Deteminantning nolga teng bo'lish sharti. Kramer formulasi

Reja:

- Determinantnolgatengbo'lishiningzarurvayetarlisharti.
- Matritsarangihaqidateorema.
- Algebraikto'ldiruvchilaryordamidateskarimatritsanitopish.
- Kramerformulalari.

$F = < F;+,\cdot,-,^{-1},0,1 >$ maydonvamaydonustida $F^{n\times n}$ matrisalarto'plamivaA= $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ berilganbo'lsin.

11.1-teorema. Kvadrat matritsaning determinanti nolga teng bo'lishi uchun uning satr (ustun)lari chiziqli bog'langan bo'lishi zarur va yetarli.[1]

2.4.4. Corollary. *Let* $A = [a_{ij}] \in \mathbf{M}_n(\mathbb{R})$. *Then*

$$\sum_{1\le j\le n} a_{tj}A_{mj} = \delta_{tm}\mathbf{det}(A) \left(and \sum_{1\le j\le n} a_{jt}A_{jm} = \delta_{tm}\mathbf{det}(A) \right),$$

for all $1 \le t, m \le n$, *where* δ_{tm} *is the Kronecker symbol.*

Isbot. 1. Matritsaning satrlari chiziqli erkli bo'lsa, $|A| \neq 0$ ekanligini isbotlaymiz.

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[1]Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya.Subbotin, "ALGEBRA AND NUMBER THEORY" pp.79-93.
*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.79-93.

Agar berilgan kvadrat matritsaning satrlari chiziqli erkli bo'lsa, u holda uni elementar matritsalar ko'paytmasi ko'rinishida ifodalash mumkin, ya'ni $A = E_1 \cdot E_2 \cdot \ldots \cdot E_k$. U holda determinant xossalariga ko'ra

$|A| = |E_1| \cdot |E_2| \cdot \ldots \cdot |E_k|$ va $|E_i| \neq 0 (i = \{1, \ldots, k\}$. Bundan $|A| \neq 0$.

To'g'ri teorema bilan teskari teoremaga qarama-qarshi teoremalar teng kuchli bo'lganligidan, $|A| = 0$ ekanligidan A matritsa chiziqli erkliligi kelib chiqadi.

2. A matritsaning satrlari chiziqli bog'liq bo'lsa, $|A| = 0$ ekanligini isbotlaymiz.

Satrlari chiziqli bog'liq matritsaning kamida bitta satri qolganlari orqali chiziqli ifodalanadi. Determinantlar xossalariga ko'ra $|A| = 0$.

Proof. Let $(c_1, c_2, \ldots, c_n)$ be an arbitrary tuple of n real numbers, and replace row t of A by this tuple to obtain a matrix that we denote by B. Thus, if $B = [b_{ij}]$, then

$$b_{ij} = \begin{cases} a_{ij}, & \text{if } i \neq t, \\ c_j, & \text{if } i = t. \end{cases}$$

By Theorem 2.4.3 we have

$$\mathbf{det}(B) = \sum_{1 \leq j \leq n} b_{jt} B_{tj}.$$

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Evidently the cofactor B_{tj} to the element b_{tj} in the matrix B coincides with A_{tj} (in order to obtain it we just cross out the tth row so we eliminate the row that makes the difference between the matrices A and B). By the definition of the elements b_{tj} we have

$$\mathbf{det}(B) = \sum_{1\le j\le n} c_j A_{tj}.$$

Now let $c_j = a_{mj}$, where $1 \le j \le n$. If $m = t$, then $B = A$, and Theorem 2.4.3 implies that

$$\sum_{1\le j\le n} a_{tj} A_{tj} = \mathbf{det}(A).$$

On the other hand, if $m \ne t$, the matrix B has two identical rows and Corollary 2.3.8 implies that its determinant is zero. Thus, $\sum_{1\le j\le n} a_{tj} A_{mj} = 0$. The Kronecker symbol allows us to write the equations we obtained as follows:

$$\sum_{1\le j\le n} a_{tj} A_{mj} = \delta_{tm}\mathbf{det}(A).$$

The second of our assertions can be obtained in a similar manner.

11.1-misol. $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 2 & 4 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 7 \\ 0 & 0 & 0 \end{vmatrix} = 0.$

11.2-teorema. Har qanday kvadrat matritsa uchun quyidagi shartlar teng kuchli:

1. $|A| \ne 0$.

2. Matritsaning satr (ustun)lari chiziqli erkli.

3. A matritsa teskarilanuvchi.

4. A matritsa elementar matritsalar yordamida ifodalanadi.

11.3-teorema. A matritsaning rangi uning noldan farqli minorlarining eng yuqori tartibiga teng.

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Isboti. Noldan farqli A=$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$matritsa berilgan

bo'lsin. U holda uning rangi $r = r(A) > 0$. Matritsaning kamida bitta noldan farqli r tartibli minori mavjudligini isbotlaymiz.

$r = r(A) > 0$ bo'lganligi uchun, A matritsaning r ta chiziqli erkli satrlari bor. Shu satrlardan tuzilgan A matritsaning $B \in F^{r \times n}$ matritsaostisini tuzamiz B=$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{r1} & a_{r2} & \dots & a_{rn} \end{pmatrix}$, bu matritsaning rangi $r(B) = r$. Matritsaning satr va ustun ranglari tengligidan $\rho(B) = r$. Demak, B matritsaning r ta chiziqli erkli ustunlari mavjud. B matritsaning r ta chiziqli erkli ustunlaridan tashkil topgan matritsaostisini C bilan belgilaymiz. U holda $C \in F^{r \times r}$ va $r(C) = r$. Yuqoridagi 11.2-teorema shartlariga ko'ra, C matritsaning ustunlari chiziqli erkli bo'lganligi uchun $|C| \neq 0$.

Demak, C matritsa A matritsaning tartibi r ga teng bo'lgan noldan farqli minori bo'ladi.

Agar $k > r(A)$ bo'lsa, A matritsaning k tartibli har qanday minori nolga teng bo'ladi.

Haqiqatdan ham, $k > r(A)$ bo'lsa, A matritsaning har qanday k ta satri chiziqli bog'langan bo'ladi. Bundan A matritsaning har qanday ($k \times k$) tartibli qismmatritsasida satrlari chiziqli bog'langan bo'ladi va 11.1-teoremaga ko'ra bunday qismmatritsalar determinanti, ya'ni A matritsaning k tartibli har qanday minori nolga teng.

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11.2-misol. $A=\begin{pmatrix} 1 & -1 & 0 & 2 \\ 2 & 1 & 1 & -2 \\ -1 & 0 & 1 & 3 \\ 2 & -2 & 0 & 4 \end{pmatrix}$ matritsa rangini minorlar yordamida aniqlang.

Yechish. Matritsa rangi haqidagi teoremaga ko'ra matritsaning noldan farqli minorlarini aniqlaymiz.

Matritsaning berilishidan, unda kamida bitta noldan farqli birinchi tartibli minor mavjud, masalan, $A_1=(1)$ matritsaostining determinanti 1ga teng, ya'ni $M_1=|1|=1\neq 0$.

Matritsaning $A_2=\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ matritsaostining determinanti

$$M_2=\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}=1-(-2)=3\neq 0.$$

Matritsaning $A_3=\begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$ matritsaostining determinanti

$$M_3=\begin{vmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix}=1+0+1-0-0-(-2)=4\neq 0.$$

Matritsaning 4-tartibli minori berilgan matritsaning determinantidan iborat, uni hisoblaymiz:

$$|A|=\begin{vmatrix} 1 & -1 & 0 & 2 \\ 2 & 1 & 1 & -2 \\ -1 & 0 & 1 & 3 \\ 2 & -2 & 0 & 4 \end{vmatrix}=\begin{vmatrix} 1 & -1 & 0 & 2 \\ 0 & 3 & 1 & -6 \\ 0 & -1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{vmatrix}=0.$$

Demak, berilgan matritsaning noldan farqli minorlari 1-tartibli, 2-tartibli va 3-tartibli. Ulardan yuqori tartibligi 3-tartibli minor bo'lganligi uchun, berilgan matritsaning rangi 3 ga teng.

11.1-ta'rif. A=$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ matritsaning a_{ij} elementining

A_{ij} $(i, j \in \{1,\dots,n\}$ algebraik to'ldiruvchilaridan iborat

$$A^* = \begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ . & . & \dots & . \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}$$ matritsaga A matritsaga biriktirilgan

matritsa deyiladi.

11.4-teorema. Agar $|A| \neq 0$ bo'lsa, u holda A matritsa teskarilanuvchi va $A^{-1} = |A|^{-1} \cdot A^*$.

Isbot. 17.3-Laplas teoremasi va 17.4-teoremalarga ko'ra

$$A_i (A^*)^j = (a_{i1},\dots,a_{in)} \cdot \begin{pmatrix} A_{j1} \\ \vdots \\ A_{jn} \end{pmatrix} = a_{i1}A_{j1} + \dots + a_{in}A_{jn} = \begin{cases} |A, agar, i = j;| \\ 0, agar, i \neq j. \end{cases}$$

Ya'ni, $A \cdot A^* = \begin{vmatrix} |A| & 0 & \dots & 0 \\ 0 & |A| & \dots & 0 \\ . & . & . & . \\ 0 & 0 & \dots & |A| \end{vmatrix} = |A| \cdot E$ gaegabo'lamiz. Bundan $|A| \neq 0$ bo'lsa,

$A \cdot (|A|^{-1} \cdot A^*) = E$ (1) hosilbo'ladi.

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Xuddishunday $A^* \cdot A = |A| \cdot E$ tenglikdan $|A| \neq 0$ bo'lsa, $\quad (|A|^{-1} \cdot A^*) \cdot A = E \quad$ (2) tenglikkaegabo'lamiz.

(1), (2) tengliklardan A va $|A|^{-1} \cdot A^*$ laro'zaroteskariekanligikelibchiqadi, ya'ni $A^{-1} = |A|^{-1} \cdot A^*$.

$F = < F;+,\cdot,-,^{-1},0,1 >$ maydonustidaquyidagi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n. \end{cases} \quad (3)$$

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$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix} \text{bo'lsin.}$$

11.3-misol. $\begin{pmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$

A=matritsagateskarimatritsanialgebraikto'ldiruvchilaryordamidatoping.

Yechish.Berilgan A matritsaningdeterminantinihisoblaymiz:

$$|A| = \begin{vmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{vmatrix} = 5(4\text{-}9)+1(2–12)–1(3–8)=\text{-}25–10+5=\text{-}30.$$

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Determinantnoldanfarqli, demak, matritsaningteskarisimavjud.Matritsaningharbirelementialgebraikto’ldiruvchisinitopamiz:

$$A_{11} = (-1)^{1+1} \text{™} M_{11} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5;$$

$$A_{12} = (-1)^{1+2} \text{™} M_{12} = \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} = 10;$$

$$A_{13} = (-1)^{1+3} \text{™} M_{13} = \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = -5;$$

$$A_{21} = (-1)^{2+1} \text{™} M_{21} = \begin{vmatrix} -1 & -1 \\ 3 & 2 \end{vmatrix} = -1;$$

$$A_{22} = (-1)^{2+2} \text{™} M_{22} = \begin{vmatrix} 5 & -1 \\ 4 & 2 \end{vmatrix} = 14;$$

$$A_{23} = (-1)^{2+3} \text{™} M_{23} = \begin{vmatrix} 5 & -1 \\ 4 & 3 \end{vmatrix} = -19;$$

$$A_{31} = (-1)^{3+1} \text{™} M_{31} = \begin{vmatrix} -1 & -1 \\ 2 & 3 \end{vmatrix} = -1;$$

$$A_{32} = (-1)^{3+2} \text{™} M_{32} = \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = -16;$$

$$A_{33} = (-1)^{3+3} \text{™} M_{33} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 11;$$

$$A^{-1} = |A|^{-1} \cdot A^{*} = \begin{pmatrix} \frac{1}{6} & \frac{1}{30} & \frac{1}{30} \\ -\frac{1}{3} & -\frac{7}{15} & \frac{8}{15} \\ \frac{1}{6} & \frac{19}{30} & -\frac{11}{30} \end{pmatrix};$$

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Tekshirish:

$$A\cdot A^{-1}=\begin{pmatrix}5 & -1 & -1\\ 1 & 2 & 3\\ 4 & 3 & 2\end{pmatrix}\begin{pmatrix}\frac{5}{30} & \frac{1}{30} & \frac{1}{30}\\ -\frac{10}{30} & -\frac{14}{30} & \frac{16}{30}\\ \frac{5}{30} & \frac{19}{30} & -\frac{11}{30}\end{pmatrix}=$$

$$=\frac{1}{30}\begin{pmatrix}25+10-5 & 5+14-19 & 5-16+11\\ 5-20+15 & 1-28+57 & 1+32-33\\ 20-30+10 & 4-42+38 & 4+48-22\end{pmatrix}=E.$$

11.5-teorema. $|A|\neq 0$ bo'lsa, uholda (3) CHTSyagona yechimga ega va uquyidagiformulalar orqaliifodalanadi:

(4) $x_1=|A|^{-1}(\beta_1 A_{11}+\ldots+\beta_n A_{n1}),\ldots,x_n=|A|^{-1}(\beta_1 A_{1n}+\ldots+\beta_n A_{nn}).$

Isbot. $\begin{cases}a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1,\\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2,\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n=b_n.\end{cases}$ sistemani

$A=\begin{pmatrix}a_{11} & a_{12} & \ldots & a_{1n}\\ a_{21} & a_{22} & \ldots & a_{2n}\\ \ldots & \ldots & \ldots & \ldots\\ a_{n1} & a_{n2} & \ldots & a_{nn}\end{pmatrix}$, $B=\begin{pmatrix}b_1\\ b_2\\ \ldots\\ b_n\end{pmatrix}$, $X=\begin{pmatrix}x_1\\ x_2\\ \ldots\\ x_n\end{pmatrix}$ belgilashlaryordamida

$AX = B$ ko'rinishgakeltiramiz. Teorema shartiga ko'ra $|A| \neq 0$ bo'lganligiuchun $AX = B$ matritsalitenglamaningyagona $X = A^{-1}B$ yechimimavjud.

11.4-teoremaga ko'ra $A^{-1} = |A|^{-1} \cdot A^{*}$ ekanligidan,

$$X = A^{-1}B = |A|^{-1} \cdot \begin{pmatrix} A_{11} & \dots & A_{n1} \\ . & . & . \\ A_{1n} & \dots & A_{nn} \end{pmatrix} \cdot \begin{pmatrix} \beta_1 \\ . \\ \beta_n \end{pmatrix} = |A|^{-1} \cdot \begin{pmatrix} \beta_1 A_{11} + & \dots & + \beta_n A_{n1} \\ . & . & . \\ \beta_1 A_{1n} + & \dots & + \beta_n A_{nn} \end{pmatrix},$$

ya'ni, $$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} |A|^{-1}(\beta_1 A_{11} + \dots + \beta_n A_{n1}) \\ \dots\dots\dots\dots\dots\dots\dots \\ |A|^{-1}(\beta_1 A_{1n} + \dots + \beta_n A_{nn}) \end{pmatrix}.$$

11.5-teorema Kramerqoidasiva (4) formulalarKramerformulalarideyiladi.

Agar $A(j)$ $j \in \{1,\dots,n\}$ orqali Amatritsaning j-ustunini (3) sistemaning ozodhadlarustunibilan almashtirishdanhosilbo'lganmatritsanibelgilasak, uholda

$$A(1) = \begin{pmatrix} \beta_1 & a_{12} & \dots & a_{1n} \\ . & . & \dots & . \\ \beta_n & a_{n2} & \dots & a_{nn} \end{pmatrix}, \dots, A(n) = \begin{pmatrix} a_{11} & \dots & a_{1n-1} & \beta_1 \\ . & . & \dots & . \\ a_{n1} & \dots & a_{nn-1} & \beta_n \end{pmatrix}$$ matritsalargaega bo'lamiz.

Laplasteoremasiniqo'llab, $A(j)$ $j \in \{1,\dots,n\}$ matritsaningdeterminantini j-ustunyoyilmasiyordamidagiifodasinihosilqilamiz:

$$|A(j)| = \beta_1 A_{1j} + \dots + \beta_n A_{nj}), (j = 1,\dots,n).$$

Hosilbo'lgantengliklaryordamida 11.5-teoremani quyidagichabayonqilishmumkin:

11.6-teorema. Agar $|A| \neq 0$ bo'lsa, u holda (3) CHTS yagonayechimgaegava u quyidagiformulalarorqaliifodalanadi:

$$x_1 = \frac{|A(1)|}{|A|}, \ldots, x_n = \frac{|A(n)|}{|A|} \quad (5).$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

chiziqlitenglamalarsistemasiningyechiminiKramerformulalariyordamidatopishuchunsistemaningasosiymatritsasiva A(1), A(2), A(3) matritsalarnituzib, ularningdeterminantlarinihisoblaymiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix};$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} -$$

$$-- a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}.$$

$$\Delta_1 = |A(1)| = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}; \quad \Delta_2 = |A(2)| = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix};$$

$$\Delta_3 = A(3) = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix};$$

U holda $x_1 = \frac{\Delta_1}{\Delta}, x_2 = \frac{\Delta_2}{\Delta}, x_3 \frac{\Delta_3}{\Delta}$.

11.4-misol. $\begin{cases} 5x - y - z = 0 \\ x + 2y + 3z = 14 \\ 4x + 3y + 2z = 16 \end{cases}$

chiziqlitenglamalarsistemasiningyechiminiKramerformulalariyordamidatoping.

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Yechish:

$$\Delta = \begin{vmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{vmatrix} =5(4–9)+(2–12)–(3–8)=-25–10+5=-30;$$

$$\Delta_1 = \begin{vmatrix} 0 & -1 & -1 \\ 14 & 2 & 3 \\ 16 & 3 & 2 \end{vmatrix} = (28 – 48) – (42 – 32) = -20 – 10 = -30.$$

$$\Delta_2 = \begin{vmatrix} 5 & 0 & -1 \\ 1 & 14 & 3 \\ 4 & 16 & 2 \end{vmatrix} = 5(28 – 48) – (16 – 56) = -100 + 40 = -60.$$

$$\Delta_3 = \begin{vmatrix} 5 & -1 & 0 \\ 1 & 2 & 14 \\ 4 & 3 & 16 \end{vmatrix} = 5(32 – 42) + (16 – 56) = -50 – 40 = -90.$$

$x_1 = \Delta_1/\Delta = 1$; $x_2 = \Delta_2/\Delta = 2$; $x_3 = \Delta_3/\Delta = 3$.

Takrorlashuchunsavollar:

1. Determinantnolgatengbo’lishiningzarurvayetarlishartiniayting.
2. Matritsarangiminorlaryordamidaqandaytopiladi?
3. Algebraikto’ldiruvchilaryordamidateskarimatritsanitopishjarayoninitushuntiring.
4. CHTSniKramerqoidasibilanyechishusulinitushuntiring.

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Foydalaniladigan adabiyotlar ro'yxati

Asosiy adabiyotlar:

1. MalikD.S., MordesonJ.N., SenM.K. Fundamentalofabstractalgebra. WCB McGrew-Hill, 1997.
2. Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" 2010.
3. Кострикин А.М. Введение в алгебру.- М.- «Мир».- 1977.
4. Под ред. Кострикина, Сборник задач по алгебре, М.Наука, 1986.
5. Хожиев Ж.Х. Файнлейб А.С. Алгебра ва сонлар назарияси курси, Тошкент, «Ўзбекистон», 2001 й.
6. Курош А.Г. Олий алгебра курси, Тошкент, «Ўқитувчи». 1975й.
7. Гельфанд И.М. Чизиқлиалгебраданлекциялар. «Олийваўртамактаб». 1964.
8. Р.Н.Назаров, Б.Т. Тошпўлатов, А.Д.Дусумбетов, Алгебра ва сонлар назарияси 1 қисм, 2 қисм, 1993й., 1995й.
9. A.Yunusov , D.Yunusova , Algebra va sonlar nazariyasi. Modultexnologiyasiasosidatuzilganmusolvamashqlarto'plami. O'quvqo'llanma. 2009.

Qo'shimcha adabiyotlar:

1. Фаддеев Д.К. Лекции по алгебре, М., "Наука"1984г.

2. Фаддеев Д.К., Соминский И.С. Сборник задач по высшей алгебре, М.: Наука, 1977 г.

3. Поскуряков И.Л. Сборник задач по линейной алгебре. «Наука», 1978г.

4. Ламбек И. Кольца и модули.- М.- «Мир».- 1971.

5. Херстейн. Некоммутативные кольца. М.- «Мир».- 1967.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.79-93.

6. VilnisDetlovs,KarlisPodnieks,Introduction to MathematicalLogic. University of Latvia. Version released: August 25, 2014.

7. А.Юнусов , Д.Юнусова, М.Маматкулова, Г.Артикова, Модул технологияси асосида тайёрланган мустақил ишлар тўплами. 1−3−қисмлар, 2010.

8. Скорняков Л.Ф. Элементи обшей алгебри. М., 1983 г.

9. Петрова В.Т. лексия по алгебре и геометрии. Ч.1,2. Москва,1999г.

10. YunusovA.S. Matematikmantiqvaalgoritmlarnazariyasielementlari. T., “Yangiasravlodi”. 2006.

11. YunusovA., YunusovaD. Sonlisistemalar. T., «Moliya−iqtisod», 2008.

12. Мазуров В.Д. и др. Краткий конспект курса висшейалгебри.

Elektron ta’lim resurslari

1. www.Ziyo.Net
2. http://vilenin.narod.ru/Mm/Books/
3. http://www.allmath.ru/
4. http://www.pedagog.uz/
5. http://www.ziyonet.uz/
6. http://window.edu.ru/window/
7. http://lib.mexmat.ru;
8. http://www.mcce.ru,
9. http://lib.mexmat.ru
10. http://techlibrary.ru;

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, “ALGEBRA AND NUMBER THEORY” pp.79-93.