19 - mavzu.Umumiy tenglamasi bilan berilgan ikkinchi tartibli chiziq. Ikkichi tartibli chiziqning to`g`ri chiziq bilan kesishishi.

Reja:

1. Umumiy tenglamasi bilan berilgan ikkinchi tartibli chiziq.
2. Ikkichi tartibli chiziqning to`g`ri chiziq bilan kesishishi.
3. Ikkinchi tartibli chiziqni uning tenglamasi bo’yicha yasash.

Ikkinchi tartibli chiziqlarning umumiy tenglamasi

Tekislikda biror affin (yoki dekart) reperda koordinatalari

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{10}x + 2a_{20}y + a_{00} = 0 \quad (57.1)$$

tenglamani qanoatlantiruvchi nuqtalar to’plami ikkinchi tartibli chiziq deb atalishi ma’lum[1] (20–§). Bunda a_{11}, a_{12}, a_{22}, a_{10}, a_{20}, a_{00} koyeffitsiyentlar haqiqiy sonlar bo’lib, a_{11}, a_{12}, a_{22} lardan kamida bittasi noldan farqlidir (bu shartni bundan buyon $a_{11}^2 + a_{12}^2 + a_{22}^2 \neq 0$ ko’rinishida yozamiz).

Biz 40 – 55 – § larda uchta chiziq ellips, giperbola va parabolani o’rgandik, bu chiziqlar ham ikkinchi tartibli chiziqlardir, chunki (57.1) tenglamada $a_{11} = \frac{1}{a^2}, a_{22} = \frac{1}{b^2}, a_{00} = 1$ bo’lib, qolgan barcha koeffitsiyentlar nol bo’lsa, u ellipsning kanonik tenglamasi, shu shartlarda yana $a_{22} = -\frac{1}{b^2}$ bo’lsa, (57.1) tenglama giperbolaning kanonik tenglamasi, $a_{10}=r$; $a_{22}=1$ bo’lib, qolgan koeffitsiyentlar nol bo’lsa, (57.1) tenglama parabolaning kanonik tenglamasidir.

Quydagi tabiiy savol tug’iladi: tekislikda ko’rilgan bu chiziqlardan boshqa yana ikkinchi tartibli chiziqlar bormi? Bu savolga quyida javob berishga harakat qilamiz. Avvalo shuni ta’kidlaymiz: 20–§ dan bizga ma’lumki, chiziqning tartibi koordinatalar sistemasining olinishiga bog’liq emas. Bundan foydalanib, koordinatalar sistemasini tegishlicha tanlash hisobiga barcha ikkinchi tartibli chiziqlarni to’la geometrik tavsiflab chiqamiz. Ikkinchi tartibli γ chiziq $Б = (O, \vec{i}, \vec{j})$ dekart reperida (57.1) umumiy tenglamasi bilan ifodalangan bo’lsin. Shunday reperni tanlaymizki, unga nisbatan γ chiziqning (57.1) tenglamasi mumkin qadar sodda – «kanonik» ko’rinishga ega bo’lsin, ya’ni

1) o’zgaruvchi koordinatalar ko’paytmasi qatnashgan had bo’lmasin;

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[1] Ikkinchi tartibli chiziqlarning umumiy nazariyasini dekart reperida qaraymiz.

2) birinchi darajali hadlar soni eng oz bo'lsin (iloji bo'lsa, ular butunlay qatnashmasin);

3) mumkin bo'lsa, ozod had qatnashmasin.

Agar (57.1) tenglamada $a_{12}\neq 0$ bo'lsa, soddalashtirishni quydagicha bajaramiz. B reperning o'qlarini 0 nuqta atrofida ixtiyoriy α burchakka burib, yangi $Б`=(O,\vec{i},\vec{j})$ Dekart reperini hosil qilamiz. $Б$ reperdan $Б`$reperga o'tish formulalari (15–§)

$$\begin{cases} x = x'\cos\alpha - y'\sin\alpha \\ y = y'\sin\alpha + y'\cos\alpha \end{cases} \qquad (57.2)$$

dan x,y ni (57.1) ga qo'ysak va o'xshash hadlarini ixchamlasak, γ chiziqni (57.1) tenglamasi B`reperda ushbu ko'rinishni oladi:

$$a'_{11}x'^2 + 2a'_{12}x'y' + a'_{22}y'^2 + 2a'_{10}x' + 2a'_{20}y' + a'_{00} = 0 \qquad (57.3)$$

бунда:

$$a`_{11} = a_{11}\cos^2\alpha + 2a_{12}\cos\alpha\sin\alpha + a_{12}\sin^2\alpha,$$

$$a`_{12} = -a_{11}\sin\alpha\cos\alpha + a_{12}\cos^2\alpha - a_{12}\sin^2\alpha + a_{22}\sin\alpha\cos\alpha, \qquad (57.4)$$

$$a`_{22} = a_{11}\sin^2\alpha - 2a_{12}\sin\alpha\cos\alpha + a_{22}\cos^2\alpha,$$

$$a`_{10} = a_{10}\cos\alpha + a_{20}\sin\alpha,\ a`_{20} = -a_{10}\sin\alpha + a_{20}\cos\alpha,\ a`_{00} = a_{00}.$$

(57.4) belgilashlardan ko'rinadiki, (57.3) tenglamadagi $a`_{11}$, $a`_{12}$, $a`_{22}$ koeffitsiyentlar (57.1) tenglamadagi $a_{11}$, $a_{12}$, $a_{22}$ koefitsiyentlarga va α burchakka bog'liq, shu bilan birga $a`_{11}$, $a`_{12}$, $a`_{22}$ ning kamida biri noldan farqli, chunki

$$\begin{vmatrix} \cos^2\alpha & 2\cos\alpha\sin\alpha & \sin^2\alpha \\ -\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha & \sin\alpha\cos\alpha \\ \sin^2\alpha & -2\sin\alpha\cos\alpha & \cos^2\alpha \end{vmatrix} = \begin{vmatrix} \cos^2\alpha & \sin 2\alpha & \sin^2\alpha \\ -\frac{1}{2}\sin 2\alpha & \cos 2\alpha & \frac{1}{2}\sin 2\alpha \\ \sin^2\alpha & -\sin 2\alpha & \cos^2\alpha \end{vmatrix} =$$

$$\begin{vmatrix} \cos^2\alpha & \sin 2\alpha & \sin^2\alpha \\ -\frac{1}{2}\sin 2\alpha & \cos 2\alpha & \frac{1}{2}\sin 2\alpha \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \cos^2\alpha & \sin 2\alpha & 1 \\ -\frac{1}{2}\sin 2\alpha & \cos 2\alpha & 0 \\ 1 & 0 & 2 \end{vmatrix} = 2\cos^2\alpha\cos 2\alpha - \cos 2\alpha + \sin^2 2\alpha =$$

$$= \cos 2\alpha(2\cos^2\alpha - \sin^2\alpha - \cos^2\alpha) + \sin^2 2\alpha = \cos 2\alpha\cdot\cos 2\alpha + \sin^2 2\alpha = 1 \neq 0$$

α burchakning ixtiyoriyligidan foydalanib, uni shunday tanlab olamizki, almashtirilgan (57.3) tenglamadagi $a`_{12}$ koeffitsiyent nolga teng bo'lsin, ya'ni

$a`_{12} = -a_{11}\sin\alpha\cos\alpha + a_{12}\cos^2\alpha - a_{12}\sin^2\alpha + a_{22}\sin\alpha\cos\alpha =$

$= -(a_{11}\cos\alpha + a_{12}\sin\alpha)\sin\alpha + (a_{21}\cos\alpha + a_{22}\sin\alpha)\cos\alpha = 0$

yoki

$$\frac{a_{11}\cos\alpha + a_{12}\sin\alpha}{\cos\alpha} = \frac{a_{21}\cos\alpha + a_{22}\sin\alpha}{\sin\alpha}. \qquad (57.5)$$

(57.5) munosabatni biror λ ga tenglab, uni quyidagi ko'rinishda yozish mumkin:

$$\begin{cases} (a_{11} - \lambda)\cos\alpha + a_{12}\sin\alpha = 0, \\ a_{21}\cos\alpha + (a_{22} - \lambda)\sin\alpha = 0. \end{cases} \qquad (57.6)$$

Bu sistema bir jinsli, shuning uchun uning determinanti nolga teng ya'ni

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0 \text{ yoki } \lambda - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}^2) = 0 \qquad (57.7)$$

bo'lgandagina sistema noldan farqli yechimga ega bo'ladi.

(57.7) tenglama γ chizisning *xarakteristik tenglamasi* deyiladi.

(57.7) tenglamaning ildizlari.

$$\lambda_{1,2} = \frac{(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}^2)}}{2} = \frac{(a_{11} + a_{22}) \pm \sqrt{D}}{2}.$$

$a_{12} \pm 0$ bo'lgani uchun uning diskriminanti:

$$D = (a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}^2) = (a_{11} - a_{22})^2 + 4a_{12}^2 > 0$$

Demak, (57.7) tenglamaning λ_1, λ_2 ildizlari turli va haqiqiydir.

(57.5) dan $$\begin{cases} a_{11}\cos\alpha + a_{12}\sin\alpha = \lambda\cos\alpha \\ a_{21}\cos\alpha + a_{22}\sin\alpha = \lambda\sin\alpha \end{cases} \qquad (57.8)$$

tengliklarni yoza olamiz. Ularning har birini cosα≠0 ga bo'lib $(\cos\alpha = 0 \Rightarrow \alpha \frac{\pi}{2})$ va

a`₁₂= – (a₁₁cosα+a₁₂sinα)sinα+(a₂₁cosα+a₂₂sinα)cosα=0⇒a₁₂=0, (ya'ni a_{12} azaldan *0* ga teng ekan) ushbuni hosil qilamiz:

$$tg\alpha = \frac{\lambda - a_{11}}{a_{12}} = \frac{a_{21}}{\lambda - a_{22}}. \qquad (57.9)$$

munosabatga navbat bilan (57.7) xarakteristik tenglamaning λ_1, λ_2 ildizlarini qo'yamiz:

$$tg\alpha_1 = \frac{\lambda_1 - a_{11}}{a_{12}}, \qquad tg\alpha_2 \frac{\lambda_2 - a_{11}}{a_{12}}. \qquad (57.10)$$

Viyet teoremasiga ko'ra (57.7) dan

$\lambda_1+\lambda_2=a_{11}+a_{22}$, $\lambda_1\lambda_2=a_{11}a_{22}-a^2_{12}$. (57.11)

(57.11) va (57.10) formulalardan ushbuga ega bo'lamiz:

$$tg\alpha_1 \cdot tg\alpha_2 = \frac{\lambda_1\lambda_2 - a_{11}(\lambda_1 - \lambda_2) + a_{11}^2}{a_{12}^2} = -1 \Rightarrow |\alpha_2 - \alpha_1| = \frac{\pi}{2}.$$

Shunga ko'ra $tg\alpha$ Ox` o'qning $B$ dagi burchak koeffitsiyenti bo'lganda $tg\alpha_2 = tg(\alpha_1 + \frac{\pi}{2})$ $Oy$` o'qining shu reperdagi burchak koeffitsiyenti bo'ladi. U holda Ox` o'qining $\vec{i}$` birlik vektorining koordinatalari bo'lmish $\cos\alpha_1$, $\sin\alpha_1$,

$$\sin\alpha_1 = \frac{tg\alpha_1}{\sqrt{1 + tg^2\alpha_1}}, \quad \cos\alpha_1 = \frac{1}{\sqrt{1 + tg^2\alpha_1}}$$

formulalardan, Oy` o'qning $\vec{j}'$ birlik vektorining koordinatalari $\cos\alpha_2$, $\sin\alpha_2$ $\sin\alpha_2 = \sin(\alpha_1 + \frac{\pi}{2}) = \cos\alpha_1, \cos\alpha_2 = \cos(\alpha_1 + \frac{\pi}{2}) = -\sin\alpha_1$ tengliklardan aniqlanadi. $\lambda=\lambda_2$ bo'lganda (57.8) dan

$a_{11}\cos\alpha_1+a_{12}\sin\alpha_1=\lambda_1\cos\alpha_1$,

$a_{21}\cos\alpha_1+ a_{22}\sin\alpha_1=\lambda_1\sin\alpha_1$,

u holda

$a`_{11}=(a_{11}\cos\alpha_1+a_{12}\sin\alpha_1)\cos\alpha_1+(a_{21}\cos\alpha_1+a_{22}\sin\alpha_1)\sin\alpha_1=\lambda\cos\alpha_1\cos\alpha_1+\lambda\sin\alpha_1\sin\alpha_1=\lambda$.

(57.4) munosabatda 1- va 3- tengliklarni hadlab qo'shsak,

$a`_{11}+ a`_{22}= =a_{11}(\sin^2\alpha+\cos^2\alpha)+ a_{22}(\sin^2\alpha+\cos^2\alpha)$ yoki $(a`_{11}+ a`_{22}= =a_{11}+ a_{22}$. (57.11) dan $a_{11}+ a_{22}=\lambda_1+\lambda_2$ va $a`_{11}=\lambda_1$ ekanini hisobga olsak, $a`_{22}=\lambda_2$ kelib chiqadi. Shunday qilib, koordinatalar sistemasini (57.10) formuladan aniqlanuvchi $\alpha=\alpha_1$ burchakka (bu yerda α_1 yangi Ox` o'qining eski Ox o'qqa og'ish burchagi) burish bilan $Б=(O,\vec{i},\vec{j})$ reperdan $Б`=(O,\vec{i}',\vec{j}')$ reperga o'tish mumkinki, unga nisbatan (57.1) tenglama soddalashib, ushbu ko'rinishga ega bo'ladi:

$\lambda_1x`^2+\lambda_2y`^2+2a`_{10}x`+2a`_{20}y`+a_{00}$. (57.12)

Agar Ox` o'qining burchak koeffitsiyenti uchun $tg\alpha_2 = \frac{\lambda_2 - \alpha_{11}}{\alpha_{12}}$ ni qabul qilinsa, u holda $a`_{11}=\lambda_2$, $a`_{22}=\lambda_1$ ekanini aynan yuqoridagi kabi ko'rsatish mumkin. Shuni aytish lozimki, agar (57.1) tenglamada $a_{12}=0$ bo'lsa, koordinatalar sistemasini burish bilan almashtirishga hojat qolmaydi.

$Б`=(O, \vec{i'}, \vec{j'})$ reperdan shunday reperga o'tamizki, unga nisbatan γ chiziqning (57.12) tenglamasida birinchi darajali hadlar qatnashmasin. Bu ishni koordinatalar boshini ko'chirish bilan bajarish mumkin.

(57.12) tenglamada λ_1, λ_2 koeffitsiyentlarning kamida biri noldan farqli, chunki agar $\lambda_1=\lambda_2=0$ bo'lsa (57.12) tenglama birinchi darajali tenglamaga aylanar edi. Demak, bu yerda quydagi uch hol bo'lishi mumkin:

1. $\lambda_1 \neq 0$, $\lambda_2 \neq 0$ $(\lambda_1\lambda_1 \neq 0)$

Bu holda $\lambda_1\lambda_1=a_{11}a_{22} - a^2_{12} \Rightarrow a_{11}a_{22} - a^2_{12} \neq 0$. (57.12) tenglamaning chap tomonidagi hadlarni $x`$, $y`$ ga nisbatan to'liq kvadratga keltiramiz:

$$\lambda_1(x^{`2} + 2\cdot\frac{a`_{10}}{\lambda_1}x` + \frac{a^{`2}_{10}}{\lambda_1^2}) + \lambda_2(y^{`2} + 2\cdot\frac{a`_{10}}{\lambda_2}y` + \frac{a^{`2}_{10}}{\lambda_2^2}) - \frac{a^{`2}_{10}}{\lambda_1} - \frac{a^{`2}_{10}}{\lambda_2} + a_{00}$$

bundan

$$\lambda_1(x` + \frac{a`_{10}}{\lambda_1})^2 + \lambda_2(y` + \frac{a`_{10}}{\lambda_2})^2 + a``_{00} = 0, \qquad (57.13)$$

bu yerda $a``_{00} = a_{00} - \frac{a^{`2}_{10}}{\lambda_1} - \frac{a^{`2}_{20}}{\lambda_2}$.

Endi $(O, \vec{i'}, \vec{j'})$ ni u quyidagi formula bilan aniqlanadigan parallel ko'chirishni bajaraylik:

$$\begin{cases} X = x` + \frac{a`_{10}}{\lambda_1}, \\ Y = y` + \frac{a`_{20}}{\lambda_2}. \end{cases} \qquad (*)$$

U holda yangi $(O`, \vec{i'}, \vec{j'})$ reper hosil bo'lib, chiziqning tenglamasi soddalashadi:

$$\lambda_1 X^2 + \lambda_2 Y^2 + a``_{10} = 0. \qquad (I)$$

2. $\lambda_1=0$ $(\lambda_2\neq 0)$, $a`_{10}\neq 0$ yoki $\lambda_2=0$ $(\lambda_1\neq 0)$, $a`_{20}\neq 0$.

Bu hollardan birini ko'rsatish yetarli; chunki

$$\begin{cases} x = y` \\ y = x` \end{cases}$$

almashtirish yordamida ularning birini ikkinchisiga keltirish mumkin.

Birinchi holni qaraymiz:

$\lambda_1=0$ $(\lambda_2\neq 0)$ ni hisobga olib, (57.11) tenglamaning chap tomonidagi hadlarini $y`$ ga nisbatan to'liq kvadratga keltiramiz:

$$\lambda_2(y`^2+2\cdot\frac{a`_{20}}{\lambda_2}y`+\frac{a`^2_{20}}{\lambda_2^2})+2a`_{10}(x`+\frac{a_{00}}{2a`_{10}}-\frac{a`^2_{20}}{2a`_{10}\lambda_2})=0,$$

yoki

$$\lambda_2(y`+\frac{a`_{20}}{\lambda_2})^2+2a`_{10}(x`+a`)^2=0,$$

bunda $a`=\frac{a_{00}}{2a`_{10}}-\frac{a`^2_{20}}{2a`_{10}\lambda_2}$ belgilashni kiritdik.

Ushbu

$$\begin{cases} X = x`+a`, \\ Y = y`+\dfrac{a`_{20}}{\lambda_2}. \end{cases}$$

formulalar bo'yicha koordinatalar sistemasini almashtiramiz, ya'ni koordinatalar boshi 0 ni $0`(-a`,\frac{a`_{20}}{\lambda_2})$ nuqtaga ko'chiramiz. U holda hosil bo'lgan

$(O`,\vec{i'},\vec{j'})$ reperga nisbatan chiziqning tenglamasi Ushbu sodda ko'rinishni qabul qiladi:

$$\lambda_2Y^2+2a`_{10}X=0. \qquad \text{(II)}$$

3. $\lambda_1=0$, $a`_{10}=0$ yoki $\lambda_2=0$, $a`_{20}=0$.

Bu hollarda ham bir-biriga o'xshash bo'lib, shuning uchun ularning birini qarash yetarli.

Birinchi holni qaraymiz. $\lambda_1=0$, $a`_{10}=0$ da (57.12) tenglama ushbu ko'rinishni oladi:

$$\lambda_2y`^2+2a`_{10}y`+a_{00}=0, \qquad (57.14)$$

bu yerda $\lambda_2\neq 0$ bo'lgani uchun (57.14) ni quydagicha yozish mumktn:

$$\lambda_2(y'^2+2\cdot\frac{a'_{20}}{\lambda_2}y'+\frac{a'^2_{10}}{\lambda_2^2})-\frac{a'^2_{20}}{\lambda_2}+a'_{00}=0$$

yoki

$$\lambda_2(y'+\frac{a'_{20}}{\lambda_2})^2+a'_{00}=0,$$

bunda

$$a'_{00}=a_{00}-\frac{a'^2_{20}}{\lambda_2}.$$

Ushbu $\begin{cases} X=x' \\ Y=y'+\dfrac{a'_{20}}{\lambda_2}. \end{cases}$ formulalar bo'yicha $(O,\vec{i}',\vec{j}')$ reperda $(O',\vec{i}',\vec{j}')$ reperga o'tamiz, ya'ni koordinatalar boshi 0 ni $0'(-a',\frac{a'_{20}}{\lambda_2})$ nuqtaga ko'chiramiz. Yangi reperda γ chiziqning sodda tenglamasi hosil bo'ladi.

$$\lambda_2Y^2+a''_{00}=0. \qquad \text{(III)}$$

X u l o s a. Agar ikkinchi tartibli γ chiziq biror dekart reperda (57.1) tenglama bilan berilgan bo'lsa, yangi dekart reperini tegishlicha tanlash bilan γ ning tenglamasini I, II, III tenglamalarning biriga keltirish mumkin.

Ikkinchi tartibli chiziqlarning tasnifi (klassifikatsiyasi).

Yuqoridagi qaralgan (I, II, III) ko'rinishdagi tenglamalarni mufassalroq tekshiramiz.

I. $\lambda_1x^2+\lambda_2y_2+a''_{00}=0$.

I tenglamada $\lambda_1\neq0$, $\lambda_2\neq0$, lekin a''_{00} – ixtiyoriy. Quydagi ikki hol bo'lishi mumkin:

a) $a''_{00}\neq0$. I dan:

$$-\frac{\lambda_1}{a''_{00}}x^2-\frac{\lambda_2}{a''_{00}}y_2=1 \text{ ёки } \frac{x^2}{-\frac{a''_{00}}{\lambda_1}-\frac{a''_{00}}{\lambda_2}}=1. \qquad (58.1)$$

Agar λ_1, λ_2 bir xil ishorali, a''_{00} esa ular bilan qarama – qarshi ishorali bo'lsa, u holda $-\frac{a''_{00}}{\lambda_1}>0$, $\frac{a''_{00}}{\lambda_2}>0$.

Endi $-\frac{a''_{00}}{\lambda_1}=a^2$, $-\frac{a''_{00}}{\lambda_1}=b^2$ belgilashni kiritsak, (58.1) dan

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

ni, ya'ni ellipsning kanonik tenglamasini hosil qilinadi.

Agar λ_1, λ_2, $a``_{00}$ ning uchvlvsi ham bir xil ishorali bo'lsa, u holda $-\frac{a``_{00}}{\lambda_1}<0$, $\frac{a``_{00}}{\lambda_2}<0$, bu yerda $-\frac{a``_{00}}{\lambda_1}=-a^2$, $-\frac{a``_{00}}{\lambda_1}=-b^2$ belgilash kiritsak, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1$ tenglamaga ega bo'lamiz. Bu tenglamani qanoatlantiruvchi bita ham haqiqiy nuqta mavjud emas, lekin bu tenglama ellips tenglamasiga o'xshashligi sababli, u *mavhum ellipsni* aniqlaydi, deb aytiladi. Agar λ_1, λ_2 qarama – qarshi ishorali va $a``_{00}\neq 0$ bo'lsa, u holda $-\frac{a``_{00}}{\lambda_1}$ va $-\frac{a``_{00}}{\lambda_2}$ lar qarama – qarshi ishorali bo'ladi. $-\frac{a``_{00}}{\lambda_1}>0$, lekin $-\frac{a``_{00}}{\lambda_2}<0$ bo'lib, ularni mos ravishda $a^2$ va $-b^2$ deb belgilasak, (58.1) tenglama $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ko'rinishda bo'lib, bu giperbolaning kanonik tenglamasidir; xudi shunga o'xshash, $-\frac{a``_{00}}{\lambda_1}<0$, $-\frac{a``_{00}}{\lambda_2}>0$ bo'lsa, ularni ham mos ravishda $-a^2$ va b^2 deb belgilasak, (58.1) tenglama ushbu ko'rinishni oladi: $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; bu ham giperbolaning kanonik tenglamasidir.

b) $a``_{00}=0$ bo'lsin. U holda

$$\mathrm{I}\Rightarrow\frac{x_2}{\frac{1}{\lambda_1}}+\frac{y_2}{\frac{1}{\lambda_2}}=0. \qquad (58.2)$$

λ_1, λ_2 qarama – qarshi ishorali bo'lsa, tegishli belgilashni kiritish bilan (58.2) ni ushbu ko'rinishda yozish mumkin:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0 \text{ ёки } \left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{a}+\frac{y}{b}\right)=0. \qquad (58.3)$$

(58.3) $\Rightarrow\frac{x}{a}+\frac{y}{b}=0,\ \frac{x}{a}-\frac{y}{b}=0,$ bu tenglamalar koordinatalar boshida kesishuvchi ikkita haqiqiy to'g'ri chiziqni aniqlaydi. Agar λ_1, λ_2 bir xil ishorali, masalan, $\lambda_1<0$,

$\lambda_2<0$ bo'lsa, u holda $\frac{1}{\lambda_1}=-a^2,\ \frac{1}{\lambda_2}-b^2$ belgilashni kiritish bilan (58.2) ni quyidagi ko'rinishda yozish mumkin:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=0 \text{ ёки} \left(\frac{x}{a}+i\frac{y}{b}\right)\left(\frac{x}{a}-i\frac{y}{b}\right)=0 \Rightarrow \frac{x}{a}+i\frac{y}{b}=0,\ \frac{x}{a}-i\frac{y}{b}=0,$$

bu tenglamalarning har biri birinchi darajali bo'lgani uchun ular to'g'ri chiziqni aniqlaydi, lekin bu ikki to'g'ri chiziq faqat bita haqiqiy nuqtaga egadir (koordinatalar boshi). Shuning uchun ularni bita haqiqiy nuqtada kesishuvchi ikkita mavhum to'g'ri chiziq tenglamasi deb aytish mumkin. Shunday qilib, ikkinchi tartibli γ chiziqning (57.6) xarakteristik tenglamasining ildizlari $\lambda_1\neq0$, $\lambda_2\neq0$ bo'lsa, quydagi besh tur chiziq hosil bo'ladi: ellips, mavhum ellips, giperbola, kesishuvchi mavhum ikki to'g'ri chiziq, kesishuvchi haqiqiy ikki to'g'ri chiziq.

2. $\lambda_2 y^2+2a`_{10}x=0$

tenglama bilan berilgan ikkinchi tartibli chiziqlarga o'tamiz. II tenglamada $\lambda_2\neq0$, $a`_{10}\neq0$ bo'lgani uchun uni quydagicha yozib olamiz: $y^2=-2\cdot\frac{a`_{10}}{\lambda_2}x;\ p=-\frac{a`_{10}}{\lambda_2}$ belgilashni kiritsak, $y^2=2px$, bu parabolaning kanonik tenglamasidir.

3. $\lambda_2 y^2+a``_{00}=0$

tenglama bilan berilgan ikkinchi tartibli chiziqlarni tasniflashga o'tamiz. Bu tenglamada $\lambda_2\neq0$, $a``_{10}$ – har qanday son. Quyidagi hollar bo'lishi mumkin.

a) $a``_{00}\neq 0\cdot\lambda_2$ bilan $a``_{00}$ har xil ishorali bo'lsa, $-\frac{a``_{00}}{\lambda_1}>0$ bo'ladi.

Tenglamani $\frac{a``_{00}}{\lambda_2}=-a^2$ faraz qilib,

$y^2=a^2$ yoki $(y-a)(y+a)=0$

ga keltiramiz. Bu tenglama esa o'zaro parallel ikki to'g'ri chiziqni aniqlaydi. λ_2 bilan $a``_0$ bir xil ishorali, ya'ni $\lambda_2>0$, $a`_{00}>0$ ($\lambda_2<0$, $a``_{00}<0$) bo'lgan holda

III $\Rightarrow y^2=-a^2$ yoki $(y-ia)(y+ia)=0$,

bu tenglama ikkita mavhum parallel to'g'ri chiziqni aniqlaydi, deb yuritiladi.

b) $a``_{00}=0$. U holda III$\Rightarrow\lambda_2 y^2=0$ va $\lambda_2\neq0$ bo'lgani uchun $y^2=0$ yoki $y=0, y=0\Rightarrow$ ikki karra olingan to'g'ri chiziq hosil qilinadi. Shunday qilib, III tenglama bilan berilgan ikkinchi tartibli chiziq quydagi uch turga bo'linadi: haqiqiy parallel ikki to'g'ri chiziq, mavhum parallel ikki to'g'ri chiziq, ustma – ust tushuvchi ikki to'g'ri chiziq.

I, II, III tenglamalar bilan berilgan ikkinchi tartibli chiziq quyidagi to'qqizta turga bo'linadi:

Kanonik tenglamalar	Chiziqlarning nomlari
1	2
1. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	ellips
2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1$	mavhum ellips
3. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\pm 1$	giperbola
4. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	kesishuvchi ikki to'g'ri chiziq
5. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$	nuqta (koordinata boshida kesishuvchi mavhum ikki to'g'ri chiziq)
6. $y^2=2px$	parabola
7. $y^2-a^2=0$	turli parallel ikki to'g'ri chiziq
8. $y^2+a^2=0$	mavhum parallel ikki to'g'ri chiziq
9. $y^2=0$	ustma – ust tushgan ikki to'g'ri chiziq

Ikkinchi tartibli chiziqni uning tenglamasi bo'yicha yasash.

Ikkinchi tartibli chiziq $(O,\vec{i},\vec{j})$ dekart reperida (57.1) umumiy tenglamasi bilan berilgan bo'lsin. Uni yasash uchun tenglamasini oldingi paragrafda bayon qilingan usullar bo'yicha soddalashtiramiz:

1) (57.1) tenglamada $a_{12}\neq 0$ bo'lsa, chiziqning

$$\lambda^2-(a_{11}+a_{22})\lambda+a_{11}a_{22}-a_{12}^2=0$$

2) tg$\alpha_1=\frac{\lambda_1-a_{11}}{a_{12}}$ formula bo'yicha tgα_1 ni, so'ngra

$$\sin\alpha_1=\frac{tg\alpha_1}{\sqrt{1+tg^2\alpha_1}},\quad \cos\alpha_1=\frac{1}{\sqrt{1+tg^2\alpha_1}}$$

ni hosil qilamiz. Bu bilan reperni α_1 burchakka burishdan hosil qilingan $(O,\vec{i}`,\vec{j}`)$ reperning $\vec{i}`,\vec{j}`$ koordinatavektorlari aniqlanadi:

$$\vec{i}`=\vec{i}\cos\alpha_1+\vec{j}\sin\alpha_1,\ ,\ \vec{j}`=-\vec{i}\sin\alpha_1+j\cos\alpha_1.$$

3) Yangi reperda chiziqning tenglamasi

$$\lambda_1 x`^2+\lambda_2 y`^2+2a`_{10}x`+2a`_{20}y`+a_{00}=0 \qquad (57.11)$$

ko'rinishda bo'lib, bunda $a`_{10}$, $a`_{20}$ koeffitsiyentlar ushbu formulalardan topiladi: B` reperning koordinatalariboshi $O$ ni 53-§ dagi (*) formuladan topiladigan $O`$ nuqtaga ko'chirish bilan B` reperdan B`` reperga o'tamiz. B`` reperda chiziqning tenglamasi kanonik ko'rinishga keladi. Agar (57.1) tenglamada a_{12}=0 bo'lsa, soddalashtirish koordinatalar boshini ko'chirishdan iborat, xolos. Bu ishlarni misollarda ko'ramiz.

Ikkinchi tartibli chiziqning to'g'ri chiziq bilan kesishishi.

Dekart reperida

$$a_{11}x^2+2a_{12}xy+a_{22}u^2+2a_{10}x+2a_{20}y+a_{00}=0 \qquad (57.1)$$

ikkinchi tartibli chiziq va

$$\begin{cases} x=x_0+a_1t \\ y=y_0+a_2t \end{cases} \qquad (61.1\ 74)$$

to'g'ri chiziq berilgan bo'lsin. Bu egri chiziqning shu to'g'ri chiziq bilan kesishish masalasiga o'tamiz. (57.1) va (61.1) dan:

$$a_{11}(x_0+a_1t)^2+2a_{12}(x_0+a_1t)(y_0+a_2t)+a_{22}(y_0+a_2t)^2+2a_{10}(x_0+a_1t)+2a_{20}(y_0+a_2t)+a_{00}=0$$

yoki

$$Pt^2+2Qt+r=0. \qquad (61.2)$$

Bu yerda quyidagi belgilashlar kiritilgan:

$$P=a_{11}a_1^2+2a_{12}a_1a_2+a_{22}a_2^2;$$

$$Q=a_{11}a_1x_0+a_{12}a_1y_0+a_{21}a_2x_0+a_{22}a_2y_0+a_{10}a_1+a_{20}a_2=$$

$$a_1(a_{11}x_0+a_{12}y_0+a_{10})+a_2(a_{21}x_0+a_{22}y_0+a_{20}); \qquad (61.3)$$

$R=a_{11}\,x_0^2+2a_{12}x_0y_0+a_{22}y_0^2+2a_{10}x_0+2a_{20}y_0+a_{00}$.

(61.2) tenglamani yechib, t ning topilgan qiymatlarini (61.1) ga qo'ysak, chiziq bilan to'g'ri chiziqning kesishgan nuqtalari topiladi. Quydagi hollarni tekshiraylik.

1. $P\neq 0$. Bu holda (61.2) tenglama ikkita ildizga ega.

$$t_{1,2}=\frac{-Q\pm\sqrt{a^2-RP}}{P}.$$

Bu yerning o'zida uchta hol bo'lishi mumkin:

a) $D=Q^2\text{-}PR>0$; (61.2) tenglama ikkita haqiqiy turli ildizlarga ega – chiziq bilan to'g'ri chiziq ikkita haqiqiy turli nuqtalarda kesishadi.

b) $D=Q^2\text{-}PR<0$; (61.2) tenglama ikkita qo'shma kompleks ildizga ega, shuning uchun (57.1) chiziq bilan (61,1) to'g'ri chiziq ikkta qo'shma kompleks nuqtalarda kesishadi, demak, to'g'ri chiziq bilan (57.1) chiziq umumiy haqiqiy nuqtalarga ega bo'lmaydi.

v) $D=Q^2\text{-}PR=0$; (61.2) tenglama ustma – ust tushgan ikkita ildizga ega – chiziq bilan to'g'ri chiziq ustma – ust tushgan ikkita nuqtada kesishadi. Bu vaqtda u to'g'ri chiziq γ chiziqqa *urinma* deb ataladai.

2. $P=0$. Bu holda (61.2) tenglama

$$2Qt+R=0 \qquad (61.4)$$

ko'rinishni oladi.

a) $Q\neq 0$, R – ixtiyoriy son. (61.4) tenglama yagona ildizga ega:

$$t=-\frac{R}{2Q};$$

b) $Q=0$, $R\neq 0$. (61.4) tenglama yechimga ega emas. Chiziq to'g'ri chiziq bilan bitta ham umumiy haqiqiy yoki mvhum nuqtaga ega emas.

v) $Q=0$, $R=0$ bu holda t ning har qanday qiymati (61.4) tenglamani qanoatlantiradi$\Rightarrow$chiziq va to'g'ri chiziq cheksiz ko'p umumiy nuqtalarga ega, ya'ni (61.1) to'g'ri barcha nuqtalari bilan (57.1) chiziqqa tegishli: $u\subset\gamma$. Shunday qilib, (61.2) tenglamada $R=0$ bo'lsa, γ chiziq u to'g'ri chiziq bilan faqat bitta umumiy nuqtaga ega yoki bitta ham umumiy nuqtaga ega emas, yoki $u\subset\gamma$.