3-mavzu: Darsda yechiladigan misollar

1-misol. $\vec{a}(1,\ 2,\ 3)$ $\vec{b}(3,\ 1,\ -3)$ $\vec{c}(2,\ 0,\ -2)$ vektorlarning qaysi jufti perpendikulyar?

Yechish $\vec{a}\cdot\vec{b}$, $\vec{a}\cdot\vec{c}$, $\vec{b}\cdot\vec{c}$ skalyar ko'paytmalarini tekshiramiz:

$\vec{a}\cdot\vec{b}=1\cdot3+2\cdot1+3\cdot(-3)=-4$ $\vec{a}\cdot\vec{c}=2+0-4=-2$ $\vec{b}\cdot\vec{c}=6+0+6=12$

Bundan $\vec{a}\perp\vec{b}$.

2-misol. $\vec{a}(1,\ -1,\ 0)$ $\vec{b}(1,\ -2,\ 2)$ vektor orasidagi burchakni toping.

Yechish (3.8) formuladan foydalanamiz.

$\cos(\vec{a}\wedge\vec{b})=\dfrac{1+2+0}{\sqrt{1+1+0}\cdot\sqrt{1+4+4}}=\dfrac{\sqrt{2}}{2}$. Bundan $(\vec{a}\wedge\vec{b})=\varphi=45^0$.

1-masala. N nuqta ABC uchburchakning og'irlik markazi bo'lsin, fazoda ixtiyoriy O nuqta olib

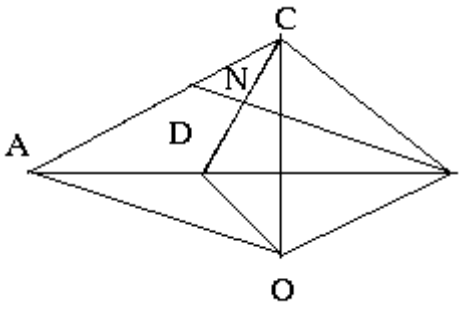
$$\overrightarrow{ON}=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}) \qquad (3.10)$$

ekanligini isbotlang.

Isboti. D nuqta AB kesmaning o'rta nuqtasi uchburchak qoidasiga asosan (13-chizma)

$\overrightarrow{ON}=\overrightarrow{OC}+\overrightarrow{CN};\quad \overrightarrow{CN}=\dfrac{2}{3}\overrightarrow{CD})$ bo'lgani uchun

$\overrightarrow{ON}=\overrightarrow{OC}+\dfrac{2}{3}\overrightarrow{CD}=\overrightarrow{OC}+\dfrac{2}{3}(\overrightarrow{OD}-\overrightarrow{OC})=\dfrac{1}{3}\overrightarrow{OC}+\dfrac{2}{3}\overrightarrow{OD}$



13-chizma

(3.9) formuladan va $\overrightarrow{OD}=\dfrac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB})$ vektor qiymatidan foydalanib, (3.10) formulani hosil qilamiz.

2-masala. C burchagi to'g'ri bo'lgan ABC uchburchak uchun Pifagor teoremasini isbotlang.

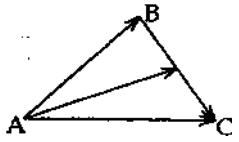
Isbot. $\overrightarrow{CB}=\vec{a},\ \overrightarrow{CA}=\vec{b},\ \overrightarrow{AB}=\vec{c}$ belgilaylik. Uchburchak qoidasiga asosan (14-chizma) $\vec{c}=\vec{a}-\vec{b}$ bu tenglikni ikala tomonini kvadratga ko'tarib topamiz.

$\vec{c}^2=\vec{a}^2-\vec{b}^2-2\vec{a}\vec{b}$ $\angle C=90^0$ bo'lgani uchun $\vec{a}\cdot\vec{b}=0$.

Shunday qilib, $\vec{c}^2=\vec{a}^2+\vec{b}^2$.

3-masala. ABC uchburchakning AB=c, AC=b va $\angle A$ berilsa m_a mediana uzunligini hisoblang.

Isbot. N nuqta $\overrightarrow{BC}$ kesmaning o'rta nuqtasi bo'lsa, (3.9) ga ko'ra bu tenglikni kvadratga ko'tarib quyidagiga ega bo'lamiz. (15-chizma)



15-chizma.

$\overrightarrow{AN}^2=\frac{1}{4}(\overrightarrow{AB}+\overrightarrow{AC})^2=\frac{1}{4}(\overrightarrow{AB}^2+\overrightarrow{AC}^2+2\overrightarrow{AB}\cdot\overrightarrow{AC})$ $|\overrightarrow{AB}|=c,\ |\overrightarrow{AC}|=b,\ |\overrightarrow{BC}|=a$

belgilasak (15-chizma).

$\overrightarrow{AN}=m_a=\sqrt{a^2+b^2+2ab\cos\varphi}$.

5 - masala. ABC uchburchak va fazoda O nuqta berilgan bo'lsin. O nuqta uchburchakning og'irlik markazi bo'lishi uchun $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\vec{0}$ bo'lishi zarur va yetarlidir.

Isbot. N nuqta ABC uchburchakning ogirlik markazi bo'lsin (13 chizma). Agar O nuqta N nuqta bilan ustma - ust tushsa, u holda $\overrightarrow{ON}=\vec{0}$, (3.10) formuladan foydalansak $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\vec{0}$ ega bo'lamiz.

3-misol. Uchlari A(1, 2), B(0, 5), C(-2, 3) nuqtalarda bo'lgan uchburchaklarning medianalar kesishgan nuqtasini toping.

Yechish AD mediana D(x, y) nuqta BC tomon o'rta nuqtasi x_D=-1, y_D=4, D(-1, 4).

Uchburchak medianalar kesishgan nuqtasi O(x, y) bo'lsin, u holda

$\frac{AO}{OD}=\lambda=2:1,\ \lambda=2$

$x=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{1+2(-1)}{3}=-\frac{1}{3}$

$y=\frac{y_1+\lambda y_2}{1+\lambda}=\frac{2+2\cdot 4}{3}=\frac{10}{3}$

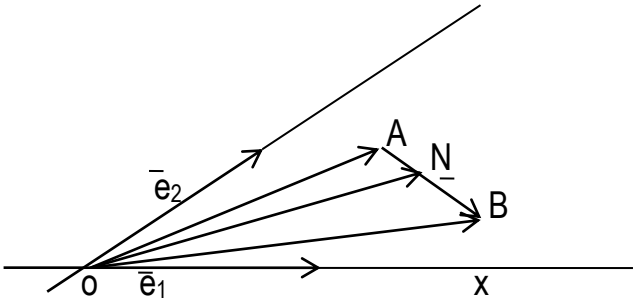
Demak, $O(-\frac{1}{3},\ \frac{10}{3})$.

5-masala. AB vektorlarining boshi $A(x_1, y_1)$ va oxiri $B(x_2, y_2)$ koordinatalari bilan berilgan bo'lsa, $\overrightarrow{AB}$ vektor koordinatasini toping.

Yechish: $\overrightarrow{OA} = x_1\vec{e}_1 + y_1\vec{e}_2$

$\overrightarrow{OB} = x_2\vec{e}_1 + y_2\vec{e}_2 \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (x_2 - x_1)\vec{e}_1 - (y_2 - y_1)\vec{e}_2$ bundan

$\overrightarrow{AB}(x_2 - x_1;\ y_2 - y_1)$



Tekshirish uchun savollar va mashqlar

1. Tekislikdagi koordinatalar sistemasi deb nimaga aytiladi?
2. Affin va Dekart koordinatalar sistemasini ta'riflang.
3. Dekart koordinatalar sistemasining affin koordinatalar sistemasidan farqini tushuntiring.
4. Affin koordinatalar sistemasini olib, ushbu nuqtalarni yasang:

   A(2, 1), B($\frac{1}{2}$, -1), C(-1, 4), D($\sqrt{2}$, -2), E(0, 1), F(2, 0), G(-3, -2).
5. A(2, 5), B(1, -1), C(2, -2), D(1, 7) nuqtalar berilgan. $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{DB}$, $\overrightarrow{BC}$ va $\overrightarrow{CA}$ vektorlarning koordinatalarini toping.
6. $\vec{a}(3, 4)$ vektorni boshi A(-2, 3) nuqtada oxiri B(x, y) nuqta koordinatasini toping.

   Javob: (1, 7)
7. Uchlari A(2, 3), B(-1, 2) nuqtalarda bo'lgan AB kesmani ushbu nisbatlarda $\lambda_1 = 1,\ \lambda_2 = -2,\ \lambda_3 = \frac{1}{2},\ \lambda_4 = 3$ bo'luvchi nuqtaning koordinatalarini toping.

   Javob: ($\frac{1}{2}$, $\frac{5}{2}$), (-4, 1), (1, $\frac{8}{3}$), ($-\frac{1}{4}$, $\frac{9}{4}$).
8. Uchburchak tomonlarining o'rtalari P(3, -2), Q(1, 6), C(-4, 2) Nuqtalarda bo'lsa, uning uchlarining koordinatalarini toping.

9. Uchburchak og’irlik markazining koordinatalari, uning uchining koordinatalari bilan qanday ifodalanadi.

Javob: $x = \frac{x_1 + x_2 + x_3}{3}$, $y = \frac{y_1 + y_2 + y_3}{3}$.