Vektorlarning chiziqli bog'liq, chiziqlibog'liq bo'lmagan sistemalari, xossalari

Reja:

- Vektorlarsistemasi.
- Vektorlarsistemasiningchiziqlikombinatsiyasi.
- Vektorlarningchiziqlibog'liqsistemasi.
- Vektorlarningchiziqlibog'liqbo'lmagansistemasi.
- Vektorlarningchiziqlibog'liq, chiziqlibog'liqbo'lmagansistemalarixossalari.

$F = < F;+,\cdot,-,^{-1},0,1 >$ maydonustidaqurilgan $\mathrm{F}^n = < F^n;+,\{\omega_\lambda \mid \lambda \in F\} >$ arifmetikvektorfazoberilganbo'lsin.

13.1-ta'rif. F^n vektorfazoningvektorlaridaniborat $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n, \ldots$ sistemagavektorlarningcheksizsistemasi; $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ sistemagavektorlarningcheklisistemasideyiladi.

13.2-ta'rif. F^n vektorfazoning $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n, \ldots$ sistemasiva F maydonning $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$ skalyarlariberilganbo'lsin. $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \ldots + \lambda_n\vec{a}_n + \ldots$ ifodaga $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n, \ldots$ vektorlarsistemasiningchiziqlikombinatsiyasideyiladi. Chiziqlikombinatsiyadagi $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$ skalyarlarchiziqlikombinatsiyaningkoeffitsientlarideyiladi.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.162-174.

13.1-misol. $\vec{a}=(1,2,3),\vec{b}=(-1,2,4),\vec{c}=(7,-5,2)$ vektorlarva $\alpha=-2,\beta=5,\gamma=9$ skalyarlarberilganbo'lsa, ularningchiziqlikombinatsiyasiniquyidagichaaniqlaymiz: $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=$

$$=(-2)(1,2,3)+5(-1,2,4)+9(7,-5,2)=(-2,-4,-6)+(-5,10,20)+$$

$$+(63,-45,18)=(56,-39,32).$$

13.3-ta'rif. F sonlarmaydoniustidaqurilgan F^n arifmetikvektorfazoningcheklisondagi $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ (1)

vektorlariuchun F maydondakamidabittasinoldanfarqlishunday $\lambda_1,\lambda_2,\ldots,\lambda_n$ skalyarlartopilib, ularuchunushbu

$$\lambda_1\vec{a}_1+\lambda_2\vec{a}_2+\ldots+\lambda_n\vec{a}_n=\vec{0} \qquad (2)$$

tenglikbajarilsa, u holda (1) sistemavektorlarningchiziqlibog'langansistemasideyiladi. Agar (2) tenglik $\lambda_1=0,\lambda_2=0,\ldots,\lambda_n=0$ bo'lgandabajarilsa, u holda (1) vektorlarningchiziqlibog'lanmagan (chiziqlierkli) sistemasideyiladi.[1]

4.2.6. Definition. *Let F be a field and let A be a vector space over F. A nonempty subset M of A is called free or linearly independent, if $x \notin \mathbf{Le}(M \setminus \{x\})$ for each element $x \in M$.*

Vektorlarningbo'shsistemasichiziqlibog'lanmagansistemahisoblanadi.

13.4-ta'rif. Agaristalgan $\lambda_i\left(i=\overline{1,n}\right)$ sonlaruchunushbu

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[1] Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya.Subbotin, "ALGEBRA AND NUMBER THEORY" pp.162-174.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.162-174.

$$\vec{a} = \lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \ldots + \lambda_n\vec{a}_n \quad (3)$$

tenglikbajarilsa, u holda $\vec{a}$ vektor $\vec{a}_i\,(i=\overline{1,n})$ vektorlarorqalichiziqliifodalanadi ($\vec{a}$ vektor $\vec{a}_i\,(i=\overline{1,n})$ vektorlarningchiziqlikombinatsiyasidaniborat) deyiladi.

13.2-misol. $\vec{e}_1$=(1,0,0), $\vec{e}_2$=(0,1,0), $\vec{e}_3$=(0,0,1) vektorlarsistemasichiziqlierklivektorlarsistemasiekanliginiisbotlang.

Haqiqatdanham, $\alpha_1\vec{e}_1+\alpha_2\vec{e}_2+\alpha_3\vec{e}_3=\alpha_1(1,0,0)+\alpha_2(0,1,0)+\alpha_3(0,0,1)=$ $=(\alpha_1,\alpha_2,\alpha_3)=(0,0,0)$ bo'lib, bundan $\alpha_1=0$, $\alpha_2=0$, $\alpha_3=0$ kelibchiqadi. Demak, $\vec{e}_1,\vec{e}_2,\vec{e}_3$ vektorlarsistemasichiziqlibog'lanmagansistemabo'ladi.

13.2-misol. F^n arifmetikvektorfazoning $\vec{e}_1=(1,0,\ldots,0),\vec{e}_2=(0,1,0,\ldots,0),\ldots,\vec{e}_n(0,\ldots,0,1)$ vektorlaridaniboratsistemachiziqlibog'lanmagan. Bu sistema n-o'lchovlibirlikvektorlardaniboratsistema.

13.1-teorema. Kamidabittanolvektorgaegavektorlarning $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ cheklisistemasichiziqlibog'langansistemabo'ladi.

Isbot. $\vec{a}_i$ vektornolvektorbo'lsin. U holdaharqandaynoldanfarqli

λ_i skalyaruchun $0\cdot\vec{a}_1+0\cdot\vec{a}_2+\ldots+\lambda_i\cdot\vec{0}+\ldots+0\cdot\vec{a}_m=\vec{0}$ tenglikbajariladi. Demak, $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ sistemachiziqlibog'langansistema.

13.2-teorema. Cheklivektorlarsistemasiningbiror-birqismichiziqlibog'langanbo'lsa, sistemaningo'zihamchiziqlibog'langanbo'ladi.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.162-174.

13.3-teorema. Vektorlarningchiziqlibog’lanmagansistemasiningharqandayqismsistemasi chiziqlibog’lanmagansistemabo’ladi.

13.4-teorema. Agar $\vec{a}_2,\ldots,\vec{a}_n$ vektorlardankamidabittasio’zidanoldingivektorlarningchiziqlikombinatsiyasidanib oratbo’lsa, u holda $\vec{a}_1 \neq \vec{0}$ bo’lgan $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ vektorlardaniboratsistemachiziqlibog’langanbo’ladi.

13.5-teorema. Agar $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ vektorlarningsistemasichiziqlibog’lanmaganbo’lib, $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n,\vec{b}$ sistemachiziqlibog’langanbo’lsa, u holda $\vec{b}$ vektor $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ vektorlarsistemasiorqaliyagonausuldachiziqliifodalanadi.

4.2.7. Proposition *(criterion for linear independence). Let F be a field, let A be a vector space over F, and let M be a subset of A.*

(i) *If M is linearly independent then every nonempty subset of M is linearly independent.*

(ii) *An infinite subset M is linearly independent if and only if every finite nonempty subset of M is linearly independent.*

(iii) *The finite subset $S = \{a_1, \ldots, a_n\}$ is linearly independent if and only if the equation $\alpha_1 a_1 + \cdots + \alpha_n a_n = 0_A$ always implies that $\alpha_1 = \cdots = \alpha_n = 0_F$.*

Proof.

(i) Suppose that M is a linearly independent subset and let W be a nonempty subset of M. Suppose, for a contradiction, that W is not linearly independent. Then, by definition, there exists an element $w \in W$ such that $w \in \mathbf{Le}(W\backslash\{w\})$. The inclusion $W \subseteq M$ implies that $W\backslash\{w\} \subseteq M\backslash\{w\}$ and Corollary 4.2.2 shows that $\mathbf{Le}(W\backslash\{w\}) \leq \mathbf{Le}(M\backslash\{w\})$. It follows that $w \in \mathbf{Le}(M\backslash\{w\})$, contradicting the fact that M is linearly independent. Thus, W must also be linearly independent.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, “ALGEBRA AND NUMBER THEORY” pp.162-174.

(ii) If M is linearly independent, then every finite nonempty subset of M is linearly independent by (i). Conversely, suppose that every nonempty finite subset of M is linearly independent, but that M is not linearly independent. Then there exists an element $x \in M$ such that $x \in \mathbf{Le}(M \setminus \{x\})$. By Corollary 4.2.4, $M \setminus \{x\}$ contains a finite subset T such that $x \in \mathbf{Le}(T)$. Let $Y = T \cup \{x\}$, and note that Y is finite, $x \in Y$ and $x \in \mathbf{Le}(Y \setminus \{x\})$. It follows that Y is linearly dependent and we obtain a contradiction. Therefore M is linearly independent.

(iii) Suppose that S is linearly independent and let $\alpha_1 a_1 + \cdots + \alpha_n a_n = 0_A$. Suppose, for a contradiction, that there is a coefficient α_j such that $\alpha_j \neq 0_F$. Then $\alpha_j a_j = \sum_{k \neq j} \alpha_k a_k$ and, since F is a field, the nonzero element α_j has a multiplicative inverse α_j^{-1}. Therefore, $a_j = \sum_{k \neq j} (\alpha_j^{-1} \alpha_k) a_k$ and it follows that $a_j \in \mathbf{Le}(S \setminus \{a_j\})$, the desired contradiction, since S is linearly independent. Consequently, $\alpha_j = 0_F$ for all j, where $1 \leq j \leq n$.

Conversely, suppose that $\alpha_1 a_1 + \cdots + \alpha_n a_n = 0_A$ always implies that $\alpha_1 = \cdots = \alpha_n = 0_F$. Assume, for a contradiction, that S is not linearly independent. Then there exists an element a_m such that $a_m \in \mathbf{Le}(S \setminus \{a_m\})$. By Proposition 4.2.3, we obtain $a_m = \sum_{k \neq m} \beta_k a_k$ for certain $\beta_k \in F$. It follows that

$$\beta_1 a_1 + \cdots + \beta_{m-1} a_{m-1} + (-e) a_m + \beta_{m+1} a_{m+1} + \cdots + \beta_n a_n = 0_A.$$

13.6-teorema. Agar $\vec{a}$ vektor $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n$ orqali va $\vec{b}_i \, (i = \overline{1,n})$ vektorlar $\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_m$ vektorlar orqali chiziqli ifodalansa, u holda $\vec{a}$ vektor $\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_m$ vektorlar orqali chiziqli ifodalanadi.

13.7-teorema. Agar $\vec{a}_1, \ldots, \vec{a}_{n+1}$ vektorlar $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n$ vektorlar orqali chiziqli ifodalansa, u holda $\vec{a}_1, \ldots, \vec{a}_{n+1}$ sistema chiziqli bog'langan bo'ladi.

13.3-misol. $\vec{a}_1 = (2,4,7), \vec{a}_2 = (3,6,11), \vec{a}_3 = (4,8,13)$ vektorlar $\vec{b}_1 = (1,2,3), \vec{b}_2 = (1,2,4)$ orqali chiziqli ifodalanadi:

$\vec{a}_1 = \vec{b}_1 + \vec{b}_2, \vec{a}_2 = \vec{b}_1 + 2\vec{b}_2, \vec{a}_3 = 3\vec{b}_1 + \vec{b}_2.$

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.162-174.

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ vektorlarsistemasiningchiziqlibog'liqliginiko'rsatamiz:

$$\begin{matrix}\vec{a}_1\\ \vec{a}_2\\ \vec{a}_3\end{matrix}\begin{pmatrix}2 & 4 & 7\\ 3 & 6 & 11\\ 4 & 8 & 13\end{pmatrix} \sim \begin{matrix}\vec{a}_1\\ 2\vec{a}_2-3\vec{a}_1\\ \vec{a}_3-2\vec{a}_1\end{matrix}\begin{pmatrix}2 & 4 & 7\\ 0 & 0 & 1\\ 0 & 0 & 1\end{pmatrix} \sim \begin{matrix}\vec{a}_1\\ 2\vec{a}_2-3\vec{a}_1\\ \vec{a}_3-2\vec{a}_1-(2\vec{a}_2-3\vec{a}_1)\end{matrix}\begin{pmatrix}2 & 4 & 7\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}.$$

Hosilbo'lganpog'onasimonmatritsadanolsatrmavjud. Bundan

$\vec{a}_3 - 2\vec{a}_1 - (2\vec{a}_2 - 3\vec{a}_1) = \vec{0}$ ifodayordamida $\vec{a}_3 = 2\vec{a}_1 + (2\vec{a}_2 - 3\vec{a}_1) = -\vec{a}_1 + 2\vec{a}_2$ tenglikni, ya'ni $\vec{a}_3$ vektorning $\vec{a}_1, \vec{a}_2$ vektorlaryordamidagiifodasinikeltiribchiqaramiz. Demak, $\vec{a}_1, \vec{a}_2, \vec{a}_3$ vektorlarsistemasichiziqlibog'langan.

13.1-natija. Agar $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ vektorlar $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_m$ sistemaorqalichiziqli

ifodalansava $n > m$ bo'lsa, u holda $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ sistemachiziqlibog'langanbo'ladi.

13.2-natija. Agar $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ vektorlar $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_m$ sistemaorqalichiziqliifodalansava $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ sistemachiziqlibog'lanmaganbo'lsa, uholda $n \le m$ bo'ladi.

13.3-natija. n-o'lchovliarifmetikvektorfazoningharqanday n danortiqvektorlardaniboratsistemasichiziqlibog'langanbo'ladi.

13.4-misol. R^3 da $\vec{a}$ (1; 2; 3), $\vec{b}$ (-1; 0; 3), $\vec{c}$ (2; 1; -1) , $\vec{d}$ (3; 2; 2) vektorlarsistemasiberilgan.
Uningchiziqlibog'langanyokichiziqlibog'lanmaganliginitekshiramiz.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.162-174.

$\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}+\delta\vec{d}=\vec{0}$ tenglamadan $\alpha=-\frac{1}{4};\beta=\frac{7}{4};\gamma=\frac{5}{2};\delta=-1$

ekanliginitopamiz. Demak, ta’rifgako’raberilgansistemachiziqlibog’langan. Haqiqatdanham, $\vec{d}=-\frac{1}{4}\vec{a}+\frac{7}{4}\vec{b}+\frac{5}{2}\vec{c}$, ya’nisistemaningbittavektoriqolganlariningchiziqlikombinatsiyasiko’rinishidaifodalanadi.

Takrorlashuchunsavollar:

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2. Vektorlarsistemasiningchiziqlikombinatsiyasigata’rifbering.
3. Vektorlarningchiziqlibog’liqsistemasidebnimagaaytiladi?.
4. Vektorlarningchiziqlibog’liqbo’lmagansistemasita’rifiniayting.
5. Vektorlarningchiziqlibog’liqsistemasixossalariniayting.
6. Vektorlarningchiziqlibog’liqbo’lmagansistemalarixossalariniayting.

Foydalaniladigan adabiyotlar ro’yxati

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