2i}^{2+2i} z^2dz$. **5.15.** $\int_{-2+i}^{1+i} zdz$.

5.16. $\int_{-2+i}^{1+i} zdz$. **5.17**. $\int_{3-2i}^{3+i} zdz$. **5.18.** $\int_{3-2i}^{3+i} z^2dz$.

5.19. $\int_{-3-i}^{4-i} zdz$. **5.20.** $\int_{-3-i}^{4-i} z^2dz$. **5.21.** $\int_{-2+i}^{1+i} z^2dz$.

6-Masala. Koshining integral formulasidan foydalanib quyidagi integrallarni hisoblang.

6.1. $\int\limits_{|z-1|=3} \frac{e^z dz}{(z-1)(z+3)(z+i)}$. **6.2.** $\int\limits_{|z+1|=3} \frac{e^z dz}{(z-3)(z+3)(z+i)}$.

6.3. $\int\limits_{|z-1|=2} \frac{\sin z}{(z^2+1)(z-2i)} dz$. **6.4.** $\int\limits_{|z-1|=2} \frac{e^z}{(z-1)(z-2)(z+2i)} dz$.

6.5. $\int\limits_{|z|=2,5} \frac{\sin z}{(z-3i)(z^2-5z+6)} dz$. **6.6.** $\int\limits_{|z-1|=2} \frac{e^z}{(z+i)(z+2)(z+2i)} dz$.

6.7. $\int\limits_{|z|=2} \frac{\cos z}{(z-i)(z+1)(z+3)} dz$ **6.8.** $\int\limits_{|z|=3} \frac{\sin z}{(z-2)(z+i)(z+4i)} dz$.

6.9. $\int\limits_{|z|=2,5} \frac{e^z dz}{(z-3i)(z^2+3z+1)}$. **6.10.** $\int\limits_{|z-i|=2} \frac{e^z}{z(z-2i)(z+2i)} dz$.

6.11. $\int\limits_{|z-i|=3} \frac{\sin z}{(z+i)(z-2i)(z+3)} dz$. **6.12.** $\int\limits_{|z-i|=2,5} \frac{\cos z}{z(z+i)(z+2i)} dz$.

6.13. $\oint\limits_{|z-i|=2} \frac{e^z}{(z-1)(z-2i)(z+2i)}dz$. **6.14.** $\oint\limits_{|z-i|=2} \frac{e^z}{z(z+1)(z+2)}dz$.

6.15. $\oint\limits_{|z-i|=2} \frac{\sin z}{(z-1)(z-2i)(z+3i)}dz$. **6.16.** $\oint\limits_{|z-i|=2} \frac{\cos z}{(z+1)(z-i)(z-2)}dz$.

6.17. $\oint\limits_{|z|=3} \frac{e^z}{(z-4i)(z-2i)(z-i)}dz$. **6.18.** $\oint\limits_{|z|=3} \frac{e^z}{(z^2+4)(z-5i)}dz$.

6.19. $\oint\limits_{|z|=3} \frac{\sin z}{(z^2-4)(z+4)}dz$. **6.20.** $\oint\limits_{|z|=3} \frac{\cos z}{(z-2)(z+2i)(z+4i)}dz$.

6.21. $\oint\limits_{|z-2|=5} \frac{e^{z^2}}{(z+4)(z^2-6z)}dz$.

7-Masala. Koshining integral formulasidan foydalanib quyidagi integrallarni hisoblang.

7.1. $\oint\limits_{|z-1|=2} \frac{z+1}{(z-1)^3(z+2)^2}dz$. **7.2.** $\oint\limits_{|z+1|=3} \frac{z-1}{(z-3)^2\cdot(z+i)^3}dz$.

7.3. $\oint\limits_{|z-1|=2} \frac{z+2}{z^2\cdot(z^2+1)}dz$. **7.4.** $\oint\limits_{|z-1|=2} \frac{z-2}{(z+i)^3\cdot(z+2)^2}dz$.

7.5. $\oint\limits_{|z|=2,5} \frac{z-1}{(z-2)^3\cdot(z-3)}dz$. **7.6.** $\oint\limits_{|z-1|=2} \frac{z+2}{z(z-1)^3\cdot(z-2)^2}dz$.

7.7. $\oint\limits_{|z|=2} \frac{z-1}{(z-i)^3\cdot(z+1)^2}dz$. **7.8.** $\oint\limits_{|z-1|=2} \frac{z+1}{(z-2)^2\cdot(z+i)^3}dz$.

7.9. $\oint\limits_{|z|=2,5} \frac{z-1}{(z+2)^2\cdot(z+1)^3}dz$. **7.10.** $\oint\limits_{|z-i|=2} \frac{z+1}{z^3\cdot(z-2i)^2}dz$.

7.11. $\oint\limits_{|z-i|=3} \frac{z+1}{(z+i)^3(z-2i)^2}dz$. **7.12.** $\oint\limits_{|z-i|=4} \frac{z-1}{z^3(z+i)^2}dz$.

7.13. $\oint\limits_{|z-i|=2} \frac{z+1}{(z-1)^3\cdot(z-2i)^2}dz$. **7.14.** $\oint\limits_{|z-i|=2} \frac{z-1}{(z+1)^3\cdot z^2}dz$.

7.15. $\oint\limits_{|z-i|=2} \frac{z-1}{(z+1)^3\cdot(z-2i)^2}dz$. **7.16.** $\oint\limits_{|z-i|=2} \frac{z-1}{(z+1)^3\cdot(z-i)}dz$.

7.17. $\int\limits_{|z|=3} \frac{z+1}{(z-2i)\cdot(z-i)^3}dz.$ **7.18.** $\int\limits_{|z|=3} \frac{z+1}{(z+2)^3\cdot(z-2)^2}dz.$

7.19. $\int\limits_{|z|=3} \frac{z+1}{(z-2i)^3\cdot(z+i)^2}dz.$ **7.20.** $\int\limits_{|z|=3} \frac{z+1}{(z-2)^3\cdot(z+2i)^2}dz.$

7.21. $\int\limits_{|z|=2} \frac{z+1}{z(z-1)^3\cdot(z-3)}dz.$

8-Masala. Quyidagi misollarda berilgan $f(z)$ funksiyani $z=a$ nuqtaning atrofida Loran qatoriga yoying va qatorning yaqinlashish sohasini toping.

8.1. $f(z)=\frac{1}{(z-1)(z+i)}, \quad a=-i.$

8.2. $f(z)=\frac{1}{(z-1)(z-2)}, \quad a=2.$

8.3. $f(z)=\frac{1}{(z-i)(z+i)}, \quad a=i.$

8.4. $f(z)=\frac{1}{(z+i)(z+2)}, \quad a=-2.$

8.5. $f(z)=\frac{1}{(z-2)(z-3)}, \quad a=2.$

8.6. $f(z)=\frac{1}{(z-3)(z-i)}, \quad a=i.$

8.7. $f(z)=\frac{1}{(z+i)(z+1)}, \quad a=-1.$

8.8. $f(z)=\frac{1}{(z-2)(z+i)}, \quad a=2.$

8.9. $f(z)=\frac{1}{(z+2)(z+1)}, \quad a=-1.$

8.10. $f(z)=\frac{1}{z(z-2i)}, \quad a=0.$

8.11. $f(z)=\dfrac{1}{(z+i)(z-2i)}, \qquad a=2i.$

8.12. $f(z)=\dfrac{1}{z(z+i)}, \qquad a=0.$

8.13. $f(z)=\dfrac{1}{(z-1)(z-2i)}, \qquad a=1.$

8.14. $f(z)=\dfrac{1}{z(z+1)}, \qquad a=0.$

8.15. $f(z)=\dfrac{1}{(z+1)(z-2i)}, \qquad a=-1.$

8.16. $f(z)=\dfrac{1}{(z+1)(z-i)}, \qquad a=i.$

8.17. $f(z)=\dfrac{1}{(z-2i)(z-i)}, \qquad a=i.$

8.18. $f(z)=\dfrac{1}{(z-2i)(z+2i)}, \qquad a=2i.$

8.19. $f(z)=\dfrac{1}{z^2+4}, \qquad a=2.$

8.20. $f(z)=\dfrac{1}{z^2-2(1-i)z-4i}, \qquad a=-2i.$

8.21. $f(z)=\dfrac{1}{z^2-3iz-2}, \qquad a=2i.$

9-Masala. Quyidagi misollarda $f(z)$ funksiyani ko‘rsatilgan xalqada Loran qatoriga yoying.

9.1. $f(z)=z^2\cdot e^{\frac{1}{z}}, \qquad V=\{0<|z|<\infty\}.$

9.2. $f(z)=\dfrac{1}{(z-1)(z-2)}, \; V=\{0<|z|<1\}.$

9.3. $f(z)=\dfrac{1}{(z-1)(z-2)}, \; V=\{2<|z|<\infty\}.$

9.4. $f(z)=\dfrac{1}{z(z-2)}$, $V=\{0<|z|<2\}$.

9.5. $f(z)=\dfrac{1}{1-z^2}$, $V=\{2<|z-1|<\infty\}$.

9.6. $f(z)=\dfrac{1}{(z-2)(z-3)}$, $V=\{2<|z|<3\}$.

9.7. $f(z)=\dfrac{1}{(z-1)(z-3)}$, $V=\{1<|z|<3\}$.

9.8. $f(z)=\dfrac{1}{z+z^2}$, $V=\{0<|z|<1\}$.

9.9. $f(z)=\dfrac{2}{z^2-1}$, $V=\{1<|z+2|<3\}$.

9.10. $f(z)=\dfrac{1}{1+z^2}$, $V=\{0<|z-i|<2\}$.

9.11. $f(z)=\dfrac{1}{1+z^2}$, $V=\{0<|z+i|<2\}$.

9.12. $f(z)=\dfrac{z+2}{z^2-4z+3}$, $V=\{2<|z-1|<\infty\}$.

9.13. $f(z)=\dfrac{1}{z^2-4z+3}$, $V=\{2<|z-1|<\infty\}$.

9.14. $f(z)=\dfrac{1}{z^2+3z+2}$, $V=\{1<|z|<2\}$.

9.15. $f(z)=\dfrac{2z+3}{z^2+3z+2}$, $V=\{1<|z|<2\}$.

9.16. $f(z)=\dfrac{z^2-z+3}{z^2-3z+2}$, $V=\{1<|z|<2\}$.

9.17. $f(z)=\dfrac{1}{z^2-4}$, $V=\{4<|z+2|<\infty\}$.

9.18. $f(z)=\dfrac{2z+1}{z^2+z-2}$, $V=\{2<|z|<\infty\}$.

9.19. $f(z)=\dfrac{1}{(z-1)(z+2)},\ V=\{1<|z|<2\}.$

9.20. $f(z)=\dfrac{1}{(z+1)(z+2)},\ V=\{1<|z|<2\}.$

9.21. $f(z)=\dfrac{2z-3}{z^2-3z+2},\ V=\{0<|z-2|<1\}.$

10-Masala. Quyidagi funksiyalarning barcha maxsus nuqtalarini toping, ularning xarakterini aniqlang va funksiyalarni $z=\infty$ nuqtada tekshiring (qutblar uchun ularning tartibini ko‘rsating).

10.1. $f(z)=ctgz-\dfrac{1}{z}.$

10.2. $f(z)=\dfrac{\cos z}{z^2(z^2+1)^2}.$

10.3. $f(z)=\dfrac{z^2}{\sin z-1}.$

10.4. $f(z)=\cos\dfrac{1}{1-z}.$

10.5. $f(z)=\dfrac{z^7}{(z^2-1)^2\cos\dfrac{1}{z-1}}.$

10.6. $f(z)=\dfrac{1}{\sin z}-\dfrac{1}{z}.$

10.7. $f(z)=\sin\dfrac{1}{z}+\dfrac{1}{z^2}.$

10.8. $f(z)=e^{-\frac{1}{z^2}}.$

10.9. $f(z)=\dfrac{1}{e^z-1}-\dfrac{1}{z}.$

10.10. $f(z)=\dfrac{e^z}{z\cdot(1-e^{-z})}.$

10.11. $f(z)=\dfrac{1}{(z+1)^2}\cdot e^{\frac{1}{z+1}}.$

10.12. $f(z)=\dfrac{e^{\frac{1}{z-1}}}{e^z-1}.$

10.13. $f(z)=\dfrac{z^2+9}{e^z}.$

10.14. $f(z)=e^{\frac{2z}{2-z}}.$

10.15. $f(z)=\dfrac{2}{(z^2-i)^3}.$

10.16. $f(z)=\dfrac{e^z}{4+z^2}.$

10.17. $f(z)=tg2z.$

10.18. $f(z)=\sin\dfrac{1}{z+i}.$

10.19. $f(z)=\dfrac{2z+3}{(z-1)^3 z\cdot(z+1)}$. **10.20.** $f(z)=e^{\frac{2}{z+3i}}$.

10.21. $f(z)=\dfrac{1}{z^3\cdot(2-\cos z)}$.

11-Masala. Quyidagi funksiyalarning barcha maxsus nuqtalaridagi va $z=\infty$ nuqtadagi chegirmalarini hisoblang (bunda $z=\infty$ nuqta maxsus nuqtalarning limit nuqtasi bo‘lmagan hol qaralsin).

11.1. $f(z)=\dfrac{\sin z}{z^3(z+4)}$. **11.2.** $f(z)=\dfrac{\sin z}{2z^2-\frac{\pi}{2}z}$.

11.3. $f(z)=\dfrac{z^2}{(1+z)^3}$. **11.4.** $f(z)=z^3\cdot\cos\dfrac{1}{z-2}$.

11.5. $f(z)=\dfrac{e^z}{z^3(z-1)}$. **11.6.** $f(z)=z^2\cdot e^{\frac{2}{z}}$**.**

11.7. $f(z)=\dfrac{tgz}{z^2-\frac{\pi z}{4}}$. **11.8.** $f(z)=\dfrac{\sin \pi z}{(z-1)^2}$.

11.9. $f(z)=\dfrac{\sin 2z}{(z+1)^3}$. **11.10.** $f(z)=z^2\cdot\cos\dfrac{1}{z-2}$.

11.11. $f(z)=\dfrac{\sin z}{(z^2+1)^2}$. **11.12.** $f(z)=\dfrac{\cos z}{(z^2+1)^2}$.

11.13. $f(z)=\dfrac{1}{z(1-e^{-z})}$. **11.14.** $f(z)=z^3\cdot\sin\dfrac{1}{z}$.

11.15. $f(z)=\dfrac{\sin z}{z^2(z^2+4)}$. **11.16.** $f(z)=\dfrac{\sin z}{z^2-z}$.

11.17. $f(z)=\dfrac{e^{2z}}{z^2\cdot(z^2+i)}$. **11.18.** $f(z)=z^3\cdot e^{\frac{1}{z}}$.

11.19. $f(z)=\dfrac{ctgz}{z^2-\frac{\pi}{4}z}$. **11.20.** $f(z)=z^2\cdot\sin\dfrac{1}{z-2}$.

11.21 $f(z)=\dfrac{e^z}{z^2\cdot(z^2+9)}$.

12-Masala. Quyidagi integrallarni chegirmalar yordamida hisoblang.

12.1. $\oint\limits_{|z|=3}\dfrac{e^z dz}{z^3\cdot(z-1)}$. **12.2.** $\oint\limits_{|z|=3}\dfrac{\sin\frac{1}{z}}{z(z+1)^2\cdot(z+2)(z+4)}dz$.

12.3. $\oint\limits_{|z|=4}\dfrac{z^2 dz}{(z^2+1)\cdot(z-3)}$. **12.4.** $\oint\limits_{|z|=3}\dfrac{z^3 e^{\frac{1}{z}}}{(z^2+1)^2}dz$.

12.5. $\oint\limits_{|z|=4}\dfrac{z^3 dz}{z^4-2}$. **12.6.** $\oint\limits_{|z|=2}\dfrac{z^3+z^5}{z^4+1}dz$.

12.7. $\oint\limits_{\left|z-\frac{1}{2}\right|=1}\dfrac{z+1}{(z-1)^2\cdot(z+i)}dz$. **12.8.** $\oint\limits_{|z|=2}\dfrac{z-1}{(z-3)\cdot(z+i)}dz$.

12.9. $\oint\limits_{|z|=1,5}\dfrac{z-2}{(z+1)^2\cdot(z+2)}dz$. **12.10.** $\oint\limits_{|z|=2,5}\dfrac{z-1}{(z-2)^2\cdot(z-3)}dz$.

12.11. $\oint\limits_{|z|=2}\dfrac{z-1}{(z-i)^2\cdot(z+i)}dz$. **12.12.** $\oint\limits_{|z-1|=2}\dfrac{z+1}{(z-2)(z+i)^2}dz$.

12.13. $\oint\limits_{|z|=1,5}\dfrac{z-1}{(z+2)\cdot(z+1)^2}dz$. **12.14.** $\oint\limits_{|z|=2}\dfrac{z-1}{z^5\cdot(z+i)^2}dz$.

12.15. $\oint\limits_{|z|=1,5}\dfrac{z+1}{z(z-2i)\cdot(z-i)^2}dz$. **12.16.** $\oint\limits_{|z+1|=1}\dfrac{1}{(z+2)^2\cdot(z-3)^2}dz$.

12.17. $\oint\limits_{|z|=1,5}\dfrac{2z+5}{(z-2i)^3\cdot(z+i)^2}dz$. **12.18.** $\oint\limits_{|z-1|=1,5}\dfrac{1}{z^2(z-2)^3\cdot(z+2i)}dz$.

12.19. $\oint\limits_{|z+1|=1}\dfrac{1}{(z^2+1)\cdot(z^4-1)}dz$. **12.20.** $\oint\limits_{|z|=2}\dfrac{z+3}{(z^3+1)\cdot(z+5)}dz$.

12.21. $\oint\limits_{|z|=2}\dfrac{1}{(z-3)\cdot(z^5-1)}dz$.

13-Masala. Quyidagi integrallarni hisoblang.

13.1. $\int\limits_{\partial D}(2z-1)\cos\dfrac{z}{z-1}\,dz, \quad D=\{|z|<2\}$.

13.2. $\int\limits_{\partial D}\dfrac{\cos z}{z^3}\,dz, \quad D=\{|z|<1\}$.

13.3. $\int\limits_{\partial D}z^2\sin\dfrac{1}{z}\,dz, \quad D=\{|z|<1\}$.

13.4. $\int\limits_{\partial D}\dfrac{1}{e^z+1}\,dz, \quad D=\{|z-2i|<2\}$.

13.5. $\int\limits_{\partial D}z^3\sin\dfrac{1}{z}\,dz, \quad D=\{|z|<2\}$.

13.6. $\int\limits_{\partial D}\dfrac{\sin\pi z}{(z^2-1)^3}\,dz\ , \quad D=\{\dfrac{x^2}{4}+y^2<1\}$.

13.7. $\int\limits_{\partial D}\dfrac{e^z}{z^4+2z^2+1}\,dz, \quad D=\{|z-i|<1\}$.

13.8. $\int\limits_{\partial D}z\cdot\sin\dfrac{z+1}{z-1}\,dz, \quad D=\{|z|<2\}$.

13.9. $\int\limits_{\partial D}\dfrac{z^3e^{\frac{1}{z}}}{z+1}\,dz, \quad D=\{|z|<2\}$.

13.10. $\int\limits_{\partial D}\dfrac{dz}{(z-1)^2(z^2+1)}\,dz, \quad D=\{|z-1-i|<2\}$.

13.11. $\int\limits_{\partial D}\dfrac{\sin z}{(z^3-z)(z-i)}\,dz, \quad D=\{|z-1|<3\}$.

13.12. $\int\limits_{\partial D}\dfrac{\sin z}{(z+1)^3}\,dz, \quad D=\{x^{\frac{2}{3}}+y^{\frac{2}{3}}<2^{\frac{2}{3}}\}$.

13.13. $\int\limits_{\partial D}\sin\dfrac{1}{z}, \quad D=\{|z|<4\}$.

13.14. $\int\limits_{\partial D}\dfrac{z}{z+2}e^{\frac{1}{2z}}\,dz, \quad D=\{|z|>4\}$.

13.15. $\int\limits_{\partial D} \frac{z}{(z-1)(z-2)^2}\,dz, \quad D = \{|z-2| < \frac{1}{2}\}.$

13.16. $\int\limits_{\partial D} \frac{z^2 \sin^2 \frac{1}{z}}{(z-1)(z-2)}\,dz, \quad D = \{|z| < 3\}.$

13.17. $\int\limits_{\partial D} \sin^2 \frac{1}{z}\,dz, \quad D = \{|z| < 2\}.$

13.18. $\int\limits_{\partial D} \sin \frac{1}{z-1}\,dz, \quad D = \{|z-1| > 1\}.$

13.19. $\int\limits_{\partial D} \frac{z}{\sin z \cdot (1-\cos z)}\,dz, \quad D = \{|z| < 5\}.$

13.20. $\int\limits_{\partial D} z \cos \frac{z}{z+1}\,dz, \quad D = \{|z| > 2\}.$

13.21. $\int\limits_{\partial D} \sin \frac{z}{z+1}\,dz, \quad D = \{|z| > 3\}.$

14-Masala. Quyidagi aniq integrallarni chegirmalar yordamida hisoblang.

14.1. $\int\limits_0^{2\pi} \frac{dx}{(2+\sin^2 x)^2}.$ **14.2.** $\int\limits_0^{2\pi} \frac{dx}{(3+2\cos^2 x)^2}.$

14.3. $\int\limits_0^{2\pi} \frac{dx}{(3+2\sin^2 x)^2}.$ **14.4.** $\int\limits_{-\pi}^{\pi} \frac{\sin^{2x} dx}{\frac{5}{4} - \cos x}.$

14.5. $\int\limits_{-\pi}^{\pi} \frac{\sin^2 x dx}{2-\cos x}.$ **14.6.** $\int\limits_{-\pi}^{\pi} \frac{\cos^2 x dx}{2+\sin x}.$

14.7. $\int\limits_0^{2\pi} \frac{dx}{(3+2\cos x)^2}.$ **14.8.** $\int\limits_0^{2\pi} \frac{dx}{(2+\cos^2 x)^2}.$

14.9. $\int\limits_0^{2\pi} \frac{dx}{(2+\sin x)^2}.$ **14.10.** $\int\limits_0^{2\pi} \frac{dx}{(3+2\cos x)^2}.$

14.11. $\int\limits_0^{\pi} \frac{\cos^4 x}{1+\sin^2 x}\,dx.$ **14.12.** $\int\limits_0^{2\pi} \frac{dx}{(2+\cos x)^2}.$

14.13. $\int\limits_{-\pi}^{\pi}\frac{dx}{5+4\sin x}$. **14.14.** $\int\limits_{0}^{2\pi}\frac{\cos^2 x}{5+4\cos x}dx$.

14.15. $\int\limits_{0}^{2\pi}\frac{dx}{3+\cos x}$. **14.16.** $\int\limits_{0}^{2\pi}\frac{dx}{4+2\cos x}$.

14.17. $\int\limits_{0}^{2\pi}\frac{dx}{\sin x+3}$. **14.18.** $\int\limits_{0}^{2\pi}\frac{dx}{\frac{5}{4}-\sin x}$.

14.19. $\int\limits_{0}^{2\pi}\frac{dx}{\cos x+2}$. **14.20.** $\int\limits_{-\pi}^{\pi}\frac{1}{4+3\cos x}dx$.

14.21. $\int\limits_{0}^{\pi}\frac{\cos^2 x dx}{2-\sin^2 x}$.

15-Masala. Quyidagi chegarasi cheksiz bo‘lgan integrallarni chegirmalar yordamida hisoblang.

15.1. $\int\limits_{-\infty}^{+\infty}\frac{dx}{(x^2+1)(x^2+4)}$. **15.2.** $\int\limits_{-\infty}^{+\infty}\frac{x^2dx}{(x^2+1)(x^2+9)}$.

15.3. $\int\limits_{0}^{+\infty}\left(\frac{x}{(x^2+1}\right)^2 dx$. **15.4.** $\int\limits_{-\infty}^{+\infty}\frac{x^2-x+2}{x^4+10x^2+9}dx$.

15.5. $\int\limits_{-\infty}^{+\infty}\frac{dx}{(x^2+2x+2)^2}$. **15.6.** $\int\limits_{-\infty}^{+\infty}\frac{x^2dx}{x^4+6x^2+25}$.

15.7. $\int\limits_{-\infty}^{+\infty}\frac{dx}{x^2-2ix-2}$. **15.8.** $\int\limits_{0}^{+\infty}\frac{x^6dx}{(x^4+8)^2}$.

15.9. $\int\limits_{-\infty}^{+\infty}\frac{x^2dx}{(x^2+4ix-5)^2}$. **15.10.** $\int\limits_{-\infty}^{+\infty}\frac{dx}{(x^2-2ix-2)^2}$.

15.11. $\int\limits_{-\infty}^{+\infty}\frac{x^2+1}{x^4+1}dx$. **15.12.** $\int\limits_{0}^{+\infty}\frac{x^4dx}{(2+3x^2)^4}$.

15.13. $\int\limits_{-\infty}^{+\infty}\frac{xdx}{(x^2+4x+13)^2}$. **15.14.** $\int\limits_{-\infty}^{+\infty}\frac{dx}{(x^2+1)(x^2+4)^2}$.

15.15. $\int_{-\infty}^{+\infty}\frac{x^2+1}{x^6+1}dx$.   **15.16.** $\int_{-\infty}^{+\infty}\frac{dx}{(x^2+1)^2(x^2+4)}$.

15.17. $\int_{0}^{+\infty}\frac{x^2dx}{(x^2+4)^2}$.   **15.18.** $\int_{0}^{+\infty}\frac{dx}{1+x^{2n}}$ $(n\in N)$.

15.19. $\int_{-\infty}^{+\infty}\frac{dx}{(x^2+4)(x^2+9)}$.   **15.20.** $\int_{-\infty}^{+\infty}\frac{dx}{(1+2x^2)^n}$ $(n\in N)$.

15.21. $\int_{0}^{+\infty}\frac{dx}{(x^2+1)^n}$ $(n\in N)$.

16-Masala. Quyidagi integrallarni Jordan lemmalaridan foydalanib hisoblang.

16.1. $\int_{0}^{+\infty}\frac{\cos x}{(x^2+4)(x^2+9)}dx$.   **16.2.** $\int_{0}^{+\infty}\frac{\cos 2x}{x^4+x^2+1}dx$.

16.3. $\int_{0}^{+\infty}\frac{x\sin 3x}{x^2+4}dx$.   **16.4.** $\int_{0}^{+\infty}\frac{\cos x}{(x^2+4)^3}dx$.

16.5. $\int_{0}^{+\infty}\frac{x\sin x}{(x^2+4)^2}dx$.   **16.6.** $\int_{0}^{+\infty}\frac{x\sin x}{(x^2+9)^2}dx$.

16.7. $\int_{0}^{+\infty}\frac{\cos 3x}{x^2+4}dx$.   **16.8.** $\int_{0}^{+\infty}\frac{x\sin x}{x^2+4}dx$.

16.9. $\int_{0}^{+\infty}\frac{\cos x}{x^2+9}dx$.   **16.10.** $\int_{0}^{+\infty}\frac{\cos 2x}{x^2+9}dx$.

16.11. $\int_{-\infty}^{+\infty}\frac{x\cos x}{x^2-2x+10}dx$.   **16.12.** $\int_{0}^{+\infty}\frac{\cos x}{x^2+4}dx$.

16.13. $\int_{-\infty}^{+\infty}\frac{(x-1)\cos 2x}{x^2-4x+5}dx$.   **16.14.** $\int_{-\infty}^{+\infty}\frac{x\sin x}{x^2+2x+10}dx$.

16.15. $\int_{-\infty}^{+\infty}\frac{(x^3+5x)\sin x}{x^4+10x^2+9}dx$.   **16.16.** $\int_{-\infty}^{+\infty}\frac{(2x^3+13x)\sin x}{x^4+13x^2+36}dx$.

16.17. $\int_{-\infty}^{+\infty}\frac{x\sin x}{x^2+4x+20}dx$.   **16.18.** $\int_{-\infty}^{+\infty}\frac{x^3\sin x}{x^4+5x^2+4}dx$.

16.19. $\int\limits_{-\infty}^{+\infty}\frac{x\cos x}{x^2-2x+10}dx$. **16.20.** $\int\limits_{-\infty}^{+\infty}\frac{x\sin x}{x^2-2x+10}dx$.

16.21. $\int\limits_{-\infty}^{+\infty}\frac{(x+1)\sin 2x}{x^2+2x+2}dx$.

- C -

NAMUNAVIY VARIANT YECHIMI.

1.21-Masala. Boshi $a=2+2i$ oxiri $b=i$ nuqtada bo‘lgan to‘g‘ri chiziq kesmasi bo‘yicha quyidagi

$$\int\limits_{\gamma}(2z-1)dz$$

integralni ta’rif yordamida hisoblang.

$\triangleleft$ γ chiziqni a dan b ga qarab $z_0, z_1, ...z_n$ nuqtalar yordamida $n-ta$ $\gamma_1,\gamma_2,...,\gamma_n$ yoylarga ajratamiz. $\forall\ \xi_k\in\gamma_k$ nuqta olib, quyidagi

$$\sigma=\sum_{k=1}^{n}f(\xi_k)\cdot(z_k-z_{k-1})$$

integral yig‘indini tuzamiz . Unda ta’rifga ko‘ra

$$\int\limits_{\gamma}(2z-1)dz=\lim_{\lambda\to 0}\sigma=\lim_{\lambda\to 0}\sum_{k=1}^{n}f(\xi_k)\cdot(z_k-z_{k-1}) \qquad (42)$$

bo‘ladi. $f(z)=2z-1\in C(\gamma)$ bo‘lgani uchun (42)-limit mavjud va bu limitning qiymati γ ning bo‘linish usuliga va ξ_k nuqtalarning tanlanishiga bog‘liq emas.

$\Rightarrow\ \xi_k=\frac{z_{k-1}+z_k}{2}$ deb olsak,

$$\sigma=\sum_{k=1}^{n}(2\xi_k-1)\cdot(z_k-z_{k-1})=\sum_{k=1}^{n}\left[2\cdot\frac{z_k+z_{k-1}}{2}-1\right]\cdot(z_k-z_{k-1})=$$

$$\sum_{k=1}^{n}[(z_k+z_{k-1})\cdot(z_k-z_{k-1})-(z_k-z_{k-1})]=\sum_{k=1}^{n}(z_k^2-z_{k-1}^2)-$$

$$-\sum_{k=1}^{n}(z_k-z_{k-1})=z_n^2-z_0^2-(z_n-z_0)=b^2-a^2-(b-a)=$$

$$=i^2-(2+2i)^2-(i-2-2i)=-1-4\cdot 2i+i+2=1-7i$$

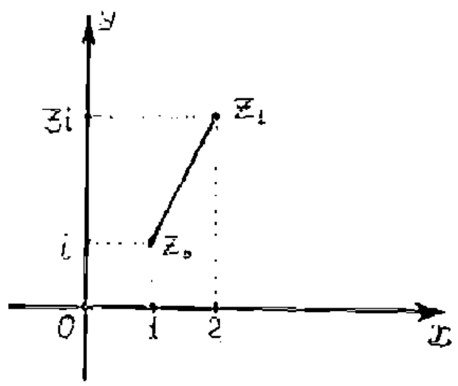
Demak,

$$\int_{\gamma}(2z-1)dz=\lim_{\lambda\to 0}\sigma=1-7i \quad \triangleright$$

2.21-Masala. Quyidagi $\int_{\gamma}(x^2+iy^2)dz$ integralni $z_0=1+i, \quad z_1=2+3i$ nuqtalarni tutashtiruvchi γ to‘g‘ri chiziq bo‘yicha hisoblang.

$\triangleleft$ Birinchi navbatda γ to‘g‘ri chiziqning tenglamasini topamiz.

$\gamma: y=2x-1, \quad 1\le x\le 2$ ekanligini ko‘rish qiyin emas.



Bu tenglama, $z=x+iy$, $dz=dx+idy$ va γ da $dy=2dx$ ekanligidan foydalanib, berilgan integralni hisoblaymiz:

$$\int_\gamma (x^2+iy^2)dz=\int_\gamma (x^2+iy^2)\cdot(dx+idy)=\int_\gamma (x^2dx-y^2dy)+$$

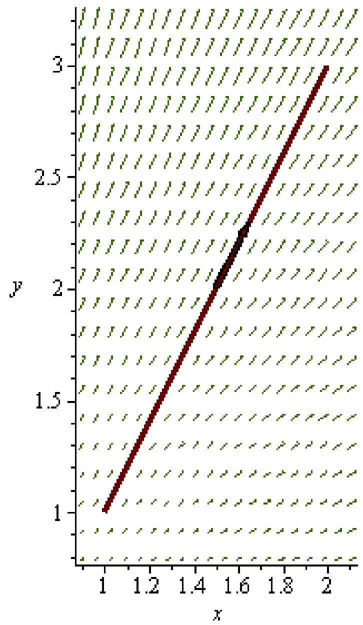
$$+i\int_\gamma (x^2dy+y^2dx)=\int_1^2[x^2-(2x-1)^2\cdot 2]dx+i\int_1^2[x^2\cdot 2+(2x-1)^2]dx=$$

$$=\int_1^2(-7x^2+8x-2)dx+i\int_1^2(6x^2-4x+1)dx=-\frac{19}{3}+9i. \qquad \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

$> with(Student[VectorCalculus]):$

$> LineInt(VectorField(\langle x, y\rangle), Line(\langle 1, 1\rangle, \langle 2, 3\rangle), output = plot)$



The path of integration, vector(s) tangent to the path, and vector-field arrows

$> LineInt(VectorField(\langle x^2, -y^2\rangle), Line(\langle 1, 1\rangle, \langle 2, 3\rangle), output = integral)$

$$\int_0^1 \left((1+t)^2-2\,(1+2\,t)^2\right)\,dt$$

$> K1 := LineInt(VectorField(\langle x^2, -y^2\rangle), Line(\langle 1, 1\rangle, \langle 2, 3\rangle))$

$$K1 := -\frac{19}{3}$$

$> LineInt(VectorField(\langle y^2, x^2\rangle), Line(\langle 1, 1\rangle, \langle 2, 3\rangle), output = integral)$

$$\int_0^1 \left((1+2t)^2 + 2(1+t)^2 \right) dt$$

> $K2 := LineInt\left(VectorField\left(\langle y^2, x^2\rangle\right), Line(\langle 1, 1\rangle, \langle 2, 3\rangle)\right)$

$$K2 := 9$$

> $K := K1 + I \cdot K2$

$$K := -\frac{19}{3} + 9\, \mathrm{I}$$

3.21-Masala. 88-chizmada tasvirlangan γ chiziq bo‘yicha olingan quydagi

$$\oint_{\gamma} \frac{7z + 6\bar{z}}{\bar{z}} dz$$

integralni hisoblang.

◁ Agar

$\gamma_1 = \{z = x + iy \in C : y = 0,\ -2 \le x \le 2\},\ \gamma_2 = \{z \in C : |z| = 2,\ \mathrm{Im}\, z > 0\}$

deb belgilansa, unda $\gamma = \gamma_1 \cup \gamma_2$ bo‘lib, integralning xossasiga ko‘ra

$$\oint_{\gamma} \frac{7z + 6\bar{z}}{\bar{z}} dz = \oint_{\gamma_1} \frac{7z + 6\bar{z}}{\bar{z}} dz + \oint_{\gamma_2} \frac{7z + 6\bar{z}}{\bar{z}} dz$$

bo‘ladi. Bu tenglikning o‘ng tomonidagi integrallarni alohida-alohida hisoblaymiz:

$$\oint_{\gamma_1}\frac{7z+6\overline{z}}{\overline{z}}dz=\int_{-2}^{2}\frac{7x+6x}{x}dx=13\int_{-2}^{2}dx=13\cdot 4=52,$$

$$\int_{\gamma_2}\frac{7z+6\overline{z}}{\overline{z}}dz=\left(\left(\begin{matrix} z=2e^{i\varphi} \quad , \quad 0\le\varphi\le\pi \\ \overline{z}=2e^{-i\varphi} \quad , \quad dz=2ie^{i\varphi}d\varphi \end{matrix}\right)\right)=$$

$$\int_{0}^{\pi}\frac{7\cdot 2e^{i\varphi}+6\cdot 2e^{-i\varphi}}{2e^{-i\varphi}}2ie^{i\varphi}d\varphi=2i\int_{0}^{\pi}(7e^{3i\varphi}+6e^{i\varphi})d\varphi=$$

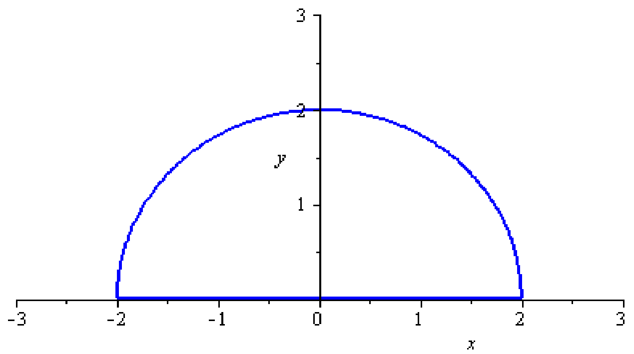
$$=2\left(\frac{7}{3}e^{3i\varphi}+6e^{i\varphi}\right)\Big|_0^{\pi}=-\frac{100}{3}.$$

Demak,

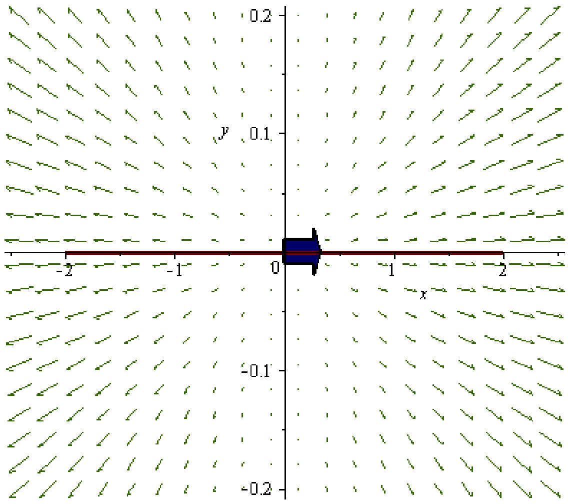
$$\oint_{\gamma}\frac{7z+6\overline{z}}{\overline{z}}dz=52-\frac{100}{3}=\frac{56}{3} \quad \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

```
> with(plots) :
> with(Student[VectorCalculus]) :
> plot([[ 2· cos(t), 2· sin(t), t = 0 .. π], [t, 0, t =-2 ..2]],
      x =-3 ..3, y = 0 ..3, color = [blue], thickness = 2, grid
      = [100, 100])
```



> $LineInt(\,VectorField(\langle x, y\rangle),\, LineSegments(\langle -2, 0\rangle, \langle 2, 0\rangle),$
$output = plot)$



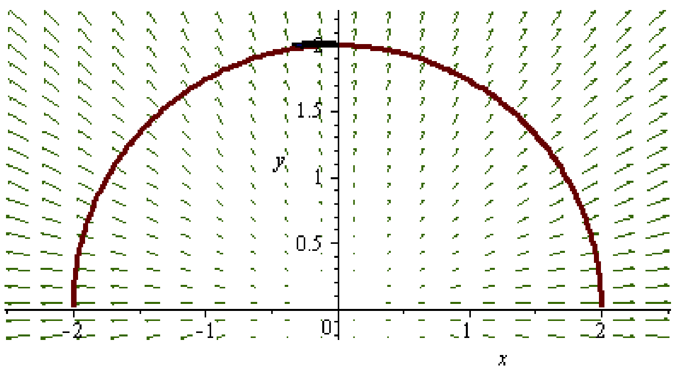
The path of integration, vector(s) tangent to the path, and vector-field arrows

>

> $$K1 := \int_{-2}^{2} \frac{7\,x + 6\,x}{x}\, \mathrm{d}x$$

$$K1 := 52$$

> $LineInt\big(\,VectorField(\langle x, y\rangle),\, Path\big(\langle 2\cdot\cos(t),\, 2\cdot\sin(t)\rangle,\, t$
$= 0\,..\pi\big),\, output = plot\big)$



The path of integration, vector(s) tangent to the path, and vector-field arrows

> $K2 := \int_0^{\pi} \frac{7 \cdot 2 \cdot e^{I \cdot t} + 6 \cdot 2 \cdot e^{-I \cdot t}}{2 \cdot e^{-I \cdot t}} \cdot 2 \cdot I \cdot e^{I \cdot t} dt$

$$K2 := -\frac{100}{3}$$

> $K := K1 + K2$

$$K := \frac{56}{3}$$

4.21-Masala. Agar $\gamma : x = 3\cos t, \quad y = 2\sin t, \ 0 \le t \le 2\pi$ ellips bo‘lsa quyidagi

$$\int_{\gamma} (4x + 3iy)dz$$

integral hisoblansin.

◁ Bu integralni (5) formuladan foydalanib hisoblaymiz:

$z = z(t) = x(t) + i \cdot y(t) = 3\cos t + i \cdot 2\sin t \Rightarrow z^1(t) = -3\sin t + 2i\cos t.$

Unda (5)–formulaga ko‘ra

$$\int_{\gamma} (4x + 3iy)dz = \int_0^{2\pi} (12\cos t + 6i\sin t) \cdot (-3\sin t + 2i\cos t)dt =$$

$$= 3\int_0^{2\pi} [-16\sin t \cdot \cos t + i(8\cos^2 t - 6\sin^2 t)]dt =$$

$$= 3\int_0^{2\pi} [-8\sin 2t + i(8 \cdot \frac{1 + \cos 2t}{2} - 6 \cdot \frac{1 - \cos 2t}{2})]dt =$$

$$= 3\int_0^{2\pi} [-8\sin 2t + i(1 + 7\cos 2t)]dt = 3 \cdot [4\cos 2t + i(t + \frac{7}{2}\sin 2t)]\Big|_0^{2\pi} =$$

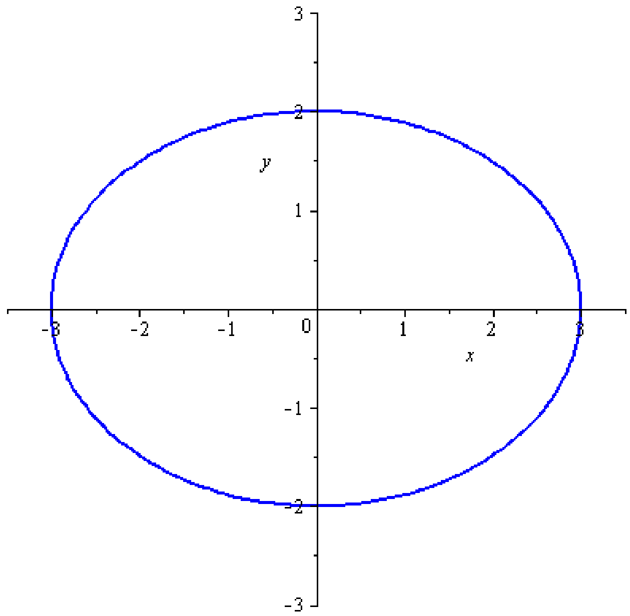
$$= 3 \cdot 2\pi i = 6\pi i \ \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(plots):$

> *with*(*Student*[*VectorCalculus*]) :

> $plot\left([3\cdot\cos(t), 2\cdot\sin(t), t = 0..2\cdot\pi], x = -3.5..3.5, y = -3..3, color = [blue], thickness = 2, grid = [100, 100]\right)$



> $$\int_0^{2\cdot\pi} (12\cdot\cos(t) + 6\cdot I\cdot\sin(t))\cdot(-3\cdot\sin(t) + 2\cdot I\cdot\cos(t))\, dt$$

$$6\, I\, \pi$$

5-21-Masala. Quyidagi

$$\int_{-2+i}^{1+i} z^2 dz$$

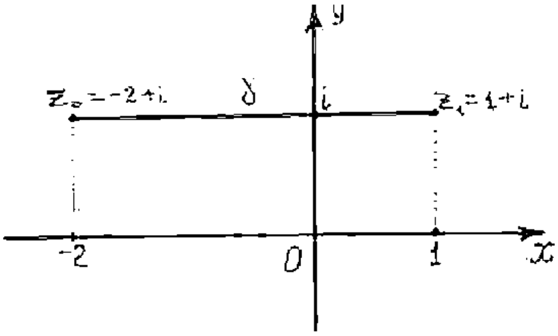
integralni hisoblang.

◃ Bu misol 2^0-punktda keltirilgan 1-misolga o‘xshash yechiladi.

$f(z) = z^2 \in O(C) \Rightarrow$ Integralning qiymati $z_0 = -2 + i$, $z_1 = 1 + i$ nuqtalarni birlashtiruvchi yo‘lga bog‘liq bo‘lmaydi. Shundan foydalanib integrallash chizig‘i γ sifatida

$$\gamma = \{z = x + iy \in C : y = 1,\ -2 \le x \le 1\}$$

to‘g‘ri chiziq kesmasini olamiz (90-chizma).



90-chizma

Bu γ chiziqda $z = x + i, \quad dz = dx$ bo‘lishidan foydalanib topamiz.

$$\int_{-2+i}^{1+i} z^2 dz = \int_{\gamma} z^2 dz = \int_{-2}^{1} (x+i)^2 dx =$$

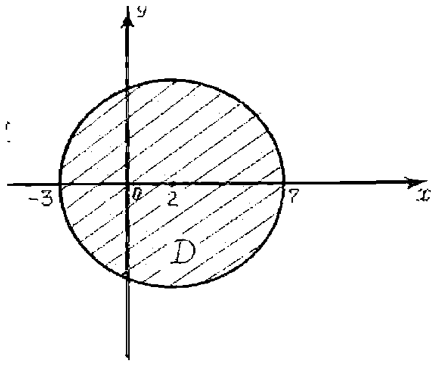
$$= \int_{-2}^{1} (x^2 + 2ix - 1) dx = \left(\frac{x^3}{3} + ix^2 - x\right)\Big|_{-2}^{1} = -3i \qquad \triangleright$$

6.21-Masala. Koshining integral formulasidan foydalanib quyidagi

$$\int_{|z-2|=5} \frac{e^{z^2} dz}{(z+4)(z^2-6z)}$$

integralni hisoblang.

◁ $|z-2| = 5$ aylana bilan chegaralangan sohani D deb belgilaymiz (91-chizma).



91-chizma

$$F(z)=\frac{e^{z^2}}{(z+4)(z^2-6z)}=\frac{e^{z^2}}{z(z-6)(z+4)}$$ deb belgilasak, $z_1=0$ *ва* $z_2=6$ nuqtalar $\in D,\ z_3=-4\notin D.$ Shu faktdan foydalanib $F(z)$ funksiyani ushbu

$$F(z)=\frac{e^{z^2}}{6(z+4)}\cdot\left(\frac{1}{z-6}-\frac{1}{z}\right)=\frac{f(z)}{z-6}-\frac{f(z)}{z}$$

ko‘rinishida ifodalab olamiz, bunda $f(z)=\dfrac{e^{z^2}}{6(z+4)}\in O(D).$

Koshining integral formulasidan foydalanib topamiz:

$$\int\limits_{|z-2|=5}\frac{e^{z^2}dz}{(z+4)(z^2-6z)}=\int\limits_{|z-2|=5}F(z)dz=\int\limits_{|z-2|=5}\frac{f(z)}{z-6}dz-$$

$$-\int\limits_{|z-2|=5}\frac{f(z)}{z}dz=2\pi i[f(6)-f(0)]=2\pi i\left(\frac{e^{36}}{60}-\frac{1}{24}\right)=$$

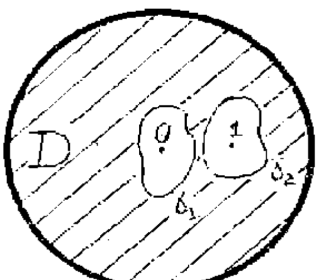
$$=\frac{\pi i}{6}\cdot\left(\frac{e^{36}}{5}-\frac{1}{2}\right).\qquad\qquad \triangleright$$

7.21-Masala. Koshining integral formulasidan foydalanib quyidagi

$$\int\limits_{|z|=2}\frac{z+1}{z(z-1)^3\cdot(z-3)}dz$$

integralni hisoblang.

$\triangleleft$ $z_0=0,\ z_1=1$ nuqtalar $\{z\in C:|z|=2\}$ aylana bilan chegaralangan $\{z\in C:|z|<2\}$ doiraga tegishli bo‘lib, $z_2=3$ nuqta esa shu doiraga tegishli emas. $z_0=0$ *ва* $z_1=1$ nuqtalarni $\{z\in C:|z|<2\}$ doiraga tegishli va o‘zaro kesishmaydigan γ_1 *ва* γ_2 yopiq chiziqlar bilan o‘raymiz. Bu γ_1 , γ_2 chiziqlar hamda $\{z\in C:|z|=2\}$ aylana bilan chegaralangan uch bog‘lamli sohani D bilan belgilaymiz.(92-chizma)



92-chizma

Berilgan integral ostidagi

$$F(z) = \frac{z+1}{z(z-1)^3 \cdot (z-3)}$$

funksiya D sohada golomorf bo‘ladi. 2^0-punktda keltirilgan ko‘p bog‘lamli soha uchun Koshi teoremasidan foydalanib topamiz:

$$\int_{|z|=2} F(z)dz = \oint_{\gamma_1} F(z)dz + \oint_{\gamma_2} F(z)dz = I_1 + I_2$$

Agar

$$I_1 = \oint_{\gamma_1} F(z)dz = \oint_{\gamma_1} \frac{z+1}{z(z-1)^3(z-3)} dz$$

integralda

$$f(z) = \frac{z+1}{(z-1)^3(z-3)}$$

deyilib, (10)-formuladan foydalanilsa

$$I_1 = \oint_{\gamma_1} \frac{f(z)}{z} dz = 2\pi i \cdot f(0) = 2\pi i \cdot \frac{1}{3} = \frac{2}{3}\pi i$$

bo‘lishi kelib chiqadi.

Endi

$$I_2 = \oint_{\gamma_2} F(z)dz = \oint_{\gamma_2} \frac{z+1}{z(z-1)^3(z-3)} dz$$

integralda

$$\varphi(z) = \frac{z+1}{z(z-3)}$$

deb va (11)-formuladan foydalanib topamiz:

$$I_2=\oint_{\gamma_2}\frac{\phi(z)}{(z-1)^3}dz=\frac{2\pi i}{2!}\phi^{(2)}(1)=((\phi(z)=\frac{z+1}{z(z-3)}=$$

$$=\frac{1}{3}(\frac{4}{z-3}-\frac{1}{z})\Rightarrow\phi^{(1)}(z)=\frac{1}{3}(-\frac{4}{(z-3)^2}+\frac{1}{z^2})\Rightarrow\phi^{(2)}(z)=$$

$$\frac{1}{3}[\frac{8}{(z-3)^3}-\frac{2}{z^3}]\Rightarrow\phi^{(2)}(1)=\frac{1}{3}(-1-2))=-\pi i.$$

Shunday qilib,

$$\int_{|z|=2}\frac{z+1}{z\cdot(z-1)^3(z-3)}dz=I_1+I_2=\frac{2}{3}\pi i-\pi i=-\frac{\pi i}{3}$$

bo‘ladi ▹

8.21-Masala. Quyidagi

$$f(z)=\frac{1}{z^2-3iz-2}$$

funksiyani $a=2i$ nuqtaning atrofida Loran qatoriga yoying va qatorning yaqinlashish sohasini toping.

◃ Oldin $f(z)$ funksiyani

$$f(z)=\frac{1}{(z-2i)(z-i)}$$

ko‘rinishda tasvirlaymiz. So‘ng uni sodda kasrlarga yoyib, cheksiz kamayuvchi geometrik progressiya yig‘indisi formulasidan foydalansak,

$$f(z)=\frac{1}{(z-2i)(z-i)}=\frac{-i}{z-2i}+\frac{i}{z-i}=\frac{-i}{z-2i}+$$

$$+\frac{1}{1+\frac{z-2i}{i}}=\frac{-i}{z-2i}+\sum_{n=0}^{\infty}(-1)^n\cdot(\frac{z-2i}{i})^n$$

Loran qatori hosil bo‘ladi va bu qator $\{0<|z-2i|<1\}$ sohada yaqinlashadi ▹

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $with(genfunc):$

> $f := \dfrac{1}{z^2 - 3\cdot I\cdot z - 2}$

$$f := \frac{1}{z^2 - 3\,\mathrm{I}\,z - 2}$$

>

>

> $f := rgf\_pfrac(\mathrm{f}, z)$

$$f := -\frac{\mathrm{I}}{z - 2\,\mathrm{I}} + \frac{\mathrm{I}}{z - \mathrm{I}}$$

> $series\left(\dfrac{I}{z - I}, z = 2\,I\right)$

$$1 + \mathrm{I}\,(z - 2\,\mathrm{I}) - (z - 2\,\mathrm{I})^2 - \mathrm{I}\,(z - 2\,\mathrm{I})^3 + (z - 2\,\mathrm{I})^4 + \mathrm{I}\,(z - 2\,\mathrm{I})^5 + \mathrm{O}\big((z - 2\,\mathrm{I})^6\big)$$

> $series(f, z = 2\,I, 8)$

$$-\frac{\mathrm{I}}{z - 2\,\mathrm{I}} + 1 + \mathrm{I}\,(z - 2\,\mathrm{I}) - (z - 2\,\mathrm{I})^2 - \mathrm{I}\,(z - 2\,\mathrm{I})^3 + (z - 2\,\mathrm{I})^4 + \mathrm{I}\,(z - 2\,\mathrm{I})^5 - (z - 2\,\mathrm{I})^6$$
$$- \mathrm{I}\,(z - 2\,\mathrm{I})^7 + \mathrm{O}\big((z - 2\,\mathrm{I})^8\big)$$

9.21-Masala. Quyidagi $f(z) = \dfrac{2z - 3}{z^2 - 3z + 2}$ funksiyani $V = \{0 < |z - 2| < 1\}$ xalqada Loran qatoriga yoying.

$$\triangleleft \qquad f(z) = \frac{2z-3}{z^2 - 3z + 2} = \frac{2z-3}{(z-2)(z-1)} = \frac{1}{z-2} + \frac{1}{z-1} =$$

$$= \frac{1}{z-2} + \frac{1}{1 + (z-2)} = \frac{1}{z-2} + \sum_{n=0}^{\infty} (-1)^n \cdot (z-2)^n$$

Hosil bo‘lgan Loran qatori berilgan $V = \{0 < |z - 2| < 1\}$ xalqada yaqinlashadi.$\triangleright$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

$> f := \dfrac{2\cdot z-3}{z^2-3\cdot z+2}$

$$f := \frac{2z-3}{z^2-3z+2}$$

$> f := rgf\_pfrac(\mathrm{f}, z)$

$$f := \frac{1}{z-2}+\frac{1}{z-1}$$

$> f := series(f, z=2, 10)$

$$f := (z-2)^{-1}+1-(z-2)+(z-2)^2-(z-2)^3+(z-2)^4-(z-2)^5+(z-2)^6-(z-2)^7+(z-2)^8-(z-2)^9+\mathrm{O}\big((z-2)^{10}\big)$$

$>$

10.21-masala. Quyidagi $f(z)=\dfrac{1}{z^3\cdot(2-\cos z)}$ funksiyaning barcha maxsus nuqtalarini toping, ularning xarakterini aniqlang va funksiyalarni $z=\infty$ nuqtada tekshiring (qutblar uchun ularning tartibini ko‘rsating).

◁ $f(z)$ funksiyaning qutb nuqtalarini topish uchun $\varphi(z)=\dfrac{1}{f(z)}=z^3\cdot(2-\cos z)$ funksiyaning nollarini topamiz. $z=0$ nuqta $\varphi(z)$ funksiyaning 3-tartibli noli bo‘lgani uchun ta’rifga ko‘ra $f(z)$ funksiyaning 3-tartibli qutb nuqtasi bo‘ladi. $\varphi(z)$ funksiyaning boshqa nollarini $2-\cos z=0$ yoki $\cos z=2$ tenglamani yechib topamiz. Bu tenglamani 2-paragrafdagi formulalardan foydalanib yechamiz:

$$z = Arc\cos 2=-iLn(2\pm\sqrt{2^2-1})=-i[\ln(2\pm\sqrt{3})+2k\pi i]=$$

$$=2k\pi-i\ln(2\pm\sqrt{3}).$$

Bu nuqtalar $\varphi(z)$ funksiya uchun 1-tartibli nol bo‘lgani uchun $f(z)$ funksiya uchun 1-tartibli qutb nuqta bo‘ladi.

$z=\infty$ nuqta $f(z)$ funksiyaning yakkalangan maxsus nuqtasi bo‘lmaydi, chunki u qutb nuqtalar uchun limit nuqta bo‘ladi.

Shunday qilib,

$z=0$ - 3-tartibli qutb;

$z_k = 2k\pi - i\ln(2\pm\sqrt{3}), \quad k=0,\pm1,\pm2,\ldots$ - 1-tartibli qutblar;

$z=\infty$ - qutblarning limit nuqtasi bo‘lar ekan ▷

11.21-masala. Quyidagi $f(z)=\dfrac{e^z}{z^2\cdot(z^2+9)}$ funksiyaning barcha maxsus nuqtalaridagi va $z=\infty$ nuqtadagi chegirmalarni hisoblang.

◁ Berilgan funksiyani

$$f(z)=\frac{e^z}{z^2\cdot(z^2+9)}=\frac{e^z}{z^2(z-3i)(z+3i)}$$

ko‘rinishda yozib, uning maxsus nuqtalari: $a_1=3i,\ a_2=-3i$ - birinchi tartibli qutb nuqtalar, $a_3=0$ - ikkinchi tartibli qutb nuqta va $z=\infty$ - *o*‘ta maxsus nuqta bo‘lishini aniqlaymiz. $\underset{z=a_1}{res} f(z)$ ва $\underset{z=a_2}{res} f(z)$ larni hisoblashda (27)-formuladan foydalanamiz:

$$\underset{z=a_1}{res} f(z)=\underset{z=3i}{res}(z-3i)f(z)=\lim_{z\to3i}\frac{e^z}{z^2\cdot(z+3i)}=$$

$$=e^{3i}\cdot\frac{1}{-9\cdot6i}=-\frac{1}{54}(\sin3-i\cos3),$$

$$\underset{z=a_2}{res} f(z)=\underset{z=-3i}{res}(z+3i)\cdot f(z)=-\frac{1}{54}(\sin3+i\cos3).$$

(29)–formulaga ko‘ra $\underset{z=a_3}{res} f(z)$ ni hisoblaymiz:

$$\underset{z=a_3}{res} f(z)=\lim_{z\to0}\frac{d}{dz}[z^2\cdot f(z)]=\lim_{z\to0}\left(\frac{e^z}{z^2+9}\right)^1=$$

$$=\lim_{z\to0}\frac{e^z\cdot(z^2-2z+9)}{(z^2+9)^2}=\frac{1}{9}.$$

$\underset{z=\infty}{res} f(z)$ ni hisoblashda esa 8^0-puktdagi 2-teorema (chegirmalarning yig‘indisi haqidagi teorema)dan foydalansa bo‘ladi:

$$\underset{z=\infty}{res}\, f(z) = -\sum_{k=1}^{3} \underset{z=a_k}{res}\, f(z) = \frac{1}{27}(\sin 3 - 3). \qquad \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $f := \dfrac{e^z}{z^2 \cdot (z^2 + 9)}$

$$f := \frac{e^z}{z^2\, (z^2 + 9)}$$

> $a1 := residue(f, z = 0)$

$$a1 := \frac{1}{9}$$

> $a2 := residue(f, z = 3 \cdot I)$

$$a2 := \frac{1}{54}\, I\, (e^I)^3$$

> **simplify( );**

$$\frac{1}{54}\, I\, e^{3\, I}$$

> $a3 := residue(f, z = -3 \cdot I)$

$$a3 := -\frac{\frac{1}{54}\, I}{(e^I)^3}$$

> $a := -(a1 + a2 + a3)$

$$a := -\frac{1}{9} - \frac{1}{54}\, I\, (e^I)^3 + \frac{\frac{1}{54}\, I}{(e^I)^3}$$

> $b := evalc(a)$

$$b := -\frac{1}{9} + \frac{1}{18}\cos(1)^2\sin(1) - \frac{1}{54}\sin(1)^3$$
$$+ \frac{1}{54}\left(3\cos(1)^2\sin(1) - \sin(1)^3\right) \Big/ \Big(\Big($$
$$-3\cos(1)\sin(1)^2 + \cos(1)^3\Big)^2$$
$$+ \left(3\cos(1)^2\sin(1) - \sin(1)^3\right)^2\Big)$$
$$+ \mathrm{I}\left(\frac{1}{18}\cos(1)\sin(1)^2 - \frac{1}{54}\cos(1)^3\right.$$
$$+ \frac{1}{54}\left(-3\cos(1)\sin(1)^2 + \cos(1)^3\right) \Big/ \Big(\Big($$
$$-3\cos(1)\sin(1)^2 + \cos(1)^3\Big)^2$$
$$\left. + \left(3\cos(1)^2\sin(1) - \sin(1)^3\right)^2\Big)\right)$$

> simplify( );

$$\frac{4}{27}\cos(1)^2\sin(1) - \frac{1}{27}\sin(1) - \frac{1}{9}$$

12.21-Masala. Quyidagi

$$\oint\limits_{|z|=2} \frac{1}{(z-3)(z^5-1)}dz$$

integralni chegirmalar yordamida hisoblang.

◁ (32)-formulaga ko‘ra $f(z) = \dfrac{1}{(z-3)(z^5-1)}$ uchun

$$\oint\limits_{|z|=2} f(z)dz = 2\pi i \sum_{k=1}^{5} \underset{z=a_k}{res}\, f(z) = -2\pi i[\underset{z=3}{res}\, f(z) + \underset{z=\infty}{res}\, f(z)]$$

bo‘ladi. Bu tenglikning o‘ng tomonidagi chegirmalarni hisoblaymiz:

$$\underset{z=3}{res}\, f(z) = \lim_{z\to 3}(z-3)f(z) = \lim_{z\to 3}\frac{1}{z^5-1} = \frac{1}{242}$$

Agar

$$f(z)=\frac{1}{(z-3)(z^5-1)}=\frac{1}{z^6}\cdot\frac{1}{(1-\frac{3}{z})(1-\frac{1}{z^5})}$$

ekanini e'tiborga olsak, unda $z=\infty$ nuqta $f(z)$ funksiyaning 6-tartibli noli bo‘lishini aniqlaymiz. Bu funksiyaning Loran qatori

$$f(z)=\frac{1}{z^6}+\frac{c_{-7}}{z^7}+\frac{c_{-8}}{z^8}+\ldots.$$

bo‘lib, $c_{-1}=0$ bo‘ladi. Demak,

$$\underset{z=\infty}{res}\, f(z)=0\,.$$

Shunday qilib,

$$\oint\limits_{|z|=2}\frac{1}{(z-3)(z^5-1)}dz=-2\pi i\cdot(\frac{1}{242}+0)=-\frac{\pi i}{121}. \qquad \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $f := \dfrac{1}{(z-3)\cdot(z^5-1)}$

$$f := \frac{1}{(z-3)\,(z^5-1)}$$

> $a1 := residue(f, z=3)$

$$a1 := \frac{1}{242}$$

> $a2 := residue(f, z=\infty)$

$$a2 := 0$$

> $a := -2\cdot\pi\cdot I\cdot(a1+a2)$

$$a := -\frac{1}{121}\,I\,\pi$$

$> solve(\{z^5 - 1 = 0\}, \{z\})$

$$\{z=1\}, \left\{z = \frac{1}{4}\sqrt{5} - \frac{1}{4} + \frac{1}{4} I\sqrt{2}\sqrt{5+\sqrt{5}}\right\}, \left\{z = -\frac{1}{4}\sqrt{5} - \frac{1}{4} + \frac{1}{4} I\sqrt{2}\sqrt{5-\sqrt{5}}\right\}, \left\{z = -\frac{1}{4}\sqrt{5} - \frac{1}{4} - \frac{1}{4} I\sqrt{2}\sqrt{5-\sqrt{5}}\right\}, \left\{z = \frac{1}{4}\sqrt{5} - \frac{1}{4} - \frac{1}{4} I\sqrt{2}\sqrt{5+\sqrt{5}}\right\}$$

13.31-Masala. Quyidagi

$$\int_{\partial D} \sin\frac{z}{z+1}dz, \qquad D = \{|z| > 3\}$$

integralni hisoblang.

$\triangleleft \quad f(z) = \sin\dfrac{z}{z+1}$ deb, so‘ng (32)-formuladan foydalanib topamiz:

$$\int_{\partial D} f(z)dz = 2\pi i \cdot \underset{z=\infty}{res}\, f(z)$$

Endi $f(z)$ funksiyaning $z = \infty$ nuqtadagi chegirmasini (30)-formulaga ko‘ra hisoblaymiz:

$$\underset{z=\infty}{res}\, f(z) = \lim_{z\to\infty} z[f(\infty) - f(z)] = \lim_{z\leftarrow\infty} z(\sin 1 - \sin\frac{z}{z+1}) =$$

$$= \lim_{z\to\infty}\left(2z\cdot\cos\frac{1+\frac{z}{z+1}}{2}\cdot\sin\frac{1-\frac{z}{z+1}}{2} \right) =$$

$$= \lim_{z\to\infty}\left[\frac{2z}{2\cdot(z+1)}\cdot\cos\frac{2z+1}{2(z+1)}\cdot\frac{\sin\frac{1}{2(z+1)}}{\frac{1}{2(z+1)}}\right] = \cos 1.$$

Demak,

$$\int_{\partial D} \sin\frac{z}{z+1}\,dz = 2\pi i\cdot\cos 1. \qquad \triangleright$$

Bu misolni Maple matematik paketida yechishni ko‘rsatamiz.

> $f := \sin\left(\frac{z}{z+1}\right)$

$$f := \sin\left(\frac{z}{z+1}\right)$$

> $a := 2\cdot\pi\cdot I\cdot \mathit{residue}(f, z=\infty)$

$$a := 2\,\mathrm{I}\,\pi\cos(1)$$

>

>

> $\lim_{z\to\infty} f$

$$\sin(1)$$

> $a1 := \lim_{z\to\infty} z\cdot(\sin(1) - f)$

$$a1 := \cos(1)$$

> $a := 2\,\pi I\cdot a1$

$$a := 2\,\pi I\cos(1)$$

14.21-Masala. Quyidagi

$$\int_0^{\pi} \frac{\cos^2 x}{2-\sin^2 x}\,dx$$

aniq integralni chegirmalar yordamida hisoblang.

$\triangleleft$ Bu integralda $e^{2ix} = z$ almashtirishni bajarsak, $x\in[0,\pi] \Rightarrow z\in\{z\in C : |z| = 1\}$,

$$dx = \frac{1}{2iz}dz,$$

$$\cos^2 x = \frac{1+\cos 2x}{2} = \frac{1+\frac{1}{2}(z+\frac{1}{z})}{2},$$

$$\sin^2 x = \frac{1-\cos 2x}{2} = \frac{1-\frac{1}{2}(z+\frac{1}{z})}{2}$$

bo‘lib,

$$\int_0^{\pi} \frac{\cos^2 x dx}{2-\sin^2 x} = \frac{1}{2i}\oint_{|z|=1} \frac{1}{z}\cdot \frac{\frac{1+\frac{1}{2}(z+\frac{1}{z})}{2}}{2-\frac{1-\frac{1}{2}(z+\frac{1}{z})}{2}} dz =$$

$$= \frac{1}{2i}\oint_{|z|=1} \frac{1}{z}\cdot \frac{(z+1)^2}{z^2+6z+1} dz$$

tenglik o‘rinlidir.

Integral ostidagi

$$f(z) = \frac{(z+1)^2}{z(z^2+6z+1)} = \frac{(z+1)^2}{z\cdot[z-(-3+2\sqrt{2})]\cdot[z-(-3-2\sqrt{2})]}$$

funksiyaning $z_0 = 0$, $z_1 = -3+2\sqrt{2}$, $z_2 = -3-2\sqrt{2}$ maxsus nuqtalari bo‘lib, ulardan $z_0 = 0$ va $z_1 = -3+2\sqrt{2}$ lar $\{|z|<1\}$ sohaga tegishli bo‘lgan qutb nuqtalaridir.

Koshi teoremasini ((32)-formulani) qo‘llab, topamiz:

$$\oint_{|z|=1} f(z)dz = 2\pi i[\underset{z=0}{res}\, f(z) + \underset{z=z_1}{res}\, f(z)] = 2\pi i[\frac{1}{z_1 z_2} +$$

$$+\frac{1}{z_1}\cdot\frac{(z_1+1)^2}{z_1-z_2}] = 2\pi i[1+\frac{1}{-3+2\sqrt{2}}\cdot\frac{(-3+2\sqrt{2}+1)^2}{4\sqrt{2}}] =$$

$$= 2\pi i(1-\frac{1}{\sqrt{2}}).$$

Demak,

$$\int_0^{\pi} \frac{\cos^2 x}{2-\sin^2 x} dx = \pi \cdot (1 - \frac{1}{\sqrt{2}}). \qquad \triangleright$$

>

> $\int_0^{\pi} \frac{(\cos(x))^2}{2-(\sin(x))^2} \, dx$

$$\pi - \frac{1}{2}\sqrt{2}\,\pi$$

>

15.21-Masala. Quyidagi chegarasi cheksiz boʻlgan

$$\int_0^{+\infty} \frac{dx}{(x^2+1)^n} \qquad (n \in N)$$

integralni chegirmalar yordamida hisoblang.

$\triangleleft$ Avvalo berilgan integralni

$$\int_0^{+\infty} \frac{dx}{(x^2+1)^n} = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^n}$$

koʻrinishda yozib olamiz.

Endi

$$f(z) = \frac{1}{(z^2+1)^n} = \frac{1}{(z+i)^n \cdot (z-i)^n}$$

desak, bu funksiya

$$\{z \in C : \operatorname{Im} z > 0\} \; da \; z = i$$

maxsus nuqtaga, n -tartibli qutbga ega.

Ravshanki,

$$\lim_{r\to\infty} r \cdot \max_{\gamma_r} f(z) = 0 \quad (\gamma_r = \{|z| = r, 0 \le \arg z \le \pi\}) \Rightarrow$$

Jordanning 1-lemmasiga ko‘ra $\lim\limits_{r\to\infty}\int\limits_{\gamma_r} f(z)dz = 0$ bo‘ladi. Unda 10^0-punktdagi teoremaga ko‘ra

$$\int\limits_{-\infty}^{+\infty}\frac{dx}{(x^2+1)^n} = 2\pi i \mathop{res}_{z=i} f(z)$$

bo‘ladi.

(29)-formuladan foydalanib topamiz:

$$\mathop{res}_{z=i} f(z) = \lim_{z\to i}\frac{1}{(n-1)!}\frac{d^{n-1}}{dz^{n-1}}[(z-i)^n\cdot f(z) =$$

$$= \frac{1}{(n-1)!}\lim_{z\to i}\frac{d^{n-1}}{dz^{n-1}}[\frac{1}{(z+i)^n}] = \frac{(2n-3)!!}{(2n-2)!!}\cdot\frac{1}{2i}.$$

Natijada

$$\int\limits_{-\infty}^{+\infty}\frac{dx}{(x^2+1)^n} = 2\pi i\cdot\frac{1}{2i}\cdot\frac{(2n-3)!!}{(2n-2)!!} = \frac{(2n-3)!!}{(2n-2)!!}\cdot\pi$$

bo‘lib, berilgan integral uchun

$$\int\limits_{0}^{+\infty}\frac{dx}{(x^2+1)^n} = \frac{(2n-3)!!}{(2n-2)!!}\cdot\frac{\pi}{2}$$

bo‘lishini topamiz ▹

>

> $\int_0^\infty \frac{1}{(x^2+1)^n}\,dx$

$$\frac{1}{2}\,\frac{\sqrt{\pi}\,\Gamma\left(-\frac{1}{2}+n\right)}{\Gamma(n)}$$

>

16.21-Masala. Quyidagi

$$\int\limits_{-\infty}^{+\infty}\frac{(x+1)\sin 2x}{x^2+2x+2}dx$$

integralni Jordan lemmalaridan foydalanib hisoblang.

◁ Bu masalani yechish uchun Jordanning 2-lemmasi va (41)-formuladan foydalanamiz. $f(z)$ funksiya deb

$$f(z) = \frac{(z+1)e^{2iz}}{z^2+2z+2} = \frac{(z+1)\cdot e^{2iz}}{[z-(-1+i)]\cdot[z-(-1-i)]}$$

funksiyani olamiz. Bu funksiyaning ikkita $z_1 = -1+i$ *va* $z_2 = -1-i$ qutb nuqtalari bo‘lib, ulardan $z_1 = -1+i \in \{\operatorname{Im} z > 0\}$ bo‘ladi.

$R(z) = \dfrac{z+1}{z^2+2z+2}$ funksiya uchun $z \to \infty$ da $R(z) \cong \dfrac{1}{z}$ bo‘lganidan Jordanning 2-lemmasi shartining bajarilishi ta’minlanadi va lemmaga ko‘ra

$$\lim_{r\to\infty} \int_{\gamma_r} R(z)e^{2iz}dz = \lim_{r\to\infty} \int_{\gamma_r} f(z)dz = 0$$

tenglik bajariladi, bunda $\gamma_r = \{|z| = r,\ 0 < \arg z < \pi\}$.

Unda (41)-formulaga ko‘ra

$$\int_{-\infty}^{+\infty} \frac{(x+1)\sin 2x}{x^2+2x+2}dx = 2\pi \cdot \operatorname{Re}[\underset{z=z_1}{res} f(z)]$$

bo‘ladi. (27)-formuladan foydalanib $\underset{z=z_1}{res} f(z)$ ni hisoblaymiz:

$$\underset{z=z_1}{res} f(z) = \lim_{z\to z_1}(z-z_1)f(z) = \lim_{z\to z_1}\left[(z-z_1)\cdot\frac{(z+1)e^{2iz}}{(z-z_1)(z-z_2)}\right] =$$

$$= \frac{(z_1+1)e^{2iz_1}}{(z_1-z_2)} = \frac{ie^{-2-2i}}{2i} = \frac{e^{-2}}{2}(\cos 2 - i\sin 2).$$

Demak,

$$\int_{-\infty}^{+\infty} \frac{(x+1)\sin 2x}{x^2+2x+2}dx = 2\pi\cdot\operatorname{Re}[\frac{e^{-2}}{2}(\cos 2 - i\sin 2)] = \pi e^{-2}\cos 2. \qquad \triangleright$$

>

> $\int_{-\infty}^{\infty} \frac{(x+1)\cdot\sin(2\cdot x)}{x^2+2\cdot x+2}\,\mathrm{d}x$

$$\frac{1}{2}\,\pi\cos(2-2\,\mathrm{I}) - \frac{1}{2}\,\mathrm{I}\,\pi\sin(2-2\,\mathrm{I}) + \frac{1}{2}\,\pi\cos(2+2\,\mathrm{I}) + \frac{1}{2}\,\mathrm{I}\,\pi\sin(2+2\,\mathrm{I})$$

> **simplify( );**

$$\pi\cos(2)\,(\cosh(2)-\sinh(2))$$

>

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Kompleks o‘zgaruvchili funksiyalar nazariyasi fanidan
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