

To'g'ri ko'paytma misollari

1. Agar 1) $A=\{a,b,s\}$, $B=\{3;5;9\}$;
2) $A = \{x;y\}$, $B = \{x,y,z\}$; 3) $A = \{3;5\}$, $B = \{1;5\}$
bo'lsa, $A \times B$, $B \times A$ to'plamni toping.

2. Quyidagi to'plamlarni Dekart koordinatalar sistemasida geometrik tasvirini toping:

- 1) $[0;1] \times [0;1]$; 2) $[-1;1] \times ([2;3])$;
3) $[1;3] \times (-\infty;3]$; 4) $[0;3] \times [1;+\infty)$;
5) $[1;4] \times (-\infty;+\infty)$; 6) $[-1;5] \times \{2,3,4\}$;
7) $[0;+\infty) \times \{1;3\}$; 8) $(-\infty;+\infty) \times \{1,2,3\}$.

3. a) Ixtiyoriy A, B va C to'plamlar uchun quyidagi tenglikni isbotlang:

- 1) $(A \cup B) \times C = (A \times C) \cup (B \times C)$; 2) $(A \cap B) \times C = (A \times C) \cap (B \times C)$
3) $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$; 4) $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$
5) $A \times (B \cup C) = (A \times B) \cup (A \times C)$; 6) $A \cup B \subset C \Rightarrow A \times B = (A \times C) \cap (C \times B)$;
7) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.

b) Ixtiyoriy A, B, C va D to'plamlar uchun quyidagi tengliklar to'g'rimi?

- 1) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \times D)$; 2) $(A \times B) \times C = A \times (B \times C)$;
3) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

4. $A_1 = \{0, \diamond\}$, $B = \{1, 2, 3\}$ to'plamlar berilgan bo'lsin, u holda

$$A \times B = \{(0, 1), (0, 2), (0, 3), (\diamond, 1), (\diamond, 2), (\diamond, 3)\}$$

$$B \times A = \{(1, 0), (1, \diamond), (2, 0), (2, \diamond), (3, 0), (3, \diamond)\}.$$

Bu misoldan $A \times B \neq B \times A$ ekanligini ko'rish mumkin, ya'ni dekart ko'paytma kommutativ emas ekan.

5. $R = A \times B$, $S = B \times A$ binar munosabatlar uchun $R \circ S$, $S \circ R$, R^2 , S^2 larni aniqlang:

5.1. $A = \{0, 2, 4\}$, $B = \{\alpha, \beta, \gamma\}$;

5.2. $A = \{\square, \diamond\}$, $B = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$;

5.3. $A = \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, $B = \{\cap, \cup, \in, \subset\}$.

5.4. $A = \{1, 3, 5\}$, $B = \{11, 13, 15\}$;

5.5. $A = \{2, 4, 6\}$, $B = \{12, 14, 16\}$;

5.6. $A = \{7, 9, 11\}$, $B = \{17, 19\}$;

5.7. $A = \{2, 3, 5\}$, $B = \{10, 13, 18\}$;

5.8. $A = \{3, 5, 7\}$, $B = \{1, 3, 5\}$;

5.9. $A = \{1, 4, 5\}$, $B = \{1, 4, 5\}$;

5.10. $A = \{11, 13, 14\}$, $B = \{11, 12, 13\}$;

5.11. $A = \{5, 6, 7\}$, $B = \{1, 11, 15\}$;

$$5.12. \quad A = \{10, 13, 15\}, \quad B = \{1, 11, 15\};$$

$$5.13. \quad A = \{4, 5\}, \quad B = \{17, 18, 19\};$$

Agar $A_1 \times \dots \times A_n$ dekart ko'paytmada $A_1 = A_2 = \dots = A_n = A$ bo'lsa, bunday dekart ko'paytma A^n ko'rinishida yoziladi va A to'plamning n-dekart darajasi deyiladi. Xususan $A^2 - A$ ning dekart kvadrati deyiladi. To'plamlarning birinchi va nolinci darajalarini $A^1 = A$, $A^0 = \emptyset$ tengliklar ko'rinishida aniqlash kelishilgan.

2-misol. N^2 ning $\{(1, 1), (2, 2), (3, 3), (4, 4), \dots\}$ to'plamostisi natural sonlar to'plamida aniqlangan tenglik munosabatidir.

3-misol. N^2 ning " $<$ " = $\{(1, 2), (1, 3), \dots, (2, 3), (2, 4), \dots, (3, 4), (3, 5), \dots\}$ to'plamostisini qaraylik. Bu munosabat tengsizlik munosabati bo'lib $(a, b) \in "<"$ bo'lishi $a < b$ orqali belgilanadi va a kichik b deb o'qiladi. 5.5-ta'rifdan ko'rinib turibdiki, A da -0 o'rinli munosabat, bu $A^0 = \{\emptyset\}$ to'plamning to'plamostilari bo'lib, faqat $\emptyset \in \{\emptyset\}$ to'plamlardan iboratdir.

Bir o'rinli munosabat esa A ning ixtiyoriy to'plamostisi bo'lar ekan. Bir o'rinli munosabat unar munosabat deyiladi.

4-misol. $A = \{a, b\}$ to'plamda aniqlangan barcha unar munosabatlar

$\emptyset, \{a\}, \{b\}, \{a, b\}$ to'plamlardan iborat.

Binar munosabatlar matematikada ko'p uchraydigan munosabatlardan biri bo'lganligi uchun u bilan to'liqroq tanishib chiqamiz.

5-misol. Z - butun sonlar to'plamida $\forall a, b \in Z$ butun sonlar ayirmasi birdan katta bo'lgan m butun songa qoldiqsiz bo'linsa, a soni b soni bilan, m -modul bo'yicha taqqoslanadi deyiladi va $a \equiv b \pmod{m}$ (mad m) deb yoziladi. Bu munosabat refleksiv munosabatidir, haqiqatdan $\forall a \in Z$ uchun $a - a = 0 : m$, ya'ni $a \equiv a \pmod{m}$; \equiv -simmetrik munosabatdir, chunki $a \equiv b \pmod{m}$ bo'lsa $a - b : m$, demak $-(b - a) : m$, ya'ni $b \equiv a \pmod{m}$; \equiv -tranzitiv munosabatdir, haqiqatdan $a \equiv b \pmod{m}$ va $b \equiv c \pmod{m}$ bo'lsa, $(a - b) : m$ va $(b - c) : m$ bo'ladi, u holda $a - c = ((a - b) + (b - c)) : m$, ya'ni $a \equiv c \pmod{m}$ bo'ladi. SHunday qilib \equiv -munosabat-refleksiv, simmetrik, tranzitiv ya'ni ekvivalentlik munosabati ekan.

6-misol. Tekislikdagi barcha to'g'ri chiziqlar to'plamida to'g'ri chiziqlarning parallel bo'lishi munosabati ekvivalentlik munosabatidir.

7-misol. Tekislikdagi barcha uchburchaklar to'plamida uchburchaklarning o'xshashlik munosabati ekvivalentlik munosabatidir.

8-misol. Z -butun sonlar to'plamida 3 modul bo'yicha taqqoslash munosabati berilgan bo'lsin, u holda $\bar{0} = \{3z / z \in Z\}$, $\bar{1} = \{3z + 1 / z \in Z\}$, $\bar{2} = \{3z + 2 / z \in Z\}$. Bu ekvivalentlik sinflari 3 modul bo'yicha chegirmalar sinflari deyiladi.