

Vektorlar vektorlar ustida amallar. Vektor fazo.

Darsda yechiladigan misollar

1. $\alpha(\vec{a} + \vec{b}) = \alpha\vec{a} + \alpha\vec{b}$ tenglikni isbotlang.
2. Berilgan $\vec{a} = 2\vec{i} - \vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} - 3\vec{j} + 2\vec{k}$, $\vec{c} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ vektorlar uchun $(\vec{x}, \vec{a}) = -5$, $(\vec{x}, \vec{b}) = -11$, $(\vec{x}, \vec{c}) = 20$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorni toping.
3. Uchburchakning $A(-1; -2; 4)$, $B(-4; -2; 0)$ va $C(3; -2; 1)$ uchlari berilgan. Uning B uchidagi burchagini toping.
4. $(\alpha + \mu)\vec{a} = \alpha\vec{a} + \mu\vec{a}$ tenglikni isbotlang.
5. Uchburchakning $A(3; 2; -3)$, $B(5; 1; -1)$ va $C(1; -2; 1)$ uchlari berilgan. A uchining tashqi burchagini toping.
6. Berilgan $\vec{a} = 2\vec{i} - \vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} - 3\vec{j} + 2\vec{k}$, $\vec{c} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ vektorlarga qurilgan parallelepiped hajmini toping.
7. $(\vec{a} + \vec{b}, \vec{c}) = (\vec{a}, \vec{c}) + (\vec{b}, \vec{c})$ tenglikni isbotlang.
8. Berilgan $\vec{a} = \alpha\vec{i} - 3\vec{j} + 2\vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - \alpha\vec{k}$ vektorlar perpendikulyar bo'lishi uchun α ning qiymati qanday bo'lishi kerak.
9. Berilgan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi φ burchak $\frac{\pi}{6}$ ga tengligi va $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 1$ ekanligi ma'lum bo'lsa, $\vec{p} = \vec{a} + \vec{b}$ va $\vec{q} = \vec{a} - \vec{b}$ vektorlarga qurilgan parallelogram yuzasi topilsin.
10. $[\lambda\vec{a}, \vec{b}] = [\vec{a}, \lambda\vec{b}] = \lambda[\vec{a}, \vec{b}]$ tenglikni isbotlang.
11. Uchburchakning $A(-1; -2; 4)$, $B(-4; -2; 0)$ va $C(3; -2; 1)$ uchlari berilgan. Uning B uchidagi tashqi burchagini toping.
12. $\vec{a} = \vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - \vec{j} + 3\vec{k}$ vektorlarga qurilgan parallelogramm yuzini toping.
13. Berilgan $\vec{a} = \{1, 0\}$, $\vec{b} = \{1, 1\}$ vektorlar orqali $\vec{c} = \{-1, 0\}$ vektorni chiziqli ifodalang.
14. Uchburchakning $A(3; 4; -1)$, $B(2; 0; 3)$ va $C(-3; 5; 4)$ uchlari berilgan. Uchburchakning yuzi hisoblansin.
15. Fazoda $M(-5; 7; -6)$ va $N(7; -9; 9)$ nuqtalar berilgan. Berilgan $\vec{a} = \{1; -3; 1\}$ vektorning $\overrightarrow{MN}$ vektor yo'nalishdagi o'qqa proeksiyasini toping.

Mavzu: 1. Vektorlar va ular ustida amallar

1.1. ([1], 1.2.1 i), p14) $\vec{a}, \vec{b}, \vec{c}$ chiziqli erkli vektorlar berilgan. i) $\vec{l} = 2\vec{b} - \vec{c} - \vec{a}$, $\vec{m} = 2\vec{a} - \vec{b} - \vec{c}$, $\vec{n} = 2\vec{c} - \vec{a} - \vec{b}$ vektorlarni chiziqli erklilikka tekshiring.

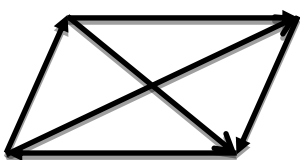
Yechilishi: Quyidagi vector tenglikni qaraymiz: $\lambda\vec{l} + \mu\vec{m} + \nu\vec{n} = \vec{a}(-\lambda + 2\mu - \nu) + \vec{b}(2\lambda - \mu - \nu) + \vec{c}(-\lambda - \mu + 2\nu) = \vec{0}$. Bu yerda $\vec{a}, \vec{b}, \vec{c}$ chiziqli erkli bo'lgani uchun

$$\begin{cases} -\lambda + 2\mu - \nu = 0 \\ 2\lambda - \mu - \nu = 0 \\ -\lambda - \mu + 2\nu = 0 \end{cases}$$

Tenglamalar sistemasiga ega bo'lamiz va noma'lumlar uchun $\lambda = \mu = \nu$ munosabat urinli bo'lib, ular noldan farqli bo'la oladi. Demak, $\vec{l}, \vec{m}, \vec{n}$ –chiziqli bog'liq vektorlar.

1.2.([2],1159) $\vec{AC} = \mathbf{a}, \vec{BD} = \mathbf{b}$ vektorlar $ABCD$ parallelogrammning diagonallari. Shu parallelogrammning tomonlari bo'lgan $\vec{AB}, \vec{BC}, \vec{CD}, \vec{DA}$ vektorlarni $\mathbf{a}, \mathbf{b}$ vektorlar orqali ifodalang.

Yechilishi: Avvalo $ABCD$ parallelogrammni chizib olamiz. AC va BD diagonallar kesishish nuqtasini O bilan belgilaymiz. Bizga ma'lumki parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo'linadi, bundan



quyidagi munosabatlarni hosil qilamiz:

$$\vec{AO} = \vec{OC} = \frac{1}{2}\mathbf{a}, \quad \vec{BO} = \vec{OD} = \frac{1}{2}\mathbf{b},$$

$$\triangle ABO \Rightarrow \vec{AB} = \vec{AO} - \vec{BO} = \frac{\mathbf{a}-\mathbf{b}}{2},$$

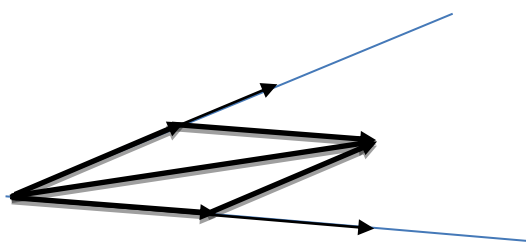
$$\triangle BCO \Rightarrow \vec{BC} = \vec{BO} - \vec{OC} = \frac{\mathbf{a}+\mathbf{b}}{2},$$

$$\triangle CDO \Rightarrow \vec{CD} = \vec{OD} - \vec{OC} = \frac{\mathbf{b}-\mathbf{a}}{2}, \quad \triangle DAO \Rightarrow \vec{DA} = -\vec{AO} - \vec{OD} = -\frac{\mathbf{a}+\mathbf{b}}{2},$$

Javob: $\vec{AB} = \frac{\mathbf{a}-\mathbf{b}}{2}, \vec{BC} = \frac{\mathbf{a}+\mathbf{b}}{2}, \vec{CD} = \frac{\mathbf{b}-\mathbf{a}}{2}, \vec{DA} = -\frac{\mathbf{a}+\mathbf{b}}{2}.$

1.3.([2],1169) O nuqtadan ikkita $\vec{AC} = \mathbf{a}, \vec{BD} = \mathbf{b}$ vektor chiqadi. AOB burchakning bissektrisasi bo'ylab yo'nalgan biror $\vec{OM}$ vektor topilsin.

Yechilishi: AOB burchakni chizib olamiz. $\mathbf{a}$ va $\mathbf{b}$ vektorlar yordamida



burchak uchidan chiqib, uning tomonlari bo'ylab yo'nalgan birlik vektorlarni topamiz. Buning uchun $\mathbf{a}$ va $\mathbf{b}$ vektorlarni mos ravishda $\frac{1}{|\mathbf{a}|}$ va $\frac{1}{|\mathbf{b}|}$ sonlarga

ko'paytiramiz. Natijada $\mathbf{e}_1 = \vec{OA_1} = \frac{\mathbf{a}}{|\mathbf{a}|}$ va $\mathbf{e}_2 = \vec{OB_1} = \frac{\mathbf{b}}{|\mathbf{b}|}$ birlik vektorlarni hosil qilamiz. $\mathbf{e}_1$ va $\mathbf{e}_2$ vektorlar orqali OA_1MB_1 rombni tuzib olamiz. Bizga ma'lumki rombning diagonallari uning burchaklari bissektrisasi bo'ladi. Demak OM nur AOB burchak bissektrisasi hamda

$\overrightarrow{OM} = \mathbf{e}_1 + \mathbf{e}_2 = \frac{a}{|a|} + \frac{b}{|b|}$ AOB burchakning bissektrisasi bo'ylab yo'nalgan vektor. **Javob:** $\overrightarrow{OM} = \frac{a}{|a|} + \frac{b}{|b|}$

Darsda yechiladigan misollar

1.1.([2], 1161) [A] ABC uchburchakda AD mediana o'tkazilgan. $\overrightarrow{AD}$ vektorni $\overrightarrow{AB}$, $\overrightarrow{AC}$ vektorlar orqali ifodalang.

1.2.([2], 1163) [A] E, F nuqtalar $ABCD$ to'rtburchak AB, CD tomonlarining o'rtalari.

$\overrightarrow{EF} = \frac{\overrightarrow{BC} + \overrightarrow{AD}}{2}$ ekanligi isbotlansin. Bundan trapetsiyaning o'rta chizig'i haqidagi teoremani keltirib chiqaring.

1.3.([2], 1165) [A] Muntazam ko'pburchak markazidan uning uchlariga qarab yo'naltirilgan vektorlar yig'indisi 0 ga tengligi isbotlansin.

1.4.([2], 1167) [A] Uchburchak tekisligida shunday nuqta topilsinki, shu nuqtadan uchburchak uchlariga yo'nalgan vektorlar yig'indisi nolga teng bo'lsin.

1.5.([1], 1.2.1 ii), p14). $\vec{a}, \vec{b}, \vec{c}$ chiziqli erkli vektorlar berilgan. $\vec{s} = \vec{a} + \vec{b} + \vec{c}$ vektorni $\vec{l}' = \vec{b} - 2\vec{c} + \vec{a}$, $\vec{m}' = \vec{a} - \vec{b}$, $\vec{n}' = 2\vec{b} + 3\vec{c}$ vektorlar orqali chiziqli ifodalang.

Uyga vazifa

1.([2], 1160)[U] K, L nuqtalar $ABCD$ parallelogrammning BC, CD tomonlarining o'rtalari. $\overrightarrow{AK} = \mathbf{k}$, $\overrightarrow{AL} = \mathbf{l}$ deb $\overrightarrow{BC}$, $\overrightarrow{CD}$ vektorlarni $\mathbf{k}$ va $\mathbf{l}$ vektorlar orqali ifodalang.

2.([2], 1162) [U] ABC uchburchakda AD, BE, CF medianalar o'tkazilgan. $\overrightarrow{AD} + \overrightarrow{BC} + \overrightarrow{CF}$ vektorlar yig'indisi topilsin.

3.([2], 1164) [U] $\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AF} = \mathbf{q}$ vektorlar muntazam $ABCDEF$ oltiburchakning ikkita qo'shni tomonlari. Bu oltiburchakning tomonlari bo'ylab qo'yilgan $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$ vektorlarni $\mathbf{p}, \mathbf{q}$ vektorlar orqali ifodalang.

4.([2], 1166) [U] Tekislikning ixtiyoriy nuqtasidan chiqib, muntazam ko'pburchak markazini tutashtiruvchi vektor shu nuqtadan chiquvchi va ko'pburchak uchlarini tutashtiruvchi vektorlarning o'rta arifmetigiga teng ekanligi isbotlansin.

5.([2], 1168) [U] 1167 masala parallelogramm uchun yechilsin.