5- Мавзу. Tekislikda affin va dekart koordinatalar sistemasini almashtirish.

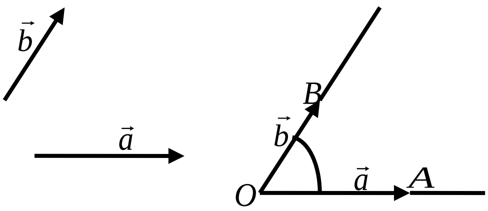
Режа:

1. Yo'nalishli tekislikdagi ikki vektor orasidagi burchak
2. Affin koordinatalar sistemasini almashtirish.
3. To'g'ri burchakli dekart koordinatalar sistemasini almashtirish

Yo'nalishli tekislikdagi ikki vektor orasidagi burchak.

Tekislikda nol bo'lmagan ikkita $\vec{a}$ va $\vec{b}$ vektorlar berilgan bo'lsa, bu vektorlarni O nuqtaga ko'chirib $\angle AOB$ ni hosil qilamiz, bu yerda $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$. Hosil bo'lgan $\overrightarrow{OA}$ va $\overrightarrow{OB}$ nurlar orasida burchak $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak deyiladi (24-chizma) va $(\vec{a}\wedge\vec{b})$ ko'rinishida belgilanadi.

Ixtiyoriy ikkita vektor uchun $0 \le (\vec{a}\wedge\vec{b}) \le \pi$ Orientatsiyalangan tekislikda yo'nalishga ega bo'lgan burchak tushunchasini



24-chizma

kiritaylik.

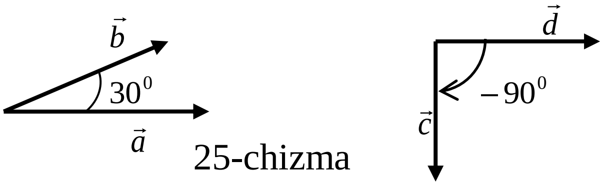
Tekislikda $\vec{a}$ va $\vec{b}$ nol bo'lmagan vektorlar berilgan bo'lsin, agar bu vektorlarni tartiblasak, ya'ni $\vec{a}$ vektorni birinchi $\vec{b}$ vektorni ikkinchi deb olsak $(\vec{a} \ne \lambda\vec{b})$, u holda $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak yo'nalgan burchak deb aytiladi va $(\vec{a}\wedge\vec{b})$ ko'rinishida yoziladi.

Agar $\vec{a}$, $\vec{b}$ vektorlar o'ng bazisni tashkil qilsa, u holda $(\vec{a}\wedge\vec{b})$>0 bo'ladi, chap bazisni tashkil qilsa -$(\vec{a}\wedge\vec{b})$ bo'ladi.

Agar $\vec{a}\uparrow\uparrow\vec{b}$ bo'lsa, $(\vec{a}\wedge\vec{b})$ =*0*, agar $\vec{a}\downarrow\uparrow\vec{b}$ bo'lsa $(\vec{a}\wedge\vec{b})=\pi$.

Shunday qilib, $\vec{a}\ne 0$ $\vec{b}\ne 0)$ vektorlar uchun $-\pi \le (\vec{a}\wedge\vec{b}) \le \pi$.

25-chizmada $\vec{a}$, $\vec{b}$ vektorlar o'ng bazisni $\vec{c}$, $\vec{d}$ vektorlar chap bazisni tashkil qiladi. $(\vec{a}\wedge\vec{b})=30^0$, $(\vec{c}\wedge\vec{d})=-90^0$ (25-chizma).



25-chizma

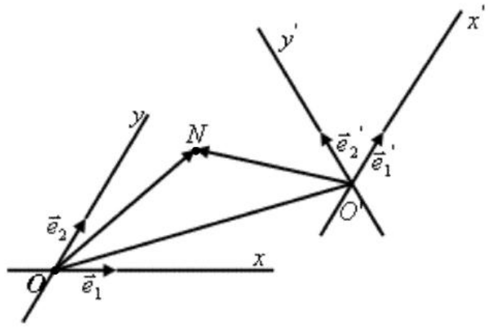
Vaholanki, $(\vec{a}\wedge\vec{b})=-(\vec{b}\wedge\vec{a})$

$sin(\vec{a}\wedge\vec{b})=-sin(\vec{b}\wedge\vec{a})$

$cos(\vec{a}\wedge\vec{b})=cos(\vec{b}\wedge\vec{a})$

Affin koordinatalar sistemasini almashtirish.

Gometrik obrazlarni soddalashtirish uchun ko'pincha bir koordinatalar sistemasidan boshqa koordinatalar sistemasiga o'tishga to'g'ri keladi. Bu esa bir nuqtaning har xil sistemadagi koordinatalarini bog'lovchi formulalarni topish masalasini keltirib chiqaradi.



27-chizma

Tekislikda ikkita $(0, \vec{e}_1, \vec{e}_2)$ va $(0', \vec{e}_1{}', \vec{e}_2{}')$ affin koordinatalar sistemasi berilgan bo'lsin (27-chizma).

Qulaylik uchun birinchisini eski, ikinchisini yangi affin koordinatalar sistemasi deb olamiz. Bundan tashqari, yangi koordinatalar sistemasining vaziyati eski koordinatalar sistemasiga nisbatan berilgan bo'lsin.

$$\vec{e}_1'(c_{11},\ c_{21}),\ \ \vec{e}_2'(c_{12},\ c_{22}),\ o'(x_0,\ y_0).\ \ C=\begin{pmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{pmatrix} \qquad (14.1)$$

Ta'rifga ko'ra ushbuni yoza olamiz.

$$\begin{aligned} \vec{e}_1' &= c_{11}\vec{e}_1 + c_{21}\vec{e}_2; \\ \vec{e}_2' &= c_{12}\vec{e}_1 + c_{22}\vec{e}_2. \qquad o' = x_0\vec{e}_1 + y_0\vec{e}_2 \end{aligned} \qquad (14.2)$$

Bizning maqsadimiz N nuqtaning eski koordinatalar sistemasidagi x, y koordinatalarini, shu nuqtaning yangi koordinatalar sistemasidagi x', y' koordinatalari orqali ifodalashdir.

Vektorlarni qo'shishdagi uchburchak qoidasiga asosan

$\overrightarrow{ON}=\overrightarrow{OO'}+\overrightarrow{O'N}$ $\quad \overrightarrow{ON}=x\vec{e}_1+y\vec{e}_2$ (26 - chizma).

Bundan, $x\vec{e}_1 + y\vec{e}_2 = \overrightarrow{OO'} + x'\vec{e}_1' + y'\vec{e}_2'$.

(14.2) dan foydalanib, $x\vec{e}_1 + y\vec{e}_2 = x_0\vec{e}_1 + y_0\vec{e}_2 + (c_{11}x' + c_{12}y')\vec{e}_1 + (c_{21}x' + c_{22}y')\vec{e}_2$

ga ega bo'lamiz. $\vec{e}_1$ va $\vec{e}_2$ vektorlar kollinear emasligidan foydalanib quyidagi

$$\begin{aligned} x &= c_{11}x' + c_{12}y' + x_0 \\ y &= c_{21}x' + c_{22}y' + y_0 \end{aligned} \qquad (14.3)$$

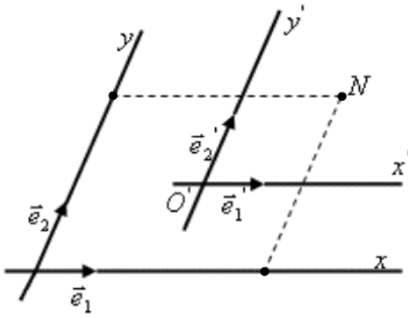
formulani yozamiz. (14.3) formulani affin koordinatalar sistemasini almashtirish formulasi deyiladi. Bu formulaning chap tomonining koeffitsientlaridan quyidagi

$$C' = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \qquad (14.4)$$

matritsani tuzaylik. *C'* matritsa *C* matritsani transponirlash natijasida hosil qilingan bo'lib,

$$\begin{vmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{vmatrix} \neq 0$$

(14.5)
chunki $\vec{e}_1'$ va $\vec{e}_2'$ vektorlar bazis vektorlar.



28-chizma

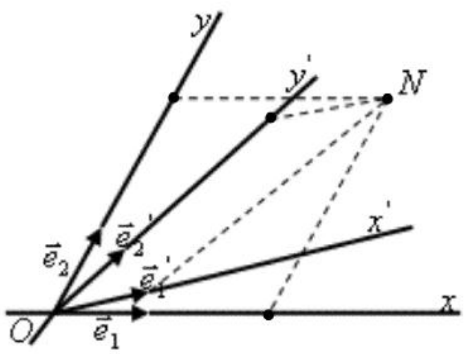
(14.3) ni hamma vaqt *x', y'* larga nisbatan yechish mumkin. Bu esa *N* nuqtaning yangi koordinatalar sistemasidagi *x', y'* koordinatalarini shu nuqtaning eski sistemasidagi *x, y* koordinatalari orqali ifodalash mumkinligini ko'rsatadi.

Quyidagi xususiy holni qaraymiz:

1. $O \neq O'$ $\vec{e}_1'(c_{11}, c_{21}) = \vec{e}_1(1,0)$ $\vec{e}_2'(c_{12},\ c_{22}) = \vec{e}_2(0,\ 1)$

bundan $c_{11} = c_{22} = 1, c_{21} = c_{12} = 0$, bo'ladi. Bu topilgan qiymatlarni (14.3) formulaga qo'yib (28-chizma)

$$\begin{aligned} x &= x' + x_0 \\ y &= y' + y_0 \end{aligned} \qquad (14.6)$$



29-chizma

koordinatalar sistemasini parallel ko'chirish formulasiga ega bo'lamiz.

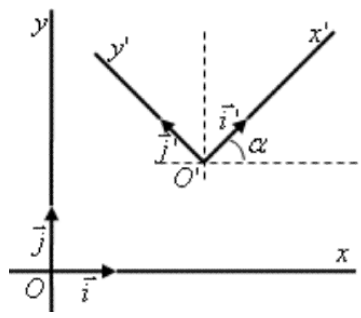
1. $O = O'$ bo'lib, bazis vektorlar turlicha bo'lsin (29-chizma), u holda $x_0 = y_0 = 0$ bo'lib,

$$x = c_{11}x' + c_{12}y'$$
$$y = c_{21}x' + c_{22}y' \qquad (14.7)$$

formulaga ega bo'lamiz.

To'g'ri burchakli dekart koordinatalar sistemasini almashtirish.

Endi dekart koordinatalar sistemasini almashtirishga to'xtaymiz. Bir to'g'ri burchakli dekart koordinatalar sistemasidan ikkinchi dekart koordinatalar sistemasiga o'tishda (14.3) formuladan foydalanamiz, lekin o'tish matritsasining c_{ij} $(i, j = 1,2)$ elementlariga qo'shimcha shartlar qo'yiladi.



30-chizma

Tekislikda $(O, \vec{i}, \vec{j})$ - eski $(O', \vec{i}', \vec{j}')$ - yangi dekart koordinatalar sistemasi bo'lsin.

$$\vec{i}' = c_{11}\vec{i} + c_{21}\vec{j}$$
$$\vec{j}' = c_{12}\vec{i} + c_{22}\vec{j} \qquad (15.1)$$

$(\vec{i} \wedge \vec{i}') = \alpha$ bo'lsin, bu yerda ikki hol o'rinli bo'ladi.

1. Eski va yangi koordinatalar sistemasi bir xil yo'nalishga ega (30-chizma).

$$(\vec{i}' \wedge \vec{j}') = 90^0 + \alpha, \quad (\vec{i}' \wedge \vec{j}) = 90^0 - \alpha, \quad (\vec{j} \wedge \vec{j}') = \alpha$$

(6.6) tenglikni navbat bilan $\vec{i}$ va $\vec{j}$ vektorlarga skalyar ko'paytirib quyidagilarga ega bo'lamiz.

$$c_{11} = \vec{i}' \cdot \vec{i} = \cos(\vec{i}' \wedge \vec{i}) = \cos\alpha \qquad c_{21} = \vec{i}' \cdot \vec{j} = \cos(\vec{i}' \wedge \vec{j}) = \cos(90^0 - \alpha) = \sin\alpha$$
$$c_{12} = \vec{j}' \cdot \vec{i} = \cos(\vec{j}' \wedge \vec{i}) = \cos(90^0 + \alpha) = -\sin\alpha, \quad c_{22} = \cos\alpha$$

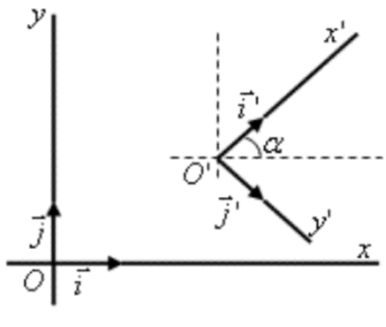
topilgan qiymatlarni (14.3) ga qo'yib,

$$x = x'\cos\alpha - y'\sin\alpha + x_0$$
$$y = x'\sin\alpha + y'\cos\alpha + y_0 \qquad (15.2)$$

Yo'nalishlari bir xil bo'lgan dekart koordinatalar sistemasini almashtirish formulasiga ega bo'lamiz.

2. Eski va yangi koordinatalar sistemasi turli yo'nalishga ega bo'lsin. (31-chizma).

$$(\vec{j}' \wedge \vec{i}) = 270^0 + \alpha, \quad (\vec{i}' \wedge \vec{j}) = 90^0 - \alpha, \quad (\vec{j} \wedge \vec{j}') = 180^0 + \alpha$$



31-chizma

Buni e'tiborga olib, (15.1 6.6) ni $\vec{i}$ va $\vec{j}$ vektorlarga navbati bilan ko'paytirsak, ushbuga ega bo'lamiz.

$$c_{11} = \vec{i}\,' \cdot \vec{i} = \cos\alpha \qquad c_{21} = \vec{i}\,' \cdot \vec{j} = \cos(\vec{i}\,' \wedge \vec{j}) = \cos(90^0 - \alpha) = \sin\alpha$$
$$c_{12} = \vec{j}\,' \cdot \vec{i} = \cos(\vec{j}\,' \wedge \vec{i}) = \cos(270^0 + \alpha) = \sin\alpha,$$
$$c_{22} = \vec{j}\,' \cdot \vec{j} = \cos(\vec{j}\,' \wedge \vec{j}) = \cos(180^0 + \alpha) = -\cos\alpha$$

Topilgan qiymatlarni (6.4) ga qo'yib,

$$\begin{aligned} x &= x'\cos\alpha + y'\sin\alpha + x_0 \\ y &= x'\sin\alpha - y'\cos\alpha + y_0 \end{aligned} \qquad (15.3)$$

Yo'nalishlari har xil bo'lgan dekart koordinatalar sistemasini almashtirish formulasiga ega bo'lamiz.

(15.2) va (15.3) formulalarni bitta

$$\begin{aligned} x &= x'\cos\alpha - \varepsilon y'\sin\alpha + x_0 \\ y &= x'\sin\alpha + \varepsilon y'\cos\alpha + y_0 \end{aligned} \qquad (15.4)$$

formulaga birlashtirish mumkin, bu yerda $\varepsilon = \pm 1$, yo'nalishlar bir xil bo'lsa $\varepsilon = +1$, agar har xil bo'lsa $\varepsilon = -1$ ga teng.

Agar (15.5) da $x_0 = y_0 = 0$ bo'lsa , u holda

$$\begin{aligned} x &= x'\cos\alpha - \varepsilon y'\sin\alpha \\ y &= x'\sin\alpha + \varepsilon y'\cos\alpha \end{aligned} \qquad (15.5)$$

formulani dekart koordinatalar sistemasini O nuqta atrofida burish formulasi deyiladi.