Vektorlar vektorlar ustida amallar. Vektor fazo.

Darsda yechiladigan misollar

1. $\alpha\left(\vec{a}+\vec{b}\right)=\alpha\vec{a}+\alpha\vec{b}$ tenglikni isbotlang.
2. Berilgan $\vec{a}=2\vec{i}-\vec{j}+3\vec{k}$, $\vec{b}=\vec{i}-3\vec{j}+2\vec{k}$, $\vec{c}=3\vec{i}+2\vec{j}-4\vec{k}$ vektorlar uchun $\left(\vec{x},\vec{a}\right)=-5$, $\left(\vec{x},\vec{b}\right)=-11$, $\left(\vec{x},\vec{c}\right)=20$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorni toping.
3. Uchburchakning $A(-1;-2;4)$, $B(-4;-2;0)$ va $C(3;-2;1)$ uchlari berilgan. Uning B uchidagi burchagini toping.
4. $(\alpha+\mu)\vec{a}=\alpha\vec{a}+\mu\vec{a}$ tenglikni isbotlang.
5. Uchburchakning $A(3;2;-3)$, $B(5;1;-1)$ va $C(1;-2;1)$ uchlari berilgan. A uchining tashqi burchagini toping.
6. Berilgan $\vec{a}=2\vec{i}-\vec{j}+3\vec{k}$, $\vec{b}=\vec{i}-3\vec{j}+2\vec{k}$, $\vec{c}=3\vec{i}+2\vec{j}-4\vec{k}$ vektorlarga qurilgan parallelipiped hajmini toping.
7. $\left(\vec{a}+\vec{b},\vec{c}\right)=\left(\vec{a},\vec{c}\right)+\left(\vec{b},\vec{c}\right)$ tenglikni isbotlang.
8. Berilgan $\vec{a}=\alpha\vec{i}-3\vec{j}+2\vec{k}$ $\vec{b}=\vec{i}+2\vec{j}-\alpha\vec{k}$ vektorlar perpendikulyar bo'lishi uchunn α ning qiymati qanday bo'lishi kerak .
9. Berilgan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi φ burchak $\frac{\pi}{6}$ ga tengligi va $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=1$ ekanligi ma'lum bo'lsa, $\vec{p}=\vec{a}+\vec{b}$ va $\vec{q}=\vec{a}-\vec{b}$ vektorlarga qurilgan parallelogram yuzasi topilsin topilsin.
10. $\left[\lambda\vec{a},\vec{b}\right]=\left[\vec{a},\lambda\vec{b}\right]=\lambda\left[\vec{a},\vec{b}\right]$ tenglikni isbotlang.
11. Uchburchakning $A(-1;-2;4)$, $B(-4;-2;0)$ va $C(3;-2;1)$ uchlari berilgan. Uning B uchidagi tashqi burchagini toping.
12. $\vec{a}=\vec{i}-3\vec{j}+\vec{k}$, $\vec{b}=2\vec{i}-\vec{j}+3\vec{k}$ vektorlarga qurilgan parallelogramm yuzini toping.
13. Berilgan $\vec{a}=\{1,0\},\vec{b}=\{1,1\}$ vektorlar orqali $\vec{c}=\{-1,0\}$ vektorni chiziqli ifodalang.
14. Uchburchakning $A(3;4;-1)$, $B(2;0;3)$ va $C(-3;5;4)$ uchlari berilgan. Uchburchakning yuzi hisoblansin.
15. Fazoda $M(-5;7;-6)$ va $N(7;-9;9)$ nuqtalar berilgan.Berilgan $\vec{a}=\{1;-3;1\}$ vektorning $\overrightarrow{MN}$ vektor yo'nalishdagi o'qqa proeksiyasini toping.

Mavzu: 1.**Vektorlar va ular ustida amallar**

1.1. ([1], 1.2.1 i),p14) .$\vec{a},\vec{b},\vec{c}$ chiziqli erkli vektorlar berilgan. i) $\vec{l}=2\vec{b}-\vec{c}-\vec{a}$, $\vec{m}=2\vec{a}-\vec{b}-\vec{c}$, $\vec{n}=2\vec{c}-\vec{a}-\vec{b}$ vektorlarni chiziqli erklilikka tekshiring.

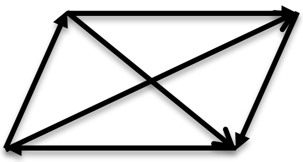
Yechilishi: Quyidagi vector tenglikni qaraymiz: $\lambda\vec{l}+\mu\vec{m}+\nu\vec{n}=\vec{a}(-\lambda+2\mu-\nu)+\vec{b}(2\lambda-\mu-\nu)+\vec{c}(-\lambda-\mu+2\nu)=\vec{0}$. Bu yerda $\vec{a},\vec{b},\vec{c}$ chiziqli erkli bo'lgani uchun

$$\begin{cases} -\lambda + 2\mu - \nu = 0 \\ 2\lambda - \mu - \nu = 0 \\ -\lambda - \mu + 2\nu = 0 \end{cases}$$

Tenglamalar sistemasiga ega bo'lamiz va noma'lumlar uchun $\lambda = \mu = \nu$ munosabat urinli bo'lib, ular noldan farqli bo'la oladi. Demak, $\vec{l},\ \vec{m},\ \vec{n}$ –chiziqli bog'liq vektorlar.

1.2.(**[2],1159**) $\overrightarrow{AC} = \boldsymbol{a}, \overrightarrow{BD} = \boldsymbol{b}$ vektorlar $ABCD$ parallelogrammning diagonallari. Shu parallelogrammning tomonlari bo'lgan $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}$ vektorlarni **a, b** vektorlar orqali ifodalang.

Yechilishi: Avvalo $ABCD$ parallelogrammni chizib olamiz. AC va BD diagonallar kesishish nuqtasini O bilan belgilaymiz.Bizga ma'lumki parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo'linadi, bundan quyidagi munosabatlarni hosil qilamiz:



$\overrightarrow{AO} = \overrightarrow{OC} = \frac{1}{2}\boldsymbol{a},\ \ \overrightarrow{BO} = \overrightarrow{OD} = \frac{1}{2}\boldsymbol{b},$

$\Delta ABO \Rightarrow \overrightarrow{AB} = \overrightarrow{AO} - \overrightarrow{BO} = \frac{\boldsymbol{a}-\boldsymbol{b}}{2},$

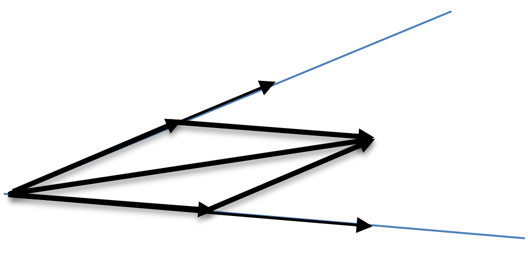
$\Delta BCO \Rightarrow \overrightarrow{BC} = \overrightarrow{BO} - \overrightarrow{OC} = \frac{\boldsymbol{a}+\boldsymbol{b}}{2},$

$\Delta CDO \Rightarrow \overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = \frac{\boldsymbol{b}-\boldsymbol{a}}{2},\quad \Delta DAO \Rightarrow \overrightarrow{DA} = -\overrightarrow{AO} - \overrightarrow{OD} = -\frac{\boldsymbol{a}+\boldsymbol{b}}{2},$

Javob: $\overrightarrow{AB} = \frac{\boldsymbol{a}-\boldsymbol{b}}{2},\ \overrightarrow{BC} = \frac{\boldsymbol{a}+\boldsymbol{b}}{2},\ \overrightarrow{CD} = \frac{\boldsymbol{b}-\boldsymbol{a}}{2},\ \overrightarrow{DA} = -\frac{\boldsymbol{a}+\boldsymbol{b}}{2}.$

1.3.(**[2],1169**) O nuqtadan ikkita $\overrightarrow{AC} = \boldsymbol{a},\ \overrightarrow{BD} = \boldsymbol{b}$ vektor chiqadi. AOB burchakning bissektrisasi bo'ylab yo'nalgan biror $\overrightarrow{OM}$ vektor topilsin.

Yechilishi: AOB burchakni chizib olamiz. a va b vektorlar yordamida burchak uchidan chiqib, uning tomonlari bo'ylab yo'nalgan birlik vektorlarni topamiz. Buning uchun $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarni mos ravishda $\frac{1}{|\boldsymbol{a}|}$ va $\frac{1}{|\boldsymbol{b}|}$ sonlarga ko'paytiramiz. Natijada $\boldsymbol{e_1} = \overrightarrow{OA_1} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$ va $\boldsymbol{e_2} = \overrightarrow{OB_2} = \frac{\boldsymbol{b}}{|\boldsymbol{b}|}$



birlik vektorlarni hosil qilamiz. $\boldsymbol{e_1}$va $\boldsymbol{e_2}$ vektorlar orqali OA_1MB_1 rombni tuzib olamiz. Bizga ma'lumki rombning dioganallari uning burchaklari bissektrisasi bo'ladi. Demak OM nur AOB burchak bissektrisasi hamda

$\overrightarrow{OM} = \boldsymbol{e_1} + \boldsymbol{e_2} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|} + \frac{\boldsymbol{b}}{|\boldsymbol{b}|}$ AOB burchakning bissektrisasi bo'ylab yo'nalgan vektor.

Javob: $\overrightarrow{OM} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|} + \frac{\boldsymbol{b}}{|\boldsymbol{b}|}$

Darsda yechiladigan misollar

1.1.([2], 1161) [A] *ABC* uchburchakda *AD* mediana o'tkazilgan. $\overrightarrow{AD}$ vektorni $\overrightarrow{AB}$, $\overrightarrow{AC}$ vektorlar orqali ifodalang.

1.2.([2], 1163) [A] *E, F* nuqtalar *ABCD* to'rtburchak *AB, CD* tomonlarining o'rtalari. $\overrightarrow{EF} = \frac{\overrightarrow{BC} + \overrightarrow{AD}}{2}$ ekanligi isbotlansin. Bundan trapetsiyaning o'rta chizig'i haqidagi teoremani keltirib chiqaring.

1.3.([2], 1165) [A] Muntazam ko'pburchak markazidan uning uchlariga qarab yo'naltirilgan vektorlar yig'indisi 0 ga tengligi isbotlansin.

1.4.([2], 1167) [A] Uchburchak tekisligida shunday nuqta topilsinki, shu nuqtadan uchburchak uchlariga yo'nalgan vektorlar yig'indisi nolga teng bo'lsin.

1.5.([1], 1.2.1 ii),p14).$\vec{a}, \vec{b}, \vec{c}$ chiziqli erkli vektorlar berilgan. $\vec{s} = \vec{a} + \vec{b} + \vec{c}$ vektorni $\vec{l'} = \vec{b} - 2\vec{c} + \vec{a}$, $\overrightarrow{m'} = \vec{a} - \vec{b}$, $\vec{n'} = 2\vec{b} + 3\vec{c}$ vektorlar orqali chiziqli ifodalang.

Uyga vazifa

1.([2], 1160)[U] *K, L* nuqtalar *ABCD* parallelogrammning *BC, CD* tomonlarining o'rtalari. $\overrightarrow{AK} = \boldsymbol{k}$, $\overrightarrow{AL} = \boldsymbol{l}$ deb $\overrightarrow{BC}, \overrightarrow{CD}$ vektorlarni **k** va **l** vektorlar orqali ifodalang.

2.([2],1162) [U] *ABC* uchburchakda *AD, BE, CF* medianalar o'tkazilgan. $\overrightarrow{AD} + \overrightarrow{BC} + \overrightarrow{CF}$ vektorlar yig'indisi topilsin.

3.([2],1164) [U] $\overrightarrow{AB}$ **=p,** $\overrightarrow{AF}$ **=q** vektorlar muntazam *ABCDEF* oltiburchakning ikkita qo'shni tomonlari. Bu oltiburchakning tomonlari bo'ylab qo'yilgan $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$ vektorlarni ***p, q*** vektorlar orqali ifodalang.

4.([2], 1166) [U] Tekislikning ixtiyoriy nuqtasidan chiqib, muntazam ko'pburchak markazini tutashtiruvchi vektor shu nuqtadan chiquvchi va ko'pburchak uchlarini tutashtiruvchi vektorlarning o'rta arifmetigiga teng ekanligi isbotlansin.

5.([2], 1168) [U] 1167 masala parallelogramm uchun yechilsin.