Determinantlar va ularning xossalari

Reja:

- Determinant.
- Determinantningasosiyxossalari.
- Matritsalarko'paytmasiningdeterminanti.

8.1-ta'rif. Kvadrat matritsaning har bir satr va har bir ustunidan bittadan elementlar olib tuzilgan ko'paytmalarning algebraik yig'indisiga berilgan kvadrat matritsaning determinanti deyiladi.

$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ matritsaning har bir satr va har bir ustunidan

bittadan element olib tuzilgan n ta elementlar ko'paytmasi $a_{1\tau_1}\cdot\ldots\cdot a_{n\tau_n}$ bilan n-darajali o'rniga qo'yish $\tau=\begin{pmatrix} 1 & \dots & n \\ \tau(1) & \dots & \tau(n) \end{pmatrix}$ larni birini ikkinchisiga mos qo'yuvchi o'zaro bir qiymatli moslik mavjud. Bu moslikdan n-tartibli kvadrat matritsaning determinantini aniqlashda foydalanamiz.

Uchinchi darajali o'rniga qo'yishlar to'plami $S_3=\{\varphi_0,\varphi_1,\varphi_2,\varphi_3,\varphi_4,\varphi_5\}$dagi o'rniga qo'yishlar quyidagicha:

$$\varphi_0=\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix},\ \varphi_1=\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix},\ \varphi_2=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix},\ \varphi_3=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix},$$

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.66-79.

$$\varphi_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \ \varphi_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

Uchinchi tartibli kvadrat matritsa determinanti $D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ ni hisoblash uchun uchinchi darajali o'rniga qo'yishlar yordamida ko'paytmalar tuzamiz. Urniga qo'yishning ishorasi u yordamida hosil qilingan ko'paytmani qo'shish yoki ayirish kerakligini aniqlab beradi. Bundan quyidagi ifodani hosil qilamiz.

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

.[1]

We will remind the reader of the definition of the determinants of second and third order and then, by analogy, we will introduce the idea of the determinant of a square matrix of arbitrary order. We have

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21},$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32} + a_{12}a_{23}a_{31} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}.$$

[1] Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya.Subbotin, "ALGEBRA AND NUMBER THEORY" pp.66-79.

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Now consider the second case where the determinants are of order 3. In this case, the set $\mathbf{S}_3$ consists of six permutations:

$$\varepsilon = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix},$$

$$\gamma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \pi = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix},$$

where $\mathbf{sign}\,\varepsilon = \mathbf{sign}\,\alpha = \mathbf{sign}\,\beta = 1$ and $\mathbf{sign}\,\gamma = \mathbf{sign}\,\pi = \mathbf{sign}\,\sigma = -1$. Now we have, by inspection,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \mathbf{sign}\,\varepsilon\, a_{1,\varepsilon(1)}a_{2,\varepsilon(2)}a_{3,\varepsilon(3)} + \mathbf{sign}\,\alpha\, a_{1,\alpha(1)}a_{2,\alpha(2)}a_{3,\alpha(3)}$$

$$+ \mathbf{sign}\,\beta\, a_{1,\beta(1)}a_{2,\beta(2)}a_{3,\beta(3)} + \mathbf{sign}\,\gamma\, a_{1,\gamma(1)}a_{2,\gamma(2)}a_{3,\gamma(3)}$$

$$+ \mathbf{sign}\,\pi\, a_{1,\pi(1)}a_{2,\pi(2)}a_{3,\pi(3)} + \mathbf{sign}\,\sigma\, a_{1,\sigma(1)}a_{2,\sigma(2)}a_{3,\sigma(3)}.$$

8.2-ta'rif. n-tartibli kvadrat matritsa $A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ ning determinanti deb $|A| = \sum_{\tau \in S_n} \operatorname{sgn}(\tau) a_{1\tau(1)} \cdot \ldots \cdot a_{n\tau(n)}$ ($n!$ qo'shiluvchilardan iborat) yig'indiga aytiladi.

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2.3.2. Definition. *Let* $A = [a_{ij}] \in \mathbf{M}_n(\mathbb{R})$. *For each permutation,* $\pi \in \mathbf{S}_n$ *form the product*

$$\mathbf{sign}\,\pi\, a_{1,\pi(1)}a_{2,\pi(2)} \dots a_{n,\pi(n)}.$$

The sum $\mathbf{det}(A)$ *of all these products is called the determinant of the matrix* A. *Thus,*

$$\mathbf{det}(A) = \sum_{\pi \in \mathbf{S}_n} \mathbf{sign}\,\pi\, a_{1,\pi(1)}a_{2,\pi(2)} \dots a_{n,\pi(n)}.$$

We will use the following expanded notation for $\mathbf{det}(A)$:

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}.$$

8.1-teorema. Nol satr yoki ustunga ega kvadrat matritsaning determinanti nolga teng.

8.2-teorema. Diagonal matritsaning determinanti asosiy diagonal elementlari ko'paytmasiga teng.

8.3-teorema. Uchburchak matritsaning determinanti asosiy diagonal elementlari ko'paytmasiga teng.

8.4-teorema. Kvadrat matritsa va unga transponirlangan matritsalar

determinantlari teng.

8.5-teorema. Kvadrat matritsaning ikkita satr (ustun)lari o'rnini almashtirish natijasida determinant ishorasi o'zgaradi.

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8.6-teorema. Ikkita bir xil satr (ustun)ga ega kvadrat matritsa determinanti nolga teng.

8.7-teorema. A kvadrat matritsaning biror bir satr (ustun) elementlarini noldan farqli λ skalyarga ko'paytirilsa, u holda A matritsaning determinanti λ skalyarga ko'paytiriladi.

8.8-teorema. Qandaydir ikkita satr (ustun)lari proporsional bo'lgan kvadrat matritsaning determinanti nolga teng.

8.9-teorema. Kvadrat matritsa i- qatori (ustuni)ning har bir elementi m ta qo'shiluvchilardan iborat bo'lsa, bunday kvadrat matritsaning determinanti m ta determinantlar yig'indisidan iborat bo'lib, birinchi determinant i- qatori (ustuni)da birinchi, ikkinchi determinantda ikkinchi qo'shiluvchilar va h.z. boshqa qatorlar A matritsanikidek bo'ladi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+a & a_{22}+b & a_{23}+c \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a & b & c \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

8.10-teorema. Kvadrat matritsaning biror-bir satr (ustun)iga noldan farqli skalyarga ko'paytirilgan boshqa satr (ustun)ni qo'shish natijasida determinant o'zgarmaydi.

8.11-teorema. Kvadrat matritsaning biror-bir satr (ustun)iga qolgan satr (ustun)lar chiziqli kombinatsiyasini qo'shish natijasida determinant o'zgarmaydi.

8.12-teorema. Kvadrat matritsaning biror-bir satri (ustuni) qolganlarining chiziqli kombinatsiyasidan iborat bo'lsa, uning determinanti nolga teng.

8.13-teorema. Har qanday elementar matritsaning determinanti noldan farqli.

8.14-teorema. Kvadrat matritsalar ko’paytmasining determinanti berilgan matritsalar determinantlari ko’paytmasiga teng.

Takrorlashuchunsavollar:

1. Kvadratmatritsadeterminantitushunchasigata’rifbering.
2. Determinantnihisoblashformulasinitushuntiring.
3. Determinantningasosiyxossalariniayting.
4. Matritsalarko’paytmasiningdeterminantinimagateng?

Foydalaniladigan adabiyotlar ro’yxati

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