

Chiziqli algebralar

Reja:

- Chiziqli algebra haqida tushunchalar.
- Chiziqli algebra misollar.
- Chiziqli operatorlar algebrasi.
- Matritsalar algebrasi.
- Chiziqli operatorlar algebrasi va matritsalar algebrasi orasidagi izomorfizm.

\mathcal{F} maydon ustida V_n vektor fazo berilgan bo'lib,

$$\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n \quad (1)$$

uning biror bazisi va φ operator berilgan V_n fazoning chiziqli operatori bo'lsin. \bar{x} va $\varphi(\bar{x})$ vektorlarning (1) bazis orqali $x = \beta_1 \bar{e}_1 + \dots + \beta_n \bar{e}_n$, $\varphi(\bar{x}) = \gamma_1 \bar{e}_1 + \dots + \gamma_n \bar{e}_n$ ko'rinishda ifodalansin.

\bar{x} va $\varphi(\bar{x})$ vektorlarning (1) bazisga nisbatan ustun koordinatalarini mos ravishda ushbu

$$M(\bar{x}) = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_n \end{bmatrix}, \quad M(\varphi(\bar{x})) = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \dots \\ \gamma_n \end{bmatrix}$$

ko'rinishlarda belgilab, ular orasidagi bog'lanish formulasini keltirib chiqaraylik.

Teorema. Agar φ operator V_n fazoda aniqlangan chiziqli operator bo'lib, $M(\varphi)$ shu φ chiziqli operatorning (1) bazisdagi matritsasi bo'lsa, u holda $\forall \bar{x} \in V_n$ uchun $M(\varphi(\bar{x})) = M(\varphi)M(\bar{x})$ tenglik bajariladi.

Ta'rif. \mathcal{F} maydon ustidagi V chiziqli fazo elementlari uchun quyidagi aksiomalar bajarilsa,

1. $\overline{xy} \in V (\forall \overline{x}, \overline{y} \in V)$;
2. $\overline{x(\overline{y}\overline{z})} = (\overline{xy})\overline{z} (\forall \overline{x}, \overline{y}, \overline{z} \in V)$;
3. $\overline{x(\overline{y} + \overline{z})} = \overline{xy} + \overline{xz}$ **ba** $(\overline{y} + \overline{z})\overline{x} = \overline{yx} + \overline{zx} (\forall \overline{x}, \overline{y}, \overline{z} \in V)$
4. $\lambda(\overline{xy}) = (\lambda\overline{x})\overline{y} = \overline{x(\lambda\overline{y})} (\lambda \in F, \forall \overline{x}, \overline{y} \in V)$

u holda V fazoni \mathcal{F} maydon ustidagi chiziqli algebra deyiladi.

Ta'rif. Agar V chiziqli algebrada $\overline{x} \bullet \overline{y} = \overline{y} \bullet \overline{x} (\forall \overline{x}, \overline{y} \in V)$ aksioma bajarilsa, V kommutativ chiziqli algebra deyiladi.

Ta'rif. V chiziqli algebraning rangi deb V fazoning o'lchoviga aytiladi.

Misol. $C = \{a+bi \mid \forall a, b \in \mathbb{R}, i^2 = -1\}$ to'plam \mathbb{R} maydon ustida rangi ikkiga teng bo'lgan chiziqli algebra tashkil etadi.

Misol. barcha n -tartibli kvadrat matritsalar to'plami $F^{n \times n}$, \mathcal{F} maydon ustida rangli n^2 bo'lgan chiziqli algebra tashkil etadi. Bunday chiziqli algebrani \mathcal{F} maydon ustidagi to'liq matritsalar algebrasi deyiladi.

Misol. \mathbb{R} maydon ustidagi kvaternionlar algebrasi \mathbb{R} maydon ustidagi to'rt o'lchovli V_4 vektor fazo bo'lib, $\overline{e}, \overline{i}, \overline{j}, \overline{k}$ vektorlar V_4 fazoning bazisi bo'lsin. V_4 fazoda ko'paytirish amali quyidagi qoida asosida kiritiladi:

$$\overline{i}^2 = \overline{j}^2 = \overline{k}^2 = -\overline{e}, \quad \overline{i} \cdot \overline{j} = -\overline{j} \cdot \overline{i} = \overline{k}, \quad \overline{j} \cdot \overline{k} = -\overline{k} \cdot \overline{j} = \overline{i}, \quad \overline{k} \cdot \overline{i} = -\overline{i} \cdot \overline{k} = \overline{j},$$

$\overline{a} \cdot \overline{e} = \overline{e} \cdot \overline{a}, \quad \overline{a} \in \{\overline{e}, \overline{i}, \overline{j}, \overline{k}\}$. U holda V_4 fazo rangi 4 ga teng bo'lgan kvaternionlar algebrasi bo'ladi.

V fazo \mathcal{F} maydon ustidagi vektor fazo bo'lib, φ, ψ lar shu vektor fazoning chiziqli operatorlari bo'lsin. φ va ψ chiziqli operatorlar ko'paytmasi quyidagicha aniqlangan bo'lsin, ya'ni $(\varphi\psi)(\overline{x}) = \varphi(\psi(\overline{x})), \forall \overline{x} \in V$.

Lemma. V vektor fazoning ixtiyoriy ikkita chiziqli operatorlari ko'paytmasi yana shu vektor fazoning chiziqli operatori bo'ladi.

Bizga ma'lumki $\text{Hom}(V, V)$ to'plam \mathcal{F} maydon ustida vektor fazo tashkil qiladi.

Ushbu algebrani $\langle \text{Hom}(V, V), +, \{\omega_\lambda | \lambda \in F\}, \bullet \rangle$ algebra V vektor fazoning chiziqli operatorlar algebrasi deyiladi va quyidagicha belgilanadi:

$$\text{End } V = \langle \text{Hom}(V, V), +, \{\omega_\lambda | \lambda \in F\}, \bullet \rangle$$

Teorema. Agar V fazo \mathcal{F} maydon ustidagi vektor fazo bo'lsa, u holda $\text{End } V$ algebra \mathcal{F} maydon ustida chiziqli algebra tashkil qiladi.

Isboti. $\text{End } V$ algebra chiziqli algebra shartlarini to'liq bajaradi. Haqiqatan,

1. $\langle \text{Hom}(V, V), +, \{\omega_\lambda | \lambda \in F\}, \bullet \rangle$ algebra \mathcal{F} maydon ustida vektor fazo tashkil qiladi;

2. $(\varphi + \psi)\lambda = \varphi\lambda + \psi\lambda;$

3. $\chi(\varphi + \psi) = \chi\varphi + \chi\psi;$

4. $\lambda(\varphi\psi) = (\lambda\varphi)\psi = \varphi(\lambda\psi), \varphi, \psi, \chi \in \text{Hom}(V, V), \text{ va } \lambda \in F.$

Ta'rif. U va U' algebralar \mathcal{F} maydon ustidagi chiziqli algebralar va $\varphi: U \rightarrow U'$ akslantirish biekktiv akslantirish bo'lib, quyidagi shartlar bajarilsa:

1. $\varphi(\bar{a} + \bar{b}) = \varphi(\bar{a}) + \varphi(\bar{b});$

2. $\varphi(\lambda\bar{a}) = \lambda\varphi(\bar{a});$

3. $\varphi(\bar{a} \cdot \bar{b}) = \varphi(\bar{a}) \cdot \varphi(\bar{b}), \forall \bar{a}, \bar{b} \in V \wedge \forall \lambda \in F$

u holda φ akslantirishga izomorfizm U va U' chiziqli algebralarga esa izomorf chiziqli algebralar deyiladi va u $U \cong U'$ ko'rinishda belgilanadi.

Misol. $S = \langle \mathbb{C}, +, \{\omega_\lambda | \lambda \in \mathbb{R}\}, \bullet \rangle$ - chiziqli algebra, $G = \left\{ \begin{pmatrix} a-b \\ b \ a \end{pmatrix} \middle| \forall a, b \in \mathbb{R} \right\}$;

$G = \langle G, +, \{\omega_\lambda | \lambda \in \mathbb{R}\}, \bullet \rangle$ - chiziqli algebra bilan izomorf, ya'ni $S \cong G$ bo'ladi (bunda

$$\varphi: a+bi \rightarrow \begin{pmatrix} a-b \\ b \ a \end{pmatrix}.$$

Agar \mathcal{F} maydon ustidagi matritsalar algebrasini $M(n, F) = \langle F^{n \times n}, +, \{\omega_\lambda | \lambda \in F\}, \bullet \rangle$ ko'rinishda belgilasak, u holda quyidagi teorema o'rinli bo'ladi:

Teorema. V fazo \mathcal{F} maydon ustidagi vektor fazo bo'lib, $\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n$ uning bazisi, $M(\varphi)$ matritsa V vektor fazoda aniqlangan φ chiziqli operatorning $\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n$ bazisga nisbatan matritsasi va $\varphi \rightarrow M(\varphi)$ akslantirish mavjud bo'lsa, u holda $\text{End } V \cong M(n, \mathcal{F})$ munosabat o'rinli bo'ladi.

Isboti. Bizga ma'lumki, $\text{End } V \rightarrow M(n, F)$ akslantirish biektiv akslantirish bo'ladi.

$$1. \quad M(\varphi + \psi) = M(\varphi) + M(\psi).$$

$$\text{Isboti. } \forall \bar{x} \in V \quad \varphi(\bar{x}) = \alpha_1 \bar{e}_1 + \dots + \alpha_n \bar{e}_n, \quad \psi(\bar{x}) = \beta_1 \bar{e}_1 + \dots + \beta_n \bar{e}_n$$

$$(\varphi + \psi)(\bar{x}) = \varphi(\bar{x}) + \psi(\bar{x}) = (\alpha_1 + \beta_1) \bar{e}_1 + \dots + (\alpha_n + \beta_n) \bar{e}_n$$

$$M((\varphi + \psi)(\bar{x})) = \begin{pmatrix} \alpha_1 + \beta_1 \\ \dots \\ \alpha_n + \beta_n \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \dots \\ \alpha_n \end{pmatrix} + \begin{pmatrix} \beta_1 \\ \dots \\ \beta_n \end{pmatrix} = M(\varphi(\bar{x})) + M(\psi(\bar{x})) \Rightarrow M((\varphi + \psi)(\bar{x})) = M(\varphi(\bar{x})) + M(\psi(\bar{x}))$$

$$M(\varphi + \psi)M(\bar{x}) = [M(\varphi) + M(\psi)]M(\bar{x}). \quad M(\varphi + \psi) = M(\varphi) + M(\psi).$$

$$2. \quad M(\lambda\varphi) = \lambda M(\varphi).$$

$$\text{Isboti. } ((\lambda\varphi)(\bar{x})) = \lambda\alpha_1 \bar{e}_1 + \dots + \lambda\alpha_n \bar{e}_n,$$

$$M((\lambda\varphi)(\bar{x})) = \begin{pmatrix} \lambda\alpha_1 \\ \dots \\ \lambda\alpha_n \end{pmatrix} = \lambda \begin{pmatrix} \alpha_1 \\ \dots \\ \alpha_n \end{pmatrix} = \lambda M(\varphi(\bar{x})),$$

$$M(\lambda\varphi)M(\bar{x}) = (\lambda M(\varphi)) M(\bar{x}). \quad M(\lambda\varphi) = \lambda M(\varphi).$$

$$3. \quad M(\varphi\psi) = M(\varphi)M(\psi) \quad (\forall \varphi, \psi \in \text{Hom}(V, V), \forall \lambda \in F)$$

$$\text{Isboti. } M((\varphi\psi)(\bar{x})) = M(\varphi(\psi(\bar{x}))) = M(\varphi) M(\psi(\bar{x})) = M(\varphi) M(\psi) M(\bar{x}).$$

$$M(\varphi\psi) M(\bar{x}) = [M(\varphi)M(\psi)] M(\bar{x}) \Rightarrow M(\varphi\psi) = M(\varphi)M(\psi)$$

Demak, ta'rifga asosan $\text{End } V \cong M(n, F)$ bo'ladi.

Takrorlash uchun savollar:

1. Chiziqli algebra deb nimaga aytiladi?
2. Chiziqli algebraga misollar keltiring.
3. Chiziqli operatorlar algebrasi deb nimaga aytiladi?
4. Matritsalar algebrasi deb nimaga aytiladi?
5. Algebralar izomorfizmi haqida teoramani bayon qiling.

Foydalaniladigan adabiyotlar ro'yxati

Asosiy adabiyotlar:

1. Malik D.S., Mordeson J.N., Sen M.K. Fundamental of abstract algebra. WCB McGraw-Hill, 1997.
2. Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" 2010.
3. Кострикин А.М. Введение в алгебру.- М.- «Мир».- 1977.

4. Под ред. Кострикина, Сборник задач по алгебре, М.Наука, 1986.
5. Хожиев Ж.Х. Файнлейб А.С. Алгебра ва сонлар назарияси курси, Тошкент, «Ўзбекистон», 2001 й.
6. Курош А.Г. Олий алгебра курси, Тошкент, «Ўқитувчи». 1975й.
7. Гельфанд И.М. Чизиқли алгебрадан лекциялар. «Олий ва ўрта мактаб». 1964.
8. Р.Н.Назаров, Б.Т. Тошпўлатов, А.Д.Дусумбетов, Алгебра ва сонлар назарияси 1 қисм, 2 қисм, 1993й., 1995й.
9. A.Yunusov , D.Yunusova , Algebra va sonlar nazariyasi. Modul texnologiyasi asosida tuzilgan musol va mashqlar to'plami. O'quv qo'llanma. 2009.

Qo'shimcha adabiyotlar:

1. Фаддеев Д.К. Лекции по алгебре, М., “Наука”1984г.
2. Фаддеев Д.К., Соминский И.С. Сборник задач по высшей алгебре, М.: Наука, 1977 г.
3. Поскуряков И.Л. Сборник задач по линейной алгебре. «Наука», 1978г.
4. Ламбек И. Кольца и модули.- М.- «Мир».- 1971.
5. Херстейн. Некоммутативные кольца. М.- «Мир».- 1967.
6. Vilnis Detlovs, Karlis Podnieks, Introduction to Mathematical Logic. [University of Latvia](#). Version released: August 25, 2014.
7. **А.Юнусов , Д.Юнусова, М.Маматқулова, Г.Артикова, Модул технологияси асосида тайёрланган мустақил ишлар тўплами. 1–3–қисмлар, 2010.**
8. Скорняков Л.Ф. Элементы общей алгебры. М., 1983 г.
9. Петрова В.Т. лексия по алгебре и геометрии. Ч.1,2. Москва,1999г.
10. Yunusov A.S. Matematik mantiq va algoritmlar nazariyasi elementlari. Т., “Yangi asr avlodi”. 2006.
11. Yunusov A., Yunusova D. Sonli sistemalar. Т., «Moliya–iqtisod», 2008.

12. Мазуров В.Д. и др. Краткий конспект курса высшей алгебры.

Elektron ta'lim resurslari

1. www.Ziyo.Net
2. <http://vilenin.narod.ru/Mm/Books/>
3. <http://www.allmath.ru/>
4. <http://www.pedagog.uz/>
5. <http://www.ziyonet.uz/>
6. <http://window.edu.ru/window/>
7. <http://lib.mexmat.ru;>
8. [http://www.mcce.ru,](http://www.mcce.ru)
9. <http://lib.mexmat.ru>
10. <http://techlibrary.ru;>