6- MAVZU: Fazoda affin va dekart koordinatalar sistemasini almashtirish.

1. Fazoda affin koordinatalar sistemasini almashtirish.

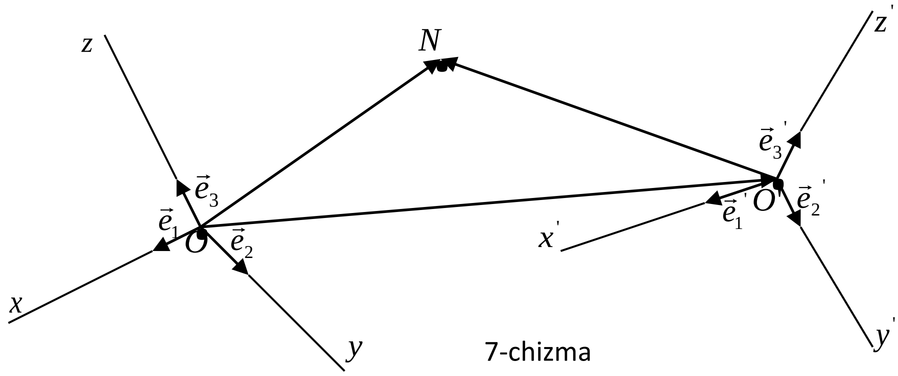
2. Fazoda to'g'ri burchakli dekart kordinatalar sistemasini almashtirish.

3. Fazoda affin koordinatalar sistemasini almashtirishning xususiy xollari.

Affin koordinatalarni almashtirish

Fazoda ikkita $(O, \vec{e}_1\, \vec{e}_2\, \vec{e}_3)$-eski, $(O', \vec{e}'_1\, \vec{e}'_2\, \vec{e}'_3)$-yangi affin koordinatalar sistemalari berilgan. Fazoda ixtiyoriy N nuqta olsak, uning eski sistemadagi x, y, z koordinatalar bilan, shu nuqtaning yangi sistemadagi x', y', z' koordinatalar orasidagi bog'lanishni aniqlash kerak. Yangi koordinatalar sistemasining boshi O' nuqta va koordinata vektorlari $\vec{e}'_1, \vec{e}'_2, \vec{e}'_3$ eski sistemaga nisbatan berilgan bo'lsin, ya'ni:

$$\begin{aligned} &\overline{O}\vec{O}'(x_0, y_0, z_0) \Leftrightarrow \overline{O}\vec{O}' = x_0\vec{e}_1 + y_0\vec{e}_2 + z_0\vec{e}_3, \\ &\vec{e}_1'(c_{11}, c_{21}, c_{31}) \Leftrightarrow \vec{e}_1' = c_{11}\vec{e}_1 + c_{21}\vec{e}_2 + c_{31}\vec{e}_3, \\ &\vec{e}_2'(c_{12}, c_{22}, c_{32}) \Leftrightarrow \vec{e}_2' = c_{12}\vec{e}_1 + c_{22}\vec{e}_2 + c_{32}\vec{e}_3, \\ &\vec{e}_3'(c_{13}, c_{23}, c_{33}) \Leftrightarrow \vec{e}_3' = c_{13}\vec{e}_1 + c_{23}\vec{e}_2 + c_{33}\vec{e}_3. \end{aligned} \quad (11)$$



7-chizma

Vektorlarni qo'shishdagi uchburchak qoidasiga ko'ra $\overrightarrow{ON} = \overrightarrow{OO'} + \overrightarrow{O'N}$, shuning uchun (7-chizma)

$$x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3 = x'\vec{e}_1' + y'\vec{e}_2' + z'\vec{e}_3' + x_0\vec{e}_1 + y_0\vec{e}_2 + z_0\vec{e}_3, \quad (12)$$

(11) dagi $\vec{e}'_1, \vec{e}'_2$ va $\vec{e}'_3$ larning ifodalarini (12) ga qo'yib, o'ng va chap tomondagi mos koeffitsientlarni tenglashtirib, quyidagilarga ega bo'lamiz:

$$
\begin{aligned}
x &= c_{11}x'+c_{21}y'+c_{31}z'+x_0,\\
y &= c_{12}x'+c_{22}y'+c_{32}z'+y_0,\\
z &= c_{13}x'+c_{23}y'+c_{33}z'+z_0.
\end{aligned}
\qquad (13)
$$

N nuqtaning eski sistemasidagi koordinatalari x, y, z lar yangi sistemadagi x', y', z' koordinatalar orqali (13) formulalar orqali ifodalanadi. (13) formula affin koordinatalar sistemasini almashtirish formulasi deyiladi. Bu almashtirish koeffitsientlaridan

$$
C'=\begin{pmatrix} c_{11} & c_{12} & c_{13}\\ c_{21} & c_{22} & c_{23}\\ c_{31} & c_{32} & c_{33}\end{pmatrix} \qquad (14)
$$

matritsa tuzilgan.

$\vec{e}'_1, \vec{e}'_2, \vec{e}'_3$ vektorlarning koordinatalaridan tuzilgan

$$
C=\begin{pmatrix} c_{11} & c_{21} & c_{31}\\ c_{12} & c_{22} & c_{32}\\ c_{13} & c_{23} & c_{33}\end{pmatrix} \qquad (15)
$$

matritsani olaylik. Bu matritsani $\vec{e}_1, \vec{e}_2, \vec{e}_3$ eski bazisdan $\vec{e}'_1, \vec{e}'_2, \vec{e}'_3$ yangi bazisga o’tish matritsasi deyiladi. Bu matritsaning determinanti

$$
\Delta=\begin{vmatrix} c_{11} & c_{12} & c_{13}\\ c_{21} & c_{22} & c_{23}\\ c_{31} & c_{32} & c_{33}\end{vmatrix} \neq 0 \qquad (15')
$$

$\vec{e}'_1, \vec{e}'_2, \vec{e}'_3$ chiziqli erkli vektorlar bo’lsa, u holda $\Delta \neq 0$.

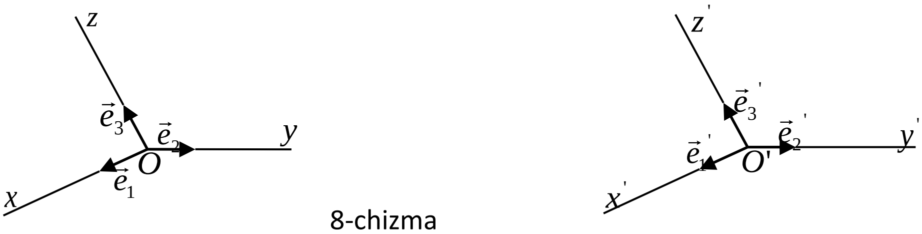
Agar $\Delta = 0$ bo’lsa, algebradan ma’lum, determinantning bitta yo’li qolgan yo’llari orqali chiziqli ifoda qilinadi. Demak, $\vec{e}'_1, \vec{e}'_2$ va $\vec{e}'_3$ vektorlar komplanar bo’ladi, bu esa zid natija. (14) va (15) matritsalarni solishtirib, C' matritsa C matritsani transponirlash bilan hosil qilingan. Demak, C' matritsa determinanti ham nolga teng emas.

Shuning uchun (13) tenglamani x', y', z' larga nisbatan bir qiymatli echib, N nuqtaning yangi koordinatalarini shu nuqtaning eski koordinatalari orqali ifodalaymiz, ya’ni:

$$
\begin{aligned}
x' &= a_{11}x + a_{12}y + c_{13}z + a, \\
y' &= a_{21}x + a_{22}y + c_{23}z + b, \qquad (16) \\
z' &= a_{31}x + a_{32}y + c_{33}z + c.
\end{aligned}
$$

Xususiy hollar:

I hol. Affin koordinatalar sistemalarining boshlari turli nuqtalarda bo'lib, bazis vektorlari mos ravishda kollinear bo'lsin. (8-chizma)



8-chizma

$$
\begin{aligned}
&c_{11} = 1,\ c_{21} = 0,\ c_{31} = 0, \\
&c_{12} = 0,\ c_{22} = 1,\ c_{32} = 0, \\
&c_{13} = 0,\ c_{23} = 0,\ c_{33} = 1,
\end{aligned}
\qquad (17)
$$

(13) va (17) larga e'tibor bersak, ushbu

$$
\begin{aligned}
x &= x' + x_0, \\
y &= y' + y_0, \qquad (18) \\
z &= z' + z_0.
\end{aligned}
$$

formulaga ega bo'lfmiz. Bu formulani koordinatalar sistemasini parallel ko'chirish formulasi deyiladi.

II hol. Eski va yangi sistemalarning koordinata boshlari bir nuqtada bo'lsin, ya'ni $x_0 = y_0 = z_0 = 0$ bo'lsin, u holda (13) dan

$$
\begin{aligned}
x &= c_{11}x' + c_{12}y' + c_{13}z', \\
y &= c_{21}x' + c_{22}y' + c_{23}z', \qquad (19) \\
z &= c_{31}x' + c_{32}y' + c_{33}z'.
\end{aligned}
$$

formulaga ega bo'lamiz.

Bir affin koordinatalar sistemasini ikkinchi affin koordinatalar sistemasiga o'tkazish formulasi (13)

$$x_0,\, y_0,\, z_0,\, c_{\beta\alpha}\left(\alpha, \beta = 1,2,3\right)$$

(12) parametrga bog'liq.

Dekart koordinatalarni almashtirish

Bir to'g'ri burchakli $(O,i\,j\,k)$ koordinatalar sistemasidan ikkinchi dekart koordinatalar $(O',i'\,j'k')$ sistemasiga o'tish formulasi (13) ko'rinishda bo'ladi, chunki to'g'ri burchakli dekart koordinatalar sistemasi affin koordinatalar sistemasining xususiy holi.

Bu formuladagi $c_{\alpha\beta}(\alpha,\beta=1,2,3)$ koeffitsientlar $\vec{i}',\vec{j}',\vec{k}'$ birlik vektorning $\vec{i},\vec{j},\vec{k}$ ortonormallashgan bazisga nisbatan koordinatalari bo'ladi:

$$\vec{i}'=c_{11}\vec{i}+c_{21}\vec{j}+c_{31}\vec{k},$$
$$\vec{j}'=c_{12}\vec{i}+c_{22}\vec{j}+c_{32}\vec{k},$$
$$\vec{k}'=c_{13}\vec{i}+c_{23}\vec{j}+c_{33}\vec{k}.$$

Bu tenglikni $\vec{i},\vec{j},\vec{k}$ vektorlarga skalyar ko'paytirib topamiz:

$$\cos\left(\widehat{\vec{i}'\vec{i}}\right)=c_{11},\ \cos\left(\widehat{\vec{i}'\vec{j}}\right)=c_{21},\ \cos\left(\widehat{\vec{i}'\vec{k}}\right)=c_{31},$$
$$\cos\left(\widehat{\vec{j}'\vec{i}}\right)=c_{12},\ \cos\left(\widehat{\vec{j}'\vec{j}}\right)=c_{22},\ \cos\left(\widehat{\vec{j}'\vec{k}}\right)=c_{32},$$
$$\cos\left(\widehat{\vec{k}'\vec{i}}\right)=c_{13},\ \cos\left(\widehat{\vec{k}'\vec{j}}\right)=c_{23},\ \cos\left(\widehat{\vec{k}'\vec{k}}\right)=c_{33}.$$

Topilgan qiymatlarni (13) formulaga qo'ysak, dekart koordinatalar sistemasini almashtirish formulasini hosil qilamiz.

$$\left(\vec{i}'\right)^2=\left(\vec{j}'\right)^2=\left(\vec{k}'\right)^2=1,\ \vec{i}'\cdot\vec{j}'=\vec{i}'\cdot\vec{k}'=\vec{j}'\cdot\vec{k}'=0,$$

$$\begin{array}{ll} c_{11}^2+c_{21}^2+c_{31}^2=1, & c_{11}c_{12}+c_{21}c_{22}+c_{31}c_{32}=0,\\ c_{12}^2+c_{22}^2+c_{32}^2=1, & c_{11}c_{13}+c_{21}c_{23}+c_{31}c_{33}=0,\\ c_{13}^2+c_{23}^2+c_{33}^2=1. & c_{12}c_{13}+c_{22}c_{23}+c_{32}c_{33}=0, \end{array} \qquad (20)$$