Vektorlar fazosining bazisi va o'lchovi

Reja:

- Vektorlar fazosining bazisi.
- Vektorlar fazosining o'lchovi.
- Vektorlar fazosining bazisi va o'lchovi haqidagi teoremalar.

23.1-Ta'rif. Agar V vektor fazoning chiziqli bog'lanmagan

$$\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n \qquad (1)$$

vektorlar sistemasi mavjud bo'lsaki, V ning qolgan barcha vektorlari (1) sistema orqali chiziqli ifodalansa, u holda (1) vektorlar sistemasi V vektorlar fazosining bazisi deyiladi.

V vektorlar fazosining bazisini $\overline{e}_1, \overline{e}_2, \ldots, \overline{e}_n$ (2)

vektorlar sistemasi ko'rinishida belgilasak, unda $\forall \overline{a} \in V$ vektorni (2) bazis orqali chiziqli ifodalash mumkin, ya'ni shunday $\alpha_1, \alpha_2, \ldots, \alpha_n \in F$ sonlar topiladiki, natijada $\overline{a} = \alpha_1 \overline{e}_1 + \alpha_2 \overline{e}_2 + \ldots + \alpha_n \overline{e}_n$ (3)

tenglik bajariladi.

23.2-Ta'rif. V vektorlar fazosining (2) bazis vektorlari uchun (3) tenglik o'rinli bo'lsa, $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ kortejga $\overline{a}$ vektorning (2) bazisga nisbatan satr koordinatalari deyiladi.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.159-174.

23.3-Ta'rif. V vektorlar fazosining bazislaridagi vektorlar soni V vektor fazoning o'lchovi deyiladi.[1]

4.2.1. Definition. *Let M be a subset of a vector space A and let $\mathfrak{S}$ be the family of subspaces containing M. The subspace $\mathbf{Le}(M) = \cap\mathfrak{S}$ is called the linear envelope of M or the subspace generated by the subset M. We also sometimes say that $\mathbf{Le}(M)$ is the subspace spanned by M. The subset M is called a set of generators or a spanning set for $\mathbf{Le}(M)$. In particular, if $\mathbf{Le}(M) = A$, then we say that*

M generates or spans A. The space A is called finitely generated, if there exists a finite subset M such that $\mathbf{Le}(M) = A$.

If B is a subspace containing M, then B contains $\mathbf{Le}(M)$, by Corollary 4.1.12. Thus $\mathbf{Le}(M)$ is the smallest subspace containing M. It is clear that if M is a subspace of A then $\mathbf{Le}(M) = M$. So we have the following.

V fazoning o'lchovi dimV orqali belgilanadi.

Agar (1) sistema V fazoning bazisi bo'lsa, V fazo n o'lchovli fazo deyiladi. n o'lchovli vektor fazo V_n yoki V^n orqali belgilanadi.

Agar (1) sistema chekli bo'lmasa, u holda bunday vektorlar fazosi cheksiz o'lchovli vektorlar fazosi deb ataladi.

23.4-Teorema. R haqiqiy sonlar maydoni ustida berilgan R^n fazoning istalgan n+1 ta vektori chiziqli bog'langan bo'ladi.

23.5-Teorema. V vektorlar fazosining ixtiyoriy vektori (2) bazis vektorlar sistemasi orqali yagona usulda chiziqli ifodalanadi.

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[1] Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.159-174.

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Isboti. V fazoda (2) sistema bazis bo'lsa, unda bazisning ta'rifiga asosan, istalgan n+1 ta vektorlar chiziqli bog'langan bo'ladi. Demak, kamida bittasi noldan farqli shunday $\alpha_1, \alpha_2, \dots, \alpha_n, \alpha_{n+1}$ sonlar mavjudki, ular uchun

$$\alpha_1 \bar{e}_1 + \alpha_2 \bar{e}_2 + \dots + \alpha_n \bar{e}_n + \alpha_{n+1} \bar{a} = \bar{0} \qquad (4)$$

tenglik bajariladi. O'z-o'zidan ma'lumki, (4) tenglikda $\alpha_{n+1} \neq 0$, aks holda

$$\alpha_1 \bar{e}_1 + \alpha_2 \bar{e}_2 + \dots + \alpha_n \bar{e}_n = \bar{0} \qquad (5)$$

bo'lib, (5) tenglik (2) ning bazis ekanligiga zid keladi. (4) tenglikning ikkala tomonini α_{n+1} ga bo'lib va $(n+1)$-haddan boshqa hadlarni qarama-qarshi ishora bilan o'ng tomonga o'tkazib,

$$\bar{a} = h_1 \bar{e}_1 + h_2 \bar{e}_2 + \dots + h_n \bar{e}_n \qquad (6)$$

tenglikni hosil qilamiz. (6) da $h_i = -\frac{\alpha_i}{\alpha_{n+1}} \quad (i = \overline{1, n})$ bo'ladi.

Endi (6) chiziqli ifodalanishning yagona ekanligini isbotlaymiz.

Teskarisini faraz qilaylik, ya'ni $\bar{a}$ vektor uchun (6) dan farqli kamida yana bitta

$$\bar{a} = \beta_1 \bar{e}_1 + \beta_2 \bar{e}_2 + \dots + \beta_n \bar{e}_n \qquad (7)$$

chiziqli ifodalanish mavjud bo'lsin.

(6) tenglikdan (7) ni hadlab ayiramiz. U holda

$$(h_1 - \beta_1)\bar{e}_1 + (h_2 - \beta_2)\bar{e}_2 + \dots + (h_n - \beta_n)\bar{e}_n = \bar{0} \qquad (8)$$

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tenglik hosil bo'ladi. (2) vektorlar sistemasi chiziqli bog'lanmagan bo'lgani uchun (8) tenglik faqat barcha koeffitsientlar nolga teng bo'lgandagina bajariladi. Demak, $h_i = \beta_i \quad (i = \overline{1, \ n})$ tengliklar o'rinli.

4.2.3. Proposition. *Let F be a field, let A be a vector space over F and let M be a subset of A. Then,* $\mathbf{Le}(M)$ *consists of all linear combinations of all finite subsets of the set M.*

Proof. Let U denote the set of all linear combinations of all finite subsets of M and let $a_1, \ldots, a_n$ be arbitrary elements of M. If B is a subspace of A containing M, then by Corollary 4.1.12, every linear combination of elements of M belongs to B. Since this is true for every subspace containing M, every linear combination of the elements $a_1, \ldots, a_n$ belongs to $\mathbf{Le}(M)$. Thus $U \leq \mathbf{Le}(M)$.

Now let $x, y \in U$ and let $\gamma \in F$. Then $x = \alpha_1 a_1 + \cdots + \alpha_n a_n$ and $y = \beta_1 b_1 + \cdots + \beta_k b_k$, where $a_1, \ldots, a_n, b_1, \ldots, b_k \in M$ and $\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_k \in F$. We have

$$\begin{aligned} x - y &= (\alpha_1 a_1 + \cdots + \alpha_n a_n) - (\beta_1 b_1 + \cdots + \beta_k b_k) \\ &= \alpha_1 a_1 + \cdots + \alpha_n a_n + (-\beta_1) b_1 + \cdots + (-\beta_k) b_k. \end{aligned}$$

Hence $x - y$ is a linear combination of $a_1, \ldots, a_n, b_1, \ldots, b_k \in M$, so that $x - y \in U$. Furthermore,

$$\begin{aligned} \gamma x &= \gamma(\alpha_1 a_1 + \cdots + \alpha_n a_n) = \gamma(\alpha_1 a_1) + \cdots + \gamma(\alpha_n a_n) \\ &= (\gamma \alpha_1) a_1 + \cdots + (\gamma \alpha_n) a_n, \end{aligned}$$

so that γx is a linear combination of the elements $a_1, \ldots, a_n \in M$ and therefore $\gamma x \in U$. Hence U satisfies conditions (**SS 1**) and (**SS 2**); Theorem 4.1.7 shows

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that U is a subspace of A. If c is an element of M then $c = ec \in U$ and it follows that $M \subseteq U$. By Proposition 4.2.2, $\mathbf{Le}(M) \subseteq U$ and, since $U \leq \mathbf{Le}(M)$, we have $\mathbf{Le}(M) = U$, which proves the result.

Takrorlash uchun savollar:

1. Vektorlar fazosining bazisi deb nimaga aytiladi?
2. Vektorlar fazosining o'lchovi deb nimaga aytiladi?
3. R^n fazoning (n+1) ta vektorlari haqidagi teoremani bayon qiling.
4. V_n fazoning ixtiyoriy vektorining bazis orqali chiziqli ifodalanishining yagonaligi haqidagi teoremani bayon qiling.

Foydalaniladigan adabiyotlar ro'yxati

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