MISOLLAR

$R, S, T \subset A \times A$ – binar munosabatlar uchun

Yechish. Binar munosabatlar tartiblangan juftliklardan iborat to'plamlar ekanligini bilgan holda to'plamlar ayirmasi, to'plamlar tengligi hamda binar munosabatlar kompozisiyasining ta'riflaridan foydalanib berilgan tenglikni isbotlaymiz:

$\forall(x,y)\in(R \circ (S \setminus T)) \Rightarrow \exists\, z \in A,\ (x,z)\in(S \setminus T)\wedge (z,y)\in R\Rightarrow$

$\Rightarrow(x,z)\in S \wedge (x,z)\notin T\wedge (z,y)\in R\Rightarrow(x,z)\in S \wedge (z,y)\in R\wedge$

$\wedge(x,z)\notin T\wedge (z,y)\in R\Rightarrow(x,y)\in(R \circ S)\wedge(x,y)\notin(R \circ T)\Rightarrow$

$\Rightarrow(x,y)\in ((R \circ S) \setminus (R \circ T))$. Demak, $R \circ (S \setminus T)\subset(R \circ S) \setminus (R \circ T)$;

2) $\forall(x',y')\in ((R \circ S) \setminus (R \circ T)) \Rightarrow(x',y')\in(R \circ S)\wedge(x',y')\notin(R \circ T)\Rightarrow$

$\Rightarrow\exists\, z' \in A,\ ((x',z')\in S \wedge (z',y')\in R)\ \wedge((x',z')\notin T\wedge (z',y')\in R)\Rightarrow$

$\Rightarrow(x',z')\in S \wedge(x',z')\notin T\wedge(z',y')\in R\Rightarrow (x',z')\in(S \setminus T)\wedge (z',y')\in R\Rightarrow$

$\Rightarrow (x',y')\in(R \circ (S \setminus T))$. Demak, $(R \circ S) \setminus (R \circ T) \subset R \circ (S \setminus T)$.

Natijada $R \circ (S \setminus T)= (R \circ S) \setminus (R \circ T)$ tenglik isbotlandi.

$M = \{1, 2, \dots, 10\}$ to'plamda berilgan

$R=\{\langle x,y\rangle \mid x, y \in M \wedge x = y -1\}$ binar munosabatning xossalarini tekshiring va grafini chizing.

Yechish. Berilgan binar munosabatni qanday xossalarga bo'ysunishini tekshiramiz: refleksivlik xossasi. $\forall(x\in M)\ (x = x-1)$ yolg'on, chunki, masalan M to'plamning 2 elementi uchun $2 \neq 2 - 1$. Demak, R- refleksiv emas.

Antirefleksivlik xossasi. $\forall(x\in M)\ \neg(x = x-1)$ rost. Demak, R- antirefleksiv.

Simmetriklik xossasi. $\forall(x,y\in M)\ (x = y -1 \Rightarrow y = x-1)$ yolg'on. Chunki, masalan $3, 4 \in M$ uchun $3 = 4-1 \Rightarrow 4 = 3-1$ da birinchi mulohaza rost va ikkinchi mulohaza yolg'on bo'lganligi uchun implikasiya yolg'on. Demak, R- simmetrik emas.

Antisimmetriklik xossasi. $\forall(x,y\in M)\ (x = y-1 \wedge y = x -1 \Rightarrow x = y)$ rost. Chunki, M to'plamning har qanday x, u elementlari uchun $x = y-1$ va $y = x-1$ mulohazalar bir vaqtda rost bo'la olmaydi. Bundan ularning kon'yunksiyasi berilgan to'plam elementlari uchun yolg'on. Birinchi mulohaza yolg'onbo'lgan implikasiya rost ekanligini e'tiborga olsak, R- antisimmetrik binar munosabat ekanligi kelib chiqadi.

Tranzitivlik xossasi.

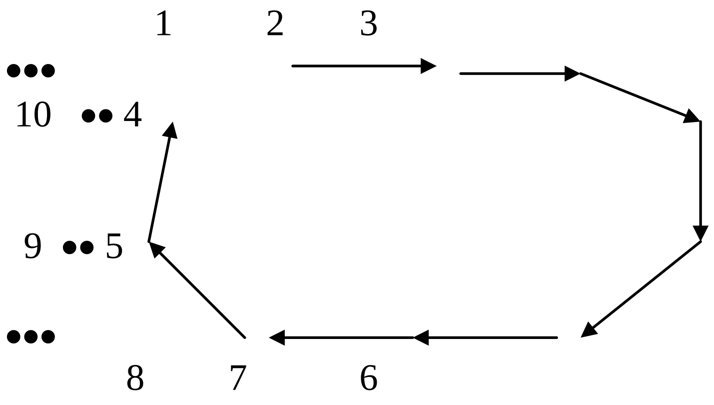
$\forall(x,y,z\in M)\ (x = y-1 \wedge y = z-1 \Rightarrow x = z-1)$ yolg'onmulohaza. Chunki, masalan M to'plamning 3,4,5 elementlariuchun

$(3 = 4-1)\wedge(4 = 5 -1)\Rightarrow (3 = 5-1)$ implikasiyadakon'yunksiyarost, lekinimplikasiyanatijasiyolg'onmulohaza. Implikasiyata'rifigako'ra, $(3 = 4-1)\wedge(4 = 5 -1)\Rightarrow (3 = 5-1)$ mulohazayolg'on. Demak, R-tranzitivemas.

6)R-ekvivalentlikmunosabatibo'laolmaydi, chunkirefleksivlik, simmetriklik, tranzitivlikxossalarigaegaemas.

7)R-tartibmunosabatibo'laolmaydi, chunkiRantisimmetrikbo'lganibilantranzitivemas.

Endiberilganbinarmunosabatninggrafinichizamiz. Buninguchun M to’plamningelementlarigatekislikda 10 tanuqtanimosqo’yamiz. Ulargrafninguchlaribo’ladi. Rmunosabatdabo’lganelementlaruchunularniifodalovchigrafuchlariniyo’naltirilgan kesmalar–grafqirralaribilantutashtiramiz. M to’plamninghechbirelementio’zi – o’zibilanR munosabatdabo’lmaganiuchungrafuchlarigahalqalarchizmaymiz. R simmetrikmunosabatbo’lmaganligiuchunqirralaryo’naltirilgan (orientirlangan) bo’ladi:



5. A = {1,2}, B = {2,5} to’plamlaruchunR = AxB ,S = BxAbinarmunosabatlarnitopib, RoS, SoR, R^2 , S^2larnianiqlang.

Yechish.To’plamlarning to’g’ri ko’paytmasi, binar munosabatlar kompozisiyasi ta’riflaridan foydalanib quyidagi to’plamlarni hosil qilamiz:

R = A x B = {(1,2), (1,5), (2,2), (2,5)};

S = B x A = {(2,1), (2,2), (5,1), (5,2)}

R o S = {(2,2), (2,5), (5,2), (5,5)}

S o R = {(1,1), (1,2), (2,1), 2,2)}

R^2= R o R = {(1,2), (1,5), (2,2), (2,5)} ;

S^2 = S o S ={(2,1), (2,2), (5,1),(5,2)}.

6.Berilgan A= {lola, shoda, olomon, osmon, olma, boshoq} so’zlaridan iborat to’plam va undagi S binar munosabat :

«x S y» ⇌ «x va y so’zlardao harfibirhil sonda qatnashgan» berilgan. $^{A}/_{S}$ faktor-to’plamnianiqlang.

Yechish. Faktor to’plam - bo’shbo’lmaganto’plamdaaniqlangan ekvivalentlikmunosabatiyordamida hosilqilingan ekvivalentliksinflaridan tuzilgan to’plam. Berilgan to’plam 6 taso’zdan iborat to’plam va undagi har qanday ikkitax,uso’zlar berilgan binar munosabatdabo’ladi, agar buso’zlar tarkibidao harfibirxil sonda qatnashgan bo’lsa.

To’plamda berilgan S binar munosabat ekvivalentlikmunosabatiekanliginiisbotlaymiz:

S – refleksivlikmunosabati, chunkiAto’plamning har birso’zinio’zibilan solishtirsak, ulardao harfibirhil sonda qatnashgan.

S – simmetriklikmunosabati, chunkiAto’plamning har qanday x, u so’zlariuchunagar xso’zbilan u so’zdao harfibirhil sonda qatnashgan bo’lsa, u holda u so’zbilan xso’zlarda ham o harfibirhil sonda qatnashadi.
S – tranzitivlikmunosabati, chunkiAto’plamning har qanday x, u, z so’zlariuchunagar xso’zbilan u so’zda va u so’zbilan z so’zdao harfibirhil sonda qatnashgan bo’lsa, u holdaxso’zbilan z so’zlarda ham o harfibirhil sonda qatnashadi.
Endi S ekvivalentlikmunosabatiyordamidaekvivalentliksinflarinituzamiz. Buninguchun «lola» so’zibilan ekvivalentlikmunosabatidabo’lganso’zlarnibirto’plamgayig’amiz:
$S/_{lola}$ = {lola, shoda, olma}. Xuddishunday yo’lbilan qolgan ekvivalentliksinflarinituzamiz:
$S/_{osmon}$ = {osmon, boshoq}, $S/_{olomon}$ = {olomon}.
U holda $A/_{S}$ = {$S/_{lola}$, $S/_{osmon}$, $S/_{olomon}$}.
R, S, T – binar munosabatlar uchun quyidagilarniisbotlang:
$(R \cap S)^{\cup} = R^{\cup} \cap S^{\cup}$.
$(R \cup S)^{\cup} = R^{\cup} \cup S^{\cup}$.
$R \circ (S \circ T) = (R \circ S) \circ T$.
$(R \circ S)^{\cup} = S^{\cup} \circ R^{\cup}$.
$(R \cup S) \circ T = R \circ T \cup S \circ T$.
$R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$.
$(R \cap S) \circ T \subset R \circ T \cap S \circ T$.
$R \circ (S \cap T) \subset R \circ S \cap R \circ T$.
Dom $(R^{\cup})$ = Im R ..
Im $(R^{\cup})$ = Dom R..
Dom $(R \circ S) \subset$ Dom S.
Im $(R \circ S) \subset$ Im R.
$(R \setminus S)^{\cup} = R^{\cup} \setminus S^{\cup}$.
R, S - tranzitiv$\Rightarrow$ $R \cup S$– tranzitiv.
S – refleksiv$\Rightarrow$ $S^{\cup}$ – refleksiv.
R, S - simmetrik$\Rightarrow$ $R \cup S$– simmetrik.
R, S - ekvivalent$\Rightarrow R^{\cup}$, $S^{\cup}$ – ekvivalent.
R, S - qat’iy tartib$\Rightarrow$ $R \cup S$ – qat’iy tartib.
S - qisman tartib$\Rightarrow$ $S^{\cup}$ – qisman tartib.
R - chiziqli tartib $R^{\cup}$ – chiziqli tartib.
R, S - antirefleksiv$\Rightarrow$ $R \cup S$– antirefleksiv.
S - antisimmetrik$\Rightarrow$ $S^{\cup}$ – antisimmetrik.
$A \subset B \Rightarrow A \times C \subset B \times C$.
$A \cup B \subset C \Rightarrow A \times B = (A \times B) \cap (C \times B)$.
$(A \times B) \cup (B \times A) = C \times C \Rightarrow A = B = C$.

R, S - tranzitiv$\Rightarrow R^{\cup}$, $S^{\cup}$ – tranzitiv.

R, S - ekvivalent$\Rightarrow$ $R \cup S$ – ekvivalent.

R - chiziqli tartib$\Rightarrow$ $R^{\cup}$– chiziqli tartib.

R , S – refleksiv$\Rightarrow$ $R^{\cup} \cup S^{\cup}$ – refleksiv.

S - antirefleksiv$\Rightarrow$ $S^{\cup}$ – antirefleksiv.

M = {1, 2, . . . , 20} to'plamda berilgan quyidagi binar munosabatlarningxossalarini tekshiring va grafinichizing:

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x^2 + y^2 = 10 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge (x - y) \vdots 3 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge (x - y) \vdots 4 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x \leq y + 2 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x \leq y + 3 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y = 15 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x \leq y + 1 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge |x| = |y| \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x \vdots y \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x < y \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x \leq y \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x \neq y \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x^2 + x = y^2 + y \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x^2 + y^2 = 1 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x \vdots y \vee x < y \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge (x - y) \vdots 2 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y = 12 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y \leq 7 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y = 20 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y \geq 20 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge (x + y) \vdots 5 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge (x > y \wedge x \vdots 3) \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y \geq 10 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x - y \geq 5 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y = 10 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x + y = 21 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x - y = 2 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x - y = -2 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x - y = 4 \}$.

$R = \{ \langle x,y \rangle \mid x,y \in M \wedge x - y = 6 \}$.

1-misol. $f=\{(1,3),(2,3),(3,6)\}$ $g=\{(1,3),(2,1),(3,4)\}$ bo'lsa, u holda $f\circ g=\{(1,6),(2,3)\}$.

2-misol. Funksiyalar kompozisiyasi quyidagi xossalarini isbotlang:

1°. $Dom\ f\circ g=\{x\ /g(x)\in Dom\ f\}$

2°. $\forall x\in Dom\ f\circ g$ *учун* $(f\circ g)(x)=f(g(x))$

3°. $f\circ g=\{(x,\ f(g(x)))\ /g(x)\in Dom\ f\}$

4°. $Dom\ f\circ g\subset Dom\ g$.

5°. $\mathrm{Im}\,(f\circ g)\subset \mathrm{Im}\ f$

6°. *Агар* $\mathrm{Im}\ g=Dom\ f$ *булса*, $Dom\ f\circ g=Dom\ g$ *ва* $\mathrm{Im}\,(f\circ g)=\mathrm{Im}\ f$

3-misol. Haqiqiysonlarto'plami R nio'zinio'zigaakslantiradigan $f(x)=x^2$ funksiyain'ektivhamemas, biektivhamemashaqiqatdanham $+2\neq -2$. Lekin $(-2)^2=2^2=4;$ $\mathrm{Im}\ f=R^+\cup\{0\};$ $[R^+\cup\{0\}]$- manfiybo'lmaganhaqiqiysonlarto'plami.

4-misol. $f(x)=x^2$ funksiyabarchahaqiqiysonlarto'plamini $R^+\cup\{0\}$ to'plamgaakslantirsin. U holda $\mathrm{Im}\ f=R^+\cup\{0\}$.Demak, f -syur'ektivakslantirish, lekinin'ektivakslantirishemas.

5-misol. $y=\sqrt{x}$ funksiya $R^+\cup\{0\}$ to'plamni R - haqiqiysonlarto'plamigaakslantiradi. Bufunksiyain'ektiv, lekinsyur'ektivemas.

6-misol. $y=x^3$ funksiya R - haqiqiysonlarto'plamini R o'zinio'zigaakslantiradiganbiektivfunksiyadir.

7-misol. $x=\{a,b\}$ to'plamberilganbo'lsin, u holda $f(a)=b;\quad f(b)=a;\quad g(a)=a;\quad g(b)=a$ shartlarbilananiqlangan f va g funksiyalarniqarasak, $((f\circ g)(a))=f(g(a))=f(a)=b$. $(f\circ g)(b)=f(g(b))=f(a)=b;(g\circ f)(a)=g(f(a))=g(b)=a\quad (g\circ f)(b)=g(f(b))=g(a)=a$ bo'ladi.

Bumisoldanko'rinadiki, $f\circ g\neq g\circ f$,ya'nifunksiyalarkompozisiyasihardoimhamkommutativbo'lavermasekan.

8-misol. $B(A)-A$ to'plamningbarchato'plamostilarito'plamibo'lsin. $B(A)$ to'plamdato'plamostibo'lishmunosabatinoqat'iytartibmunosabtidir.

9-misol. $A=\{4,12,36,72\}$ to'plamdabo'linishmunosabatinoqat'iytartibmunosabatidir.

10-misol. N -naturalsonlarto'plamida $R=\{(x,y)\,|\,\forall x,y\in N\ x\vdots y\}$ munosabatqismantartibmunosabatbo'ladi.

$"<"=\{(x,y)\,|\,\forall x,\ y\in N\ \exists k\in N\ y=x+k\}$

munosabatesachiziqlitartibmunosabatdir.

11-misol. $(N,<)$-juftlikchiziqlitartiblanganto'plamdir. Kelgisida $a<b$ yozuvniodatdagidek $a<b$, $a\leq b$ yozuvniesa a kichikyokiteng b debo'qiymizva $a\leq b$

ni $(a < b) \vee (a = b)$ mulohazama'nosidatushunamiz. Xususan $4 \le 4, 3 \le 4$ mulohazalaraynanrostmulohazalardir.

$(A, <)$- tartiblanganto'plamberilganbo'lsin, u holda $a \in A$ elementdankichikelementmavjudbo'lmasa a - minimalelement, agar a dankattaelementmavjudbo'lmasa a -maksimalelementdeyiladi. A dagio'zidanboshqabarchaelementlaridankichikbo'lgan a element A ningengkichikelementi, A dagio'zidanboshqabarchaelementlaridankattabo'lgan b element A ningengkattaelementideyiladi.

12-misol. $A = \{1, 2, 3, 4, 12\}$ to'plamida, agar $a \vdots b$ bo'lsa, $b < a$ deylik, u holda 1 engkichikelement, 12 engkattaelementbo'ladi.

13–misol. N -naturalsonlarto'plamida< -tabiiytartibmunosabatibo'lsin. Ya'niagar $\forall a, в \in N$ uchunshunday R topilib, $a = в + к$ bo'lsa, $в < a$ deymiz. U holda $(N, <)$ to'plamto'liqtartiblanganto'plamdir.

14–misol. R - haqiqiysonlarto'plamitabiiytartibmunosabatganisbatanto'liqtartiblanganbo'laolmaydi. Chunki R ningengkichikelementiyo'q.