Minorlar va algebraik to'ldiruvchilar

Reja:

- Qismmatritsa.
- n-tartibli minor.
- Algebraik to'ldiruvchi.
- Determinantni algebraik to'ldiruvchi yordamida aniqlash.
- Laplas teoremasi.

$F=<F;+,\cdot,-,^{-1},0,1>$ maydonvamaydonustida $F^{m\times n}$ matritsalarto'plamiberilganbo'lsin.

9.1-ta'rif. $A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$ matritsaning matritsaosti deb, uning qandaydir satr va ustunlarini o'chirishdan hosil bo'lgan matritsaga aytiladi.

9.2-ta'rif. k ta satr va k ta ustundan iborat matritsaosti k-tartibli matritsaosti deyiladi.

9.1-misol. $\begin{pmatrix} 1 & 1 & 8 & 2 \\ 0 & 3 & 7 & 3 \\ 9 & 2 & -6 & 1 \end{pmatrix}$ matritsaning 3-tartibli qismmatritsasini hosil qilish uchun ixtiyoriy bitta ustunini o'chirish mumkin, masalan $\begin{pmatrix} 1 & 1 & 8 \\ 0 & 3 & 7 \\ 9 & 2 & -6 \end{pmatrix}$.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.66-79.

9.3-ta'rif. k-tartibli matritsaosti determinanti A matritsaning k-tartibli minori deyiladi.

Matritsaning har bir elementi 1-tartibli minor bo'ladi.

9.4-ta'rif. Kvadrat matritsaning i- qatori j-ustunini o'chirishdan hosil bo'lgan matritsaosti determinanti a_{ij} elementning **minori** deyiladi va M_{ij} ko'rinishda belgilanadi.[1]

2.4.1. Definition. *Let* $A = [a_{ij}] \in \mathbf{M}_n(\mathbb{R})$ *and let* $1 \leq t \leq n$. *Select* t *rows and* t *columns in the matrix* A *and form the* $t \times t$ *submatrix* B *consisting of the elements situated at the intersections of these chosen rows and columns. Suppose that the selected rows are those numbered* $k(1), k(2), \dots, k(t)$ *and that the selected columns are those numbered* $j(1), j(2), \dots, j(t)$. *The determinant of* B *is called* the minor of degree t *corresponding to rows* $k(1), k(2), \dots, k(t)$ *and columns* $j(1), j(2), \dots, j(t)$, *and it will be denoted by*

$$\mathbf{minor}\{\mathbf{k(1), k(2), \dots, k(t); j(1), j(2), \dots, j(t)}\}.$$

9.5-ta'rif. $A_{ij} = (-1)^{i+j} \cdot M_{ij}$ ko'paytmaga a_{ij} elementning algebraik to'ldiruvchisi deyiladi.

9.1-teorema.A= $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ kvadrat matritsaning n-satr (ustun) elementi a_{nn} dan boshqa hammasi nolga teng bo'lsa, u holda $|A| = a_{nn} \cdot M_{nn}$ bo'ladi.

[1]Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya.Subbotin, "ALGEBRA AND NUMBER THEORY" pp.66-79.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.66-79.

9.2-teorema. A= $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ kvadrat matritsaning qandaydir satr (ustun) elementlaridan bittasidan boshqa hammasi nolga teng bo'lsa, u holda berilgan matritsa determinanti shu elementni uning algebraik to'ldiruvchisi bilan ko'paytmasiga teng.

9.3-teorema (Laplas teoremasi). A= $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ kvadrat matritsaning determinanti biror-bir satr (ustun) elementlari bilan ularning algebraik to'ldiruvchilari ko'paytmalarining yig'indisiga, ya'ni

$|A| = a_{1j}A_{1j} + \ldots + a_{nj}A_{nj} (|A| = a_{i1}A_{i1} + \ldots + a_{in}A_{in}), i, j \in \{1, \ldots, n\}$ ga teng.

2.4.7. Theorem *(Pierre–Simon Laplace). Let* $A = [a_{ij}] \in \mathbf{M}_n(\mathbb{R})$. *In the matrix A choose t rows (respectively, t columns). Multiply every minor of dimension t corresponding to the chosen rows (respectively, columns) by its algebraic complement. The sum of all these products is equal to* **det**(*A*).

Isbot. A= $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ matritsaning j-ustunini n ta ustunlar yig'indisi ko'rinishida ifodalaymiz:

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.66-79.

$$A^j = \begin{pmatrix} a_{1j} \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a_{2j} \\ \vdots \\ 0 \end{pmatrix} + \ldots + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ a_{nj} \end{pmatrix}.$$

U holda kvadrat matritsa determinanti xossalariga (16.9-teorema) ko’ra

$$|A| = \begin{vmatrix} a_{11} & \ldots & a_{1j} & \ldots & a_{1n} \\ a_{21} & \ldots & 0 & \ldots & a_{2n} \\ . & . & . & . & . \\ a_{n1} & \ldots & 0 & \ldots & a_{nn} \end{vmatrix} + \ldots + \begin{vmatrix} a_{11} & \ldots & 0 & \ldots & a_{1n} \\ a_{21} & \ldots & 0 & \ldots & a_{2n} \\ . & . & . & . & . \\ a_{n1} & \ldots & a_{nj} & \ldots & a_{nn} \end{vmatrix}$$

ifodagaegabo’lamiz. 9.2-teoremaga ko’ra

(1) $|A| = a_{1j}A_{1j} + a_{2j}A_{2j} + \ldots + a_{nj}A_{nj}$.

(2) $|A| = a_{i1}A_{i1} + \ldots + a_{in}A_{in}, i \in \{1, \ldots, n\}$ ekanligiyuqoridagikabiisbotlanadi.

Proof. By using Proposition 2.3.3 we need to consider only the case with rows. Let the selected rows be the rows numbered $k(1), k(2), \ldots, k(t)$. We recall that

$$\mathbf{det}(A) = \sum_{\pi \in \mathbf{S}_n} \mathbf{sign}\, \pi\, a_{1,\pi(1)} a_{2,\pi(2)} \ldots a_{n,\pi(n)}.$$

Consider an arbitrary term $\mathbf{sign}\, \pi\, a_{1,\pi(1)} a_{2,\pi(2)} \ldots a_{n,\pi(n)}$ from this sum and within this consider the terms whose first indices belong to the selected rows.

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Thus, we consider $\mathbf{sign}\,\pi\, a_{k(1),\pi(k(1))}a_{k(2),\pi(k(2))}\dots a_{k(t),\pi(k(t))}$. This product together with the sign + or − belongs to the decomposition

$$\mathbf{minor}\{\mathbf{k(1)}, \mathbf{k(2)}, \dots, \mathbf{k(t)}; \pi(\mathbf{k(1)}), \pi(\mathbf{k(2)}), \dots, \pi(\mathbf{k(t)})\}.$$

Clearly, the product of all other elements $a_{j,\pi(j)}$, where $j \notin \{k(1), \dots, k(t)\}$ (again with the sign + or −) belongs to the decomposition

$$\mathbf{comp}\{\mathbf{k(1)}, \mathbf{k(2)}, \dots, \mathbf{k(t)}; \pi(\mathbf{k(1)}), \pi(\mathbf{k(2)}), \dots, \pi(\mathbf{k(t)})\}.$$

By Theorem 2.4.2, the term

$$\mathbf{sign}\,\pi\, a_{1,\pi(1)}a_{2,\pi(2)}\dots a_{n,\pi(n)}$$

belongs to the decomposition of the product of

$$\mathbf{minor}\{\mathbf{k(1)}, \mathbf{k(2)}, \dots, \mathbf{k(t)}; \pi(\mathbf{k(1)}), \pi(\mathbf{k(2)}), \dots, \pi(\mathbf{k(t)})\}$$

and

$$\mathbf{A}_{\mathbf{k(1)},\mathbf{k(2)},\dots,\mathbf{k(t)};\pi(\mathbf{k(1)}),\pi(\mathbf{k(2)}),\dots,\pi(\mathbf{k(t)})}.$$

include $t!(n-t)!$ terms.

Next, we show that the decompositions of the products of two distinct minors corresponding to the chosen rows by their algebraic complements do not include identical terms. Let

$$\{j(1), j(2), \dots, j(t)\} \neq \{s(1), s(2), \dots, s(t)\}$$

and let $\mathbf{sign}\,\pi\, a_{1,\pi(1)}a_{2,\pi(2)}\dots a_{n,\pi(n)}$ belong to a decomposition of the product

$$\mathbf{minor}\{\mathbf{k(1)}, \dots, \mathbf{k(t)}; \mathbf{j(1)}, \dots, \mathbf{j(t)}\}\mathbf{A}_{\mathbf{k(1)},\dots,\mathbf{k(t)};\mathbf{j(1)},\dots,\mathbf{j(t)}};$$

let $\mathbf{sign}\,\sigma\, a_{1,\sigma\pi(1)}a_{2,\sigma(2)}\dots a_{n,\sigma(n)}$ belong to a decomposition of the product

$$\mathbf{minor}\{\mathbf{k(1)}, \dots, \mathbf{k(t)}; \mathbf{s(1)}, \dots, \mathbf{s(t)}\}\mathbf{A}_{\mathbf{k(1)},\dots,\mathbf{k(t)};\mathbf{s(1)},\dots,\mathbf{s(t)}}.$$

This means that

$$\{\pi(k(1)), \pi(k(2)), \dots, \pi(k(t))\} = \{j(1), j(2), \dots, j(t)\} \neq \{s(1), s(2), \dots, s(t)\} = \{\sigma(k(1)), \sigma(k(2)), \dots, \sigma(k(t))\}.$$

The total number of minors of dimension t, which corresponds to the selected rows, is equal to the number of combinations $\binom{n}{t} = \frac{n!}{t!(n-t)!}$. Thus the sum of the products of all the minors of dimension t that corresponds to the selected t rows by their algebraic complements gives us $t!(n-t)! \cdot \frac{n!}{t!(n-t)!} = n!$ terms from the decomposition of $\mathbf{det}(A)$. Since the decomposition of $\mathbf{det}(A)$ includes exactly $n!$ terms, we see that the sum of the products of all the minors of dimension t that corresponds to the selected t rows by their algebraic complements is $\mathbf{det}(A)$.

(1) formulagadeterminantni j –ustunbo'yicha, 2-formulaga i-satrbo'yichayoyilmasideyiladi.

9.4-teorema. $a_{1j}A_{1k} + a_{2j}A_{2k} + \dots + a_{nj}A_{nk} = 0, (j \neq k)$ va $a_{i1}A_{m1} + \dots + a_{in}A_{mn} = 0, (i \neq m)$, ya'ni A matritsaningbiror-birsatr (ustun) elementlariniboshqabirsatr (ustun) elementlarialgebraikto'ldiruvchilarigako'paytmalariningyig'indisinolgateng.

9.1-misol. A = $\begin{pmatrix} 1 & 2 & 1 \\ 0 & -2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ matritsadeterminantinihisoblang.

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Yechish. $\begin{vmatrix} 1 & 2 & 1 \\ 0 & -2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 0 & 3 \\ 3 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 0 & -2 \\ 3 & 1 \end{vmatrix} =$

$(-2 \cdot 1 - 1 \cdot 3) - 2(0 \cdot 1 - 3 \cdot 3) + (0 \cdot 1 + 3 \cdot 2) =$ -5 + 18 + 6 = 19.

9.2-misol. $\begin{vmatrix} -1 & 0 & 3 & 4 \\ 2 & -1 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 2 & 1 & 4 & 3 \end{vmatrix}$ determinantnihisoblang.

Yechish. $\begin{vmatrix} -1 & 0 & 3 & 4 \\ 2 & -1 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 2 & 1 & 4 & 3 \end{vmatrix} = -1 \begin{vmatrix} -1 & 1 & 2 \\ 3 & 2 & 1 \\ 1 & 4 & 3 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & -1 & 2 \\ 0 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} - 4 \cdot \begin{vmatrix} 2 & -1 & 1 \\ 0 & 3 & 2 \\ 2 & 1 & 4 \end{vmatrix}.$

$\begin{vmatrix} -1 & 1 & 2 \\ 3 & 2 & 1 \\ 1 & 4 & 3 \end{vmatrix}$ = -1(6 – 4) – 1(9 – 1) + 2(12 – 2) = -2 – 8 + 20 = 10.

$\begin{vmatrix} 2 & -1 & 2 \\ 0 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & -2 & -1 \\ 0 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix}$ = 2(0 – 2) – 1(0 – 6) = 2.

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & 3 & 2 \\ 2 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 0 & -2 & -3 \\ 0 & 3 & 2 \\ 2 & 1 & 4 \end{vmatrix} = 2(-4) – 3(-6) = -8 + 18 = 10.$$

Demak, determinant -10 + 6 – 40 = -44 gateng.

Takrorlashuchunsavollar:

1. Matritsaostigata'rifbering.
2. n-tartibliminordebnimagaaytiladi?.
3. Matritsabirorbirelementiningalgebraikto'ldiruvchisinima?
4. Determinantnialgebraikto'ldiruvchiyordamidaaniqlashjarayoninitus huntiring.
5. Laplasteoremasiniayting.

Foydalaniladigan adabiyotlar ro'yxati

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