ARIFMETIKA, ALGEBRA, TRIGONOMETRIYA

ELEMENTAR MATEMATIKA

PLANIMETRIYA, STEREOMETRIYA

MA'LUMOTNOMA

- oliy o'quv yurtlariga kiruvchilar
- akademik litsey va kasb-hunar kolleji o'quvchilari
- umumiy o'rta ta'lim maktabi o'quvchi va o'qituvchilari uchun

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ALGEBRA

To‘plamlar nazariyasi elementlari

To‘plam tushunchasi matematikaning boshlang‘ich (ta’riflanmaydigan) tushunchalaridan biridir. U chekli yoki cheksiz ko‘p obyektlar (narsalar, buyumlar, shaxslar va h.k.)ni birgalikda bir butun deb qarash natijasida vujudga keladi.
Masalan: Butun sonlar to‘plami, molekulalar to‘plami, sinfdagi o‘quvchilar to‘plami, geometrik figuralar to‘plami va hokazo.

To‘plam tushunchasini matematikaga birinchi bo‘lib, nemis matematigi ***G.Kantor*** (1845-1918 yy.) kiritgan.

To‘plamni tashkil etgan obyektlar uning ***elementlari*** deyiladi.

To‘plamlar odatda lotin alifbosining bosh harflari bilan, uning elementlari esa kichik harflari bilan belgilanadi.
Masalan: $A=\{a,b,c\}$ yozuv A to‘plam a, b, c elementlardan tashkil topgan ekanligini bildiradi.

Ta’rif: a element A to‘plamga ***tegishliligi*** $a\in A$ ko‘rinishida, ***tegishli emasligi*** esa $a\notin A$ ko‘rinishida belgilanadi.
Masalan: $3\in N$; $-2\in Z$; $1{,}3\notin N$; $1{,}15\notin Z$.

Ta’rif: A va B to‘plamlarning har ikkisida ham mavjud bo‘lgan x elementga shu to‘plamlarning ***umumiy elementi*** deyiladi.

Ta’rif: Elementlari soni chekli bo‘lgan to‘plam ***chekli to‘plam***, elementlari soni cheksiz bo‘lgan to‘plam esa ***cheksiz to‘plam*** deyiladi.

Ta’rif: Birorta ham elementga ega bo‘lmagan to‘plam ***bo‘sh to‘plam*** deyiladi va $\varnothing$ kabi belgilanadi.

Ta’rif: Ayni bir xil elementlardan tuzilgan to‘plamlar ***teng to‘plamlar*** deyiladi.

Ta’rif: A va B to‘plamlarning ***birlashmasi*** (yoki ***yig‘indisi***) deb, ularning kamida bittasida mavjud bo‘lgan barcha elementlaridan tuzilgan to‘plamga aytiladi va $A\cup B$ ko‘rinishida belgilanadi.

Xossalari:

$A\cup B=B\cup A,\qquad A\cup A=A,$

$A\cup\varnothing=A,\qquad A\cup(B\cup C)=(A\cup B)\cup C.$

Ta’rif: A va B to‘plamlarning ***kesishmasi*** (yoki ***ko‘paytmasi***) deb, ularning barcha umumiy elementlaridan tuzilgan to‘plamga aytiladi va $A\cap B$ ko‘rinishida belgilanadi.

Xossalari:

$A\cap B=B\cap A,\qquad A\cap A=A,$

$A\cap\varnothing=\varnothing,\qquad A\cap(B\cap C)=(A\cap B)\cap C.$

Ta’rif: A va B to‘plamlarning ***ayirmasi*** deb, A to‘plamning B to‘plamda mavjud bo‘lmagan barcha elementlaridan tuzilgan to‘plamga aytiladi va $A\setminus B$ ko‘rinishida belgilanadi.

Teorema (jamlash qoidasi): Kesishmaydigan A va B chekli to'plamlarning birlashmasidagi elementlar soni A va B to'plamlar elementlari sonlarining yig'indisiga teng.

$$n(A\cup B)=n(A)+n(B)$$

Teorema: Ixtiyoriy A va B chekli to'plamlar uchun ushbu tenglik o'rinli:

$$n(A\cup B)=n(A)+n(B)-n(A\cap B).$$

Ta'rif: Agar B to'plamning har bir elementi A to'plamning ham elementi bo'lsa, B to'plam A to'plamning ***qism-to'plami*** deyiladi va $B\subset A$ ko'rinishida belgilanadi.

Xossalari:

$A\subset A\cup B,\quad B\subset A\cup B,\quad A\cap B\subset A,\quad A\cap B\subset B.$

Izoh: *Bo'sh* to'plam har qanday to'plamga qism-to'plam bo'ladi.

Eslatma: Har qanday chekli to'plamni 2^n ta (n-to'plamning elementlari soni) qism-to'plamlarga ajratish mumkin.

Masalan: $A=\{a,b,c,d,e\}$ to'plamning $2^5=32$ ta qism-to'plamlari mavjud.

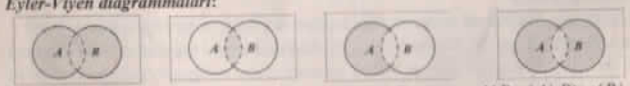
Eslatma: Har qanday chekli to'plamni 2^{n-1} ta usul bilan ikkita kesishmaydigan qism-to'plamlarga ajratish mumkin.

Qism-to'plamlar ikki turga bo'linadi: xos va xosmas qism-to'plamlar.

Ta'rif: To'plamning o'zi va bo'sh to'plam ***xosmas qism-to'plam*** deyiladi. Ulardan boshqa qism to'plamlar ***xos qism-to'plam*** deyiladi.

Masalan: $A=\{a,b,c\}$ to'plamning *xos qism-to'plamlari:* $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$; *xosmas qism-to'plamlari:* $\{a,b,c\}$ va $\varnothing$.

Eyler-Viyen diagrammalari:



$A\cup B$ $\quad$ $A\cap B$ $\quad$ $A\setminus B$ $\quad$ $A\Delta B=(A\setminus B)\cup(B\setminus A)$

Sonli to'plamlar

Ta'rif: Barcha elementlari sonlardan iborat bo'lgan to'plam ***sonli to'plam*** deyiladi.

Ta'rif: Narsa va buyumlarni sanash uchun ishlatiladigan sonlar ***natural sonlar*** deyiladi va $\mathbb{N}=\{1,2,3,...,n,...\}$ ko'rinishida belgilanadi.

Eslatma: $n_1,n_2\in\mathbb{N}$ bo'lsa, $n_1+n_2\in\mathbb{N}$, $n_1\cdot n_2\in\mathbb{N}$.

Ta'rif: Natural sonlar va ularga qarama-qarshi sonlar hamda nol soni birgalikda ***butun sonlar*** deyiladi va $\mathbb{Z}=\{...,-n,...,-2,-1,0,1,2,...,n,...\}$ ko'rinishida belgilanadi.

$\mathbb{Z}^-=\{...,-n,...,-2,-1\}$ – ***manfiy butun sonlar*** to'plami.

$\mathbb{Z}^+=\mathbb{N}=\{1,2,...,n,...\}$ – ***musbat butun sonlar*** to'plami.

0 (*nol*) manfiy son ham, musbat son ham emas.

$\mathbb{Z}_1=\{\pm1,\pm3,\pm5,...,\pm(2n-1),...\}$ – ***toq butun sonlar*** to'plami.

$\mathbb{Z}_2=\{0,\pm2,\pm4,\pm6,...,\pm(2n),...\}$ – ***juft butun sonlar*** to'plami.

Eslatma: $m_1,m_2\in\mathbb{Z}$ bo'lsa, $m_1-m_2\in\mathbb{Z}$, $m_1+m_2\in\mathbb{Z}$, $m_1\cdot m_2\in\mathbb{Z}$.

Ta'rif: Butun va kasr sonlar birgalikda ***ratsional sonlar*** deyiladi va $\mathbb{Q}=\left\{r\,\middle|\,r=\frac{m}{n},\ m\in\mathbb{Z},\ n\in\mathbb{N}\right\}$ ko'rinishida belgilanadi.

Eslatma: $r_1,r_2\in\mathbb{Q}$ bo'lsa, $r_1-r_2\in\mathbb{Q}$, $r_1+r_2\in\mathbb{Q}$, $r_1\cdot r_2\in\mathbb{Q}$, $r_1:r_2\in\mathbb{Q}$ $r_2\neq0$.

Ta'rif: Davriy bo'lmagan cheksiz o'nli kasr sonlar ***irratsional sonlar*** deyiladi va I ko'rinishida belgilanadi.

Masalan: $\pi\approx3{,}14$, $e\approx2{,}7$; $\sqrt{2}$; ... irratsional sonlar.

Ta'rif: Ratsional va irratsional sonlar birgalikda ***haqiqiy sonlar*** to'plamini tashkil etadi va $\mathbb{R}=\{x|\ -\infty<x<+\infty\}$ ko'rinishida belgilanadi.

Eslatma: $\mathbb{N}\subset\mathbb{Z}\subset\mathbb{Q}\subset\mathbb{R}$.

Qoldiqsiz bo'linish qoidalari

- Agar sonning oxirgi raqami juft yoki 0 bo'lsa, u holda bu son **2** ga bo'linadi.
- Agar sonning raqamlari yig'indisi 3 (9) ga bo'linsa, u holda bu son **3** **(9)** ga bo'linadi.
- Agar sonning oxirgi ikki raqamidan tuzilgan ikki xonali son 4 (25) ga bo'linsa, u holda bu son **4** **(25)** ga bo'linadi.
- Agar sonning oxirgi raqami 5 yoki 0 bo'lsa, u holda bu son **5** ga bo'linadi.
- Agar sonning oxirgi raqami juft yoki 0 bo'lib, raqamlari yig'indisi 3 ga bo'linsa, u holda bu son **6** ga bo'linadi.
- Agar sonning oxirgi uchta raqamidan tuzilgan sondan qolgan raqamlaridan tuzilgan sonning (yoki aksincha) ayirmasi 7 (11, 13) ga bo'linsa, u holda bu son **7** **(11, 13)** ga bo'linadi.
- Agar sonning oxirgi uchta raqamidan tuzilgan uch xonali son 8 ga bo'linsa, u holda bu son **8** ga bo'linadi.
- Agar sonning oxirgi raqami 0 bo'lsa, u holda bu son **10** ga bo'linadi.

Eslatma: Qolgan sonlar uchun qoldiqsiz bo'linish qoidalari ularni tub ko'paytuvchilarga ajratish usuli orqali aniqlanadi.

Masalan: Biror son **12** ga bo'linishi uchun, u son bir vaqtning o'zida ham 3 ga, ham 4 ga bo'linishi kerak. Chunki, $12=2^2\cdot3$.

Qo'shimcha:

Biror sonni **4** yoki **8** ga qoldiqsiz bo'linishini quyidagicha aniqlash mumkin.

1-*misol*: 2396 sonining 4 ga qoldiqsiz bo'linishini tekshirish?

Yechish: Oxirgi ikkita raqamini olib, quyidagicha hisoblaymiz: $9+\frac{6}{2}=12$. Natija juft son, demak, 2396 soni 4 ga qoldiqsiz bo'linadi.

2-*misol*: 7152 sonining 8 ga qoldiqsiz bo'linishini tekshirish?

Yechish: Oxirgi uchta raqamini olib, quyidagicha hisoblaymiz: $1+\frac{5}{2}+\frac{2}{4}=4$. Natija juft son, demak, 7152 soni 8 ga qoldiqsiz bo'linadi.

***Izoh*:** Agar natija toq son yoki kasr son chiqsa, u holda berilgan son **4 (8)** ga qoldiqli bo'linadi.

Tub va murakkab sonlar

Ta'rif: Faqat birga va o'zigagina bo'linadigan natural sonlar ***tub sonlar*** deyiladi.

Masalan: 2, 3, 5, 7, 11, 13, 17, ...

Ta'rif: Uch va undan ortiq natural bo'luvchiga ega bo'lgan natural sonlar (yoki tub bo'lmagan sonlar) ***murakkab sonlar*** deyiladi.

Masalan: 4, 6, 8, 9, 10, 12, 14, 16, ...

***Eslatma*: 1 (*bir*)** – tub son ham, murakkab son ham emas.

Teorema: Agar n sonining $\sqrt{n}$ dan katta bo'lmagan tub bo'luvchisi mavjud bo'lmasa, u holda n tub son bo'ladi.

Masalan: 89 tub son, chunki, $\sqrt{89}$ dan kichik tub sonlar 2, 3, 5 va 7 lar bo'lib, 89 ularning hech biriga bo'linmaydi.

***Eslatma*:**

a) *Ikkita juft natural son hech qachon o'zaro tub bo'la olmaydi.*

b) *Ketma-ket keluvchi ikkita natural son har doim o'zaro tub bo'ladi.*

c) *Ketma-ket keluvchi ikkita toq natural son har doim o'zaro tub bo'ladi.*

d) *Ixtiyoriy ikkita tub son har doim o'zaro tub bo'ladi.*

***Tub sonlar jadvali* (1000 *gacha*)**

2	47	109	191	269	353	439	523	617	709	**811**	**907**
3	53	113	193	271	359	443	541	619	719	821	911
5	59	127	197	277	367	449	547	631	727	823	919
7	61	131	199	281	373	457	557	641	733	827	929
11	67	137	**211**	283	379	461	563	643	739	829	937
13	71	139	223	293	383	463	569	647	743	839	941
17	73	149	227	**307**	389	467	571	653	751	853	947
19	79	151	229	311	397	479	577	659	757	857	953
23	83	157	233	313	**401**	487	587	661	761	857	967
29	89	163	239	317	409	491	593	673	769	863	971
31	97	167	241	331	419	499	599	677	773	877	977
37	**101**	173	251	337	421	**503**	**601**	683	787	881	983
41	103	179	257	347	431	509	607	691	797	883	991
43	107	181	263	349	433	521	613	**701**	**809**	887	997

Natural sonning kanonik yoyilmasi

Ta'rif: Har qanday a natural sonning ***kanonik yoyilmasi*** deb, shu sonni tub sonlar ko'paytmasi ko'rinishida tasvirlashga aytiladi va

$$a=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdot\ldots\cdot p_k^{\alpha_k}$$

kabi yoziladi. Bu yerda $p_1, p_2, \ldots, p_k$ – tub sonlar.

Masalan: $72=2^3\cdot 3^2$ ($p_1=2,\ p_2=3,\ \alpha_1=3,\ \alpha_2=2$).

1. $a=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdot\ldots\cdot p_k^{\alpha_k}$ sonning ***natural bo'luvchilari soni*** (*NBS*).

$$NBS(a)=(\alpha_1+1)\cdot(\alpha_2+1)\cdot\ldots\cdot(\alpha_k+1)$$

2. $a=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdot\ldots\cdot p_k^{\alpha_k}$ sonning ***natural bo'luvchilari yig'indisi*** (*NBY*):

$$NBY(a)=\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_2+1}-1}{p_2-1}\cdot\ldots\cdot\frac{p_k^{\alpha_k+1}-1}{p_k-1}$$

3. $a=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdot\ldots\cdot p_k^{\alpha_k}$ sonning ***natural bo'luvchilari ko'paytmasi*** (*NBK*)

$$NBK(a)=a^{\frac{NBS(a)}{2}}$$

4. n ta raqamdan $n!=1\cdot2\cdot3\cdot\ldots\cdot n$ (!-***faktorial***) ta n xonali son tuzish mumkin

5. $n!$ soni k ta *nol* bilan tugaydi, ya'ni

$$k=\left[\frac{n}{5}\right]+\left[\frac{n}{5^2}\right]+\left[\frac{n}{5^3}\right]+\ldots$$

6. $n!$ sonning kanonik yoyilmasida p tub son θ_p ***daraja*** bilan qatnashadi, ya'ni

$$\theta_p=\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots$$

7. 1 dan n gacha natural sonlar ichida a ga ham, b ga ham ***bo'linadiganlari*** (***qoldiqsiz***) soni m ta, ya'ni

$$m=\left[\frac{n}{a}\right]+\left[\frac{n}{b}\right]-\left[\frac{n}{c}\right]$$

bu yerda $c=EKUK(a,b)$. ***Bo'linmaydiganlari*** soni esa $(n-m)$ ta.

Sonlarning EKUB va EKUKi

Ta'rif: Ikki yoki undan ortiq sonlarning har biri bo'linadigan (qoldiqsiz) son shu sonlarning ***umumiy bo'luvchisi*** deyiladi.

Ta'rif: Ikki yoki undan ortiq sonlarning umumiy bo'luvchilari ichida eng kattasi shu sonlarning ***eng katta umumiy bo'luvchisi*** (***EKUB***) deyiladi.

Ta'rif: Birdan boshqa umumiy bo'luvchilarga ega bo'lmagan natural sonlar ***o'zaro tub sonlar*** deyiladi.

Masalan: (4, 5); (7, 13); (14, 45); ...

Ta'rif: Ikki yoki undan ortiq sonlarning har biriga bo'linadigan son shu sonlarning ***umumiy bo'linuvchisi*** (***karralisi***) deyiladi.

Ta'rif: Ikki yoki undan ortiq sonlarning ***eng kichik umumiy bo'linuvchisi*** (***EKUK***) deb, bu sonlarga bo'linadigan eng kichik songa aytiladi.

Ikki sonning EKUB va EKUKini topish:

Misol: 120 va 180 sonlarining EKUB va EKUKini toping?

1-usul:

120	2	180	2
60	2	90	2
30	2	45	3
15	3	15	3
5	5	5	5
1		1	

$120 = 2^3 \cdot 3^1 \cdot 5^1$ $\qquad 180 = 2^2 \cdot 3^2 \cdot 5^1$

$EKUB(120;180) = 2^2 \cdot 3^1 \cdot 5^1 = 4 \cdot 3 \cdot 5 = 60$

$EKUK(120;180) = 2^3 \cdot 3^2 \cdot 5^1 = 8 \cdot 9 \cdot 5 = 360$

2-usul:

120	180	2
60	90	2
30	45	3
10	15	5
2	3	

$(2;3)$ *o'zaro tub sonlar*

$EKUB(120;180) = 2^2 \cdot 3^1 \cdot 5^1 = 4 \cdot 3 \cdot 5 = 60$

$EKUK(120;180) = 60 \cdot 2 \cdot 3 = 360$

Eslatma: a va b natural sonlar uchun $a \cdot b = EKUB(a;b) \cdot EKUK(a;b)$ bo'ladi.

a va b natural sonlarning ***umumiy bo'luvchilari soni (UBS)***:

$$UBS(a,b) = (\beta_1+1)\cdot(\beta_2+1)\cdot\ldots\cdot(\beta_k+1)$$

a va b natural sonlarning ***umumiy bo'luvchilari yig'indisi (UBY)***:

$$UBY(a,b) = \frac{q_1^{\beta_1+1}-1}{q_1-1}\cdot\frac{q_2^{\beta_2+1}-1}{q_2-1}\cdot\ldots\cdot\frac{q_k^{\beta_k+1}-1}{q_k-1}$$

bu yerda $EKUB(a;b) = q_1^{\beta_1}\cdot q_2^{\beta_2}\cdot\ldots\cdot q_k^{\beta_k}$.

Sonning butun va kasr qismi

Ta'rif: a sonning ***butun qismi*** deb, o'zidan katta bo'lmagan eng katta butun songa aytiladi va $[a]$ kabi belgilanadi. Masalan: $[2,4]=2$; $[-3,7]=-4$.

Eslatma: $a-1<[a]\le a$.

Ta'rif: a sonning ***kasr qismi*** deb, shu sondan o'zining butun qismini ayirish natijasida hosil bo'lgan songa aytiladi va $\{a\}$ kabi belgilanadi. Demak, $\{a\}=a-[a]$.

Masalan: $\{-0,3\}=0,7$; $\{2,6\}=0,6$.

Qoldiqli bo'lish

Ta'rif: a,b,c,d natural sonlar uchun $a=b\cdot c+d \quad (0\le d<b)$ ifoda ***qoldiqli bo'lishni*** ifodalaydi. Bu yerda a-bo'linuvchi, b-bo'luvchi, c-bo'linma, d-qoldiq.

Ta'rif: $(a\cdot b+c)^n$ sonni b songa bo'lgandagi qoldiq, c^n sonni b songa bo'lgandagi qoldiqqa teng.

Sonning mezoni

Ta'rif: Biror sonning ***mezoni*** deb, shu sonning raqamlari yig'indisini 9 ga bo'lganda hosil bo'lgan ***qoldiqqa*** aytiladi.

Masalan: 1523748 sonining mezoni $1+5+2+3+7+4+8=30$, u holda 30 ni 9 ga bo'lsak qoldiq 3 ga teng bo'ladi. Demak, berilgan sonning mezoni 3 bo'ladi.

Izoh: Ikki yoki undan ortiq sonlar ***yig'indisi (ayirmasi, ko'paytmasi, bo'linmasi*** va ***ildizi***)ning mezoni deb, har bir qo'shiluvchilardan olingan mezonlar yig'indisi (ayirmasi, ko'paytmasi, bo'linmasi va ildizi)ning mezoniga teng.

Kasr sonlar

Ta'rif: Birlikning bir yoki bir necha ulushini ifodalovchi son ***kasr son*** deyiladi.

* ***Kasr sonlar ikki xil ko'rinishda bo'ladi:***

1. Oddiy kasr $\qquad$ 2. O'nli kasr

Ta'rif: ***Oddiy kasr*** sonning umumiy ko'rinishi $\frac{m}{n}$ shaklida bo'lib, bunda m ***kasrning surati***, n esa ***kasrning maxraji*** deyiladi.

* ***Oddiy kasr ikki xil ko'rinishda bo'ladi:***

1. To'g'ri kasr ($m<n$) $\qquad$ 2. Noto'g'ri kasr ($m\ge n$)

Masalan: $\frac{1}{4}, \frac{12}{47}, \frac{145}{1231}$ – to'g'ri kasrlar, $\quad \frac{12}{5}, \frac{143}{85}, \frac{1864}{397}$ – noto'g'ri kasrlar.

Ta'rif: Agar noto'g'ri kasrning suratini maxrajiga bo'lib butun qismi ajratib yozilsa, bunday kasr ***aralash kasr*** deyiladi.

Masalan: $\frac{12}{5}=2\frac{2}{5}$, $\frac{145}{19}=7\frac{12}{19}$.

Eslatma: *Aralash kasrni noto'g'ri kasr ko'rinishida yozish uchun uning butun qismini maxrajiga ko'paytirib surati qo'shiladi va natija suratga yoziladi, maxraj esa o'zgarmay qoldiriladi.*

Ta'rif: Maxraji 10 yoki uning darajalaridan iborat bo'lgan kasr ***o'nli kasr*** deyiladi. Demak, $n=10, 100, 1000, \ldots$

Masalan: $\frac{12}{10}=1,2$; $\frac{123}{100}=1,23$; $\frac{169}{1000}=0,169$.

* ***O'nli kasr ikki xil ko'rinishda bo'ladi:***

1. Chekli o'nli kasr $\qquad$ 2. Cheksiz o'nli (davriy) kasr

Eslatma: Agar qisqarmas oddiy kasrning maxraji tub ko'paytuvchilarga ajratilganda tub ko'paytuvchilar faqat **2 *lar***, faqat **5 *lar*** yoki **2** va **5 *lardan*** iborat bo'lsa, bunday oddiy kasrni ***chekli o'nli kasr*** ko'rinishida ifodalab bo'ladi (aks holda ***cheksiz o'nli kasr*** hosil bo'ladi).

Qoida: *Oddiy kasrni o'nli kasr ko'rinishida yozish uchun uning suratini maxrajiga bo'lish kerak.*

Qoida: *O'nli kasrni oddiy kasr ko'rinishida yozish uchun uning o'qilish tartibiga rioya qilgan holda yozish kerak.*

Eslatma: Oddiy kasrni o'nli kasr ko'rinishida yozishda ba'zan, sonning kasr qismida bir xil son takrorlanib keladi. Bunday o'nli kasr ***davriy kasr*** deyiladi.

* ***Davriy kasrlar ikki xil ko'rinishda bo'ladi:***

1. Sof davriy kasr   2. Aralash davriy kasr

Masalan. $\frac{1}{3}=0,333...=0,(3)$ sof davriy kasr, $1\frac{5}{18}=1,2(7)$ aralash davriy kasr.

Qoida: *Sof davriy kasrni oddiy kasr ko'rinishida yozish uchun davrdagi son nechta raqamdan iborat bo'lsa, shuncha 9 ni maxrajga yozib, suratiga davrni yozish kifoya.*

Qoida: *Aralash davriy kasrni oddiy kasr ko'rinishida yozish uchun maxrajga kasrning davrida nechta raqam bo'lsa, shuncha 9 va davrgacha nechta raqam bo'lsa, shuncha nol (9 dan keyin) yozish kerak, suratga esa verguldan keyingi sonlardan (davrga e'tibor qilmay) davrgacha bo'lgan sonni ayirib yozish kerak.*

Davriy o'nli kasrlarni oddiy kasr ko'rinishida yozish

Sof davriy kasrlar uchun: $\overline{a_0,(b_1...b_n)}=a_0\frac{\overline{b_1...b_n}}{10^n-1}$

Aralash davriy kasrlar uchun:

$$\overline{a_0,a_1...a_k(b_1...b_n)}=a_0\frac{\overline{a_1...a_kb_1...b_n}-\overline{a_1...a_k}}{(10^n-1)\cdot 10^k}$$

Oddiy kasrlar ustida amallar

1. $\frac{a}{b}\pm\frac{c}{b}=\frac{a\pm c}{b}$
2. $\frac{a}{b}\pm\frac{c}{d}=\frac{a\cdot\alpha\pm c\cdot\beta}{P}$
3. $a\frac{c}{b}=a+\frac{c}{b}$
4. $a\frac{c}{b}\pm d\frac{m}{n}=(a\pm d)+\left(\frac{c}{b}\pm\frac{m}{n}\right)$
5. $\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$
6. $\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}$

bu yerda $P=EKUK(b;d)$, $\alpha=\frac{P}{b}$, $\beta=\frac{P}{d}$.

Kasr sonlarni taqqoslash

1. Maxraji bir xil bo'lgan musbat oddiy kasrlarda surati kattasi ***katta*** hisoblanadi, ya'ni $a>b$ bo'lsa, $\frac{a}{c}>\frac{b}{c}$

2. Surati bir xil bo'lgan musbat oddiy kasrlarda maxraji kichigi ***katta*** hisoblanadi, ya'ni $b>c$ bo'lsa, $\frac{a}{b}<\frac{a}{c}$

Eslatma: Surat va maxraji har xil bo'lgan musbat oddiy kasrlarni taqqoslashda surati yoki maxrajini bir xilga keltirib olinadi.

3. Musbat o'nli kasrlarda butun qismi kattasi ***katta*** hisoblanadi. Agar butun qismlari teng bo'lsa, kasr qismidagi raqamlar chapdan o'ngga qarab taqqoslanib boriladi.

Eslatma: Manfiy kasr sonlarni taqqoslashda yuqoridagilar aksincha bo'ladi.

Haqiqiy sonning moduli

Ta'rif: a sonning ***absolyut qiymati (moduli)*** deb, agar u son nomanfiy bo'lsa, a sonning o'ziga, agar u son manfiy bo'lsa, $-a$ songa aytiladi va $|a|$ ko'rinishida belgilanadi. Demak,

$$|a|=\begin{cases}a, & \text{agar } a\geq 0 \text{ bo'lsa,}\\ -a, & \text{agar } a<0 \text{ bo'lsa.}\end{cases}$$

Xossalari:

1. $|a|\geq a$
2. $|a|^2=a^2$
3. $|a\cdot b|=|a|\cdot|b|$
4. $\left|\frac{a}{b}\right|=\frac{|a|}{|b|}\ (b\neq 0)$
5. $|a+b|\leq|a|+|b|$
6. $|a-b|\geq|a|-|b|$

Daraja va uning asosiy xossalari

Ta'rif: a sonning n***-darajasi*** $(n\in\mathbb{N})$ deb, a sonni n marta o'zini o'ziga ko'paytirishga aytiladi, ya'ni $a^n=\underbrace{a\cdot a\cdot...\cdot a}_{n}$.

Xossalari:

1. $a^0=1,\ a^1=a,\ a^{-p}=\frac{1}{a^p}$
2. $(a^p)^q=a^{p\cdot q}$
3. $a^p\cdot a^q=a^{p+q}$
4. $a^p:a^q=a^{p-q}$
5. $(a\cdot b)^p=a^p\cdot b^p$
6. $\left(\frac{a}{b}\right)^p=\frac{a^p}{b^p}$

bu yerda $a>0,\ b>0,\ p,q\in\mathbb{R}$.

Arifmetik ildiz va uning asosiy xossalari

Ta'rif: a sonning n***-darajali ildizi*** $(n\in\mathbb{N})$ deb, n-darajasi a songa teng bo'lgan b songa aytiladi, ya'ni $\sqrt[n]{a}=b \Rightarrow a=b^n,\ n\geq 2$.

$$\sqrt[n]{a^n}=\begin{cases}|a|, & \text{agar } n=2k \text{ bo'lsa,}\\ a, & \text{agar } n=2k+1 \text{ bo'lsa}\end{cases}\quad (k\in\mathbb{N})$$

Xossalari $(a>0, b>0)$:

1. $\sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}$

2. $\sqrt[n]{\dfrac{a}{b}}=\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}\ (b\neq 0)$

3. $\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}=a^{\frac{m}{n}}$

4. $\sqrt[n]{a^m}=\sqrt[nk]{a^{mk}}$

5. $a\sqrt[n]{b}=\sqrt[n]{a^n\cdot b}$

6. $-a\sqrt[n]{b}=-\sqrt[n]{a^n\cdot b}$

7. $\sqrt[n]{a\sqrt[m]{b}}=\sqrt[nm]{a^m\cdot b}$

8. $\sqrt[n]{a\sqrt[m]{b\sqrt[n]{a\sqrt[m]{b\ldots}}}}=\sqrt[nm-1]{a^m\cdot b}$

9. $\sqrt[n]{a\cdot\sqrt[n]{a\cdot\sqrt[n]{a\cdot\ldots}}}=\sqrt[n-1]{a}$

10. $\sqrt[n]{a:\sqrt[n]{a:\sqrt[n]{a:\ldots}}}=\sqrt[n+1]{a}$

11. $\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}}=\dfrac{\sqrt{1+4a}+1}{2}$

12. $\sqrt{a-\sqrt{a-\sqrt{a-\ldots}}}=\dfrac{\sqrt{1+4a}-1}{2}$

13. $\sqrt{a+b\pm 2\sqrt{a\cdot b}}=\sqrt{a}\pm\sqrt{b}\ (a\geq b\geq 0)$

Musbat sonning standart shakli

$$a=b\cdot 10^n$$

bu yerda $1\leq|b|<10$, b – sonning mantissasi, $n\in\mathbb{Z}$.

Proporsiya

Ta'rif: Ikki nisbatning tengligi ***proporsiya*** deyiladi, ya'ni

$$\frac{a}{b}=\frac{c}{d} \qquad \text{yoki} \qquad a:b=c:d$$

bu yerda a va d ***chetki hadlar***, b va c ***o'rta hadlar***.

Asosiy xossasi: Proporsiyaning chetki hadlari ko'paytmasi o'rta hadlari ko'paytmasiga teng, ya'ni $a\cdot d=b\cdot c$.

1. $\dfrac{a}{c}=\dfrac{b}{d},\ \dfrac{d}{b}=\dfrac{c}{a},\ \dfrac{d}{c}=\dfrac{b}{a}$

2. $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{x}{y}=\ldots\Rightarrow\dfrac{a+c+x+\ldots}{b+d+y+\ldots}=\dfrac{a}{b}$

3. $\dfrac{a+b}{a-b}=\dfrac{c+d}{c-d}$

4. $\dfrac{a+c}{a-c}=\dfrac{b+d}{b-d}$

5. $\begin{matrix}a\leftrightarrow p\\ b\leftrightarrow q\end{matrix}\Rightarrow a\cdot q=b\cdot p$ o'rinli bo'lsa, ***to'g'ri proporsiya***.

6. $\begin{matrix}a\leftrightarrow p\\ b\leftrightarrow q\end{matrix}\Rightarrow a\cdot p=b\cdot q$ o'rinli bo'lsa, ***teskari proporsiya***.

To'g'ri va teskari proporsional bog'lanishlar

1. A sonni $a_1, a_2, \ldots, a_n$ sonlarga ***to'g'ri proporsional (mutanosib) bo'laklarga*** ajratish $(a_1:a_2:\ldots:a_n$ ***nisbatda bo'lish***)

$$A=(a_1+a_2+\ldots+a_n)x, \qquad a_1x,\ a_2x,\ \ldots,\ a_nx \text{ – bo'laklar.}$$

2. A sonni $a_1, a_2, \ldots, a_n$ sonlarga ***teskari proporsional bo'laklarga*** ajratish $(a_1:a_2:\ldots:a_n$ ***nisbatda bo'lish***)

$$A=(a_1+a_2+\ldots+a_n)x, \qquad a_nx,\ a_{n-1}x,\ \ldots,\ a_1x \text{ – bo'laklar.}$$

3. a son b songa ***to'g'ri proporsional:***

$$a=b\cdot k \qquad (k\text{-proporsionallik koeffisiyenti})$$

4. a son b songa ***teskari proporsional:***

$$a=\frac{k}{b} \qquad (k\text{-proporsionallik koeffisiyenti})$$

O'rta qiymatlar

Agar $a_1, a_2, \ldots, a_n$ musbat sonlar bo'lsa, ularning

O'rta arifmetik: $A=\dfrac{a_1+a_2+\ldots+a_n}{n}$

O'rta geometrik: $G=\sqrt[n]{a_1\cdot a_2\cdot\ldots\cdot a_n}$

O'rta garmonik: $H=\dfrac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}}$

O'rta kvadratik: $K=\sqrt{\dfrac{a_1^2+a_2^2+\ldots+a_n^2}{n}}$

O'rta qiymatlar orasidagi bog'lanish: $H\leq G\leq A\leq K$

Izoh: Ikki sonning ***o'rta proporsional*** qiymati: $F=\sqrt{a_1\cdot a_2}$

Foiz

Ta'rif: Har qanday miqdor (son)ning ***yuzdan bir qismi*** shu miqdorning ***bir foizi (prosenti)*** deyiladi.

Ta'rif: Har qanday miqdor (son)ning ***mingdan bir qismi*** shu miqdorning ***bir promili*** deyiladi.

$$\frac{1}{100}=1\%, \qquad \frac{1}{1000}=1‰$$

1. a sonning p foizi $a\cdot\dfrac{p}{100}$ ga teng.

2. p foizi b bo'lgan son $\dfrac{b}{p}\cdot 100$ ga teng.

3. b son a sonning $\frac{b}{a}\cdot 100$ foiziga teng.

4. a somi p foizga n marta orttirilsa (kamaytirilsa), $a\cdot\left(1\pm\frac{p}{100}\right)^n$ ga teng.

5. a son birinchi safar x foizga, ikkinchi safar y foizga orttirilsa (kamaytirilsa),

$$a\cdot\frac{100\pm x}{100}\cdot\frac{100\pm y}{100} \text{ ga teng.}$$

6. a son birinchi safar x foizga orttirilsa, ikkinchi safar y foizga kamaytirilsa (va aksincha),

$$a\cdot\frac{100\pm x}{100}\cdot\frac{100\mp y}{100} \text{ ga teng.}$$

Qisqa ko'paytirish formulalari

1. $(a+b)^2=a^2+2ab+b^2$
2. $(a-b)^2=a^2-2ab+b^2$
3. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$
4. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$
5. $a^2-b^2=(a-b)(a+b)$
6. $a^3+b^3=(a+b)(a^2-ab+b^2)$
7. $a^3-b^3=(a-b)(a^2+ab+b^2)$

Kombinatorika elementlari

1. n ta elementdan barcha ***o'rin almashtirishlar*** soni: $P_n=n!$
2. n ta elementdan m tadan barcha ***o'rinlashtirishlar*** soni: $A_n^m=\frac{n!}{(n-m)!}$
3. n ta elementdan m tadan barcha ***gruppalashlar*** soni: $C_n^m=\frac{n!}{(n-m)!\cdot m!}$

Xossalari:

$$C_m^0=C_m^m=1 \qquad C_n^m=C_n^{n-m}$$

$$C_n^m=C_{n-1}^{m-1}+C_{n-1}^m \qquad C_n^0+C_n^1+C_n^2+...+C_n^n=2^n$$

$$C_n^0+C_n^2+C_n^4+...=C_n^1+C_n^3+C_n^5+...=2^{n-1}$$

$$C_n^n+C_{n+1}^n+C_{n+2}^n+...+C_{n+m-1}^n=C_{n+m}^{n+1}$$

Paskal uchburchagi

$$C_0^0$$
$$C_1^0 \quad C_1^1$$
$$C_2^0 \quad C_2^1 \quad C_2^2$$
$$C_3^0 \quad C_3^1 \quad C_3^2 \quad C_3^3$$
$$C_4^0 \quad C_4^1 \quad C_4^2 \quad C_4^3 \quad C_4^4$$
$$C_5^0 \quad C_5^1 \quad C_5^2 \quad C_5^3 \quad C_5^4 \quad C_5^5$$
$$\dots \qquad \dots \qquad \dots$$

$n=0$: 1
$n=1$: 1 1
$n=2$: 1 2 1
$n=3$: 1 3 3 1
$n=4$: 1 4 6 4 1
$n=5$: 1 5 10 10 5 1
$n=6$: 1 6 15 20 15 6 1
$n=7$: 1 7 21 35 35 21 7 1
$n=8$: 1 8 28 56 70 56 28 8 1
... ...

Nyuton binomi:

$$(a+b)^n=C_n^0a^nb^0+C_n^1a^{n-1}b^1+...+C_n^ma^{n-m}b^m+...+C_n^na^0b^n$$

Masalan:

$$(a+b)^5=a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$$

Logarifm va uning asosiy xossalari

Ta'rif: Biror b sonning a asosga ko'ra ***logarifmi*** deb, b sonini hosil qilish uchun a sonini ko'tarish kerak bo'lgan daraja ko'rsatkichiga aytiladi, ya'ni

$$\log_a b=c \quad\Leftrightarrow\quad b=a^c$$

bu yerda $b>0,\ a>0,\ a\neq 1$.

Xossalari:

$$\log_a 1=0 \qquad \log_a a=1 \qquad \log_a b^m=m\log_a b \qquad \log_{a^n} b=\frac{1}{n}\log_a b$$

$$\log_a b=\frac{1}{\log_b a} \qquad \log_a b=\frac{\log_c b}{\log_c a} \qquad a^{\log_a b}=b \qquad a^{\log_b c}=c^{\log_b a}$$

$$\log_a(b\cdot c)=\log_a b+\log_a c \qquad \log_a\left(\frac{b}{c}\right)=\log_a b-\log_a c$$

$$\begin{cases}a>1\\ b>1\end{cases} \Rightarrow \log_a b>0 \qquad \begin{cases}a>1\\ 0<b<1\end{cases} \Rightarrow \log_a b<0$$

$\begin{cases} 0<a<1 \\ 0<b<1 \end{cases} \Rightarrow \log_a b>0$ $\qquad$ $\begin{cases} 0<a<1 \\ b>1 \end{cases} \Rightarrow \log_a b<0$

$\log_{10} b=\lg b$ – ***o'nli logarifm*** $\qquad$ $\log_e b=\ln b$ – ***natural logarifm***

Arifmetik progressiya

Ta'rif: Sonli ketma-ketlikning ikkinchi hadidan boshlab har bir hadi o'zidan oldingi hadga biror d (*o'zgarmas*) sonni qo'shish natijasida hosil bo'lsa, bunday sonli ketma-ketlik ***arifmetik progressiya*** deyiladi, ya'ni

$$a_{n+1}=a_n+d \qquad (n\in\mathbb{N},\ d\text{-ayirma}).$$

Eslatma: $d\neq 0$

Agar $d>0$ bo'lsa, arifmetik progressiya ***o'suvchi*** bo'ladi.

Agar $d<0$ bo'lsa, arifmetik progressiya ***kamayuvchi*** bo'ladi.

1. n-hadni topish: $\quad a_n=a_1+(n-1)d$

2. Ayirmani topish: $\quad d=a_{n+1}-a_n,\quad d=\frac{a_n-a_m}{n-m}\ (n>m)$

3. O'rta hadni topish: $\quad a_n=\frac{a_{n-m}+a_{n+m}}{2}\ (n>m)$

4. Hadlar orasidagi bog'lanish: $a_m+a_n=a_p+a_q$, bunda $m+n=p+q$

5. Dastlabki n ta hadning yig'indisi: $S_n=a_1+a_2+...+a_n$

$$S_n=\frac{a_1+a_n}{2}\cdot n, \qquad S_n=\frac{2a_1+(n-1)d}{2}\cdot n,$$

$$S_n=a_k\cdot n\ (a_k\text{-o'rta had}), \qquad S_{n+1}-S_n=a_{n+1}$$

6. Dastlabki n ta hadning yig'indisi S_n, dastlabki m ta hadning yig'indisi S_m bo'lsa, dastlabki $n+m$ ta hadning yig'indisi $S_{n+m}=\frac{n+m}{n-m}(S_n-S_m)$, $n>m$.

7. Agar $a_1, a_2, ..., a_{2n}$ arifmetik progressiyada $a_1+a_3+...+a_{2n-1}=S_1$, $a_2+a_4+...+a_{2n}=S_2$ va d berilgan bo'lsa, $n=\frac{2}{d}(S_2-S_1)$.

Geometrik progressiya

Ta'rif: Sonli ketma-ketlikning ikkinchi hadidan boshlab har bir hadi o'zidan oldingi hadni biror q (*o'zgarmas*) songa ko'paytirish natijasida hosil bo'lsa, bunday sonli ketma-ketlik ***geometrik progressiya*** deyiladi, ya'ni

$$b_{n+1}=b_n\cdot q \quad (n\in\mathbb{N},\ q\text{-maxraj}).$$

Eslatma: $q\neq 0,\ q\neq \pm 1$

Agar $q>1$ bo'lsa, geometrik progressiya ***o'suvchi*** bo'ladi.

Agar $q<0$ bo'lsa, geometrik progressiya ***ishorasi almashinuvchi*** bo'ladi.

Agar $|q|<1$ bo'lsa, geometrik progressiya ***cheksiz kamayuvchi*** bo'ladi.

1. n-hadni topish: $\quad b_n=b_1\cdot q^{n-1}$

2. Maxrajni topish: $\quad q=\frac{b_{n+1}}{b_n}=\sqrt[n-1]{\frac{b_n}{b_1}},\quad q^{n-m}=\frac{b_n}{b_m}\ (n>m)$

3. O'rta hadni topish: $\quad b_n=\sqrt{b_{n-m}\cdot b_{n+m}}\ (n>m)$

4. Hadlar orasidagi bog'lanish: $b_m\cdot b_n=b_p\cdot b_q$, bunda $m+n=p+q$

5. Dastlabki n ta hadning yig'indisi: $S_n=b_1+b_2+...+b_n$

$$S_n=\frac{b_n\cdot q-b_1}{q-1}, \qquad S_n=\frac{b_1(q^n-1)}{q-1}, \qquad S_{n+1}-S_n=b_{n+1}$$

6. Cheksiz kamayuvchi geometrik progressiya hadlari yig'indisi:

$$S=\frac{b_1}{1-q} \quad (|q|<1)$$

TENGLAMALAR

Ta'rif: ***Tenglama deb***, noma'lum son qatnashgan tenglikka aytiladi.

Agar tenglamada bitta noma'lum qatnashsa, bu tenglama ***bir noma'lumli***, ikkita noma'lum qatnashsa, ***ikki noma'lumli*** va hokazo noma'lumli tenglama deyiladi.

Tenglamaning ildizi deb, noma'lumning shu tenglamani to'g'ri tenglikka aylantiradigan qiymatiga aytiladi.

Tenglamani yechish deb, uning barcha ildizlarini topish yoki ularning yo'qligini aniqlashga aytiladi.

Xossasi:

1. Tenglamaning istalgan hadini uning bir qismidan ikkinchi qismiga ishorasini qarama-qarshisiga o'zgartirgan holda olib o'tish mumkin.

2. Tenglamaning har ikki qismini nolga teng bo'lmagan ayni bir xil songa ko'paytirish yoki bo'lish mumkin.

Chiziqli tenglama

Chiziqli tenglama deb,

$$ax+b=0$$

ko'rinishidagi yoki shu ko'rinishga keltirish mumkin bo'lgan tenglamaga aytiladi. Bunda a va b haqiqiy sonlar, x esa noma'lum son.

1. Agar $a\neq 0$ bo'lganda $b\in\mathbb{R}$ bo'lsa, tenglama ***yagona yechimga ega***.

2. Agar $a=0$ bo'lganda $b\neq 0$ bo'lsa, tenglama ***yechimga ega emas***.

3. Agar $a=0$ … ***ega***.

Chiziqli tenglamalar sistemasi

Chiziqli tenglamalar sistemasi deb,

$$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases}$$

ko'rinishidagi ***ikki noma'lumli ikkita tenglamadan iborat sistemaga*** aytiladi. Bunda a_i, b_i va c_i $(i=1,2)$ haqiqiy sonlar, x va y lar esa noma'lum sonlar.

1. Agar $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ bo'lsa, tenglamalar sistemasi ***yagona yechimga ega***.

2. Agar $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ bo'lsa, tenglamalar sistemasi ***yechimga ega emas***.

3. Agar $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ bo'lsa, tenglamalar sistemasi ***cheksiz ko'p yechimga ega***.

Kvadrat tenglama

Kvadrat tenglama deb,

$$ax^2+bx+c=0$$

ko'rinishidagi tenglamaga aytiladi. Bunda a, b va c haqiqiy sonlar kvadrat tenglamaning koeffisiyentlari $(a \neq 0)$, x esa noma'lum son.

Ildizlarini topish formulasi ($D=b^2-4ac$ *diskriminant*):

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}.$$

1. Agar $D>0$ bo'lsa, tenglama ***2 ta har xil haqiqiy yechimga ega:***

$$x_1=\frac{-b-\sqrt{D}}{2a} \text{ va } x_2=\frac{-b+\sqrt{D}}{2a};$$

2. Agar $D=0$ bo'lsa, tenglama ***1 ta ikki karrali haqiqiy yechimga ega:***

$$x_1=x_2=-\frac{b}{2a};$$

3. Agar $D<0$ bo'lsa, tenglama ***haqiqiy yechimga ega emas***.

Qo'shimcha:

Tenglama bitta yechimga ega bo'lsa: $a=0$ yoki $D=0$.

Chala kvadrat tenglamalar

$ax^2=0$ $(b=0,\ c=0)$ *yechimi:* $x_1=x_2=0$;

$ax^2+bx=0$ $(c=0)$ *yechimi:* $x_1=0$, $x_2=-\frac{b}{a}$;

$ax^2+c=0$ $(b=0)$ *yechimi:* $x_{1,2}=\pm\sqrt{-\frac{c}{a}}$, $\frac{a}{c}<0$.

Kvadrat tenglama yechimlarining xossalari ($D>0$)

Agar x_1 va x_2 sonlar $x^2+px+q=0$ $\left(p=\frac{b}{a},\ q=\frac{c}{a}\right)$ ***keltirilgan kvadrat tenglamaning ildizlari*** bo'lsa, u holda:

Viyet teoremasi: $x_1+x_2=-p$, $x_1\cdot x_2=q$

Ko'paytuvchilarga ajratish: $x^2+px+q=(x-x_1)(x-x_2)$

Kvadrat tenglamaning ildizlari va koeffisiyentlari orasidagi bog'lanish:

$\frac{1}{x_1}+\frac{1}{x_2}=-\frac{p}{q}$ $\frac{x_1}{x_2}+\frac{x_2}{x_1}=\frac{p^2-2q}{q}$ $x_1^2+x_2^2=p^2-2q$

$\frac{1}{x_1^2}+\frac{1}{x_2^2}=\frac{p^2-2q}{q^2}$ $\frac{x_1}{x_2^2}+\frac{x_2}{x_1^2}=\frac{3pq-p^3}{q^2}$ $x_1^3+x_2^3=3pq-p^3$

$\frac{1}{x_1^3}+\frac{1}{x_2^3}=\frac{3pq-p^3}{q^3}$ $x_1x_2^2+x_1^2x_2=-pq$ $x_1^4+x_2^4=(p^2-2q)^2-2q^2$

Parametrga bog'liq keltirilgan kvadrat tenglamalar ($D>0$)

1. Agar $x^2+px+q=0$ kvadrat tenglamada $q<0$ bo'lsa, u holda kvadrat tenglama ***ikkita turli ishorali ildizlarga*** ega.

2. Agar $x^2+px+q=0$ kvadrat tenglamada $p>0$ va $q>0$ bo'lsa, u holda kvadrat tenglama ***ikkita manfiy ildizlarga*** ega.

3. Agar $x^2+px+q=0$ kvadrat tenglamada $p<0$ va $q>0$ bo'lsa, u holda kvadrat tenglama ***ikkita musbat ildizlarga*** ega.

4. Agar $x^2+px+q=0$ kvadrat tenglamada $p=0$ va $q<0$ bo'lsa, u holda kvadrat tenglama ***ikkita qarama-qarshi ildizlarga*** ega.

5. Agar $x^2+px+q=0$ kvadrat tenglamada $q=1$ bo'lsa, u holda kvadrat tenglama ***ikkita o'zaro teskari ildizlarga*** ega.

6. Agar $x^2+px+q=0$ kvadrat tenglamaning koeffisiyentlari yig'indisi nolga teng $(1+p+q=0)$ bo'lsa, u holda $x_1=1$ va $x_2=q$ bo'ladi.

7. Agar $x^2+px+q=0$ kvadrat tenglamaning ildizlaridan biri ikkinchisidan n ga katta (yoki kichik) bo'lsa, u holda $x_1=y$ va $x_2=y+n$ bo'ladi.

8. Agar $x^2+px+q=0$ kvadrat tenglamaning ildizlaridan biri ikkinchisidan n marta katta (yoki kichik) bo'lsa, u holda $x_1=y$ va $x_2=ny$ bo'ladi.

9. Agar $x^2+px+q=0$ kvadrat tenglamaning ildizlaridan biri ikkinchisining n-darajasiga teng bo'lsa, u holda $x_1=y$ va $x_2=y^n$ bo'ladi.

Qo'shimcha:

1. $x^2+px+q=0$ kvadrat tenglama ildizlariga ***qarama-qarshi ildizli*** kvadrat tenglama $x^2-px+q=0$ bo'ladi.

2. $x^2+px+q=0$ kvadrat tenglama ildizlariga ***teskari ildizli*** kvadrat tenglama $qx^2+px+1=0$ bo'ladi.

3. To'la kvadratga ajratish: $x^2+px+q=\left(x+\frac{p}{2}\right)^2-\frac{p^2-4q}{4}$.

4. $A^2(x)+B^2(y)=0$ tenglama $A(x)=0$ va $B(y)=0$ bo'lsagina yechimga ega.

Eslatma: Yuqoridagi barcha munosabatlarni umumiy holdagi $ax^2+bx+c=0$ kvadrat tenglama uchun ham qo'llash mumkin.

Kub tenglama

Kub tenglama deb,

$$ax^3+bx^2+cx+d=0$$

ko'rinishidagi tenglamaga aytiladi. Bunda a, b, c va d haqiqiy sonlar kub tenglamaning koeffisiyentlari $(a\neq 0)$, x esa noma'lum son.

Kub tenglama yechimlarining xossalari

Agar x_1, x_2 va x_3 sonlar $x^3+px^2+qx+k=0$ $\left(p=\frac{b}{a},\ q=\frac{c}{a},\ k=\frac{d}{a}\right)$ ***keltirilgan kub tenglamaning ildizlari*** bo'lsa, u holda:

Viyet teoremasi:

$x_1+x_2+x_3=-p$, $\quad x_1\cdot x_2+x_1\cdot x_3+x_2\cdot x_3=q$, $\quad x_1\cdot x_2\cdot x_3=-k$

Ko'paytuvchilarga ajratish:

$$x^3+px^2+qx+k=(x-x_1)(x-x_2)(x-x_3)$$

Bikvadrat tenglama

Bikvadrat tenglama deb,

$$ax^4+bx^2+c=0$$

ko'rinishidagi tenglamaga aytiladi. Bunda a, b va c haqiqiy sonlar bikvadrat tenglamaning koeffisiyentlari $(a\neq 0)$, x esa noma'lum son.

Ildizlarini topish formulasi ($D=b^2-4ac$ *diskriminant*):

$$x_{1,2,3,4}=\pm\sqrt{\frac{-b\pm\sqrt{D}}{2a}}.$$

Bikvadrat tenglama yechimlarining xossalari

Agar x_1, x_2, x_3 va x_4 sonlar $x^4+px^2+q=0$ $\left(p=\frac{b}{a},\ q=\frac{c}{a}\right)$ ***keltirilgan bikvadrat tenglamaning ildizlari*** bo'lsa, u holda:

Viyet teoremasi:

$x_1+x_2+x_3+x_4=0$, $\quad x_1\cdot x_2\cdot x_3\cdot x_4=q$

Ko'paytuvchilarga ajratish:

$$x^4+px^2+q=(x-x_1)(x-x_2)(x-x_3)(x-x_4)$$

Izoh: Bikvadrat tenglama $x^2=y$ $(y\geq 0)$ belgilash orqali $ay^2+by+c=0$ kvadrat tenglamaga keltirib ishlanadi.

n-darajali tenglama

n-darajali tenglama deb,

$$a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots+a_1x+a_0=0$$

ko'rinishidagi tenglamaga aytiladi. Bunda a_n, a_{n-1}, a_{n-2}, ... a_1, a_0 haqiqiy sonlar n-darajali tenglamaning koeffisiyentlari $(a_n\neq 0)$, x esa noma'lum son.

n-darajali tenglama yechimlarining xossalari

$x_1+x_2+x_3+\ldots+x_n=-\frac{a_{n-1}}{a_n}$, $\quad x_1\cdot x_2\cdot x_3\cdot\ldots\cdot x_n=(-1)^n\frac{a_0}{a_n}$

<u>**Teorema:**</u> n-darajali tenglama ko'pi bilan n ta haqiqiy yechimga ega.

Ratsional tenglamalar

1. $f(x)\cdot g(x)=0 \quad\Rightarrow\quad \begin{cases} f(x)=0\\ g(x)=0\end{cases}$

2. $\frac{f(x)}{g(x)}=0 \quad\Rightarrow\quad \begin{cases} f(x)=0\\ g(x)\neq 0\end{cases}$

Irratsional tenglamalar

1. $\sqrt[n]{f(x)}=0 \quad\Rightarrow\quad f(x)=0$

2. $\sqrt[2k]{f(x)}=C \quad\Rightarrow\quad \begin{cases} C>0, & f(x)=C^{2k}\\ C<0, & \varnothing\end{cases} \quad (C=const)$

3. $\sqrt[2k+1]{f(x)}=C \quad\Rightarrow\quad f(x)=C^{2k+1} \quad (C=const)$

4. $\sqrt[2k]{f(x)}=g(x) \quad\Rightarrow\quad \begin{cases} g(x)\geq 0\\ f(x)=g^{2k}(x)\end{cases}$

5. $\sqrt[2k+1]{f(x)}=g(x) \Rightarrow f(x)=g^{2k+1}(x)$

6. $\sqrt[2k]{f(x)}=\sqrt[2k]{g(x)} \Rightarrow \begin{cases} f(x)\ge 0 \\ g(x)\ge 0 \\ f(x)=g(x) \end{cases}$

7. $\sqrt[2k+1]{f(x)}=\sqrt[2k+1]{g(x)} \Rightarrow f(x)=g(x)$

8. $\sqrt[2k]{f(x)}+\sqrt[2k]{g(x)}=0 \Rightarrow \begin{cases} f(x)=0 \\ g(x)=0 \end{cases}$

9. $a\sqrt[k]{f(x)}+b\sqrt[2k]{f(x)}+c=0 \Rightarrow \begin{cases} \sqrt[2k]{f(x)}=y,\ y\ge 0 \\ ay^2+by+c=0 \end{cases}$

Modul qatnashgan tenglamalar

1. $|f(x)|=0 \Rightarrow f(x)=0$

2. $|f(x)|=C \Rightarrow \begin{cases} C>0,\ f(x)=\pm C \\ C<0,\ \varnothing \end{cases} \quad (C=const)$

3. $|f(x)|=f(x) \Rightarrow f(x)\ge 0$

4. $|f(x)|=g(x) \Rightarrow \begin{cases} g(x)\ge 0 \\ f(x)=\pm g(x) \end{cases}$

5. $|f(x)|=|g(x)| \Rightarrow f(x)=\pm g(x)$

6. $|f(x)|+|g(x)|=0 \Rightarrow f(x)=0,\ g(x)=0$

7. $|f(x)+g(x)|=|f(x)|+|g(x)| \Rightarrow f(x)\cdot g(x)\ge 0$

Ko'rsatkichli tenglamalar

1. $a^{f(x)}=1 \Rightarrow f(x)=0\ (a>0)$

2. $a^{f(x)}=b \Rightarrow \begin{cases} b>0,\ f(x)=\log_a b \\ b<0,\ \varnothing \end{cases},\ (a>0)$

3. $a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x)\ (a>0)$

4. $a^{f(x)}=b^{f(x)} \Rightarrow f(x)=0\ (a\cdot b\ne 0)$

5. $a^{f(x)}=b^{g(x)} \Rightarrow f(x)=g(x)\cdot\log_a b\ (a,b>0)$

6. $[f(x)]^{g(x)}=1 \Rightarrow \begin{cases} f(x)=1 \\ g(x)\in\mathbb{R} \end{cases} \cup \begin{cases} f(x)\ne 0 \\ g(x)=0 \end{cases}$

7. $[\varphi(x)]^{f(x)}=[\varphi(x)]^{g(x)} \Rightarrow \begin{cases} \varphi(x)=1;\ f(x),g(x)\in\mathbb{R} \\ \varphi(x)=0;\ f(x)>0,g(x)>0 \\ \varphi(x)\ne 0;\ f(x)=g(x) \end{cases}$

8. $[f(x)]^{\varphi(x)}=[g(x)]^{\varphi(x)} \Rightarrow \begin{cases} \varphi(x)=0;\ f(x)\ne 0,g(x)\ne 0 \\ \varphi(x)\ne 0;\ f(x)=g(x) \end{cases}$

9. $|\varphi(x)|^{f(x)}=|\varphi(x)|^{g(x)} \Rightarrow \begin{cases} \varphi(x)=0;\ f(x)>0,g(x)>0 \\ \varphi(x)\ne 0;\ f(x)=g(x) \\ \varphi(x)=\pm 1 \end{cases}$

10. $a\cdot k^{2f(x)}+b\cdot k^{f(x)}+c=0 \Rightarrow \begin{cases} k^{f(x)}=y,\ y>0 \\ ay^2+by+c=0 \end{cases}$

Logarifmik tenglamalar

1. $\log_a f(x)=b \Rightarrow \begin{cases} f(x)>0 \\ f(x)=a^b \end{cases} \quad (0<a\ne 1)$

2. $\log_{f(x)} a=b \Rightarrow \begin{cases} 0<f(x)\ne 1 \\ f(x)=a^{\frac{1}{b}} \end{cases} \quad (a>0)$

3. $\log_a f(x)=\log_a g(x) \Rightarrow \begin{cases} f(x)>0,\ g(x)>0 \\ f(x)=g(x) \end{cases} \quad (0<a\ne 1)$

4. $\log_{f(x)} g(x)=b \Rightarrow \begin{cases} 0<f(x)\ne 1,\ g(x)>0 \\ g(x)=f^b(x) \end{cases}$

5. $\log_{\varphi(x)} f(x)=\log_{\varphi(x)} g(x) \Rightarrow \begin{cases} 0<\varphi(x)\ne 1 \\ f(x)>0,\ g(x)>0 \\ f(x)=g(x) \end{cases}$

6. $a\cdot\log_k^2 f(x)+b\cdot\log_k f(x)+c=0 \Rightarrow \begin{cases} \log_k f(x)=y \\ ay^2+by+c=0 \end{cases}$

TENGSIZLIKLAR

<u>**Ta'rif**</u>: ***Tengsizlik deb***, $>$ yoki $<$ ($\ge$ yoki $\le$) belgilari bilan birlashtirilgan ikkita sonli yoki algebraik ifodaga aytiladi.

Tengsizlikning yechimi deb, noma'lumning shu tengsizlikni to'g'ri sonli tengsizlikka aylantiradigan qiymatiga aytiladi.

Tengsizlikni yechish deb, uning barcha yechimlarini topish yoki ularning yo'qligini aniqlashga aytiladi.

Xossasi:

1. Tengsizlikning istalgan hadini uning bir qismidan ikkinchi qismiga ishorasini qarama-qarshisiga o'zgartirgan holda olib o'tish mumkin, bunda tengsizlik ishorasi o'zgarmaydi.

2. Tengsizlikning har ikki qismini nolga teng bo'lmagan ayni bir xil songa ko'paytirish yoki bo'lish mumkin: agar bu son musbat bo'lsa, tengsizlik ishorasi o'zgarmaydi, bordi-yu, bu son manfiy bo'lsa, u holda tengsizlik ishorasi qarama-qarshisiga o'zgaradi.

Sonli tengsizliklar

1. $a>b \Leftrightarrow a-b>0$ **2.** $a<b \Leftrightarrow a-b<0$

3. $a>b,\ b>c \Leftrightarrow a>c$ **4.** $a>b \Leftrightarrow a\pm c>b\pm c$

5. $a>b,\ c>d \Leftrightarrow a+c>b+d$ **6.** $a>b,\ c>d \Leftrightarrow a\cdot c>b\cdot d$

7. $a>b,\ c>0 \Leftrightarrow a\cdot c>b\cdot c,\ \frac{a}{c}>\frac{b}{c}$

8. $a>b,\ c<0 \Leftrightarrow a\cdot c<b\cdot c,\ \frac{a}{c}<\frac{b}{c}$

9. $a>b>0 \Leftrightarrow a^n>b^n\ (n\in\mathbb{N})$

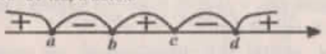
10. $a>b \Leftrightarrow \frac{1}{a}<\frac{1}{b}\ \begin{pmatrix} a\neq 0, \\ b\neq 0 \end{pmatrix}$

Sonli oraliqlar

1. Ochiq oraliq:	$(a;b)$	*yoki*	$a<x<b,\ x\in\mathbb{R}$
2. Yarim ochiq oraliq:	$[a;b)$	*yoki*	$a\le x<b,\ x\in\mathbb{R}$
3. Yopiq oraliq:	$[a;b]$	*yoki*	$a\le x\le b,\ x\in\mathbb{R}$
4. Yarim yopiq oraliq:	$(a;b]$	*yoki*	$a<x\le b,\ x\in\mathbb{R}$
5. Cheksiz oraliq:	$(-\infty;+\infty)$	*yoki*	$-\infty<x<+\infty,\ x\in\mathbb{R}$
6. Yarim cheksiz oraliq:	$(-\infty;b)$	*yoki*	$-\infty<x<b,\ x\in\mathbb{R}$
	$(a;+\infty)$	*yoki*	$a<x<+\infty,\ x\in\mathbb{R}$

Oraliqlar usuli

Agar $a<b<c<d$ bo'lsa, u holda:



1. $(x-a)(x-b)(x-c)(x-d)>0$ tengsizlikning yechimi $(-\infty;a)\cup(b;c)\cup(d;+\infty)$

2. $(x-a)(x-b)(x-c)(x-d)<0$ tengsizlikning yechimi $(a;b)\cup(c;d)$

3. $(x-a)(x-b)(x-c)(x-d)\ge 0$ tengsizlikning yechimi $(-\infty;a]\cup[b;c]\cup[d;+\infty)$

4. $(x-a)(x-b)(x-c)(x-d)\le 0$ tengsizlikning yechimi $[a;b]\cup[c;d]$

Chiziqli tengsizliklar

Chiziqli tengsizlik deb,

$$ax+b>0\ \ (\ge,<,\le)$$

ko'rinishidagi yoki shu ko'rinishga keltirish mumkin bo'lgan tengsizlikka ayt... Bunda a va b haqiqiy sonlar, x esa noma'lum son.

$a>0$ bo'lganda:

$ax+b>0 \quad x\in\left(-\frac{b}{a};+\infty\right)$ $\qquad ax+b<0 \quad x\in\left(-\infty;-\frac{b}{a}\right)$

$ax+b\ge 0 \quad x\in\left[-\frac{b}{a};+\infty\right)$ $\qquad ax+b\le 0 \quad x\in\left(-\infty;-\frac{b}{a}\right]$

Agar $a<0$ bo'lsa, tengsizlikning har ikki tomonini minus birga ko'paytirgan tengsizlik belgisi qarama-qarshisiga o'zgaradi.

Kvadrat tengsizliklar

Kvadrat tengsizlik deb,

$$ax^2+bx+c>0\ \ (\ge,<,\le)$$

ko'rinishidagi tengsizlikka aytiladi. Bunda a, b va c haqiqiy sonlar kvadrat tengsizlikning koeffisiyentlari $(a\neq 0)$, x esa noma'lum son.

$a>0$ bo'lganda:

	$ax^2+bx+c>0$	$ax^2+bx+c\ge 0$
$D>0$	$x\in(-\infty;x_1)\cup(x_2;+\infty)$	$x\in(-\infty;x_1]\cup[x_2;+\infty)$
$D=0$	$x\in(-\infty;x_0)\cup(x_0;+\infty)$	$x\in(-\infty;+\infty)$
$D<0$	$x\in(-\infty;+\infty)$	$x\in(-\infty;+\infty)$
	$ax^2+bx+c<0$	$ax^2+bx+c\le 0$
$D>0$	$x\in(x_1;x_2)$	$x\in[x_1;x_2]$
$D=0$	$x\in\varnothing$	$x=x_0$
$D<0$	$x\in\varnothing$	$x\in\varnothing$

Bu yerda $x_0=-\frac{b}{2a}$. Agar $a<0$ bo'lsa, tengsizlikning har ikki tomonini minus birga ko'paytirganda tengsizlik belgisi qarama-qarshisiga o'zgaradi.

Ratsional tengsizliklar

1. $\frac{f(x)}{g(x)}>0 \Leftrightarrow f(x)\cdot g(x)>0$ **2.** $\frac{f(x)}{g(x)}<0 \Leftrightarrow f(x)\cdot g(x)<0$

3. $\frac{f(x)}{g(x)}\ge 0 \Leftrightarrow \begin{cases} f(x)\cdot g(x)\ge 0 \\ g(x)\neq 0 \end{cases}$ **4.** $\frac{f(x)}{g(x)}\le 0 \Leftrightarrow \begin{cases} f(x)\cdot g(x)\le 0 \\ g(x)\neq 0 \end{cases}$

Irratsional tengsizliklar

1. $\sqrt[2k]{f(x)} > C \quad \Leftrightarrow \quad \begin{cases} C<0, & f(x)\ge 0 \\ C=0, & f(x)>0 \\ C>0, & f(x)>C^{2k} \end{cases} \quad (C=const)$

2. $\sqrt[2k]{f(x)} \ge C \quad \Leftrightarrow \quad \begin{cases} C<0, & f(x)\ge 0 \\ C=0, & f(x)\ge 0 \\ C>0, & f(x)\ge C^{2k} \end{cases} \quad (C=const)$

3. $\sqrt[2k]{f(x)} < C \quad \Leftrightarrow \quad \begin{cases} C<0, & \varnothing \\ C=0, & \varnothing \\ C>0, & 0\le f(x)<C^{2k} \end{cases} \quad (C=const)$

4. $\sqrt[2k]{f(x)} \le C \quad \Leftrightarrow \quad \begin{cases} C<0, & \varnothing \\ C=0, & f(x)=0 \\ C>0, & 0\le f(x)\le C^{2k} \end{cases} \quad (C=const)$

5. $\sqrt[2k]{f(x)} > g(x) \quad \Leftrightarrow \quad \begin{cases} g(x)\ge 0 \\ f(x)>g^{2k}(x) \end{cases}$ *yoki* $\begin{cases} g(x)<0 \\ f(x)\ge 0 \end{cases}$

6. $\sqrt[2k]{f(x)} \ge g(x) \quad \Leftrightarrow \quad \begin{cases} g(x)\ge 0 \\ f(x)\ge g^{2k}(x) \end{cases}$ *yoki* $\begin{cases} g(x)<0 \\ f(x)\ge 0 \end{cases}$

7. $\sqrt[2k]{f(x)} < g(x) \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ g(x)>0 \\ f(x)<g^{2k}(x) \end{cases}$

8. $\sqrt[2k]{f(x)} \le g(x) \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ g(x)\ge 0 \\ f(x)\le g^{2k}(x) \end{cases}$

9. $\sqrt[2k]{f(x)} > \sqrt[2k]{g(x)} \quad \Leftrightarrow \quad \begin{cases} g(x)\ge 0 \\ f(x)>g(x) \end{cases}$

10. $\sqrt[2k]{f(x)} \ge \sqrt[2k]{g(x)} \quad \Leftrightarrow \quad \begin{cases} g(x)\ge 0 \\ f(x)\ge g(x) \end{cases}$

11. $\sqrt[2k]{f(x)} < \sqrt[2k]{g(x)} \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ f(x)<g(x) \end{cases}$

12. $\sqrt[2k]{f(x)} \le \sqrt[2k]{g(x)} \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ f(x)\le g(x) \end{cases}$

13. $\sqrt[2k]{f(x)} + \sqrt[2k]{g(x)} > 0 \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ g(x)\ge 0 \end{cases} \setminus \begin{cases} f(x)=0 \\ g(x)=0 \end{cases}$

14. $\sqrt[2k]{f(x)} + \sqrt[2k]{g(x)} \ge 0 \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ g(x)\ge 0 \end{cases}$

15. $\sqrt[2k]{f(x)} + \sqrt[2k]{g(x)} < 0 \quad \Leftrightarrow \quad \varnothing$

16. $\sqrt[2k]{f(x)} + \sqrt[2k]{g(x)} \le 0 \quad \Leftrightarrow \quad \begin{cases} f(x)=0 \\ g(x)=0 \end{cases}$

17. $g(x)\cdot\sqrt[2k]{f(x)} > 0 \quad \Leftrightarrow \quad \begin{cases} f(x)>0 \\ g(x)>0 \end{cases}$

18. $g(x)\cdot\sqrt[2k]{f(x)} \ge 0 \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ g(x)\ge 0 \end{cases}$

19. $g(x)\cdot\sqrt[2k]{f(x)} < 0 \quad \Leftrightarrow \quad \begin{cases} f(x)>0 \\ g(x)<0 \end{cases}$

20. $g(x)\cdot\sqrt[2k]{f(x)} \le 0 \quad \Leftrightarrow \quad \begin{cases} f(x)\ge 0 \\ g(x)\le 0 \end{cases}$

Izoh: Toq darajali irratsional tengsizliklarni yechishda hech qanday shart qo'yi-la'lab etilmaydi, ya'ni qanday berilgan bo'lsa, shu holda ishlanadi.

Modul qatnashgan tengsizliklar

1. $|f(x)|>C \quad \Leftrightarrow \quad C\ge 0 \text{ da } \left[\begin{array}{l} f(x)>C \\ f(x)<-C \end{array}\right.$

   $C<0 \text{ da } (-\infty;\ +\infty) \quad (C=const)$

2. $|f(x)|\ge C \quad \Leftrightarrow \quad C\ge 0 \text{ da } \left[\begin{array}{l} f(x)\ge C \\ f(x)\le -C \end{array}\right.$

   $C<0 \text{ da } (-\infty;\ +\infty) \quad (C=const)$

3. $|f(x)|<C \quad \Leftrightarrow \quad C>0 \text{ da } -C<f(x)<C$

   $C\le 0 \text{ da } \varnothing \quad (C=const)$

4. $|f(x)|\le C \quad \Leftrightarrow \quad C\ge 0 \text{ da } -C\le f(x)\le C$

   $C<0 \text{ da } \varnothing \quad (C=const)$

5. $|f(x)|>g(x) \quad \Leftrightarrow \quad \left[\begin{array}{l} f(x)>g(x) \\ f(x)<-g(x) \end{array}\right.$

6. $|f(x)|\ge g(x) \quad \Leftrightarrow \quad \left[\begin{array}{l} f(x)\ge g(x) \\ f(x)\le -g(x) \end{array}\right.$

7. $|f(x)|<g(x) \quad \Leftrightarrow \quad g(x)>0 \text{ da } \begin{cases} f(x)>-g(x) \\ f(x)<g(x) \end{cases}$

8. $|f(x)| \le g(x) \quad \Leftrightarrow \quad g(x) \ge 0$ da $\begin{cases} f(x) \ge -g(x) \\ f(x) \le g(x) \end{cases}$

9. $|f(x)| > |g(x)|$, $|f(x)| \ge |g(x)|$, $|f(x)| < |g(x)|$, $|f(x)| \le |g(x)|$ kabi tengsizliklarni yechishda tengsizlikning har ikki tomonini kvadratga ko'tarib yoki oraliqlar usuli orqali ishlanadi.

Ko'rsatkichli tengsizliklar

1. $a^{f(x)} > a^{g(x)} \quad \Leftrightarrow \quad \left[\begin{array}{ll} a > 1, & f(x) > g(x) \\ 0 < a < 1, & f(x) < g(x) \end{array}\right.$

2. $a^{f(x)} > b \quad \Leftrightarrow \quad \left[\begin{array}{ll} b > 0,\ a > 1, & f(x) > \log_a b \\ b > 0,\ 0 < a < 1, & f(x) < \log_a b \end{array}\right.$

$b \le 0$ da $a^{f(x)} > b$ tengsizlik ***cheksiz ko'p yechimga ega***.

$b \le 0$ da $a^{f(x)} < b$ tengsizlik ***yechimga ega emas***.

Izoh: $>$, $\ge$, $\le$ belgilari bilan berilgan tengsizliklar ham yuqoridagi kabi ishlanadi.

Logarifmik tengsizliklar

1. $\log_a f(x) > b \quad \Leftrightarrow \quad \left[\begin{array}{ll} a > 1, & f(x) > a^b \\ 0 < a < 1, & 0 < f(x) < a^b \end{array}\right.$

2. $\log_a f(x) \ge b \quad \Leftrightarrow \quad \left[\begin{array}{ll} a > 1, & f(x) \ge a^b \\ 0 < a < 1, & 0 < f(x) \le a^b \end{array}\right.$

3. $\log_a f(x) > \log_a g(x) \quad \Leftrightarrow \quad \left[\begin{array}{ll} a > 1, & \begin{cases} f(x) > g(x) \\ g(x) > 0 \end{cases} \\ 0 < a < 1, & \begin{cases} f(x) < g(x) \\ f(x) > 0 \end{cases} \end{array}\right.$

4. $\log_a f(x) \ge \log_a g(x) \quad \Leftrightarrow \quad \left[\begin{array}{ll} a > 1, & \begin{cases} f(x) \ge g(x) \\ g(x) > 0 \end{cases} \\ 0 < a < 1, & \begin{cases} f(x) \le g(x) \\ f(x) > 0 \end{cases} \end{array}\right.$

5. $\log_{g(x)} f(x) > 0 \quad \Leftrightarrow \quad \begin{cases} f(x) > 0 \\ g(x) > 0,\ g(x) \ne 1 \\ (f(x)-1)(g(x)-1) > 0 \end{cases}$

6. $\log_{g(x)} f(x) \ge 0 \quad \Leftrightarrow \quad \begin{cases} f(x) > 0 \\ g(x) > 0,\ g(x) \ne 1 \\ (f(x)-1)(g(x)-1) \ge 0 \end{cases}$

7. $\log_{f(x)} g(x) > b \quad \Leftrightarrow \quad \begin{cases} 0 < f(x) < 1 \\ g(x) > 0 \\ g(x) < f^b(x) \end{cases} \cup \begin{cases} f(x) > 1 \\ g(x) > 0 \\ g(x) > f^b(x) \end{cases}$

8. $\log_{f(x)} g(x) \ge b \quad \Leftrightarrow \quad \begin{cases} 0 < f(x) < 1 \\ g(x) > 0 \\ g(x) \le f^b(x) \end{cases} \cup \begin{cases} f(x) > 1 \\ g(x) > 0 \\ g(x) \ge f^b(x) \end{cases}$

9. $\log_{\varphi(x)} f(x) > \log_{\varphi(x)} g(x) \quad \Leftrightarrow \quad \begin{cases} 0 < \varphi(x) < 1 \\ f(x) > 0 \\ f(x) < g(x) \end{cases} \cup \begin{cases} \varphi(x) > 1 \\ g(x) > 0 \\ f(x) > g(x) \end{cases}$

10. $\log_{\varphi(x)} f(x) \ge \log_{\varphi(x)} g(x) \quad \Leftrightarrow \quad \begin{cases} 0 < \varphi(x) < 1 \\ f(x) > 0 \\ f(x) \le g(x) \end{cases} \cup \begin{cases} \varphi(x) > 1 \\ g(x) > 0 \\ f(x) \ge g(x) \end{cases}$

Izoh: $<$, $\le$ belgilari bilan berilgan tengsizliklar ham yuqoridagi kabi ishlanadi.

Ba'zi sonli tengsizliklar

1. ***Koshi tengsizligi***: *($a_1, a_2, ..., a_n$ nomanfiy sonlar)*

$$\frac{a_1 + a_2 + ... + a_n}{n} \ge \sqrt[n]{a_1 \cdot a_2 \cdot ... \cdot a_n}$$

2. ***Bernulli tengsizligi***: $(1+a)^n \ge 1 + n \cdot a$ $(a > -1, n \ge 0)$.

3. Agar $a > 0$, $b > 0$ bo'lsa, $\frac{a}{b} + \frac{b}{a} \ge 2$ o'rinli.

4. Agar $a \ge 0$ bo'lsa, $\sqrt{a} + \sqrt{a+2} \le 2\sqrt{a+1}$ o'rinli.

5. Agar $\forall n \in N$ bo'lsa, $\sqrt{n^n} \le n! < \left(\frac{n+1}{2}\right)^n$ o'rinli.

TRIGONOMETRIYA

$\alpha^0 = \frac{180^0}{\pi} \cdot \alpha_{rad}$

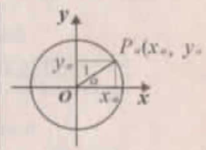
$1 rad \approx 57^0 17' 15''$

$\sin\alpha = y_\alpha, \quad \cos\alpha = x_\alpha$

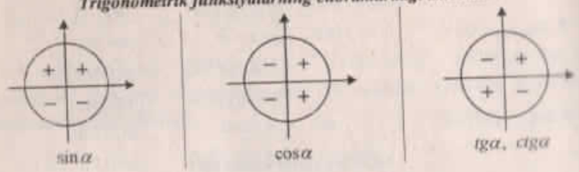
$\alpha_{rad} = \frac{\pi}{180^0} \cdot \alpha^0$

$\pi = 3,141592...$

$tg\alpha = \frac{y_\alpha}{x_\alpha}, \quad ctg\alpha = \frac{x_\alpha}{y_\alpha}$



Trigonometrik funksiyalarning choraklardagi ishoralari



Asosiy trigonometrik ayniyatlar

$\sin^2\alpha+\cos^2\alpha=1$ $\quad tg\alpha=\dfrac{\sin\alpha}{\cos\alpha}$ $\quad ctg\alpha=\dfrac{\cos\alpha}{\sin\alpha}$

$tg\alpha\cdot ctg\alpha=1$ $\quad 1+tg^2\alpha=\dfrac{1}{\cos^2\alpha}$ $\quad 1+ctg^2\alpha=\dfrac{1}{\sin^2\alpha}$

Qo'shish formulalari

$\sin(\alpha\pm\beta)=\sin\alpha\cdot\cos\beta\pm\cos\alpha\cdot\sin\beta$ $\quad tg(\alpha\pm\beta)=\dfrac{tg\alpha\pm tg\beta}{1\mp tg\alpha\cdot tg\beta}$

$\cos(\alpha\pm\beta)=\cos\alpha\cdot\cos\beta\mp\sin\alpha\cdot\sin\beta$ $\quad ctg(\alpha\pm\beta)=\dfrac{ctg\alpha\cdot ctg\beta\mp1}{ctg\alpha\pm ctg\beta}$

Ikkilangan burchak formulalari

$\sin2\alpha=2\sin\alpha\cdot\cos\alpha$ $\quad \cos2\alpha=\cos^2\alpha-\sin^2\alpha$

$tg2\alpha=\dfrac{2tg\alpha}{1-tg^2\alpha}$ $\quad ctg2\alpha=\dfrac{ctg^2\alpha-1}{2ctg\alpha}$

o'shimcha:

$1+\cos2\alpha=2\cos^2\alpha$ $\quad 1-\cos2\alpha=2\sin^2\alpha$

$1+\sin2\alpha=(\sin\alpha+\cos\alpha)^2$ $\quad 1-\sin2\alpha=(\sin\alpha-\cos\alpha)^2$

$\sin2\alpha=\dfrac{2}{ctg\alpha+tg\alpha}$ $\quad tg2\alpha=\dfrac{2}{ctg\alpha-tg\alpha}$

Uchlangan burchak formulalari

$\sin3\alpha=3\sin\alpha\cdot\cos^2\alpha-\sin^3\alpha=3\sin\alpha-4\sin^3\alpha$

$\cos3\alpha=\cos^3\alpha-3\cos\alpha\cdot\sin^2\alpha=4\cos^3\alpha-3\cos\alpha$

$$tg3\alpha=\frac{tg\alpha+tg2\alpha}{1-tg\alpha\cdot tg2\alpha}=\frac{3tg\alpha-tg^3\alpha}{1-3tg^2\alpha}$$

$$ctg3\alpha=\frac{ctg\alpha+ctg2\alpha}{ctg\alpha\cdot ctg2\alpha-1}=\frac{1-3ctg^2\alpha}{3ctg\alpha-ctg^3\alpha}$$

Yarim burchak formulalari

$\sin\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{2}}$ $\quad tg\dfrac{\alpha}{2}=\dfrac{1-\cos\alpha}{\sin\alpha}=\dfrac{\sin\alpha}{1+\cos\alpha}=\pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

$\cos\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1+\cos\alpha}{2}}$ $\quad ctg\dfrac{\alpha}{2}=\dfrac{1+\cos\alpha}{\sin\alpha}=\dfrac{\sin\alpha}{1-\cos\alpha}=\pm\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}$

Darajani pasaytirish formulalari

$\cos^2\alpha=\dfrac{1+\cos2\alpha}{2}$ $\quad \sin^2\alpha=\dfrac{1-\cos2\alpha}{2}$

$\cos^3\alpha=\dfrac{3\cos\alpha+\cos3\alpha}{4}$ $\quad \sin^3\alpha=\dfrac{3\sin\alpha-\sin3\alpha}{4}$

$\cos^4\alpha=\dfrac{3+4\cos2\alpha+\cos4\alpha}{8}$ $\quad \sin^4\alpha=\dfrac{3-4\cos2\alpha+\cos4\alpha}{8}$

$\sin\alpha$, $\cos\alpha$, $tg\alpha$ va $ctg\alpha$ larni $tg\frac{\alpha}{2}$ orqali ifodalash

$\sin\alpha=\dfrac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$ $\quad \cos\alpha=\dfrac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$ $\quad tg\alpha=\dfrac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}$ $\quad ctg\alpha=\dfrac{1-tg^2\frac{\alpha}{2}}{2tg\frac{\alpha}{2}}$

Yig'indini ko'paytmaga keltirish formulalari

$\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$ $\quad \sin\alpha-\sin\beta=2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$

$\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$ $\quad \cos\alpha-\cos\beta=-2\sin\dfrac{\alpha-\beta}{2}\sin\dfrac{\alpha+\beta}{2}$

$tg\alpha\pm tg\beta=\dfrac{\sin(\alpha\pm\beta)}{\cos\alpha\cdot\cos\beta}$ $\quad ctg\alpha\pm ctg\beta=\dfrac{\sin(\alpha\pm\beta)}{\sin\alpha\cdot\sin\beta}$

Qo'shimcha:

$$a\sin\alpha\pm b\cos\alpha=\sqrt{a^2+b^2}\cdot\sin(\alpha\pm\varphi)$$

bu yerda, $\varphi=arctg\dfrac{b}{a}$, $\sin\varphi=\dfrac{b}{\sqrt{a^2+b^2}}$, $\cos\varphi=\dfrac{a}{\sqrt{a^2+b^2}}$.

Ko'paytmani yig'indiga keltirish formulalari

$\sin\alpha\cdot\sin\beta=\dfrac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

$\cos\alpha\cdot\cos\beta=\dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$ $\quad tg\alpha\cdot ctg\beta=\dfrac{tg\alpha+ctg\beta}{ctg\alpha+tg\beta}$

$\sin\alpha\cdot\cos\beta=\dfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

Trigonometrik funksiyalarning birini ikkinchisi orqali ifodalash

$$\sin\alpha = \pm\sqrt{1-\cos^2\alpha} = \frac{tg\alpha}{\pm\sqrt{1+tg^2\alpha}} = \frac{1}{\pm\sqrt{1+ctg^2\alpha}}$$

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha} = \frac{1}{\pm\sqrt{1+tg^2\alpha}} = \frac{ctg\alpha}{\pm\sqrt{1+ctg^2\alpha}}$$

$$tg\alpha = \frac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}} = \frac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha} = \frac{1}{ctg\alpha}$$

$$ctg\alpha = \frac{\pm\sqrt{1-\sin^2\alpha}}{\sin\alpha} = \frac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}} = \frac{1}{tg\alpha}$$

Keltirish formulalari

$\sin\left(\frac{\pi}{2}\pm x\right)=\cos x$ $\qquad$ $\cos\left(\frac{\pi}{2}\pm x\right)=\mp\sin x$

$\sin(\pi\pm x)=\mp\sin x$ $\qquad$ $\cos(\pi\pm x)=-\cos x$

$\sin\left(\frac{3\pi}{2}\pm x\right)=-\cos x$ $\qquad$ $\cos\left(\frac{3\pi}{2}\pm x\right)=\pm\sin x$

$\sin(2\pi\pm x)=\pm\sin x$ $\qquad$ $\cos(2\pi\pm x)=\cos x$

$tg\left(\frac{\pi}{2}\pm x\right)=\mp ctgx$ $\qquad$ $ctg\left(\frac{\pi}{2}\pm x\right)=\mp tgx$

$tg(\pi\pm x)=\pm tgx$ $\qquad$ $ctg(\pi\pm x)=\pm ctgx$

$tg\left(\frac{3\pi}{2}\pm x\right)=\mp ctgx$ $\qquad$ $ctg\left(\frac{3\pi}{2}\pm x\right)=\mp tgx$

$tg(2\pi\pm x)=\pm tgx$ $\qquad$ $ctg(2\pi\pm x)=\pm ctgx$

Eslatma: Agar argument $(90^0\pm x)$ va $(270^0\pm x)$ bo'lsa, u holda trigonometrik nksiyalarning nomi o'zgarib (mos holda sinus kosinusga, tangens esa kotangensga va sincha), ishorasi choraklardan aniqlanadi. Agar argument $(180^0\pm x)$ va $(360^0\pm x)$ 'lsa, u holda trigonometrik funksiyalarning nomi o'zgarmaydi, ishorasi choraklardan uqlanadi.

Trigonometrik funksiyalarning ayrim burchaklardagi qiymatlari

Gradus	Radian	$\sin\alpha$	$\cos\alpha$	$tg\alpha$	$ctg\alpha$
0^0	0	0	1	0	–
30^0	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45^0	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60^0	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90^0	$\frac{\pi}{2}$	1	0	–	0
120^0	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
135^0	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	–1	–1
150^0	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
180^0	π	0	–1	0	–
270^0	$\frac{3\pi}{2}$	–1	0	–	0
360^0	2π	0	1	0	–

Gradus	Radian	$\sin\alpha$	$\cos\alpha$	$tg\alpha$	$ctg\alpha$
15^0	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18^0	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$
36^0	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
54^0	$\frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
72^0	$\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$
75^0	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$

Trigonometrik tenglamalar

. $\sin x = a$

$-1 < a < 0$	bo'lsa,	$x = (-1)^{n+1}\arcsin\lvert a\rvert + \pi n,$	$n \in \mathbb{Z}$
$0 < a < 1$	bo'lsa,	$x = (-1)^n \arcsin a + \pi n,$	$n \in \mathbb{Z}$

2. $\cos x = a$

$-1 < a < 0$	bo'lsa,	$x = \pm(\pi - \arccos\lvert a\rvert) + 2\pi n,$	$n \in \mathbb{Z}$
$0 < a < 1$	bo'lsa,	$x = \pm\arccos a + 2\pi n,$	$n \in \mathbb{Z}$

Izoh: $\lvert a\rvert > 1$ bo'lganda yuqoridagi tenglamalar yechimga ega emas.

Qo'shimcha:

$\sin^2 x = a$	$(0 \le a \le 1),$	$x = \pm\arcsin\sqrt{a} + \pi n,$	$n \in \mathbb{Z}$
$\cos^2 x = a$	$(0 \le a \le 1),$	$x = \pm\arccos\sqrt{a} + \pi n,$	$n \in \mathbb{Z}$

a	$\sin x = a$	$\cos x = a$
-1	$x = -\dfrac{\pi}{2} + 2\pi n$	$x = \pi + 2\pi n$
0	$x = \pi n$	$x = \dfrac{\pi}{2} + \pi n$
1	$x = \dfrac{\pi}{2} + 2\pi n$	$x = 2\pi n$

3. $tgx = a, \quad a \in \mathbb{R}$:

$a \ge 0$	bo'lsa,	$x = arctga + \pi n,$	$n \in \mathbb{Z}$
$a < 0$	bo'lsa,	$x = -arctg\lvert a\rvert + \pi n,$	$n \in \mathbb{Z}$

4. $ctgx = a, \quad a \in \mathbb{R}$:

$a \ge 0$	bo'lsa,	$x = arcctga + \pi n,$	$n \in \mathbb{Z}$
$a < 0$	bo'lsa,	$x = \pi - arcctg\lvert a\rvert + \pi n,$	$n \in \mathbb{Z}$

Qo'shimcha:

$tg^2 x = a$	$(0 \le a < +\infty),$	$x = \pm arctg\sqrt{a} + \pi n,$	$n \in \mathbb{Z}$
$ctg^2 x = a$	$(0 \le a < +\infty),$	$x = \pm arcctg\sqrt{a} + \pi n,$	$n \in \mathbb{Z}$

Ikki o'zgaruvchili trigonometrik tenglamalar

$$\sin x = \sin y \quad \Rightarrow \quad \left[\begin{array}{l} x + y = \pi + 2\pi n \\ x - y = 2\pi n \end{array}\right. \quad n \in \mathbb{Z}$$

$$\cos x = \cos y \quad \Rightarrow \quad \left[\begin{array}{l} x + y = 2\pi n \\ x - y = 2\pi n \end{array}\right. \quad n \in \mathbb{Z}$$

$$\sin x = \cos y \quad \Rightarrow \quad \left[\begin{array}{l} x + y = -\dfrac{\pi}{2} + 2\pi n \\ x - y = \dfrac{\pi}{2} + 2\pi n \end{array}\right. \quad n \in \mathbb{Z}$$

$$tgx = tgy \quad \Rightarrow \quad \left[\begin{array}{l} x, y \ne \dfrac{\pi}{2} + \pi n \\ x - y = \pi n \end{array}\right. \quad n \in \mathbb{Z}$$

$$ctgx = ctgy \quad \Rightarrow \quad \left[\begin{array}{l} x, y \ne \pi + 2\pi n \\ x - y = \pi n \end{array}\right. \quad n \in \mathbb{Z}$$

$$tgx = ctgy \quad \Rightarrow \quad \left[\begin{array}{l} x \ne \dfrac{\pi}{2} + \pi n, \ y \ne \pi n \\ x + y = \dfrac{\pi}{2} + \pi n \end{array}\right. \quad n \in \mathbb{Z}$$

Ba'zi trigonometrik ayniyatlar

$4\sin\alpha \cdot \sin(60^0 - \alpha) \cdot \sin(60^0 + \alpha) = \sin 3\alpha$	$16\sin 10^0 \cdot \sin 30^0 \cdot \sin 50^0 \cdot \sin 70^0 = 1$
$4\cos\alpha \cdot \cos(60^0 - \alpha) \cdot \cos(60^0 + \alpha) = \cos 3\alpha$	$16\cos 10^0 \cdot \cos 30^0 \cdot \cos 50^0 \cdot \cos 70^0 = 3$
$tg\alpha \cdot tg(60^0 - \alpha) \cdot tg(60^0 + \alpha) = tg3\alpha$	$16\sin 20^0 \cdot \sin 40^0 \cdot \sin 60^0 \cdot \sin 80^0 = 3$
$ctg\alpha \cdot ctg(60^0 - \alpha) \cdot ctg(60^0 + \alpha) = ctg3\alpha$	$16\cos 20^0 \cdot \cos 40^0 \cdot \cos 60^0 \cdot \cos 80^0 = 1$

$$\cos\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \ldots \cdot \cos 2^n\alpha = \frac{1}{2^{n+1}} \cdot \frac{\sin 2^{n+1}\alpha}{\sin\alpha}$$

$$\sin\alpha + \sin 2\alpha + \sin 3\alpha + \ldots + \sin n\alpha = \frac{\sin\dfrac{n\alpha}{2} \cdot \sin\dfrac{(n+1)\alpha}{2}}{\sin\dfrac{\alpha}{2}}$$

$$\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots + \cos n\alpha = \frac{\sin\dfrac{n\alpha}{2} \cdot \cos\dfrac{(n+1)\alpha}{2}}{\sin\dfrac{\alpha}{2}}$$

$\alpha + \beta + \gamma = 180^0$ bo'lsa:

$\dfrac{tg\alpha + tg\beta + tg\gamma}{tg\alpha \cdot tg\beta \cdot tg\gamma} = 1$	$\sin\dfrac{\alpha}{2} \cdot \sin\dfrac{\beta}{2} \cdot \sin\dfrac{\gamma}{2} \le \dfrac{1}{8}$
$\cos\alpha + \cos\beta + \cos\gamma \le \dfrac{3}{2}$	$\cos\dfrac{\alpha}{2} \cdot \cos\dfrac{\beta}{2} \cdot \cos\dfrac{\gamma}{2} \le \dfrac{3\sqrt{3}}{8}$
$\sin^2\alpha + \sin^2\beta + \sin^2\gamma \le \dfrac{9}{4}$	$\cos^2\alpha + \cos^2\beta + \cos^2\gamma \le 2$

$$\sin\frac{\alpha}{2} \cdot \sin\frac{\beta}{2} \cdot \sin\frac{\gamma}{2} = a \quad \Rightarrow \quad \cos\alpha + \cos\beta + \cos\gamma = 4a + 1$$

$$\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}=a \;\Rightarrow\; \sin\alpha+\sin\beta+\sin\gamma=4a$$

$$\cos\alpha\cdot\cos\beta\cdot\cos\gamma=a \;\Rightarrow\; \sin^2\alpha+\sin^2\beta+\sin^2\gamma=2(a+1)$$

$$\cos\alpha\cdot\cos\beta\cdot\cos\gamma=a \;\Rightarrow\; \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1-a$$

Trigonometrik tengsizliklar

1. $\sin x>a$
   - **a)** $-1\le a<1$ bo'lsa, $\arcsin a+2\pi n<x<\pi-\arcsin a+2\pi n,\ n\in\mathbb{Z}$
   - **b)** $a<-1$ bo'lsa, $x\in\mathbb{R}$
   - **c)** $a\ge 1$ bo'lsa, $x\in\varnothing$
2. $\sin x<a$
   - **a)** $-1<a\le 1$ bo'lsa, $-\pi-\arcsin a+2\pi n<x<\arcsin a+2\pi n,\ n\in\mathbb{Z}$
   - **b)** $a\le -1$ bo'lsa, $x\in\varnothing$
   - **c)** $a>1$ bo'lsa, $x\in\mathbb{R}$
3. $\cos x>a$
   - **a)** $-1\le a<1$ bo'lsa, $-\arccos a+2\pi n<x<\arccos a+2\pi n,\ n\in\mathbb{Z}$
   - **b)** $a<-1$ bo'lsa, $x\in\mathbb{R}$
   - **c)** $a\ge 1$ bo'lsa, $x\in\varnothing$
4. $\cos x<a$
   - **a)** $-1<a\le 1$ bo'lsa, $\arccos a+2\pi n<x<2\pi-\arccos a+2\pi n,\ n\in\mathbb{Z}$
   - **b)** $a\le -1$ bo'lsa, $x\in\varnothing$
   - **c)** $a>1$ bo'lsa, $x\in\mathbb{R}$
5. $tgx>a \;\Rightarrow\; arctga+\pi n<x<\frac{\pi}{2}+\pi n,\quad n\in\mathbb{Z}$
6. $tgx<a \;\Rightarrow\; -\frac{\pi}{2}+\pi n<x<arctga+\pi n,\quad n\in\mathbb{Z}$
7. $ctgx>a \;\Rightarrow\; \pi n<x<arcctga+\pi n,\quad n\in\mathbb{Z}$
8. $ctgx<a \;\Rightarrow\; arcctga+\pi n<x<\pi+\pi n,\quad n\in\mathbb{Z}$

Teskari trigonometrik funksiyalar orasidagi bog'lanishlar

$\arcsin x+\arccos x=\pi/2$ $\qquad arctgx+arcctgx=\pi/2$

$\arcsin(-x)=-\arcsin x$ $\qquad arctg(-x)=-arctgx$

$\arccos(-x)=\pi-\arccos x$ $\qquad arcctg(-x)=\pi-arcctgx$

To'g'ri va teskari trigonometrik funksiyalar orasidagi bog'lanishlar

$\sin(\arcsin x)=x,\quad x\in[-1;1]$ $\qquad \arcsin(\sin x)=x,\quad x\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$

$\cos(\arccos x)=x,\quad x\in[-1;1]$ $\qquad \arccos(\cos x)=x,\quad x\in[0;\pi]$

$tg(arctgx)=x,\quad x\in\mathbb{R}$ $\qquad arctg(tgx)=x,\quad x\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$

$ctg(arcctgx)=x,\quad x\in\mathbb{R}$ $\qquad arcctg(ctgx)=x,\quad x\in(0;\pi)$

Teskari trigonometrik funksiyalarning birini ikkinchisi orqali ifodalash

$$\arcsin x=\arccos\sqrt{1-x^2}=arctg\frac{x}{\sqrt{1-x^2}}=arcctg\frac{\sqrt{1-x^2}}{x}$$

$$\arccos x=\arcsin\sqrt{1-x^2}=arctg\frac{\sqrt{1-x^2}}{x}=arcctg\frac{x}{\sqrt{1-x^2}}$$

$$arctgx=\arcsin\frac{x}{\sqrt{1+x^2}}=\arccos\frac{1}{\sqrt{1+x^2}}=arcctg\frac{1}{x}$$

$$arcctgx=\arcsin\frac{1}{\sqrt{1+x^2}}=\arccos\frac{x}{\sqrt{1+x^2}}=arctg\frac{1}{x}$$

Teskari trigonometrik funksiyalarning yig'indisi

$$\arcsin x+\arcsin y=\arccos\left(\sqrt{1-x^2}\sqrt{1-y^2}-xy\right)$$

$$\arcsin x-\arcsin y=\arcsin\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$$

$$\arccos x+\arccos y=\arccos\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\arccos x-\arccos y=\arcsin\left(y\sqrt{1-x^2}-x\sqrt{1-y^2}\right)$$

$$arctgx\pm arctgy=arctg\frac{x\pm y}{1\mp xy},\quad xy\ne\pm1$$

$$arcctgx\pm arcctgy=arcctg\frac{xy\mp1}{x\pm y},\quad x\ne\pm y$$

FUNKSIYA VA UNING ASOSIY XOSSALARI

Ta'rif: X sonlar to'plamidan olingan x ning har bir qiymatiga biror qonuniyat yoki qoida yordamida Y sonlar to'plamidan olingan yagona y qiymat mos kelsa, bunday moslik ***funksiya*** deyiladi va $y=f(x)$ ko'rinishida belgilanadi.

Ta'rif: Funksiya argumentining (ya'ni, x ning) qabul qilishi mumkin bo'lgan barcha qiymatlari to'plami ***funksiyaning aniqlanish sohasi*** deyiladi va $D(f)$ ko'rinishida belgilanadi.

Ta'rif: Funksiyaning argumentlarga mos qiymatlari to'plami ***funksiyaning qiymatlar sohasi*** deyiladi va $E(f)$ ko'rinishida belgilanadi.

Ba'zi funksiyalarning aniqlanish sohasini (a.s.) topish

1. $y=\sqrt[2n]{f(x)}$ funksiyaning a.s. $f(x)\geq 0$ tengsizlikning yechimi bo'ladi.

2. $y=\dfrac{1}{f(x)}$ funksiyaning a.s. $f(x)\neq 0$ tenglikning yechimi bo'ladi.

3. $y=\log_{g(x)}f(x)$ funksiyaning a.s. $\begin{cases} f(x)>0 \\ g(x)>0 \\ g(x)\neq 1 \end{cases}$ sistemaning yechimi bo'ladi.

Ba'zi funksiyalarning qiymatlar sohasini (q.s.) topish

1. $y=\sqrt{ax^2+bx+c}$ funksiyaning q.s.: $y_0=c-\dfrac{b^2}{4a}$

a) agar $a>0$ bo'lsa, $E(y)=\left[\sqrt{y_0};+\infty\right)$

b) agar $a<0$ bo'lsa, $E(y)=\left[0;\sqrt{y_0}\right]$

2. $y=a\sin kx+b\cos kx$ funksiyaning q.s.: $E(y)=\left[-\sqrt{a^2+b^2};\ \sqrt{a^2+b^2}\right]$

3. $y=\dfrac{a}{x^2}+\dfrac{x^2}{b}$ $(a,b>0)$ funksiyaning q.s.: $E(y)=\left[2\sqrt{\dfrac{a}{b}};\ +\infty\right)$

Funksiyaning juft va toqligi

Ta'rif: Agar $\forall x\in D(f)$ uchun $-x\in D(f)$ va $f(-x)=f(x)$ tenglik o'rinli bo'lsa, $y=f(x)$ funksiya ***juft***, $f(-x)=-f(x)$ bo'lsa, $y=f(x)$ funksiya ***toq*** deyiladi, aks holda ***juft ham emas, toq ham emas***.

Masalan: $y=x^2$ – juft funksiya, $y=x^3$ – toq funksiya.

Agar $f(x)$, $g(x)$ – juft funksiyalar va $\varphi(x)$, $\phi(x)$ – toq funksiyalar bo'lsa, u holda

$f(x)\pm g(x)$ – juft	$\varphi(x)\pm\phi(x)$ – toq	$g(x)\pm\varphi(x)$ – j.t.e.
$f(x)\cdot g(x)$ – juft	$\varphi(x)\cdot\phi(x)$ – juft	$g(x)\cdot\varphi(x)$ – toq
$f(x):g(x)$ – juft	$\varphi(x):\phi(x)$ – juft	$g(x):\varphi(x)$ – toq

Eslatma: Juft funksiyaning grafigi Oy o'qiga nisbatan simmetrik, toq funksiyaning grafigi esa koordinatalar boshiga nisbatan simmetrik bo'ladi.

Funksiyaning davriyligi

Ta'rif: Agar $\forall x\in D(f)$ uchun $(x\pm T)\in D(f)$ $(T>0)$ va $f(x\pm T)=f(x)$ bo'lsa, $y=f(x)$ ***davriy funksiya***, T soni esa uning ***davri*** deyiladi.

Agar $T>0$ soni $y=f(x)$ funksiyaning davri bo'lsa, nT $(n\in\mathbb{Z})$ soni ham $y=f(x)$ funksiyaning davri bo'ladi.

Agar $y=f(x)$ funksiyaning eng kichik musbat (e.k.m.) davri T bo'lsa, u holda $y=f(kx+b)$ funksiyaning e.k.m. davri $\dfrac{T}{k}$ bo'ladi.

Eslatma: Bir nechta davriy funksiyalar yig'indisidan iborat funksiyaning e.k.m. davri qo'shiluvchi funksiyalar e.k.m. davrlarining EKUKiga teng.

Funksiyaning o'sishi va kamayishi (monotonligi)

Ta'rif: Agar $y=f(x)$ funksiya $(a;b)$ oraliqda aniqlangan va shu oraliqdan olingan ixtiyoriy x_1 va x_2 lar uchun $x_1<x_2$ bo'lganda:

1) Agar $f(x_1)<f(x_2)$ bo'lsa, $y=f(x)$ funksiya $(a;b)$ oraliqda ***qat'iy o'suvchi*** deyiladi.

2) Agar $f(x_1)>f(x_2)$ bo'lsa, $y=f(x)$ funksiya $(a;b)$ oraliqda ***qat'iy kamayuvchi*** deyiladi.

Teskari funksiyani topish

$y=f(x)$ funksiyaga teskari funksiyani topish uchun:

1) $y=f(x)$ tenglamadan $D(f)$ ni hisobga olgan holda x topiladi.

2) Hosil bo'lgan tenglikda x va y larning o'rni almashtiriladi.

Masalan: $y=\dfrac{2x+3}{x-1}$ funksiyaga teskari funksiyani toping.

Yechish: avvalo x ga nisbatan tenglama qilib yechamiz va x ni topamiz:

$$yx-y=2x+3\ \Rightarrow\ yx-2x=y+3\ \Rightarrow\ x(y-2)=y+3\ \Rightarrow\ x=\frac{y+3}{y-2}.$$

Oxirgi tenglikda x va y larning o'rnini almashtiramiz: $y=\dfrac{x+3}{x-2}$. Demak, topilgan funksiya berilgan funksiyaga teskari funksiya bo'ladi.

Funksiya grafigini parallel ko'chirish va cho'zish (siqish)

1) $y=f(x)$ funksiya grafigini Ox o'qidan a birlik, Oy o'qidan b birlik ***parallel ko'chirish*** orqali $y=f(x-a)+b$ funksiya grafigi hosil qilinadi.

2) $y=f(x)$ funksiya grafigini Ox o'qidan a marta (Oy o'qi bo'yicha), Oy o'qidan b marta (Ox o'qi bo'yicha) ***cho'zish (siqish)*** orqali $y=af\left(\dfrac{x}{b}\right)$ funksiya grafigi hosil qilinadi.

Izoh: $a>1$, $b>1$ bo'lganda funksiya cho'ziladi, $0<a<1$, $0<b<1$ bo'lganda esa funksiya siqiladi.

Funksiyaning quyidan va yuqoridan chegaralanganligi

Biror K haqiqiy son berilgan bo'lsin, u holda

1) ixtiyoriy $x \in R$ uchun $f(x) > K$ munosabat o'rinli bo'lsa, $y = f(x)$ funksiya ***quyidan*** chegaralangan;

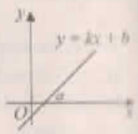
2) ixtiyoriy $x \in R$ uchun $f(x) < K$ munosabat o'rinli bo'lsa, $y = f(x)$ funksiya ***yuqoridan*** chegaralangan;

3) ixtiyoriy $x \in R$ uchun $|f(x)| < K$ munosabat o'rinli bo'lsa, $y = f(x)$ funksiya ***ham quyidan, ham yuqoridan*** chegaralangan;

4) ixtiyoriy $x \in R$ uchun $|f(x)| > K$ munosabat o'rinli bo'lsa, $y = f(x)$ funksiya chegaralanmagan deyiladi.

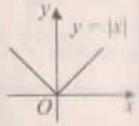
BA'ZI ELEMENTAR FUNKSIYALAR VA ULARNING ASOSIY XOSSALARI

$y = kx + b$ *chiziqli funksiya*



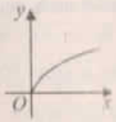
- Aniqlanish sohasi: $D(y) = \mathbb{R}$
- Qiymatlar sohasi: $E(y) = \mathbb{R}$
- $k > 0$ bo'lsa, son o'qida o'suvchi
- $k < 0$ bo'lsa, son o'qida kamayuvchi
- k – burchak koeffisiyenti
- α – to'g'ri chiziqning Ox o'qi bilan musbat yo'nalishda hosil qilgan burchak tangensi: $k = tg\alpha$

$y = |x|$ *funksiya*



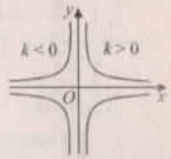
- Aniqlanish sohasi: $D(y) = \mathbb{R}$
- Qiymatlar sohasi: $E(y) = [0; +\infty)$
- $x \in [0; +\infty)$ oraliqda o'suvchi
- $x \in (-\infty; 0]$ oraliqda kamayuvchi
- Juft funksiya

$y = \sqrt{x}$ *funksiya*



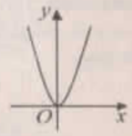
- Aniqlanish sohasi: $D(y) = [0; +\infty)$
- Qiymatlar sohasi: $E(y) = [0; +\infty)$
- $x \in [0; +\infty)$ oraliqda o'suvchi

$y = \frac{k}{x}$ *funksiya*



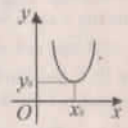
- Aniqlanish sohasi: $D(y) = (-\infty; 0) \cup (0; +\infty)$
- Qiymatlar sohasi: $E(y) = (-\infty; 0) \cup (0; +\infty)$
- $k > 0$ bo'lsa, $x \in (-\infty; 0) \cup (0; +\infty)$ da kamayuvchi
- $k < 0$ bo'lsa, $x \in (-\infty; 0) \cup (0; +\infty)$ da o'suvchi
- Toq funksiya
- Asimptotalari: $x = 0$, $y = 0$

$y = x^2$ *funksiya*



- Aniqlanish sohasi: $D(y) = \mathbb{R}$
- Qiymatlar sohasi: $E(y) = [0; +\infty)$
- $x \in [0; +\infty)$ oraliqda o'suvchi
- $x \in (-\infty; 0]$ oraliqda kamayuvchi
- Juft funksiya

$y = ax^2 + bx + c$ $(a \neq 0)$ *kvadrat funksiya*



$y = a(x - x_0)^2 + y_0$

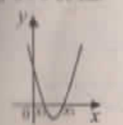
- Aniqlanish sohasi: $D(y) = \mathbb{R}$
- Qiymatlar sohasi: $a > 0$ bo'lsa, $E(y) = [y_0; +\infty)$
  $a < 0$ bo'lsa, $E(y) = (-\infty; y_0]$
- Parabola uchining koordinatalari:

$$x_0 = -\frac{b}{2a}, \qquad y_0 = ax_0^2 + bx_0 + c$$

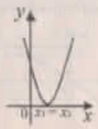
- Simmetriya o'qi tenglamasi: $x = x_0$
- $a > 0$ bo'lsa, $x \in [x_0; +\infty)$ oraliqda o'suvchi,
  $x \in (-\infty; x_0]$ oraliqda kamayuvchi
- $a < 0$ bo'lsa, $x \in (-\infty; x_0]$ oraliqda o'suvchi,
  $x \in [x_0; +\infty)$ oraliqda kamayuvchi

Kvadrat funksiya grafigini koordinatalar tekisligida joylashuvi

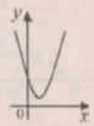
- Agar $a > 0$ bo'lsa:



$D > 0$

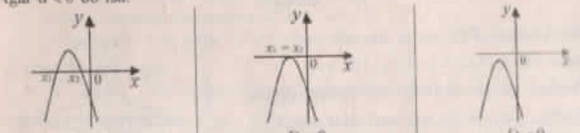


$D = 0$

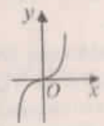


$D < 0$

Agar $a<0$ bo'lsa:

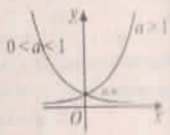


$y=x^3$ ***funksiya***



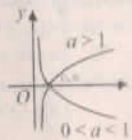
Aniqlanish sohasi: $D(y)=\mathbb{R}$
Qiymatlar sohasi: $E(y)=\mathbb{R}$
$x\in(-\infty;+\infty)$ oraliqda o'suvchi
Toq funksiya

$y=a^x$ $(a>0,\ a\neq1)$ ***ko'rsatkichli funksiya***



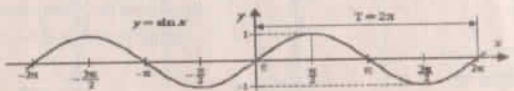
Aniqlanish sohasi: $D(y)=\mathbb{R}$
Qiymatlar sohasi: $E(y)=(0;+\infty)$
$a>1$ bo'lsa, $x\in(-\infty;+\infty)$ oraliqda o'suvchi
$0<a<1$ bo'lsa, $x\in(-\infty;+\infty)$ oraliqda kamayuvchi
Grafigi $(0;1)$ nuqtadan o'tadi
Asimptotasi: $y=0$

$y=\log_a x$ $(a>0,\ a\neq1)$ ***logarifmik funksiya***



Aniqlanish sohasi: $D(y)=(0;+\infty)$
Qiymatlar sohasi: $E(y)=\mathbb{R}$
$a>1$ bo'lsa, $x\in(0;+\infty)$ oraliqda o'suvchi
$0<a<1$ bo'lsa, $x\in(0;+\infty)$ oraliqda kamayuvchi
Grafigi $(1;0)$ nuqtadan o'tadi
Asimptotasi: $x=0$

$y=\sin x$ ***funksiya***



- Aniqlanish sohasi: $D(y)=\mathbb{R}$
- Qiymatlar sohasi: $E(y)=[-1;1]$
- Toq funksiya: $\sin(-x)=-\sin x$
- Ishora o'zgarmas oraliqlar:

$x\in(2\pi n;\ \pi+2\pi n)$ $(n\in\mathbb{Z})$ da, $\sin x>0$
$x\in(-\pi+2\pi n;\ 2\pi n)$ $(n\in\mathbb{Z})$ da, $\sin x<0$

- Eng katta qiymati 1
- Eng kichik qiymati -1
- Eng kichik musbat davri: $T=2\pi$
- Nollari: $x_0=\pi n,\ n\in\mathbb{Z}$

- $x\in\left[-\frac{\pi}{2}+2\pi n;\ \frac{\pi}{2}+2\pi n\right]$ $(n\in\mathbb{Z})$ – oraliqlarda o'suvchi
- $x\in\left[\frac{\pi}{2}+2\pi n;\ \frac{3\pi}{2}+2\pi n\right]$ $(n\in\mathbb{Z})$ – oraliqlarda kamayuvchi
- $x\in[2\pi n;\ \pi+2\pi n]$ $(n\in\mathbb{Z})$ – oraliqlarda qavariq
- $x\in[\pi+2\pi n;\ 2\pi+2\pi n]$ $(n\in\mathbb{Z})$ – oraliqlarda botiq

$y=\cos x$ ***funksiya***

- Aniqlanish sohasi: $D(y)=\mathbb{R}$
- Qiymatlar sohasi: $E(y)=[-1;1]$
- Juft funksiya: $\cos(-x)=\cos x$
- Ishora o'zgarmas oraliqlar:

- Eng katta qiymati 1
- Eng kichik qiymati -1
- Eng kichik musbat davri: $T=2\pi$
- Nollari: $x_0=\frac{\pi}{2}+\pi n,\ n\in\mathbb{Z}$

$x\in\left(-\frac{\pi}{2}+2\pi n;\ \frac{\pi}{2}+2\pi n\right)$ $(n\in\mathbb{Z})$ da, $\cos x>0$
$x\in\left(\frac{\pi}{2}+2\pi n;\ \frac{3\pi}{2}+2\pi n\right)$ $(n\in\mathbb{Z})$ da, $\cos x<0$

- $x\in[-\pi+2\pi n;\ 2\pi n]$ $(n\in\mathbb{Z})$ – oraliqlarda o'suvchi
- $x\in[2\pi n;\ \pi+2\pi n]$ $(n\in\mathbb{Z})$ – oraliqlarda kamayuvchi
- $x\in\left[-\frac{\pi}{2}+2\pi n;\ \frac{\pi}{2}+2\pi n\right]$ $(n\in\mathbb{Z})$ – oraliqlarda qavariq
- $x\in\left[\frac{\pi}{2}+2\pi n;\ \frac{3\pi}{2}+2\pi n\right]$ $(n\in\mathbb{Z})$ – oraliqlarda botiq

$y = tgx$ *funksiya*

Aniqlanish sohasi: $x \neq \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$

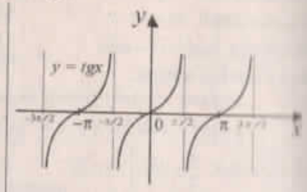
Qiymatlar sohasi: $E(y) = \mathbb{R}$

Toq funksiya: $tg(-x) = -tgx$

Eng kichik musbat davri: $T = \pi$

Nollari: $x_0 = \pi n,\ n \in \mathbb{Z}$

Asimptotalari: $x = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$



Ishora o'zgarmas oraliqlar:

$$x \in \left(\pi n;\ \frac{\pi}{2} + \pi n\right)\ (n \in \mathbb{Z}) \text{ bo'lsa, } tgx > 0$$

$$x \in \left(-\frac{\pi}{2} + \pi n,\ \pi n\right)\ (n \in \mathbb{Z}) \text{ bo'lsa, } tgx < 0$$

$x \in \left(-\frac{\pi}{2} + \pi n;\ \frac{\pi}{2} + \pi n\right)\ (n \in \mathbb{Z})$ oraliqlarda o'suvchi

$y = ctgx$ *funksiya*

Aniqlanish sohasi: $x \neq \pi n,\ n \in \mathbb{Z}$

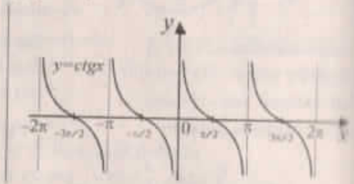
Qiymatlar sohasi: $E(y) = \mathbb{R}$

Toq funksiya: $ctg(-x) = -ctgx$

Eng kichik musbat davri: $T = \pi$

Nollari: $x_0 = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$

Asimptotalari: $x = \pi n,\ n \in \mathbb{Z}$



Ishora o'zgarmas oraliqlar: $x \in \left(\pi n;\ \frac{\pi}{2} + \pi n\right)\ (n \in \mathbb{Z})$ bo'lsa, $ctgx > 0$

$$x \in \left(-\frac{\pi}{2} + \pi n;\ \pi n\right)\ (n \in \mathbb{Z}) \text{ bo'lsa, } ctgx < 0$$

$x \in [\pi n;\ \pi + \pi n]$, $(n \in \mathbb{Z})$ oraliqlarda kamayuvchi

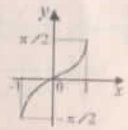
$y = \arcsin x$ *funksiya*

Aniqlanish sohasi: $D(y) = [-1; 1]$

Qiymatlar sohasi: $E(y) = \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

Toq funksiya: $\arcsin(-x) = -\arcsin x$

Aniqlanish sohasida o'suvchi



$y = \arccos x$ *funksiya*

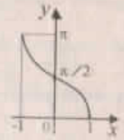
Aniqlanish sohasi: $D(y) = [-1; 1]$

Qiymatlar sohasi: $E(y) = [0; \pi]$

Toq ham, juft ham emas:

$$\arccos(-x) = \pi - \arccos x$$

Aniqlanish sohasida kamayuvchi



$y = arctgx$ *funksiya*

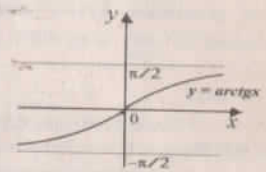
Aniqlanish sohasi: $D(y) = \mathbb{R}$

Qiymatlar sohasi: $E(y) = \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

Toq funksiya: $arctg(-x) = -arctgx$

Aniqlanish sohasida o'suvchi

Asimtotalari: $y = \pm\frac{\pi}{2}$



$y = arcctgx$ *funksiya*

Aniqlanish sohasi: $D(y) = \mathbb{R}$

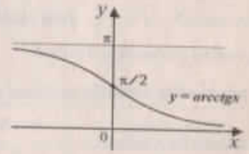
Qiymatlar sohasi: $E(y) = (0; \pi)$

Toq ham emas, juft ham emas:

$$arcctg(-x) = \pi - arcctgx$$

Aniqlanish sohasida kamayuvchi

Asimptotalari: $y = 0,\ y = \pi$



HOSILA

Ta'rif: $y = f(x)$ funksiyaning $x = x_0$ nuqtadagi hosilasi

$$y' = f'(x_0) = \lim_{\Delta x \to x_0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to x_0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Hosilaning geometrik ma'nosi: $y = f(x)$ funksiya grafigiga $x = x_0$ nuqtad o'tkazilgan urinmaning burchak koeffisiyenti $k = tg\alpha = f'(x_0)$.

Hosilaning fizik ma'nosi: Harakatlanayotgan jismning t_0 vaqtda bosib o'tga yo'li $S = f(t)$, uning t_0 vaqtdagi tezligi $V(t_0)$, tezlanishi esa $a(t_0)$ bo'lsa, u holda

$$V(t_0) = f'(t_0), \quad a(t_0) = V'(t_0) = f''(t_0)$$

Urinma tenglamasi: $y = f(x_0) + f'(x_0)(x - x_0)$

Normal tenglamasi: $y = f(x_0) - \frac{x - x_0}{f'(x_0)}$

Ba'zi elementar funksiyalarning hosilalari

funksiya	*hosilasi*	*funksiya*	*hosilasi*
$y=C,\ y=x$	$y'=0,\ y'=1$	$y=\sin x$	$y'=\cos x$
$y=x^n$	$y'=n\cdot x^{n-1}$	$y=\cos x$	$y'=-\sin x$
$y=\sqrt{x}$	$y'=\frac{1}{2\sqrt{x}}$	$y=tgx$	$y'=\frac{1}{\cos^2 x}$
$y=a^x$	$y'=a^x\cdot\ln a$	$y=ctgx$	$y'=-\frac{1}{\sin^2 x}$
$y=e^x$	$y'=e^x$	$y=\arcsin x$	$y'=\frac{1}{\sqrt{1-x^2}}$
$y=\log_a x$	$y'=\frac{1}{x\cdot\ln a}$	$y=\arccos x$	$y'=-\frac{1}{\sqrt{1-x^2}}$
$y=\ln x$	$y'=\frac{1}{x}$	$y=arctgx$	$y'=\frac{1}{1+x^2}$
		$y=arcctgx$	$y'=-\frac{1}{1+x^2}$

Hosila olishning asosiy qoidalari

'g'indi va ayirmaning hosilasi: $[f(x)\pm g(x)]'=f'(x)\pm g'(x)$

'paytmaning hosilasi: $[f(x)\cdot g(x)]'=f'(x)\cdot g(x)+f(x)\cdot g'(x)$

o'linmaning hosilasi: $\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)},\ g(x)\neq 0$

'urakkab funksiyaning hosilasi: $[f(g(x))]'=f'(g(x))\cdot g'(x)$

Funksiyani hosila yordamida tekshirish

$y=f(x)$ funksiya (a,b) oraliqda aniqlangan bo'lsin.

1. Agar (a,b) oraliqda $y'>0$ bo'lsa, u holda funksiya shu oraliqda *qat'iy 'suvchi*.

2. Agar (a,b) oraliqda $y'<0$ bo'lsa, u holda funksiya shu oraliqda *qat'iy ımayuvchi*.

3. Agar (a,b) oraliqda $y''>0$ bo'lsa, u holda funksiya shu oraliqda *botiq*.

4. Agar (a,b) oraliqda $y''<0$ bo'lsa, u holda funksiya shu oraliqda *qavariq*.

Izoh: Agar funksiya hosilasining noldagi qiymatlari aniqlanish sohasiga tegishli ɔ'lsa, u holda funksiya shu oraliqda ***noqat'iy o'suvchi (noqat'iy kamayuvchi)*** bo'ladi.

Funksiyaning eng katta va eng kichik qiymatlari

Ta'rif: Funksiyaning hosilasi nolga teng bo'ladigan nuqtalar ***stasionar nuqtal*** deyiladi.

Ta'rif: Funksiyaning hosilasi nolga teng yoki mavjud bo'lmagan nuqtalar ***kritik nuqtalari*** deyiladi.

$y=f(x)$ funksiyaning $[a;b]$ kesmadagi ***eng katta*** va ***eng kichik qiymatlari*** topish uchun:

1) funksiyaning kritik nuqtalardagi qiymatlari topiladi;

2) funksiyani $[a;b]$ kesma chegarasidagi qiymatlari topiladi, yani $f(a)$ va $f(b$

3) topilgan barcha qiymatlar taqqoslanib, $y=f(x)$ funksiyaning $[a;b$ kesmadagi eng katta, ya'ni $\max\limits_{x\in[a;b]} f(x)$ va eng kichik, ya'ni $\min\limits_{x\in[a;b]} f(x)$ qiymatla aniqlanadi.

BOSHLANG'ICH FUNKSIYA (ANIQMAS INTEGRAL)

Ta'rif: Agar berilgan oraliqdagi barcha x uchun $F'(x)=f(x)$ tenglik bajarilsa u holda $F(x)$ funksiya shu oraliqda $f(x)$ funksiyaning ***boshlang'ich funksiyas*** deyiladi.

Agar $F(x)$ funksiya $y=f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, ha qanday o'zgarmas C uchun $F(x)+C$ funksiya ham $y=f(x)$ funksiyanin boshlang'ich funksiyasi bo'ladi.

Ba'zi elementar funksiyalarning boshlang'ichlari

funksiya	*boshlang'ichi*	*funksiya*	*boshlang'ichi*
$y=x^m$	$Y=\frac{x^{m+1}}{m+1}+C,\ m\neq -1$	$y=\frac{1}{\sin x}$	$Y=\ln\left\lvert tg\frac{x}{2}\right\rvert+C$
$y=a^x$	$Y=\frac{a^x}{\ln a}+C$	$y=\frac{1}{\cos x}$	$Y=-\ln\left\lvert tg\frac{x}{2}\right\rvert+C$
$y=e^x$	$Y=e^x+C$	$y=\frac{1}{\sin^2 x}$	$Y=-ctgx+C$
$y=\frac{1}{x}$	$Y=\ln\lvert x\rvert+C$	$y=\frac{1}{\cos^2 x}$	$Y=tgx+C$
$y=\sin kx$	$Y=-\frac{1}{k}\cdot\cos kx+C$	$y=\frac{1}{a^2+x^2}$	$Y=\frac{1}{a}\cdot arctg\frac{x}{a}+C$
$y=\cos kx$	$Y=\frac{1}{k}\cdot\sin kx+C$	$y=\frac{1}{x^2-a^2}$	$Y=\frac{1}{2a}\cdot\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$

$y=tgkx$	$Y=-\frac{1}{k}\cdot\ln\lvert\cos kx\rvert+C$	$y=\frac{1}{\sqrt{a^2-x^2}}$	$Y=\arcsin\frac{x}{a}+C$
$y=ctgkx$	$Y=\frac{1}{k}\cdot\ln\lvert\sin kx\rvert+C$	$y=\frac{1}{\sqrt{x^2\pm a^2}}$	$Y=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$

Nyuton-Leybnits formulasi

$[a,b]$ kesmada $F(x)$ funksiya berilgan $f(x)$ uzluksiz funksiyaning boshlang'ich funksiyasi bo'lsa, bu funksiyaning $[a,b]$ kesmadagi ***aniq integral qiymati***:

$$\int_a^b f(x)dx=F(x)\Big|_a^b=F(b)-F(a)$$

formula bilan topiladi.

xossalari:

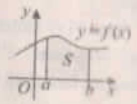
$\int_a^b kf(x)dx=k\int_a^b f(x)dx$ $\qquad$ $\int_a^b f(x)dx=-\int_b^a f(x)dx$

$\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$ $\qquad$ $\int_a^b[f(x)\pm g(x)]dx=\int_a^b f(x)dx\pm\int_a^b g(x)dx$

Egri chiziqli trapetsiyaning yuzi

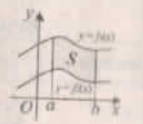
1. $y=f(x)$ egri chiziq va $y=0$, $x=a$, $x=b$ to'g'ri chiziqlar bilan chegaralangan ***soha yuzi***:

$$S=\int_a^b f(x)dx$$



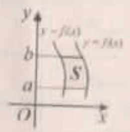
2. $y=f_1(x)$ va $y=f_2(x)$ $(f_1(x)>f_2(x))$ egri chiziqlar hamda $x=a$, $x=b$ to'g'ri chiziqlar bilan chegaralangan ***soha yuzi***:

$$S=\int_a^b[f_1(x)-f_2(x)]dx$$



3. $x=f_1(y)$ va $x=f_2(y)$ $(f_2(y)>f_1(y))$ egri chiziqlar hamda $y=a$, $y=b$ to'g'ri chiziqlar bilan chegaralangan ***soha yuzi***:

$$S=\int_a^b[f_2(y)-f_1(y)]dy$$



Aylanma jismning hajmi

1. Egri chiziqli trapetsiyani Ox o'qi atrofida aylanishidan hosil bo'lgan ***jism hajmi***: $\qquad V=\pi\int_a^b f^2(x)dx$

2. Egri chiziqli trapetsiyani Oy o'qi atrofida aylanishidan hosil bo'lgan ***jism hajmi***: $\qquad V=2\pi\int_a^b xf(x)dx$

PLANIMETRIYA

Planimetriya – geometriyaning bir bo'limi bo'lib, unda tekislikdagi geometrik shakllarning xossalari o'rganiladi.

Tekislikdagi asosiy geometrik shakllar ***nuqta*** va ***to'g'ri chiziq*** hisoblanadi.

Nuqtalar lotin alifbosining katta harflari A, B, C,... bilan, to'g'ri chiziqlar esa kichik harflari a, b, c,... bilan belgilanadi.

* Har qanday geometrik shakl nuqtalardan tashkil topgan bo'ladi.
* Har qanday geometrik shaklning qismi ham geometrik shakl bo'ladi.
* Bir nechta geometrik shakllarning birlashmasi ham geometrik shakl bo'ladi.
* Geometrik shakllarning xossalari to'g'riligini ifodalovchi mulohazalarga ***jumlalar*** deyiladi.
* Geometrik shakllarning xossalari to'g'riligini isbotlab beruvchi tasdiq ***teorema*** deyiladi.
* Geometrik shakllarning xossalari isbotsiz qabul qilinadigan tasdiq ***aksioma*** deyiladi.

__Teorema__: Ikkita turli to'g'ri chiziq faqat bitta nuqtada ***kesishadi*** yoki umuman ***kesishmaydi***.

__Ta'rif__: Bitta umumiy nuqtaga ega bo'lgan to'g'ri chiziqlar ***kesishuvchi to'g'ri chiziqlar*** deyiladi.

__Ta'rif__: To'g'ri chiziqdagi ikkita nuqta orasida yotuvchi barcha nuqtalar to'plamiga ***kesma*** deyiladi.

1-aksioma: To'g'ri chiziqni har qanday olmaylik, shu to'g'ri chiziqqa ***tegishli nuqtalar*** ham, unga ***tegishli bo'lmagan nuqtalar*** ham mavjud.

2-aksioma: Har qanday ikkita turli nuqta orqali ***faqat va faqat bitta*** to'g'ri chiziq o'tkazish mumkin.

3-aksioma: To'g'ri chiziqdagi uchta nuqtadan ***bittasi va faqat bittasi*** qolgan ikkitasining orasida yotadi.

4-aksioma: To'g'ri chiziq tekislikni ***ikkita yarimtekislikka*** ajratadi.

5-aksioma: Har bir kesma noldan katta ***tayin uzunlikka*** ega.

6-aksioma: Istalgan yarimto'g'ri chiziqda uning boshlang'ich nuqtasidan berilgan uzunlikda ***yagona kesma*** qo'yish mumkin.

Tekislikda Dekart koordinatalar sistemasi

__Ta'rif__: Tekislikda o'zaro perpendikulyar o'qlardan iborat masshtabli figuraga ***to'g'ri burchakli (Dekart)*** koordinatalar sistemasi deyiladi.

Vertikal joylashgan o'q ***ordinatalar o'qi*** (Oy), gorizontal joylashgan o'q ***absissalar o'qi*** (Ox) va ular kesishgan nuqta esa ***koordinatalar boshi*** $(O(0;0))$ deyiladi.

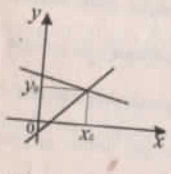
Dekart koordinatalar sistemasida $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar berilgan bo'lsin:

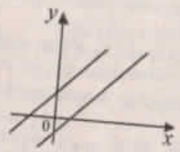
1. *AB* ***kesma uzunligi:***	$\lvert AB\rvert=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
2. *AB* ***kesma o'rtasining koordinatasi*** $C(x;y)$:	$x=\frac{x_1+x_2}{2},\quad y=\frac{y_1+y_2}{2}$
3. *AB* ***kesmani*** $\frac{\lambda}{\mu}$ ***nisbatda bo'lish nuqtasining koordinatasi*** $C(x;y)$:	$x=\frac{\mu x_1+\lambda x_2}{\mu+\lambda},\quad y=\frac{\mu y_1+\lambda y_2}{\mu+\lambda}$

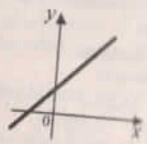
Izoh: *A* va *B* nuqtalar fazoda Dekart koordinatalar sistemasida berilganda ham huddi tekislikdagi kabi xossalarga ega.

Tekislikda to'g'ri chiziq tenglamalari

Tekislikda to'g'ri chiziq tenglamalarining berilish usullari va xossalari







I. To'g'ri chiziqning ***umumiy tenglamasi:*** $\qquad ax+by+c=0$

1. $ax+by+c=0$ to'g'ri chiziq:

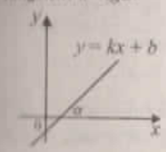
- $a=0$ da Ox o'qiga ***parallel***
- $b=0$ da Oy o'qiga ***parallel***
- $c=0$ da ***koordinata boshidan o'tadi***
- $a=c=0$ da Ox o'qi bilan ***ustma-ust tushadi***
- $b=c=0$ da Oy o'qi bilan ***ustma-ust tushadi***

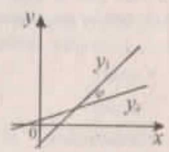
2. Ikkita $a_1x+b_1y+c_1=0$ va $a_2x+b_2y+c_2=0$ to'g'ri chiziqlar Dekart koordinatalar sistemasida quyidagicha joylashadi:

- $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ bo'lsa, ***kesishadi***
- $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ bo'lsa, ***parallel (kesishmaydi)***
- $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ bo'lsa, ***ustma-ust tushadi***

3. $M(x_0;y_0)$ nuqtadan o'tib, $ax+by+c=0$ to'g'ri chiziqqa ***perpendikulyar to'g'ri chiziq tenglamasi*** ($\vec{n}(a;b)$ vektorga ***parallel***):	$\frac{x-x_0}{a}=\frac{y-y_0}{b}$
4. $M(x_0;y_0)$ nuqtadan o'tib, $ax+by+c=0$ to'g'ri chiziqqa ***parallel to'g'ri chiziq tenglamasi*** ($\vec{n}(a;b)$ vektorga ***perpendikulyar***):	$\frac{x-x_0}{b}=-\frac{y-y_0}{a}$
II. Ikki $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi ***to'g'ri chiziq tenglamasi:***	$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$
III. To'g'ri chiziqning ***kanonik*** (Ox va Oy o'qlardan ajratgan kesmalari bo'yicha) ***tenglamasi:***	$\frac{x}{a}+\frac{y}{b}=1$
IV. To'g'ri chiziqning ***burchak koeffisiyentli tenglamasi:***	$y=kx+b$

Bu yerda k – ***burchak koeffisiyent***, α – Ox o'qi bilan musbat yo'nalishda hosil qilgan ***burchak tangensi:*** $k=tg\alpha$.





1. $y_1=k_1x+b_1$ va $y_2=k_2x+b_2$ to'g'ri chiziqlar orasidagi burchak tangensi: $\qquad tg\varphi=\frac{k_1-k_2}{1+k_1\cdot k_2}$

2. $y_1=k_1x+b_1$ va $y_2=k_2x+b_2$ to'g'ri chiziqlarning koordinatalar tekisligida o'zaro joylashuvi:

a) $k_1\neq k_2$ bo'lsa, ***kesishuvchi*** bo'ladi:

- $b_1=b_2>0$ da Oy o'qining ***musbat*** qismida
- $b_1=b_2<0$ da Oy o'qining ***manfiy*** qismida
- $b_1=b_2=0$ da koordinata ***boshida***
- $b_1\neq b_2$ da to'rt chorakdan birida
- $k_1\cdot b_2=b_1\cdot k_2>0$ da Ox o'qining ***manfiy*** qismida
- $k_1\cdot b_2=b_1\cdot k_2<0$ da Ox o'qining ***musbat*** qismida

b) $k_1=k_2$, $b_1\neq b_2$ bo'lsa, ***parallel (kesishmaydi)*** bo'ladi.

c) $k_1=k_2$, $b_1=b_2$ bo'lsa, ***ustma-ust*** tushadi.

d) $k_1\cdot k_2=-1$ bo'lsa, ***perpendikulyar*** bo'ladi.

Qo'shimcha:

1. $A(x_0; y_0)$ nuqtadan $ax+by+c=0$ to'g'ri chiziqqacha bo'lgan ***masofa:***

$$d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$

2. $ax+by+c_1=0$ va $ax+by+c_2=0$ parallel to'g'ri chiziqlar orasidagi ***masofa:*** .

$$h=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$$

3. $ax+by+c=0$ to'g'ri chiziqqa ***perpendikulyar vektorlardan biri:***

$$\vec{n}(a; b)$$

4. $ax+by+c=0$ to'g'ri chiziqqa ***parallel vektorlardan biri:***

$$\vec{m}(b; -a) \text{ yoki } \vec{m}(-b; a)$$

5. Uchlari $A(x_1; y_1)$, $B(x_2; y_2)$ va $C(x_3; y_3)$ nuqtalarda bo'lgan ABC ***uchburchakning yuzi:***

$$S_{ABC}=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\frac{1}{2}|(x_2-x_1)(y_3-y_1)-(x_3-x_1)(y_2-y_1)|$$

6. Markazi $O(a; b)$ nuqtada, radiusi R ga teng bo'lgan ***aylana tenglamasi:***

$$(x-a)^2+(y-b)^2=R^2$$

7. $M(x_0; y_0)$ nuqta koordinatalar tekisligida aylananing:

- *ichida:* $(x_0-a)^2+(y_0-b)^2<R^2$
- *ustida:* $(x_0-a)^2+(y_0-b)^2=R^2$
- *tashqarisida:* $(x_0-a)^2+(y_0-b)^2>R^2$ *yotadi.*

8. Markazi $O(a; b)$ nuqtada, radiuslari R_1 va R_2 ga teng bo'lgan ikkita aylanalar ***konsentrik aylanalar*** deyiladi:

$$(x-a)^2+(y-b)^2=R_1^2$$

va

$$(x-a)^2+(y-b)^2=R_2^2$$

9. Markazi $O(a; b; c)$ nuqtada, radiusi R ga teng bo'lgan ***sfera tenglamasi:***

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2$$

Tekislik tenglamalari

1. Tekislikning ***umumiy tenglamasi:***

$$ax+by+cz+d=0$$

2. Tekislikning ***kanonik*** (Ox, Oy va Oz o'qlardan ajratgan kesmalari bo'yicha) ***tenglamasi:***

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

3. $M(x_0; y_0; z_0)$ nuqtadan o'tib, $\vec{n}(a; b; c)$ vektorga ***perpendikulyar tekislik tenglamasi:***

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

4. $a_1x+b_1y+c_1z+d_1=0$ va $a_2x+b_2y+c_2z+d_2=0$ tekisliklarning koordinatalar tekisligida o'zaro joylashuvi:

- $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\neq\frac{d_1}{d_2}$ bo'lsa, ***parallel*** bo'ladi.
- $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$ bo'lsa, ***ustma-ust*** tushadi.
- $a_1\cdot a_2+b_1\cdot b_2+c_1\cdot c_2=0$ bo'lsa, ***perpendikulyar*** bo'ladi.

Qo'shimcha:

1. $M(x_0; y_0; z_0)$ nuqtadan $ax+by+cz+d=0$ tekislikkacha bo'lgan ***masofa:***

$$h=\left|\frac{ax_0+by_0+cz_0+d}{\sqrt{a^2+b^2+c^2}}\right|$$

2. $ax+by+cz+d=0$ tekislikka ***normal vektor:***

$$\vec{n}(a; b; c)$$

3. $a_1x+b_1y+c_1z+d_1=0$ va $a_2x+b_2y+c_2z+d_2=0$ tekislik orasidagi burchak:

$$\cos\varphi=\frac{a_1\cdot a_2+b_1\cdot b_2+c_1\cdot c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$$

4. Tekislikda istalgan uchtasi bir to'g'ri chiziqda yotmaydigan n $(n>2)$ ta nuqtadan $\frac{n(n-1)}{2}$ ta ***to'g'ri chiziq*** o'tkazish mumkin va bu to'g'ri chiziqlar tekislikni ko'pi bilan $\frac{n^2+n+2}{2}$ ta ***tekislikka*** ajratadi.

Vektorlar

Ta'rif: Yo'naltirilgan kesma ***vektor*** deyiladi.

Vektorlar lotin alifbosining kichik harflari $\vec{a}$, $\vec{b}$, $\vec{c}$, ... bilan yoki $\overline{AB}$ ko'rinishida belgilanadi. $\overline{AB}$ belgilashda A nuqta vektorning ***boshi*** B nuqta esa vektorning ***oxiri*** deyiladi.

Boshi $A(x_1; y_1)$ va oxiri $B(x_2; y_2)$ nuqtalarda bo'lgan $\overline{AB}$ vektor:

$$\overline{AB}(x_2-x_1;\ y_2-y_1)=\vec{a}(a_1; a_2)$$

bu yerda $x_2-x_1=a_1$ va $y_2-y_1=a_2$. a_1 va a_2 sonlar $\vec{a}$ ***vektorning koordinatalari*** deyiladi.

1. Koordinata o'qlaridagi birlik vektorlar (ortlar): $\vec{i}(1; 0)$ – Ox o'qi yo'nalishida, $\vec{j}(0; 1)$ – Oy o'qi yo'nalishida.

2. $\vec{a}(a_1; a_2)$ vektor yo'nalishidagi birlik vektor:

$$\vec{e}\left(\frac{a_1}{\sqrt{a_1^2+a_2^2}}; \frac{a_2}{\sqrt{a_1^2+a_2^2}}\right)$$

3. Nol vektor: $\vec{O}(0; 0)$

4. $\vec{a}(a_1; a_2)$ vektorga qarama-qarshi vektor: $\vec{a}(-a_1; -a_2)$

5. $\vec{a}(a_1; a_2)$ vektorni koordinata o'qlaridagi birlik vektorlar orqali ifodalash: $\vec{a}(a_1; a_2)=a_1\vec{i}+a_2\vec{j}$

6. $\vec{a}(a_1; a_2)$ va $\vec{b}(b_1; b_2)$ vektorlar hamda ular orasidagi burchak φ berilgan bo'lsa, shu vektorlarga qurilgan uchburchakning yuzi: $S=\frac{1}{2}|\vec{a}|\cdot|\vec{b}|\cdot\sin\varphi=\frac{1}{2}\vec{a}\cdot\vec{b}\cdot\mathrm{tg}\varphi$

Eslatma: Uzunligi nolga teng vektor ***nol vektor***, uzunligi birga teng vektor ***birlik vektor*** va bir xil yo'nalgan hamda uzunliklari teng bo'lgan vektorlar ***teng vektorlar*** deyiladi.

Ta'rif: Bir to'g'ri chiziqqa parallel bo'lgan vektorlar ***kollinear vektorlar*** deyiladi. Kollinear vektorlar bir xil yo'nalgan yoki qarama-qarshi yo'nalgan bo'ladi.

Ta'rif: Bir tekislikka parallel bo'lgan uchta vektor ***komplanar vektorlar*** deyiladi.

Vektorlar ustida amallar

Tekislikda ikkita $\vec{a}(a_1; a_2)$ va $\vec{b}(b_1; b_2)$ vektorlar berilgan bo'lsa:

1. Vektorning ***absolyut qiymati (uzunligi)***:	$\lvert\vec{a}\rvert=\sqrt{a_1^2+a_2^2}$
2. Ikki vektorning ***yig'indisi***:	$\vec{a}+\vec{b}=\vec{c}(a_1+b_1; a_2+b_2)$
3. Ikki vektorning ***ayirmasi***:	$\vec{a}-\vec{b}=\vec{c}(a_1-b_1; a_2-b_2)$
4. Vektorni ***songa ko'paytirish***:	$\lambda\vec{a}=\vec{c}(\lambda a_1; \lambda a_2),\ \lambda\in\mathbb{R}$
5. Vektorning ***darajasi***:	$\vec{a}\cdot\vec{a}=(\vec{a})^2=\lvert\vec{a}\rvert^2$
6. Ikki vektorning ***skalyar ko'paytmasi***: bu yerda α – $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak.	$\vec{a}\cdot\vec{b}=a_1\cdot b_1+a_2\cdot b_2$ $\vec{a}\cdot\vec{b}=\lvert\vec{a}\rvert\cdot\lvert\vec{b}\rvert\cdot\cos\alpha$
7. Ikki vektor orasidagi ***burchak kosinusi***:	$\cos\alpha=\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\cdot\lvert\vec{b}\rvert}=\frac{a_1\cdot b_1+a_2\cdot b_2}{\sqrt{a_1^2+a_2^2}\cdot\sqrt{b_1^2+b_2^2}}$
8. Ikki vektorning ***parallellik sharti***:	$\frac{a_1}{b_1}=\frac{a_2}{b_2}$
9. Ikki vektorning ***perpendikulyarlik sharti***:	$\vec{a}\cdot\vec{b}=0$

Tekislikda simmetriya

I. $M(x; y)$ nuqtaning koordinata o'qlariga nisbatan simmetriyasi:

• Ox o'qqa nisbatan simmetrik nuqta:	$M'(x; -y)$
• Oy o'qqa nisbatan simmetrik nuqta:	$M'(-x; y)$
• Koor. boshiga nisbatan simmetrik nuqta:	$M'(-x; -y)$

II. $y=kx+b$ $(ax+by+c=0)$ t/chning koordinata o'qlariga nisbatan simmetriyasi:

• Ox o'qqa nisbatan simmetrik t/ch:	$y=-kx-b$ $(ax-by+c=0)$
• Oy o'qqa nisbatan simmetrik t/ch:	$y=-kx+b$ $(ax-by-c=0)$
• Koordinata boshiga nis. sim. t/ch:	$y=kx-b$ $(ax+by-c=0)$

III. $y=kx+b$ $(ax+by+c=0)$ t/chning $y=m$ t/chga nisbatan simmetriyasi:	$y=-kx-b+2m$ $(ax-by+c+2bm=0)$
IV. $y=kx+b$ $(ax+by+c=0)$ t/chning $x=n$ t/chga nisbatan simmetriyasi:	$y=-kx+b+2kn$ $(ax-by-c-2an=0)$

[illegible]:

* O'zaro ***teskari funksiyalar*** grafigi $y=x$ t/chqa nisbatan simmetrik bo'ladi.

Burchaklar

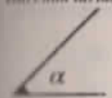
Ta'rif: To'g'ri chiziqning berilgan nuqtasidan bir tomonda yotgan nuqtalaridan [illegible] ***yarimto'g'ri chiziq*** yoki ***nur*** deyiladi.

Ta'rif: Boshi bir nuqtada bo'lgan ikkita nurdan tashkil topgan shakl ***burchak*** deyiladi.

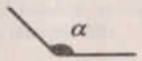
Berilgan nuqta ***burchakning uchi***, nurlar esa ***burchakning tomonlari***.

Burchak kattaligi yunon alifbosining kichik harflari α, β, γ,... bilan belgilanadi va uning qiymati graduslarda o'lchanadi.

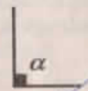
Burchak turlari:



$0<\alpha<90^0$
O'tkir burchak

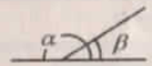


$90^0<\alpha<180^0$
O'tmas burchak



$\alpha=90^0$
To'g'ri burchak

$\alpha=180^0$
Yoyiq burchak



$\alpha+\beta=180^0$
Qo'shni burchaklar

$\alpha=\beta,\ \gamma=\delta$
Vertikal burchaklar

1-aksioma: Har qanday burchak noldan katta tayin gradus o'lchovga ega.

2-aksioma: Istalgan yarimto'g'ri chiziqdan berilgan yarimtekislikka gradus o'lchovi 180^0 dan kichik yagona burchakni qo'yish mumkin.

Ta'rif: Burchakning uchidan chiqib, uni teng ikkiga ajratgan nur ***bissektrisa*** deyiladi.

Parallel to'g'ri chiziqlar

Ta'rif: Agar tekislikdagi ikkita to'g'ri chiziq kesishmasa, ular ***parallel*** $(a\parallel b)$ ***to'g'ri chiziqlar*** deyiladi.

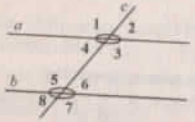
Aksioma: Berilgan to'g'ri chiziqda yotmaydigan nuqta orqali tekislikda berilgan to'g'ri chiziqqa bittadan ortiq bo'lmagan ***parallel to'g'ri chiziq*** o'tkazish mumkin.

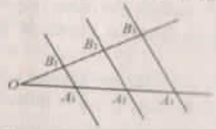
Teorema: Uchinchi to'g'ri chiziqqa parallel ikkita to'g'ri chiziq *o'zaro parallel* bo'ladi.

Ikki parallel to'g'ri chiziqni uchinchi chiziq kesib o'tganda hosil bo'lgan burchaklar:

- Mos burchaklar: 1 va 3; 5 va 7; 2 va 4; 6 va 8
- Ichki almashinuvchi burchaklar: 3 va 5; 4 va 6
- Tashqi almashinuvchi burchaklar: 1 va 7; 2 va 8
- Ichki bir tomonli burchaklar: 4 va 5; 3 va 6
- Tashqi bir tomonli burchaklar: 1 va 8; 2 va 7

$\angle 1 = \angle 3 = \angle 5 = \angle 7$, $\angle 2 = \angle 4 = \angle 6 = \angle 8$, $\angle 1 + \angle 4 = \angle 2 + \angle 3 = 180^0$





Fales teoremasi: Agar burchak tomonlarini kesuvchi parallel to'g'ri chiziqlar burchakning bir tomonidan ***teng kesmalar*** ajratsa, ular uning ikkinchi tomonidan ham ***teng kesmalar*** ajratadi:

$A_1B_1 \parallel A_2B_2 \parallel A_3B_3$, $OA_1 : OB_1 = OA_2 : OB_2 = OA_3 : OB_3$

Perpendikulyar to'g'ri chiziqlar

Ta'rif: Agar ikkita to'g'ri chiziq to'g'ri burchak ostida kesishsa, ular ***perpendikulyar*** ($a \perp b$) ***to'g'ri chiziqlar*** deyiladi.

1-teorema: To'g'ri chiziqning har bir nuqtasidan unga perpendikulyar to'g'ri chiziq o'tkazish mumkin va ***faqat birgina***.

2-teorema: Biror to'g'ri chiziqda yotmagan istalgan nuqtadan bu to'g'ri chiziqqa perpendikulyar tushirish mumkin va ***faqat birgina***.

Ta'rif: Biror nuqtadan to'g'ri chiziqqa tushirilgan perpendikulyarning uzunligi ***nuqtadan to'g'ri chiziqqacha masofa*** deyiladi.

Aylana va doira

Ta'rif: Tekislikda bir nuqtadan teng uzoqlikda yotgan barcha nuqtalarning geometrik o'rniga ***aylana*** deyiladi.

Berilgan O nuqta ***aylana markazi***, aylanadagi bir nuqtadan markazgacha bo'lgan masofa ***aylana radiusi*** deyiladi.

Ta'rif: Tekislikning aylana bilan chegaralangan qismi ***doira*** deyiladi.

Aylana uzunligi $l = 2\pi R$

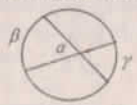
Yoy uzunligi $l_{yoy} = \dfrac{\pi R \alpha}{180^0}$

Doira yuzi $S = \pi R^2$

- Aylananing ikki nuqtasini tutashtiruvchi kesma ***vatar*** (CD), aylana markazidan o'tuvchi vatar esa ***diametr*** (eng katta vatar $AB = 2R$) deyiladi.
- Uchi aylana markazida, tomonlari esa aylana radiuslaridan iborat burchak ***markaziy burchak*** ($\alpha = \angle AOB$) deyiladi va u o'zi tiralgan yoy bilan o'lchanadi.
- Uchi aylanada yotgan, tomonlari esa shu aylanani kesib o'tuvchi burchakka ***aylanaga ichki chizilgan burchak*** ($\beta = \angle ACB$) deyiladi va u markaziy burchakning yarmiga teng $\beta = \dfrac{\alpha}{2}$.

Aylanadagi burchaklar

$x = \dfrac{\beta - \alpha}{2}$

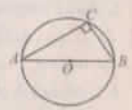


$\alpha = \dfrac{\beta + \gamma}{2}$

$\alpha = \dfrac{\beta - \gamma}{2}$

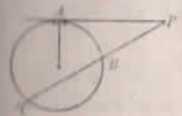
$\alpha = x + y$

$\alpha = \beta = \gamma$

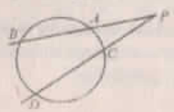


$\angle ACB = 90^0$

Aylanadagi teoremalar



$PA^2 = PB \cdot PC$



$PA \cdot PB = PC \cdot PD$

$PA \cdot PB = PC \cdot PD$

Doiraviy sektor va segment

Ta'rif: ***Doiraviy sektor*** deb, doiraning mos markaziy burchak ichida yotgan qismiga aytiladi

$P = 2R + \frac{\pi R\alpha}{180^0}$ $S_{sekt} = \frac{\pi R^2\alpha}{360^0}$ yoki $S_{sekt} = \frac{R^2\beta}{2}$

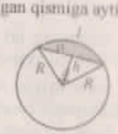
bu yerda α-gradus, β-radian.

Ta'rif: ***Doiraviy segment*** deb, doiraning bitta vatari ajratgan qismiga aytiladi.

$S_{segm} = \frac{\pi R^2\alpha}{360^0} - \frac{R^2}{2}\sin\alpha$, $S_{segm} = \frac{1}{2}R^2(\beta - \sin\beta)$,

$S_{segm} = \frac{R(l-a)+ah}{2}$

bu yerda α-gradus, β-radian.

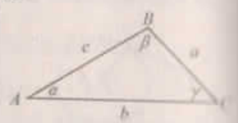


UCHBURCHAKLAR

Ta'rif: Bir to'g'ri chiziqda yotmaydigan uchta nuqtani ketma-ket tutashtirish natijasida hosil bo'lgan yopiq shaklga ***uchburchak*** deyiladi.

ABC ***uchburchakda:***

* mos tomonlar: a, b, c
* mos ichki burchaklar: α, β, γ
* mos tashqi burchaklar: $\alpha_1, \beta_1, \gamma_1$

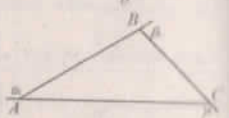


Xossalari:

1. Perimetri: $P = a + b + c$
2. Ichki burchaklari: $\alpha + \beta + \gamma = 180^0$
3. Tashqi burchaklari:

$\alpha_1 + \beta_1 + \gamma_1 = 360^0$

$\alpha_1 = \beta + \gamma$, $\beta_1 = \alpha + \gamma$, $\gamma_1 = \alpha + \beta$



4. Uchburchak tengsizligi:

$a + b > c > |a - b|$, $b + c > a > |b - c|$, $a + c > b > |a - c|$

5. Uchburchakning katta burchagi qarshisida katta tomoni, kichik burchagi qarshisida esa kichik tomoni yotadi.

Uchburchaklarning turlari

1. Uchburchaklar tomonlariga ko'ra ***uch turga*** bo'linadi:

* ***Turli tomonli*** (ixtiyoriy) uchburchak: α, β, γ va a, b, c
* ***Teng yonli*** uchburchak: α, β, β va a, b, b
* ***Teng tomonli*** (muntazam) uchburchak: $\alpha, \alpha, \alpha = 60^0$ va a, a, a

2. Uchburchaklar burchaklariga ko'ra ***uch turga*** bo'linadi ($c > a, b$):

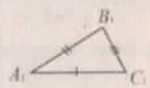
* ***O'tkir burchakli*** uchburchak: $\alpha, \beta, \gamma < 90^0$ va $a^2 + b^2 > c^2$
* ***To'g'ri burchakli*** uchburchak: $\alpha + \beta = 90^0$, $\gamma = 90^0$ va $a^2 + b^2 = c^2$
* ***O'tmas burchakli*** uchburchak: $\alpha + \beta < 90^0$, $\gamma > 90^0$ va $a^2 + b^2 < c^2$

Uchburchaklarning tengligi

Ta'rif: $\Delta A_1B_1C_1$ va $\Delta A_2B_2C_2$ ***teng*** deyiladi, agar

1) mos ravishda ***uchta tomoni*** teng:

$A_1B_1 = A_2B_2$, $A_1C_1 = A_2C_2$, $B_1C_1 = B_2C_2$



2) ***ikki tomoni*** va ular orasidagi ***burchaklari*** teng:

$A_1B_1 = A_2B_2$, $A_1C_1 = A_2C_2$, $\angle A_1 = \angle A_2$

3) ***bir tomoni*** va unga yopishgan ***ikki burchagi*** teng:

$A_1B_1 = A_2B_2$, $\angle A_1 = \angle A_2$, $\angle B_1 = \angle B_2$

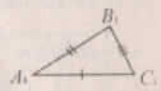
Eslatma: Teng uchburchaklarning ***yuzalari*** ham, ***barcha chiziqli elementlari*** ham teng bo'ladi.

Uchburchaklarning o'xshashligi

Ta'rif: $\Delta A_1B_1C_1$ va $\Delta A_2B_2C_2$ ***o'xshash*** deyiladi, agar

1) ***ikki burchagi*** mos holda teng:

$\angle A_1 = \angle A_2$, $\angle B_1 = \angle B_2$



2) ***ikki tomoni*** mos holda proporsional va ular orasidagi ***burchagi*** teng:

$\frac{A_1B_1}{A_2B_2} = \frac{A_1C_1}{A_2C_2}$, $\angle A_1 = \angle A_2$

3) ***barcha tomonlari*** mos holda proporsional:

$\frac{A_1B_1}{A_2B_2} = \frac{A_1C_1}{A_2C_2} = \frac{B_1C_1}{B_2C_2}$

Eslatma: O'xshash uchburchaklarning burchaklari o'zgarmaydi, lekin barcha ***chiziqli elementlari*** mos holda ***proporsional*** bo'ladi, ya'ni

$\frac{A_1B_1}{A_2B_2} = \frac{A_1C_1}{A_2C_2} = \frac{B_1C_1}{B_2C_2} = \frac{P_{A_1B_1C_1}}{P_{A_2B_2C_2}} = \frac{l_1}{l_2} = \frac{h_1}{h_2} = \frac{m_1}{m_2} = \frac{r_1}{r_2} = \frac{R_1}{R_2}$, $\frac{S_{A_1B_1C_1}}{S_{A_2B_2C_2}} = \left(\frac{P_{A_1B_1C_1}}{P_{A_2B_2C_2}}\right)^2$

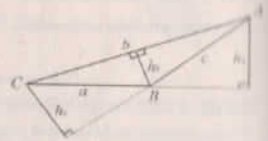
Uchburchakning balandligi

Ta'rif: Uchburchakning uchidan shu uch qarshisidagi tomonga tushirilgan perpendikulyar ***balandlik*** deyiladi va mos holda h_a, h_b, h_c bilan belgilanadi.

$\angle A=\alpha,\quad \angle B=\beta,\quad \angle C=\gamma$

$$h_a=\frac{2S}{a}=b\cdot\sin\gamma=c\cdot\sin\beta$$

$$h_a:h_b:h_c=\frac{1}{a}:\frac{1}{b}:\frac{1}{c}=bc:ac:ab$$



Eslatma: Uchburchakning eng katta balandligi uning eng kichik tomoniga tushadi, va aksincha eng kichik balandligi eng katta tomoniga tushadi.

Uchburchakning medianasi

Ta'rif: Uchburchak uchi bilan shu uch qarshisidagi tomon o'rtasini tutashtiruvchi kesma ***mediana*** deyiladi va mos holda m_a, m_b, m_c bilan belgilanadi.

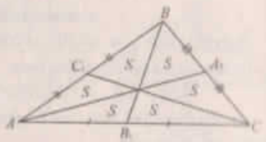
$\angle A=\alpha,\quad \angle B=\beta,\quad \angle C=\gamma,\qquad AB=c,\quad BC=a,\quad AC=b$

$$m_a=\frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$$

$$m_a=\frac{1}{2}\sqrt{b^2+c^2+2bc\cdot\cos\alpha}$$

$$a=\frac{2}{3}\sqrt{2(m_b^{\,2}+m_c^{\,2})-m_a^{\,2}}$$

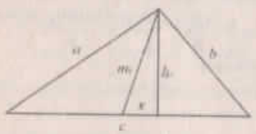
$$m_a^{\,2}+m_b^{\,2}+m_c^{\,2}=\frac{3}{4}(a^2+b^2+c^2)$$



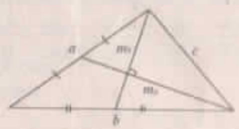
Eslatma: Uchburchakning uchala medianasi bir nuqtada kesishadi va har biri uchidan boshlab hisoblaganda shu nuqtada **2:1** ***nisbatda*** bo'linadi.

* Medianalar kesishish nuqtasi ***uchburchakning og'irlik markazi*** deyiladi.
* Uchburchakning medianasi uning ***yuzini*** teng ikkiga bo'ladi.
* Uchburchakning eng katta medianasi uning eng kichik tomoniga tushadi, va aksincha, eng kichik medianasi eng katta tomoniga tushadi.

Qo'shimcha:



$$x=\frac{|a^2-b^2|}{2c}$$



$$m_a\perp m_b,\quad m_c^2=m_a^2+m_b^2,$$

$$m_c^2=\frac{3}{8}(a^2+b^2+c^2)$$

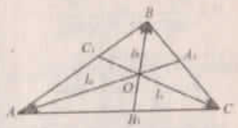
Uchburchakning bissektrisasi

Ta'rif: Uchburchak uchidan chiqib, shu uchidagi burchakni teng ikkiga bo'luvchi va uchi shu burchak qarshisidagi tomonda yotuvchi kesma ***bissektrisa*** deyiladi va l_a, l_b, l_c bilan belgilanadi.

$\angle A=\alpha,\quad \angle B=\beta,\quad \angle C=\gamma$

$AB=c,\quad BC=a,\quad AC=b$

$$\frac{AB}{AC}=\frac{BA_1}{A_1C},\qquad \frac{S_{ABA_1}}{S_{ACA_1}}=\frac{AB}{AC}$$



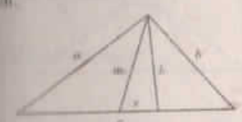
$$l_a=\frac{1}{b+c}\sqrt{bc(b+c+a)(b+c-a)},\quad l_a=\frac{2bc}{b+c}\cos\frac{\alpha}{2},\quad l_a=\sqrt{AB\cdot AC-BA_1\cdot A_1C}$$

Eslatma: Ucburchakning uchala bissektrisasi bir nuqtada kesishadi va har biri uchidan boshlab hisoblaganda shu nuqtada mos ravishda quyidagicha ***nisbatda*** bo'linadi.

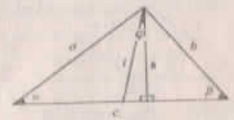
* l_a bissektrisa $AO:OA_1=(b+c):a$ nisbatda
* l_b bissektrisa $BO:OB_1=(a+c):b$ nisbatda
* l_c bissektrisa $CO:OC_1=(a+b):c$ nisbatda

Qo'shimcha:

Uchburchaklarda balandlik, bissektrisa va mediana $h\le l\le m$ ***munosabatda*** bo'ladi.



$$x=\frac{c}{2}\left|\frac{a-b}{a+b}\right|$$



$$\varphi=\frac{\beta-\alpha}{2}$$

Uchburchakka ichki va tashqi chizilgan aylanalar

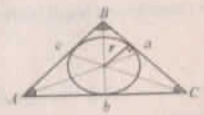
Ta'rif: Uchburchakka ichki chizilgan aylana markazi ***bissektrisalari*** kesishgan nuqtada bo'ladi.

Ichki chizilgan ***aylana radiusi*** r bilan belgilanadi.

$\angle A=\alpha,\quad \angle B=\beta,\quad \angle C=\gamma$

$$r=\frac{S}{p},\quad r=(p-a)tg\frac{\alpha}{2},\quad r=a\cdot\frac{\cos\frac{\beta}{2}\cos\frac{\gamma}{2}}{\cos\frac{\alpha}{2}},$$

$$r=p\cdot tg\frac{\alpha}{2}\cdot tg\frac{\beta}{2}\cdot tg\frac{\gamma}{2},\qquad p=\frac{a+b+c}{2}$$



$$\frac{1}{r}=\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}$$

Ta'rif: Uchburchakka tashqi chizilgan aylana markazi uchburchak tomonlarining ***o'rta perpendikulyarlari*** kesishgan nuqtada bo'ladi.

Tashqi chizilgan ***aylana radiusi*** R bilan belgilanadi.

$$R=\frac{abc}{4S},\quad R=\frac{2S^2}{h_a h_b h_c},$$

$$R=\frac{a+b+c}{8\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}}$$

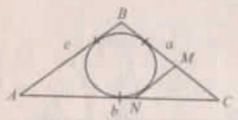
Eslatma: Tashqi chizilgan aylana markazi:

* o'tkir burchakli uchburchakda ***uchburchak ichida,***
* to'g'ri burchakli uchburchakda ***gipotenuzaning o'rtasida,***
* o'tmas burchakli uchburchakda esa ***uchburchak tashqarisida*** bo'ladi.

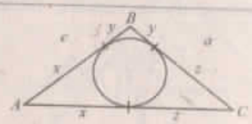
Qo'shimcha:

$$O_1O_2=\sqrt{R^2-2Rr},\qquad \frac{r}{R}\le\frac{1}{2},$$

$$\frac{r}{R}=4\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}$$



$MN\parallel AB,\quad P_{MNC}=a+b-c$



$$x=\frac{b+c-a}{2},\quad y=\frac{a+c-b}{2},\quad z=\frac{a+b-c}{2}$$

Uchburchakning yuzi

* Bir tomon va unga tushirilgan balandligi orqali: $S=\dfrac{a\cdot h_a}{2}$
* Ikki tomon va ular orasidagi burchagi orqali: $S=\dfrac{1}{2}ab\cdot\sin\gamma$
* Uchta tomoni orqali (***Geron formulasi***): $S=\sqrt{p(p-a)(p-b)(p-c)},$ $p=\dfrac{a+b+c}{2}$
* Ichki va tashqi chizilgan aylana radiuslari orqali: $S=\dfrac{abc}{4R}\quad S=\dfrac{a+b+c}{2}r$

* Medianalari orqali:

$$S=\frac{4}{3}\sqrt{m(m-m_a)(m-m_b)(m-m_c)},$$

$$m=\frac{m_a+m_b+m_c}{2}$$

* Ikki medianasi va ular orasidagi burchagi orqali: $S=\dfrac{2}{3}m_a m_b\sin\varphi$
* Ikki tomoni va ular orasidagi burchak bissektrisasi orqali: $S=\dfrac{(a+b)l_c}{4ab}\sqrt{4a^2b^2-l_c^2(a+b)^2}$
* Perimetri va ichki burchaklari orqali: $S=p^2\cdot\mathrm{tg}\dfrac{\alpha}{2}\cdot\mathrm{tg}\dfrac{\beta}{2}\cdot\mathrm{tg}\dfrac{\gamma}{2},$ $p=\dfrac{a+b+c}{2}$
* Uchta balandligi orqali:

$$S=\frac{1}{\sqrt{\left(\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}\right)\left(-\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}\right)\left(\frac{1}{h_a}-\frac{1}{h_b}+\frac{1}{h_c}\right)\left(\frac{1}{h_a}+\frac{1}{h_b}-\frac{1}{h_c}\right)}}$$

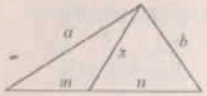
Uchburchakdagi asosiy teoremalar

* ***Sinuslar teoremasi:*** $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}=\dfrac{c}{\sin\gamma}=2R$

bu yerda R-tashqi chizilgan aylana radiusi.

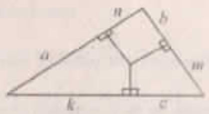
* ***Kosinuslar teoremasi:*** $a^2=b^2+c^2-2bc\cdot\cos\alpha,$ $a=b\cos\gamma+c\cos\beta$
* ***Tangenslar teoremasi:*** $\dfrac{a+b}{a-b}=\dfrac{\mathrm{tg}\frac{\alpha+\beta}{2}}{\mathrm{tg}\frac{\alpha-\beta}{2}}$
* ***Molveyde formulasi:*** $\dfrac{a+b}{c}=\dfrac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}}$

Styuart teoremasi



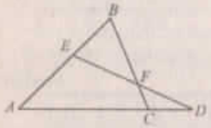
$$x^2 = \frac{a^2 \cdot n + b^2 \cdot m}{n+m} - n \cdot m$$

Karno teoremasi



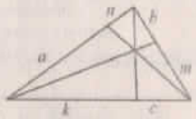
$$a^2 + b^2 + c^2 = m^2 + n^2 + k^2$$

Menelay teoremasi

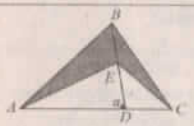


$$\frac{CD}{AD} \cdot \frac{AE}{EB} \cdot \frac{BF}{FC} = 1$$

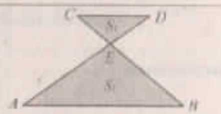
Cheva teoremasi



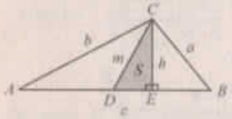
$$a \cdot b \cdot c = m \cdot n \cdot k$$



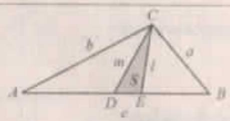
$$S_{ABCE} = \frac{AC \cdot BE}{2} \sin \alpha$$



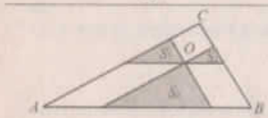
$$\frac{S_1}{S_2} = \frac{AE \cdot BE}{CE \cdot DE}$$



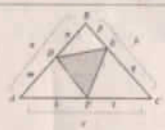
$$S_{CDE} = \frac{|a^2 - b^2|}{2c^2} S_{ABC}$$



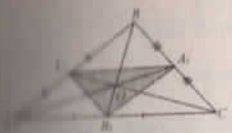
$$S_{CDE} = \left|\frac{a-b}{2(a+b)}\right| S_{ABC}$$



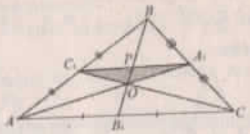
$$S_{ABC} = \left(\sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3}\right)^2$$



$$\frac{S_{PQR}}{S_{ABC}} = \frac{nqk + mpt}{abc}$$

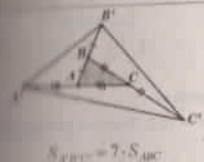


$$S_{A_1B_1C_1} = \frac{1}{4} S_{ABC}$$

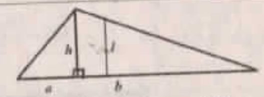


$BB_1 = 6OP$, $S_{AOP} = S_{C_1OP}$, $S_{BPC_1} = S_{BPA_1}$

$S_{AOP} = \frac{1}{24} S_{ABC}$, $S_{BPA_1} = \frac{1}{8} S_{ABC}$

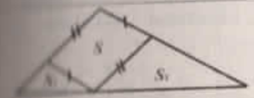


$$S_{A'B'C'} = 7 \cdot S_{ABC}$$



l to'g'ri chiziq h ga parallel va

Δ yuzini teng ikkiga bo'lsa, $l = h\sqrt{\frac{a+b}{2b}}$

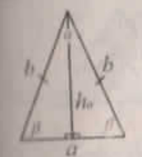


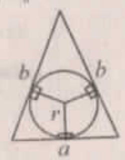
$$S = 2\sqrt{S_1 \cdot S_2}$$

Teng yonli uchburchak

Ta'rif: Faqat ikki tomoni teng bo'lgan uchburchakka ***teng yonli uchburchak*** deyiladi.

- Teng tomonlar ***yon tomon***, uchinchi tomon esa ***asos*** deb ataladi.
- Teng yonli uchburchakning asosdagi ***burchaklari teng*** bo'ladi.
- Teng yonli uchburchakning asosiga tushirilgan balandligi, medianasi va bissektrisasi teng bo'ladi: $h_a = m_a = l_a$



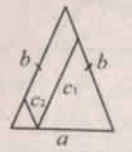


$P = a + 2b$, $S = \frac{1}{2} b^2 \sin \alpha$, $S = \frac{1}{2} ab \sin \beta$, $S = \frac{ah_a}{2}$, $S = \frac{bh_b}{2}$

$h_a = \frac{1}{2}\sqrt{4b^2 - a^2}$, $R = \frac{b^2}{2h_a}$, $r = \frac{a(2b-a)}{4h_a}$, $r = \frac{ah_a}{a+2b}$

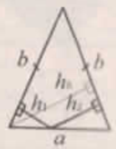
$$h_b = \frac{a}{2b}\sqrt{4b^2 - a^2}, \qquad m_b = \frac{1}{2}\sqrt{2a^2 + b^2}, \qquad l_b = \frac{a}{a+b}\sqrt{2b^2 + ab}$$

Qo'shimcha:



$c_1 + c_2 = b$
$(c_1 \parallel b,\ c_2 \parallel b)$

$MN = \frac{|m-n|}{2}$.

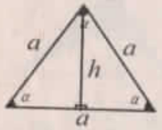


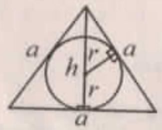
$h_1 + h_2 = h_b$
$(h_1 \perp b,\ h_2 \perp b)$

Teng tomonli (muntazam) uchburchak

Ta'rif: Barcha tomonlari teng bo'lgan uchburchakka ***teng tomonli (muntazam) uchburchak*** deyiladi.

* Teng tomonli uchburchakning barcha ***burchaklari teng***, ya'ni $\alpha = 60^0$ bo'ladi.
* Teng tomonli uchburchakning barcha balandliklari, medianalari va bissektrisalari teng bo'ladi.

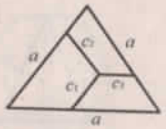




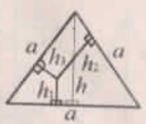
$$P = 3a, \qquad h = \frac{\sqrt{3}a}{2}, \quad R = \frac{\sqrt{3}a}{3}, \quad r = \frac{\sqrt{3}a}{6}, \quad h = r + R$$

$$S = \frac{\sqrt{3}}{4}a^2, \quad S = \frac{\sqrt{3}}{3}h^2, \quad S = \frac{3\sqrt{3}}{4}R^2, \quad S = 3\sqrt{3}r^2$$

Qo'shimcha:



$c_1 + c_2 + c_3 = a$
$(c_1 \parallel a,\ c_2 \parallel a,\ c_3 \parallel a)$



$h_1 + h_2 + h_3 = h = \frac{a\sqrt{3}}{2}$
$(h_1 \perp a,\ h_2 \perp a,\ h_3 \perp a)$

To'g'ri burchakli uchburchak

Ta'rif: Ikki tomoni perpendikulyar bo'lgan uchburchakka ***to'g'ri burchakli uchburchak*** deyiladi.

To'g'ri burchakli uchburchakda: $BC = a$ va $AC = b$ ***katetlar***, $AB = c$ esa ***gipotenuza*** deyiladi.

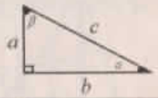
Pifagor teoremasi: Katetlari kvadratlarining yig'indisi gipotenuzaning kvadratiga teng, ya'ni

$$a^2 + b^2 = c^2.$$

Burchak ***sinusi, kosinusi, tangensi*** va ***kotangensi***:

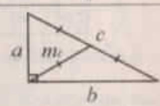
$$\sin\alpha = \frac{a}{c}, \qquad \cos\alpha = \frac{b}{c}$$

$$tg\alpha = \frac{a}{b}, \qquad ctg\alpha = \frac{b}{a}$$



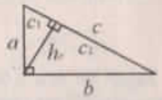
$$m_c = \frac{c}{2} = R, \qquad m_a^2 + m_b^2 = 5m_c^2$$

$$l_a = b\sqrt{\frac{2c}{b+c}}, \quad l_b = a\sqrt{\frac{2c}{a+c}}, \quad l_c = \frac{ab\sqrt{2}}{a+b}$$



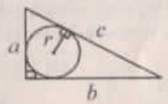
$$a^2 = c \cdot c_1, \quad b^2 = c \cdot c_2, \quad h_c^2 = c_1 \cdot c_2, \quad h_c = \frac{ab}{c}$$

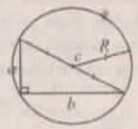
bu yerda c_1 va c_2 lar, a va b katetlarning c gipotenuzadagi ***proeksiyalari***: $c_1 + c_2 = c$.



$$r = \frac{a+b-c}{2} = \frac{ab}{a+b+c},$$

$$R = \frac{c}{2} = m_c$$

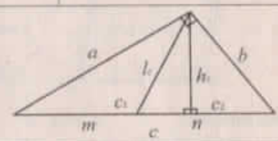




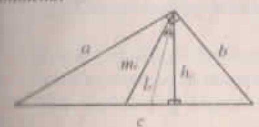
$$\left(\frac{a}{b}\right)^2 = \left(\frac{m}{n}\right)^2 = \frac{c_1}{c_2}, \qquad S = \frac{h_c^2 \cdot l_c^2}{2h_c^2 - l_c^2}$$

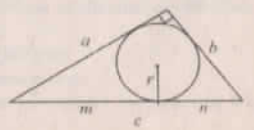
$$S = \frac{ab}{2}, \quad S = \frac{ch_c}{2}, \quad S = Rh_c,$$

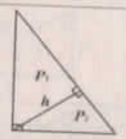
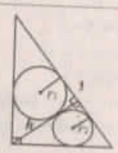
$$S = m_c h_c, \quad S = r^2 + 2rR$$



Qo'shimcha:





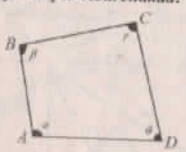
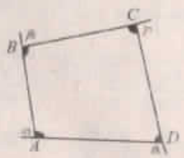
To'g'ri burchakli uchburchakning ***medianalari kesishgan nuqtasidan*** uning ***bissektrisalari kesishgan nuqtasigacha*** bo'lgan masofa: (d)	$S=m\cdot n$ $d=\sqrt{\left(\frac{a}{3}-r\right)^2+\left(\frac{b}{3}-r\right)^2}$
To'g'ri burchakli uchburchakka ***ichki chizilgan aylana markazidan*** unga ***tashqi chizilgan aylana markazigacha*** bo'lgan masofa: (O_1O_2)	$O_1O_2=\sqrt{\left(\frac{a}{2}-r\right)^2+\left(\frac{b}{2}-r\right)^2}$
 P_1 va P_2 berilgan bo'lsa, $P^2=P_1^2+P_2^2,\quad h=\frac{P_1+P_2-\sqrt{P_1^2+P_2^2}}{2}$	 r_1 va r_2 berilgan bo'lsa, $r^2=r_1^2+r_2^2,\quad h=r_1+r_2+\sqrt{r_1^2+r_2^2}$

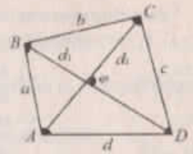
TO'RTBURCHAKLAR

Ta'rif: Bir to'g'ri chiziqda yotmaydigan to'rtta nuqtani ketma-ket tutashtirish natijasida hosil bo'lgan yopiq shaklga ***to'rtburchak*** deyiladi.

Ta'rif: Agar to'rtburchak o'zining istalgan tomoni yotgan to'g'ri chiziqqa nisbatan bir tomonda yotsa, bunday to'rtburchak ***qavariq to'rtburchak*** deyiladi.

ABCD ***qavariq to'rtburchakda:***

	
* mos tomonlar: a, b, c, d * mos ichki burchaklar: $\alpha, \beta, \gamma, \theta$	* mos diagonallar: d_1, d_2 * mos tashqi burchaklar: $\alpha_1, \beta_1, \gamma_1, \theta_1$

Formulalar:		
1. Perimetri:	$P=a+b+c+d$	
2. Ichki burchaklari:	$\alpha+\beta+\gamma+\theta=360^0$	
3. Yuzasi:	$S=\frac{1}{2}d_1d_2\sin\varphi$	
4. Tashqi burchaklari:	$\alpha_1+\beta_1+\gamma_1+\theta_1=360^0$	

To'rtburchakka ichki va tashqi chizilgan aylanalar

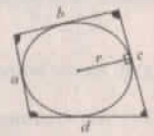
Ta'rif: To'rtburchakning qarama-qarshi ***tomonlari yig'indisi teng*** bo'lsa, unga ichki aylana chizish mumkin.

$a+c=b+d$

$S=pr,\quad S=(a+c)r,\quad S=(b+d)r$

$S=\sqrt{(p-a)(p-b)(p-c)(p-d)}$

$p=\frac{a+b+c+d}{2}$



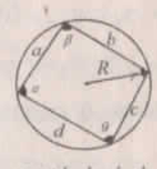
Ta'rif: To'rtburchakning qarama-qarshi ***burchaklari yig'indisi teng*** va 180^0 bo'lsa, unga tashqi aylana chizish mumkin.

$\alpha+\beta=\gamma+\varphi=180^0$

$R=\frac{1}{4S}\sqrt{(ab+cd)(ac+bd)(ad+bc)}$

$d_1=\sqrt{\frac{(a^2+b^2)cd+(c^2+d^2)ab}{ab+cd}},$

$d_2=\sqrt{\frac{(a^2+d^2)bc+(b^2+c^2)ad}{ad+bc}}$



$a\cdot c+b\cdot d=d_1\cdot d_2$

(Ptolomey teoremasi)

Qo'shimcha:

To'rtburchakka ***ichki va tashqi aylana*** chizilgan bo'lsa:	$S=\sqrt{a\cdot b\cdot c\cdot d}$
To'rtburchakning ***diagonallari perpendikulyar*** bo'lsa:	$a^2+c^2=b^2+d^2$

Parallelogramm

Ta'rif: Qarama-qarshi tomonlari parallel bo'lgan to'rtburchak ***parallelogramm*** deyiladi.

* Parallelogrammning qarama-qarshi ***tomonlari teng***.
* Parallelogrammning qarama-qarshi ***burchaklari teng***.
* Parallelogrammning diagonallari ***kesishadi*** va kesishish nuqtasida ***teng ikkiga*** bo'linadi.

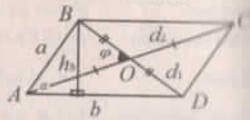
* Parallelogrammning bissektrisalari ***to'g'ri burchak ostida kesishadi.***

$P = 2(a+b)$, $d_1^2 + d_2^2 = 2(a^2+b^2)$

$S = \frac{d_1 d_2}{2}\sin\varphi$, $S = \frac{h_a \cdot h_b}{\sin\alpha}$

$S = bh_b$, $S = ab \cdot \sin\alpha$

$S = \frac{|a^2-b^2|}{2}tg\varphi$, $S = \frac{|d_1^2-d_2^2|}{4}tg\alpha$



Eslatma: Parallelogrammga ichki va tashqi aylana chizib bo'lmaydi.

Izoh: Ichki aylana chizilgan parallelogramm ***romb***, tashqi aylana chizilgan parallelogramm esa ***to'g'ri to'rtburchak*** deyiladi.

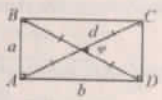
To'g'ri to'rtburchak

Ta'rif: Barcha burchaklari to'g'ri bo'lgan parallelogramm ***to'g'ri to'rtburchak*** deyiladi.

* To'g'ri to'rtburchakning ***diagonallari teng.***
* To'g'ri to'rtburchakning diagonallari ***kesishadi*** va kesishish nuqtasida ***teng ikkiga*** bo'linadi.

$P = 2(a+b)$, $d = \sqrt{a^2+b^2}$, $d = 2R$

$S = ab$, $S = \frac{d^2}{2}\sin\varphi$



Eslatma: To'g'ri to'rtburchakka ichki aylana chizib bo'lmaydi.

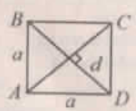
Kvadrat

Ta'rif: Barcha tomonlari teng bo'lgan to'g'ri to'rtburchakka ***kvadrat*** deyiladi.

* Kvadratning ***diagonallari teng.***
* Kvadratning diagonallari ***to'g'ri burchak ostida kesishadi*** va kesishish nuqtasida ***teng ikkiga*** bo'linadi.

$P = 4a$, $a = \frac{d}{\sqrt{2}}$, $a = 2r$, $a = \sqrt{2}R$

$S = a^2$, $S = \frac{d^2}{2}$, $S = 4r^2$, $S = 2R^2$



Qo'shimcha:

Kvadratning ichida olingan ixtiyoriy O nuqtadan uning uchlarigacha bo'lgan ***masofalar:***

$OA^2 + OC^2 = OB^2 + OD^2$

Romb

Ta'rif: Barcha tomonlari teng bo'lgan parallelogramm ***romb*** deyiladi.

* Rombning qarama-qarshi ***burchaklari teng.***
* Rombning diagonallari ***to'g'ri burchak ostida kesishadi*** va kesishish nuqtasida ***teng ikkiga*** bo'linadi.
* Rombning diagonallari burchaklarining ***bissektrisalari*** bo'ladi.

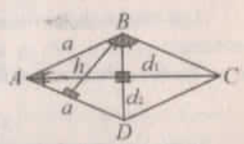
$P = 4a$, $h = 2r$, $d_1^2 + d_2^2 = 4a^2$

$d_1 = a\sqrt{2(1-\cos\alpha)}$, $d_2 = a\sqrt{2(1-\cos\beta)}$

$d_1 + d_2 = 2\sqrt{a^2+S}$, $d_1 + d_2 = 2a\sqrt{1+\sin\alpha}$

$S = a^2 \cdot \sin\alpha$, $S = ah$, $S = \frac{d_1 d_2}{2}$

$S = 2ar$, $S = \frac{h^2}{\sin\alpha}$



Eslatma: Rombga tashqi aylana chizib bo'lmaydi.

Qo'shimcha:

Rombning ichida olingan ixtiyoriy O nuqtadan uning tomonlarigacha bo'lgan ***masofalar:***

$h_1^2 + h_2^2 + h_3^2 + h_4^2 = 2h$

Trapetsiya

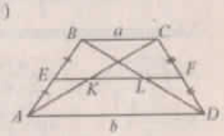
Ta'rif: Ikkita qarama-qarshi tomonlarigina parallel bo'lgan to'rtburchakka ***trapetsiya*** deyiladi.

Parallel tomonlar ***trepetsiyaning asoslari***, parallel bo'lmagan tomonlar esa ***yon tomonlari*** deyiladi.

Ta'rif: Trapetsiyaning asoslariga parallel va yon tomonlarini teng ikkiga bo'luvchi chiziq, uning ***o'rta chizig'i*** deyiladi. ($m = EF$)

$EF = \frac{a+b}{2}$, $AK = KC$, $EK = LF = \frac{a}{2}$

$KL = \frac{b-a}{2}$, $DL = LB$, $EL = KF = \frac{b}{2}$

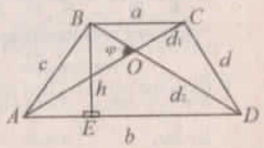


$d_1^2 + d_2^2 = c^2 + d^2 + 2ab$

$d_1 = \sqrt{ab + \frac{a \cdot c^2 - b \cdot d^2}{a-b}}$, $d_2 = \sqrt{ab + \frac{a \cdot d^2 - b \cdot c^2}{a-b}}$

$S = \frac{a+b}{2}h$, $S = \frac{d_1 d_2}{2}\sin\varphi$

$S = \frac{h}{2}\left(\sqrt{d_1^2 - h^2} + \sqrt{d_2^2 - h^2}\right)$



Izoh: Trapetsiyaning diagonallari bir nuqtada (O) kesishadi va kesishish nuqtasida quyidagicha ***nisbatda bo'linadi:*** $\frac{AO}{OC}=\frac{OD}{OB}=\frac{AD}{BC}$

Qo'shimcha:

Agar trapetsiyaning $d_1 \perp d_2$ bo'lsa:

$$d_1^2+d_2^2=(a+b)^2$$
$$S=\frac{h}{2}\sqrt{d_1^2+d_2^2}$$

Agar trapetsiyaning katta asosiga yopishgan burchaklari yig'indisi 90^0 ga teng, ya'ni $\alpha+\beta=90^0$ bo'lsa, ***asoslari o'rtalarini tutashtiruvchi chiziq:*** (l)

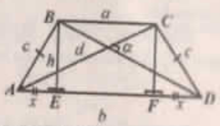
$$l=\frac{b-a}{2}$$

Teng yonli trapetsiya

Ta'rif: Yon tomonlari teng bo'lgan trapetsiya ***teng yonli trapetsiya*** deyiladi.

Teng yonli ***trapetsiyaning diagonallari*** ham, ***asoslariga yopishgan burchaklari*** ham teng bo'ladi.

$$P=a+b+2c,\quad d=\sqrt{ab+c^2}$$
$$S=\frac{a+b}{2}h,\quad S=\frac{d^2}{2}\sin\alpha$$



Eslatma: Teng yonli trapetsiyaga tashqi aylana chizish mumkin.

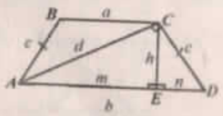
Qo'shimcha:

* Agar teng yonli trapetsiyada $\alpha=90^0$ bo'lsa:

$$h=m=\frac{a+b}{2}=\frac{d}{\sqrt{2}},\qquad S=h^2=m^2=\left(\frac{a+b}{2}\right)^2=\frac{d^2}{2}$$

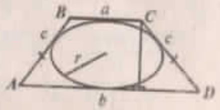
* Agar teng yonli trapetsiyaning diagonali yon tomoniga ***perpendikulyar*** bo'lsa:

$$h=\frac{\sqrt{b^2-a^2}}{2},\qquad S=\frac{a+b}{4}\sqrt{b^2-a^2}$$
$$m=\frac{a+b}{2},\qquad n=\frac{b-a}{2}$$
$$d=\sqrt{\frac{b^2+ab}{2}},\qquad c=\sqrt{\frac{b^2-ab}{2}},$$



* Agar teng yonli trapetsiyaga ***ichki aylana*** chizilgan bo'lsa:

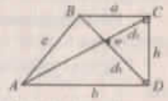
$$h=2r=\sqrt{ab}$$
$$S=\frac{a+b}{2}\sqrt{ab},\qquad S=(a+b)r$$
$$S=2cr,\qquad S=ch$$



To'g'ri burchakli trapetsiya

Ta'rif: Bitta yon tomoni asoslariga perpendikulyar bo'lgan trapetsiya ***to'g'ri burchakli trapetsiya*** deyiladi.

$$P=a+b+c+h,\quad S=\frac{a+b}{2}h,\quad S=\frac{d_1d_2}{2}\sin\varphi$$
$$d_1^2-d_2^2=b^2-a^2,\quad h=\sqrt{d_2^2-b^2}=\sqrt{d_1^2-a^2}$$



Eslatma: To'g'ri burchakli trapetsiyaga tashqi aylana chizib bo'lmaydi.

Qo'shimcha:

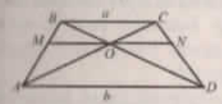
* Agar to'g'ri burchakli trapetsiyaga ichki aylana chizilgan bo'lsa:

$$b=\frac{ar}{a-r},\quad r=\frac{ab}{a+b},$$
$$S=(a+b)r,\quad S=ab$$

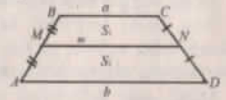
* Agar to'g'ri burchakli trapetsiyada diagonallari orasidagi burchak, ya'ni $\varphi=90^0$ bo'lsa:

$$h=\sqrt{ab}$$

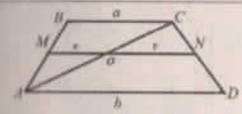
To'rtburchakdagi ajoyib topilmalar



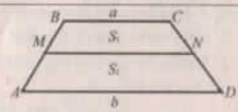
$BC \parallel MN \parallel AD,\quad MO=ON=\frac{ab}{a+b}$



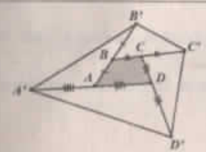
$MN=m=\frac{a+b}{2},\quad \frac{S_1}{S_2}=\frac{a+m}{b+m}$



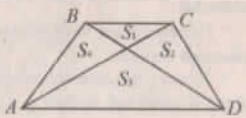
$BC \parallel MN \parallel AD,\quad \frac{x}{a}+\frac{y}{b}=1$



$S_1=S_2,\quad MN=\sqrt{\frac{a^2+b^2}{2}}$



$S_{A'B'C'D'}=5\cdot S_{ABCD}$



$S_2=S_4=\sqrt{S_1\cdot S_3},\quad S=\left(\sqrt{S_1}+\sqrt{S_3}\right)^2$

Qo'shimcha:

* ***Qavariq to'rtburchak*** tomonlarining o'rtalarini tutashtirish natijasida ***parallelogramm*** hosil bo'ladi.
* ***Parallelogramm*** tomonlarining o'rtalarini tutashtirish natijasida yana ***parallelogramm*** hosil bo'ladi.
* ***To'g'ri to'rtburchak*** tomonlarining o'rtalarini tutashtirish natijasida ***romb*** hosil bo'ladi.
* ***Kvadrat*** tomonlarining o'rtalarini tutashtirish natijasida yana ***kvadrat*** hosil bo'ladi.
* ***Romb*** tomonlarining o'rtalarini tutashtirish natijasida ***to'g'ri to'rtburchak*** hosil bo'ladi.
* ***Teng yonli trapetsiya*** tomonlarining o'rtalarini tutashtirish natijasida ***romb*** hosil bo'ladi.

KO'PBURCHAKLAR

Ta'rif: O'z-o'zi bilan kesishmaydigan yopiq siniq chiziq ***ko'pburchak*** deyiladi.

Siniq chiziq bo'g'inlari ***ko'pburchakning tomonlari***, siniq chiziq uchlari ***ko'pburchakning uchlari***, qo'shni bo'lmagan uchlarini tutashtiruvchi kesmalar esa ***ko'pburchakning diagonallari*** deyiladi.

Ta'rif: Agar ko'pburchak o'zining istalgan tomoni yotgan to'g'ri chiziqqa nisbatan bir tomonda yotsa, bunday ko'pburchak ***qavariq ko'pburchak*** deyiladi.

Qavariq n-burchakning xossalari:

1. Ichki burchaklari yig'indisi:	$(n-2)\pi$
2. Tashqi burchaklari yig'indisi:	2π
3. Diagonallari soni:	$\frac{n(n-3)}{2}$
4. Bir uchdan chiquvchi diagonallari soni:	$n-3$

Qo'shimcha:

* Hamma uchlari biror aylanada yotgan ko'pburchak ***aylanaga ichki chizilgan ko'pburchak*** deyiladi.
* Hamma tomonlari biror aylanaga urinadigan ko'pburchak ***aylanaga tashqi chizilgan ko'pburchak*** deyiladi.

Muntazam ko'pburchaklar

Ta'rif: Barcha tomonlari va barcha burchaklari teng bo'lgan qavariq ko'pburchak ***muntazam ko'pburchak*** deyiladi.

Ichki burchagi: $\frac{(n-2)\pi}{n}$ ***Tashqi burchagi:*** $\frac{2\pi}{n}$

$$r=\frac{a}{2tg\frac{\pi}{n}},\quad r=\sqrt{R^2-\frac{a^2}{4}} \qquad R=\frac{a}{2\sin\frac{\pi}{n}},\quad R=\sqrt{r^2+\frac{a^2}{4}}$$

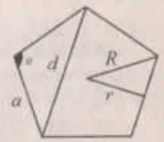
$$a=2\sqrt{R^2-r^2},\quad S=\frac{n\cdot a\cdot r}{2},\quad S=n\cdot r^2\cdot tg\frac{\pi}{n},\quad S=\frac{n}{2}\cdot R^2\cdot\sin\frac{2\pi}{n}$$

Qo'shimcha:

* Muntazam ko'pburchakning (***juft tomonli***) bir uchidan chiquvchi eng katta va eng kichik ***diagonallari*** orasidagi burchak (φ): $\varphi=\alpha-90^0$ (α-bitta ichki burchak)
* Radiusi R ga teng bo'lgan aylanaga tashqi chizilgan muntazam n-burchakning tomoni b ga teng bo'lsa, shu aylanaga ichki chizilgan muntazam n-burchakning tomoni (a): $a=\frac{2bR}{\sqrt{4R^2+b^2}}$

Muntazam beshburchak

Ichki burchaklari yig'indisi:	540^0
Ichki burchagi:	108^0
Tashqi burchagi:	72^0
Diagonallar soni:	5



$$a=\sqrt{\frac{5-\sqrt5}{2}}R,\quad a=2\sqrt{5-2\sqrt5}\,r,\quad a=\frac{\sqrt5-1}{2}d$$

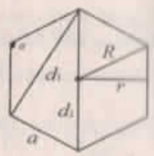
$$d=\frac{\sqrt5+1}{2}a,\quad d=\sqrt{\frac{5+\sqrt5}{2}}R\quad d=\sqrt{10-2\sqrt5}\,r$$

$$R=\sqrt{\frac{5+\sqrt5}{10}}a,\quad R=\left(\sqrt5-1\right)r,\quad r=\sqrt{\frac{5+2\sqrt5}{20}}a,\quad r=\frac{\sqrt5+1}{4}R$$

$$S=\frac{a^2}{4}\sqrt{25+10\sqrt5},\quad S=\frac{5}{8}R^2\sqrt{10+2\sqrt5},\quad S=5r^2\sqrt{5-2\sqrt5}$$

Muntazam oltiburchak

Ichki burchaklari yig'indisi:	720^0
Ichki burchagi:	120^0
Tashqi burchagi:	60^0
Diagonallar soni:	9



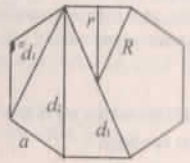
$$a=R,\quad a=\frac{2r}{\sqrt{3}},\qquad d_1^2+d_2^2=7a^2$$

$$d_1=a\sqrt{3}=2r=R\sqrt{3}=\frac{\sqrt{3}}{2}d_2,\qquad d_2=2a=\frac{4r}{\sqrt{3}}=2R=\frac{2d_1}{\sqrt{3}}$$

$$S=\frac{3\sqrt{3}}{2}a^2,\quad S=\frac{3\sqrt{3}}{2}R^2,\quad S=2\sqrt{3}r^2,\quad S=\frac{\sqrt{3}}{2}d_1^2,\quad S=\frac{3\sqrt{3}}{8}d_2^2$$

Muntazam sakkizburchak

Ichki burchaklari yig'indisi:	1080^0
Ichki burchagi:	135^0
Tashqi burchagi:	45^0
Diagonallar soni:	20



$$a=R\sqrt{2-\sqrt{2}},\quad a=2r\left(\sqrt{2}-1\right)$$

$$d_1=a\sqrt{2+\sqrt{2}},\qquad d_1=R\sqrt{2},\qquad d_1=2r\sqrt{2-\sqrt{2}}$$

$$d_2=a\left(1+\sqrt{2}\right),\qquad d_2=R\sqrt{2+\sqrt{2}},\qquad d_2=2r$$

$$d_3=a\sqrt{4+2\sqrt{2}},\qquad d_3=2R,\qquad d_3=2r\sqrt{4-2\sqrt{2}}$$

$$R=\frac{a}{2}\sqrt{4+2\sqrt{2}},\quad R=r\sqrt{4-2\sqrt{2}},\quad R=\frac{d_1}{\sqrt{2}},\quad R=\frac{d_2}{\sqrt{2+\sqrt{2}}},\quad R=\frac{d_3}{2}$$

$$r=\frac{a}{2}\left(\sqrt{2}+1\right),\quad r=\frac{R}{2}\sqrt{2+\sqrt{2}},\quad r=\frac{d_1}{2\sqrt{2-\sqrt{2}}},\quad r=\frac{d_2}{2},\quad r=\frac{d_3}{2\sqrt{4-2\sqrt{2}}}$$

$$S=2\left(\sqrt{2}+1\right)a^2,\quad S=2\sqrt{2}R^2,\quad S=8\left(\sqrt{2}-1\right)r^2,$$

$$S=\sqrt{2}d_1^2,\quad S=2(\sqrt{2}-1)d_2^2,\quad S=\frac{d_3^2}{\sqrt{2}}$$

STEREOMETRIYA

Stereometriya – geometriyaning bir bo'limi bo'lib, unda fazoviy shakllarning xossalari o'rganiladi.

1-***aksioma:*** Tekislik qanday bo'lmasin, shu tekislikka tegishli nuqtalar va unga tegishli bo'lmagan nuqtalar mavjud.

2-***aksioma:*** Agar ikkita turli tekislik umumiy nuqtaga ega bo'lsa, ular shu nuqtadan o'tuvchi to'g'ri chiziq bo'yicha kesishadi.

3-***aksioma:*** Bitta to'g'ri chiziqda yotmaydigan uchta nuqtadan bitta va faqat bitta tekislik o'tkazish mumkin.

1-teorema: Agar ikkita turli to'g'ri chiziq umumiy nuqtaga ega bo'lsa, ular orqali ***bitta va faqat bitta*** tekislik o'tkazish mumkin.

2-teorema: To'g'ri chiziq va unda yotmaydigan nuqta orqali ***bitta va faqat bitta*** tekislik o'tkazish mumkin.

3-teorema: To'g'ri chiziqning ikkita nuqtasi tekislikka tegishli bo'lsa, u holda to'g'ri chiziqning o'zi ham tekislikka tegishli bo'ladi.

Fazoda to'g'ri chiziqlar

Ta'rif: Fazodagi ikkita to'g'ri chiziq bir tekislikda yotsa va kesishmasa, ular ***parallel to'g'ri chiziqlar*** deyiladi.

Ta'rif: Fazodagi o'zaro parallel bo'lmagan va kesishmaydigan to'g'ri chiziqlar ***ayqash to'g'ri chiziqlar*** deyiladi.

* Ayqash to'g'ri chiziqlar orasidagi burchak deb, berilgan ayqash to'g'ri chiziqlarga parallel kesishuvchi to'g'ri chiziqlar orasidagi burchakka aytiladi.

* Ikkita ayqash to'g'ri chiziq bitta va faqat bitta umumiy perpendikulyarga ega.

* Perpendikulyar to'g'ri chiziqlarga mos ravishda parallel bo'lgan kesishuvchi to'g'ri chiziqlarning o'zlari ham perpendikulyardir.

Fazoda to'g'ri chiziq va tekisliklar

Teorema: Agar to'g'ri chiziq tekislikda yotgan biror to'g'ri chiziqqa parallel bo'lsa, u tekislikning o'ziga ham ***parallel*** bo'ladi.

Teorema: Agar to'g'ri chiziq tekislikdagi kesishuvchi ikki to'g'ri chiziqqa perpendikulyar bo'lsa, bu to'g'ri chiziq tekislikka ham ***perpendikulyar*** bo'ladi.

Uch perpendikulyar haqidagi teorema: Tekislikda og'maning asosidan uning proeksiyasiga perpendikulyar qilib o'tkazilgan to'g'ri chiziq og'maning o'ziga ham ***perpendikulyardir***.

Teorema: Agar bir tekislikning kesishuvchi ikki to'g'ri chizig'i ikkinchi tekislikdagi ikki to'g'ri chiziqqa mos holda parallel bo'lsa, bu ***tekisliklar parallel*** bo'ladi.

Teorema: Ikkita parallel tekislik orasiga joylashgan parallel to'g'ri chiziqlarning ***kesmalari teng*** bo'ladi.

Teorema: Agar to'g'ri chiziq ikkita parallel tekisliklarning biriga perpendikulyar bo'lsa, ularning ikkinchisiga ham ***perpendikulyar*** bo'ladi (agar ikki tekislik bitta to'g'ri chiziqqa perpendikulyar bo'lsa, ular o'zaro parallel bo'ladi).

Teorema: Agar tekislik o'zaro parallel ikki to'g'ri chiziqlarning biriga perpendikulyar bo'lsa, ularning ikkinchisiga ham ***perpendikulyar*** bo'ladi (agar ikki to'g'ri chiziq bitta tekislikka perpendikulyar bo'lsa, ular o'zaro parallel bo'ladi).

Ta'rif: Tekislikni kesib o'tib, unga perpendikulyar bo'lmagan to'g'ri chiziq, bu tekislikka ***og'ma*** deyiladi.

Ta'rif: To'g'ri chiziq bilan uning tekislikdagi proeksiyasi orasidagi burchakka to'g'ri chiziq bilan tekislik orasidagi ***burchak*** deyiladi.

KO'PYOQLAR

Ta'rif: Bitta AB to'g'ri chiziqdan chiquvchi ikkita α va β yarimtekislikdan tashkil topgan shakl ***ikki yoqli burchak*** deyiladi.

AB to'g'ri chiziq ikki yoqli burchakning ***qirrasi***, α va β yarimtekisliklar esa ikki yoqli burchakning ***yoqlari*** yoki ***tomonlari*** (***qirrasi***) deyiladi.

Ta'rif: Bir nuqtadan chiquvchi va bitta tekislikda yotmagan uchta a, b va c nur yordamida hosil qilingan uchta yassi (ab), (bc) va (ac) burchakdan tashkil topgan shakl ***uch yoqli burchak*** deyiladi.

* Uch yoqli burchakning har bir yassi burchagi uning qolgan ikkita yassi burchagi yig'indisidan kichik.
* Uch yoqli burchak yassi burchaklarining yig'indisi 360^0 dan kichik.

Ta'rif: Sirti chekli sondagi ko'pburchaklardan (yassi tekisliklardan) iborat jism ***ko'pyoq*** deyiladi.

Ko'pyoqni chegaralovchi ko'pburchaklar uning ***yoqlari***, qo'shni yoqlarining umumiy tomonlari uning ***qirralari*** deyiladi.

Ko'pyoqning bitta nuqtada uchrashadigan yoqlari ko'pyoqli burchak tashkil qiladi va bunday ko'pyoqli burchaklarning uchlari ***ko'pyoqning uchlari*** deyiladi.

Ko'pyoqning bitta yog'ida yotmagan ixtiyoriy ikkita uchini tutashtiruvchi to'g'ri chiziqlar ***ko'pyoqning diagonallari*** deyiladi.

Ta'rif: O'zining har bir yog'i tekisligining bir tomonida joylashgan ko'pyoq ***qavariq ko'pyoq*** deyiladi.

1-xossa: Qavariq ko'pyoqning barcha yoqlari qavariq ko'pburchaklardan iborat.

2-xossa: Qavariq ko'pyoq umumiy uchga ega bo'lgan va asoslari ko'pyoq yuzasini hosil qiladigan piramidalardan tashkil topishi mumkin.

3-xossa: Ko'pyoq sirtiga tegishli istalgan tekislikning bir tomonida yotadi.

Masalan: Prizma, parallelepiped, kub, piramidalar qavariq ko'pyoqlardir.

Eyler teoremasi: Ixtiyoriy n ***yoq*** uchun $U+Y-Q=2$ munosabat bajariladi.

Bu yerda U – ko'pyoqning uchlari soni, Y – yoqlari soni, Q esa qirralari soni.

Teorema: Ko'pyoq tekis burchaklarining soni uning qirralari sonidan ***ikki marta*** ko'p.

1-natija: Ko'pyoq tekis burchaklarining soni har doim juftdir.

2-natija: Agar ko'pyoqning har bir uchida bir xil k sondagi qirralar tutashsa, $U \cdot k = 2Q$ munosabat o'rinli.

3-natija: Agar ko'pyoqning barcha yoqlari bir xil n tomonli ko'pburchaklardan tashkil topgan bo'lsa, $Y \cdot n = 2Q$ munosabat o'rinli.

Teorema: Yoqlari soni Y va qirralari soni Q bo'lgan ko'pyoqning tekis burchaklari yig'indisi $(Y-Q)\cdot 360^0$ ga teng.

Prizma

Ta'rif: Ikki yog'i o'zaro parallel tekis ko'pburchaklardan, qolgan yoqlari esa parallelogrammlardan iborat bo'lgan ko'pyoq ***prizma*** deyiladi.

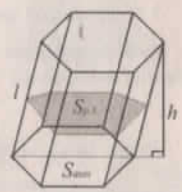
* Parallel ko'pburchaklar ***prizmaning asoslari***, parallelogrammlar esa uning ***yon yoqlari*** deyiladi.
* Prizma asoslarining tekisliklari orasidagi masofaga ***prizmaning balandligi*** deyiladi.
* Prizmaning bitta yog'iga tegishli bo'lmagan ikki uchini tutashtiruvchi kesmaga ***prizmaning diagonali*** deyiladi.

Ta'rif: Asoslari muntazam ko'pburchaklardan iborat bo'lgan va balandligi asosi markazidan o'tgan ko'pyoq ***muntazam ko'pyoq*** deyiladi.

Ta'rif: Yon yoqlari asos tekisliklariga perpendikulyar bo'lsa ***to'g'ri prizma***, aks holda ***og'ma prizma*** deyiladi.

Prizmaning parametrlari:

* Yon sirti (og'ma prizma): $S_{yon} = P_{p.k} \cdot l$
* Yon sirti (to'g'ri prizma): $S_{yon} = P_{asos} \cdot h$
* To'la sirti: $S_{to'la} = S_{yon} + 2S_{asos}$
* Hajmi (og'ma prizma): $V = S_{p.k} \cdot l$
* Hajmi (to'g'ri prizma): $V = S_{asos} \cdot h$
* Yoqlari soni: $Y = n + 2$
* Qirralari soni: $Q = 3n$
* Diagonallari soni: $D = n(n-3)$
* Ichki chizilgan shar radiusi: $r_{shar} = \frac{h}{2}$



$Y = \frac{Q}{3} + 2$

bu yerda l-qirra, h-balandlik, n-tomonlar soni, $P_{p.k}$-perpendikulyar kesim perimetri, $S_{p.k}$-perpendikulyar kesim yuzi.

Parallelepiped

Ta'rif: Prizmaning asosi parallelogramm bo'lsa, bunday prizmaga ***parallelepiped*** deyiladi.

Parallelepipedlar 3 xil:

1) Barcha yoqlari parallelogrammlardan iborat parallelepiped ***o'g'ma parallelepiped***.

2) Yon yoqlari to'g'ri to'rtburchaklardan iborat parallelepiped ***to'g'ri parallelepiped***.

3) Barcha yoqlari to'g'ri to'rtburchaklardan iborat parallelepiped esa ***to'g'ri burchakli parallelepiped***.

Har qanday parallelepipedda:

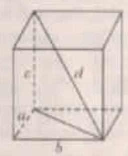
1) Qarama-qarshi yoqlar teng va parallel.

2) Barcha diagonallar bir nuqtada kesishadi va shu nuqtada har qaysi diagonal teng ikkiga bo'linadi.

Natija: Parallelepipedning diagonallari kesishadigan nuqta uning ***simmetriya markazidan*** iborat.

Ta'rif: To'g'ri burchakli parallelepipedning bir uchidan chiqqan uchta qirrasi uning ***uch o'lchovi*** deyiladi va $d^2 = a^2 + b^2 + c^2$.

To'g'ri burchakli parallelepipedning parametrlari:



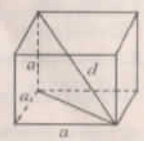
* Yon sirti: $S_{yon} = 2(ac + bc)$
* To'la sirti: $S_{to'la} = 2(ab + ac + bc)$
* Hajmi: $V = abc$
* Tashqi chizilgan shar radiusi: $R_{shar} = \frac{d}{2}$

Izoh: Papallelepiped 8 ta ***uch***, 12 ta ***qirra***, 6 ta ***yoq***, 4 ta ***diagonal*** va 5 ta ***simmetriya tekisligiga*** ega.

Kub

Ta'rif: Barcha qirralari teng bo'lgan to'g'ri burchakli parallelepiped ***kub*** deyiladi.

Kubning parametrlari:



* Yon sirti: $S_{yon} = 4a^2$
* To'la sirti: $S_{to'la} = 6a^2$
* Hajmi: $V = a^3$
* Diagonali: $d = a\sqrt{3}$
* Tashqi va ichki chizilgan shar radiusi: $r_{shar} = \frac{a}{2}$, $R_{shar} = \frac{a\sqrt{3}}{2} = \frac{d}{2}$

Izoh: Kub 8 ta ***uch***, 12 ta ***qirra***, 6 ta ***yoq***, 4 ta ***diagonal*** va 9 ta ***simmetriya tekisligiga*** ega.

Piramida

Ta'rif: Bitta yog'i ixtiyoriy qavariq ko'pburchakdan, qolgan yoqlari esa umumiy uchga ega bo'lgan uchburchaklardan iborat ko'pyoq ***piramida*** deyiladi.

* Uchburchaklarning umumiy nuqtasi S-***piramidaning uchi***, ko'pburchak ***piramidaning asosi***, uchburchaklar esa ***piramidaning yon yoqlari*** deyiladi.
* Piramidaning uchidan uning asosiga tushirilgan perpendikulyar ***piramidaning balandligi*** ($SO = h$) deyiladi.
* Piramidaning uchi va asosining diagonali orqali o'tkazilgan kesim ***piramidaning diagonal kesimi*** deyiladi.

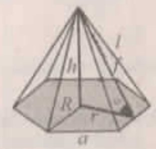
Ta'rif: Piramida yon yoqlari yuzlarining yig'indisi uning ***yon sirti yuzi*** yoki ***yon sirti*** deyiladi va S_{yon} kabi belgilanadi.

* ***To'la sirti:*** $S_{to'la} = S_{yon} + S_{asos}$
* ***Hajmi:*** $V = \frac{1}{3} S_{asos} \cdot h$

Ta'rif: Agar piramidaning asosi muntazam ko'pburchak bo'lib, balandligi asosining markazidan o'tsa, bunday piramida ***muntazam piramida*** deyiladi.

Muntazam piramida yon yog'ining balandligi ***piramidaning apofemasi*** deyiladi.

Muntazam piramidaning parametrlari:



* Asosining perimetri: $P_{asos} = n \cdot a$
* Asosining yuzi: $S_{asos} = \frac{n \cdot a \cdot r}{2}$
* Yon qirra: $l = \sqrt{R^2 + h^2}$
* Apofema: $f = \sqrt{r^2 + h^2}$
* Yon sirti: $S_{yon} = \frac{1}{2} P_{asos} \cdot f$

bu yerda φ-ikki yoqli burchak.

1-eslatma: Agar piramidaning yon yoqlari asos tekisligi bilan bir xil burchak hosil qilsa yoki apofemalari teng bo'lsa, u holda ***piramida balandligi asosga ichki chizilgan aylana markaziga*** tushadi. φ-ikki yoqli burchak esa

$$\cos\varphi = \frac{S_{asos}}{S_{yon}}$$

2-eslatma: Agar piramidaning yon qirralari asos tekisligi bilan bir xil burchak hosil qilsa yoki yon qirralari teng bo'lsa, u holda ***piramida balandligi asosga tashqi chizilgan aylana markaziga*** tushadi.

3-eslatma: Muntazam piramidaning yon qirralari ham, yon yoqlari ham asos tekisligi bilan ***bir xil burchak*** tashkil qiladi.

Kesik piramida

<u>**Ta'rif:**</u> Asoslari ikkita parallel ko'pburchaklardan, yon yoqlari esa trapetsiyalardan iborat ko'pyoq ***kesik piramida*** deyiladi.

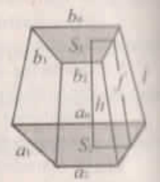
* ***To'la sirti:*** $S_{to'la} = S_{yon} + S_{1asos} + S_{2asos}$

* ***Hajmi:*** $V = \frac{1}{3}h\left(S_1 + \sqrt{S_1S_2} + S_2\right)$

Muntazam kesik piramidaning parametrlari:

* Yon sirti: $S_{yon} = \frac{1}{2}(P_1 + P_2)\cdot f$

bu yerda f-apofema, P_1 va P_2-asoslarining perimetrlari.



Xususiy hollar

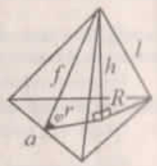
1. *Muntazam uchburchakli piramida*

* Asosi muntazam uchburchakdan iborat piramida ***muntazam uchburchakli piramida*** deyiladi.

$f = \sqrt{r^2 + h^2}$, $\quad l = \sqrt{R^2 + h^2}$

$S_{yon} = \frac{3}{2}af$ $\quad S_{asos} = \frac{a^2\sqrt{3}}{4}$

$V = \frac{1}{3}S_{asos}h$, $\quad V = \frac{a^2\sqrt{3}}{12}h$



bu yerda a-asos tomoni, l-yon qirra, f-apofema, h-balandlik.

2. *Muntazam to'rtburchakli piramida*

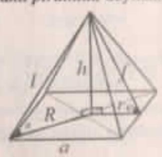
* Asosi kvadratdan iborat piramida ***muntazam to'rtburchakli piramida*** deyiladi.

$f = \sqrt{r^2 + h^2}$, $\quad l = \sqrt{R^2 + h^2}$

$r = \frac{a}{2\sqrt{3}}$, $\quad R = \frac{a}{\sqrt{3}}$

$S_{yon} = 2af$, $\quad S_{asos} = a^2$

$V = \frac{1}{3}S_{asos}h$, $\quad V = \frac{a^2\sqrt{3}}{12}h$



bu yerda a-asos tomoni, l-yon qirra, f-apofema, h-balandlik, φ-ikki yoqli burchak

Qo'shimcha:

* Apofemalari teng bo'lgan uchburchakli piramida hajmi (S-asos yuzi, p-yarim perimetr, φ-ikki yoqli burchak): $\quad V = \frac{S^2}{3p}tg\varphi$

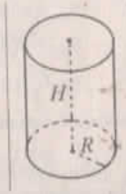
* Yon qirralari teng bo'lgan uchburchakli piramida hajmi (a, b va c-asos tomonlari, α-qirra va asos tekisligi orasidagi burchak): $\quad V = \frac{abc}{12}tg\alpha$

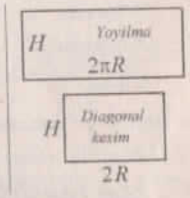
Silindr

<u>**Ta'rif:**</u> To'g'ri to'rtburchakni biror tomoni atrofida aylantirishdan hosil bo'lgan aylanma jismga ***silindr*** deyiladi.

* Asosi yuzi: $S_{asos} = \pi R^2$
* Yon sirti: $S_{yon} = 2\pi RH$
* To'la sirti: $S_{to'la} = 2\pi R(R + H)$
* Hajmi: $V = \pi R^2 H$
* Diagonal kesim yuzi: $S_d = 2RH$

bu yerda R-asos radiusi, H-balandlik.



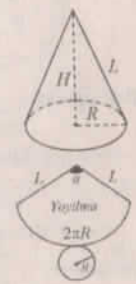


Konus

<u>**Ta'rif:**</u> To'g'ri burchakli uchburchakni biror kateti atrofida aylantirishdan hosil bo'lgan aylanma jismga ***konus*** deyiladi.

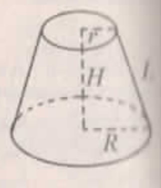
* Yasovchisi: $L = \sqrt{R^2 + H^2}$
* Asosi yuzi: $S_{asos} = \pi R^2$
* O'q kesim yuzi: $S_{o'q} = RH$
* Yon sirti: $S_{yon} = \pi RL$
* To'la sirti: $S_{to'la} = \pi R(R + L)$
* Hajmi: $V = \frac{1}{3}\pi R^2 H$
* Yoyilmaning uchidagi burchak: $\alpha = \frac{2\pi R}{L}$

bu yerda R-asos radiusi, H-balandlik, L-yasovchi.



Kesik konus

Ta'rif: Teng yonli trapetsiyani simmetriya o'qi atrofida aylantirishdan hosil bo'lgan aylanma jismga ***kesik konus*** deyiladi.



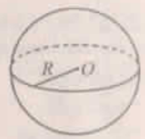
- Yasovchisi: $L=\sqrt{(R-r)^2+H^2}$
- Asosi yuzi: $S_1=\pi R^2,\quad S_2=\pi r^2$
- O'q kesim yuzi: $S_{o'q}=(R+r)H$
- Yon sirti: $S_{yon}=\pi L(R+r)$
- To'la sirti: $S_{to'la}=\pi[R^2+r^2+L(R+r)]$
- Hajmi: $V=\frac{\pi H}{3}(R^2+Rr+r^2)$

bu yerda R va r-asos radiuslari, H-balandlik, L-yasovchi.

Shar, sfera va shar qismlari

Ta'rif: Aylanani (doirani) diametri atrofida aylantirishdan hosil bo'lgan aylanma jismga ***sfera (shar)*** deyiladi.

$$S=4\pi R^2=\pi D^2$$
$$V=\frac{4\pi}{3}R^3=\frac{\pi}{6}D^3$$

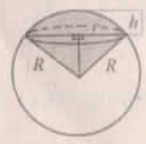


1. *Shar segmenti:*

$$r=\sqrt{h(2R-h)}$$
$$S_{yon}=2\pi Rh,\qquad S_{yon}=\pi(r^2+h^2)$$
$$S_{to'la}=\pi(2Rh+r^2)$$
$$V=\frac{\pi}{3}h^2(3R-h),\qquad V=\frac{\pi}{6}h(3r^2+h^2)$$

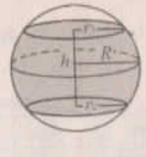
2. *Shar sektori:*

$$S_{to'la}=\pi R(2h+r)$$
$$V=\frac{2\pi}{3}R^2h$$



3. *Shar halqasi:*

$$S_{yon}=2\pi Rh$$
$$S_{to'la}=\pi(2Rh+r_1^2+r_2^2)$$
$$V=\frac{\pi}{6}h(3r_1^2+3r_2^2+h^2)$$



Shar va silindr

1. Silindrga tashqi chizilgan shar radiusi: $R_{shar}=\frac{1}{2}\sqrt{H^2+4R_s^2}$
2. Silindrga ichki chizilgan shar radiusi: $r_{shar}=\frac{H}{2}$

Eslatma: Silindrga har doim tashqi shar (sfera) chizish mumkin, lekin ichki shar (...ra) chizib bo'lavermaydi. Silindrga ichki shar chizish uchun uning o'q kesimi ...lari ($H=2R_s$) bo'lishi shart.

Shar va konus

1. Konusga tashqi chizilgan shar radiusi: $R_{shar}=\frac{L^2}{2H}$
2. Konusga ichki chizilgan shar radiusi: $r_{shar}=\frac{R_k H}{L+R_k}$
3. Kesik konusga ichki chizilgan shar radiusi: $r_{shar}=\sqrt{R_k\cdot r_k}$

Qo'shimcha:

O'q kesimlari muntazam uchburchak va kvadratdan iborat konus va silindrda:

1. $S^k_{to'la}=S^s_{to'la}$ bo'lsa, $\frac{V_k}{V_s}=\frac{\sqrt{2}}{\sqrt{3}}$, $\frac{R_k}{R_s}=\sqrt{2}:1$
2. $V_k=V_s$ bo'lsa, $\frac{S^k_{to'la}}{S^s_{to'la}}=\frac{\sqrt[3]{3}}{\sqrt[3]{2}}$, $\frac{R_k}{R_s}=\sqrt[6]{12}:1$

Shar va ko'pyoq

1. Sharga tashqi chizilgan prizma ($H=2r_{shar}$): $V_{pr}=\frac{r_{shar}}{3}S_{to'la_{pr}}$
2. Sharga tashqi chizilgan piramida: $V_{pir}=\frac{r_{shar}}{3}S_{to'la_{pir}}$
3. Sharga ichki chizilgan piramida: $L=2HR_{shar}$

Qo'shimcha:

O'xshash ko'pyoqlar uchun: $\frac{S_1}{S_2}=\left(\frac{P_1}{P_2}\right)^2$, $\frac{V_1}{V_2}=\left(\frac{P_1}{P_2}\right)^3$

Muntazam ko'pyoqlar haqida ma'lumotlar

Nomi	R_{shar}	r_{shar}	$\cos\varphi$	$S_{to'la}$	V
Muntazam tetraedr	$\frac{a\sqrt{6}}{4}$	$\frac{a\sqrt{6}}{12}$	$\frac{1}{3}$	$a^2\sqrt{3}$	$\frac{\sqrt{2}}{12}a^3$
Kub	$\frac{a\sqrt{3}}{2}$	$\frac{a}{2}$	0	$6a^2$	a^3
Oktaedr	$\frac{a}{\sqrt{2}}$	$\frac{a}{\sqrt{6}}$	$-\frac{1}{3}$	$2a^2\sqrt{3}$	$\frac{\sqrt{2}}{3}a^3$
Dodekaedr	$\frac{a\sqrt{3}}{\sqrt{5}-1}$	$\frac{a}{2}\sqrt{\frac{25+11\sqrt{5}}{10}}$	$-\frac{1}{\sqrt{5}}$	$3a^2\sqrt{25+10\sqrt{5}}$	$\frac{a^3}{4}\sqrt{15+7\sqrt{5}}$
Ikosaedr	$\frac{a}{4}\sqrt{10+2\sqrt{5}}$	$\frac{a(3+\sqrt{5})}{4\sqrt{3}}$	$-\frac{1}{\sqrt{5}}$	$5a^2\sqrt{3}$	$\frac{5a^3}{12}(3+\sqrt{5})$

Ba'zi yig'indilar

1. $1^2+2^2+3^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$
2. $1^2+3^2+5^2+...+(2n-1)^2=\frac{n(4n^2-1)}{3}$
3. $2^2+4^2+6^2+...+(2n)^2=\frac{2n(n+1)(2n+1)}{3}$
4. $1^3+2^3+3^3+...+n^3=\frac{n^2(n+1)^2}{4}$
5. $1^3+3^3+5^3+...+(2n-1)^3=n^2(2n^2-1)$
6. $1^4+2^4+3^4+...+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
7. $1\cdot2+2\cdot3+3\cdot4+...+n(n+1)=\frac{n(n+1)(n+2)}{3}$
8. $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+...+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$
9. $1\cdot4+2\cdot7+3\cdot10+...+n(3n+1)=n(n+1)^2$
10. $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+...+\frac{1}{n(n+1)}=\frac{n}{n+1}$

11. $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+...+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$
12. $\frac{1^2}{1\cdot3}+\frac{2^2}{3\cdot5}+\frac{3^2}{5\cdot7}+...+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}$
13. $\frac{1}{1\cdot2}+\frac{7}{2\cdot3}+\frac{13}{3\cdot4}+...+\frac{n^2+n+1}{n(n+1)}=\frac{n(n+2)}{n+1}$
14. $\frac{1}{2^1}+\frac{2}{2^2}+\frac{3}{2^3}+...+\frac{n}{2^n}=2-\frac{n+2}{2^n}$
15. $\frac{1}{2^1}+\frac{3}{2^2}+\frac{5}{2^3}+...+\frac{2n-1}{2^n}=3-\frac{2n-3}{2^n}$
16. $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+...+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$
17. $\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+...+\frac{1}{n(n+1)(n+2)(n+3)}=\frac{n(n^2+6n+11)}{18(n+1)(n+2)(n+3)}$
18. $\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}+...+\frac{1}{(2n-1)(2n+1)(2n+3)}=\frac{n(n+1)}{2(2n+1)(2n+3)}$
19. $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdot...\cdot\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+2}{2(n+1)}$

QO'SHIMCHA MA'LUMOTLAR

1. O'lchov birliklari:

a) uzunlik o'lchovlari (*metr*):

1 km = 1 000 m = 10 000 dm = 100 000 sm 1 000 000 mm;
1 m = 10 dm = 100 sm = 1 000 mm;
1 dm = 10 sm = 100 mm;
1 sm = 10 mm; 1 dyum = 25,4 mm.

b) yuza o'lchovlari (*metr kvadrat*):

1 km² = 1 000 000 m²; 100 m² = 1 *sotx*;
1 m² = 100 dm²; 10 000 m² = 1 *gektar* = 100 *ar*;
1 dm² = 100 sm²;
1 sm² = 100 mm².

c) hajm o'lchovlari (*metr kub*):

1 m³ = 1 000 dm³; 1 dm³ = 1 *litr*;
1 dm³ = 1 000 sm³;
1 sm³ = 1 000 mm³.

d) og'irlik o'lchovlari (*kilogramm*):

1 *tonna* = 10 *sentner* = 1 000 kg = 1 000 000 g;
1 *sentner* = 100 kg = 100 000 g;

1 kg = 1 000 g;

1 g = 1 000 mg; 1 *pud* = 10 kg 9 g.

e) vaqt o‘lchovlari (*minut*):

1 *sutka* = 24 *soat*;

1 *soat* = 60 *minut*;

1 *minut* = 60 *sekund*.

2. Oxirgi raqamni topish qoidalari ($n \in \mathbb{N}$)

$...0^n = ...0$	$...5^n = ...5$	$...1^n = ...1$	$...6^n = ...6$
$...4^{2n} = ...6$	$...4^{2n+1} = ...4$	$...9^{2n} = ...1$	$...9^{2n+1} = ...9$
$...2^{4n+1} = ...2$	$...3^{4n+1} = ...3$	$...7^{4n+1} = ...7$	$...8^{4n+1} = ...8$
$...2^{4n+2} = ...4$	$...3^{4n+2} = ...9$	$...7^{4n+2} = ...9$	$...8^{4n+2} = ...4$
$...2^{4n+3} = ...8$	$...3^{4n+3} = ...7$	$...7^{4n+3} = ...3$	$...8^{4n+3} = ...2$
$...2^{4n} = ...6$	$...3^{4n} = ...1$	$...7^{4n} = ...1$	$...8^{4n} = ...6$

3. (***Aralashmalar uchun***). Konsentrasiyasi $p\%$ bo‘lgan a litr eritma, konsentrasiyasi $q\%$ bo‘lgan b litr eritmaga aralashtirilsa, konsentrasiyasi $x\%$ bo‘lgan $a+b$ litr eritma hosil bo‘ladi:

$$x = \frac{a \cdot p + b \cdot q}{a+b}$$

4. (***Ish-harakat uchun***). x, y yakka holda qilingan ishlar, z esa birgalikda qilingan ishlar:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

5. (***Harakat uchun***). Bir-biridan S masofada turgan ikki (V_1 va V_2) jism bir-biri tomon (bir tomonga) harakatlanganda (t_0-uchrashuv vaqti, $\bar{t}_0$-quvib yetish vaqti):

$$S = (V_1 + V_2) \cdot t_0 \qquad (S = (V_1 - V_2) \cdot \bar{t}_0)$$

6. Agar $a \le x \le y \le z \le t \le b$ bo‘lsa, $\frac{x}{y} + \frac{z}{t}$ ning

$$\min\left(\frac{x}{y} + \frac{z}{t}\right) = 2 \cdot \sqrt{\frac{a}{b}} \quad \text{va} \quad \max\left(\frac{x}{y} + \frac{z}{t}\right) = 2$$

7. Agar $ax + by = c$ bo‘lib,

a) $a \cdot b > 0$ bo‘lsa, $x \cdot y$ ning ***eng katta qiymati***: $\max(x \cdot y) = \frac{c^2}{4ab}$

b) $a \cdot b < 0$ bo‘lsa, $x \cdot y$ ning ***eng kichik qiymati***: $\min(x \cdot y) = \frac{c^2}{4ab}$

8. Agar $x^2 + y^2 = c$ bo‘lsa,

a) $x \cdot y$ ning ***eng katta qiymati***: $\max(x \cdot y) = \frac{c}{2}$

b) $x \cdot y$ ning ***eng kichik qiymati***: $\min(x \cdot y) = -\frac{c}{2}$

9. $x(x+a)(x+b)(x+a+b) + m^2$ ifodani $m = \pm\frac{ab}{2}$ bo‘lganda ***to‘la kvadrat*** ko‘rinishida ifodalash mumkin.

10. $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ... + a_2 x^2 + a_1 x + a_0$ **ko‘phadning:**

1) barcha koeffisiyentlari yig‘indisi: $P(1)$

2) barcha toq koeffisiyentlari yig‘indisi: $\frac{1}{2}[P(1) - P(-1)]$

3) barcha juft koeffisiyentlari yig‘indisi: $\frac{1}{2}[P(1) + P(-1)]$

4) ozod hadi: $P(0)$

11. Ba’zi ayniyatlar:

$$(x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$$

$$\frac{a(b-c)^3 + b(c-a)^3 + c(a-b)^3}{a^2(c-b) + b^2(a-c) + c^2(b-a)} = a+b+c$$

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ac} = a+b+c$$

12. $f(x) = a^2\frac{(x-b)(x-c)}{(a-b)(a-c)} + b^2\frac{(x-a)(x-c)}{(b-a)(b-c)} + c^2\frac{(x-a)(x-b)}{(c-a)(c-b)}$ funksiya $f(x) = x^2$ funksiyaga teng kuchli. $x = x_0$ nuqtadagi qiymati $f(x_0) = x_0^2$ ga teng.

13. $f(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}$ funksiya $f(x) = 1$ funksiyaga teng kuchli. $x = x_0$ nuqtadagi qiymati $f(x_0) = 1$ ga teng.

14. Agar $y = f(x)$ funksiya monoton o‘suvchi bo‘lsa, u holda $\forall k \in \mathbb{N}$ son uchun $f(f(...f(x))...) = x$ va $f(x) = x$ funksiyalar teng kuchli bo‘ladi. Masalan,

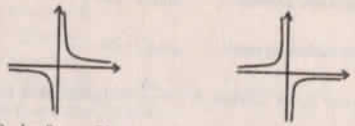
$$(x^2 + px + q)^2 + p(x^2 + px + q) + q = x \quad \text{va} \quad x^2 + px + q = x$$

15. Quyidagi tengsizliklarning butun yechimlari yig‘indisi:

$\frac{1}{|x-a|} > \frac{1}{b}$ $\quad 2a(b-1)$; $\qquad \frac{1}{|x-a|} \ge \frac{1}{b}$ $\quad 2ab$

Bu yerda a va b natural sonlar.

16. $y = a + \dfrac{b}{x+c}$ funksiya grafigi



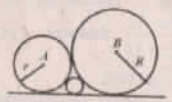
$(a=0,\ b>0,\ c=0)$ $(a=0,\ b<0,\ c=0)$

1) agar $a>0$ bo'lsa, Oy o'qi bo'yicha (Ox o'qiga parallel) yuqoriga suriladi.
agar $a<0$ bo'lsa, Oy o'qi bo'yicha (Ox o'qiga parallel) pastga suriladi.
2) agar $c>0$ bo'lsa, Ox o'qi bo'yicha (Oy o'qiga parallel) chapga suriladi.
agar $c<0$ bo'lsa, Ox o'qi bo'yicha (Oy o'qiga parallel) o'ngga suriladi.

17. AB kesma K aylananing diametri bo'lsin. L aylana K aylanaga hamda AB to'g'ri chiziqqa K aylananing markaziga urinadi. M aylana L aylana K aylanalarga hamda AB to'g'ri chiziqqa urinadi. Bunda

$R_M = r, \quad R_L = 2r, \quad R_K = 4r$

18. R va r radiusli ikkita aylana bir-biriga va to'g'ri chiziqqa urinadi. Shu to'g'ri chiziqqa va aylanalarga urinadigan kichik aylana radiusi (r_0):



$$r_0 = \frac{R\cdot r}{(\sqrt{R}+\sqrt{r})^2} \text{ yoki } \frac{1}{\sqrt{r_0}} = \frac{1}{\sqrt{R}} + \frac{1}{\sqrt{r}}$$

19. Parallelepipedning asoslari tomoni a ga teng kvadratlardan, barcha yon yoqlari romblardan iborat. Yuqori asosining uchlaridan biri ostki asosining barcha uchlaridan baravar uzoqlikda joylashgan. Parallelepipedning hajmi $V = \dfrac{a^3}{\sqrt{2}}$ ga teng.

20. $ABCD$ tetraedrning D uchidagi barcha yassi burchaklar to'g'ri. Shu tetraedrga kub shunday ichki chizilganki, kubning bitta uchi D nuqtada, unga qarama-qarshi uchi esa ABC yoqda yotibdi. Agar $DA=a,\ DB=b$ va $DC=c$ bo'lsa, kub qirrasining uzunligi $l = \dfrac{abc}{ab+bc+ac}$ ga teng.

MUNDARIJA

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