O'ZBEKISTON RESPUBLIKASI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

SAMARQAND DAVLAT UNIVERSITETI

MATEMATIKA

informatika o'qitish metodikasi yo'nalishi uchun amaliy mashg'ulotlar (1-qism)

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SAMARQAND – 2019

Matematika fani bo'yicha informatika o'qitish metodikasi yo'nalishi uchun amaliy mashg'ulotlar (5110700–informatika o'qitish metodikasi bakalavr ta'lim yo'nalishi talabalari uchun) – Samarqand: 2019. – 155 bet

O'quv qo'llanma Tahrir hayatining tavsiyasiga asosan, Samarkand davlat universiteti 2016 yil 18 noyabrdagi Ilmiy kengashinig №3 qaroriga asosan chop etilgan.

Hozirda ma'lumki, har bir ta'lim yo'nalishi bo'yicha bilim olayotgan talabaga boshqa bir fanni o'qitishda bu fanlarning o'zaro munosabatiga, o'qitilayotgan fanning kasbga yo'nalgan sohaga tatbiqlariga jiddiy e'tibor berilmayotganligini ko'rishimiz mumkin. Shu bilan bir qatorda bakalavriatning turli ta'lim yo'nalishlarida ta'lim olayotgan fanni bir xil o'qitilishi, ularga o'rganilayotgan fan uning kelgusi kasbiy faoliyatida qay darajada kerak bo'lishi to'g'risidagi tushunchaga va uning o'rnini sezishga imkon bermaydi. Shuning uchun ham o'qitilayotgan fanning talabaga chuqurroq singdirish maqsadida bu fanni kasbga yo'naltirilgan holda o'qitishni maqsadga muvofiq deb hisoblaymiz.

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ISBN-978-9943-6179-3-3

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MUNDARIJA

SO'Z BOSHI

Matematika fani yoshlarning mantiqiy fikrlash qobiliyatini o'stiruvchi vosita sifatida qadimgi Yunoniston maktablarida o'qita boshlangan. Yangi era boshlarida Xitoyda sonlar nazariyasi, Hindistonda o'nli sanoq sistemasi, O'rta Yer dengiz sohillarida trigonometriya yaratila boshlangan. VII-VIII asrlardan boshlab ilm-fan taraqqiyotining markazi O'rta Osiyoga ko'chdi. O'z ilmiy ishlari bilan butun dunyoga tanilgan Muhammad Muso al-Xorazmiy, Ahmad Farg'oniy, Abu Rayhon Beruniy, Abu Ali ibn Sino, Abu Nasr Forobiy, Ismoil Buxoriy, Umar Hayyom, Ulug'bek va boshqalar O'rta Osiyoda yashab ijod qilganlar.

Ma'lumki real obyektlar juda murakkab bo'ladi. Ularni o'rganish uchun modellar yasaladi. Modellarni o'rganish natijasida obyektlarga nisbatan xulosalar chiqariladi.

Matematik modellarni qurish *matematik modellashtirish* deb ataladi. Bu modellarni qurishda matamatika asosiy rolni o'ynaydi. Asosan matematik modellashtirish orqali boshqa fanlarni ilmiy izlanishlarida matematika qo'llaniladi. Bu informatikada yaqqol ko'zga tashlanadi.

Bu kitob matematika fanidan amaliy mashg'ulotlar bo'yicha qo'llanma bo'lib, universitet va institutlarning informatika o'qitish metodikasi ta'lim yo'nalishlari bo'yicha bilim olayotgan talabalariga mo'ljallagan, lekin matematikaning asosiy tushunchalari bilan mustaqil ravishda tanishmoqchi bo'lgan kitobxonlar uchun ham foydali.

Matematika bo'yicha juda ko'p darsliklar, masalalar to'plamlari bor bo'lsada, maxsus yo'nalish bo'yicha bilim olayotgan talabalar uchun, jumladan, informatika o'qitish metodikasi yo'nalishi bo'yicha ta'lim olayotgan talabalar uchun matematika masalalarini informatika va texnikaga qo'llab yechishga o'rgatadigan, ko'nikma hosil qiladigan, o'zbek tilida yozilgan kitoblarning kamligi sezilib turadi.

Bu kitobni yozishdan maqsad talabalar bu fanni o'rganishda qiziqishini oshirish, mustaqil ravishda masalalar yechishdagi aktivligini oshirish, ularning kelgusi kasbiy faoliyatining har bir qadamida kerak ekanligini tushuntirishdan iborat.

Ushbu informatika o'qitish metodikasi uchun matematikadan amaliy mashg'ulotlar bakalavriatning 5110700 – "Informatika o'qitish metodikasi" ta'lim yo'nalishi uchun mo'ljallangan bo'lib, u amaldagi davlat ta'lim standartlari va «Matematika» fani namunaviy dasturiga asosan tuzildi.

Yuqoridagi maqsadlarni nazarda tutgan holda taqdim etilayotgan mazkur kitob birinchi marta yozilayotgani uchun u kamchiliklardan xoli emas. Mualliflar o'quvchilar tomonidan ko'rsatilgan har qanday kamchilik va takliflarni mamnuniyat bilan qabul qiladilar.

I BOB. OLIY ALGEBRA

1.1. Determinantlar va determinantlarning asosiy xossalari. Yuqori tartibli determinantlar.

To'rtta sondan iborat

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

kvadrat jadval *ikkinchi tartibli kvadrat matritsa* deyiladi.

Ikkinchi tartibli kvadrat matritsaga mos keluvchi *ikkinchi tartibli determinant* deb quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

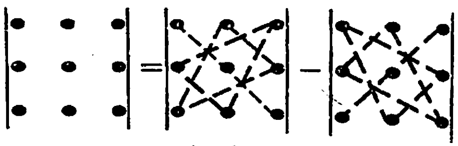
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}.$$

Shunga o'xshash ushbu

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} -$$

$$- a_{31}a_{22}a_{13} - a_{21}a_{12}a_{33} - a_{32}a_{23}a_{11}.$$

ifoda *uchinchi tartibli determinant* deyiladi. Bu ifodaga musbat ishora bilan kiradigan har bir ko'paytma, hamda manfiy ishorali ko'paytmalar ko'paytuvchilarini alohida-alohida punktir chiziqlar yordamida tutashtirib, uchinchi tartibli determinantlarni hisoblash uchun xotirada oson saqlanadigan «uchburchaklar qoidasi»ga ega bo'lamiz (1-shakl).



1- shakl

Determinant a_{ik} elementining M_{ik} *minori* deb, shu determinantdan bu element turgan qator va ustunni o'chirish natijasida hosil bo'lgan determinantga aytiladi.

Determinant a_{ik} elementining algebraik to'ldiruvchisi

$$A_{ik} = (-1)^{i+k} M_{ik}$$

munosabat bilan aniqlanadi.

Determinantlarning asosiy xossalari:

a) agar determinantning barcha satrlari mos ustunlari bilan almashtirilsa, uning qiymati o'zgarmaydi;

Keyingi xossalarni ta'riflashda satrlar va ustunlarni bir so'z bilan *qator* deb ataymiz.

b) agar determinant nollardan iborat qatorga ega bo'lsa, uning qiymati nolga teng bo'ladi;

c) agar determinant ikkita bir xil parallel qatorga ega bo'lsa, uning qiymati nolga teng bo'ladi;

d) agar determinant ikkita parallel qatorining mos elementlari mo'tanosib (proporsional) bo'lsa, uning qiymati nolga teng bo'ladi;

e) biror qator elementlarining umumiy ko'paytuvchisini determinant belgisidan tashqariga chiqarish mumkin;

f) agar determinant ikkita parallel qatorining o'rinlari almashtirilsa, determinant ishorasini qarama-qarshisiga o'zgartiradi;

g) determinantning qiymati biror qator elementlari bilan shu elementlarga tegishli algebraik to'ldiruvchilari ko'paytmalari yig'indisiga teng.

Bu xossa *determinantni* qator elementlari bo'yicha *yoyish* deyiladi. Undan determinantlarni hisoblashda foydalaniladi.

h) biror qator elementlari bilan parallel qator mos elementlari algebraik to'ldiruvchilari ko'paytmalarining yig'indisi nolga teng.

i) agar determinant biror qatorining har bir elementi ikki qo'shiluvchining yig'indisidan iborat bo'lsa, u holda determinant ikki *determinant yig'indisiga teng bo'lib, ularning biri tegishli qator birinchi* qo'shiluvchilardan, ikkinchisi esa ikkinchi qo'shiluvchilardan iborat bo'ladi. Masalan,

$$\begin{vmatrix} a_{11}a_{12} & + & b_1a_{13} \\ a_{21}a_{22} & + & b_2a_{23} \\ a_{31}a_{32} & + & b_3a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

j) agar determinantning biror qatori elementlariga parallel qatorning mos elementlarini biror o'zgarmas songa ko'paytirib qo'shilsa, determinantning qiymati o'zgarmaydi. Masalan:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31}+\lambda a_{11} & a_{32}+\lambda a_{12} & a_{33}+\lambda a_{13} \end{vmatrix}$$

$(n\times n)$ ta sondan iborat ushbu

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & .. & a_{nn} \end{vmatrix}$$

jadval n- *tartibli kvadrat matritsa* deyiladi. Uning n- *tartibli determinanti* deb quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots \\ a_{n2} & a_{n2} & .. & a_{nn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}.$$

Bandda keltirilgan xossalarning hammasi istalgan tartibli determinantga tegishlidir. Ixtiyoriy tartibli determinantni hisoblashning ikkita usulini keltiramiz:

1. *Determinant tartibini pasaytirish usuli* — determinant biror qatori elementlarining bittasidan boshqalarini oldindan nolga aylantirib olib, shu qator bo'yicha yoyish usuli.

1-misol.

$$\Delta = \begin{vmatrix} 3 & -1 & 12 & 8 \\ -5 & 3 & -34 & -23 \\ 1 & 1 & 3 & -7 \\ -9 & 2 & 8 & -15 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 12 & 8 \\ 4 & 0 & 2 & 1 \\ 4 & 0 & 15 & 1 \\ -3 & 0 & 32 & 1 \end{vmatrix} =$$

$$= -(-1)^3 \begin{vmatrix} 4 & 2 & 1 \\ 4 & 15 & 1 \\ -3 & 32 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 1 \\ 0 & 13 & 0 \\ -7 & 30 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 13 \\ -7 & 30 \end{vmatrix} = 91.$$

Berilgan misolni maple orqali yechamiz:

with(*LinearAlgebra*) :
A := *Matrix*(4, [[3, -1, 12, 8], [-5, 3, -34, -23], [1, 1, 3, -7], [-9, 2, 8, -15]])
;

$$A := \begin{bmatrix} 3 & -1 & 12 & 8 \\ -5 & 3 & -34 & -23 \\ 1 & 1 & 3 & -7 \\ -9 & 2 & 8 & -15 \end{bmatrix}$$

> *Determinant*(*A*);

91

2. *Determinantni uchburchak ko'rinishga keltirish usuli* determinantni shunday almashtirishdan iboratki, uning bosh diagonalidan bir tomonida yotuvchi hamma elementlari nolga aylantiriladi va uchburchaksimon shaklga keltiriladi, masalan

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{vmatrix}$$

Ravshanki, uchburchak shaklidagi determinantning qiymati bosh diagonallari elementlari ko'paytmasiga teng:

$$\Delta = a_{11}\cdot a_{22}\cdot\ldots\cdot a_{nn}.$$

2- misol.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ -2 & -4 & -6 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 8 \end{vmatrix} = 1\cdot 2\cdot 3\cdot 8.$$

1.2. Mustaqil ishlash uchun topshiriqlar

Qaysidir satr yoki ustun bo'yicha yoyib detrminantni hisoblang.

1. $\begin{vmatrix} 3 & -1 & 2 & 0 \\ 1 & -2 & 4 & 3 \\ 5 & 3 & 6 & 2 \\ 4 & 1 & 1 & -3 \end{vmatrix}$;

2. $\begin{vmatrix} 2 & 1 & -1 & 3 \\ 4 & 3 & 5 & 0 \\ -1 & 2 & 6 & 7 \\ -3 & -5 & 2 & -2 \end{vmatrix}$;

3. $\begin{vmatrix} 4 & 1 & -2 & 5 \\ 3 & 0 & -1 & 2 \\ 1 & 7 & -4 & 1 \\ -3 & 6 & 1 & 2 \end{vmatrix}$;

4. $\begin{vmatrix} -3 & 5 & 0 & 2 \\ 1 & 3 & 4 & -3 \\ 2 & -1 & 6 & 1 \\ -4 & 2 & 4 & 7 \end{vmatrix}$;

5. $\begin{vmatrix} -4 & -5 & 2 & -2 \\ 2 & 6 & 3 & 5 \\ -3 & 1 & 4 & 3 \\ -1 & 0 & 1 & 7 \end{vmatrix}$;

6. $\begin{vmatrix} 5 & 1 & -1 & -5 \\ -4 & 2 & 7 & -3 \\ 3 & 6 & -2 & 1 \\ 1 & 1 & 0 & 4 \end{vmatrix}$;

7. $\begin{vmatrix} 6 & 2 & 1 & 7 \\ 4 & -2 & 0 & -6 \\ -1 & 3 & -5 & -3 \\ 3 & 5 & 2 & 1 \end{vmatrix}$;

8. $\begin{vmatrix} -5 & 3 & -3 & 0 \\ 2 & -4 & -1 & 2 \\ 1 & 6 & 5 & 3 \\ 7 & -2 & 4 & 1 \end{vmatrix}$;

9. $\begin{vmatrix} 2 & 1 & -3 & 6 \\ 1 & 4 & 8 & -4 \\ -1 & 5 & -2 & 1 \\ 3 & 0 & -1 & 1 \end{vmatrix}$;

10. $\begin{vmatrix} -3 & 5 & 6 & -2 \\ 1 & 3 & -7 & 0 \\ 2 & -1 & 4 & -5 \\ 4 & 2 & -4 & -1 \end{vmatrix}$;

11. $\begin{vmatrix} -4 & -3 & 6 & 7 \\ -5 & 4 & 1 & 2 \\ 1 & -2 & -1 & 3 \\ 3 & 2 & -4 & 0 \end{vmatrix}$;

12. $\begin{vmatrix} 6 & 4 & 5 & 2 \\ -1 & -3 & 0 & -1 \\ -2 & -1 & 3 & -4 \\ 1 & 2 & 1 & 1 \end{vmatrix}$;

13. $\begin{vmatrix} 2 & 3 & -4 & 5 \\ 4 & -2 & 6 & -1 \\ 2 & 0 & 3 & 1 \\ -1 & -5 & 1 & 4 \end{vmatrix}$;

14. $\begin{vmatrix} -3 & 4 & 0 & -5 \\ -2 & 1 & -6 & -1 \\ 1 & 2 & 1 & -4 \\ -1 & -2 & 8 & 7 \end{vmatrix}$;

15. $\begin{vmatrix} -2 & 7 & 1 & 0 \\ -1 & -4 & 2 & -3 \\ 3 & -3 & 4 & 5 \\ 1 & 5 & 3 & 6 \end{vmatrix}$;

16. $\begin{vmatrix} 5 & -2 & -3 & -6 \\ -1 & 7 & 2 & -3 \\ 1 & 0 & -4 & 1 \\ 3 & 4 & 1 & 8 \end{vmatrix}$;

17. $\begin{vmatrix} 4 & -2 & 1 & -1 \\ 2 & 3 & 0 & 7 \\ -3 & 1 & 5 & -1 \\ -1 & 2 & -4 & 2 \end{vmatrix}$;

18. $\begin{vmatrix} 3 & 0 & 5 & 6 \\ -3 & 4 & -1 & 2 \\ 1 & 2 & -4 & -5 \\ -2 & -1 & -3 & 1 \end{vmatrix}$;

19. $\begin{vmatrix} -4 & -2 & -1 & 3 \\ 1 & 5 & -3 & 0 \\ -1 & 2 & 4 & 6 \\ 3 & 1 & 7 & -1 \end{vmatrix}$;

20. $\begin{vmatrix} 2 & 4 & -1 & -5 \\ -1 & 2 & 1 & -3 \\ 3 & -2 & 6 & 3 \\ 5 & 1 & -4 & 0 \end{vmatrix}$;

21. $\begin{vmatrix} -5 & -2 & -4 & -3 \\ 3 & 7 & 0 & -1 \\ 4 & 1 & 2 & 2 \\ 1 & -3 & 2 & 3 \end{vmatrix}$;

22. $\begin{vmatrix} -3 & 5 & -4 & -1 \\ 3 & 6 & -3 & -5 \\ -2 & 1 & -1 & 2 \\ -1 & 2 & 3 & 0 \end{vmatrix}$;

23. $\begin{vmatrix} -2 & 4 & 2 & -3 \\ 1 & 1 & -4 & 3 \\ -1 & 3 & 1 & -2 \\ -5 & 6 & -1 & 0 \end{vmatrix}$;

24. $\begin{vmatrix} 6 & -4 & -5 & 1 \\ 2 & 3 & 2 & -1 \\ -1 & 0 & 4 & 7 \\ 2 & -3 & 1 & -2 \end{vmatrix}$;

25. $\begin{vmatrix} 7 & 3 & -5 & -2 \\ -2 & 4 & 0 & -4 \\ 1 & 2 & -1 & -3 \\ -3 & -1 & 1 & 2 \end{vmatrix}$;

26. $\begin{vmatrix} 4 & 3 & -1 & 0 \\ -5 & 1 & -2 & -3 \\ 2 & 2 & 3 & -1 \\ -1 & 1 & 4 & -2 \end{vmatrix}$;

27. $\begin{vmatrix} -6 & 0 & 4 & -1 \\ 2 & 1 & 2 & 5 \\ -3 & -4 & -1 & -3 \\ -1 & -2 & 1 & -1 \end{vmatrix}$; 28. $\begin{vmatrix} 4 & -1 & 1 & -6 \\ -3 & 0 & -2 & 1 \\ 1 & 3 & -4 & -5 \\ -3 & 5 & -3 & -1 \end{vmatrix}$;

29. $\begin{vmatrix} 3 & -4 & 2 & -3 \\ -2 & -1 & -6 & 2 \\ 4 & 5 & 1 & -4 \\ 1 & -3 & -1 & 0 \end{vmatrix}$; 30. $\begin{vmatrix} -6 & 3 & 2 & -4 \\ 1 & -2 & -5 & -3 \\ 2 & -4 & 1 & 1 \\ -1 & 0 & 1 & -2 \end{vmatrix}$.

1.3. Ikki va uch noma'lumli chiziqli tenglamalar sistemasi. Kramer qoidasi. Gauss usuli

Ikki noma'lumli ikkita chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

ning bosh determinanti $\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ bo'lganda, yagona yechimga ega va u Kramer qoidasi bo'yicha quyidagi formulalar bilan hisoblanadi:

$$x_1 = \frac{\Delta_{x_1}}{\Delta}, \quad x_2 = \frac{\Delta_{x_2}}{\Delta},$$

bu yerda

$$\Delta_{x_1} = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}, \quad \Delta_{x_2} = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}.$$

Agar $\Delta = 0$ va shu bilan birga Δ_{x_1}, Δ_{x_2} lardan aqalli bittasi nolga teng bo'lmasa, sistema yechimga ega emas.

Agar $\Delta = \Delta_{x_1} = \Delta_{x_2} = 0$ bo'lsa, u holda berilgan sistema cheksiz ko'p yechimga ega bo'ladi.

Uch noma'lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

ning bosh determinanti

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

bo'lganda yagona yechimga ega bo'lib, bu yechim Kramer formulalari bilan hisoblanadi:

$$x_1 = \frac{\Delta_{x_1}}{\Delta}, \quad x_2 = \frac{\Delta_{x_2}}{\Delta}, \quad x_3 = \frac{\Delta_{x_3}}{\Delta},$$

bunda

$$\Delta_{x_1} = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_{x_2} = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_{x_3} = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

Agar $\Delta = 0$ va Δ_{x_1}, Δ_{x_2}, Δ_{x_3} determinantlardan aqalli bittasi noldan farqli bo'lsa, u holda berilgan sistema yechimga ega bo'lmaydi va bu sistema *birgalikda bo'lmagan sistema* deb ataladi. Kamida bitta yechimga ega bo'lgan sistema *birgalikdagi sistema* deb ataladi.

1-misol. Chiziqli tenglamalar sistemasini yeching:

$$\begin{cases} x_1 - 2x_2 + x_3 = -4, \\ 3x_1 + 2x_2 - x_3 = 8, \\ 2x_1 - 3x_2 + 2x_3 = -6. \end{cases}$$

Yechish. Determinantlarni topamiz:

$$\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ 2 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 \\ 0 & 8 & -4 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 8 & -4 \\ 1 & 0 \end{vmatrix} = 4.$$

Determinant $\Delta = 4 \neq 0$ bo'lgani uchun sistema yagona yechimga ega va Kramer formulasini qo'llab, uni topamiz:

$$\Delta_{x_1} = \begin{vmatrix} -4 & -2 & 1 \\ 8 & 2 & -1 \\ -6 & -3 & 2 \end{vmatrix} = \begin{vmatrix} -4 & -2 & 1 \\ 4 & 0 & 0 \\ 2 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} = 4,$$

$$\Delta_{x_2}=\begin{vmatrix}1&-4&1\\3&8&-1\\2&-6&2\end{vmatrix}=\begin{vmatrix}1&-4&1\\0&20&-4\\0&2&0\end{vmatrix}=\begin{vmatrix}20&-4\\2&0\end{vmatrix}=8,$$

$$\Delta_{x_3}=\begin{vmatrix}1&-2&-4\\3&2&8\\2&-3&-6\end{vmatrix}=\begin{vmatrix}1&-2&-4\\0&8&20\\0&1&2\end{vmatrix}=\begin{vmatrix}8&20\\1&2\end{vmatrix}=-4,$$

$$x_1=\frac{\Delta_{x_1}}{\Delta}=1,\quad x_2=\frac{\Delta_{x_2}}{\Delta}=2,\quad x_3=\frac{\Delta_{x_3}}{\Delta}=-1.$$

Berilgan misolni maple orqali yechamiz:

```
> with(LinearAlgebra) :
> A := ⟨⟨1, 3, 2⟩|⟨-2, 2, -3⟩|⟨1, -1, 2⟩⟩;
```

$$A:=\begin{bmatrix}1&-2&1\\3&2&-1\\2&-3&2\end{bmatrix}$$

```
> d := Determinant(A);
```

$$d:=4$$

```
> with(LinearAlgebra) :
> A1 := ⟨⟨-4, 8, 6⟩|⟨-2, 2, -3⟩|⟨1, -1, 2⟩⟩;
```

$$A1:=\begin{bmatrix}-4&-2&1\\8&2&-1\\6&-3&2\end{bmatrix}$$

```
> x := Determinant(A1);
```

$$x:=4$$

```
> with(LinearAlgebra) :
> A2 := ⟨⟨1, 3, 2⟩|⟨-4, 8, -6⟩|⟨1, -1, 2⟩⟩;
```

$$A2:=\begin{bmatrix}1&-4&1\\3&8&-1\\2&-6&2\end{bmatrix}$$

```
> y := Determinant(A2);
```

$$y:=8$$

```
> with(LinearAlgebra) :
> A3 := ⟨⟨1, 3, 2⟩|⟨-2, 2, -3⟩|⟨-4, 8, -6⟩⟩;
```


$$A3:=\begin{bmatrix}1&-2&-4\\3&2&8\\2&-3&-6\end{bmatrix}$$

```
> z := Determinant(A3);
```

$$z:=-4$$

```
> a1 := x/d; a2 := y/d; a3 := z/d;
```

$$a1:=1$$

$$a2:=2$$

$$a3:=-1$$

n ta noma'lumli n ta chiziqli tenglamalar sistemasini n ning katta $(n\geq 4)$ qiymatlarida Kramer qoidasi bilan yechish bir nechta yuqori tartibli diterminantlarni hisoblashni talab etadi. Shu sababli, bunday sistemalarni yechishda Gauss usulidan foydalanish maqsadga muvofiq. Bu usulning mohiyati shundan iboratki, unda noma'lumlar ketma-ket yo'qotilib, sistema uchburchaksimon shaklga keltiriladi. Agar sistema uchburchaksimon shaklga kelsa, u yagona yechimga ega bo'ladi va uning noma'lumlari oxirgi tenglamadan boshlab topib boriladi. (Sistema cheksiz ko'p yechimga ega bo'lsa, noma'lumlar ketma-ket yo'qotilgach, u trapesiyasimon shaklga keladi.)

2- misol. Ushbu

$$\begin{cases}x_1+x_2+5x_3+2x_4=1,\\x_1+x_2+3x_3+4x_4=-3,\\2x_1+3x_2+11x_3+5x_4=2,\\2x_1+x_2+3x_3+2x_4=-3,\end{cases}$$

chiziqli tenglamalar sistemasini Gauss usuli bilan yeching.

Yechish. Ikkinchi, uchinchi, to'rtinchi tenglamalardan x_1 larni yo'qotamiz. Buning uchun birinchi tenglamani ketma-ket -1,-2, - 2 ga ko'paytiramiz va mos ravishda ikkinchi, uchinchi, to'rtinchi tenglamalar bilan qo'shamiz. Natijada ushbu sistemaga ega bo'lamiz:

$$\begin{cases} x_1 + x_2 + 5x_3 + 2x_4 = 1, \\ 2x_3 - 2x_4 = 4, \\ x_2 + x_3 + x_4 = 0, \\ -x_2 - 7x_3 - 2x_4 = -5, \end{cases}$$

yoki

$$\begin{cases} x_1 + x_2 + 5x_3 + 2x_4 = 1, \\ x_2 + x_3 + x_4 = 0, \\ x_2 + 7x_3 + 2x_4 = 5, \\ x_3 - x_4 = 2. \end{cases}$$

Uchinchi tenglamadan ikkinchi tenglamani ayiramiz:

$$\begin{cases} x_1 + x_2 + 5x_3 + 2x_4 = 1, \\ x_2 + x_3 + x_4 = 0, \\ 6x_3 + x_4 = 5, \\ x_3 - x_4 = 2, \end{cases}$$

so'ngra to'rtinchi tenglamani -6 ga ko'paytirib, uchinchi tenglamaga qo'shsak, uchburchakli sistema hosil bo'ladi:

$$\begin{cases} x_1 + x_2 + 5x_3 + 2x_4 = 1, \\ x_2 + x_3 + x_4 = 0, \\ x_3 - x_4 = 2, \\ 7x_4 = -7. \end{cases}$$

Bundan,

$x_4 = 1,$

$x_3 = 2 + x_4 = 1,$

$x_2 = -x_3 - x_4 = 0,$

$x_1 = 1 - x_2 - 5x_3 - 2x_4 = -2.$

Shunday qilib,

$x_1 = -2,\ x_2 = 0,\ x_3 = 1,\ x_4 = 1.$

Berilgan misolni maple orqali yechamiz:

```
> with(LinearAlgebra) :
> A := ⟨⟨1, 1, 5, 2⟩|⟨1, 1, 3, 4⟩|⟨2, 3, 11, 5⟩|⟨2, 1, 3, 2⟩⟩;
```


$$A := \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 3 & 1 \\ 5 & 3 & 11 & 3 \\ 2 & 4 & 5 & 2 \end{bmatrix}$$

```
> d := Determinant(A);
```

$d := 66$

```
> with(LinearAlgebra);
> A1 := ⟨⟨1,-3, 2, 2⟩|⟨1, 1, 3, 4⟩|⟨2, 3, 11, 5⟩|⟨2, 1, 3, 2⟩⟩;
```

$$A1 := \begin{bmatrix} 1 & 1 & 2 & 2 \\ -3 & 1 & 3 & 1 \\ 2 & 3 & 11 & 3 \\ 2 & 4 & 5 & 2 \end{bmatrix}$$

```
> d1 := Determinant(A1);
```

$d1 := -143$

```
> with(LinearAlgebra) :
> A2 := ⟨⟨1, 1, 5, 2⟩|⟨1,-3, 2, 2⟩|⟨2, 3, 11, 5⟩|⟨2, 1, 3,
    2⟩⟩;
```

$$A2 := \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & -3 & 3 & 1 \\ 5 & 2 & 11 & 3 \\ 2 & 2 & 5 & 2 \end{bmatrix}$$

```
> d2 := Determinant(A2);
```

$d2 := -23$

```
> with(LinearAlgebra) :
> A3 := ⟨⟨1, 1, 5, 2⟩|⟨1, 1, 3, 4⟩|⟨1,-3, 2, 2⟩|⟨2, 1, 3, 2⟩⟩;
```

$$A3 := \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & -3 & 1 \\ 5 & 3 & 2 & 3 \\ 2 & 4 & 2 & 2 \end{bmatrix}$$

> *d3* := *Determinant*(*A3*);

$$d3 := 66$$

> *with*(*LinearAlgebra*) :

> *A4* := ⟨⟨1, 1, 5, 2⟩|⟨1, 1, 3, 4⟩|⟨2, 3, 11, 5⟩|⟨1, -3, 2, 2⟩⟩;

$$A4 := \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 3 & -3 \\ 5 & 3 & 11 & 2 \\ 2 & 4 & 5 & 2 \end{bmatrix}$$

> *d4* := *Determinant*(*A4*);

$$d4 := 10$$

> *x4* := −1;

$$x4 := -1$$

> *x3* := 2 + *x4*;

$$x3 := 1$$

> *x2* := −*x3* − *x4*;

$$x2 := 0$$

> *x1* := 1 − *x2* − 5·*x3* − 2·*x4*;

$$x1 := -2$$

1.4. Mustaqil ishlash uchun topshiriqlar

1–topshiriq. a) Kramer qiodasi bo'yicha; b) Gauss usuli; c) matritsalar usuli bilan tenglamalar sistemasini hisoblang.

1) $\begin{cases} 2x_1 + 3x_2 - x_3 = -6, \\ -x_1 + 2x_2 + x_3 = 5, \\ x_1 + 6x_2 + 3x_3 = -1. \end{cases}$

2) $\begin{cases} x_1 - 2x_2 - x_3 = -5, \\ 3x_1 + 4x_2 + 2x_3 = 0, \\ -2x_1 + 5x_2 + x_3 = 7. \end{cases}$

3) $\begin{cases} 4x_1 + 2x_2 - x_3 = -1, \\ -3x_1 - x_2 + x_3 = -1, \\ -x_1 + 4x_2 + 5x_3 = -8. \end{cases}$

4) $\begin{cases} -2x_1 + x_2 - 3x_3 = -11, \\ x_1 + 3x_2 = 6, \\ 3x_1 - 5x_2 - x_3 = 3. \end{cases}$

5) $\begin{cases} -2x_1 - x_2 + x_3 = 10, \\ 3x_1 + 2x_2 - x_3 = -14, \\ -x_1 + 3x_2 + 2x_3 = 6. \end{cases}$

6) $\begin{cases} 4x_1 - x_2 + 3x_3 = 5, \\ x_1 + 2x_2 + 4x_3 = 0, \\ -3x_1 + 3x_2 - 5x_3 = -11. \end{cases}$

7) $\begin{cases} -5x_1 - 2x_2 + x_3 = -10, \\ 4x_1 + 3x_2 - 2x_3 = 7, \\ -x_1 - 6x_2 + 5x_3 = 2. \end{cases}$

8) $\begin{cases} x_1 + 3x_2 - x_3 = 2, \\ 5x_1 + 2x_3 = 18, \\ -3x_1 + x_2 - 6x_3 = -7. \end{cases}$

9) $\begin{cases} 2x_1 - 2x_2 + 5x_3 = -12, \\ x_1 + 3x_2 + 7x_3 = 2, \\ -x_1 - 5x_2 + x_3 = -6. \end{cases}$

10) $\begin{cases} 5x_1 + 4x_2 - x_3 = -5, \\ -3x_1 - 6x_2 + 2x_3 = 5, \\ 2x_1 - 3x_2 - 4x_3 = -21. \end{cases}$

11) $\begin{cases} 4x_1 - 3x_2 - x_3 = 2, \\ -3x_1 + 7x_2 - x_3 = -5, \\ 2x_1 + 8x_2 + 5x_3 = 12. \end{cases}$

12) $\begin{cases} -3x_1 + 7x_2 - x_3 = -2, \\ 8x_1 - 3x_2 + 4x = -9, \\ 2x_1 + 5x_2 + 6x_3 = -3. \end{cases}$

13) $\begin{cases} -2x_1 + 4x_2 + 7x_3 = -21, \\ 5x_1 + 3x_2 + x_3 = 8, \\ 4x_1 + 9x_2 + 2x_3 = -8. \end{cases}$

14) $\begin{cases} 3x_1 + 6x_2 + x_3 = 1, \\ -4x_1 - 5x_2 + 2x_3 = -2, \\ 3x_2 + 4x_3 = -2. \end{cases}$

15) $\begin{cases} -7x_1 - 2x_2 + 5x_3 = -11, \\ x_1 + 4x_2 + 3x_3 = 9, \\ 3x_1 - x_2 + 4x_3 = -7. \end{cases}$

16) $\begin{cases} -8x_1 + x_2 + 5x_3 = 4, \\ 3x_1 - 2x_2 - x_3 = 2, \\ -4x_1 + 5x_2 - 3x_3 = -20. \end{cases}$

17) $\begin{cases} 4x_1 + x_2 - 5x_3 = 7, \\ -3x_1 + 2x_2 + 4x_3 = -5, \\ 2x_1 + 9x_2 - x_3 = 5. \end{cases}$

18) $\begin{cases} -x_1 + 3x_2 + 2x_3 = -1, \\ 3x_1 - 5x_2 - x_3 = 0, \\ 4x_1 - 8x_2 + x_3 = 5. \end{cases}$

19) $\begin{cases} 7x_1 - 2x_2 - 3x_3 = 0, \\ 5x_1 + 4x_2 + 2x_3 = -16, \\ -4x_1 + 5x_2 - 6x_3 = -3. \end{cases}$

20) $\begin{cases} -4x_1 + 5x_2 - x_3 = 11, \\ 2x_1 - 3x_2 + 7x_3 = -7, \\ -x_1 + 4x_2 - 6x_3 = 11. \end{cases}$

$$21)\begin{cases} x_1-4x_2+3x_3=1, \\ -5x_1-x_2+6x_3=21, \\ 2x_1+3x_2-7x_3=-17. \end{cases}$$

$$22)\begin{cases} 3x_1+5x_2-x_3=-13, \\ 2x_1-x_2+6x_3=14, \\ -2x_1+7x_2+4x_3=-4. \end{cases}$$

$$23)\begin{cases} -x_1+3x_2-x_3=2, \\ 6x_1-4x_2+3x_3=-7, \\ 3x_1+5x_2-x_3=-4. \end{cases}$$

$$24)\begin{cases} -7x_1+8x_2-x_3=-22, \\ 3x_1+x_2-4x_3=5, \\ 2x_1+4x_2+3x_3=-2. \end{cases}$$

$$25)\begin{cases} 2x_1+4x_2-x_3=-1, \\ 3x_1+5x_2+3x_3=5, \\ -x_1+6x_2-4x_3=-20. \end{cases}$$

$$26)\begin{cases} 2x_1+x_2-4x_3=-9, \\ -3x+5x_2-x_3=26, \\ x_1-6x_2-7x_3=-29. \end{cases}$$

$$27)\begin{cases} -8x_1-7x_2+x_3=-9, \\ 2x_1+3x_2-4x_3=16, \\ 5x_1+4x_2-x_3=6. \end{cases}$$

$$28)\begin{cases} 5x_1-x_2-10x_3=-2, \\ x_1+2x_2-x_3=-5, \\ 4x_1+3x_2+x_3=5. \end{cases}$$

$$29)\begin{cases} 6x_1-x_2+8x_3=7, \\ -x_1+3x_2-x_3=-10, \\ 4x_1+2x_2+5x_3=-3. \end{cases}$$

$$30)\begin{cases} x_1+2x_2+3x_3=-1, \\ -2x_1+x_2+5x_3=4, \\ 6x_1-x_2+2x_3=14. \end{cases}$$

2 – topshiriq. Tenglamalar sistemasini yeching.

$$1.\begin{cases} 4x_1-2x+x_3-x_4+3x_5=1, \\ -x_1+3x_2-x_3+5x_4+2x_5=-2, \\ 3x_1+11x_2-3x_3+2x_4-x_5=0; \end{cases}$$

$$2.\begin{cases} 2x_1-4x_2-3x_3+2x_4-x_5=-6, \\ 3x_1+x_2+4x_3+x_4-2x_5=3, \\ 3x_1-13x_2-13x_3+5x_4+x_5=2. \end{cases}$$

$$3.\begin{cases} 3x_1+x_2-4x_3+x_4+2x_5=5, \\ -2x_1-3x_2+6x_3-x_4-5x_5=-1, \\ x_1-2x_2+2x_3-3x_5=2; \end{cases}$$

$$4.\begin{cases} x_1+2x_2-x_3-3x_4+x_5=4, \\ -3x_1-6x_2+2x_3+3x_4-x_5=-2, \\ -x_2-2x_2-3x_4+x_5=6. \end{cases}$$

$$5.\begin{cases} 5x_1+3x_2-x_3+2x_4+x_5=3, \\ x_1-4x_2+2x_3+x_4+x_5=0, \\ 2x_1+x_2+x_3-3x_4-x_5=1; \end{cases}$$

$$6.\begin{cases} -2x_1+5x_2+3x_3-x_4+x_5=-2, \\ x_1-4x_2-2x_3+5x_4+x_5=3, \\ -3x_1+6x_2+4x_3+3x_4+3x_5=4. \end{cases}$$

$$7.\begin{cases} -4x_1+2x_2+3x_3-x_4+5x_5=-2, \\ 3x_1-x_2+5x_3+2x_4=1, \\ 2x_1 13x_3+x_4+5x_5=-3; \end{cases}$$

$$8.\begin{cases} 2x_1+2x_2-3x_3+x_4-x_5=-4, \\ x_1+5x_2+x_3-2x_4+3x_5=0, \\ x_1-11x_2-9x_3-x_4+2x_5=1. \end{cases}$$

$$9.\begin{cases} 6x_1-x_2-3x_3-x_4+2x_5=2, \\ -x_1+2x_2-x_3+4x_4-x_5=-1, \\ 2x_1+7x_2-7x_3+15x_4+2x_5=0; \end{cases}$$

$$10.\begin{cases} -5x_1+2x_2-x_3+4x_4-x_5=-2, \\ 3x_1+x_2+2x_3-x_4+4x_5=-1, \\ x_1+4x_2+3x_3+2x_4+7x_5=3. \end{cases}$$

$$11.\begin{cases} -7x_1+2x_2+3x_3-x_4+4x_5=5, \\ 2x_1-x_2+2x_3-5x_4-x_5=-3, \\ -x_1-x_2+9x_3-16x_4+x_5=0; \end{cases}$$

$$12.\begin{cases} 3x_1+2x_2-4x_3-5x_4+x_5=1, \\ 4x_1-x_2-2x_3+3x_4-x_5=-2, \\ -x_1-7x_2+10x_3+2x_4+3x_5=4. \end{cases}$$

13. $\begin{cases} 2x_1 - x_2 + 6x_3 + 4x_4 - 3x_5 = -4, \\ -x_1 + 3x_2 + 4x_3 - 5x_4 + 2x_5 = 3, \\ 3x_1 + x_2 + 16x_3 + 2x_4 - x_5 = 0; \end{cases}$

14. $\begin{cases} 8x_1 - x_2 + 2x_3 + 4x_4 + 3x_5 = -1, \\ -3x_1 - x_2 + 5x_3 - 2x_4 + 6x_5 = 2, \\ -x_1 - 4x_2 + 17x_3 - 2x_4 + 21x_5 = -2. \end{cases}$

15. $\begin{cases} x_1 - 7x_2 - x_3 + 5x_4 - x_5 = 8, \\ 3x_1 + 2x_2 + 6x_3 - x_4 + 4x_5 = -3, \\ 5x_1 - 12x_2 + 4x_3 + 9x_4 + 2x_5 = 2; \end{cases}$

16. $\begin{cases} 9x_1 + 4x_2 - 5x_3 - 7x_4 + x_5 = 0, \\ -4x_1 + 2x_2 + 3x_3 - 2x_4 + 3x_5 = -2, \\ x_1 + 8x_2 + x_3 - 11x_4 + 2x_5 = 3. \end{cases}$

17. $\begin{cases} 10x_1 - 2x_2 + 5x_3 - x_4 + 2x_5 = -3, \\ 3x_1 + 5x_2 - 3x_3 - 2x_4 - x_5 = 2, \\ -x_1 - 6x_2 + x_3 + x_4 + 3x_5 = 0; \end{cases}$

18. $\begin{cases} -6x_1 + 4x_2 - 7x_3 + x_4 - 2x_5 = -1, \\ 5x_1 - 2x_2 + 4x_3 + 2x_4 - x_5 = 2, \\ 3x_1 + 2x_2 - 2x_3 + 8x_4 - 7x_5 = -3. \end{cases}$

19. $\begin{cases} -6x_1 + 5x_2 - 3x_3 + 2x_4 - x_5 = 2, \\ 7x_1 - x_2 - 2x_3 - 5x_4 + x_5 = -1, \\ x_1 + 4x_2 - 5x_3 - 3x_4 + 2x_5 = 5; \end{cases}$

20. $\begin{cases} -4x_1 + 7x_2 + x_3 - 5x_4 + x_5 = -4, \\ 3x_1 + 6x_2 - x_3 + 2x_4 + 2x_5 = -3, \\ 2x_1 + 20x_2 - x_3 + 9x_4 + 5x_5 = 0. \end{cases}$

21. $\begin{cases} 5x_1 - 3x_2 - 6x_3 + x_4 - 4x_5 = 1, \\ -2x_1 - x_2 + 4x_3 - x_4 + 2x_5 = -1, \\ 4x_1 - 9x_2 - x_4 - 2x_5 = 5; \end{cases}$

22. $\begin{cases} -2x_1 + 7x_2 - 4x_3 - x_4 + 6x_5 = -2, \\ -3x_1 + 5x_2 + 4x_3 - x_4 - 2x_5 = 1, \\ -x_1 + 13x_2 - 28x_3 + 2x_4 + x_5 = 3. \end{cases}$

23. $\begin{cases} -x_1 + 3x_2 + 7x_3 - 2x_4 - x_5 = -5, \\ 2x_1 + 4x_2 - 5x_3 + x_4 + 3x_5 = 2, \\ 4x_1 - 6x_2 - x_3 + 2x_4 - 5x; \end{cases}$

24. $\begin{cases} 3x_1 - 2x_2 + 4x_3 - x_4 + 6x_5 = 4, \\ 4x_1 - x_2 - 2x_3 + 5x_4 - x_5 = -2, \\ -x_1 - x_2 - 6x_3 - 6x_4 + 7x_5 = -1. \end{cases}$

25. $\begin{cases} 8x_1 - x_2 + 6x_3 + 2x_4 + 3x_5 = -3, \\ 3x_1 + 2x_2 - x_3 + 5x_4 - x_5 = 2, \\ x_1 + 7x_2 - 9x_3 + x_4 + 6x_5 = 0; \end{cases}$

26. $\begin{cases} 4x_1 - x_2 - x_3 + 2x_4 - 3x_5 = -6, \\ 5x_1 + 2x_2 + 2x_3 - 2x_4 + x_5 = 2, \\ 3x_1 - 4x_2 - 4x_3 + 6x_4 - 7x_5 = -1. \end{cases}$

27. $\begin{cases} -9x_1 + 5x_2 - x_3 + 2x_4 - 7x_5 = 3, \\ 4x_1 + 3x_2 - 8x_3 - x_4 + x_5 = -2, \\ -x_1 + 11x_2 - 17x_3 - 5x_5 = -1; \end{cases}$

28. $\begin{cases} 2x_1 - x_2 - x_3 - 2x_4 + 7x_5 = 6, \\ 5x_1 + 2x_2 + x_3 - 6x_4 - 3x_5 = -2, \\ x_1 - 5x_2 - 4x_3 + 24x_5 = 2. \end{cases}$

29. $\begin{cases} 5x_1 - 4x_2 - 4x_3 + 3x_4 + 2x_5 = 7, \\ 2x_1 - x_2 + 2x_3 - 5x_4 + x_5 = -5, \\ x_1 - 2x_2 - 8x_3 + 13x_4 = 4; \end{cases}$

30. $\begin{cases} 3x_1 - 7x_2 + 5x_3 - 2x_4 - x_5 = 8, \\ -x_1 + 4x_2 + 2x_3 - 3x_4 + 3x_5 = -3, \\ 5x_1 - 15x_2 + x_3 + 4x_4 + 2x_5 = 1. \end{cases}$

1.5. Matritsalar va matritsalar ustida amallar. Matritsaning rangi.

Sonlarning m ta satr va n ta qatordan iborat to'g'ri to'rtburchakli jadvali *$m\times n$ o'lchamli matritsa* deyiladi. Bu matritsa

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

ko'rinishda yoziladi.

Agar $m=1$ bo'lsa, *satr matritsa,* $n=1$ bo'lsa *ustup matritsa,* $m=n$ bo'lsa, *kvadrat matritsa* hosil bo'ladi. Kvadrat A matritsa uchun shu matritsaning elementlaridan tuzilgan $n-$tartibli determinantni hisoblash mumkin. Bu determinant $\det A$ yoki $|A|$ orqali belgilanadi:

$$\det A=|A|=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix}.$$

Agar $\det A=0$ bo'lsa, u holda A matritsa *maxsus,* $\det A\neq 0$ bo'lsa, *maxsusmas* deyiladi.

Bosh diagonalida turgan elementlari birga, qolgan elementlari nolga teng bo'lgan kvadrat matritsa *birlik matritsa* deb ataladi va E bilan belgilanadi:

$$E=\begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

Ravshanki, $\det E=1$.

Agar o'lchamlari bir xil $m\times n$ bo'lgan ikki matritsaning barcha mos elementlari o'zaro teng bo'lsa, bu matritsalar *teng* deyiladi.

Bir xil $m\times n$ o'lchamli A va B matritsaning *yig'indisi* deb o'sha o'lchamli shunday $C=A+B$ matritsaga aytiladiki, uning har bir elementi *A va B* matritsalarning mos elementlari yig'indisidan iborat bo'ladi.

$m\times n$ o'lchamli A matritsaning *λ songa ko'paytmasi* deb, o'sha o'lchamdagi $B=\lambda\cdot A$ matritsaga aytiladiki, bu matritsa elementlari A matritsa elementlarini λ ga ko'paytirishdan hosil bo'ladi.

$m\times n$ o'lchamli A matritsaning $k\times n$ o'lchamli B *matritsaga ko'paytmasi* deb, $m\times n$ o'lchamli shunday $C=A\cdot B$ matritsaga aytiladiki, uning c_{ij} elementi A matritsaning i- satri elementlarini B matritsaning j-ustunidagi mos elementlariga ko'paytmalari yig'indisiga teng, ya'ni

$$c_{ij}=a_{i1}b_{1j}+a2b_{2j}+\dots+a_{ik}b_{kj}.$$

Agar $AB=BA$ bo'lsa, u holda A va B matritsalar *o'rni almashinadigan* yoki *kommutativ matritsalar* deyiladi.

1 - misol. Ushbu

$$A=\begin{pmatrix} 3 & 1 & -2 \\ 2 & -4 & 5 \end{pmatrix},\quad B=\begin{pmatrix} 2 & -1 \\ -1 & 3 \\ 4 & -5 \end{pmatrix}$$

matritsalarning AB va BA ko'paytmalarini toping.

Yechish. AB matritsa 2×2 o'lchamga ega bo'ladi:

$$AB=\begin{pmatrix} 3 & 1 & -2 \\ 2 & -4 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 3 \\ 4 & -5 \end{pmatrix}=\begin{pmatrix} 3\cdot 2+1\cdot(-1)+(-2)\cdot 4 & 3\cdot(-1)+1\cdot 3+(-2)\cdot(-5) \\ 2\cdot 2+(-4)\cdot(-1)+5\cdot 4 & 2\cdot(-1)+(-4)\cdot(-1)+5\cdot(-5) \end{pmatrix}=$$

$$=\begin{pmatrix} -3 & 10 \\ 28 & -39 \end{pmatrix}.$$

Berilgan misolni maple orqali yechamiz:

$$> P:=\begin{bmatrix} 3 & 1 & -2 \\ 2 & -4 & 5 \end{bmatrix};\#P:=matrix(3, 3, [3, 1, -2, 2, -4, 5, ; 0, 9, 6])$$

;

$$P:=\begin{bmatrix} 3 & 1 & -2 \\ 2 & -4 & 5 \end{bmatrix}$$

$$> Q := \begin{bmatrix} 2 & -1 \\ -1 & 3 \\ 4 & -5 \end{bmatrix} \#Q:=matrix(3,3,[2,-1,-1,3,4,-5,\;;$$

8, 5, 3])

;

$$Q := \begin{bmatrix} 2 & -1 \\ -1 & 3 \\ 4 & -5 \end{bmatrix}$$

> R := P.Q; #S := Q; W := R.S;

$$R := \begin{bmatrix} -3 & 10 \\ 28 & -39 \end{bmatrix}$$

BA matritsa 3×3 o'lchamga ega bo'ladi:

$$BA=\begin{pmatrix} 2 & -1 \\ -1 & 3 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} 3 & 1 & -2 \\ 2 & -4 & 5 \end{pmatrix}=\begin{pmatrix} 2\cdot3+(-1)\cdot2 & 2\cdot1+(-1)\cdot(-4) & 2\cdot(-2)+(-1)\cdot5 \\ (-1)\cdot3+3\cdot2 & (-1)\cdot1+3\cdot(-4) & (-1)\cdot(-2)+3\cdot5 \\ 4\cdot3+(-5)\cdot2 & 4\cdot1+(-5)\cdot(-4) & 4\cdot(-2)+(-5)\cdot5 \end{pmatrix}=$$

$$=\begin{pmatrix} 4 & 6 & -9 \\ 3 & -13 & 17 \\ 2 & 24 & -33 \end{pmatrix}$$

$AB \neq BA$ bo'lganligi sababli A *va* B matritsalar kommutativ emas.

Agar kvadrat matritsa maxsusmas bo'lsa, u holda $AA^{-1}=A^{-1}A=E$ tenglikni qanoatlantiruvchi yagona A^{-1} matritsa mavjud bo'ladi va u A matritsaga *teskari matritsa* deyiladi. A matritsaning A^{-1} teskari matritsasi quyidagicha aniqlanadi:

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}.$$

Bu yerda A_{ik} A matritsa determinanti a_{ik} elementining algebraik to'ldiruvchisi.

2- misol. Berilgan

$$A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & -2 & 5 \end{pmatrix}$$

matritsaga teskari matritsani toping.

Yechish. Matritsaning determinantini hisoblaymiz:

$$\det A=\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & -2 & 5 \end{vmatrix}=1\cdot4-2\cdot7-1\cdot(-6)=-4\neq0$$

Demak, A matritsa maxsusmas matritsa ekan. Endi A_{ik} algebraik to'ldiruvchilarni hisoblaymiz:

$$A_{11}=\begin{vmatrix} 0 & 2 \\ -2 & 5 \end{vmatrix}=4,\quad A_{12}=-\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}=-7,\quad A_{13}=\begin{vmatrix} 3 & 0 \\ 4 & -2 \end{vmatrix}=-6,$$

$$A_{21}=-\begin{vmatrix} 2 & -1 \\ -2 & 5 \end{vmatrix}=-8,\quad A_{22}=\begin{vmatrix} 1 & -1 \\ 4 & 5 \end{vmatrix}=9,\quad A_{23}=-\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}=-5,$$

$$A_{31}=\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix}=4,\quad A_{23}=-\begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix}=10,\quad A_{33}=\begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}=-6.$$

Teskari matritsani tuzamiz:

$$A^{-1}=-\frac{1}{4}\begin{pmatrix} 4 & -8 & 4 \\ -7 & 9 & -5 \\ -6 & 10 & -6 \end{pmatrix}=\begin{pmatrix} -1 & 2 & -1 \\ \frac{7}{4} & \frac{-9}{4} & \frac{5}{4} \\ \frac{3}{2} & \frac{-5}{2} & \frac{3}{2} \end{pmatrix}$$

$AA^{-1}=A^{-1}A=E$ ekanini tekshirish mumkin. n ta noma'lumli n ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+\dots+a_{2n}x_n=b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1+a_{n2}x_2+\dots+a_{nn}x_n=b_n \end{cases}$$

ni matritsa ko'rinishda

$$AX = B$$

kabi yozish mumkin, bunda

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{pmatrix},$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}.$$

Agar A maxsusmas matritsa, ya'ni $\det A \neq 0$ bo'lsa, u holda bu sistemaning matritsa shaklidagi yechimi ushbu ko'rinishga ega bo'ladi:

$$X = A^{-1}B.$$

3-misol. Matritsa rangini toping:

$$A = \begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$$

Yechish. Matritsa ustida elementar almashtirishlarni bajaramiz:

$$A = \begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & -5 \\ 2 & 3 & 0 \\ 3 & 1 & 7 \\ 0 & 2 & -4 \\ 0 & 5 & -10 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 4 & -5 \\ 0 & -5 & 10 \\ 0 & -11 & 22 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & -5 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Hosil qilingan matritsaning rangi 2 ga teng. Demak, berilgan A matritsaning rangi ham 2 ga teng bo'ladi.

Kroneker-Kapelli teoremasi. n ta noma'lumli m ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

birgalikda bo'lishi uchun

$$rang A = rang B$$

bo'lishi zarur va yetarlidir. Bu yerda

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}.$$

1.6. Mustaqil ishlash uchun topshiriqlar.

1- topshiriq. A, B, C, D matritsalar, α va β sonlar berilgan:

a) $\alpha A^2 + \beta BC$ ni hisoblang;

b) $B \times D$ ni hisoblang.

1. $A = \begin{pmatrix} -5 & 1 & 2 \\ 3 & -1 & 4 \\ -1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 & -2 & 6 \\ 0 & -1 & 1 & -3 \\ -2 & 5 & 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 & 3 \\ 2 & 5 & 1 \\ -4 & 4 & 7 \\ 1 & 6 & 5 \end{pmatrix}$,

$D = \begin{pmatrix} -3 & 3 & 2 & 1 & 5 \\ 1 & 5 & 0 & 13 & 5 \\ 2 & 1 & -1 & 6 & 0 \\ -1 & 13 & 2 & -4 & 2 \end{pmatrix}$, $\alpha = -2$, $\beta = 3$;

2. $$A=\begin{pmatrix}2&-1&5\\1&6&-2\\3&0&4\end{pmatrix},\quad B=\begin{pmatrix}-2&1&4&-2\\2&3&-5&1\\7&0&-1&1\end{pmatrix},\quad C=\begin{pmatrix}1&-1&3\\4&0&2\\5&-1&3\\2&3&6\end{pmatrix},$$
$$D=\begin{pmatrix}-1&3&5&2&1\\0&1&-1&2&4\\-2&7&9&6&6\\1&1&-2&3&0\end{pmatrix},\quad \alpha=2,\ \beta=-3;$$

3. $$A=\begin{pmatrix}-4&1&-2\\0&-1&1\\2&3&-1\end{pmatrix},\quad B=\begin{pmatrix}5&0&-1&2\\-2&3&1&1\\1&2&4&0\end{pmatrix},\quad C=\begin{pmatrix}3&1&-1\\2&-1&1\\0&4&-2\\5&0&-3\end{pmatrix},$$
$$D=\begin{pmatrix}-1&3&5&2&1\\0&1&-1&2&4\\-2&7&9&6&6\\1&1&-2&3&0\end{pmatrix},\quad \alpha=2,\ \beta=-3;$$

4. $$A=\begin{pmatrix}-3&0&2\\4&1&-3\\1&-1&3\end{pmatrix},\quad B=\begin{pmatrix}2&1&-1&4\\1&3&5&-1\\0&-2&2&-3\end{pmatrix},\quad C=\begin{pmatrix}1&-1&3\\-2&4&0\\-5&2&1\\-1&3&-4\end{pmatrix},$$
$$D=\begin{pmatrix}3&-2&2&0&1\\5&6&-2&2&4\\1&4&-2&1&-1\\2&-4&4&-1&7\end{pmatrix},\quad \alpha=3,\ \beta=-2;$$

5. $$A=\begin{pmatrix}-5&-2&3\\1&0&-1\\4&1&2\end{pmatrix},\quad B=\begin{pmatrix}-3&2&1&7\\-1&0&-2&1\\4&5&0&3\end{pmatrix},\quad C=\begin{pmatrix}1&-2&4\\3&-1&0\\5&-3&1\\2&-4&3\end{pmatrix},$$
$$D=\begin{pmatrix}2&-1&3&0\\-3&2&1&-1\\1&0&7&-2\\-1&1&3&-3\\3&-1&10&-2\end{pmatrix},\quad \alpha=-1,\ \beta=-2;$$

6. $$A=\begin{pmatrix}-2&6&1\\3&1&5\\0&4&2\end{pmatrix},\quad B=\begin{pmatrix}0&2&-1&3\\1&4&5&1\\-2&1&2&-3\end{pmatrix},\quad C=\begin{pmatrix}1&-1&2\\3&2&4\\5&0&1\\-1&1&-4\end{pmatrix},$$
$$D=\begin{pmatrix}-2&1&3&5&1\\-1&0&7&11&6\\3&-2&1&1&4\\1&-1&4&6&5\end{pmatrix},\quad \alpha=2,\ \beta=-1;$$

7. $$A=\begin{pmatrix}-4&-1&2\\1&1&3\\-2&0&1\end{pmatrix},\quad B=\begin{pmatrix}2&-1&3&1\\0&4&1&5\\1&0&-2&-1\end{pmatrix},\quad C=\begin{pmatrix}0&1&-2\\2&-1&3\\-3&4&0\\5&1&1\end{pmatrix},$$
$$D=\begin{pmatrix}3&-1&-3&1&2\\-2&-1&1&0&4\\1&3&1&-1&-2\\1&-2&-2&1&6\end{pmatrix},\quad \alpha=1,\ \beta=4;$$

8. $$A=\begin{pmatrix}0&2&-1\\3&1&1\\-2&4&0\end{pmatrix},\quad B=\begin{pmatrix}2&-1&3&2\\3&1&-2&0\\1&4&1&-1\end{pmatrix},\quad C=\begin{pmatrix}1&-1&3\\2&1&5\\0&4&-1\\-3&2&2\end{pmatrix},$$
$$D=\begin{pmatrix}2&4&1&3&-1\\-1&3&2&0&1\\3&1&-1&2&0\\5&5&0&5&-1\end{pmatrix},\quad \alpha=2,\ \beta=-2;$$

9. $$A=\begin{pmatrix}-2&5&-1\\3&1&1\\4&-1&2\end{pmatrix},\quad B=\begin{pmatrix}2&-4&1&3\\-1&3&5&1\\-3&1&1&4\end{pmatrix},\quad C=\begin{pmatrix}3&2&5\\1&-1&1\\0&4&-1\\-1&1&0\end{pmatrix},$$
$$D=\begin{pmatrix}1&-1&1&2&4\\2&3&-2&-1&1\\4&1&0&3&9\\-1&-1&5&-1&3\end{pmatrix},\quad \alpha=-3,\ \beta=1;$$

10. $A=\begin{pmatrix}1&-4&1\\2&5&0\\-3&1&-2\end{pmatrix}$, $B=\begin{pmatrix}-2&3&1&4\\1&0&2&3\\-1&4&5&1\end{pmatrix}$, $C=\begin{pmatrix}2&1&0\\3&2&1\\-4&0&1\\1&5&2\end{pmatrix}$,

$D=\begin{pmatrix}3&2&4&-1&0\\1&1&7&-1&-1\\2&1&-3&0&1\\4&3&11&-2&3\end{pmatrix}$, $\alpha=2$, $\beta=-1$;

11. $A=\begin{pmatrix}-5&4&1\\1&-2&3\\0&6&2\end{pmatrix}$, $B=\begin{pmatrix}4&-1&1&5\\-3&2&0&1\\-2&1&2&3\end{pmatrix}$, $C=\begin{pmatrix}-1&1&6\\2&1&3\\0&4&2\\5&1&-1\end{pmatrix}$,

$D=\begin{pmatrix}1&-2&-1&3&4\\1&7&6&-9&-15\\4&1&3&0&-3\\2&-1&4&1&0\end{pmatrix}$, $\alpha=3$, $\beta=-2$;

12. $A=\begin{pmatrix}-2&3&4\\-2&1&-1\\1&0&5\end{pmatrix}$, $B=\begin{pmatrix}-6&1&-1&2\\0&3&1&-1\\2&-4&1&5\end{pmatrix}$, $C=\begin{pmatrix}3&-2&-1\\1&-3&0\\4&1&1\\5&1&-1\end{pmatrix}$,

$D=\begin{pmatrix}5&2&3&1&0\\1&8&11&0&3\\2&-3&4&2&1\\4&13&26&2&7\end{pmatrix}$, $\alpha=2$, $\beta=-2$;

13. $A=\begin{pmatrix}1&3&2\\-2&1&0\\3&5&-4\end{pmatrix}$, $B=\begin{pmatrix}4&-1&1&2\\-3&1&2&-2\\0&5&-1&4\end{pmatrix}$, $C=\begin{pmatrix}0&2&5\\-1&1&3\\-2&0&6\\4&-1&1\end{pmatrix}$,

$D=\begin{pmatrix}-2&1&4&0&3\\1&1&9&1&-1\\5&-1&1&2&4\\2&-4&-26&-1&7\end{pmatrix}$, $\alpha=2$, $\beta=-3$;

14. $A=\begin{pmatrix}3&5&-1\\2&0&5\\4&-2&-6\end{pmatrix}$, $B=\begin{pmatrix}-7&2&0&3\\1&1&4&-2\\6&3&5&-1\end{pmatrix}$, $C=\begin{pmatrix}2&1&-2\\4&3&0\\5&2&7\\-1&-1&3\end{pmatrix}$,

$D=\begin{pmatrix}-3&2&1&4&5\\1&6&-5&10&7\\2&2&-3&3&1\\-5&0&4&1&4\end{pmatrix}$, $\alpha=-2$, $\beta=-1$;

15. $A=\begin{pmatrix}-5&-3&1\\-1&2&0\\0&1&4\end{pmatrix}$, $B=\begin{pmatrix}2&-3&1&4\\-1&0&-4&1\\3&5&-2&3\end{pmatrix}$, $C=\begin{pmatrix}2&-7&1\\3&-2&4\\-1&1&-5\\2&1&3\end{pmatrix}$,

$D=\begin{pmatrix}-5&2&1&3&0\\-1&0&3&11&6\\2&-1&1&4&3\\3&-3&2&0&1\end{pmatrix}$, $\alpha=-3$, $\beta=1$;

16. $A=\begin{pmatrix}-4&3&2\\-1&0&1\\2&1&5\end{pmatrix}$, $B=\begin{pmatrix}-1&1&3&4\\2&-3&5&0\\-4&0&1&-3\end{pmatrix}$, $C=\begin{pmatrix}-2&2&1\\3&0&-1\\4&-1&1\\0&-3&5\end{pmatrix}$,

$D=\begin{pmatrix}3&-1&1&0&2\\-1&7&-1&2&2\\-2&4&-1&1&0\\4&2&2&3&1\end{pmatrix}$, $\alpha=1$, $\beta=-2$;

17. $A=\begin{pmatrix}-3&2&4\\0&1&-1\\-2&3&1\end{pmatrix}$, $B=\begin{pmatrix}4&1&2&0\\-1&1&3&-2\\0&-3&1&1\end{pmatrix}$, $C=\begin{pmatrix}-2&1&-1\\3&-1&4\\5&0&1\\3&2&6\end{pmatrix}$,

$D=\begin{pmatrix}2&1&-1&3&4\\1&3&-4&1&-1\\3&-1&2&1&0\\4&7&-9&5&2\end{pmatrix}$, $\alpha=2$, $\beta=-3$;

18. $A=\begin{pmatrix}2&3&0\\7&4&-2\\-1&1&5\end{pmatrix}$, $B=\begin{pmatrix}-6&1&-1&3\\2&0&4&-3\\1&-1&5&2\end{pmatrix}$, $C=\begin{pmatrix}5&1&-1\\4&2&3\\-3&1&0\\-2&1&-1\end{pmatrix}$,

$D=\begin{pmatrix}2&-2&5&1&0\\3&-5&9&1&2\\1&-3&4&0&2\\1&1&1&1&-2\end{pmatrix}$, $\alpha=3,\ \beta=2$;

19. $A=\begin{pmatrix}-1&3&1\\2&0&1\\4&1&-3\end{pmatrix}$, $B=\begin{pmatrix}3&1&-1&1\\2&4&0&5\\6&-1&1&2\end{pmatrix}$, $C=\begin{pmatrix}0&1&-1\\2&4&3\\1&-2&2\\3&1&4\end{pmatrix}$,

$D=\begin{pmatrix}3&1&-1&2&0\\1&7&3&6&-1\\-1&3&2&2&5\\0&10&5&8&15\end{pmatrix}$, $\alpha=1,\ \beta=-2$;

20. $A=\begin{pmatrix}-4&-2&-1\\1&0&3\\-3&1&2\end{pmatrix}$, $B=\begin{pmatrix}-3&2&-1&1\\1&0&5&3\\-2&4&1&-1\end{pmatrix}$, $C=\begin{pmatrix}3&1&0\\-1&2&-3\\0&4&1\\2&-1&5\end{pmatrix}$,

$D=\begin{pmatrix}-1&2&3&1&0\\1&3&10&-3&2\\3&-1&4&-5&2\\4&-1&1&3&1\end{pmatrix}$, $\alpha=2,\ \beta=-2$;

21. $A=\begin{pmatrix}-2&1&2\\1&-1&4\\3&5&7\end{pmatrix}$, $B=\begin{pmatrix}5&1&-1&0\\2&0&3&-1\\1&1&4&6\end{pmatrix}$, $C=\begin{pmatrix}0&2&-1\\3&5&2\\4&-2&1\\6&0&-3\end{pmatrix}$,

$D=\begin{pmatrix}5&-1&2&2&0\\1&3&0&-6&-6\\2&-2&1&4&3\\4&4&1&-8&-9\end{pmatrix}$, $\alpha=1,\ \beta=4$;

22. $A=\begin{pmatrix}-1&3&4\\-1&2&1\\2&0&-1\end{pmatrix}$, $B=\begin{pmatrix}1&-1&3&2\\0&5&-1&4\\-1&-2&4&-3\end{pmatrix}$, $C=\begin{pmatrix}5&-1&1\\2&1&0\\3&-4&1\\1&1&-2\end{pmatrix}$,

$D=\begin{pmatrix}2&-1&3&0&-2\\-1&2&-7&1&0\\3&2&-1&1&1\\2&4&-8&2&1\end{pmatrix}$, $\alpha=2,\ \beta=-1$;

23. $A=\begin{pmatrix}0&2&-1\\1&3&4\\5&4&-5\end{pmatrix}$, $B=\begin{pmatrix}3&2&4&1\\0&1&-3&-1\\1&0&5&6\end{pmatrix}$, $C=\begin{pmatrix}2&3&-1\\1&4&5\\2&-6&1\\0&1&-3\end{pmatrix}$,

$D=\begin{pmatrix}-3&2&1&4&1\\-1&1&2&8&4\\5&-3&2&-1&2\\2&0&8&2&-1\end{pmatrix}$, $\alpha=-2,\ \beta=1$;

24. $A=\begin{pmatrix}-4&-3&1\\0&2&5\\1&-1&3\end{pmatrix}$, $B=\begin{pmatrix}1&-1&2&1\\0&3&-4&-1\\-2&1&3&5\end{pmatrix}$, $C=\begin{pmatrix}3&1&-2\\2&1&-1\\0&2&-3\\1&4&0\end{pmatrix}$,

$D=\begin{pmatrix}2&2&-1&-3\\3&-1&1&3\\1&8&-1&4\\-3&4&1&8\\5&-3&0&-3\end{pmatrix}$, $\alpha=-1,\ \beta=3$;

25. $A=\begin{pmatrix}-4&2&0\\-3&3&-1\\1&5&1\end{pmatrix}$, $B=\begin{pmatrix}4&0&1&-2\\1&3&-1&5\\2&7&0&3\end{pmatrix}$, $C=\begin{pmatrix}-2&1&-1\\3&0&2\\1&4&-3\\-5&1&1\end{pmatrix}$,

$D=\begin{pmatrix}-3&1&2&1&4\\1&-5&10&1&3\\2&-3&4&0&-1\\4&-13&24&2&5\end{pmatrix}$, $\alpha=-2,\ \beta=1$;

18. $A=\begin{pmatrix}2&3&0\\7&4&-2\\-1&1&5\end{pmatrix}$, $B=\begin{pmatrix}-6&1&-1&3\\2&0&4&-3\\1&-1&5&2\end{pmatrix}$, $C=\begin{pmatrix}5&1&-1\\4&2&3\\-3&1&0\\-2&1&-1\end{pmatrix}$,

$D=\begin{pmatrix}2&-2&5&1&0\\3&-5&9&1&2\\1&-3&4&0&2\\1&1&1&1&-2\end{pmatrix}$, $\alpha=3,\ \beta=2;$

19. $A=\begin{pmatrix}-1&3&1\\2&0&1\\4&1&-3\end{pmatrix}$, $B=\begin{pmatrix}3&1&-1&1\\2&4&0&5\\6&-1&1&2\end{pmatrix}$, $C=\begin{pmatrix}0&1&-1\\2&4&3\\1&-2&2\\3&1&4\end{pmatrix}$,

$D=\begin{pmatrix}3&1&-1&2&0\\1&7&3&6&-1\\-1&3&2&2&5\\0&10&5&8&15\end{pmatrix}$, $\alpha=1,\ \beta=-2;$

20. $A=\begin{pmatrix}-4&-2&-1\\1&0&3\\-3&1&2\end{pmatrix}$, $B=\begin{pmatrix}-3&2&-1&1\\1&0&5&3\\-2&4&1&-1\end{pmatrix}$, $C=\begin{pmatrix}3&1&0\\-1&2&-3\\0&4&1\\2&-1&5\end{pmatrix}$,

$D=\begin{pmatrix}-1&2&3&1&0\\1&3&10&-3&2\\3&-1&4&-5&2\\4&-1&1&3&1\end{pmatrix}$, $\alpha=2,\ \beta=-2;$

21. $A=\begin{pmatrix}-2&1&2\\1&-1&4\\3&5&7\end{pmatrix}$, $B=\begin{pmatrix}5&1&-1&0\\2&0&3&-1\\1&1&4&6\end{pmatrix}$, $C=\begin{pmatrix}0&2&-1\\3&5&2\\4&-2&1\\6&0&-3\end{pmatrix}$,

$D=\begin{pmatrix}5&-1&2&2&0\\1&3&0&-6&-6\\2&-2&1&4&3\\4&4&1&-8&-9\end{pmatrix}$, $\alpha=1,\ \beta=4;$

22. $A=\begin{pmatrix}-1&3&4\\-1&2&1\\2&0&-1\end{pmatrix}$, $B=\begin{pmatrix}1&-1&3&2\\0&5&-1&4\\-1&-2&4&-3\end{pmatrix}$, $C=\begin{pmatrix}5&-1&1\\2&1&0\\3&-4&1\\1&1&-2\end{pmatrix}$,

$D=\begin{pmatrix}2&-1&3&0&-2\\-1&2&-7&1&0\\3&2&-1&1&1\\2&4&-8&2&1\end{pmatrix}$, $\alpha=2,\ \beta=-1;$

23. $A=\begin{pmatrix}0&2&-1\\1&3&4\\5&4&-5\end{pmatrix}$, $B=\begin{pmatrix}3&2&4&1\\0&1&-3&-1\\1&0&5&6\end{pmatrix}$, $C=\begin{pmatrix}2&3&-1\\1&4&5\\2&-6&1\\0&1&-3\end{pmatrix}$,

$D=\begin{pmatrix}-3&2&1&4&1\\-1&1&2&8&4\\5&-3&2&-1&2\\2&0&8&2&-1\end{pmatrix}$, $\alpha=-2,\ \beta=1;$

24. $A=\begin{pmatrix}-4&-3&1\\0&2&5\\1&-1&3\end{pmatrix}$, $B=\begin{pmatrix}1&-1&2&1\\0&3&-4&-1\\-2&1&3&5\end{pmatrix}$, $C=\begin{pmatrix}3&1&-2\\2&1&-1\\0&2&-3\\1&4&0\end{pmatrix}$,

$D=\begin{pmatrix}2&2&-1&-3\\3&-1&1&3\\1&8&-1&4\\-3&4&1&8\\5&-3&0&-3\end{pmatrix}$, $\alpha=-1,\ \beta=3;$

25. $A=\begin{pmatrix}-4&2&0\\-3&3&-1\\1&5&1\end{pmatrix}$, $B=\begin{pmatrix}4&0&1&-2\\1&3&-1&5\\2&7&0&3\end{pmatrix}$, $C=\begin{pmatrix}-2&1&-1\\3&0&2\\1&4&-3\\-5&1&1\end{pmatrix}$,

$D=\begin{pmatrix}-3&1&2&1&4\\1&-5&10&1&3\\2&-3&4&0&-1\\4&-13&24&2&5\end{pmatrix}$, $\alpha=-2,\ \beta=1;$

26. $A=\begin{pmatrix}-1 & 2 & 1\\ 3 & -2 & -1\\ 0 & 1 & 5\end{pmatrix}$, $B=\begin{pmatrix}0 & 1 & -2 & 3\\ 1 & 5 & 1 & 2\\ 4 & 1 & -1 & 3\end{pmatrix}$, $C=\begin{pmatrix}-2 & 2 & 1\\ 1 & 0 & 3\\ 1 & 6 & -4\\ 5 & 0 & 1\end{pmatrix}$,

$D=\begin{pmatrix}4 & 1 & -1 & 2 & 0\\ 2 & 7 & -5 & 1 & 3\\ 1 & -3 & 2 & -1 & 1\\ 3 & 4 & -3 & 0 & 4\end{pmatrix}$, $\alpha=1,\quad \beta=-1;$

27. $A=\begin{pmatrix}1 & 2 & 4\\ 1 & 5 & -2\\ -3 & 0 & 7\end{pmatrix}$, $B=\begin{pmatrix}5 & -1 & 0 & 1\\ 2 & 3 & -1 & 4\\ 1 & -2 & 6 & -2\end{pmatrix}$, $C=\begin{pmatrix}1 & -1 & 3\\ 2 & 2 & 5\\ 3 & 3 & -1\\ 4 & 0 & 2\end{pmatrix}$,

$D=\begin{pmatrix}4 & -2 & 3 & 0 & 5\\ 1 & -3 & 4 & -2 & 2\\ 3 & 1 & -1 & 2 & 3\\ 2 & 7 & -9 & 6 & -1\end{pmatrix}$, $\alpha=1,\quad \beta=2;$

28. $A=\begin{pmatrix}-3 & -1 & 1\\ 2 & 0 & 3\\ 1 & 4 & -1\end{pmatrix}$, $B=\begin{pmatrix}3 & 1 & -1 & 2\\ 4 & 1 & 0 & 3\\ 2 & -2 & 1 & 1\end{pmatrix}$, $C=\begin{pmatrix}0 & 1 & -1\\ 2 & 4 & 3\\ 5 & -1 & -2\\ 3 & -3 & 1\end{pmatrix}$,

$D=\begin{pmatrix}4 & 1 & -1 & 3 & 0\\ -2 & -3 & -3 & 3 & 5\\ 3 & 2 & 1 & 0 & 1\\ 1 & -1 & -2 & 3 & 6\end{pmatrix}$, $\alpha=-2,\quad \beta=-3;$

29. $A=\begin{pmatrix}-3 & 4 & 2\\ -2 & 1 & 0\\ 1 & 0 & -1\end{pmatrix}$, $B=\begin{pmatrix}-2 & 1 & 3 & 4\\ 0 & -1 & 1 & -1\\ 3 & 2 & -2 & 5\end{pmatrix}$, $C=\begin{pmatrix}4 & 1 & -1\\ 2 & 3 & 1\\ -1 & 0 & 5\\ -2 & 1 & 0\end{pmatrix}$,

$D=\begin{pmatrix}-3 & 2 & 0 & 1 & 4\\ -1 & 0 & 6 & 5 & 0\\ 1 & -1 & 3 & 2 & -1\\ 1 & -2 & 12 & 9 & -2\end{pmatrix}$, $\alpha=1,\quad \beta=-3;$

30. $A=\begin{pmatrix}0 & -2 & 1\\ 5 & 4 & 3\\ -3 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}3 & 0 & -2 & 1\\ 4 & 1 & 0 & -3\\ -1 & 5 & 1 & -1\end{pmatrix}$, $C=\begin{pmatrix}1 & 1 & -2\\ 3 & 0 & 4\\ 5 & 2 & 6\\ -4 & 3 & -1\end{pmatrix}$,

$D=\begin{pmatrix}-3 & 2 & 4 & -1 & 2\\ 3 & 2 & 5 & 2 & -4\\ 2 & 0 & -3 & 1 & -2\\ -1 & 2 & 1 & 0 & 1\end{pmatrix}$, $\alpha=2,\quad \beta=-1;$

2–topshiriq. Matritsali tenglamalarni yeching: a) $AX=B$; b) $XA=B$, *c)* $AXC=B$.

1) $A=\begin{pmatrix}-4 & 1\\ 2 & 3\end{pmatrix}$, $B=\begin{pmatrix}0 & 1\\ 2 & -3\end{pmatrix}$, $C=\begin{pmatrix}-3 & 1\\ -1 & 2\end{pmatrix}$;

2) $A=\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}-2 & 1\\ 1 & -1\end{pmatrix}$, $C=\begin{pmatrix}-1 & 3\\ -2 & 4\end{pmatrix}$;

3) $A=\begin{pmatrix}-5 & 0\\ 2 & 1\end{pmatrix}$, $B=\begin{pmatrix}-3 & 3\\ 2 & 1\end{pmatrix}$, $C=\begin{pmatrix}-1 & 4\\ 1 & 2\end{pmatrix}$;

4) $A=\begin{pmatrix}1 & 3\\ -6 & -1\end{pmatrix}$, $B=\begin{pmatrix}4 & -3\\ -2 & -1\end{pmatrix}$, $C=\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix}$;

5) $A=\begin{pmatrix}2 & 0\\ -1 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & -1\\ -2 & 1\end{pmatrix}$, $C=\begin{pmatrix}-4 & -2\\ 1 & 3\end{pmatrix}$;

6) $A=\begin{pmatrix}-9 & 2\\ 4 & -1\end{pmatrix}$, $B=\begin{pmatrix}3 & -8\\ 2 & 7\end{pmatrix}$, $C=\begin{pmatrix}1 & -4\\ 2 & 1\end{pmatrix}$;

7) $A=\begin{pmatrix}2 & 4\\ -1 & -3\end{pmatrix}$, $B=\begin{pmatrix}3 & -8\\ 2 & 7\end{pmatrix}$, $C=\begin{pmatrix}-2 & 4\\ 3 & -5\end{pmatrix}$;

8) $A=\begin{pmatrix}5 & -7\\ -1 & 2\end{pmatrix}$, $B=\begin{pmatrix}4 & 2\\ 1 & 0\end{pmatrix}$, $C=\begin{pmatrix}-3 & 2\\ 1 & -1\end{pmatrix}$;

9) $A=\begin{pmatrix}10 & 2\\ 4 & 1\end{pmatrix}$, $B=\begin{pmatrix}-7 & 5\\ 2 & -1\end{pmatrix}$, $C=\begin{pmatrix}-1 & -2\\ 2 & 3\end{pmatrix}$;

10) $A=\begin{pmatrix}-2 & 5\\ 1 & -3\end{pmatrix}$, $B=\begin{pmatrix}2 & 0\\ -1 & -3\end{pmatrix}$, $C=\begin{pmatrix}1 & -1\\ 2 & 4\end{pmatrix}$;

11) $A=\begin{pmatrix}-6 & -2\\ 3 & 2\end{pmatrix}$, $B=\begin{pmatrix}4 & 5\\ 2 & 3\end{pmatrix}$, $C=\begin{pmatrix}-4 & 2\\ 3 & -1\end{pmatrix}$;

12) $A=\begin{pmatrix}-1 & 9\\ 1 & -10\end{pmatrix}$, $B=\begin{pmatrix}-3 & 0\\ 1 & 2\end{pmatrix}$, $C=\begin{pmatrix}2 & 4\\ 1 & 3\end{pmatrix}$;

13) $A=\begin{pmatrix}7 & 2\\ 3 & 4\end{pmatrix}$, $B=\begin{pmatrix}5 & -1\\ 1 & 2\end{pmatrix}$, $C=\begin{pmatrix}2 & -3\\ 1 & -1\end{pmatrix}$;

14) $A=\begin{pmatrix}-4 & 3\\ -1 & 2\end{pmatrix}$, $B=\begin{pmatrix}-1 & 8\\ -2 & 9\end{pmatrix}$, $C=\begin{pmatrix}3 & -1\\ 4 & 2\end{pmatrix}$;

15) $A=\begin{pmatrix}0 & -2\\ 1 & 3\end{pmatrix}$; $B=\begin{pmatrix}6 & 1\\ -1 & 2\end{pmatrix}$, $C=\begin{pmatrix}2 & -1\\ 1 & 2\end{pmatrix}$;

16) $A=\begin{pmatrix}9 & 2\\ 5 & 1\end{pmatrix}$, $B=\begin{pmatrix}3 & -1\\ 1 & 3\end{pmatrix}$, $C=\begin{pmatrix}-3 & -2\\ 2 & 1\end{pmatrix}$;

17) $A=\begin{pmatrix}-8 & -2\\ 5 & 1\end{pmatrix}$, $B=\begin{pmatrix}5 & -1\\ 1 & 1\end{pmatrix}$, $C=\begin{pmatrix}-3 & -1\\ 2 & 3\end{pmatrix}$;

18) $A=\begin{pmatrix}-7 & 2\\ 4 & -3\end{pmatrix}$, $B=\begin{pmatrix}-2 & -1\\ 3 & 1\end{pmatrix}$, $C=\begin{pmatrix}1 & 5\\ -1 & -1\end{pmatrix}$;

19) $A=\begin{pmatrix}6 & 5\\ -3 & -2\end{pmatrix}$, $B=\begin{pmatrix}7 & 2\\ -1 & 1\end{pmatrix}$, $C=\begin{pmatrix}-1 & 3\\ 1 & -1\end{pmatrix}$;

20) $A=\begin{pmatrix}-5 & 2\\ 8 & -3\end{pmatrix}$, $B=\begin{pmatrix}4 & -4\\ 1 & 2\end{pmatrix}$, $C=\begin{pmatrix}4 & 1\\ 3 & 2\end{pmatrix}$;

21) $A=\begin{pmatrix}-2 & 5\\ -3 & 7\end{pmatrix}$, $B=\begin{pmatrix}1 & 2\\ -3 & 4\end{pmatrix}$, $C=\begin{pmatrix}2 & 4\\ -1 & 3\end{pmatrix}$;

22) $A=\begin{pmatrix}1 & -1\\ 3 & 5\end{pmatrix}$, $B=\begin{pmatrix}-4 & 1\\ -1 & 3\end{pmatrix}$, $C=\begin{pmatrix}-4 & 2\\ 1 & 1\end{pmatrix}$;

23) $A=\begin{pmatrix}2 & -1\\ 3 & -5\end{pmatrix}$, $B=\begin{pmatrix}-6 & 7\\ -2 & 1\end{pmatrix}$, $C=\begin{pmatrix}-3 & 4\\ 1 & -2\end{pmatrix}$;

24) $A=\begin{pmatrix}-9 & -1\\ 5 & 1\end{pmatrix}$, $B=\begin{pmatrix}-3 & 5\\ 2 & 4\end{pmatrix}$, $C=\begin{pmatrix}-2 & 1\\ 1 & 3\end{pmatrix}$;

25) $A=\begin{pmatrix}-4 & 3\\ -2 & -1\end{pmatrix}$, $B=\begin{pmatrix}11 & 2\\ 5 & 1\end{pmatrix}$, $C=\begin{pmatrix}2 & -3\\ 1 & -1\end{pmatrix}$;

26) $A=\begin{pmatrix}12 & 3\\ 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}-2 & 5\\ 6 & 9\end{pmatrix}$, $C=\begin{pmatrix}1 & -2\\ -1 & 1\end{pmatrix}$;

27) $A=\begin{pmatrix}-10 & 3\\ 5 & -1\end{pmatrix}$, $B=\begin{pmatrix}2 & 4\\ 7 & 3\end{pmatrix}$, $C=\begin{pmatrix}-2 & -4\\ 1 & -3\end{pmatrix}$;

28) $A=\begin{pmatrix}3 & -5\\ -1 & 4\end{pmatrix}$, $B=\begin{pmatrix}2 & -5\\ 1 & 9\end{pmatrix}$, $C=\begin{pmatrix}1 & -2\\ 3 & 1\end{pmatrix}$;

29) $A=\begin{pmatrix}6 & -5\\ -2 & 2\end{pmatrix}$, $B=\begin{pmatrix}-3 & 0\\ 2 & 1\end{pmatrix}$, $C=\begin{pmatrix}-1 & 0\\ 2 & 3\end{pmatrix}$;

30) $A=\begin{pmatrix}5 & -1\\ 2 & -1\end{pmatrix}$, $B=\begin{pmatrix}-1 & 2\\ -3 & 4\end{pmatrix}$, $C=\begin{pmatrix}-4 & -1\\ 6 & 1\end{pmatrix}$.

3 – topshiriq. Matritsaning rangini toping.

1. $D=\begin{pmatrix}-3 & 3 & 2 & 1 & 5\\ 1 & 5 & 0 & 13 & 5\\ 2 & 1 & -1 & 6 & 0\\ -1 & 13 & 2 & -4 & 2\end{pmatrix}$;

2. $D=\begin{pmatrix}-1 & 3 & 5 & 2 & 1\\ 0 & 1 & -1 & 2 & 4\\ -2 & 7 & 9 & 6 & 6\\ 1 & 1 & -2 & 3 & 0\end{pmatrix}$;

3. $D=\begin{pmatrix}-1 & 3 & 5 & 2 & 1\\ 0 & 1 & -1 & 2 & 4\\ -2 & 7 & 9 & 6 & 6\\ 1 & 1 & -2 & 3 & 0\end{pmatrix}$;

4. $D=\begin{pmatrix}3 & -2 & 2 & 0 & 1\\ 5 & 6 & -2 & 2 & 4\\ 1 & 4 & -2 & 1 & -1\\ 2 & -4 & 4 & -1 & 7\end{pmatrix}$;

5. $D=\begin{pmatrix}2 & -1 & 3 & 0\\ -3 & 2 & 1 & -1\\ 1 & 0 & 7 & -2\\ -1 & 1 & 3 & -3\\ 3 & -1 & 10 & -2\end{pmatrix}$;

6. $D=\begin{pmatrix}-2 & 1 & 3 & 5 & 1\\ -1 & 0 & 7 & 11 & 6\\ 3 & -2 & 1 & 1 & 4\\ 1 & -1 & 4 & 6 & 5\end{pmatrix}$;

7. $D=\begin{pmatrix} 3 & -1 & -3 & 1 & 2 \\ -2 & -1 & 1 & 0 & 4 \\ 1 & 3 & 1 & -1 & -2 \\ 1 & -2 & -2 & 1 & 6 \end{pmatrix}$;

8. $D=\begin{pmatrix} 2 & 4 & 1 & 3 & -1 \\ -1 & 3 & 2 & 0 & 1 \\ 3 & 1 & -1 & 2 & 0 \\ 5 & 5 & 0 & 5 & -1 \end{pmatrix}$;

9. $D=\begin{pmatrix} 1 & -1 & 1 & 2 & 4 \\ 2 & 3 & -2 & -1 & 1 \\ 4 & 1 & 0 & 3 & 9 \\ -1 & -1 & 5 & -1 & 3 \end{pmatrix}$;

10. $D=\begin{pmatrix} 3 & 2 & 4 & -1 & 0 \\ 1 & 1 & 7 & -1 & -1 \\ 2 & 1 & -3 & 0 & 1 \\ 4 & 3 & 11 & -2 & 3 \end{pmatrix}$;

11. $D=\begin{pmatrix} 1 & -2 & -1 & 3 & 4 \\ 1 & 7 & 6 & -9 & -15 \\ 4 & 1 & 3 & 0 & -3 \\ 2 & -1 & 4 & 1 & 0 \end{pmatrix}$;

12. $D=\begin{pmatrix} 5 & 2 & 3 & 1 & 0 \\ 1 & 8 & 11 & 0 & 3 \\ 2 & -3 & 4 & 2 & 1 \\ 4 & 13 & 26 & 2 & 7 \end{pmatrix}$;

13. $D=\begin{pmatrix} -2 & 1 & 4 & 0 & 3 \\ 1 & 1 & 9 & 1 & -1 \\ 5 & -1 & 1 & 2 & 4 \\ 2 & -4 & -26 & -1 & 7 \end{pmatrix}$;

14. $D=\begin{pmatrix} -3 & 2 & 1 & 4 & 5 \\ 1 & 6 & -5 & 10 & 7 \\ 2 & 2 & -3 & 3 & 1 \\ -5 & 0 & 4 & 1 & 4 \end{pmatrix}$;

15. $D=\begin{pmatrix} -5 & 2 & 1 & 3 & 0 \\ -1 & 0 & 3 & 11 & 6 \\ 2 & -1 & 1 & 4 & 3 \\ 3 & -3 & 2 & 0 & 1 \end{pmatrix}$;

16. $D=\begin{pmatrix} 3 & -1 & 1 & 0 & 2 \\ -1 & 7 & -1 & 2 & 2 \\ -2 & 4 & -1 & 1 & 0 \\ 4 & 2 & 2 & 3 & 1 \end{pmatrix}$;

17. $D=\begin{pmatrix} 2 & 1 & -1 & 3 & 4 \\ 1 & 3 & -4 & 1 & -1 \\ 3 & -1 & 2 & 1 & 0 \\ 4 & 7 & -9 & 5 & 2 \end{pmatrix}$;

18. $D=\begin{pmatrix} 2 & -2 & 5 & 1 & 0 \\ 3 & -5 & 9 & 1 & 2 \\ 1 & -3 & 4 & 0 & 2 \\ 1 & 1 & 1 & 1 & -2 \end{pmatrix}$;

19. $D=\begin{pmatrix} 3 & 1 & -1 & 2 & 0 \\ 1 & 7 & 3 & 6 & -1 \\ -1 & 3 & 2 & 2 & 5 \\ 0 & 10 & 5 & 8 & 15 \end{pmatrix}$;

20. $D=\begin{pmatrix} -1 & 2 & 3 & 1 & 0 \\ 1 & 3 & 10 & -3 & 2 \\ 3 & -1 & 4 & -5 & 2 \\ 4 & -1 & 1 & 3 & 1 \end{pmatrix}$;

21. $D=\begin{pmatrix} 5 & -1 & 2 & 2 & 0 \\ 1 & 3 & 0 & -6 & -6 \\ 2 & -2 & 1 & 4 & 3 \\ 4 & 4 & 1 & -8 & -9 \end{pmatrix}$;

22. $D=\begin{pmatrix} 2 & -1 & 3 & 0 & -2 \\ -1 & 2 & -7 & 1 & 0 \\ 3 & 2 & -1 & 1 & 1 \\ 2 & 4 & -8 & 2 & 1 \end{pmatrix}$;

23. $D=\begin{pmatrix} -3 & 2 & 1 & 4 & 1 \\ -1 & 1 & 2 & 8 & 4 \\ 5 & -3 & 2 & -1 & 2 \\ 2 & 0 & 8 & 2 & -1 \end{pmatrix}$;

24. $D=\begin{pmatrix} 2 & 2 & -1 & -3 \\ 3 & -1 & 1 & 3 \\ 1 & 8 & -1 & 4 \\ -3 & 4 & 1 & 8 \\ 5 & -3 & 0 & -3 \end{pmatrix}$;

25. $D=\begin{pmatrix} -3 & 1 & 2 & 1 & 4 \\ 1 & -5 & 10 & 1 & 3 \\ 2 & -3 & 4 & 0 & -1 \\ 4 & -13 & 24 & 2 & 5 \end{pmatrix}$;

26. $D=\begin{pmatrix} 4 & 1 & -1 & 2 & 0 \\ 2 & 7 & -5 & 1 & 3 \\ 1 & -3 & 2 & -1 & 1 \\ 3 & 4 & -3 & 0 & 4 \end{pmatrix}$;

27. $D=\begin{pmatrix} 4 & -2 & 3 & 0 & 5 \\ 1 & -3 & 4 & -2 & 2 \\ 3 & 1 & -1 & 2 & 3 \\ 2 & 7 & -9 & 6 & -1 \end{pmatrix}$;

28. $D=\begin{pmatrix} 4 & 1 & -1 & 3 & 0 \\ -2 & -3 & -3 & 3 & 5 \\ 3 & 2 & 1 & 0 & 1 \\ 1 & -1 & -2 & 3 & 6 \end{pmatrix}$;

29. $D=\begin{pmatrix} -3 & 2 & 0 & 1 & 4 \\ -1 & 0 & 6 & 5 & 0 \\ 1 & -1 & 3 & 2 & -1 \\ 1 & -2 & 12 & 9 & -2 \end{pmatrix}$;

30. $D=\begin{pmatrix} -3 & 2 & 4 & -1 & 2 \\ 3 & 2 & 5 & 2 & -4 \\ 2 & 0 & -3 & 1 & -2 \\ -1 & 2 & 1 & 0 & 1 \end{pmatrix}$.

II BOB. TEKISLIKDA VA FAZODA ANALITIK GEOMETRIYA

2.1. Tekislikning tenglamasi. Tekislikning umumiy tenglamasini tekshirish. To'g'ri chiziqning tenglamasi

Oxyz to'g'ri burchakli koordinatalar sistemasida har qanday tekislik tenglamasini *x, y, z* o'zgaruvchilarga nisbatan quyidagi chiziqli tenglama shaklida yozish mumkin:

$$Ax + By + Cz + D = 0.$$

Bu tenglama tekislikning *umumiy tenglamasi* deyiladi. Bu yerda *A, B, S* koeffisentlar berilgan tekislikka perpendikulyar bo'lgan va uning *normal vektori* deb ataluvchi $\vec{n} = |A, B, C|$ vektorning koordinatalaridir. Tekislikning fazodagi holati *A, B, C* koeffisentlari va ozod hadining qiymatlariga bog'liq. Xususan, agar:

I. $D = 0$ bo'lsa, u holda $Ax + By + Cz = 0$ va tekislik koordinatalar boshidan o'tadi.

II. a) $C = 0$ bo'lsa, u holda $Ax + By + D = 0$ va tekislik *Oz* o'qiga parallel bo'ladi;

b) $B = 0$ bo'lsa, u holda $Ax + Cz + D = 0$ va tekislik *Oy* o'qiga parallel bo'ladi;

d) $A = 0$ bo'lsa, u holda $By + Cz + D = 0$ va tekislik *Ox* o'qiga parallel bo'ladi.

III. a) $D = 0$, $C = 0$ bo'lsa, u holda $Ax + By = 0$ va tekislik *Oz* o'qi orqali o'tadi.

b) $D = 0$, $B = 0$ bo'lsa, u holda $Ax + Cz = 0$ va tekislik *Oy* o'qi orqali o'tadi.

d) $D = 0$, $A = 0$ bo'lsa, u holda $By + Cz = 0$ va tekislik *Ox* o'qi orqali o'tadi.

IV. a) $C = 0$, $B = 0$ bo'lsa, u holda, $Ax + D = 0$ va tekislik *Oyz* koordnnatalar tekisligiga parallel (yoki *Ox* o'qqa perpendikulyar) bo'ladi;

b) $C = 0$, $A = 0$ bo'lsa, u holda $By + D = 0$ va tekislik *Oxz* koordinatalar tekisligiga parallel (yoki *Oy* o'qqa perpendikulyar) bo'ladi;

d) $A = 0$, $B = 0$ bo'lsa, u holda $Cz + D = 0$ va tekislik *Oxy* koordinatalar tekisligiga parallel (yoki *Oz* o'qqa perpendikulyar) bo'ladi.

V. a) $D = 0$, $A = 0$ va $B = 0$ bo'lsa, u holda $Cz = 0$ yoki $z = 0$ va tekislik *Oxy* koordinatalar tekisligi bilan ustma-ust tushadi;

b) $D = 0$, $A = 0$ va $C = 0$ bo'lsa, u holda $By = 0$ yoki $y = 0$ va tekislik *Oxz* koordinatalar tekisligi bilan ustma-ust tushadi;

d) $D = 0$, $B = 0$ va $C = 0$ bo'lsa, u holda $Ax = 0$ yoki $x = 0$ va tekislik *Oyz* tekislik bilan ustma-ust tushadi.

Quyida ma'lum shartlarni qanoatlantiruvchi tekisliklar tenglamalari keltnrnlgan:

a) berilgan $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi va berilgan $\vec{n} = |A, B, C|$ normal vektorga ega tekislik tenglamasi:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0;$$

b) tekislikning kesmalarga nisbatan tenglamasi

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1,$$

bunda *a, b, c* — tekislikning mos koordinata o'qlaridan kesadigan kesmalari;

d) berilgan uchta $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ va $M_3(x_3, y_3, z_3)$ nuqtadan o'tuvchi tekislik tenglamasi:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

To'g'ri chiziqning fazoda berilish usuliga qarab uning tenglamasi turlicha bo'lishi mumkin:

a) berilgan $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi va $\vec{s} = (l, m, p)$ yo'naltiruvchi vektorga ega bo'lgan to'gri chiziqning kanonik shakldagi tenglamalari

$$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}.$$

b) to'g'ri chiziqning parametrik tenglamalari

$$\begin{cases} x=x_0+lt, \\ y=y_0+mt, \\ z=z_0+pt \end{cases}$$

bunda t — parametr;

d) berilgan ikki $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x-x_0}{x_2-x_1}=\frac{y-y_0}{y_2-y_1}=\frac{z-z_0}{z_2-z_1};$$

g) fazodagi to'g'ri chiziqning umumiy tenglamalari:

$$\begin{cases} A_1x+B_1y+C_1z+D_1=0, \\ A_2x+B_2y+C_2z+D_2=0. \end{cases}$$

Bunda

$$\frac{A_1}{A_2}\neq\frac{B_1}{B_2}\neq\frac{C_1}{C_2}.$$

Bu to'g'ri chiziqning yo'naltiruvchi vektori $\vec{s}$ ushbu

$$\vec{s}=\vec{n}\times\vec{n}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

formula bo'yicha aniqlanadi.

$Ax+By+Cz+D=0$ va $z=0$ tekisliklarning kesishish chizig'i Oxy tekislikda yotuvchi

$$Ax+By+C=0$$

to'g'ri chiziqdan iborat bo'ladi. Bu tenglama tekislikdagi *to'g'ri chiziqning umumiy tenglamasi* deyiladi. Berilgan to'g'ri chiziqqa perpendikulyar bo'lgan $\vec{n}=|A,B|$ vektor to'g'ri chiziqning *normal vektori* deyiladi. Tekislikdagi to'g'ri chiziqning tenglamalari:

a) berilgan $M_0(x_0,y_0)$ nuqtadan o'tuvchi va berilgan $\vec{n}=|A,B|$ normal vektorga ega to'g'ri chiziq tenglamasi

$$A(x-x_0)+B(y-y_0)=0;$$

b) to'g'ri chiziqning kanonik tenglamasi

$$\frac{x-x_0}{l}=\frac{y-y_0}{m},$$

bunda $\vec{s}=(l,m)$ — to'g'ri chiziqning yo'naltiruvchi vektori, $M_0(x_0,y_0)$ — to'g'ri chiziqda yotuvchi berilgan nuqta;

c) to'g'ri chiziqning burchak koeffisentli tenglamasi

$$y=kx+b,$$

bunda b — to'g'ri chiziqning Oy o'qdan kesadigan kesmasi; k — to'g'ri chiziqning burchak koeffisenti; $k=tg\alpha$ (α - to'g'ri chiziq bilan Ox o'qning musbat yo'nalishi orasidagi burchak);

d) $M_0(x_0,y_0)$ nuqtadan o'tuvchi va k burchak koeffisentli to'g'ri chiziqning tenglamasi

$$y-y_0=k(x-x_0);$$

e) to'g'ri chiziqning kesmalarga nisbatan tenglamasi

$$\frac{x}{a}+\frac{y}{b}=1,$$

bunda a va b - to'g'ri chiziqning koordinatalar o'qlarida kesadigan kesmasi;

f) berilgan ikki $M_1(x_1,y_1)$ va $M_2(x_2,y_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x-x_0}{x_2-x_1}=\frac{y-y_0}{y_2-y_1}.$$

Misol. $M_1(-2;1;-1)$ nuqtadan o'tuvchi $\vec{s}=\{1,\ -1,\ 2\}$ vektorga parallel to'g'ri chiziq tenglamasini toping.

Yechish. $\vec{S}$ vektor to'g'ri chiziqqa parallel bo'lgani uchun u to'g'ri chiziqning yo'naltiruvchi vektori bo'ladi. Shu sababli to'g'ri chiziqning kanonik tenglamalari $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ ga asosan izlanayotgan to'g'ri chiziq tenglamalari

$$\frac{x+2}{1}=\frac{y-1}{-1}=\frac{z+1}{2}$$

ko'rinishda bo'ladi.

$$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}.$$

b) to'g'ri chiziqning parametrik tenglamalari

$$\begin{cases} x=x_0+lt, \\ y=y_0+mt, \\ z=z_0+pt \end{cases}$$

bunda t — parametr;

d) berilgan ikki $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x-x_0}{x_2-x_1}=\frac{y-y_0}{y_2-y_1}=\frac{z-z_0}{z_2-z_1};$$

g) fazodagi to'g'ri chiziqning umumiy tenglamalari:

$$\begin{cases} A_1x+B_1y+C_1z+D_1=0, \\ A_2x+B_2y+C_2z+D_2=0. \end{cases}$$

Bunda

$$\frac{A_1}{A_2}\neq\frac{B_1}{B_2}\neq\frac{C_1}{C_2}.$$

Bu to'g'ri chiziqning yo'naltiruvchi vektori $\vec{s}$ ushbu

$$\vec{s}=\vec{n}\times\vec{n}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

formula bo'yicha aniqlanadi.

$Ax+By+Cz+D=0$ va $z=0$ tekisliklarning kesishish chizig'i Oxy tekislikda yotuvchi

$$Ax+By+C=0$$

to'g'ri chiziqdan iborat bo'ladi. Bu tenglama tekislikdagi *to'g'ri chiziqning umumiy tenglamasi* deyiladi. Berilgan to'g'ri chiziqqa perpendikulyar bo'lgan $\vec{n}=|A,B|$ vektor to'g'ri chiziqning *normal vektori* deyiladi. Tekislikdagi to'g'ri chiziqning tenglamalari:

a) berilgan $M_0(x_0,y_0)$ nuqtadan o'tuvchi va berilgan $\vec{n}=|A,B|$ normal vektorga ega to'g'ri chiziq tenglamasi

$$A(x-x_0)+B(y-y_0)=0;$$

b) to'g'ri chiziqning kanonik tenglamasi

$$\frac{x-x_0}{l}=\frac{y-y_0}{m},$$

bunda $\vec{s}=(l,m)$ — to'g'ri chiziqning yo'naltiruvchi vektori, $M_0(x_0,y_0)$ — to'g'ri chiziqda yotuvchi berilgan nuqta;

c) to'g'ri chiziqning burchak koeffisentli tenglamasi

$$y=kx+b,$$

bunda b — to'g'ri chiziqning Oy o'qdan kesadigan kesmasi; k — to'g'ri chiziqning burchak koeffisenti; $k=tg\alpha$ (α - to'g'ri chiziq bilan Ox o'qning musbat yo'nalishi orasidagi burchak);

d) $M_0(x_0,y_0)$ nuqtadan o'tuvchi va k burchak koeffisentli to'g'ri chiziqning tenglamasi

$$y-y_0=k(x-x_0);$$

e) to'g'ri chiziqning kesmalarga nisbatan tenglamasi

$$\frac{x}{a}+\frac{y}{b}=1,$$

bunda a va b - to'g'ri chiziqning koordinatalar o'qlarida kesadigan kesmasi;

f) berilgan ikki $M_1(x_1,y_1)$ va $M_2(x_2,y_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x-x_0}{x_2-x_1}=\frac{y-y_0}{y_2-y_1}.$$

Misol. $M_1(-2;1;-1)$ nuqtadan o'tuvchi $\vec{s}=\{1,\ -1,\ 2\}$ vektorga parallel to'g'ri chiziq tenglamasini toping.

Yechish. $\vec{S}$ vektor to'g'ri chiziqqa parallel bo'lgani uchun u to'g'ri chiziqning yo'naltiruvchi vektori bo'ladi. Shu sababli to'g'ri chiziqning kanonik tenglamalari $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ ga asosan izlanayotgan to'g'ri chiziq tenglamalari

$$\frac{x+2}{1}=\frac{y-1}{-1}=\frac{z+1}{2}$$

ko'rinishda bo'ladi.

2.2. Tekisliklar va to'g'ri chiziqlarning o'zaro joylashuvi. Tekisliklar va to'g'ri chiziqlar orasidagi burchak. Nuqtadan to'g'ri chiziqqacha va tekislikkacha bo'lgan masofa

Tekisliklar $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi burchak quyidagi formula asosida hisoblanadi:

$$\cos\varphi=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1|\cdot|\vec{n}_2|}=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}.$$

bunda $\vec{n}_1=\{A_1,\ B_1,\ C_1\}$ va $\vec{n}_2=\{A_2,\ B_2,\ C_2\}$— berilgan tekisliklarning normal vektorlari.

a) Agar tekisliklar perpendikulyar bo'lsa, u holda $\vec{n}_1\cdot\vec{n}_2=0$ yoki

$$A_1A_2+B_1B_2+C_1C_2=0.$$

b) Agar tekisliklar parallel bo'lsa, u holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}\neq\frac{D_1}{D_2}.$$

c) Agar tekisliklar ustma-ust tushsa, u holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}.$$

d) $M_0(x_0,y_0,z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikgacha bo'lgan d masofa:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

formula bo'yicha hisoblanadi.

To'g'ri chiziqlar

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{p_1}$$

va

$$\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{p_2}$$

kanonik tenglamalar bilan berilgan bo'lsin. Bu to'g'ri chiziqlar orasidagi φ burchak quyidagi formuladan topiladi:

$$\cos\varphi=\frac{\vec{s}_1\cdot\vec{s}_2}{|\vec{s}|_1\cdot|\vec{s}_2|}=\frac{l_1l_2+m_1m_2+p_1p_2}{\sqrt{l_1^2+m_1^2+p_1^2}\cdot\sqrt{l_2^2+m_2^2+p_2^2}}.$$

a) Agar to'g'ri chiziqlar perpendikulyar bo'lsa, u holda

$$\vec{S}_1\cdot\vec{S}_2=0$$

yoki

$$l_1l_2+m_1m_2+p_1p_2=0.$$

b) Agar to'g'ri chiziqlar parallel bo'lsa, u holda $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{p_1}{p_2}.$

c) Agar to'g'ri chiziqlar ustma-ust tushsa, u holda $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{p_1}{p_2}$

,

shu bilan birga

$$\frac{x_2-x_1}{l_1}=\frac{y_2-y_1}{m_1}=\frac{z_2-z_1}{p_1}.$$

d) Agar to'g'ri chiziqlar kesishsa, u holda

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & p_1 \\ l_2 & m_2 & p_2 \end{vmatrix}=0.$$

e) Agar to'g'ri chiziqlar ayqash bo'lsa, u holda

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & p_1 \\ l_2 & m_2 & p_2 \end{vmatrix}\neq 0.$$

$M_1(x_1,y_1,z_1)$ nuqtadan $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ to'g'ri chiziqqacha bo'lgan masofa quyidagi formula bo'yicha hisoblanadi:

$$d=\frac{|\vec{s}\times\overline{M_1M_0}|}{|\vec{s}|},$$

bunda $M_0(x_0,y_0,z_0)$ nuqta shu to'g'ri chiziqqa tegishli va $\vec{s}=(l,m,p)$ uning yo'naltiruvchi vektori. Ikki ayqash

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{p_1}$$

va

2.2. Tekisliklar va to'g'ri chiziqlarning o'zaro joylashuvi. Tekisliklar va to'g'ri chiziqlar orasidagi burchak. Nuqtadan to'g'ri chiziqqacha va tekislikkacha bo'lgan masofa

Tekisliklar $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tenglamalar bilan berilgan bo'lsin. Ular orasidagi burchak quyidagi formula asosida hisoblanadi:

$$\cos\varphi=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1|\cdot|\vec{n}_2|}=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}.$$

bunda $\vec{n}_1=|A_1,\ B_1,\ C_1|$ va $\vec{n}_2=|A_2,\ B_2,\ C_2|$— berilgan tekisliklarning normal vektorlari.

a) Agar tekisliklar perpendikulyar bo'lsa, u holda $\vec{n}_1\cdot\vec{n}_2=0$ yoki

$$A_1A_2+B_1B_2+C_1C_2=0.$$

b) Agar tekisliklar parallel bo'lsa, u holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}\neq\frac{D_1}{D_2}.$$

c) Agar tekisliklar ustma-ust tushsa, u holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}.$$

d) $M_0(x_0,y_0,z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikgacha bo'lgan d masofa:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

formula bo'yicha hisoblanadi.

To'g'ri chiziqlar

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{p_1}$$

va

$$\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{p_2}$$

kanonik tenglamalar bilan berilgan bo'lsin. Bu to'g'ri chiziqlar orasidagi φ burchak quyidagi formuladan topiladi:

$$\cos\varphi=\frac{\bar{s}_1\cdot\bar{s}_2}{|\bar{s}|_1\cdot|\bar{s}_2|}=\frac{l_1l_2+m_1m_2+p_1p_2}{\sqrt{l_1^2+m_1^2+p_1^2}\cdot\sqrt{l_2^2+m_2^2+p_2^2}}.$$

a) Agar to'g'ri chiziqlar perpendikulyar bo'lsa, u holda

$$\bar{S}_1\cdot\bar{S}_2=0$$

yoki

$$l_1l_2+m_1m_2+p_1p_2=0.$$

b) Agar to'g'ri chiziqlar parallel bo'lsa, u holda $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{p_1}{p_2}.$

c) Agar to'g'ri chiziqlar ustma-ust tushsa, u holda $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{p_1}{p_2}$

, shu bilan birga

$$\frac{x_2-x_1}{l_1}=\frac{y_2-y_1}{m_1}=\frac{z_2-z_1}{p_1}.$$

d) Agar to'g'ri chiziqlar kesishsa, u holda

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & p_1 \\ l_2 & m_2 & p_2 \end{vmatrix}=0.$$

e) Agar to'g'ri chiziqlar ayqash bo'lsa, u holda

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & p_1 \\ l_2 & m_2 & p_2 \end{vmatrix}\neq 0.$$

$M_1(x_1,y_1,z_1)$ nuqtadan $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ to'g'ri chiziqqacha bo'lgan masofa quyidagi formula bo'yicha hisoblanadi:

$$d=\frac{|\bar{s}\times\overline{M_1M_0}|}{|\bar{s}|},$$

bunda $M_0(x_0,y_0,z_0)$ nuqta shu to'g'ri chiziqqa tegishli va $\bar{s}=(l,m,p)$ uning yo'naltiruvchi vektori. Ikki ayqash

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{p_1}$$

va

$$\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{p_2}$$

to'g'ri chiziqlar orasidagi eng qisqa d masofa quyidagicha aniqlanadi:

$$d=\frac{|\overline{M_1M_2}\cdot\bar{s}_1\cdot\bar{s}_2|}{|\bar{s}_1\times\bar{s}_2|},$$

bunda $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtalar mos ravishda bu to'g'ri chiziqlarga tegishli. $\bar{s}_1=(l_1,m_1,p_1)$ va $\bar{s}_2=(l_2,m_2,p_2)$ lar esa ularning yo'naltiruvchi vektorlari.

Misol. $x-2y+2z-8=0$ va $x+z-6=0$ tekisliklar orasidagi burchakni toping.

Yechish. Ikki tekislik orasidagi burchak formulasiga ko'ra:

$$\cos\varphi=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}=\frac{1\cdot1+(-2)\cdot0+2\cdot1}{\sqrt{1+4+4}\cdot\sqrt{1+1}}=\frac{\sqrt{2}}{2}.$$

Bundan $\varphi=\arccos\frac{\sqrt{2}}{2}=45^\circ$ kelib chiqadi.

$Ax+By+Cz+D=0$ tekislik bilan $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ to'g'ri chiziq orasidagi φ burchak ushbu formula bo'yicha hisoblanadi:

$$\sin\varphi=\frac{\bar{n}\cdot\bar{s}}{|\bar{n}|\cdot|\bar{s}|}=\frac{Al+Bm+Cp}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{l^2+m^2+p^2}}.$$

Bunda $\bar{n}=|A,B,C|$— tekislikning normal vektori $\bar{s}=\{A,B,C\}$ to'g'ri chiziqning yo'naltiruvchi vektori.

a) Agar tekislik bilan to'g'ri chiziq perpendikulyar bo'lsa, $\bar{n}$ va $\bar{s}$ vektorlar kollinear yoki $\frac{A}{l}=\frac{B}{m}=\frac{C}{p}$ bo'ladi.

b) Agar tekislik bilan to'g'ri chiziq parallel bo'lsa, u holda $\bar{n}$ va $\bar{s}$ vektorlar perpendikulyar yoki $Al+Bm+Cp\neq0$ bo'ladi.

c) Agar tekislik bilan to'g'ri chiziq ustma-ust tushsa, u holda $Al+Bm+Cp=0$, shu bilan birga $Ax_0+By_0+Cz_0+D=0$ bo'ladi.

d) Agar tekislik bilan to'g'ri chiziq kesishsa, u holda

$$Al+Bm+Cp\neq0.$$

Tekislikdagi to'g'ri chiziqlar

$$A_1x+B_1y+C_1=0 \text{ va } A_2x+B_2y+C_2=0$$

tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchak ushbu formula bo'yicha hisoblanadi:

$$\cos\varphi=\frac{\bar{n}_1\cdot\bar{n}_2}{|\bar{n}|\cdot|\bar{n}_2|}=\frac{A_1A_2+B_1B_2}{\sqrt{A_1^2+B_1^2}\cdot\sqrt{A_2^2+B_2^2}}.$$

bunda $\bar{n}_1=|A_1,\ B_1|$ va $\bar{n}_2=|A_2,\ B_2|$— mos ravishda berilgan to'g'ri chiziqlarning normal vektorlari.

a) Agar bu to'g'ri chiziqlar o'zaro perpendikulyar bo'lsa, u holda

$$A_1A_2+B_1B_2=0.$$

b) Agar bu to'g'ri chiziqlar parallel bo'lsa, u holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}.$$

c) Agar bu to'g'ri chiziqlar ustma-ust tushsa, u holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}.$$

Tekislikdagi to'g'ri chiziqlar

$$y=k_1x+b_1 \text{ va } y=k_2x+b_2$$

tenglamalar bilan berilgan bo'lsin. Ular orasidagi φ burchak ushbu formula bo'yicha hisoblanadi:

$$tg\varphi=\frac{k_1+k_2}{1+k_1k_2}.$$

Bu to'g'ri chiziqlarning perpendikulyarlik sharti $k_1k_2=-1$ dan iborat, parallellik sharti esa $k_1=k_2$ bo'ladi.

$M_0(x_0,y_0)$ nuqtadan $Ax+By+C=0$ to'g'ri chiziqqacha bo'lgan d masofa ushbu

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$$

formula bo'yicha hisoblanadi.

2.3. Mustaqil ishlash uchun topshiriqlar.

1 – topshiriq. A, B, C, D nuqtalar berilgan:

a) AB kesma uzunligini;

b) ABC uchburchakning B burchagining kosinusini;

d) $\overline{AB}^0$ va $\overline{AB}$ vektorning yo'naltiruvchi kosinuslarini;

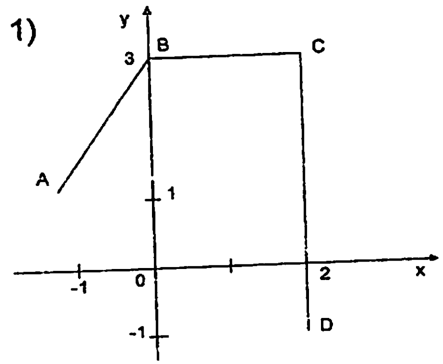
e) ABC uchburchakning yuzini;

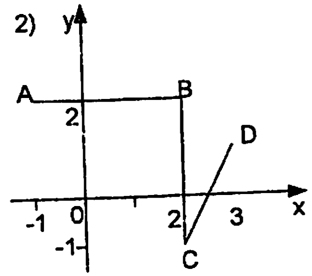
f) ABC uchburchakning C uchdan AB tomonga tushirilgan h balandligini toping.

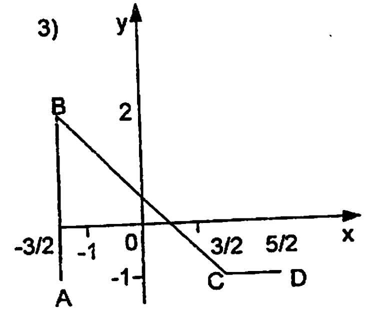
№	A	B	C	D	α	β
1	(–3;2;–1)	(1;–1;4)	(2;0;1)	(1;–3;5)	2	–1
2	(1;–2;1)	(3;0;2)	(–4;2;–1)	(–1;–1;3)	–2	3
3	(–4;–1;1)	(–2;0;–1)	(–1;–2;3)	(1;–3;–1)	–2	1
4	(2;0;–3)	(1;–1;2)	(3;1;–1)	(–2;–1;–1)	3	–2
5	(–1;–1;1)	(2;–2;0)	(3;1;–4)	(–2;1;3)	4	–1
6	(–2;2;1)	(3;0;–1)	(2;1;–4)	(3;2;–2)	–2	–3
7	(1;–1;–1)	(2;–1;0)	(4;1;–2)	(3;0;1)	1	2
8	(4;1;–1)	(–2;–1;1)	(0;2;–1)	(3;1;–2)	–3	1
9	(0;–2;–1)	(3;1;–2)	(4;2;1)	(1;–1;4)	2	5
10	(1;3;–3)	(2;1;0)	(–1;2;–1)	(3;2;1)	–2	–1
11	(–2;1;1)	(1;–1;0)	(2;3;–1)	(–1;–2;1)	3	2
12	(–3;1;2)	(–2;3;1)	(–1;4;1)	(1;0;3)	–1	–3
13	(2;1;–5)	(3;0;–2)	(1;–1;0)	(–1;2;–4)	–3	2
14	(0;–1;4)	(2;–2;5)	(4;1;0)	(–2;2;3)	4	–2
15	(3;–2;1)	(5;–3;4)	(2;1;1)	(–1;2;3)	2	–3
16	(–3;5;–1)	(–2;3;2)	(0;1;–2)	(–1;1–1)	5	3
17	(2;–1;–4)	(–1;–1;–2)	(1;0;1)	(3;1;2)	4	–3
18	(3;5;2)	(0;4;1)	(2;–1;–1)	(4;2;–3)	–2	5
19	(–4;–1;2)	(–2;0;5)	(–1;1;3)	(–3;4;7)	1	3
20	(6;–1;1)	(4;0;5)	(3;–2;1)	(1;–4;4)	–2	4
21	(5;2;–3)	(1;3;–1)	(2;4;–5)	(4;–1;1)	–5	2
22	(–1;–1;7)	(1;–3;5)	(2;–4;3)	(3;1;–1)	–4	3
23	(2;–7;–5)	(1;–4;–6)	(–1;–8;–3)	(5;–4;–2)	5	–3
24	(–3;2;8)	(1;1;5)	(–1;3;3)	(0;4;1)	3	4

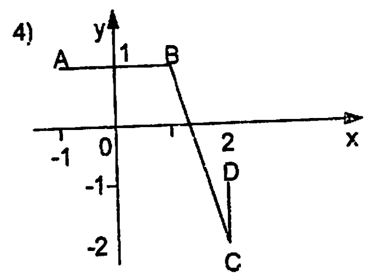
25	(6;–1;–1)	(4;–2;0)	(7;0;1)	(2;–3;2)	–2	–5
26	(–5;2;–4)	(–3;1;–6)	(0;–1;–1)	(–1;–2;2)	4	5
27	(4;–2;–3)	(2;1;–2)	(–1;0;–1)	(3;2;–4)	3	5
28	(–1;–1;4)	(2;1;3)	(–3;2;1)	(0;1;–1)	–3	4
29	(–5;–3;1)	(–6;–2;2)	(–1;–4;1)	(–4;1;–1)	1	5
30	(–6;–2;1)	(–8;0;1)	(–4;–3;2)	(–5;3;–1)	5	4

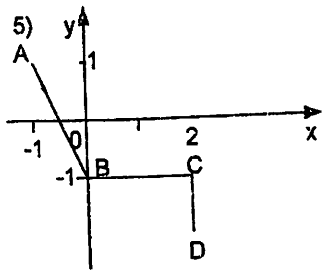
2 – topshiriq. To'g'ri chiziqlarni tenglamalarini tuzing: AB, BC, CD.

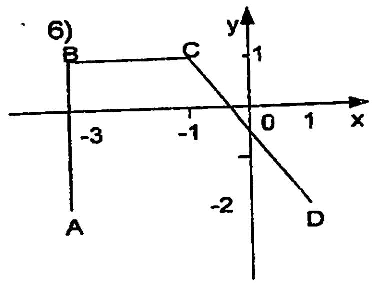












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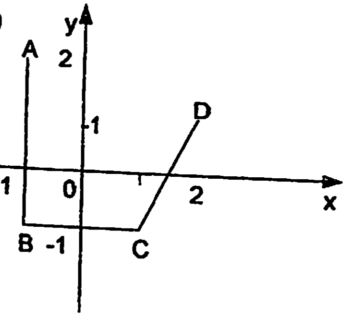

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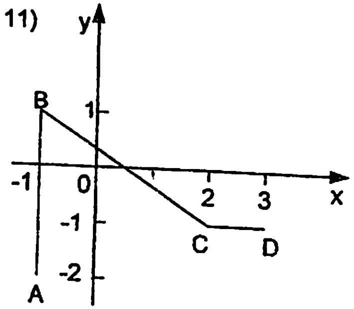

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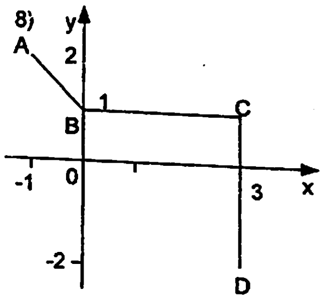

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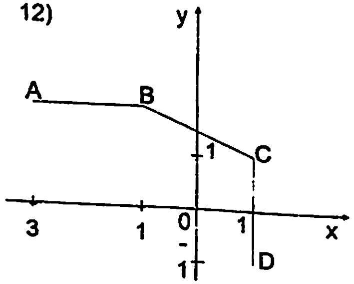

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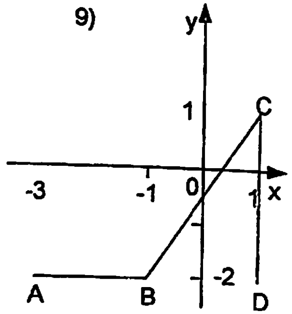

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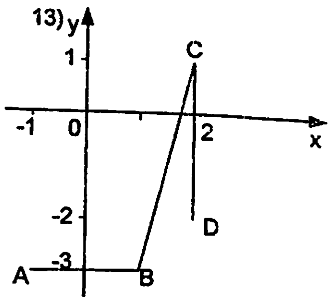

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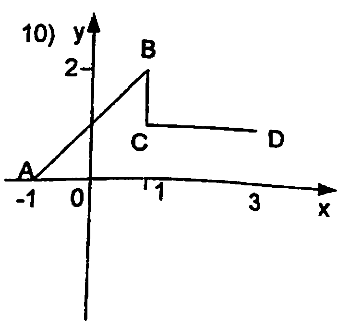

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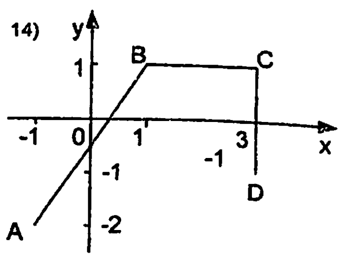

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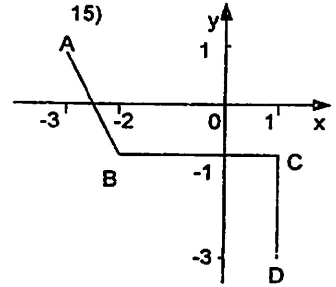

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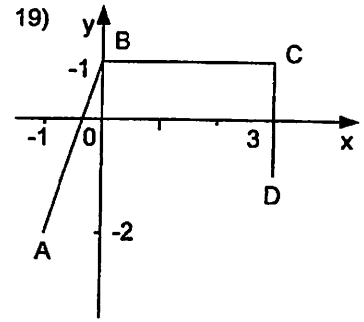

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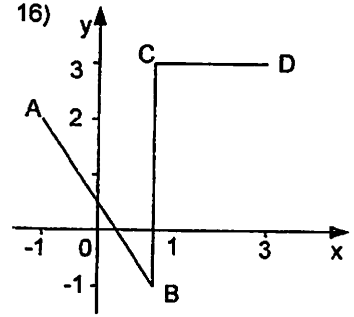

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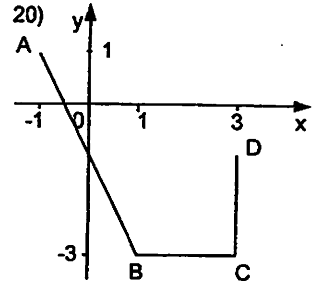

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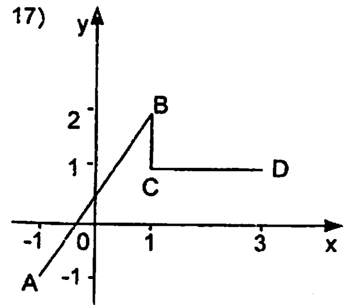

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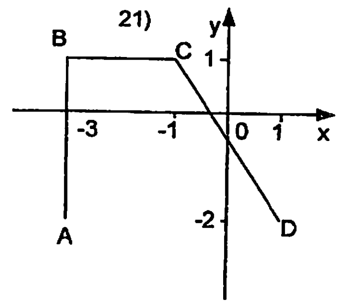

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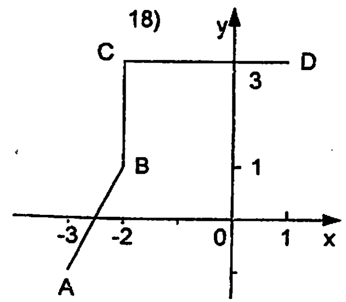

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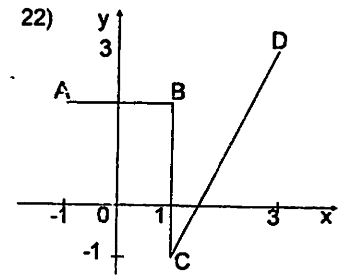
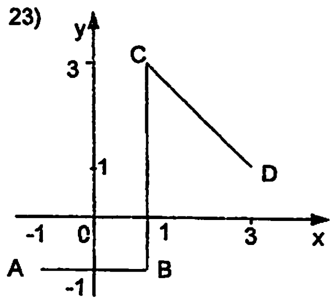
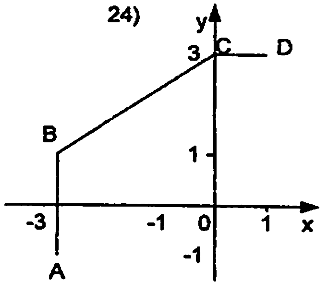
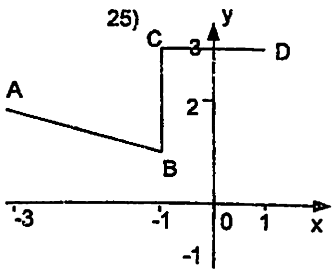
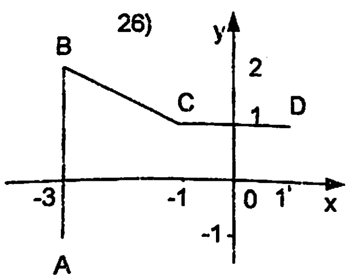
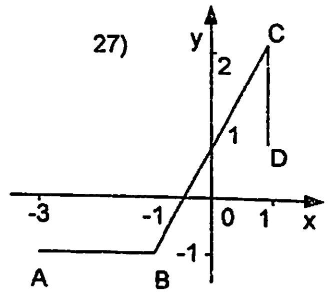


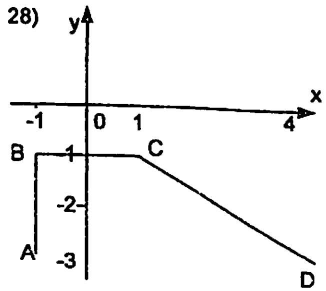


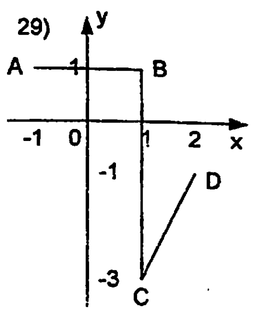


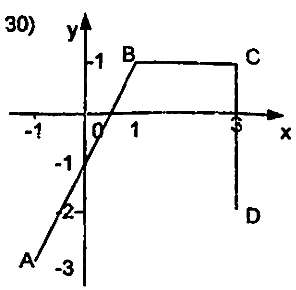












3–topshiriq.

$(\Pi): Ax+By+Cz+D=0$, $(\Pi'): A'x+B'y+C'z+D'=0$, $(\Pi''): A''x+B''y+C''z+D''=0$ tekisliklar, $(L): \dfrac{x-x_0}{l}=\dfrac{y-y_0}{m}=\dfrac{z-z_0}{n}$ to'g'ri chiziq va $M(x';y';z')$ nuqta berilgan.

a) M nuqtadan o'tuvchi va (П) tekislikka parallel bo'lgan tekislik tenglamasini tuzing;

b) M nuqtadan o'tuvchi va (L) tekislikka parallel bo'lgan tekislik tenglamasini tuzing;

c) M nuqtadan o'tuvchi va (L) tekislikka perpendikulyar bo'lgan tekislik tenglamasini tuzing;

d) M nuqtadan o'tuvchi va (П) tekislikka perpendikulyar bo'lgan tekislik tenglamasini tuzing;

e) M nuqtadan va (L) to'g'ri chiziqdan o'tuvchi tekislik tenglamasini tuzing;

f) (L) to'g'ri chiziq va (П) tekisliklarning kesishish nuqtalarini toping;

g) M nuqtadan o'tuvchi, (Π') va (Π'') tekislikka perpendikulyar bo'lgan tekislik tenglamasini tuzing;

h) $\begin{cases} A'x+B'y+C'z+D'=0, \\ A''x+B''y+C''z+D''=0 \end{cases}$ to'g'ri chiziqlarning kanonik tenglamasini tuzing;

i) M nuqtadan (П) tekislikkacha bo'lgan masofani toping.

№	(A;B;C;D)	(A';B';C';D')	(A";B";C";D")	$(x_0;y_0;z_0)$	(l; m; n)	(x';y';z')
1	(–5;2;–1;3)	(2;1;–1;4)	(–3;–1;0;1)	(2;–2;3)	(1;5;–4)	(2;–1;–3)
2	(1;3;–4;1)	(–3;–1;2;6)	(2;5;–1;–4)	(–1;–4;2)	(2;–1;3)	(–2;1;1)
3	(–2;2;3;7)	(1;–1;3;4)	(–1;–2;3;5)	(2;7;–9)	(3;–2;–1)	(3;–1;1)
4	(–1;5;–3;8)	(2;7;–1;3)	(–2;1;–1;6)	(–9;–6;1)	(–4;–2;–5)	(2;1;–1)
5	(4;–2;1;3)	(–1;–3;5;0)	(3;2;–1;9)	(3;5;–7)	(2;1;–4)	(3;–1;1)
6	(1;–3;0;5)	(–2;6;1;–7)	(2;4;–3;–8)	(7;–7;5)	(3;–3;2)	(–1;–2;–3)
7	(3;–2;1;–5)	(1;–1;–4;6)	(–6;2;1;7)	(3;0;1)	(2;–3;0)	(4;1;1)
8	(–4;1;–3;–6)	(0;2;1;–8)	(3;5;–1;1)	(7;–9;–6)	(5;–2;1)	(–3;–2;1)

9	(0;3;–2;3)	(–4;–1;2;7)	(1;3;–5;6)	(3;–3;4)	(–1;3;7)	(2;–2;1)
10	(2;–3;1;5)	(–1;–1;3;4)	(–4;1;–1;5)	(4;9;–9)	(2;7;3)	(1;–2;–2)
11	(–1;–5;3;0)	(1;2;–1;4)	(–3;–1;1;8)	(5;–3;–2)	(2;–1;–4)	(–5;–2;–3)
12	(6;–2;–1;3)	(–2;1;–3;5)	(–1;3;4;1)	(6;2;–1)	(5;–1;2)	(1;–1;4)
13	(4;–3;1;2)	(0;3;–2;6)	(2;1;–1;3)	(–3;–5;1)	(0;2;–1)	(1;–1;3)
14	(–3;0;2;–6)	(1;–2;4;–5)	(–2;3;1;3)	(2;–8;5)	(1;–2;–3)	(–1;1;–2)
15	(–4;–3;1;9)	(2;–2;5;1)	(1;3;–1;4)	(3;–2;–4)	(1;2;–3)	(–1;–2;3)
16	(–2;5;–1;2)	(–3;1;3;4)	(1;–2;–1;6)	(–4;–2;3)	(2;–1;6)	(2;1;–1)
17	(4;1;–1;5)	(3;–3;1;1)	(2;0;1;–4)	(7;–5;2)	(3;–4;1)	(–1;2;0)
18	(5;1;–1;8)	(–2;1;–3;4)	(3;–1;2;9)	(5;2;–4)	(–1;2;–1)	(1;2;3)
19	(1;–2;4;3)	(3;–1;0;6)	(–2;–1;3;4)	(2;–7;9)	(2;–1;2)	(1;3;–1)
20	(3;–2;–1;7)	(2;1;3;–8)	(1;–3;–3;5)	(6;–1;8)	(–2;0;1)	(–1;2;1)
21	(2;2;5;–1)	(1;–1;4;7)	(0;2;1;3)	(2;9;3)	(1;–4;1)	(4;–3;–1)
22	(–3;5;1;4)	(2;1;–1;3)	(4;–2;–1;5)	(6;8;–1)	(2;1;–2)	(1;–1;2)
23	(0;2;–1;7)	(3;–4;1;6)	(2;1;–3;7)	(2;–4;–6)	(3;–4;1)	(–4;2;1)
24	(2;–2;3;1)	(4;–1;1;3)	(–3;3;2;5)	(6;–2;4)	(2;–1;1)	(3;3;1)
25	(–5;1;–1;3)	(2;1;–2;5)	(4;–1;3;0)	(2;–5;4)	(1;–2;3)	(2;–2;3)
26	(3;4;–5;1)	(–2;3;4;6)	(2;0;1;9)	(3;–4;1)	(2;–3;2)	(–1;–1;–2)
27	(4;3;–1;8)	(–3;1;2;5)	(1;–1;5;4)	(4;–2;1)	(–3;1;3)	(2;1;–5)
28	(1;–4;1;5)	(2;1;4;–6)	(3;–3;2;1)	(5;0;2)	(–1;2;–4)	(–3;–2;1)
29	(2;1;–3;4)	(–5;2;3;4)	(1;–1;3;–2)	(–8;1;3)	(–1;3;2)	(4;–2;1)
30	(5;–2;1;3)	(2;1;–1;3)	(–4;2;3;1)	(3;–2;7)	(1;–2;–4)	(–1;–2;4)

III BOB. MATEMATIK ANALIZGA KIRISH

3.1. To'plamlar va ular haqida asosiy tushunchalar.

To'plam tushunchasi matematikaning boshlang'ich va muhim tushunchalaridan biridir. Masalan: Natural sonlar to'plami, auditoriyadagi talabalar to'plami, kutubxonadagi kitoblar to'plami, bir nuqtadan o'tuvchi to'g'ri chiziqlar to'plami biror xildagi mahsulot ishlab chiqaruvchi korxonalar to'plami va boshqalar.

To'plamni tashkil etgan narsalar to'plamning elementlari deyiladi. Matematikada to'plamlar bosh harflar bilan, masalan: $A, B, X, Y, \ldots\ldots$ uning elementlari esa kichik harflar, masalan: $a, b, x, y, \ldots\ldots$ bilan belgilanadi.

To'plam chekli sondagi elementlardan tashkil topgan bo'lsa, unga ***chekli to'plam*** deb ataladi. Masalan, kutubxonadagi kitoblar soni yoki guruhdagi talabalar soni chekli bo'ladi. Cheksiz elementlardan tashkil topgan to'plam ***cheksiz to'plam*** deb ataladi. Masalan, natural sonlar to'plami, bitta nuqtadan o'tuvchi to'g'ri chiziqlar to'plami va boshqalar.

x element X to'plamga tegishli bo'lsa, $x \in X$ deb belgilanadi, aks holda $x \notin X$ yoziladi. $\{x \in X / P(x)\}$ belgi P xossaga ega bo'lgan $x \in X$ lar to'plamini bildiradi. Bo'sh to'plamni

$$\varnothing = \{x \in \varnothing / x \neq x\}$$

deb yozish mumkin.

1-misol. Quyidagi xossalarga ega bo'lgan to'plamlar elementlarini aniqlang.

1) $A = \{x \in N | x \leq 5\}$;

2) $B = \{x \in N | x \leq 0\}$;

3) $C = \{x \in Z | |x| \leq 2\}$

Yechish. 1) To'plam 5 dan kichik va teng bo'lgan natural sonlardan iboratligini bildiradi, ya'ni $A = \{1,2,3,4,5\}$.

2) manfiy natural son yo'q shuning uchun $B = \varnothing$.

3) bu holda $|x| \leq 2$ tengsizlikni qanoatlantiruvchi faqat butun sonlar olinadi, bu $[-2; 2]$ kesmada bo'ladi. Shunday qilib,

$C=\{-2;-1;0;1;2\}$.

Qavariq to'plam.

1-ta'rif. Istalgan ikki nuqta shu to'plamga tegishli bo'lganda, bu nuqtalarni tutashtiruvchi to'g'ri chiziq kesmasi ham shu to'plamga tegishli bo'lsa, bunday to'plamga ***qavariq to'plam*** deyiladi.

Nuqtaning atrofi.

2-ta'rif. r biror musbat son bo'lsin. $M_0 \in R^n$ fazoning nuqtasi uchun $\rho(M, M_0)<r$ tengsizlikni qanoatlantiruvchi hamma $M \in R^n$ nuqtalar to'plamiga M_0 ***nuqtaning*** r***-atrofi*** deyiladi va $S_r(M_0)$ bilan belgilanadi, ya'ni

$$S_r(M_0)=\{M \in R^n |\ \rho(M, M_0)<r\}.$$

Masalan, $M_1(2;3;-1;3) \in S_2(M_0)$, $M_0(1;2;-1;2)$ nuqtaning $S_r(M_0)$ atrofiga tegishli, chunki

$$\rho(M,M_0)=\sqrt{(1-2)^2+(2-3)^2+(-1+1)^2+(2-3)^2}=\sqrt{3}$$

bo'lib, $\sqrt{3}<2$ bo'ladi. $M_2(3;3;-1;3)$ nuqta $S_2(M_0)$ atrofga tegishli emas, chunki $\rho(M_2, M_0)=\sqrt{(1-3)^2+(2-3)^2+(-1+1)^2+(2-3)^2}=\sqrt{6}$ bo'lib, $\sqrt{6}>2$ bo'ladi.

R^1 (sonlar o'qi) fazoda $M_0(a)$ nuqtaning r atrofi $(a-r, a+r)$ intervaldan iborat.

R^2 (tekislik) fazoda $M_0(a,b)$ nuqtaning r atrofi, radiusi r, markazi $M_0(a,b)$ nuqtada bo'lgan doiraning ichki nuqtalaridan iborat bo'ladi.

R^3 fazoda esa, $M_0(a,b,c)$ nuqtaning r atrofi, radiusi r, markazi $M_0(a,b,c)$ nuqtada bo'lgan sharning ichki qismidan iborat bo'ladi.

To'plamning chegaralanganligi.

3-ta'rif. R^n fazoning B to'plamning istalgan $M(x_1,x_2,....,x_n) \in B$ nuqtasi uchun shunday $A>0$ son mavjud bo'lib,

$$|x_1| \le A, \quad |x_2| \le A, ..., |x_n| \le A$$

munosabatlar bajarilsa, B to'plamga ***chegaralangan to'plam*** deyiladi. Masalan, n o'lchovli fazoda istalgan nuqtaning r atrofi chegaralangan to'plamdir.

To'plamning ichki va chegaraviy nuqtalari.

4-ta'rif. $M_0 \in B$ nuqta B to'plamga o'zining biror r atrofi bilan kirsa, unga B ***to'plamning ichki nuqtasi*** deyiladi .

5-ta'rif. $M_0 \in B$ nuqta o'zining har bir atrofida B to'plamga tegishli bo'lgan hamda tegishli bo'lmagan nuqtalar bilan kirsa, M_0 nuqtaga B to'plamning ***chegaraviy nuqtasi*** deyiladi .

To'plamning quyuqlanish nuqtasi.

6-ta'rif. M_0 nuqtaning ixtiyoriy atrofi B to'plamning M_0 nuqtadan farqli cheksiz ko'p nuqtalari (M_0 nuqtadan farqli)ni o'z ichiga olsa, M_0 nuqta B to'plamning ***quyuqlanish nuqtasi*** deyiladi. Quyuqlanish nuqtasi to'plamning o'ziga qarashli bo'lishi ham, qarashli bo'lmasligi ham mumkin. Masalan, $B=[a,b]$ yoki $B=(a,b]$ bo'lsa, ikkala holda ham a nuqta B uchun quyuqlanish nuqtasi bo'ladi, lekin birinchi holda bu nuqta B to'plamda yotadi, ikkinchi holda esa u B to'plamda yotmaydi.

Yopiq va ochiq to'plamlar.

7-ta'rif. B to'plam o'zining hamma quyuqlanish nuqtalarini o'zida saqlasa, unga ***yopiq to'plam*** deyiladi. Masalan, $[a,b]$ kesma R^1 sonlar o'qida, $\{M(x,y) \in R^2 |\ x^2+y^2 \le r^2\}$ R^2 doira tekislikda yopiq to'plamlardir.

8-ta'rif. B to'plamning hamma nuqtalari ichki nuqtalar bo'lsa, bunday to'plamga ***ochiq to'plam*** deyiladi. Masalan, (a,b) R^1 da, $\{M(x,y) \in R^2 |\ x^2+y^2<r^2\}$ R^2 da ochiq to'plamlardir. R^n fazoda istalgan nuqtaning r atrofi ochiq to'plamdir.

R^n fazoda chegaralangan yopiq to'plamga ***kompakt*** deb ataladi.

To'plamlar ustida amallar. B to'plamning har bir elementi A

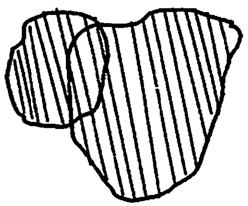
to'plamning ham elementi bo'lsa, B to'plamga A to'plamning ***qism to'plami*** deyiladi va $B \subset A$ yoki $A \supset B$ bilan belgilanadi. $A \subset B$ va $B \subset A$ bo'lsa, A va B to'plamlar teng deyiladi va $A = B$ bilan belgilanadi.

1) A va B ***to'plamlarning birlashmasi*** **(yig'indisi)** deb uchinchi bir C to'plamga aytiladiki, bu to'plamning istalgan elementi A yoki B to'plamga tegishlai bo'ladi va $A \cup B$ bilan belgilanadi, ya'ni

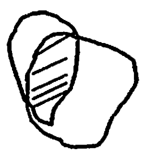
$$C = A \cup B = \{x | \ x \in A \quad yoki \quad x \in B\} \text{ (1 -chizma).}$$

2) A va B ***to'plamlarning kesishmasi*** **(ko'paytmasi)** deb, uchunchi bir C to'plamga aytiladiki, uning har bir elementi A to'plamga ham, B to'plamga ham tegishli bo'ladi va $A \cap B$ bilan belgilanadi, ya'ni $C = A \cap B = \{x | \ x \in A \quad va \quad x \in B\}$ (2 -chizma).

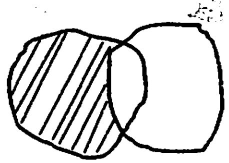
3) A to'plamdan B to'plamning **farqi (ayirmasi)** deb shunday uchinchi bir C to'plamga aytiladiki, uning har bir elementi A ga tegishli bo'lsa, B ga tegishli bo'lmaydi, va uni $A / B = \{x | \ x \in A \quad va \quad x \notin B\}$ (3-chizma).



1-chizma



2-chizma



3-chizma

2-misol. $A = \{1, \ 2\}$ to'plamning hamma qism to'plamlaridan iborat bo'lgan B to'plamni tuzing.

Yechish. Qism to'plam ta'rifiga asosan, $\varnothing \in A$, $\{1\} \in A$, $\{1, \ 2\} \in A$, $\{2\} \in A$. Demak, $B = \{\varnothing, \{1\}, \{2\}, \{1, \ 2\}\}$.

3-misol. $A = (4, \ 8)$ va $B = (1, \ 4]$ bo'lsa, ularning birlashmasini va kesishmasini toping.

Yechish. Birlashmaning ta'rifidan $A \cup B = (1{,}8)$ bo'lib,

kesishmaning ta'rifidan $A \cap B = \varnothing$ bo'ladi.

4-misol. $A = (-3, \ 7]$ va $B[5, \ 6]$ bo'lsa, ularning birlashmasi va kesishmasini toping.

Yechish. Ta'rifga asosan $A \cup B = (-3, \ 7]$, $A \cap B = [5{,}6]$ bo'ladi.

5- misol. Ushbu

$$A = \{1{,}2{,}3{,}4{,}5{,}6\}, \quad B = \{2{,}4{,}6{,}8\}, \quad C = \{1{,}3\}$$

to'plamlarni qaraylik. Bu to'plamlar uchun

$$A \cup B = \{1{,}2{,}3{,}4{,}5{,}6{,}8\},$$
$$A \cap B = \{2{,}4{,}6\},$$
$$A \backslash B = \{1{,}3{,}5\},$$
$$B \setminus A = \{8\},$$
$$A \cup C = \{1{,}2{,}3{,}4{,}5{,}6\},$$
$$A \cap C = \{1{,}3\},$$
$$B \cap C = \varnothing,$$
$$B \times C = \{(2{,}1), (2{,}3), (4{,}1), (4{,}3), (6{,}3)(8{,}1), (8{,}3)\}.$$

bo'ladi.

Yuqorida keltirilgan ta'riflardan

$$E \cup E = E, \quad E \cap E = E, \quad E \setminus E = \varnothing,$$

shuningdek $E \subset F$ bo'lganda

$$E \cup F = F, \quad E \cap F = E$$

bo'lishi kelib chiqadi.

Barcha 1, 2, 3, ..., n, ... - natural sonlardan iborat to'plam natural sonlar to'plami deyiladi va u N harfi bilan belgilanadi:

$$N = \{1, \ 2, \ 3, \dots, \ n, \dots\}.$$

Barcha ..., -2, -1, 0, 1, 2, ... - butun sonlardan iborat to'plam ***butun sonlar to'plami*** deyiladi va u Z harfi bilan belgilanadi:

$$Z = \{\dots, -2, \ -1, \ 0, \ 1, \ 2, \dots\}$$

Ravshanki,

$$N \subset Z$$

bo'ladi.

Tartiblangan to'plamlar haqida

Agar biror E to'plamning elementlari uchun quyidagi tasdiqlar:

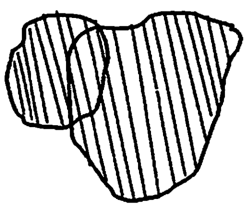
to'plamning ham elementi bo'lsa, B to'plamga A to'plamning ***qism to'plami*** deyiladi va $B \subset A$ yoki $A \supset B$ bilan belgilanadi. $A \subset B$ va $B \subset A$ bo'lsa, A va B to'plamlar teng deyiladi va $A = B$ bilan belgilanadi.

1) A va B ***to'plamlarning birlashmasi*** **(yig'indisi)** deb uchinchi bir C to'plamga aytiladiki, bu to'plamning istalgan elementi A yoki B to'plamga tegishlai bo'ladi va $A \cup B$ bilan belgilanadi, ya'ni

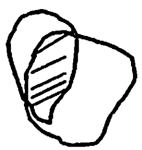
$$C = A \cup B = \{x |\ x \in A \quad yoki \quad x \in B\} \text{ (1 -chizma).}$$

2) A va B ***to'plamlarning kesishmasi*** **(ko'paytmasi)** deb, uchunchi bir C to'plamga aytiladiki, uning har bir elementi A to'plamga ham, B to'plamga ham tegishli bo'ladi va $A \cap B$ bilan belgilanadi, ya'ni $C = A \cap B = \{x |\ x \in A \quad va \quad x \in B\}$ (2 -chizma).

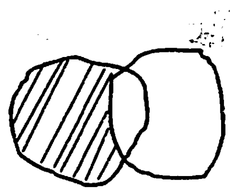
3) A to'plamdan B to'plamning **farqi (ayirmasi)** deb shunday uchinchi bir C to'plamga aytiladiki, uning har bir elementi A ga tegishli bo'lsa, B ga tegishli bo'lmaydi, va uni $A / B = \{x |\ x \in A \quad va \quad x \notin B\}$ (3-chizma).



1-chizma



2-chizma



3-chizma

2-misol. $A = \{1,\ 2\}$ to'plamning hamma qism to'plamlaridan iborat bo'lgan B to'plamni tuzing.

Yechish. Qism to'plam ta'rifiga asosan, $\varnothing \in A$, $\{1\} \in A$, $\{1,\ 2\} \in A$, $\{2\} \in A$. Demak, $B = \{\varnothing, \{1\}, \{2\}, \{1,\ 2\}\}$.

3-misol. $A = (4,\ 8)$ va $B = (1,\ 4]$ bo'lsa, ularning birlashmasini va kesishmasini toping.

Yechish. Birlashmaning ta'rifidan $A \cup B = (1,8)$ bo'lib,

kesishmaning ta'rifidan $A \cap B = \varnothing$ bo'ladi.

4-misol. $A = (-3,\ 7]$ va $B[5,\ 6]$ bo'lsa, ularning birlashmasi va kesishmasini toping.

Yechish. Ta'rifga asosan $A \cup B = (-3,\ 7]$, $A \cap B = [5,6]$ bo'ladi.

5- misol. Ushbu

$$A = \{1,2,3,4,5,6\}, \quad B = \{2,4,6,8\}, \quad C = \{1,3\}$$

to'plamlarni qaraylik. Bu to'plamlar uchun

$$A \cup B = \{1,2,3,4,5,6,8\},$$
$$A \cap B = \{2,4,6\},$$
$$A \backslash B = \{1,3,5\},$$
$$B \backslash A = \{8\},$$
$$A \cup C = \{1,2,3,4,5,6\},$$
$$A \cap C = \{1,3\},$$
$$B \cap C = \varnothing,$$
$$B \times C = \{(2,1),(2,3),(4,1),(4,3),(6,3)(8,1),(8,3)\}.$$

bo'ladi.

Yuqorida keltirilgan ta'riflardan

$$E \cup E = E, \quad E \cap E = E, \quad E \backslash E = \varnothing,$$

shuningdek $E \subset F$ bo'lganda

$$E \cup F = F, \quad E \cap F = E$$

bo'lishi kelib chiqadi.

Barcha 1, 2, 3, ..., n, ... - natural sonlardan iborat to'plam natural sonlar to'plami deyiladi va u N harfi bilan belgilanadi:

$$N = \{1,\ 2, 3, ..., n, ...\}.$$

Barcha ..., -2, -1, 0, 1, 2, ... - butun sonlardan iborat to'plam ***butun sonlar to'plami*** deyiladi va u Z harfi bilan belgilanadi:

$$Z = \{..., -2,\ -1,\ 0,\ 1,\ 2, ...\}$$

Ravshanki,

$$N \subset Z$$

bo'ladi.

Tartiblangan to'plamlar haqida

Agar biror E to'plamning elementlari uchun quyidagi tasdiqlar:

1) $n=m$, $n>m$, $n<m$ munosabatlardan bittasi va faqat bittasi o'rinli;

2) $n<m$, $m<p$ tengsizliklardan $n<p$ tengsizlik o'rinli bo'lsa, E to'plam tartiblangan to'plam deyiladi.

Tartiblangan to'plamlarga dastlabki misol, $N=\{1, 2, 3,..., n,...\}$ natural sonlar to'plami bo'ladi. Bundan tashqari butun, ratsional, haqiqiy sonlar to'plamlari ham tartiblangan to'plamlarga misol bo'la oladi.

To'plamlarning ekvivalentligi

Ixtiyoriy ikkita E va F to'plamlar berilgan holda, tabiiyki, ularning qaysi birining elementi «ko'p» degan savol tug'iladi. Natijada to'plamlarni solishtirish (elementlar soni jihatidan solishtirish) masalasi yuzaga keladi. Odatda bu masala ikki usul bilan hal qilinadi:

1) to'plamlarning elementlarini bevosita sanash bilan ularning elementlari soni solishtiriladi;

2) biror qoidaga ko'ra bir to'plamning elementlariga ikkinchi to'plamning elementlarini mos qo'yish yo'li bilan ularning elementlari solishtiriladi.

Masalan, $E=\{1, 2, 3\}$, $F=\{1, 4, 9, 16\}$ to'plamlarning elementlari sonini solishtirib, F to'plamning elementlari soni E to'plamning elementlari sonidan ko'p ekanligini aniqlaymiz yoki E to'plamning har bir elementiga F to'plamning bitta elementini

$$1\to 1, 2\to 4, 3\to 9$$

tarzda mos qo'yib, F to'plamda E to'plam elementiga mos qo'yilmay qolgan element borligini (u 16) hisobga olib, yana F ning elementlari soni E ning elementlari sonidan ko'p degan xulosaga kelamiz. Agar to'plamlar cheksiz bo'lsa, ravshanki, ularni 1- usul bilan solishtirib bo'lmaydi. Bunday vaziyatda faqat 2 - usul bilangina ish ko'riladi. Masalan, $N=\{1, 2,..., n,...\}$ natural sonlar to'plamining har bir n elementiga $(n=1, 2,...)$ juft sonlar to'plami $N_1=\{2, 4,..., 2n,...\}$ ning $2n$ elementini $(n=1, 2,...)$ mos qo'yish bilan $(n\to 2n)$ solishtirib, ularning elementlari soni «teng» degan xulosaga kelamiz.

1–ta'rif. Agar E to'plamning har bir a elementiga F to'plamning bitta b elementi mos qo'yilgan bo'lib, bunda F

to'plamning har bir elementi uchun E to'plamda unga mos keladigan bittagina element bor bo'lsa, u holda E va F to'plamlar elementlari orasida **o'zaro bir qiymatli moslik o'rnatilgan** deyiladi.

2–ta'rif. Agar E va F to'plam elementlari orasida o'zaro bir qiymatli moslik o'rnatish mumkin bo'lsa, ular bir-biriga **ekvivalent to'plamlar** deb ataladi va

$$E\sim F$$

kabi belgilanadi.

6-misol. Ushbu

$$E=\{1, 2, 3, 4, 5\},\quad F=\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$$

to'plamlar ekvivalent to'plamlar bo'ladi. Bu to'plam elementlari orasida o'zaro bir qiymatli moslik mavjud. Uni quyidagicha

$$1\leftrightarrow 1,\quad 2\leftrightarrow\frac{1}{2},\quad 3\leftrightarrow\frac{1}{3},\quad 4\leftrightarrow\frac{1}{4},\quad 5\leftrightarrow\frac{1}{5},$$

o'rnatish mumkin. Demak, $E\sim F$.

7-misol. Ushbu

$$E=\{2, 4, 6, 8\},\quad F=\{2, 4, 6, 8, 10\},$$

to'plamlar ekvivalent to'plamlar bo'lmaydi. Chunki bu to'plam elementlari orasida o'zaro bir qiymatli moslik o'rnatib bo'lmaydi.

8-misol. Ushbu

$$E=N=\{1, 2, 3,....., n....\},\quad F=\left\{1, \frac{1}{2}, \frac{1}{3}, ...\frac{1}{n}....\right\},$$

to'plamlar ekvivalent to'plamlar bo'ladi. Bu to'plam elementlari orasidagi o'zaro bir qiymatli moslik har bir n ga $(n\in N)\frac{1}{n}$ ni $(\frac{1}{n}\in F)$ mos qo'yish bilan o'rnatiladi. Demak, $E\sim F$.

9-misol. Ushbu

$$E=N=\{1,2,3,...n,,,\},\quad N_1=\{2,4,6,....,2n,....\}$$

to'plamlar o'zaro ekvivalent bo'ladi. Bu to'plam elementlari orasida o'zaro bir qiymatili moslikni quyidagicha o'rnatish mumkin: har bir natural n $(n\in N)$ songa $2n$ son $(2n\in N_1)$ mos qo'yiladi $n\leftrightarrow 2n)$. Demak, $E=N\sim N_1$.

Ravshanki, $N_1\subset N$. Bu esa to'plamning qismi o'ziga ekvivalent bo'lishi mumkin ekanligini ko'rsatadi. Bunday holat faqat cheksiz to'plamlargina xosdir.

Yuqorida keltirilgan ta'rif va misollardan ikki chekli to'plamning o'zaro ekvivalent bo'lishi uchun ularning elementlari soni bir-biriga teng bo'lishi zarur va yetarli ekanligini ko'ramiz.

Ekvivalentlik munosabati quyidagi xossalariga ega:

1) $E \sim E$ (refleksivlik xossasi);

2) $E \sim F$ bo'lsa, $F \sim E$ bo'ladi (simmetrik xossasi);

3) $E \sim F$, $F \sim G$ bo'ladi (tranzitivlik xossasi).

To'plamlarning ekvivalentlik tushunchasi to'plamlarni sinflarga ajratish imkonini beradi.

Masalan,

$$N_1 = \{2,4,6,....,2n,...\},$$
$$N_2 = \{1,3,5,...2n-1,...\},$$
$$N_3 = \left\{1,\frac{1}{2},\frac{1}{3},...,\frac{1}{n},...\right\}$$

to'plamlar sanoqli to'plamlardir, chunki

$$N_1 \sim N \quad (2n \leftrightarrow n, n=1,2,3,...),$$
$$N_2 \sim N \quad (2n-1 \leftrightarrow n, n=1,2,3,...),$$
$$N_3 \sim N \, (\frac{1}{n} \leftrightarrow n, n=1,2,3,....).$$

To'plamning quvvati. To'plamning quvvati, to'plam "elementlarining soni" tushunchasining ixtiyoriy (chekli va cheksiz) to'plamlar uchun umumlashtirilganidir. To'plamning quvvati berilgan to'plamga ekvivalent bo'lgan barcha to'plamlarga, ya'ni elementlari berilgan to'plamning elementlari bilan o'zaro bir qiymatli moslikda bo'la oladigan barcha to'plamlarga umumiy bo'lgan narsa sifatida aniqlanadi.

To'plam quvvati tushunchasini matematikaga to'plamlar nazariyasining asoschisi nemis matematigi G.Kantor (1845-1918) kiritgan (1879 yilda). Kantor cheksiz to'plamlar uchun ***har xil quvvatlar*** *mavjudligini isbotlagan.*

3-ta'rif. Natural sonlar qatoriga ekvivalent bo'lgan to'plam, ya'ni hamma elementlarini natural sonlar bilan raqamlab (belgilab) chiqish

mumkin bo'lgan to'plamga ***sanoqli to'plam*** deyiladi. Masalan,

$$N_1 = \{2,4,6,....,2n,...\},$$
$$N_2 = \{1,3,5,...2n-1,...\},$$
$$N_3 = \left\{1,\frac{1}{2},\frac{1}{3},...,\frac{1}{n},...\right\}$$

to'plamlar sanoqli to'plamlardir, chunki

$$N_1 \sim N \quad (2n \leftrightarrow n, n=1,2,3,...),$$
$$N_2 \sim N \quad (2n-1 \leftrightarrow n, n=1,2,3,...),$$
$$N_3 \sim N \quad (\frac{1}{n} \leftrightarrow n, n=1,2,3,....).$$

Sanoqli to'plamning quvvati cheksiz to'plamlar quvvati orasida eng kichigi bo'lib hisoblanadi.

Sanoqli bo'lmagan to'plam sanoqsiz to'plam deb ataladi.

$0 \le x \le 1$ kesmadagi sonlarning L to'plamining quvvati nomi ***kontinuum*** deyiladi. L ni natural sonlar to'plamiga o'zaro bir qiymatli akslantirish mumkin emas. **"Kontinuum matematikasi"** termini uzluksizlik tushunchasi bilan bog'liq bo'lgan nazariyalarda qo'llanilib, u diskret matematikaga qarama-qarshi qo'yiladi. Kontinuum quvvat sanoqli to'plam quvvatidan katta. Bir necha o'n yil muqaddam sanoqli to'plam quvvatidan katta va kontinuum quvvatdan kichik bo'lgan to'plam mavjudmi? degan muammo qo'yilgan.

Matematik belgilar haqida. Matematikada tez-tez uchraydigan so'z va so'z birikmalari o'rniga maxsus belgilar ishlatiladi. Ulardan eng muhimlarini keltiramiz:

1) «Agar .... bo'lsa, u holda ..... bo'ladi» iborasi «$\Rightarrow$» belgisi orqali yoziladi;

2) ikki iboraning ekvivalentligi ushbu «$\Leftrightarrow$» belgisi orqali yoziladi;

3) «Har qanday», «ixtiyoriy», «barchasi uchun» so'zlari o'rniga «$\forall$» umumiylik belgisi ishlatiladi;

4) «Mavjudki», «topiladiki» so'zlari o'rniga «$\exists$» mavjudlik belgisi ishlatiladi.

3.2. Sonli ketma-ketliklar

Sonli ketma-ketlik ta'rifi va umumiy tushunchalar

1-ta'rif. Natural sonlar qatoridagi

$$1,2,3, \ldots, n, \ldots$$

har bir n songa haqiqiy x_n son mos qo'yilgan bo'lsa,

$$x_1, x_2, \ldots, x_n, \ldots \qquad (1)$$

(1) haqiqiy sonlar to'plamiga sonli ketma-ketlik yoki qisqacha ketma-ketlik deyiladi.

$x_1, x_2, \ldots, x_n, \ldots$ sonlarga sonli ketma-ketlikning hadlari deyilib, x_n ga ketma – ketlikning umumiy hadi yoki n – hadi deb ataladi, (1) sonli ketma-ketlikni qisqacha $\{x_n\}$ simvol bilan belgilanadi. Masalan, 1) $\left\{\frac{1}{n}\right\}$ sonlar ketma-ketligi

$$1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n} \ldots$$

bo'ladi;

2) $\left\{\frac{n}{n+1}\right\}$ sonlar ketma-ketligi $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$ bo'ladi.

Sonli ketma-ketlikning umumiy hadini olish usuli ko'rsatilgan bo'lsa, u berilgan deyiladi. Misol uchun, 1) $x_n = 2 + (-1)^n$ bo'lsa, u 1, 3, 1, 3, 1, 3, ..., 1, 3, ... ;

3) $\frac{2}{3}$ kasrni o'nli kasrga aylantirganda verguldan keyin bitta, ikkita, uchta va hokazo raqamlarni olib,

$$x_1 = 0,6,\ x_2 = 0,66,\ x_3 = 0,666, \ldots$$

sonlar ketma-ketligini olish mumkin;

4) $a_1, a_1 + d, a_1 + 2d, \ldots, a_1 + (n-1)d, \ldots$

arifmetik progressiya ham sonli ketma-ketlikdir, bunda a_1 birinchi had, d arifmetik progressiya ayirmasi;

4) $b_1, b_1 q, b_1 q^2, \ldots, b_1 q^{n-1}, \ldots$

sonlar ketma-ketligi ham ketma-ketlikka misol bo'ladi, bu birinchi hadi

b_1 maxraji q bo'lgan geometrik progressiyadir.

Sonli ketma-ketlikning ta'rifidan ma'lumki, u cheksiz sondagi elementlarga ega bo'lib, ular hech bo'lmaganda o'zlarining tartib raqami bilan farq qiladi.

Sonlar ketma-ketligining geometrik tasviri sonlar o'qidagi nuqtalar bilan ifodalanadi.

Sonli ketma-ketliklar ustida ushbu arifmetik amallarini bajarish mumkin: 1) $\{x_n\}$ sonlar ketma-ketligini songa ko'paytirish,

$$mx_1,\ mx_2,\ mx_3, \ldots, mx_n, \ldots.$$

ko'rinishda bo'ladi;

2) ikkita $\{x_n\}$ va $\{y_n\}$ sonlar ketma-ketligining yig'indisi

$$x_1 + y_1,\ x_2 + y_2, \ldots, x_n + y_n, \ldots;$$

ko'rinishda aniqlanadi;

3) ikkita $\{x_n\}$ va $\{y_n\}$sonlar ketma-ketiligini ayirmasi

$$x_1 - y_1,\ x_2 - y_2, \ldots, x_n - y_n, \ldots.$$

ko'rinishda bo'ladi;

4) ikkita $\{x_n\}$ va $\{y_n\}$ sonlar ketma-ketligi ko'paytmasi

$$x_1 \cdot y_1,\ x_2 \cdot y_2, \ldots, x_n \cdot y_n, \ldots;$$

kabi aniqlanadi;

5) ikkita $\{x_n\}$ va $\{y_n\}$ sonlar ketma-ketligining nisbati, maxraj 0 dan farqli bo'lganda,

$$\frac{x_1}{y_1}, \frac{x_2}{y_2}, \ldots, \frac{x_n}{y_n}, \ldots.$$

ko'rinishda bo'ladi hamda mos ravishda $\{mx_n\}$, $\{x_n + y_n\}$, $\{x_n - y_n\}$, $\{x_n \cdot y_n\}$, $\left\{\frac{x_n}{y_n}\right\}$ simvollar bilan belgilanadi.

Chegaralangan va chegaralanmagan sonli ketma-ketliklar.

1-ta'rif. $\{x_n\}$ sonlar ketma – ketligi uchun shunday M (m son) son mavjud bo'lib, ketma-ketlikning istalgan elementi uchun $x_n \le M\ (x_n \ge m)$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik yuqoridan

Isbot. $\{\alpha_n\}$ va $\{\beta_n\}$ cheksiz kichik ketma-ketliklar bo'lsin. Bu cheksiz kichik ketma-ketliklar uchun, istalgan ε son uchun N_1 raqam topiladiki, $n > N_1$ lar uchun, $|\alpha_n| < \frac{\varepsilon}{2}$ tengsizlik, N_2 raqam topiladiki, $n > N_2$ lar uchun $|\beta_n| < \frac{\varepsilon}{2}$ tengsizliklar bajariladi. $N = \max\{N_1, N_2\}$ desak, $n > N$ lar uchun birdaniga $|\alpha_n| < \frac{\varepsilon}{2}$, $|\beta_n| < \frac{\varepsilon}{2}$ tengsizliklar bajariladi. Shunday qilib,

$$|\alpha_n \pm \beta_n| \le |\alpha_n| + |\beta_n| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

bo'ladi.

Bu $\{\alpha_n \pm \beta_n\}$ ketma-ketlikning cheksiz kichik ekanligini bildiradi.

Natija. Istalgan chekli sondagi cheksiz kichiklarning algebraik yig'indisi yana cheksiz kichik ketma-ketlikdir.

3-teorema. Ikkita cheksiz kichik ketma-ketlikning ko'paytmasi, cheksiz kichik ketma-ketlik bo'ladi.

Isbot. $\{\alpha_n\}$ va $\{\beta_n\}$ lar cheksiz kichik ketma-ketliklar bo'lsin. $\{\alpha_n \cdot \beta_n\}$ ketma-ketlikning cheksiz kichikligini isbotlash talab etiladi. $\{\alpha_n\}$ cheksiz kichik bo'lganligi uchun, istalgan $\varepsilon > 0$ son uchun shunday N_1 raqam topiladiki, $n > N_1$ lar uchun $|\alpha_n| < \varepsilon$, $\{\beta_n\}$ cheksiz kichik ketma-ketlik bo'lganligi uchun $\varepsilon = 1$ uchun shunday N_2 topiladiki $n > N_2$ lar uchun $|\beta_n| < 1$ bajariladi. $N = \max\{N_1, N_2\}$ deb olsak, $n > N$ lar uchun ikkala tengsizlik ham bajarilib,

$$|\alpha_n \cdot \beta_n| \le |\alpha_n| \cdot |\beta_n| < \varepsilon \cdot 1 = \varepsilon$$

bo'ladi. Bu $\{\alpha_n \cdot \beta_n\}$ ketma-ketlikning cheksiz kichikligini bildiradi.

Natija. Istalgan sondagi cheksiz kichiklarning ko'paytmasi yana cheksiz kichik bo'ladi.

Eslatma. Ikkita cheksiz kichiklarning nisbati cheksiz kichik bo'lmasligi mumkin, masalan, $\alpha_n = \frac{1}{n}$, $\beta_n = \frac{1}{n}$ cheksiz kichiklarning nisbati hamma elementlari 1 lardan iborat chegaralanlan ketma-ketlikdir.

$\alpha_n = \frac{1}{n}$, $\beta_n = \frac{1}{n^2}$ cheksiz kichik ketma-ketliklarning nisbati $\left\{\frac{\alpha_n}{\beta_n}\right\} = \{n\}$ bo'lib, cheksiz katta ketma-ketlik hosil bo'ladi. $\alpha_n = \frac{1}{n^2}$, $\beta_n = \frac{1}{n}$ bo'lsa, ularning nisbati $\left\{\frac{\alpha_n}{\beta_n}\right\} = \left\{\frac{1}{n}\right\}$ cheksiz kichik bo'ladi.

4-teorema. Chegaralangan ketma-ketlikning cheksiz kichik ketma-ketlikka ko'paytmasi cheksiz kichik ketma-ketlik bo'ladi. (Bu teoremaning isbotini o'quvchiga havola qilamiz).

Sonli ketma-ketlikning limiti va uning xossalari

1-ta'rif. Istalgan $\varepsilon > 0$ son uchun unga bog'liq bo'lgan N son topilsaki, barcha $n > N$ lar uchun $|x_n - a| < \varepsilon$ tengsizlik bajarilsa, a songa $\{x_n\}$ ketma-ketlikning $n \to \infty$ dagi limiti deyiladi va

$$\lim_{n\to\infty} x_n = a \quad yoki \quad n \to \infty \text{ да } x_n \to a$$

simvollar bilan belgilanadi. Chekli limitga ega sonli ketma-ketlikka yaqinlashuvchi ketma-ketlik deyiladi.

Limitning ta'rifiga misol qaraymiz.

Limitning ta'rifidan foydalanib, $\lim_{n\to\infty} \frac{n}{n+1} = 1$ ekanligini ko'rsatamiz. Istalgan $\varepsilon > 0$ son olamiz.

$$|x_n - 1| = \left|\frac{n}{n+1} - 1\right| = \left|\frac{n-n-1}{n+1}\right| = \frac{1}{n+1}$$

bo'lganligi uchun, $|x_n - 1| < \varepsilon$ tengsizlikni qanoatlantiruvchi n larning qiymatini topish, $\frac{1}{n+1} < \varepsilon$ tengsizlik bilan bog'liq va $1 < \varepsilon(n+1)$ *yoki* $n > \frac{1-\varepsilon}{\varepsilon}$ bo'ladi. Shuning uchun N sifatida $\frac{1-\varepsilon}{\varepsilon}$ sonning butun qismini olish mumkin, ya'ni $N = \left[\frac{1-\varepsilon}{\varepsilon}\right]$ bo'ladi. Bu holda $|x_n - 1| < \varepsilon$ tengsizlik hamma $n > N$ lar uchun bajariladi. Masalan, $\varepsilon = 0{,}1$ bo'lsin, bu holda

$$N=\left[\frac{1-0{,}1}{0{,}1}\right]=\left[\frac{0{,}9}{0{,}1}\right]=9. \quad n=10>N=9$$

bo'lsin. Bunda

$$x_{10}=\frac{10}{10+1}=\frac{10}{11}$$

bo'lib,

$$|x_{10}-1|=\left|\frac{10}{11}-1\right|=\frac{1}{11}<\varepsilon=0{,}1.$$

Shunday qilib, n=10 dan boshlab hamma n lar uchun $|x_{10}-1|<0{,}1$ tengsizlik bajariladi.

Demak, $\lim\limits_{n\to\infty}\frac{n}{n+1}=1$ tenglik o'rinli bo'ladi.

Boshqa bir necha $\varepsilon>0$ lar olib, qaysi raqamlardan boshlab tengsizlikning bajarilishini ko'rsatishni o'quvchiga havola etamiz.

Eslatma 1. $\{x_n\}$ sonlar ketma-ketligi biror a limitga ega bo'lsa, uni $\alpha_n=x_n-a$ cheksiz kichik miqdor ko'rinishida ifodalash mumkin, chunki $\varepsilon>0$ son uchun shunday N topiladiki, $n>N$ lar uchun

$$|\alpha_n|=|x_n-a|<\varepsilon$$

tengsizlik bajariladi. Shuning uchun a limitga ega bo'lgan $\{x_n\}$ sonlar ketma-ketligini

$$x_n-a=\alpha_n$$

ko'rinishda ifodalash mumkin, bunda α_n cheksiz kichik ketma-ketlik.

2-ta'rif. $\varepsilon>0$ biror musbat son bo'lsin. $|x_n-a|<\varepsilon$ tengsizlik hamma n lar uchun bajarilsa, $\{x_n\}$ sonlar ketma-ketligi a nuqtaning ε atrofida deyiladi.

2-eslatma. Ma'lumki $|x_n-a|<\varepsilon$ tengsizligi

$$-\varepsilon<x_n-a<\varepsilon \text{ yoki } a-\varepsilon<x_n<a+\varepsilon$$

tengsizlik bilan teng kuchli bo'lib, x_n element a nuqtaning ε atrofida bo'ladi. Shuning uchun, $\{x_n\}$ ketma-ketlikning limitini quyidagicha ham ta'riflash mumkin: a nuqtaning ε atrofi uchun shunday N raqamni ko'rsatish mumkin bo'lsaki, hamma $n>N$ lardan boshlab, hamma x_n

elementlar a nuqtaning ε atrofida bo'lsa, a songa $\{x_n\}$ ketma-ketlikning limiti deyiladi.

3-eslatma. Ma'lumki cheksiz katta ketma-ketlik limitga ega emas yoki uni cheksiz limitga ega deyiladi va

$$\lim_{n\to\infty}x_n=\infty$$

bilan belgilanadi. Ketma-ketlikning limitini cheksiz limitdan farq qilishi uchun chekli limit ham deb yuritiladi.

Eslatma. Tushunarliki, har bir cheksiz kichik ketma-ketlik yaqinlashuvchi va uning limiti $a=0$ ga teng.

Yaqinlashuvchi ketma-ketliklar quyidagi xossalarga ega:

1. Yaqinlashuvchi ketma-ketlikning limiti yagonadir.
2. Yaqinlashuvchi ketma-ketlik chegaralangan.

Eslatma. Chegaralangan ketma-ketlik yaqinlashuvchi bo'lmasligi mumkin. Masalan,

$$-1, 1, -1, \ldots, (-1)^n, \ldots$$

ketma-ketlik, chegaralangan, lekin limitga ega emas.

3. $\{x_n\}$ va $\{y_n\}$ sonli ketma-ketliklar yaqinlashuvchi bo'lib, mos ravishda a va b limitlarga ega bo'lsa, ularning algebraik yig'indisi ham yaqinlashuvchi bo'lib, $a\pm b$ limitga ega bo'ladi.

4. $\{x_n\}$ va $\{y_n\}$ sonli ketma-ketliklar yaqinlashuvchi bo'lib, mos ravishda a va b limitlarga ega bo'lsa, ularning ko'paytmasi ham yaqinlashuvchi bo'lib, limiti $a\cdot b$ ga teng bo'ladi.

5. $\{x_n\}$ va $\{y_n\}$ sonli ketma-ketliklar yaqinlashuvchi bo'lib, mos ravishda a va b limitlarga ega bo'lsa, ularning nisbati ham maxrajning limiti noldan farqli bo'lganda, yaqinlashuvchi bo'lib, uning limiti $\frac{a}{b}$ ga teng bo'ladi.

Bu xossalarni, ketma-ketlikning limiti va cheksiz kichik ketma-ketliklarning xossalaridan foydalanib isbotlash mumkin. Masalan, 4-xossani isbotlaylik. Ketma-ketliklar yaqilashuvchi bo'lganligi uchun

$$x_n=a+\alpha_n, \quad y_n=b+\beta_n$$

ko'rinishda ifodalanadi, bunda α_n, β_n lar cheksiz kichik ketma-ketliklar. Bu holda

$$x_n \cdot y_n - ab = a\beta_n + b\alpha_n + \alpha_n \cdot \beta_n$$

bo'ladi. $(a\beta_n + b\alpha_n + \alpha_n \cdot \beta_n)$ ifoda cheksiz kichik ketma-ketlikning xossalariga asosan cheksiz kichik ketma-ketlikdir. Demak, $x_n y_n - ab$ ham cheksiz kichikdir, ya'ni

$$\lim_{n\to\infty}(x_n y_n - ab) = 0 \quad yoki \quad \lim_{n\to\infty} x_n y_n = ab$$

bo'ladi.

1-misol. Ushbu limitni hisoblang.

$$\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^2-5}.$$

Yechish. $n \to \infty$ surat ham maxraj ham cheksiz katta bo'lib, nisbatning limiti haqidagi xossani qo'llash mumkin emas, chunki bu xossada surat va maxrajning limiti mavjud bo'lishi kerak edi. Shuning uchun, bu ketma-ketliklarni n^2 ga bo'lib, shaklini o'zgartiramiz hamda limitlarning xossalarini qo'llab, ushbuni hosil qilamiz:

$$\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^2-5} = \lim_{n\to\infty} \frac{3+\frac{2}{n}-\frac{1}{n^2}}{4-\frac{5}{n^2}} = \frac{\lim\limits_{n\to\infty}(3+\frac{2}{n}+\frac{1}{n^2})}{\lim\limits_{n\to\infty}(4-\frac{5}{n^2})} =$$

$$= \frac{\lim\limits_{n\to\infty} 3 + \lim\limits_{n\to\infty}\frac{2}{n} - \lim\limits_{n\to\infty}\frac{1}{n^2}}{\lim\limits_{n\to\infty} 4 - \lim\limits_{n\to\infty}\frac{5}{n^2}} = \frac{3+0-0}{4-0} = \frac{3}{4}.$$

Berilgan misolni maple orqali yechamiz:

$$> Limit\left(\left(\frac{3\cdot n\cdot n+2\cdot n-1}{4\cdot n-5}\right), n=\infty\right)$$

$$= limit\left(\left(\frac{3\cdot n\cdot n+2\cdot n-1}{4\cdot n\cdot n-5}\right), n=\infty\right);$$

$$\lim_{n\to\infty} \frac{3n^2+2n-1}{4n-5} = \frac{3}{4}$$

3.3. Mustaqil ish uchun topshiriqlar

1. Ushbu

$$x_n = \frac{1}{3n},\ x_n = \frac{n}{5n-1},\ x_n = \frac{1}{4n-1},\ x_n = 3n$$

sonli ketma-ketliklarning n=1,2,3,4,5, bo'lgandagi qiymatlarini yozing?

2. $x_n = \dfrac{n}{n+2}$ sonli ketma-ketlikning chegaralanganligini ko'rsating.

3. $x_n = \dfrac{3}{n}$, $x_n = \dfrac{3(-1)^n}{2n}$, $x_n = 3+(-1)^n$ sonlar ketma-ketligining geometrik tasvirini n=1,2,3,4,5,6 bo'lganda ko'rsating.

4. Bir necha arifmetik va geometrik progressiyalarning umumiy (n-hadi) ni yozing va n=1,2,3,4,5,6 bo'lgandagi qiymatlarini yozing.

5. Ushbu

$$x_n = 3n,\ \ x_n = -5n+1,\ \ x_n = \frac{1}{y_n+1},\ \ x_n = (-1)^n 3n$$

sonlar ketma-ketliklari chegaralanganmi va qanday?

6. Bir necha cheksiz katta va cheksiz kichik sonlar ketma-ketliklarini yozing.

7. Ushbu tengliklar

$$\lim_{n\to\infty}\frac{2n}{n+1} = 2,\ \ \lim_{n\to\infty}\frac{1}{3n} = 0,\ \ \lim_{n\to\infty}\frac{3n+1}{n} = 3$$

ning to'g'riligini sonli ketma-ketlikning limiti ta'rifidan foydalanib isbotlang va har biri uchun $\varepsilon > 0$ ni aniqlab qanday raqamdan boshlab tengsizlikning bajarilishini ko'rsating?

8. Ushbu sonlar ketma-ketliklarining limitlarga ega ekanligi yoki ega emasligi va u nimaga tengligini ko'rsating?

1) $x_n = \dfrac{n}{3n+1}$, 2) $x_n = \dfrac{5n-1}{2n}$, 3) $x_n = \dfrac{(-1)^n n}{4n+1}$,

4) $x_n = \dfrac{3n+1}{n^2}$, 5) $x_n = \dfrac{n}{4n-3}$, 6) $x_n = \dfrac{(-1)^n n}{3n-1}$,

7) $x_n = \dfrac{3n^2}{n+1}$, 8) $x_n = \dfrac{2n+5}{n^3}$.

3.4. Elementar funksiyalar

Agar x miqdorning biror D to'plamdan olingan har bir qiymatiga biror E to'plamdan olingan y miqdorning birdan-bir aniq qiymati mos qo'yilgan bo'lsa, u holda y o'zgaruvchi miqdor x o'zgaruvchi miqdorning *funksiyasi* deyiladi.

x miqdor erkli o'zgaruvchi yoki *argument*, y miqdor esa bog'liq o'zgaruvchi yoki *funksiya* deyiladi. Funksiyani belgilash uchun ushbu yozuvlardan foydalaniladi:

$$y = f(x), \quad y = y(x), \quad y = \varphi(x)$$

va h. k.

x o'zgaruvchining $f(x)$ funksiya ma'noga ega bo'ladigan qiymatlari to'plami funksiyaning qiymatlar *sohasi* deyiladi va $D(f)$ ko'rinishda belgilanadi. $y = f(x)$ funksiyaning $f(x)$ dagi qiymati. Bunda $x_0 \in D(f)$, funksiyaning *xususiy qiymati* deyiladi va y_0 yoki $f(x_0)$ ko'rinishda belgilanadi. Shunday qilib,

$$y_0 = f(x_0) \quad \text{yoki} \quad y|_{x=x_0} = y_0.$$

Funksiyaning qabul qiladigan qiymatlari to'plami uning *o'zgarish sohasi* deyiladi va $E(f)$ bilan belgilanadi.

Oxy tekislikning $y = f(x)$ munosabatni qanoatlantiruvchi $M(x,y)$ nuqtalari to'plami $y = f(x)$ funksiyaning *grafigi* deyiladi.

Agar $y = f(x)$ funksiya $D(f)$ sohani $E(f)$ sohaga o'zaro bir qiymatli akslantirsa, u holda x ni y orqali bir qiymatli ifodalash mumkin:

$$x = \varphi(y).$$

Hosil bo'lgan funksiya $y = f(x)$ funksiyaga nisbatan *teskari funksiya* deyiladi.

$y = f(x)$ va $x = \varphi(y)$ funksiyalar *o'zaro teskari* funksiyalardir.

$x = \varphi(y)$ teskari funksiyani odatda x va y larning o'rinlarini almashtirish bilan standart ko'rinishla yoziladi.

$$y = \varphi(x).$$

O'zaro teskari $y = f(x)$ va $y = \varphi(x)$ funksiyalarning grafiklari birinchi va uchinchi koordinata choraklarining bissektrisasiga nisbatan simmetrik. $y = f(x)$ funksiyaning aniqlanish sohasi $y = \varphi(x)$ teskari funksiyaning qiymatlari sohasi bo'ladi.

$u = \varphi(x)$ funksiyaning aniqlanish sohasi D, qiymatlar sohasi B bo'lsin. $y = f(u)$ funksiyaning aniqlanish sohasi B bo'lib, o'zgarish sohasi l bo'lsin, u holda $y = f(\varphi(x))$ aniqlanish sohasi D va o'zgarish sohasi l bo'lgan murakkab funksiya yoki f va φ funksiyalarning *kompozisiyasi* deyiladi. u o'zgaruvchi *oraliq o'zgaruvchi* deyiladi.

$y = f(x)$ ko'rinishidagi funksiya *oshkor funksiya* deyiladi. $F(x,y) = 0$ ko'rinishdagi tenglama ham, umuman aytganda x va y o'zgaruvchilar orasidagi funksional bog'lanishni beradi. Bu holda ta'rifga ko'ra y o'zgaruvchi x ning *oshkormas* funksiyasi bo'ladi. Masalan, $x^2 + y^2 = 4$ tenglama y ni x ning oshkormas funksiyasi sifatida aniqlaydi. Aniqlanish sohasi $D(f)$ koordinatalar boshiga nisbatan simmetrik bo'lgan $f(x)$ funksiya x ning har qanday $x_0 \in D(f)$ qiymati uchun $f(-x) = f(x)$ (yoki $f(-x) = -f(x)$) munosabat bajarilsa, *juft* (yoki *toq)* funksiya deyiladi.

Juft funksiya grafigi ordinatlar o'qiga nisbatan simmetrik, toq funksiya grafigi esa koordinatlar boshiga nisbatan simmetrikdir.

Agar $T > 0$ o'zgarmas son mavjud bo'lib, har bir $x \in D(f)$ va $(x+T) \in D(f)$ da $f(x+T) = f(x)$ tenglik bajarilsa, $f(x)$ funksiya *davriy funksiya* deyiladi.

Aytilgan xossaga ega bo'lgan T larning eng kichigi T_0 *funksiyaning davri* deyiladi.

Quyidagi funksiyalar *asosiy elementar funksiyalar* deyiladi:

a) $y = x^\alpha$ darajali funksiya, bunda $\alpha \in R$; $D(f)$ va $E(f)$ lar α ga bog'liq;

b) $y = a^x$ ko'rsatkichli funksiya, bunda $a > 0$ va $a \neq 1$; $D(f) = R$ va $E(f) = (0, +\infty)$;

c) $y=\log_a x$ logarifmik funksiya, bunda $a>0, a\neq 1$; $D(f)=(0,+\infty)$ va $E(f)=R$

d) trigonometrik funksiyalar:

$y=\sin x,\ D(f)=R\ va\ E(f)=[-1;1];\ T_0=2\pi;$

$y=\cos x, D(f)=R\ va\ E(f)=[-1;1];\ T_0=2\pi;$

$$y=tgx, D(f)=\left\{x\neq\frac{\pi}{2}+\pi k,\ k\in Z\right\}\ va\ E(f)=R;\ T_0=\pi;$$

$$y=ctgx,\ D(f)=\{x\neq\pi k,\ k\in Z\}\ va\ E(f)=R;\ T_0=\pi;$$

$$y=\sec x,\ D(f)=\left\{x\neq\frac{\pi}{2}+\pi k,\ k\in Z\right\}\ va$$

$E(f)=(-\infty;-1]\cup[1;+\infty);\ T_0=2\pi;$

$y=\operatorname{cosec} x,\ D(f)=\{x\neq\pi k, k\in Z\}\ va$

$E(f)=(-\infty,-1]\cup[1,+\infty);\quad T_0=2\pi.$

e) teskari trigonometrik funksiyalar:

$$y=\arcsin x,\ D(f)=[-1;1]\ va\ E(f)=[-\frac{\pi}{2};\frac{\pi}{2}];$$

$$y=\arccos x,\ D(f)=[-1;1]\ va\ E(f)=[0;\pi];$$

$$y=arctgx,\ D(f)=R\ va\ E(f)=\left(-\frac{\pi}{2};\frac{\pi}{2}\right);$$

$$y=arcctgx,\ D(f)=R\ va\ E(f)=(0;\pi);$$

$$y=arc\sec x,\ D(f)=(-\infty;-1]\cup[1;+\infty)\ va\ E(f)=\left[0;\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2};\pi\right];$$

$$y=\operatorname{arccosec} x,\ D(f)=(-\infty;-1]\cup[1;+\infty)\ va\ E(f)=\left[-\frac{\pi}{2};0\right)\cup\left(0;\frac{\pi}{2}\right].$$

Elementar funksiya deb asosiy elementar funksiyalardan chekli sondagi arifmetik amallar yordamida tuzilgan murakkab funksiyalarga aytiladi.

Elementar funksiyalarning grafiklari.

$f(x)$ funksiya grafigini chizishda har xil usullar qo'llaniladi: nuqtalar bo'yicha, grafiklar bilan amallar bajarish, grafiklarni almashtirish. $f(x)$ funksiya grafigidan foydalanib sodda almashtirishlar yordamida murakkabroq funksiyalar grafiklarini hosil qilish mumkin.

a) $y=f(x-a)$ funksiyaning grafigi $y=f(x)$ funksiya grafigidan, bu grafikni Ox o'q bo'ylab a>0 da o'ngga, a<0 bo'lganda esa chapga a birlik surish bilan hosil qilinadi.

b) $y=f(x)+b$ funksiya grafigi $y=f(x)$ funksiya grafigidan, bu grafikni Oy o'q bo'ylab $b>0$ da yuqoriga, $b<0$ da pastga b birlik surish bilan hosil qilinadi.

c) $y=f(kx)$ $(k\neq 0, k\neq 1)$ funksiyaning grafigi $y=f(x)$ funksiya grafigidan, uning nuqtalari ordinatalarini saqlagan holda $|k|<1$ da abssissalarini $\frac{1}{|k|}$ marta cho'zish bilan, $|k|>1$ da esa abssissalarini $|k|$ marta siqish bilan hosil qilinadi.

d) $y=mf(x)$ $(m\neq 0, m\neq 1)$ funksiya grafigi $y=f(x)$ funksiya grafigidan, uning nuqtalari mos abssissalarini saqlagan holda ordinatalarini $|m|<1$ da $\frac{1}{|m|}$ marta qisish, $|m|>1$ da esa $|m|$ marta cho'zish orqali hosil qilinadi.

e) $y=f(-x)$ funksiya grafigi $y=f(x)$ funksiya grafigidan, bu grafikni Oy o'qqa nisbatan simmetrik akslantirish yordamida hosil qilinadi.

f) $y=-f(x)$ funksiya grafigi $y=f(x)$ funksiya grafigidan, bu grafikni Ox o'qqa nisbatan simmetrik akslantirish yordamida hosil qilinadi.

g) $y=|f(x)|$ funksiya grafigi Ox o'qning $f(x)\geq 0$ bo'ladigan qismlarida $y=f(x)$ funksiya grafigi bilan bir xil bo'ladi. Ox o'qning

$f(x)<0$ bo'ladigan qismida bu grafikni $y=f(x)$ funksiya grafigini Ox o'qqa nisbatan simmetrik akslantirish yordamida hosil qilinadi.

Misol. $y=-2\sin(2x+2)$ funksiyaning grafigini $y=\sin x$ funksiya grafigidan foydalanib chizing.

Yechish. $y=\sin x$ funksiya grafigidan foydalanib, $y=-2\sin(2x+2)$ funksiya grafigini chizish quyidagi shakl almashtirishlar orqali amalga oshiriladi:

$$y_1=\sin 2x_1,\ y_2=-2\sin 2x_2,$$
$$y=-2\sin 2(x+1)=-2\sin(2x+2).$$

Geometrik nuqtai nazardan bu shakl yasashlarga olib keladi.

1. $0\le x\le 2\pi$ oraliqda $y=\sin x$ sinusoidani chizamiz.

2. Sinusoidada bir nechta nuqta belgilaymiz va ordinatalarini o'zgartirmay, abssissalarini ikki marta kamaytiramiz: $x_1=\frac{1}{2}x,\quad y_1=y.$ Hosil bo'lgan nuqtalarni silliq chiziq bilan birlashtirib, $y_1=\sin 2x_1$ funksiyaning grafigini chizamiz.

3. Hosil bo'lgan grafikdagi nuqtalar abssissalarini o'zgartirmay, ordinatalarini 2 marta orttiramiz va ularning ishoralarini almashtiramiz: $y_2=-2y_1,\ x_2=x_1$. Hosil bo'lgan nuqtalarni silliq chiziq bilan birlashtirib, $y_2=-2\sin x_2$ funksiyaning grafigini chizamiz.

4. Oxirgi grafikni abssissalar o'qi bo'yicha (-1) ga ko'chiramiz: $x=x_2-1,\ \ y=y_2$. Hosil qilingan nuqtalarni silliq chiziq bilan birlashtirib, $y=-2\sin(2x+2)$ funksiya grafigini chizamiz.

3.5. Ketma-ketlikning limiti. Funksiyaning limiti

Natural sonlar to'plamida aniqlangan funksiya sonli *ketma-ketlik* deyiladi va $\{x_n\}$ ko'rinishda belgilanadi.

Agar shunday M musbat son manjud bo'lib, har qanday natural son n uchun

$$|x_n|\le M$$

tengsizlik o'rinli bo'lsa, x_n *chegaralangan ketma-ketlik* deyiladi. Agar har qanday natural son n uchun

$$x_{n+1}>x_n$$

tengsizlik bajarilsa, x_n *o'suvchi ketma-ketlik* deyiladi. Agar har qanday natural son n uchun

$$x_{n+1}<x_n$$

tengsizlik bajarilsa, x_n *kamayuvchi ketma-ketlik* deyiladi.

Faqat o'suvchi yoki kamayuvchi ketma-ketlik *monaton ketma-ketlik* deyiladi.

Agar istalgan $\varepsilon>0$ son uchun shunday $N=N(\varepsilon)>0$ son mavjud bo'lsaki, barcha $n\ge N$ lar uchun

$$|x_n-a|<\varepsilon$$

tengsizlik bajarilsa, o'zgarmas a son x_n ketma-ketlikning *limiti* deyiladi va bu quyidagicha yoziladi:

$$\lim_{n\to\infty}x_n=a.$$

Agar x_n ketma-ketlik limitga ega bo'lsa, u *yaqinlashuvchi*, aks holda *uzoqlashuvchi ketma-ketlik* deyiladi.

Har qanday chegaralangan va monoton ketma-ketlik limitga ega.

1- misol. $\lim\limits_{n\to\infty}\frac{2n+3}{2n+1}=1$ ekamligini isbot qiling va $N(\varepsilon)$ ni aniqlang.

Yechish. Agar ixtiyoriy $\varepsilon>0$ uchun shunday $N(\varepsilon)$ soni mavjud bo'lsaki, barcha $n\ge N(\varepsilon)$ lar uchun

$$|x_n-a|=\left|\frac{2n+3}{2n+1}-1\right|<\varepsilon$$

tengsizlik bajarilsa, limitning ta'rifiga ko'ra quyidagi masala hal bo'ladi. Yuqoridagi tengsizlik quyidagiga teng kuchli:

$$\frac{2}{2n+1}<\varepsilon,$$

bundan

$$2n+1>\frac{2}{\varepsilon}\quad yoki\quad n>\frac{2-\varepsilon}{2\varepsilon}$$

$f(x)<0$ bo'ladigan qismida bu grafikni $y=f(x)$ funksiya grafigini Ox o'qqa nisbatan simmetrik akslantirish yordamida hosil qilinadi.

Misol. $y=-2\sin(2x+2)$ funksiyaning grafigini $y=\sin x$ funksiya grafigidan foydalanib chizing.

Yechish. $y=\sin x$ funksiya grafigidan foydalanib, $y=-2\sin(2x+2)$ funksiya grafigini chizish quyidagi shakl almashtirishlar orqali amalga oshiriladi:

$$y_1=\sin 2x_1,\ y_2=-2\sin 2x_2,$$

$$y=-2\sin 2(x+1)=-2\sin(2x+2).$$

Geometrik nuqtai nazardan bu shakl yasashlarga olib keladi.

1. $0\le x\le 2\pi$ oraliqda $y=\sin x$ sinusoidani chizamiz.

2. Sinusoidada bir nechta nuqta belgilaymiz va ordinatalarini o'zgartirmay, abssissalarini ikki marta kamaytiramiz: $x_1=\frac{1}{2}x,\quad y_1=y.$ Hosil bo'lgan nuqtalarni silliq chiziq bilan birlashtirib, $y_1=\sin 2x_1$ funksiyaning grafigini chizamiz.

3. Hosil bo'lgan grafikdagi nuqtalar abssissalarini o'zgartirmay, ordinatalarini 2 marta orttiramiz va ularning ishoralarini almashtiramiz: $y_2=-2y_1,\ x_2=x_1$. Hosil bo'lgan nuqtalarni silliq chiziq bilan birlashtirib, $y_2=-2\sin x_2$ funksiyaning grafigini chizamiz.

4. Oxirgi grafikni abssissalar o'qi bo'yicha (-1) ga ko'chiramiz: $x=x_2-1,\ y=y_2$. Hosil qilingan nuqtalarni silliq chiziq bilan birlashtirib, $y=-2\sin(2x+2)$ funksiya grafigini chizamiz.

3.5. Ketma-ketlikning limiti. Funksiyaning limiti

Natural sonlar to'plamida aniqlangan funksiya sonli *ketma-ketlik* deyiladi va $\{x_n\}$ ko'rinishda belgilanadi.

Agar shunday M musbat son manjud bo'lib, har qanday natural son n uchun

$$|x_n|\le M$$

tengsizlik o'rinli bo'lsa, x_n *chegaralangan ketma-ketlik* deyiladi. Agar har qanday natural son n uchun

$$x_{n+1}>x_n$$

tengsizlik bajarilsa, x_n *o'suvchi ketma-ketlik* deyiladi. Agar har qanday natural son n uchun

$$x_{n+1}<x_n$$

tengsizlik bajarilsa, x_n *kamayuvchi ketma-ketlik* deyiladi.

Faqat o'suvchi yoki kamayuvchi ketma-ketlik *monaton ketma-ketlik* deyiladi.

Agar istalgan $\varepsilon>0$ son uchun shunday $N=N(\varepsilon)>0$ son mavjud bo'lsaki, barcha $n\ge N$ lar uchun

$$|x_n-a|<\varepsilon$$

tengsizlik bajarilsa, o'zgarmas a son x_n ketma-ketlikning *limiti* deyiladi va bu quyidagicha yoziladi:

$$\lim_{n\to\infty}x_n=a.$$

Agar x_n ketma-ketlik limitga ega bo'lsa, u *yaqinlashuvchi*, aks holda *uzoqlashuvchi ketma-ketlik* deyiladi.

Har qanday chegaralangan va monoton ketma-ketlik limitga ega.

1- misol. $\lim\limits_{n\to\infty}\frac{2n+3}{2n+1}=1$ ekamligini isbot qiling va $N(\varepsilon)$ ni aniqlang.

Yechish. Agar ixtiyoriy $\varepsilon>0$ uchun shunday $N(\varepsilon)$ soni mavjud bo'lsaki, barcha $n\ge N(\varepsilon)$ lar uchun

$$|x_n-a|=\left|\frac{2n+3}{2n+1}-1\right|<\varepsilon$$

tengsizlik bajarilsa, limitning ta'rifiga ko'ra quyidagi masala hal bo'ladi. Yuqoridagi tengsizlik quyidagiga teng kuchli:

$$\frac{2}{2n+1}<\varepsilon,$$

bundan

$$2n+1>\frac{2}{\varepsilon}\quad yoki\quad n>\frac{2-\varepsilon}{2\varepsilon}$$

tengsizlikka ega bo'lamiz. Demak, $N = N(\varepsilon) = \frac{2-\varepsilon}{2\varepsilon}$.

Shunday qilib,

$$\lim_{n\to\infty}\frac{2n+3}{2n+1}=1.$$

Agar har qanday $\varepsilon>0$ son uchun shunday $\delta=\delta(\varepsilon)>0$ son mavjud bo'lib, $|x-a|<\delta$ da $|f(x)-b|<\varepsilon$ tengsizlik bajarilsa, b soni $f(x)$ funksiyaning $x\to a$ dagi limiti deyiladi va quyidagicha yoziladi:

$$\lim_{x\to a} f(x)=b.$$

Agar ixtiyoriy $\varepsilon>0$ uchun shunday $N=N(\varepsilon)>0$ son mavjud bo'lib, barcha $|x|>N$ lar uchun $|f(x)-b|<\varepsilon$ tengsizlik bajarilsa, b soni $f(x)$ funksiyaning $x\to\infty$ dagi limiti deyiladi va quyidagicha yoziladi:

$$\lim_{x\to\infty} f(x)=b.$$

Agar ixtiyoriy $M>0$ uchun shunday $\delta=\delta(M)>0$ mavjud bo'lib, $|x-a|<\delta$ da $|f(x)|>M$ tengsizlik bajarilsa, $f(x)$ funksiya $f(x)$ da *cheksiz katta* deyiladi va bunday yoziladi:

$$\lim_{x\to a} f(x)=\infty.$$

Agar $x\to a$ da $x>a$ bo'lsa, u holda $x\to a+0$ belgi, agar $x\to a$ da $x<a$ bo'lsa, u holda $x\to a-0$ belgi ko'llaniladi. $f(x)$ funksiyaning a nuqtadagi *chap* va *o'ng limitlari* deb mos ravishda

$$f(a-0)=\lim_{x\to a-0} f(x)=f(a+0)=\lim_{x\to a+0} f(x)$$

sonlarga aytiladi.

$f(x)$ funksiyaning $x\to a$ dagi limiti mavjud bo'lishi uchun $f(a-0)=f(a+0)$ bo'lishi zarur va yetarli.

Limitlar haqida quyidagi teoremalar o'rinli (limitga o'tish qoidalari)

a) Agar C o'zgarmas bo'lsa,

$$\lim_{x\to a} C=C.$$

b) Agar $\lim_{x\to a} f(x)$ va $\lim_{x\to a}\varphi(x)$ mavjud bo'lsa,

$$\lim_{x\to a}(f(x)+\varphi(x))=\lim_{x\to a} f(x)+\lim_{x\to a}\varphi(x).$$

c) Agar $\lim_{x\to a} f(x)$ va $\lim_{x\to a}\varphi(x)$ limitlar mavjud bo'lsa, u holda

$$\lim_{x\to a}(f(x)\cdot\varphi(x))=\lim_{x\to a} f(x)\cdot\lim_{x\to a}\varphi(x).$$

tenglik o'rinli.

d) Agar $\lim_{x\to a} f(x)$ va $\lim_{x\to a}\varphi(x)\neq 0$ bo'lsa, u holda

$$\lim_{x\to a}\frac{f(x)}{\varphi(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a}\varphi(x)}.$$

tenglik o'rinli.

Agar bu teoremalarning shartlari bajarilmasa, u holda $\frac{\infty}{\infty}$, $\frac{0}{0}$, ∞, 0 ko'rinishidagi *aniqmasliklar* paydo bo'lishi mumkin.

Bu aniqmasliklar ba'zi hollarda algebraik almashtirishlar yordamida ochiladi.

2-misol. Limitni hisoblang:

$$\lim_{n\to\infty}\frac{3n^2+5n-1}{n^2+3}.$$

Yechish. Bu misolda kasrning surat va maxraji cheksizlikka intiladi, ya'ni $\frac{\infty}{\infty}$ — ko'rinishdagi aniqmaslikka egamiz.

Kasrning surat va maxrajini n^2 ga bo'lsak:

$$\lim_{n\to\infty}\frac{3n^2+5n-1}{n^2+3}=\lim_{n\to\infty}\frac{3+\frac{5}{n}-\frac{1}{n^2}}{1+\frac{3}{n^2}}=\frac{3}{1}=3.$$

Berilgan misolni maple orqali yechamiz:

$$> limit\left(\frac{3\cdot n^2+5\cdot n-1}{n^2+3},\ n=\infty\right);$$

$$3$$

3-misol. Limitni hisoblang:

$$\lim_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+3)!}.$$

tengsizlikka ega bo'lamiz. Demak, $N = N(\varepsilon) = \frac{2-\varepsilon}{2\varepsilon}$.

Shunday qilib,

$$\lim_{n\to\infty}\frac{2n+3}{2n+1} = 1.$$

Agar har qanday $\varepsilon > 0$ son uchun shunday $\delta = \delta(\varepsilon) > 0$ son mavjud bo'lib, $|x-a| < \delta$ da $|f(x)-b| < \varepsilon$ tengsizlik bajarilsa, b soni $f(x)$ funksiyaning $x \to a$ dagi limiti deyiladi va quyidagicha yoziladi:

$$\lim_{x\to a} f(x) = b.$$

Agar ixtiyoriy $\varepsilon > 0$ uchun shunday $N = N(\varepsilon) > 0$ son mavjud bo'lib, barcha $|x| > N$ lar uchun $|f(x)-b| < \varepsilon$ tengsizlik bajarilsa, b soni $f(x)$ funksiyaning $x \to \infty$ dagi limiti deyiladi va quyidagicha yoziladi:

$$\lim_{x\to\infty} f(x) = b.$$

Agar ixtiyoriy $M > 0$ uchun shunday $\delta = \delta(M) > 0$ mavjud bo'lib, $|x-a| < \delta$ da $|f(x)| > M$ tengsizlik bajarilsa, $f(x)$ funksiya $f(x)$ da *cheksiz katta* deyiladi va bunday yoziladi:

$$\lim_{x\to a} f(x) = \infty.$$

Agar $x \to a$ da $x > a$ bo'lsa, u holda $x \to a+0$ belgi, agar $x \to a$ da $x < a$ bo'lsa, u holda $x \to a-0$ belgi ko'llaniladi. $f(x)$ funksiyaning a nuqtadagi *chap* va *o'ng limitlari* deb mos ravishda

$$f(a-0) = \lim_{x\to a-0} f(x) = f(a+0) = \lim_{x\to a+0} f(x)$$

sonlarga aytiladi.

$f(x)$ funksiyaning $x \to a$ dagi limiti mavjud bo'lishi uchun $f(a-0) = f(a+0)$ bo'lishi zarur va yetarli.

Limitlar haqida quyidagi teoremalar o'rinli (limitga o'tish qoidalari)

a) Agar C o'zgarmas bo'lsa,

$$\lim_{x\to a} C = C.$$

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$$\lim_{x\to a}(f(x)+\varphi(x)) = \lim_{x\to a} f(x) + \lim_{x\to a}\varphi(x).$$

c) Agar $\lim_{x\to a} f(x)$ va $\lim_{x\to a}\varphi(x)$ limitlar mavjud bo'lsa, u holda

$$\lim_{x\to a}(f(x)\cdot\varphi(x)) = \lim_{x\to a} f(x)\cdot\lim_{x\to a}\varphi(x).$$

tenglik o'rinli.

d) Agar $\lim_{x\to a} f(x)$ va $\lim_{x\to a}\varphi(x) \neq 0$ bo'lsa, u holda

$$\lim_{x\to a}\frac{f(x)}{\varphi(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a}\varphi(x)}.$$

tenglik o'rinli.

Agar bu teoremalarning shartlari bajarilmasa, u holda $\frac{\infty}{\infty}$, $\frac{0}{0}$, ∞, 0 ko'rinishidagi *aniqmasliklar* paydo bo'lishi mumkin.

Bu aniqmasliklar ba'zi hollarda algebraik almashtirishlar yordamida ochiladi.

2-misol. Limitni hisoblang:

$$\lim_{n\to\infty}\frac{3n^2+5n-1}{n^2+3}.$$

Yechish. Bu misolda kasrning surat va maxraji cheksizlikka intiladi, ya'ni $\frac{\infty}{\infty}$ — ko'rinishdagi aniqmaslikka egamiz.

Kasrning surat va maxrajini n^2 ga bo'lsak:

$$\lim_{n\to\infty}\frac{3n^2+5n-1}{n^2+3} = \lim_{n\to\infty}\frac{3+\frac{5}{n}-\frac{1}{n^2}}{1+\frac{3}{n^2}} = \frac{3}{1} = 3.$$

Berilgan misolni maple orqali yechamiz:

$$> limit\left(\frac{3\cdot n^2+5\cdot n-1}{n^2+3},\ n=\infty\right);$$

$$3$$

3-misol. Limitni hisoblang:

$$\lim_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+3)!}.$$

Yechish. Bunda $\frac{\infty}{\infty}$ — ko'rinishdagi aniqmaslikka egamiz. $(n+2)! = (n+1)!(n+2)$ va $(n+3)! = (n+1)!(n+3)(n+2)$ almashtirishlarni bajarsak,

$$\lim_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+3)!} = \lim_{n\to\infty}\frac{(n+1)!+(n+3)}{(n+1)!(n+2)(n+3)} = \lim_{n\to\infty}\frac{1}{(n+2)} = 0.$$

Berilgan misolni maple orqali yechamiz:

$$> limit\left(\frac{(n+2)!+(n+1)!}{(n+3)!}, n=\infty\right);$$

$$0$$

Funksiyaning limitini hisoblash

Funksiyaning limitini amalda hisoblash oldingi paragrafda bayon qilingan teoremalar va ba'zi shakl almashtirishlarga asoslanadi.

1-misol. Limitni hisoblang:

$$\lim_{x\to 2}\frac{3x-2}{x^2+1}.$$

Yechish. $x\to 2$ da kasrning surati $3\cdot 2-2=4$ ga, maxraji esa $2^2+1=5$ ga intiladi. Demak,

$$\lim_{x\to 2}\frac{3x-2}{x^2+1}=\frac{4}{5}.$$

2-misol. Limitni hisoblang:

$$\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3+x^2-x-1}.$$

Yechish. Bu misolda kasrning surati ham, maxraji ham $n\to 1$ da nolga intiladi. $\frac{0}{0}$ ko'rinishdagi aniqmaslikka egamiz. Kasrning surat va maxrajini ko'paytuvchilarga ajratsak:

$$\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3+x^2-x-1} = \lim_{x\to 1}\frac{x^2(x-1)-(x-1)}{x^2(x+1)-(x+1)} =$$

$$\lim_{x\to 1}\frac{(x-1)(x^2-1)}{(x+1)(x^2-1)} = \lim_{x\to 1}\frac{(x-1)}{(x+1)} = \frac{0}{2} = 0.$$

3-misol. Limitni hisoblang:

$$\lim_{x\to 2}\frac{4}{x^2-4}-\frac{1}{x-2}.$$

Yechish. $\infty-\infty$ ko'rinishdagi aniqmaslikka egamiz. Hisoblaymiz:

$$\lim_{x\to 2}\frac{4}{x^2-4}-\frac{1}{x-2} = \lim_{x\to 2}\frac{4-(x+2)}{x^2-4} = \lim_{x\to 2}\frac{2-x}{x^2-4} =$$

$$= \lim_{x\to 2}\frac{-(x-2)}{(x+2)(x-2)} = -\lim_{x\to 2}\frac{1}{x+2} = -\frac{1}{4}.$$

4-misol. Limitni hisoblang.

$$\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x^2+2x}.$$

Yechish. $\frac{0}{0}$ ko'rinishdagi aniqmaslikka egamiz. Kasrning surati va maxrajini $\sqrt{2+x}-\sqrt{2}$ ifodaga ko'paytirsak

$$\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x^2+2x} = \lim_{x\to 0}\frac{(\sqrt{2+x}-\sqrt{2})(\sqrt{2+x}+\sqrt{2})}{x(x+2)(\sqrt{2+x}+\sqrt{2})} =$$

$$= \lim_{x\to 0}\frac{2+x-2}{x(x+2)(\sqrt{2+x}+\sqrt{2})} = \lim_{x\to 0}\frac{x}{x(x+2)(\sqrt{2+x}+\sqrt{2})} =$$

$$= \lim_{x\to 0}\frac{1}{(x+2)(\sqrt{2+x}+\sqrt{2})} = \frac{1}{2\cdot 2\sqrt{2}} = \frac{\sqrt{2}}{8}.$$

Berilgan misoni maple orqali yechamiz:

$$> Limit\left(\frac{sqrt(2+x)-sqrt(2)}{x\cdot x+2\cdot x}, x=0\right) = limit\left(\frac{sqrt(2+x)-sqrt(2)}{x\cdot x+2\cdot x}, x=0\right);$$

$$\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x^2+2x} = \frac{1}{8}\sqrt{2}$$

3.6. Birinchi va ikkinchi ajoyib limitlar

Ko'pgina limitlarni topishda quyidagi ma'lum formulalardan foydalaniladi:

$$\lim_{a\to 0}\frac{\sin a}{a} = 1 \text{ — } \textit{birinchi ajoyib limit;}$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{a\to\infty}(1+a)^{\frac{1}{a}} = e \text{ — } \textit{ikkinchi ajoyib limit.}$$

Misollar yechganda quyidagi tengliklarni nazarda tutish foydali:

$$\lim_{\alpha\to 0}(1+k\alpha)^{\frac{1}{\alpha}}=\lim_{x\to\infty}(1+\frac{k}{x})^x=e^k;$$

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=1;$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1;$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a,\ (a>0).$$

1-misol. Limitni hisoblang:

$$\lim_{x\to 0}\frac{\sin 3x}{x}.$$

Yechish. $\frac{0}{0}$ — ko'rinishdagi aniqmaslikka egamiz. Birinchi ajoyib limitdan foydalanamiz:

$$\lim_{x\to 0}\frac{\sin 3x}{x}=\lim_{x\to 0}\frac{3\sin 3x}{3x}=3\lim_{x\to 0}\frac{\sin 3x}{3x}=3.$$

Berilgan misolni maple orqali yechamiz:

$$> limit\left(\frac{\sin(3\cdot x)}{x},x=0\right);$$

$$3$$

2-misol. Limitni hisoblang:

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\pi-2x}.$$

Yechish. $\frac{0}{0}$ — ko'rinishdagi aniqmaslikka egamiz. $\frac{\pi}{2}-x=z$ belgilash kiritsak, u holda $x\to\frac{\pi}{2}$ da $z\to 0$ bo'ladi. Hisoblaymiz:

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\pi-2x}=\lim_{z\to 0}\frac{\cos(\frac{\pi}{2}-z)}{\pi-2(\frac{\pi}{2}-z)}=\lim_{z\to 0}\frac{\sin z}{\pi-\pi+2z}=$$

$$=\lim_{z\to 0}\frac{\sin z}{2z}=\frac{1}{2}\lim_{z\to 0}\frac{\sin z}{z}=\frac{1}{2}.$$

Berilgan misolni maple orqali yechamiz:

$$> limit\left(\frac{\cos(x)}{\pi-2\cdot x},x=\frac{\pi}{2}\right)=limit\left(\frac{\cos(x)}{\pi-2\cdot x},x=\frac{\pi}{2}\right);$$

$$\lim_{x\to\frac{1}{2}\pi}\frac{\cos(x)}{\pi-2x}=\frac{1}{2}$$

3- misol. Limitni hisoblang:

$$\lim_{x\to+\infty}\left(\frac{2x+1}{2x-3}\right)^{4x-1}.$$

Yechish. Kasrning suratini maxrajiga bo'lib, butun qismini ajratib apamiz:

$$\frac{2x+1}{2x-3}=\frac{(2x-3)+4}{2x-3}=1+\frac{4}{2x-3}.$$

Shunday qilib, $x\to\infty$ da berilgan funksiya asosi birga intiluvchi, ko'rsatkichi esa cheksizlikka intiluvchi darajani ifodalaydi, ya'ni 1^∞ ko'rinishdagi aniqmaslikka egamiz. Funksiyani ikkinchi ajoyib limitdan foydalanish mumkin bo'ladigan qilib o'zgartiramiz:

$$\left(\frac{2x+1}{2x-3}\right)^{4x-1}=\left(1+\frac{4}{2x-3}\right)^{4x-1}=\left[\left(1+\frac{4}{2x-3}\right)^{\frac{2x-3}{4}}\right]^{\frac{4(4x-1)}{2x-3}}=$$

$$=\left[\left(1+\frac{4}{2x-3}\right)^{\frac{2x-3}{4}}\right]^{\frac{4(4-\frac{1}{x})}{2-\frac{3}{x}}}.$$

$x\to\infty$ da $\frac{4}{2x-3}=0$ bo'lgani sababli ikkinchi ajoyib limitga ko'ra:

$$\lim_{x\to+\infty}\left(1+\frac{4}{2x-3}\right)^{\frac{2x-3}{4}}=e.$$

$\lim\limits_{x\to+\infty}\frac{4(4-\frac{1}{x})}{2-\frac{3}{x}}=8$ ekanini hisobga olib, uzil-kesil

$\lim\limits_{x\to+\infty}\left(\frac{2x+1}{2x-3}\right)^{4x-1}=e^8$ ekanini topamiz.

Berilgan misolni maple orqali yechamiz:

$$> limit\left(\left(\frac{(2\cdot x+1)}{(2\cdot x-3)}\right)^{4\cdot x-1}, x=\infty\right);$$

$$e^8$$

3.7. Mustaqil ishlash uchun topshiriqlar

1-topshiriq. To'plamlar ustida quyidagi amallarni bajaring.

1. N natural sonlar to'plami va Z butun sonlar to'plami birlashmasini toping.
2. G ratsional sonlar to'plami, R haqiqiy sonlar to'plami bo'lsa $G\cap R$ ni toping.
3. Ratsional va irratsional sonlar to'plami birlashmasini toping.
4. A to'g'ri to'rtburchaklar to'plami, B romblar to'plami bo'lsa, $A\cap B$ ni toping.
5. A juft sonlar to'plami Z butun sonlar to'plami bo'lsa, ularning kesishmasini toping.
6. A juft sonlar to'plami B toq sonlar to'plami bo'lsa, A va B larning kesishmasini toping.
7. $\{0;\ 1,2\}$ bo'lsa, hamma qism to'plamlar to'plamini toping.
8. A juft sonlar to'plami, B toq sonlar to'plami, C tub sonlar to'plami bo'lsa, $A\cup B$, $A\cap B$, $A\cap C$ toping.
9. $A=\{1,2,4,6,9\}$, $B=\{3,4,5,8,10\}$ bo'lsa $A\setminus B$ va $B\setminus A$ larni toping.
10. G ratsional sonlar to'plami, R haqiqiy sonlar to'plami bo'lsa, $G\setminus R$ ni toping.
11. A= $\{a,b,d,c\}$ B= $\{b,c,e,k\}$ to'plamlar kesishmasini ko'rsating.
12. A=(26,39,5), B=(26,39,5,40) to'plamlar birlashmasini ko'rsating.
13. Agar A=(-2;3) va B=[-4;1] bo'lsa, A∩B ni toping.
14. A=[-3,5; - 2,5] va B= (-3; 0) to'plamlar berilgan. AUB ni toping.
15. A= [-2; -1] va B= (0;2) bo'lsa, A∩B ni toping.
16. A= (4; 5] va B=[2;3) bo'lsa, A \ B ni toping.
17. A=[-5;0) va B=[-3;-1) to'plamlar berilgan. B\A ni toping.
18. A = (4; 5] va B=[2;3) bo'lsa, A \ B ni toping.
19. [1;5] va [3;7] kesmalarning kesishmasini toping.
20. A={1,2,3}, B={1,3,5}, C={1,5,9} to'plamlar berilgan. Shu to'plamlarga universal to'plamni aniqlang.
21. $A=\{1,2,3,5\}$ va $B=\{1;5\}$ to'plamlar berilgan bo'lsa, A/B ni toping.

$\lim_{x\to+\infty}\frac{4(4-\frac{1}{x})}{2-\frac{3}{x}}=8$ ekanini hisobga olib, uzil-kesil

$\lim_{x\to+\infty}\left(\frac{2x+1}{2x-3}\right)^{4x-1}=e^8$ ekanini topamiz.

Berilgan misolni maple orqali yechamiz:

$$> limit\left(\left(\frac{(2\cdot x+1)}{(2\cdot x-3)}\right)^{4\cdot x-1}, x=\infty\right);$$

$$e^8$$

3.7. Mustaqil ishlash uchun topshiriqlar

1-topshiriq. To'plamlar ustida quyidagi amallarni bajaring.

1. N natural sonlar to'plami va Z butun sonlar to'plami birlashmasini toping.
2. G ratsional sonlar to'plami, R haqiqiy sonlar to'plami bo'lsa $G\cap R$ ni toping.
3. Ratsional va irratsional sonlar to'plami birlashmasini toping.
4. A to'g'ri to'rtburchaklar to'plami, B romblar to'plami bo'lsa, $A\cap B$ ni toping.
5. A juft sonlar to'plami Z butun sonlar to'plami bo'lsa, ularning kesishmasini toping.
6. A juft sonlar to'plami B toq sonlar to'plami bo'lsa, A va B larning kesishmasini toping.
7. $\{0;\ 1,2\}$ bo'lsa, hamma qism to'plamlar to'plamini toping.
8. A juft sonlar to'plami, B toq sonlar to'plami, C tub sonlar to'plami bo'lsa, $A\cup B$, $A\cap B$, $A\cap C$ toping.
9. $A=\{1,2,4,6,9\}$, $B=\{3,4,5,8,10\}$ bo'lsa $A\setminus B$ va $B\setminus A$ larni toping.
10. G ratsional sonlar to'plami, R haqiqiy sonlar to'plami bo'lsa, $G\setminus R$ ni toping.
11. A= $\{a,b,d,c\}$ B= $\{b,c,e,k\}$ to'plamlar kesishmasini ko'rsating.
12. A=(26,39,5); B=(26,39,5,40) to'plamlar birlashmasini ko'rsating.
13. Agar A=(-2;3) va B=[-4;1] bo'lsa, A∩B ni toping.
14. A=[-3,5; - 2,5] va B= (-3; 0) to'plamlar berilgan. AUB ni toping.
15. A= [-2; -1] va B= (0;2) bo'lsa, A∩B ni toping.
16. A= (4; 5] va B=[2;3) bo'lsa, A \ B ni toping.
17. A=[-5;0) va B=[-3;-1) to'plamlar berilgan. B\A ni toping.
18. A = (4; 5] va B=[2;3) bo'lsa, A \ B ni toping.
19. [1;5] va [3;7] kesmalarning kesishmasini toping.
20. A={1,2,3}, B={1,3,5}, C={1,5,9} to'plamlar berilgan. Shu to'plamlarga universal to'plamni aniqlang.
21. $A=\{1,2,3,5\}$ va $B=\{1;5\}$ to'plamlar berilgan bo'lsa, A/B ni toping.

22. $A=\{2;\ 5;\ 7;\ 9\}$ va $B=\{2;\ 4;\ 7\}$ to'plamlar berilgan bo'lsa, u holda $A\cap B$ ni toping.

23. $A=\{2;\ 5;\ 7;\ 9\}$ va $B=\{2;\ 4;\ 7\}$ to'plamlar berilgan bo'lsin, u holda A/B ni toping.

24. Agar $A=\{1;\ 2;\ 3;\ 4\}$ va $B=\{1;\ 2\}$ to'plamlar berilgan bo'lsa, u holda A/B ni toping.

25. Quyidagi to'plamni sonlar o'qida tasvirlang:

1) $\{x/x\in N,\ \ x\le 3\}$; 2) $\{x/x\in R,\ \ x<-7\}$;

26. Quyidagi to'plamni sonlar o'qida tasvirlang:

1) $\{x/x\in Z,\ \ -2\le x\le 2\}$; 2) $\{x/x\in R,\ \ -2{,}7\le x\le 0\}$;

27. Quyidagi to'plamni sonlar o'qida tasvirlang:

1) $\{x/x\in R,\ \ x>3{,}2\}$; 2) $\{x/x\in R,\ \ 3{,}4<x\le 8\}$.

28. Quyidagi tenglamalar yechimlarining to'plamini toping:

$$4x+5=4(x-7),\quad x\in R;$$

29. Quyidagi tenglamalar yechimlarining to'plamini toping:

$$2x+3=3,\quad x\in R;$$

30. Quyidagi tenglamalar yechimlarining to'plamini toping:

$$x^2-4=0,\quad x\in Z;$$

2 – topshiriq. Berilgan $\{x_n\}_{n=1}^{\infty}$ ketma-ketlik uchun quyidagilarni toping:

a) $\lim\limits_{n\to\infty} x_n = a$;

b) shunday n_0 mavjud bo'lsaki, barcha $n>n_0$ lar uchun $|x_n-a|<0{,}001$ tengsizlik bajariladi.

1. $x_n=\dfrac{3n+1}{-2n-1}$;
2. $x_n=\dfrac{-2n+5}{n+1}$;
3. $x_n=\dfrac{n+2}{4n-1}$;
4. $x_n=\dfrac{5n-11}{-2n+7}$;
5. $x_n=\dfrac{6n+1}{-n-3}$;
6. $x_n=\dfrac{6n-5}{3n+2}$;
7. $x_n=\dfrac{4n-2}{-5n+3}$;
8. $x_n=\dfrac{-5n+3}{-2n+7}$;
9. $x_n=\dfrac{-3n+4}{5n-2}$;
10. $x_n=\dfrac{4n-6}{-3n+5}$;

11. $x_n=\dfrac{-3n+2}{-n+3}$;
12. $x_n=\dfrac{2n-9}{-7n+10}$;
13. $x_n=\dfrac{-2n+3}{-3n+1}$;
14. $x_n=\dfrac{6n-5}{4n-3}$;
15. $x_n=\dfrac{n+1}{-3n-2}$;
16. $x_n=\dfrac{3n-7}{4n+5}$;
17. $x_n=\dfrac{-5n+1}{-2n-3}$;
18. $x_n=\dfrac{n+12}{-5n+2}$;
19. $x_n=\dfrac{-n+8}{-5n+4}$;
20. $x_n=\dfrac{5n-4}{-4n+11}$;
21. $x_n=\dfrac{4n-11}{2n+9}$;
22. $x_n=\dfrac{4n+9}{-n+5}$;
23. $x_n=\dfrac{-2n+11}{4n+7}$;
24. $x_n=\dfrac{-4n+11}{3n-2}$;
25. $x_n=\dfrac{-5n+1}{-4n-3}$;
26. $x_n=\dfrac{-3n+10}{-5n+6}$;
27. $x_n=\dfrac{-3n+2}{2n+11}$;
28. $x_n=\dfrac{2n-7}{3n-8}$;
29. $x_n=\dfrac{2n+5}{-3n+7}$;
30. $x_n=\dfrac{5n+8}{-6n-1}$.

3 – topshiriq. Limit ta'rifidan foydalanib $\lim\limits_{x\to x_0} f(x)=A$. Berilgan $\varepsilon=0{,}01$ shunday $\delta>0$ mavjud bo'lsaki, $|x-x_0|<\delta$ tengsizlikdan $|f(x)-A|<0{,}01$ tengsizlikning bajarilishi kelib chiqadi.

№	$f(x)$	x_0	A
1	$7x-1$	1	6
2	$9x+1$	–1	–8
3	$3x+4$	2	10
4	$5x+3$	–2	–7
5	$8x-2$	2	14
6	x^2-9	2	–5
7	$6x-7$	2	5
8	$4x^2-1$	1	3
9	$-3x+5$	–1	8

22. $A=\{2; 5; 7; 9\}$ va $B=\{2; 4; 7\}$ to'plamlar berilgan bo'lsa, u holda $A\cap B$ ni toping.

23. $A=\{2; 5; 7; 9\}$ va $B=\{2; 4; 7\}$ to'plamlar berilgan bo'lsin, u holda A/B ni toping.

24. Agar $A=\{1; 2; 3; 4\}$ va $B=\{1; 2\}$ to'plamlar berilgan bo'lsa, u holda A/B ni toping.

25. Quyidagi to'plamni sonlar o'qida tasvirlang:

1) $\{x/x\in N,\ \ x\le 3\}$; 2) $\{x/x\in R,\ \ x<-7\}$;

26. Quyidagi to'plamni sonlar o'qida tasvirlang:

1) $\{x/x\in Z,\ \ -2\le x\le 2\}$; 2) $\{x/x\in R,\ \ -2{,}7\le x\le 0\}$;

27. Quyidagi to'plamni sonlar o'qida tasvirlang:

1) $\{x/x\in R,\ \ x>3{,}2\}$; 2) $\{x/x\in R,\ \ 3{,}4<x\le 8\}$.

28. Quyidagi tenglamalar yechimlarining to'plamini toping:

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29. Quyidagi tenglamalar yechimlarining to'plamini toping:

$$2x+3=3,\quad x\in R;$$

30. Quyidagi tenglamalar yechimlarining to'plamini toping:

$$x^2-4=0,\quad x\in Z;$$

2 – topshiriq. Berilgan $\{x_n\}_{n=1}^{\infty}$ ketma-ketlik uchun quyidagilarni toping:

a) $\lim\limits_{n\to\infty} x_n = a$;

b) shunday n_0 mavjud bo'lsaki, barcha $n>n_0$ lar uchun $|x_n-a|<0{,}001$ tengsizlik bajariladi.

1. $x_n=\dfrac{3n+1}{-2n-1}$;
2. $x_n=\dfrac{-2n+5}{n+1}$;
3. $x_n=\dfrac{n+2}{4n-1}$;
4. $x_n=\dfrac{5n-11}{-2n+7}$;
5. $x_n=\dfrac{6n+1}{-n-3}$;
6. $x_n=\dfrac{6n-5}{3n+2}$;
7. $x_n=\dfrac{4n-2}{-5n+3}$;
8. $x_n=\dfrac{-5n+3}{-2n+7}$;
9. $x_n=\dfrac{-3n+4}{5n-2}$;
10. $x_n=\dfrac{4n-6}{-3n+5}$;

11. $x_n=\dfrac{-3n+2}{-n+3}$;
12. $x_n=\dfrac{2n-9}{-7n+10}$;
13. $x_n=\dfrac{-2n+3}{-3n+1}$;
14. $x_n=\dfrac{6n-5}{4n-3}$;
15. $x_n=\dfrac{n+1}{-3n-2}$;
16. $x_n=\dfrac{3n-7}{4n+5}$;
17. $x_n=\dfrac{-5n+1}{-2n-3}$;
18. $x_n=\dfrac{n+12}{-5n+2}$;
19. $x_n=\dfrac{-n+8}{-5n+4}$;
20. $x_n=\dfrac{5n-4}{-4n+11}$;
21. $x_n=\dfrac{4n-11}{2n+9}$;
22. $x_n=\dfrac{4n+9}{-n+5}$;
23. $x_n=\dfrac{-2n+11}{4n+7}$;
24. $x_n=\dfrac{-4n+11}{3n-2}$;
25. $x_n=\dfrac{-5n+1}{-4n-3}$;
26. $x_n=\dfrac{-3n+10}{-5n+6}$;
27. $x_n=\dfrac{-3n+2}{2n+11}$;
28. $x_n=\dfrac{2n-7}{3n-8}$;
29. $x_n=\dfrac{2n+5}{-3n+7}$;
30. $x_n=\dfrac{5n+8}{-6n-1}$.

3 – topshiriq. Limit ta'rifidan foydalanib $\lim\limits_{x\to x_0} f(x)=A$. Berilgan $\varepsilon=0{,}01$ shunday $\delta>0$ mavjud bo'lsaki, $|x-x_0|<\delta$ tengsizlikdan $|f(x)-A|<0{,}01$ tengsizlikning bajarilishi kelib chiqadi.

№	$f(x)$	x_0	A
1	$7x-1$	1	6
2	$9x+1$	–1	–8
3	$3x+4$	2	10
4	$5x+3$	–2	–7
5	$8x-2$	2	14
6	x^2-9	2	–5
7	$6x-7$	2	5
8	$4x^2-1$	1	3
9	$-3x+5$	–1	8

10	8x–4	2	12
11	4x–3	1	1
12	x^2–1	1	0
13	x^2–4	3	5
14	6x+1	1	7
15	–x+4	2	2
16	–2x+1	1	–1
17	–3x–3	1	–6
18	x–5	4	–1
19	–3x+4	2	–2
20	7x–2	2	12
21	10x+1	1	11
22	12x–5	2	19
23	11x+3	–1	–8
24	–6x+5	–1	11
25	–x+7	1	6
26	–x^2+1	1	0
27	–x^2–5	3	–14
28	3x–9	3	0
29	2x+7	–1	5
30	–4x+3	2	–5

4 – topshiriq. Limitlarni toping.

1. $\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{x^{16}+3x}+1}{\sqrt[8]{x^{32}+x^2+x}+x^4}$;

2. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^{20}+x^5+x}+3}{\sqrt[3]{x^{15}+3x+2}}$;

3. $\lim\limits_{x\to\infty}\dfrac{\sqrt[5]{x^{10}+4x^2+9}}{\sqrt[5]{x^5+7x}+5x^2}$;

4. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^7+x^6+5x}+2x^3}{\sqrt[9]{x^{27}+6x^{20}}+7}$;

5. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^6+2x^3}+3x^2}{\sqrt[7]{x^{21}+5x^2+x}}$;

6. $\lim\limits_{x\to\infty}\dfrac{\sqrt[15]{x^{30}+5x^{10}}+10x}{\sqrt[10]{x^{20}+7x^6+9}+x^2}$;

7. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^{12}+4x+7}+4x^2}{\sqrt[5]{x^{20}+x^{11}+x^2}+9x^4}$;

8. $\lim\limits_{x\to\infty}\dfrac{\sqrt[30]{x^{60}+5x^{10}+x^{5}}}{\sqrt[8]{x^8+5x^7}+3x^2}$;

9. $\lim\limits_{x\to\infty}\dfrac{\sqrt[18]{x^{36}+x^{10}}+7x^6}{\sqrt[5]{x^{40}+x^{20}+10x}}$;

10. $\lim\limits_{x\to\infty}\dfrac{\sqrt[8]{x^{72}+x^{15}+5x}-15}{\sqrt[4]{x^{16}+5}+3x^9}$;

11. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{14}+5x^{10}+9}}{\sqrt[6]{x^{12}+x^5+3}+8x}$;

12. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{14}+x^{13}+5}-7x^5}{\sqrt{x^{10}+5x^5+x}+2x}$;

13. $\lim\limits_{x\to\infty}\dfrac{\sqrt[5]{x^{30}+7x^{20}+x^3}}{\sqrt[10]{x^{10}+5x^6+10}+8x^6}$;

14. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^{30}+10x^{15}+3}}{\sqrt{5x^{20}+10x}-12}$;

15. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{24}+7x^2+x}+2x^3}{\sqrt[8]{x^{24}+5x^{10}+3}+10}$;

16. $\lim\limits_{x\to\infty}\dfrac{\sqrt[20]{x^{40}+4x^{30}-3}}{\sqrt[3]{x^3+x^2-3x}+5x^2}$;

17. $\lim\limits_{x\to\infty}\dfrac{\sqrt[10]{x^{10}+x^9+7}+3}{\sqrt[5]{x^5+x^4+x}+2x}$;

18. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^8+7x^6+x}-10}{\sqrt[30]{x^{10}+2x^7+5}+3x^4}$;

19. $\lim\limits_{x\to\infty}\dfrac{\sqrt[8]{x^{40}+x^{10}+10}}{\sqrt{x^{10}+x^9+x}+15}$;

20. $\lim\limits_{x\to\infty}\dfrac{\sqrt[5]{x^{20}+4x^3+7}}{\sqrt[8]{x^{32}+x}-9x^2}$;

21. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{21}+x^{20}+5x}+8x^6}{\sqrt{x^{40}+x^{10}+x^3}}$;

22. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{28}+5x^{20}+x}+7}{\sqrt[5]{x^{40}+x^{25}+3}}$;

23. $\lim\limits_{x\to\infty}\dfrac{\sqrt[6]{x^{12}+3x-4}+x^2}{\sqrt[5]{x^{10}+x^2+6}+7x}$;

24. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^{30}+2x-5}+2x^{10}}{\sqrt{x^{20}+x^{10}+x}+4x^5}$;

25. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{49}+x^3+x}+20x}{2x^7+\sqrt{x^6+3x^2+9}}$;

26. $\lim\limits_{x\to\infty}\dfrac{\sqrt[10]{x^{10}+5x^9+4}}{\sqrt[15]{x^{15}+x^{10}+x}+9x}$;

27. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^3+5x^2+x}+2x^6}{\sqrt[3]{x^{18}+4x^6+3}-7}$;

28. $\lim\limits_{x\to\infty}\dfrac{\sqrt[11]{x^{33}+5x}-7}{\sqrt[5]{x^{10}+x^9+4}+3x^3}$;

29. $\lim\limits_{x\to\infty}\dfrac{\sqrt[16]{x^{16}+x^5+3}+2x}{\sqrt{3x^2+2x+5}}$;

30. $\lim\limits_{x\to\infty}\dfrac{\sqrt[12]{x^{24}+x^{20}+x}}{\sqrt[10]{x^{20}+x^8+4}+20x^2}$.

5 – topshiriq. Limitlarni toping.

1. $\lim\limits_{x\to-2}\dfrac{x^3+3x^2+2x}{x^2-x-6}$;

2. $\lim\limits_{x\to2}\dfrac{x^2-2x}{3x^2-7x-2}$;

3. $\lim\limits_{x\to1}\dfrac{x^3+x^2-2}{x^2-1}$;

4. $\lim\limits_{x\to1/2}\dfrac{8x^3-1}{6x^2-5x+1}$;

5. $\lim\limits_{x\to-1}\dfrac{x^3+x^2}{x^2-1}$;

6. $\lim\limits_{x\to1}\dfrac{x^4-3x+2}{x^4-4x+3}$;

10	8x–4	2	12
11	4x–3	1	1
12	x^2–1	1	0
13	x^2–4	3	5
14	6x+1	1	7
15	–x+4	2	2
16	–2x+1	1	–1
17	–3x–3	1	–6
18	x–5	4	–1
19	–3x+4	2	–2
20	7x–2	2	12
21	10x+1	1	11
22	12x–5	2	19
23	11x+3	–1	–8
24	–6x+5	–1	11
25	–x+7	1	6
26	–x^2+1	1	0
27	–x^2–5	3	–14
28	3x–9	3	0
29	2x+7	–1	5
30	–4x+3	2	–5

4 – topshiriq. Limitlarni toping.

1. $\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{x^{16}+3x}+1}{\sqrt[8]{x^{32}+x^2+x}+x^4}$;

2. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^{20}+x^5+x}+3}{\sqrt[3]{x^{15}+3x}+2}$;

3. $\lim\limits_{x\to\infty}\dfrac{\sqrt[5]{x^{10}+4x^2+9}}{\sqrt[5]{x^5+7x}+5x^2}$;

4. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^7+x^6+5x}+2x^3}{\sqrt[9]{x^{27}+6x^{20}}+7}$;

5. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^6+2x^3}+3x^2}{\sqrt[7]{x^{21}+5x^2+x}}$;

6. $\lim\limits_{x\to\infty}\dfrac{\sqrt[15]{x^{30}+5x^{10}}+10x}{\sqrt[10]{x^{20}+7x^6+9}+x^2}$;

7. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^{12}+4x+7}+4x^2}{\sqrt[5]{x^{20}+x^{11}+x^2}+9x^4}$;

8. $\lim\limits_{x\to\infty}\dfrac{\sqrt[30]{x^{60}+5x^{10}+x}}{\sqrt[8]{x^8+5x^7}+3x^2}$;

9. $\lim\limits_{x\to\infty}\dfrac{\sqrt[18]{x^{36}+x^{10}}+7x^6}{\sqrt[5]{x^{40}+x^{20}+10x}}$;

10. $\lim\limits_{x\to\infty}\dfrac{\sqrt[8]{x^{72}+x^{15}+5x}-15}{\sqrt[4]{x^{16}+5}+3x^9}$;

11. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{14}+5x^{10}+9}}{\sqrt[6]{x^{12}+x^5+3}+8x}$;

12. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{14}+x^{13}+5}-7x^5}{\sqrt{x^{10}+5x^5+x}+2x}$;

13. $\lim\limits_{x\to\infty}\dfrac{\sqrt[5]{x^{30}+7x^{20}+x^3}}{\sqrt[10]{x^{10}+5x^6+10}+8x^6}$;

14. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^{30}+10x^{15}+3}}{\sqrt{5x^{20}+10x}-12}$;

15. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{24}+7x^2+x}+2x^3}{\sqrt[8]{x^{24}+5x^{10}+3}+10}$;

16. $\lim\limits_{x\to\infty}\dfrac{\sqrt[20]{x^{40}+4x^{30}-3}}{\sqrt[3]{x^3+x^2-3x}+5x^2}$;

17. $\lim\limits_{x\to\infty}\dfrac{\sqrt[10]{x^{10}+x^9+7}+3}{\sqrt[5]{x^5+x^4+x}+2x}$;

18. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^8+7x^6+x}-10}{\sqrt[30]{x^{10}+2x^7+5}+3x^4}$;

19. $\lim\limits_{x\to\infty}\dfrac{\sqrt[8]{x^{40}+x^{10}+10}}{\sqrt{x^{10}+x^9+x+15}}$;

20. $\lim\limits_{x\to\infty}\dfrac{\sqrt[5]{x^{20}+4x^3+7}}{\sqrt[8]{x^{32}+x}-9x^2}$;

21. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{21}+x^{20}+5x}+8x^6}{\sqrt{x^{40}+x^{10}+x^3}}$;

22. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{28}+5x^{20}+x}+7}{\sqrt[5]{x^{40}+x^{25}+3}}$;

23. $\lim\limits_{x\to\infty}\dfrac{\sqrt[6]{x^{12}+3x-4}+x^2}{\sqrt[5]{x^{10}+x^2+6}+7x}$;

24. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^{30}+2x-5}+2x^{10}}{\sqrt{x^{20}+x^{10}+x}+4x^5}$;

25. $\lim\limits_{x\to\infty}\dfrac{\sqrt[7]{x^{49}+x^3+x}+20x}{2x^7+\sqrt{x^6+3x^2+9}}$;

26. $\lim\limits_{x\to\infty}\dfrac{\sqrt[10]{x^{10}+5x^9+4}}{\sqrt[15]{x^{15}+x^{10}+x}+9x}$;

27. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^3+5x^2+x}+2x^6}{\sqrt[3]{x^{18}+4x^6+3}-7}$;

28. $\lim\limits_{x\to\infty}\dfrac{\sqrt[11]{x^{33}+5x}-7}{\sqrt[5]{x^{10}+x^9+4}+3x^3}$;

29. $\lim\limits_{x\to\infty}\dfrac{\sqrt[16]{x^{16}+x^5+3}+2x}{\sqrt{3x^2+2x+5}}$;

30. $\lim\limits_{x\to\infty}\dfrac{\sqrt[12]{x^{24}+x^{20}+x}}{\sqrt[10]{x^{20}+x^8+4}+20x^2}$.

5 – topshiriq. Limitlarni toping.

1. $\lim\limits_{x\to-2}\dfrac{x^3+3x^2+2x}{x^2-x-6}$;

2. $\lim\limits_{x\to2}\dfrac{x^2-2x}{3x^2-7x-2}$;

3. $\lim\limits_{x\to1}\dfrac{x^3+x^2-2}{x^2-1}$;

4. $\lim\limits_{x\to1/2}\dfrac{8x^3-1}{6x^2-5x+1}$;

5. $\lim\limits_{x\to-1}\dfrac{x^3+x^2}{x^2-1}$;

6. $\lim\limits_{x\to1}\dfrac{x^4-3x+2}{x^4-4x+3}$;

7. $\lim\limits_{x\to -1}\dfrac{(x^3-2x-1)(x+1)}{x^4+4x^2-5}$;

8. $\lim\limits_{x\to 2}\dfrac{x^2-4x+4}{x^3-2x^2-x+2}$;

9. $\lim\limits_{x\to 1}\dfrac{x^4-x}{x^2+x-2}$;

10. $\lim\limits_{x\to 2}\dfrac{3x^4-12x^2+x+2}{x^2-4}$;

11. $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x^3-x^2-x+1}$;

12. $\lim\limits_{x\to 5}\dfrac{x^3-125}{x^3-2x^2-15x}$;

13. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^3-5x^2+6x}$;

14. $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{x^3+2x^2-x-2}$;

15. $\lim\limits_{x\to 0}\dfrac{(1+x)^3-(1+3x)}{x^2+x^5}$;

16. $\lim\limits_{x\to 3}\dfrac{x^2-6x+9}{3x^2-10x+3}$;

17. $\lim\limits_{x\to 1}\dfrac{2x^3-2x^2+x-1}{x^3-x^2+3x-3}$;

18. $\lim\limits_{x\to 4}\dfrac{x^2-8x+16}{x^3-4x^2-2x+8}$;

19. $\lim\limits_{x\to -1}\dfrac{(x^2+3x+2)^2}{x^3+2x^2-x-2}$;

20. $\lim\limits_{x\to 2}\dfrac{4x^2-7x-2}{5x^2-11x+2}$;

21. $\lim\limits_{x\to 2}\dfrac{x^4-3x^2-4}{x^4-16}$;

22. $\lim\limits_{x\to 4}\dfrac{x^3-64}{3x^2-11x-4}$;

23. $\lim\limits_{x\to 1}\dfrac{x^4-1}{2x^4-x^2-1}$;

24. $\lim\limits_{x\to -1}\dfrac{(x^2+2x+1)^2}{x^5+x^2}$;

25. $\lim\limits_{x\to 2}\dfrac{x^3-2x^2-4x+8}{x^2-4x+4}$;

26. $\lim\limits_{x\to -2}\dfrac{x^3+3x^2+2x}{x^2-x-6}$;

27. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-3x+2}$;

28. $\lim\limits_{x\to 1}\dfrac{(2x^2-x-1)^2}{x^3+2x^2-x-2}$;

29. $\lim\limits_{x\to 3}\dfrac{x^4-81}{x^4+2x^3-15x^2}$;

30. $\lim\limits_{x\to 0}\dfrac{(1+x)^3-(1+3x)}{x^2+x^6}$.

6 – topshiriq. Limitlarni toping.

1. $\lim\limits_{x\to 3}\dfrac{x^2+x-12}{\sqrt{x-2}-\sqrt{4-x}}$;

2. $\lim\limits_{x\to 0}\dfrac{x}{\sqrt[3]{x+1}-1}$;

3) $\lim\limits_{x\to -5}\dfrac{\sqrt{3x+17}-\sqrt{2x+12}}{x^2+8x+15}$;

4) $\lim\limits_{x\to 5}\dfrac{\sqrt{x-1}-2}{x-5}$;

5. $\lim\limits_{x\to -1}\dfrac{3x^2+4x+1}{\sqrt{x+3}-\sqrt{5+3x}}$;

6. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$;

7. $\lim\limits_{x\to -2}\dfrac{\sqrt{2-x}-\sqrt{x+6}}{x^2-x-6}$;

8. $\lim\limits_{x\to 4}\dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2}$;

9. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x+2}$;

10. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{x}}$;

11. $\lim\limits_{x\to -3}\dfrac{\sqrt{x+10}-\sqrt{4-x}}{2x^2-x-21}$;

12. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$;

13. $\lim\limits_{x\to 4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$;

14. $\lim\limits_{x\to 1}\dfrac{\sqrt{3+2x}-\sqrt{x+4}}{3x^2-4x+1}$;

15. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{\sqrt{2x+1}-3}$;

16. $\lim\limits_{x\to -5}\dfrac{\sqrt{x+14}-\sqrt{4-x}}{2x^2+11x+5}$;

17. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x-8}+\sqrt[3]{x+8}}{x}$;

18. $\lim\limits_{x\to 1}\dfrac{x^2-1}{\sqrt{x+5}-\sqrt{2x+4}}$;

19. $\lim\limits_{x\to 0}\dfrac{5x^2+6x+1}{\sqrt{x+9}-2\sqrt{1-x}}$;

20. $\lim\limits_{x\to 1}\dfrac{\sqrt{5-x}-2}{\sqrt{2-x}-1}$;

21. $\lim\limits_{x\to 4}\dfrac{\sqrt{5-x}-\sqrt{x-3}}{2x^2-9x+4}$;

22. $\lim\limits_{x\to 4}\dfrac{x^2-16}{\sqrt{2x+1}-3}$;

23. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$;

24. $\lim\limits_{x\to 0}\dfrac{\sqrt{9+5x+4x^2}-3}{x^3+9x}$;

25. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{x^2+9}-3}$;

26. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}$;

27. $\lim\limits_{x\to -1}\dfrac{x+1}{\sqrt{6x^2+3}+3x}$;

28. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-2x}-\sqrt{1-3x}}{x^3+6x^2+9x}$;

29. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x+x^2}-2}{x+x^2}$;

30. $\lim\limits_{x\to 5}\dfrac{1-\sqrt{x-4}}{2-\sqrt{2x-6}}$.

7 - topshiriq. Limitlarni toping.

1. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^2 x}{x^2}$;

2. $\lim\limits_{x\to 0}\dfrac{\sin x-\cos x}{1-\mathrm{tg}x}$;

3. $\lim\limits_{x\to 0}\dfrac{\mathrm{arctg}2x}{\sin 3x}$;

4. $\lim\limits_{x\to 0}\dfrac{x-\sin 2x}{x+\sin 3x}$;

5. $\lim\limits_{x\to -1}\dfrac{3x^2+4x+1}{\sqrt{x+3}-\sqrt{5+3x}}$;

6. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$;

7. $\lim\limits_{x\to -2}\dfrac{\sqrt{2-x}-\sqrt{x+6}}{x^2-x-6}$;

8. $\lim\limits_{x\to 4}\dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2}$;

9. $\lim\limits_{x\to -2}\dfrac{\sqrt[3]{x-6}+2}{x+2}$;

10. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{x}}$;

11. $\lim\limits_{x\to -3}\dfrac{\sqrt{x+10}-\sqrt{4-x}}{2x^2-x-21}$;

12. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$;

13. $\lim\limits_{x\to 4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$;

14. $\lim\limits_{x\to 1}\dfrac{\sqrt{3+2x}-\sqrt{x+4}}{3x^2-4x+1}$;

15. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{\sqrt{2x+1}-3}$;

16. $\lim\limits_{x\to -5}\dfrac{\sqrt{x+14}-\sqrt{4-x}}{2x^2+11x+5}$;

17. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x-8}+\sqrt[3]{x+8}}{x}$;

18. $\lim\limits_{x\to 1}\dfrac{x^2-1}{\sqrt{x+5}-\sqrt{2x+4}}$;

19. $\lim\limits_{x\to 0}\dfrac{5x^2+6x+1}{\sqrt{x+9}-2\sqrt{1-x}}$;

20. $\lim\limits_{x\to 1}\dfrac{\sqrt{5-x}-2}{\sqrt{2-x}-1}$;

21. $\lim\limits_{x\to 4}\dfrac{\sqrt{5-x}-\sqrt{x-3}}{2x^2-9x+4}$;

22. $\lim\limits_{x\to 4}\dfrac{x^2-16}{\sqrt{2x+1}-3}$;

23. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$;

24. $\lim\limits_{x\to 0}\dfrac{\sqrt{9+5x+4x^2}-3}{x^3+9x}$;

25. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{x^2+9}-3}$;

26. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}$;

27. $\lim\limits_{x\to -1}\dfrac{x+1}{\sqrt{6x^2+3}+3x}$;

28. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-2x}-\sqrt{1-3x}}{x^3+6x^2+9x}$;

29. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x+x^2}-2}{x+x^2}$;

30. $\lim\limits_{x\to 5}\dfrac{1-\sqrt{x-4}}{2-\sqrt{2x-6}}$.

7 - topshiriq. Limitlarni toping.

1. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^2 x}{x^2}$;

2. $\lim\limits_{x\to 0}\dfrac{\sin x-\cos x}{1-\operatorname{tg} x}$;

3. $\lim\limits_{x\to 0}\dfrac{\operatorname{arctg} 2x}{\sin 3x}$;

4. $\lim\limits_{x\to 0}\dfrac{x-\sin 2x}{x+\sin 3x}$;

5. $\lim\limits_{x\to 0} ctg2x\cdot ctg\left(\frac{\pi}{2}-x\right)$;

6. $\lim\limits_{x\to 0}\frac{1-\sqrt{\cos x}}{x^2}$;

7. $\lim\limits_{x\to 0}\frac{\cos x-\cos^5 x}{x^2}$;

8. $\lim\limits_{x\to 0}\frac{\sin 6x-\sin 2x}{5x}$;

9. $\lim\limits_{x\to 0}\frac{5\sin^2 3x}{x\cdot arctg2x}$;

10. $\lim\limits_{x\to 0}\frac{\sin 5x-\sin x}{tg5x}$;

11. $\lim\limits_{x\to 0}\frac{1-\cos\frac{x}{4}}{tg^2 5x}$;

12. $\lim\limits_{x\to 0}\frac{tg^3 5x}{\sin 8x^3}$;

13. $\lim\limits_{x\to 0}\frac{\arcsin^3 2x}{arctg\, x^3}$;

14. $\lim\limits_{x\to 0}\frac{\cos 6x-\cos 10x}{tg^2 3x}$;

15. $\lim\limits_{x\to 0}\frac{\sin 10x-\sin 2x}{\arcsin 3x}$;

16. $\lim\limits_{x\to 0}\frac{tgx-\sin x}{\sin^3 x}$;

17. $\lim\limits_{x\to 0}\frac{\cos x-\cos 3x}{x^2}$;

18. $\lim\limits_{x\to 0}\frac{1-\cos 5x}{1-\cos 3x}$;

19. $\lim\limits_{x\to 0}\frac{tgx-\sin x}{x(1-\cos 2x)}$;

20. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{\cos 7x-\cos 3x}$;

21. $\lim\limits_{x\to 0}\frac{1-\cos^3 x}{4x^2}$;

22. $\lim\limits_{x\to 0}\frac{2x\sin x}{1-\cos x}$;

23. $\lim\limits_{x\to 0}\frac{1-\cos 4x}{2x\cdot tg2x}$;

24. $\lim\limits_{x\to 0}\frac{\sqrt{1-\cos 2x}}{x}$;

25. $\lim\limits_{x\to 0}\frac{\cos x-\cos^3 x}{x^2}$;

26. $\lim\limits_{x\to 0}\frac{x^2 ctg2x}{\sin 3x}$;

27. $\lim\limits_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x}$;

28. $\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$;

29. $\lim\limits_{x\to 0}\frac{(1-\cos x)^2}{tgx-\sin x}$;

30. $\lim\limits_{x\to 0}\frac{2x-\arcsin x}{2x+arctgx}$.

8-topshiriq. Ajoyib limitlardan foydalanib limitlarni hisoblang.

1. $\lim\limits_{x\to\infty}(x+7)[\ln(x+1)-\ln(x+3)]$;

2. $\lim\limits_{x\to\infty}\left(\frac{3x^2+1}{3x^2-1}\right)^{2x}$;

3. $\lim\limits_{x\to\infty}(2x+1)[\ln(3x+1)-\ln 3x]$;

4. $\lim\limits_{x\to\infty}\left(\frac{3x^2+2}{3x^2-2}\right)^{x^2}$;

5. $\lim\limits_{x\to\infty}(x-5)[\ln(2x-3)-\ln(2x-1)]$;

6. $\lim\limits_{x\to 0}\left(\frac{1+3x^2}{1-2x^2}\right)^{-\frac{1}{x^2}}$;

7. $\lim\limits_{x\to\infty}(6x+3)[\ln(5x+2)-\ln(5x-1)]$;

8. $\lim\limits_{x\to 0}\left(\frac{2-x^2}{2+x^2}\right)^{\frac{3}{x^2}}$;

9. $\lim\limits_{x\to\infty}(2x-7)[\ln(x+4)-\ln(x+5)]$;

10. $\lim\limits_{x\to 0}\left(\frac{1+4x^2}{1+2x^2}\right)^{\frac{2+x}{x^2}}$;

11. $\lim\limits_{x\to\infty}(x+3)[\ln(2x-3)-\ln 2x]$;

12. $\lim\limits_{x\to\infty}\left(\frac{4+x^2}{2+x^2}\right)^{x^2}$;

13. $\lim\limits_{x\to\infty}(2x-5)[\ln(3x+4)-\ln(3x-2)]$;

14. $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+3}\right)^{\frac{3x^2}{(x+1)}}$;

15. $\lim\limits_{x\to\infty}(3x+2)[\ln(4x+2)-\ln(4x-1)]$;

16. $\lim\limits_{x\to\infty}\left(\frac{3x+1}{3x-5}\right)^{\frac{x^2-1}{x+2}}$;

17. $\lim\limits_{x\to\infty}(x+4)[\ln(2x+7)-\ln(2x+2)]$;

18. $\lim\limits_{x\to\infty}\left(\frac{-3+2x}{1-4x}\right)^{\frac{3x^2-1}{x+1}}$;

19. $\lim\limits_{x\to\infty}(x+2)[\ln(3+2x)-\ln(2x-1)]$;

20. $\lim\limits_{x\to\infty}\left(\frac{4x+1}{4x-1}\right)^{\frac{5x^2+1}{4x-1}}$;

21. $\lim\limits_{x\to\infty}(2x-9)[\ln(6x+1)-\ln 6x]$;

22. $\lim\limits_{x\to 0}\left(\frac{3-2x^2}{3+x^2}\right)^{\frac{2x+1}{x^2}}$;

23. $\lim\limits_{x\to\infty}(x+1)[\ln(2x+10)-\ln(2x-3)]$;

24. $\lim\limits_{x\to\infty}\left(\frac{4-2x^2}{1-2x^2}\right)^{2x^2+1}$;

25. $\lim\limits_{x\to\infty}(8x-1)[\ln(9x+2)-\ln 9x]$;

26. $\lim\limits_{x\to 0}\left(\frac{4x^2+1}{x^2+1}\right)^{\frac{x^2+1}{x^3}}$;

27. $\lim\limits_{x\to\infty}(6x-2)[\ln(2x-3)-\ln(2x+5)]$;

28. $\lim\limits_{x\to 0}\left(\frac{2x^2-4}{3x^2-4}\right)^{\frac{2x-1}{2x^2}}$;

29. $\lim\limits_{x\to\infty}(2x+8)[\ln(x+2)-\ln(x-5)]$;

30. $\lim\limits_{x\to 0}\left(\frac{4x^2-1}{x^2-1}\right)^{\frac{x+3}{4x^2}}$.

9 – topshiriq. Ajoyib limitlardan foydalanib limitlarni hisoblang.

1. $\lim\limits_{x\to 0}(1+\sin x)^{1/\operatorname{tg}2x}$;

2. $\lim\limits_{x\to 1}\left(1+\operatorname{ctg}\frac{\pi x}{2}\right)^{1/\sqrt{x^2-1}}$;

3. $\lim\limits_{x\to 0}(1+\operatorname{tg}\pi x)^{1/\arcsin x}$;

4. $\lim\limits_{x\to 1+0}\left(1+\sqrt{x+1}\right)^{1/\cos(\pi x/2)}$;

5. $\lim\limits_{x\to 1}\left(1+\cos\frac{\pi x}{2}\right)^{1/2\sin\pi x}$;

6. $\lim\limits_{x\to 0}(1-\arcsin x)^{1/\sin(\pi/2)}$;

7. $\lim\limits_{x\to 3+0}\left(1+\sqrt{x^2-9}\right)^{1/\operatorname{tg}\pi x}$;

8. $\lim\limits_{x\to 1-0}(1-\arccos x)^{1/x-1}$;

9. $\lim\limits_{x\to 0}(1+\sin 3x)^{\operatorname{ctg}2x}$;

10. $\lim\limits_{x\to -3-0}\left(1+\sqrt{x^2-9}\right)^{\operatorname{ctg}\pi x}$;

11. $\lim\limits_{x\to 1}(1+\sin \pi x)^{1/\arccos x}$;

12. $\lim\limits_{x\to 0}(1-\operatorname{arctg}x)^{1/\cos x-1}$;

13. $\lim\limits_{x\to 2}\left(1+\sqrt{x^2-4}\right)^{1/\sin\pi x}$;

14. $\lim\limits_{x\to\frac{\pi}{3}}(1+\sin 3x)^{\operatorname{ctg}3x}$

15. $\lim\limits_{x\to 0}\left(1+\operatorname{tg}\frac{x}{2}\right)^{1/\sin 2x}$;

16. $\lim\limits_{x\to 0}(1+\operatorname{tg}2x)^{1/\sin 3x}$;

17. $\lim\limits_{x\to 4+0}\left(1+\sqrt{x^2-16}\right)^{1/\operatorname{tg}\pi x}$;

18. $\lim\limits_{x\to\frac{\pi}{3}}(4+3\cos 3x)^{1/\operatorname{tg}3x}$;

19. $\lim\limits_{x\to\frac{\pi}{4}}(2-\operatorname{tg}x)^{1/\cos 2x}$;

20. $\lim\limits_{x\to 2+0}\left(1+\sqrt{x^2-4}\right)^{1/\sin(\pi x/2)}$;

21. $\lim\limits_{x\to 1}(1-\operatorname{tg}2\pi x)^{1/\sin\pi x}$;

22. $\lim\limits_{x\to\pi+0}\left(1+\sqrt{x-\pi}\right)^{\operatorname{ctg}3x}$;

23. $\lim\limits_{x\to 2+0}\left(1+\sqrt{x-2}\right)^{1/\cos\frac{\pi x}{4}}$;

24. $\lim\limits_{x\to\pi}(1-\sin 2x)^{\operatorname{ctg}x}$;

25. $\lim\limits_{x\to\frac{\pi}{4}}(2-\operatorname{ctg}x)^{1/\sin 4x}$;

26. $\lim\limits_{x\to\frac{\pi}{2}}(1+\operatorname{ctg}3x)^{1/\cos x}$;

27. $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos x)^{1/2x-\pi}$;

28. $\lim\limits_{x\to 1-0}\left(1-\cos\frac{\pi x}{2}\right)^{1/\arccos x}$;

29. $\lim\limits_{x\to 3+0}\left(1-\sqrt{x-3}\right)^{1/\sin\pi x}$;

30. $\lim\limits_{x\to\frac{\pi}{4}}(1+\cos 2x)^{1/4x-\pi}$.

3.8. Hosila. Hosilalar jadvali

$y=f(x)$ funksiyaning x_0 nuqtadagi orttirmasi Δy ning argument orttirmasi Δx ga nisbatining Δx nolga intilgandagi limiti mavjud bo'lsa, bu limit $y=f(x)$ funksiyaning x_0 nuqtadagi *hosilasi* deyiladi.

Hosilaning belgilanishi:

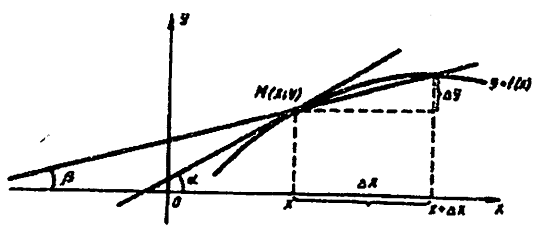
$$y' \quad yoki \quad f'(x_0) \quad yoki \quad \frac{dy}{dx} \quad yoki \quad \frac{df}{dx}.$$

Shunday qilib, ta'rifga ko'ra:

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}.$$

Agar $y=f(x)$ funksiya x_0 nuqtada hosilaga ega bo'lsa, u holda $f(x)$ funksiya x_0 nuqtada *differensiallanuvchi* deyiladi, hosilasi topish jarayoni *differensiallash* deyiladi.

Geometrik nuqtan nazardan $y=f(x)$ funksiyaning x_0 nuqtadagi hosilasi uning grafigiga $M(x_0, f(x_0))$ nuqtada o'tkazilgan urinmaning Ox o'qining musbat yo'nalishi bilan hosil qilingan burchagining tangensiga teng (1- shakl).



1- shakl

Yuqori tartibli hosilalar

$y=f(x)$ funksiyaning *ikkinchi tartibli* yoki *ikkinchi hosilasi* deb uning birinchi tartibli hosilasidan olingan hosilaga, ya'ni $(y')'$ ga aytiladi.

Ikkinchi tartibli hosila quyidagilarning biri bilan belgilanadi:

$$y'', \quad f''(x), \quad \frac{d^2y}{dx^2}.$$

$y=f(x)$ funksiyaning *n-tartibli* yoki *n-hosilasi* deb uning $(n-1)$ - tartibli hosilasidan olingan hosilaga aytiladi. n – tartibli hosila uchun ushbu belgilashlardan biri qo'llaniladi:

$$y^{(n)}, \quad f^{(n)}(x), \quad \frac{d^{(n)}y}{dx^{(n)}}.$$

Belgilashga ko'ra

$$y^{(n)}=(y^{(n-1)})'.$$

1-misol. $y=\ln x$ funksiyaning n – tartibli hosilasini toping.

Yechish. n marta ketma-ket differensiallab, quyidagiga ega bo'lamiz:

$$y'=\frac{1}{x}, \quad y''=-\frac{1}{x^2}, \quad y'''=\frac{2}{x^3}, \quad y^{IV}=-\frac{2\cdot 3}{x^4}, \ldots,$$

$$y^{(n)}=\frac{(-1)^{n+1}}{x^n}(n-1)!$$

x o'zgaruvchining y funksiyasi oshkormas shaklda $F(x,y)=0$ tenglama bilan berilgan bo'lsa, u holda y' hosilani topish uchun $F(x,y)=0$ tenglikning ikkala qismini x bo'yicha differensiallab, so'ngra hosil bo'lgan y' ga nisbatan chiziqli tenglamadan hosilani topish kerak. Ikkinchi va undan yuqoriroq tartibli hosilalar ham shu kabi topiladi.

Berilgan misolning birinchi tartibli hosilasini maple orqali topamiz:

> *diff* (ln(x), x);

$$\frac{1}{x}$$

2-misol. Oshkormas holda

$$x^2+y^2=64$$

tenglama bilan berilgan y funksiyaning y' va y'' hosilalarini toping.

Yechish. y o'zgaruvchi x ning funksiyasi deb hisoblab, berilgan tenglamaning ikkala qismini x bo'yicha differensiallaymiz:

$$2x+2y\cdot y'=0.$$

Bundan $y'=-\dfrac{x}{y}$ topilgan birinchi y' hosilani yana x bo'yicha differensiallaymiz:

$$y''=(y')'=-\frac{y-xy'}{y^2}.$$

Endi ekanini hisobga olib,

$$y''=-\frac{y-x\left(-\dfrac{x}{y}\right)}{y^2}$$

ni hosil qilamiz.

Shunday qilib, $y''=-\dfrac{y^2+x^2}{y^3}$ yoki $y''=-\dfrac{64}{y^3}$, chunki shartga ko'ra $x^2+y^2=64$.

Agar y funksiyaning x argumentga bog'liqligi

$$\begin{cases} x=x(t), \\ y=y(t) \end{cases}$$

tenglamalar bilan parametrik shaklda berilgan bo'lsa, u holda

$$y_x'=\frac{y'_t}{x'_t},\quad y''_{x^2}=\left(\frac{y'_t}{x'_t}\right)'_t\cdot\frac{1}{x'_t}=\frac{y''_{tt}x'_t-x''_{tt}y'_t}{(x'_t)^3}.$$

3- misol. Ushbu

$$\begin{cases} x=8\cos t, \\ y=8\sin t \end{cases}$$

parametrik tenglamalar bilan berilgan funksiyaning birinchi va ikkinchi tartibli hosilalarini toping.

Yechish. Yuqorida keltirilgan formuladan foydalanib, quyidagilarni oson topamiz:

$$x'_t=-8\sin t,\quad y'_t=8\cos t;$$

$$y_x'=\frac{y_t'}{x_t'}=\frac{8\cos t}{-8\sin t}=-ctgt;$$

$$y''_{x^2}=\left(\frac{y_t'}{x_t'}\right)'_t\cdot\frac{1}{x_t'}=(-ctgt)'_t\cdot\frac{1}{-8\sin t}=\frac{1}{-8\sin^3 t}.$$

Berilgan misolni maple orqali yechamiz:

> $x := 8\cdot\cos(t);$

$$x := 8\cos(t)$$

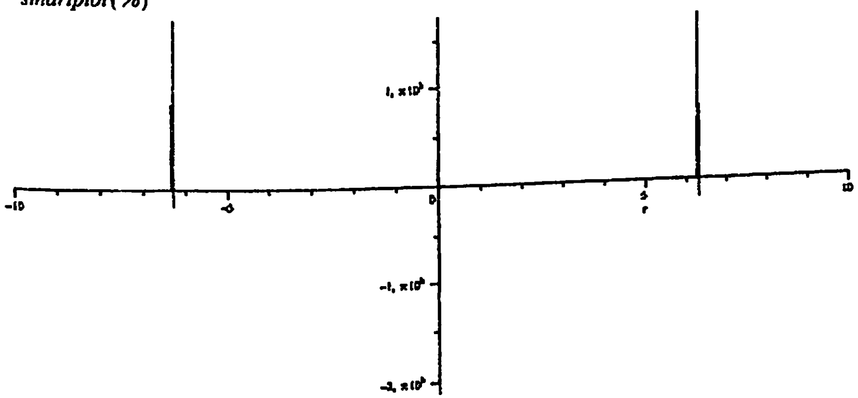
> $y := 8\cdot\sin(t);$

$$y := 8\sin(t)$$

> $\dfrac{diff\left(\dfrac{diff(y,t)}{diff(x,t)},t\right)}{diff(x,t)};$

$$-\frac{1}{8}\,\frac{1+\dfrac{\cos(t)^2}{\sin(t)^2}}{\sin(t)}$$

> $smartplot(\%)$



3.9. Funksiyaning differensiali

$y=f(x)$ funksiyaning differensiali deb, uning orttirmasining erkli o'zgaruvchi x ning orttirmasiga nisbatan chiziqli bo'lgan bosh qismiga aytiladi.

$y=f(x)$ funksiyaning differensiali dy bilan belgilanadi. Funksiyaning differensiali uning hosilasi bilan erkli o'zgaruvchi orttirmasining ko'paytmasiga teng:

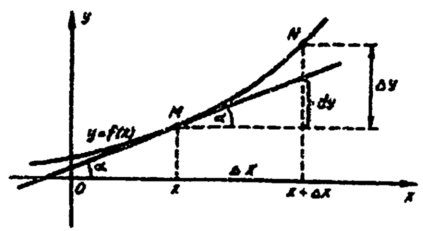
$$dy = f'(x)\Delta x \quad \text{yoki} \quad dy = y'\Delta x.$$

Ravshanki, $dx = \Delta x$. Shu sababli

$$dy = f'(x)dx \quad \text{yoki} \quad dy = y'dx.$$

Differensial geometrik jixatdan $y = f(x)$ funksiya grafigiga $M(x,y)$ nuqtada o'tkazilgan urinma ordinatasining orttirmasiga teng (2-shakl).

Funksiyaning differensiali dy uning Δy orttirmasidan Δx ga nisbatan yuqori tartibli cheksiz kichik miqdorga farq qiladi.



2- shakl

Agar $u = u(x)$ va $v = v(x)$ funksiyalar differensiallanuvchi bo'lsa, u holda differensialning ta'rifi va differensiallash qoidalaridan bevosita differensialning asosiy xossalariga ega bo'lamiz:

1. $d(C) = 0$, bunda C - o'zgarmas.
2. $d(Cu) = Cdu$.
3. $d(u \pm v) = du \pm dv$.
4. $d(u \cdot v) = du \cdot dv$.
5. $d(\frac{u}{v}) = \frac{vdu - udv}{v^2}$, bunda $v \neq 0$.
6. $df(u) = f'(u) \cdot u'dx = f'(u)du$.

1-Misol. $y = tg^4 2x$ funksiya differensialini toping.

Yechish. Oldin berilgan funksiyaning hosilasini topamiz:

$$y = 8tg^3 2x \frac{1}{\cos^2 2x} = 8tg^3 2x \sec^2 2x.$$

U holda

$$dy = 8\operatorname{tg}^3 2x\sec^2 2xdx.$$

Berilgan misolni maple orqali yechamiz:

$> diff(\tan^4(2\cdot x), x);$

$$4\tan(2x)^3(2 + 2\tan(2x)^2)$$

$y = f(x)$ funksiyaning ikkinchi tartibli differensiali deb birinchi tartibli differensialdan olingan differensialga aytiladi va

$$d^2y = d(dy)$$

kabi belgilanadi.

$y = f(x)$ funksiyaning n- tartibli differensiali deb $(n-1)$-tartibli differensialdan olingan differensialga aytiladi, ya'ni:

$$d^n y = d(d^{n-1}y).$$

$y = f(x)$ funksiya berilgan bo'lib, bunda x — erkli o'zgaruvchi bo'lsa, u holda uning yuqori tartibli differensiallari ushbu formulalar bo'yicha hisoblanadi:

$$d^2y = y''dx^2, \quad d^3y = y'''dx^3, ..., d^n y = y^{(n)}dx^n.$$

2-misol. $y = x(\ln x - 1)$ funksiyaning ikkinchi tartibli differensialini toping.

Yechish. Berilgan funksiyaning birinchi va ikkinchi tartibli hosilalarini topamiz:

$$y' = \ln x - 1 + x \cdot \frac{1}{x} = \ln x, \quad y'' = \frac{1}{x}.$$

Demak, $dy = \ln x dx$, $d^2y = \frac{1}{x}dx^2$.

Funksiyaning dy differensiali uning Δy orttirmasidan $\Delta x = dx$ ga nisbatan yuqori tartibli cheksiz kichik miqdorga farq qiladi, shu sababli $\Delta y \approx dy$ yoki

$$f(x + \Delta x) - f(x) \approx f'(x)\Delta x,$$

bundan

$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x$$

formulaga ega bo'lamiz, bu formula funksiya qiymatlarini taqribiy hisoblashlarda qo'llaniladi.

3-misol. arcsin 0,15 ning taqribiy qiymatini hisoblang.

Yechish. $y = \arcsin x$ funksiyani qaraymiz: $x = 0{,}15$, $\Delta x = 0{,}01$ deb olib va $\arcsin(x+\Delta x) \approx \arcsin x + (\arcsin x)'\Delta x$ formuladan foydalanib topamiz:

$$\arcsin 0.51 \approx \arcsin 0.5 + \frac{1}{\sqrt{1-(0.5)^2}} \cdot 0.01 =$$

$$= \frac{\pi}{6} + \cdot 0.011 \approx 0.534.$$

Shunday qilib, $\arcsin 0{,}15 \approx 0{,}534$ radian.

Roll, Lagranj, Koshi teoremalari. Lopital qoidasi

Roll teoremasi. Agar $y = f(x)$ funksiya $[a,b]$ kesmada uzluksiz, (a,b) oraliqda differensiallanuvchi va $f(a) = f(b)$ bo'lsa, u holda aqalli bitta shunday $x = c$ $(a < c < b)$ nuqta mavjudki, unda $f'(c) = 0$ bo'ladi.

Bu teorema hosilaning nollari yoki ildizlari haqidagi teorema ham deyiladi.

Lagranj teoremasi. Agar $y = f(x)$ funksiya $[a,b]$ kesmada uzluksiz, (a,b) oraliqda differensiallanuvchi bo'lsa, u holda aqalli bitta shunday $x = c$ $(a < c < b)$ nuqta mavjudki,

$$f(b) - f(a) = f'(c)(b-a)$$

bo'ladi.

Bu teorema chekli ayirmalar haqidagi teorema ham deyiladi.

Koshi teoremasi. Agar $y = f(x)$ va $y = \varphi(x)$ funksiyalar $[a,b]$ kesmada uzluksiz, (a,b) oraliqda differensiallanuvchi, shu bilan birga bu oraliqda $\varphi'(x) \neq 0$ bo'lsa, u holda aqalli bitta shunday $x = c$ $(a < c < b)$ nuqta mavjudki,

$$\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)} = \frac{f'(c)}{\varphi'(c)}$$

bo'ladi, bunda $\varphi(b) \neq \varphi(a)$.

1-misol. [1,5] kesmada $f(x) = x^2 - 6x + 100$ funksiya uchun Roll teoremasi o'rinlimi? x ning qanday qiymatida $f'(x) = 0$ bo'ladi?

Yechish. $f(x)$ funksiya x ning barcha qiymatlarida uzluksiz, differensiallanuvchi va uning [1; 5] kesma oxirlaridagi qiymatlari teng: $f(1)=f(5)=95$ bo'lgani uchun Roll teoremasi shartlari bajariladi. x ning $f'(x) = 0$ bo'ladigan qiymati $f'(x) = 2x - 6 = 0$ tenglamadan aniqlanadi, ya'ni $x = 3$.

2-misol. $f(x) = 2x - x^2$ egri chiziqning AB yoyida shunday M nuqtani topingki, bu nuqtada egri chiziqqa o'tkazilgan o'rinma AB vatarga parallel bo'lsin, bunda $A(1,1)$ va $B(3,-3)$.

Yechish. $f(x) = 2x - x^2$ funksiya x ning barcha qiymatlarida uzluksiz va differensiallanuvchi. Izlanayotgan M nuqtada o'tkazilgan o'rinmaning burchak koeffisenti shartga ko'ra $\frac{f(b)-f(a)}{b-a}$ ga teng, ikkinchi tomondan, Lagranj teoremasiga ko'ra ikkita $a=1$ va $b=3$ qiymat orasida

$$f(b) - f(a) = f'(c)(b-a)$$

tenglikni qanoatlantiruvchi $x = c$ qiymat mavjud, bunda $f'(x) = 2 - 2x$.

Tegishli qiymatlarni qo'ysak,

$$f(3) - f(1) = f'(c)(3-1)$$

yoki

$$(2\cdot 3 - 3^2) - (2\cdot 1 - 1^2) = (3-1)(2-2c).$$

Bu oxirgi tenglamani c ga nisbatan yechsak, $c = 2$, $f(2) = 0$.

Shunday qilib, M nuqta (2, 0) koordinatalarga ega.

3-misol. $f(x) = \sqrt[3]{(x-8)^2}$ funksiya uchun [0; 10] kesmada Lagranj teoremasi o'rinlimi?

Yechish. $f(x)$ funksiya x ning barcha qiymatlarida uzluksiz, ammo uning $f'(x)=\frac{2}{3\sqrt[3]{(x-8)}}$ hosilasi (0; 10) oraliqning $x=8$ nuqtasida mavjud emas, shunga ko'ra Lagranj teoremasi o'rinli emas.

Aniqmasliklarni ochishning Lopital qoidsi ($\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarni ochish). $f(x)$ va $\varphi(x)$ funksiyalar x_0 nuqtaning biror atrofida (x_0 nuqtaning o'zidan tashqari) differensiallanuvchi va $\varphi'(x)\neq 0$ bo'lsin. Agar $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0}\varphi(x)=0$ yoki $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0}\varphi(x)=\infty$ bo'lib, $\lim\limits_{x\to x_0}\frac{f(x)}{\varphi(x)}$ mavjud bo'lsa, u holda $\lim\limits_{x\to x_0}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to x_0}\frac{f'(x)}{\varphi'(x)}$ bo'ladi. $x\to\infty$ da ham Lopital qoidasi o'rinli.

$0\cdot\infty$ yoki $\infty-\infty$ shaklidagi aniqmasliklar algebraik almashtirishlar orqali $\frac{0}{0}$ yeki $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltirilib, so'ngra Lopital qoidasidan foydalaniladi.

0^0, ∞^0 yoki 1^∞ ko'rinishdagi aniqmasliklar logarifmlash orqali $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltiriladi.

4-misol. $\lim\limits_{x\to 0}\frac{x-\sin x}{x^3}$ ni toping.

Yechish. Ifodaning surati va maxraji $x\to 0$ da nolga intiladi, shu sababli $\frac{0}{0}$ shakldagi aniqmaslikka egamiz. Lopital qoidasidan foydalansak.

$$\lim_{x\to 0}\frac{x-\sin x}{x^3}=\lim_{x\to 0}\frac{1-\cos x}{3x^2}=\lim_{x\to 0}\frac{\sin x}{6x}=\frac{1}{6};$$

Bu yerda Lopital qoidasi ikki marta qo'llanildi.

Berilgan misolni maple orqali yechamiz:

$> limit\left(\frac{(x-\sin(x))}{x^3}, x=0\right);$

$$\frac{1}{6}$$

5-misol. $\lim\limits_{x\to 0} x^2\ln x$ ni toping.

Yechish. $0\cdot\infty$ shaklidagi aniqmaslikka egamiz, $x^2\ln x$ ko'paytmani $\frac{\ln x}{\frac{1}{x^2}}$ bo'linma shaklida ifodalasak, natijada $\frac{\infty}{\infty}$ shaklidagi aniqmaslikka ega bo'lamiz. Lopital qoidasini qo'llaymiz:

$$\lim_{x\to 0} x^2\ln x=\lim_{x\to 0}\frac{\ln x}{\frac{1}{x^2}}=\lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{2}{x^3}}=-\frac{1}{2}\lim_{x\to 0}x^2=0.$$

Berilgan misolni maple orqali yechamiz:

$> limit(x\cdot x\cdot\ln(x), x=0);$

$$0$$

6-misol. $\lim\limits_{x\to 0}(\sin x)^x$ ni toping.

Yechish. 0^0 shaklidagi aniqmaslikka egamiz. Berilgan funksiyani $y=(\sin x)^x$ bilan belgilab, uni logarifmlaymiz:

$$\ln y = x\ln\sin x=\frac{\ln\sin x}{\frac{1}{x}}.$$

$$\lim_{x\to 0}\ln y=\lim_{x\to 0}\frac{\ln\sin x}{\frac{1}{x}}=\lim_{x\to 0}\frac{\frac{\cos x}{\sin x}}{-\frac{1}{x^2}}=$$

$$=-\lim_{x\to 0}\frac{x^2\cos x}{\sin x}=-\lim_{x\to 0}\left(x\cdot\frac{x}{\sin x}\cdot\cos x\right)=0.$$

Lopital qoidasini qo'llaymiz: Shunday qilib, $\lim\limits_{x\to 0}y=e^0=1$.

3.10. Mustaqil ishlash uchun topshiriqlar.

1 – topshiriq. Funksiyaning birinchi tartibli hosilasini toping.

1. $y = \text{arcctg}^5\left(\frac{1-2x}{3+x^3}\right) - \log_3^4\left(\arcsin\frac{1}{x}\right)4^{\sqrt[6]{1-2x^2}}$;

2. $y = 2^{\text{arctg}\left(\frac{x}{\cos x}\right)} + \text{ctg}\sqrt[4]{(1-x^2)^3}\sin\frac{4}{\sqrt{x}}$;

3. $y = \frac{3^{\text{ctg}\sqrt{1-x}} - \cos(1-x^2)}{\log_4 \text{ch}\frac{x}{2}} + \arcsin^2\left(e^{2x}\sqrt[3]{x^2}\right)$;

4. $y = e^{\arccos^3\left(\frac{x^2}{1-2x}\right)} + \text{ctg}\left(4^{x^3}\cdot\sqrt{x+\sqrt{x}}\right) + \sin^3\cos 2$;

5. $y = \arcsin^2\left(x\cdot 5^{\sqrt{2-2x}}\right) + \text{ctg}\left(\sqrt[3]{\frac{x-1}{x+1}}\right)$;

6. $y = 5^{\cos^2 3x}\left(x^3+4\right)^3 + \frac{\sqrt{x^4+2x}}{x} + \arccos\frac{3}{2x-1}$;

7. $y = \ln^2\left(e^{3x} + \sqrt{e^{6x}-1}\right) + \arcsin^3\left(\frac{\sqrt{x}-2}{\sqrt{5x}}\right)$;

8. $y = \arcsin\left(\frac{x\sqrt{2}}{1+x}\right)\sqrt{1+2x-x^2} + \log_3^3 \text{ctg}\frac{1}{x\sqrt{x}}$;

9. $y = \ln\frac{1+\sqrt{2x-x^2}}{x-1} + \text{arctg}\left(\frac{1}{x}\right)e^{\sin^3 x}$;

10. $y = \frac{\text{ctg}5x + x}{1 - x\,\text{ctg}5x} + 3^{\arcsin^3\frac{1}{2x+3}} + \cos\left(\sqrt[3]{\text{ctg}2}\right)$;

11. $y = \text{arctg}^3\frac{\cos x}{\sqrt[4]{\cos 2x}} + e^{x^3\cdot\sqrt[4]{1-x^2}} + \ln\cos\frac{1}{3}$;

12. $y = \arcsin^2\frac{3+\text{ch}x}{1+3\text{ch}x} + 5^{x^4\cdot\text{ctg}\frac{1}{x^3}}$;

13. $y = \left(\text{arcctg}\frac{\sqrt{1-x}}{1-\sqrt{x}}\right)^3 + \ln\left(x+\sqrt{1+x^2}\right)$;

14. $y = 3^{\arccos^3\frac{x}{1-2x}} + \left(3x^3+1\right)\cos\frac{3}{\sqrt[3]{x^2}}$;

15. $y = \arcsin^4\sqrt{\frac{x}{x+1}} + 10^{x\cdot\text{tg}x^2}$;

16. $y = \text{ctg}^3\left(x\cdot 2^{\sqrt{1-x}}\right) + \ln\sqrt[3]{\frac{x^2+1}{x^2-1}}$;

17. $y = \left(\text{arctg}\sqrt{x^2-1}\right)^5 + 2^{\frac{x}{\ln x}} + \cos\ln 2$;

18. $y = \sqrt{\cos x}\cdot 3^{\sqrt{\cos x}} + \log_3^2\sin\frac{e^3+e^{-3x}}{2}$;

19. $y = \frac{1-e^{2x}}{1+e^{2x}}\text{arctg}\,e^{-x} + \ln^3\left(\cos^2 x + \sqrt{1+\cos^2 x}\right)$;

20. $y = \log_3^3 \text{arcctg}\sqrt{1+x^2} + \sqrt{x}\,e^{\frac{x}{2}} + \ln\sin\frac{1}{2}$;

21. $y = \arcsin\left(\frac{1}{2x-1}\right)(2x-1)^4 + e^{\text{ctg}^2 3x^3} + \sqrt{\text{ctg}4}$;

22. $y = \left(1+4x^2\right)e^{\text{arctg}2x} + \sqrt[4]{\left(1+\sin^2 2x\right)^3}$;

23. $y = \text{arctg}\frac{\sqrt{2\text{tg}x}}{1-\text{tg}x} + \sqrt[5]{\log_5^7\left(x^4-2x\right)}$;

24. $y = \ln\frac{\sin x}{\cos x + \sqrt{\cos 2x}} + \sqrt{\text{arctg}\frac{2}{x} + \text{ctg}\sqrt[3]{5}}$;

25. $y = \arccos^3\frac{x^2-4}{\sqrt{x^4+16}} + \ln\left(2^{3-x^3} + \sqrt{6x}\right)$;

26. $y = \log_3^2\left(\frac{\ln x}{\sin\frac{1}{x}}\right) + \frac{2}{3}\sqrt{\left(\text{arctg}\,e^{2x}\right)^3}$;

27. $y=\ln^3\cos\frac{2x+4}{5x+1}+4^{\operatorname{arctg}\sqrt{x}}+\cos\sqrt[3]{\operatorname{tg}2}$;

28. $y=\operatorname{arctg}\left(\frac{5x}{3}-x^3\sin\frac{1}{x^2}\right)+\log_3^3(x+\cos3x)$;

29. $y=\operatorname{ctg}^3\left(2^{x^2}\cdot\cos\frac{1}{3x}\right)+\ln\arcsin\sqrt{1-e^{2x}}$;

30. $y=\sqrt{1+\ln\left(1+x^2\cos\frac{1}{x}\right)^2}+\left(\arccos\sqrt{1-e^{4x}}\right)^3$.

2 – topshiriq. y' hosilani toping.

1. $y=x-\ln\left(2+e^x+2\sqrt{e^{2x}+e^x+1}\right)$;
2. $y=e^{2x}(2-\sin2x-\cos2x)/8$;
3. $y=\frac{1}{2}\operatorname{arctg}\frac{e^x-3}{2}$;
4. $y=\frac{1}{\ln4}\ln\frac{1+2^x}{1-2^x}$;
5. $y=2\sqrt{e^x+1}+\ln\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$;
6. $y=\frac{2}{3}\sqrt{\left(\operatorname{arctg}e^x\right)^3}$;
7. $y=\frac{1}{2}\ln\left(e^{2x}+1\right)-2\operatorname{arctg}e^x$;
8. $y=\ln\left(e^x+1\right)+\frac{18e^{2x}+27e^x+11}{6\left(e^x+1\right)^3}$;
9. $y=2\left(\sqrt{2^x-1}-\operatorname{arctg}\sqrt{2^x-1}\right)/\ln2$;
10. $y=2(x-2)\sqrt{1+e^x}-2\ln\left(\left(\sqrt{1+e^x}-1\right)/\left(\sqrt{1+e^x}+1\right)\right)$;
11. $y=e^{\alpha x}(\alpha\sin\beta x-\beta\cos\beta x)/\left(\alpha^2+\beta^2\right)$;
12. $y=e^{\alpha x}(\beta\sin\beta x+\alpha\cos\beta x)/\left(\alpha^2+\beta^2\right)$;

13. $y=e^{ax}\left[\frac{1}{2a}+\frac{a\cos2bx+2b\sin2bx}{2\left(a^2+4b^2\right)}\right]$;
14. $y=x+1/\left(1+e^x\right)-\ln\left(1+e^x\right)$;
15. $y=x-3\ln\left[\left(1+e^{x/6}\right)\sqrt{1+e^{x/3}}\right]-3\operatorname{arctg}e^{x/6}$;
16. $y=x+\frac{8}{1+e^{x/4}}$;
17. $y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}$;
18. $y=x-e^{-x}\arcsin e^x-\ln\left(1+\sqrt{1-e^{2x}}\right)$;
19. $y=x-\ln\left(1+e^x\right)-2e^{-x/2}\operatorname{arctg}e^{x/2}-\left(\operatorname{arctg}e^{x/2}\right)^2$;
20. $y=\frac{e^{x^3}}{1+x^3}$;
21. $y=\frac{1}{m\sqrt{ab}}\operatorname{arctg}\left(e^{mx}\sqrt{\frac{a}{b}}\right)$;
22. $y=3e^{\sqrt[3]{x}}\left(\sqrt[3]{x^2}-2\sqrt[3]{x}+2\right)$;
23. $y=\ln\frac{\sqrt{1+e^x+e^{2x}}-e^x-1}{\sqrt{1+e^x+e^{2x}}-e^x+1}$;
24. $y=e^{\sin x}\left(x-\frac{1}{\cos x}\right)$;
25. $y=\frac{e^x}{2}\left[\left(x^2-1\right)\cos x+(x-1)^2\sin x\right]$;
26. $y=\operatorname{arctg}\left(e^x-e^{-x}\right)$;
27. $y=3e^{\sqrt[3]{x}}\left[\sqrt[3]{x^5}-5\sqrt[3]{x^4}+20x-60\sqrt[3]{x^2}+120\sqrt[3]{x}-120\right]$;
28. $y=-e^{3x}/\left(3\operatorname{sh}^3x\right)$;
29. $y=\arcsin e^x-\sqrt{1-e^{2x}}$;
30. $y=-\frac{1}{2}e^{-x^2}\left(x^4+2x^2+2\right)$.

27. $y=\ln^3\cos\dfrac{2x+4}{5x+1}+4^{\operatorname{arctg}\sqrt{x}}+\cos\sqrt[3]{\operatorname{tg}2}$;

28. $y=\operatorname{arctg}\left(\dfrac{5x}{3}-x^3\sin\dfrac{1}{x^2}\right)+\log_3^3(x+\cos 3x)$;

29. $y=\operatorname{ctg}^3\left(2^{x^2}\cdot\cos\dfrac{1}{3x}\right)+\ln\arcsin\sqrt{1-e^{2x}}$;

30. $y=\sqrt{1+\ln\left(1+x^2\cos\dfrac{1}{x}\right)^2}+\left(\arccos\sqrt{1-e^{4x}}\right)^3$.

2 – topshiriq. y' hosilani toping.

1. $y=x-\ln\left(2+e^x+2\sqrt{e^{2x}+e^x+1}\right)$;
2. $y=e^{2x}(2-\sin 2x-\cos 2x)/8$;
3. $y=\dfrac{1}{2}\operatorname{arctg}\dfrac{e^x-3}{2}$;
4. $y=\dfrac{1}{\ln 4}\ln\dfrac{1+2^x}{1-2^x}$;
5. $y=2\sqrt{e^x+1}+\ln\dfrac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$;
6. $y=\dfrac{2}{3}\sqrt{\left(\operatorname{arctg}e^x\right)^3}$;
7. $y=\dfrac{1}{2}\ln\left(e^{2x}+1\right)-2\operatorname{arctg}e^x$;
8. $y=\ln\left(e^x+1\right)+\dfrac{18e^{2x}+27e^x+11}{6\left(e^x+1\right)^3}$;
9. $y=2\left(\sqrt{2^x-1}-\operatorname{arctg}\sqrt{2^x-1}\right)/\ln 2$;
10. $y=2(x-2)\sqrt{1+e^x}-2\ln\left(\left(\sqrt{1+e^x}-1\right)/\left(\sqrt{1+e^x}+1\right)\right)$;
11. $y=e^{\alpha x}(\alpha\sin\beta x-\beta\cos\beta x)/\left(\alpha^2+\beta^2\right)$;
12. $y=e^{\alpha x}(\beta\sin\beta x+\alpha\cos\beta x)/\left(\alpha^2+\beta^2\right)$;

13. $y=e^{ax}\left[\dfrac{1}{2a}+\dfrac{a\cos 2bx+2b\sin 2bx}{2\left(a^2+4b^2\right)}\right]$;
14. $y=x+1/\left(1+e^x\right)-\ln\left(1+e^x\right)$;
15. $y=x-3\ln\left[\left(1+e^{x/6}\right)\sqrt{1+e^{x/3}}\right]-3\operatorname{arctg}e^{x/6}$;
16. $y=x+\dfrac{8}{1+e^{x/4}}$;
17. $y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}$;
18. $y=x-e^{-x}\arcsin e^x-\ln\left(1+\sqrt{1-e^{2x}}\right)$;
19. $y=x-\ln\left(1+e^x\right)-2e^{-x/2}\operatorname{arctg}e^{x/2}-\left(\operatorname{arctg}e^{x/2}\right)^2$;
20. $y=\dfrac{e^{x^3}}{1+x^3}$;
21. $y=\dfrac{1}{m\sqrt{ab}}\operatorname{arctg}\left(e^{mx}\sqrt{\dfrac{a}{b}}\right)$;
22. $y=3e^{\sqrt[3]{x}}\left(\sqrt[3]{x^2}-2\sqrt[3]{x}+2\right)$;
23. $y=\ln\dfrac{\sqrt{1+e^x+e^{2x}}-e^x-1}{\sqrt{1+e^x+e^{2x}}-e^x+1}$;
24. $y=e^{\sin x}\left(x-\dfrac{1}{\cos x}\right)$;
25. $y=\dfrac{e^x}{2}\left[\left(x^2-1\right)\cos x+(x-1)^2\sin x\right]$;
26. $y=\operatorname{arctg}\left(e^x-e^{-x}\right)$;
27. $y=3e^{\sqrt[3]{x}}\left[\sqrt[3]{x^5}-5\sqrt[3]{x^4}+20x-60\sqrt[3]{x^2}+120\sqrt[3]{x}-120\right]$;
28. $y=-e^{3x}/\left(3\operatorname{sh}^3 x\right)$;
29. $y=\arcsin e^x-\sqrt{1-e^{2x}}$;
30. $y=-\dfrac{1}{2}e^{-x^2}\left(x^4+2x^2+2\right)$.

3- topshiriq. y' hosilani toping.

1. $y=\frac{1}{3}\frac{\sin^2 3x}{\cos 6x}$;
2. $y=\frac{\sin^2 15x}{15\cos 30x}$;
3. $y=-\frac{1}{3}\frac{\cos^2 3x}{\sin 6x}$;
4. $y=\frac{\cos^2 16x}{32\sin 32x}$;
5. $y=\frac{1}{4}\frac{\sin^2 4x}{\cos 8x}$;
6. $y=\frac{\sin^2 17x}{17\cos 34x}$;
7. $y=-\frac{1}{8}\frac{\cos^2 4x}{\sin 8x}$;
8. $y=\frac{\cos^2 18x}{36\sin 36x}$;
9. $y=\frac{\sin^2 2x}{2\cos 4x}$;
10. $y=\frac{\sin^2 19x}{19\cos 38x}$;
11. $y=\frac{\cos^2 2x}{4\sin 4x}$;
12. $y=-\frac{1}{40}\frac{\cos^2 20x}{\sin 40x}$;
13. $y=\frac{\sin^2 7x}{7\cos 14x}$;
14. $y=\frac{\sin^2 21x}{21\cos 42x}$;
15. $y=-\frac{1}{16}\frac{\cos^2 8x}{\sin 16x}$;
16. $y=-\frac{1}{44}\frac{\cos^2 22x}{\sin 44x}$;
17. $y=\frac{1}{6}\frac{\sin^2 6x}{\cos 12x}$;
18. $y=\frac{\sin^2 23x}{23\cos 46x}$;
19. $y=-\frac{1}{20}\frac{\cos^2 10x}{\sin 20x}$;
20. $y=-\frac{1}{48}\frac{\cos^2 24x}{\sin 48x}$;
21. $y=\frac{1}{10}\frac{\sin^2 10x}{\cos 20x}$;
22. $y=\frac{\sin^2 25x}{25\cos 50x}$;
23. $y=-\frac{1}{24}\frac{\cos^2 12x}{\sin 24x}$;
24. $y=-\frac{1}{52}\frac{\cos^2 26x}{\sin 52x}$;
25. $y=\frac{1}{5}\frac{\sin^2 5x}{\cos 10x}$;
26. $y=\frac{\sin^2 27x}{27\cos 54x}$;
27. $y=\frac{\cos^2 14x}{28\sin 28x}$;
28. $y=\frac{\cos^2 28x}{56\sin 56x}$;
29. $y=\frac{\sin^2 29x}{29\cos 58x}$;
30. $y=\frac{\cos^2 30x}{60\sin 60x}$.

4 - topshiriq. y' hosilani toping

1. $y=x\arcsin(1/x)+\ln\left|x+\sqrt{x^2-1}\right|,\ x>0$;
2. $y=\mathrm{tg}\left(2\arccos\sqrt{1-2x^2}\right),\ x>0$;
3. $y=\sqrt{1+2x}-\ln\left(x+\sqrt{1+2x}\right)$;
4. $y=x^2\mathrm{arctg}\sqrt{x^2-1}-\sqrt{x^2-1}$;
5. $y=\arccos\left(1/\sqrt{1+2x^2}\right),x>0$;
6. $y=x\ln\left|x+\sqrt{x^2+3}\right|-\sqrt{x^2+3}$;
7. $y=\mathrm{arctg}(\mathrm{sh}\,x)+(\mathrm{sh}\,x)\ln\mathrm{ch}\,x$;
8. $y=\arccos\left(\left(x^2-1\right)/\left(x^2\sqrt{2}\right)\right)$;
9. $y=\ln\left(\cos^2 x+\sqrt{1+\cos^4 x}\right)$;
10. $y=\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}\,\mathrm{arctg}\,x$;
11. $y=\frac{\ln|x|}{1+x^2}-\frac{1}{2}\ln\frac{x^2}{1+x^2}$;
12. $y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}$;
13. $y=x\sqrt{4-x^2}+4\arcsin(x/2)$;
14. $y=\ln\mathrm{tg}(x/2)-x/\sin x$;
15. $y=2x+\ln|\sin x+2\cos x|$;
16. $y=\sqrt{\mathrm{ctg}\,x}-\sqrt{\mathrm{tg}^3 x}/3$;
17. $y=\ln\left|\frac{x+\sqrt{x^2+1}}{2x}\right|$;
18. $y=\sqrt[3]{\frac{x+2}{x-2}}$;
19. $y=\mathrm{arctg}\frac{x^2-1}{x}$;

3- topshiriq. y' hosilani toping.

1. $y=\frac{1}{3}\frac{\sin^2 3x}{\cos 6x}$;
2. $y=\frac{\sin^2 15x}{15\cos 30x}$;
3. $y=-\frac{1}{3}\frac{\cos^2 3x}{\sin 6x}$;
4. $y=\frac{\cos^2 16x}{32\sin 32x}$;
5. $y=\frac{1}{4}\frac{\sin^2 4x}{\cos 8x}$;
6. $y=\frac{\sin^2 17x}{17\cos 34x}$;
7. $y=-\frac{1}{8}\frac{\cos^2 4x}{\sin 8x}$;
8. $y=\frac{\cos^2 18x}{36\sin 36x}$;
9. $y=\frac{\sin^2 2x}{2\cos 4x}$;
10. $y=\frac{\sin^2 19x}{19\cos 38x}$;
11. $y=\frac{\cos^2 2x}{4\sin 4x}$;
12. $y=-\frac{1}{40}\frac{\cos^2 20x}{\sin 40x}$;
13. $y=\frac{\sin^2 7x}{7\cos 14x}$;
14. $y=\frac{\sin^2 21x}{21\cos 42x}$;
15. $y=-\frac{1}{16}\frac{\cos^2 8x}{\sin 16x}$;
16. $y=-\frac{1}{44}\frac{\cos^2 22x}{\sin 44x}$;
17. $y=\frac{1}{6}\frac{\sin^2 6x}{\cos 12x}$;
18. $y=\frac{\sin^2 23x}{23\cos 46x}$;
19. $y=-\frac{1}{20}\frac{\cos^2 10x}{\sin 20x}$;
20. $y=-\frac{1}{48}\frac{\cos^2 24x}{\sin 48x}$;
21. $y=\frac{1}{10}\frac{\sin^2 10x}{\cos 20x}$;
22. $y=\frac{\sin^2 25x}{25\cos 50x}$;
23. $y=-\frac{1}{24}\frac{\cos^2 12x}{\sin 24x}$;
24. $y=-\frac{1}{52}\frac{\cos^2 26x}{\sin 52x}$;
25. $y=\frac{1}{5}\frac{\sin^2 5x}{\cos 10x}$;
26. $y=\frac{\sin^2 27x}{27\cos 54x}$;
27. $y=\frac{\cos^2 14x}{28\sin 28x}$;
28. $y=\frac{\cos^2 28x}{56\sin 56x}$;
29. $y=\frac{\sin^2 29x}{29\cos 58x}$;
30. $y=\frac{\cos^2 30x}{60\sin 60x}$.

4 - topshiriq. y' hosilani toping

1. $y = x\arcsin(1/x)+\ln\left|x+\sqrt{x^2-1}\right|,\ x>0$;
2. $y=\mathrm{tg}\left(2\arccos\sqrt{1-2x^2}\right),\ x>0$;
3. $y=\sqrt{1+2x}-\ln\left(x+\sqrt{1+2x}\right)$;
4. $y=x^2\mathrm{arctg}\sqrt{x^2-1}-\sqrt{x^2-1}$;
5. $y=\arccos\left(1/\sqrt{1+2x^2}\right), x>0$;
6. $y=x\ln\left|x+\sqrt{x^2+3}\right|-\sqrt{x^2+3}$;
7. $y=\mathrm{arctg}(\mathrm{sh}\,x)+(\mathrm{sh}\,x)\ln\mathrm{ch}\,x$;
8. $y=\arccos\left((x^2-1)/(x^2\sqrt{2})\right)$;
9. $y=\ln\left(\cos^2 x+\sqrt{1+\cos^4 x}\right)$;
10. $y=\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}\,\mathrm{arctg}\,x$;
11. $y=\frac{\ln|x|}{1+x^2}-\frac{1}{2}\ln\frac{x^2}{1+x^2}$;
12. $y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}$;
13. $y=x\sqrt{4-x^2}+4\arcsin(x/2)$;
14. $y=\ln\mathrm{tg}(x/2)-x/\sin x$;
15. $y=2x+\ln|\sin x+2\cos x|$;
16. $y=\sqrt{\mathrm{ctg}\,x}-\sqrt{\mathrm{tg}^3 x}/3$;
17. $y=\ln\left|\frac{x+\sqrt{x^2+1}}{2x}\right|$;
18. $y=\sqrt[3]{\frac{x+2}{x-2}}$;
19. $y=\mathrm{arctg}\frac{x^2-1}{x}$;

20. $y=\ln\left|x^2-1\right|-\dfrac{1}{x^2-1}$;

21. $y=\text{arctg}\left(\text{tg}\dfrac{x}{2}+1\right)$;

22. $y=\ln\left|2x+2\sqrt{x^2+x}+1\right|$;

23. $y=\ln\left|\cos\sqrt{x}\right|+\sqrt{x}\,\text{tg}\sqrt{x}$;

24. $y=e^x(\cos 2x+2\sin 2x)$;

25. $y=x(\sin\ln x-\cos\ln x)$;

26. $y=\left(\sqrt{x-1}-\dfrac{1}{2}\right)e^{2\sqrt{x-1}}$;

27. $y=\cos x\cdot\ln\text{tg}\,x-\ln\text{tg}\dfrac{x}{2}$;

28. $y=\sqrt{3+x^2}-x\ln\left|x+\sqrt{3+x^2}\right|$;

29. $y=\sqrt{x}-(1+x)\,\text{arctg}\sqrt{x}$;

30. $y=x\,\text{arctg}\,x-\ln\sqrt{1+x^2}$.

5 – topshiriq. $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ differensiallarni toping.

1. $y=\ln\text{tg}x$;
2. $y=\ln\sin(2x+5)$;
3. $y=\ln\text{ctg}2x$;
4. $y=2^{x^2}$;
5. $y=\sin^3\dfrac{x}{2}$;
6. $y=\ln(x^2+5)$;
7. $y=\ln\text{tg}\dfrac{x}{2}$;
8. $y=\sqrt{1-3x^2}$;
9. $y=e^{\sqrt{2x}}\left(\sqrt{2x}-1\right)$;
10. $y=\sin^2\dfrac{x}{2}$;
11. $y=\cos^3\dfrac{x}{3}$;

12. $y=\sqrt{2x^2+1}$;
13. $y=\ln\text{ctg}\dfrac{x}{2}$;
14. $y=\text{tg}\dfrac{3}{x^3}$;
15. $y=\arcsin\sqrt{2x}$;
16. $y=\text{arctg}\sqrt{3x}$;
17. $y=\text{ctg}\dfrac{1}{x^2}$;
18. $y=\text{ctg}\sqrt{\dfrac{x}{2}}$;
19. $y=\dfrac{1}{\sqrt{\sin 2x}}$;
20. $y=\dfrac{1}{\sqrt{\cos 3x}}$;
21. $y=\ln\cos 2x$;
22. $y=\cos\dfrac{2}{x^2}$;
23. $y=\ln\cos\dfrac{x}{2}$;
24. $y=\arccos\sqrt{x}$;
25. $y=\text{arcctg}\sqrt{2x}$;
26. $y=\text{tg}^2\dfrac{x}{2}$;
27. $y=\text{ctg}^3\dfrac{x}{3}$;
28. $y=\text{arctg}e^{2x}$;
29. $y=3^{x^3}$;
30. $y=e^{\frac{1}{x^2}}$.

JAVOBLAR VA KO'RSATMALAR

1.6. Paragrafdagi misollarning javoblari.

1 – Topshiriq.

1. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 14 & 147 & 95 \\ 17 & -65 & -39 \\ 1 & 125 & 56 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -15 & 105 & 2019 & 47 \\ 4 & -43 & -7\;5 & -11 \\ 14 & 34 & -471 & 17 \end{pmatrix}$

2. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -18 & 8 & 76 \\ 31 & 52 & -39 \\ 24 & 3 & 10 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -8 & 21 & 29\;16 & 26 \\ 9 & -25 & -40-17 & -16 \\ -4 & 15 & 24\;11 & 1 \end{pmatrix}$

3. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -51 & -25 & 49 \\ -11 & 11 & -4 \\ -41 & -53 & 21 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -1 & 10 & 1210 & -1 \\ 1 & 5 & -611 & 16 \\ -9 & 33 & 3930 & 33 \end{pmatrix}$

4. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 31 & -30 & 22 \\ 25 & -24 & -36 \\ -6 & 14 & 14 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 18 & -18 & 20\;-3 & 35 \\ 21 & 80 & -1812 & 1 \\ -14 & 8 & -12\;1 & -31 \end{pmatrix}$

5. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -79 & 41 & -13 \\ 27 & -5 & 5 \\ -39 & 56 & -65 \end{pmatrix}$

b. $\emptyset$

6. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 46 & -11 & 65 \\ -43 & 70 & 17 \\ 10 & 23 & 34 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -2 & -1 & 25\;39 & 23 \\ 10 & -10 & 40\;60 & 50 \\ 6 & -3 & -9-15 & -3 \end{pmatrix}$

7. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -13 & 51 & -1 \\ 111 & 20 & 76 \\ 10 & -38 & 1 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 12 & 6 & -60 & 0 \\ -2 & -11 & -54 & 44 \\ 0 & -5 & -32 & 0 \end{pmatrix}$

8. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 28 & -30 & 0 \\ -8 & 42 & -36 \\ 0 & -10 & -28 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 24 & 18 & -322 & -5 \\ -1 & 13 & 7\;5 & -2 \\ -4 & 12 & 8\;0 & 4 \end{pmatrix}$

9. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -46 & 27 & -10 \\ -4 & -29 & -7 \\ -3 & -50 & -12 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -5 & -10 & 25\;8 & 22 \\ 24 & 16 & -2\;9 & 47 \\ -1 & 11 & 15-8 & 10 \end{pmatrix}$

10. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -25 & -70 & -14 \\ 27 & 18 & -4 \\ 19 & 18 & -9 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 15 & 12 & 54-9 & 10 \\ 19 & 13 & 31-7 & 11 \\ 15 & 10 & 20-5 & 4 \end{pmatrix}$

11. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 49 & -90 & -9 \\ -45 & 78 & 29 \\ -20 & -20 & 89 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 17 & -19 & 13 & 26 & 28 \\ 1 & 19 & 19 & -26 & -42 \\ 13 & 10 & 26 & -12 & -29 \end{pmatrix}$

12. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 26 & -26 & 12 \\ -2 & 8 & -32 \\ 56 & -22 & 70 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -23 & 25 & 41 & -4 & 16 \\ 1 & 8 & 11 & 0 & 3 \\ 28 & 34 & 96 & 14 & 24 \end{pmatrix}$

13. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -19 & 17 & -87 \\ 31 & -1 & -2 \\ -77 & -15 & 5 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 0 & -6 & -44 & -1 & 32 \\ 13 & 4 & 51 & 7 & -16 \\ 8 & -10 & -60 & -1 & 19 \end{pmatrix}$

14. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -21 & -30 & -79 \\ -80 & -14 & 44 \\ -18 & -90 & -64 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 8 & -2 & -5 & -5 & -9 \\ 16 & 16 & -24 & 24 & 8 \\ 0 & 40 & -28 & 68 & 52 \end{pmatrix}$

15. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -82 & -33 & 0 \\ -5 & -17 & 25 \\ 32 & -48 & -6 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 7 & -9 & 2 & -23 & -11 \\ 0 & -1 & -3 & 19 & -11 \\ -15 & -1 & 22 & 56 & 27 \end{pmatrix}$

16. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -17 & 24 & -37 \\ -8 & 0 & -17 \\ -23 & 11 & 66 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 6 & 22 & 3 & 17 & 4 \\ -1 & 15 & 0 & -1 & -2 \\ -26 & 2 & -11 & -8 & -11 \end{pmatrix}$

17. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -13 & 7 & -26 \\ -38 & 14 & 8 \\ 11 & -11 & -5 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 15 & 5 & -4 & 15 & 15 \\ 0 & -15 & 21 & -9 & -9 \\ 4 & -3 & 5 & 3 & 5 \end{pmatrix}$

18. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 17 & 50 & -6 \\ 140 & 111 & -52 \\ -36 & 30 & 57 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -7 & 13 & -22 & -2 & -6 \\ 5 & -19 & 23 & -1 & 14 \\ 6 & -10 & 18 & 2 & 4 \end{pmatrix}$

19. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 3 & -22 & -5 \\ -44 & -39 & -61 \\ -24 & 5 & 12 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 11 & 17 & 3 & 18 & 9 \\ 10 & 80 & 35 & 68 & 71 \\ 16 & 22 & 3 & 24 & 36 \end{pmatrix}$

20. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 52 & 22 & -4 \\ -44 & -34 & -30 \\ 38 & -6 & 52 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 6 & 0 & 8 & -1 & 3 \\ 26 & -6 & 26 & -15 & 13 \\ 5 & 8 & 37 & 22 & 9 \end{pmatrix}$

21. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 7 & 75 & -2 \\ 33 & 14 & 42 \\ 240 & 29 & 23 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 24 & 0 & 9 & 0 & -9 \\ 13 & -12 & 6 & 24 & 18 \\ 38 & 18 & 12 & -36 & -48 \end{pmatrix}$

22. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -2 & 18 & -10 \\ -9 & -11 & 3 \\ -8 & 32 & 9 \end{pmatrix}$

b. $B\times D=\begin{pmatrix}16 & 11 & -9 & 6 & 3\\ 0 & 24 & -66 & 12 & 3\\ 6 & -7 & 31 & -4 & 3\end{pmatrix}$

23. a. $\alpha A^2+\beta BC=\begin{pmatrix}22 & -14 & -18\\ -51 & -17 & 23\\ 58 & -33 & -70\end{pmatrix}$

b. $B\times D=\begin{pmatrix}11 & -4 & 23 & 26 & 18\\ -18 & 10 & -12 & 9 & -1\\ 34 & -13 & 59 & 11 & 5\end{pmatrix}$

24. a. $\alpha A^2+\beta BC=\begin{pmatrix}-11 & 19 & -5\\ 10 & -26 & 2\\ 4 & 83 & -23\end{pmatrix}$

25. a. $\alpha A^2+\beta BC=\begin{pmatrix}-17 & 10 & -5\\ -23 & 6 & 21\\ 38 & -39 & 23\end{pmatrix}$

b. $B\times D=\begin{pmatrix}-18 & 27 & -36 & 0 & 5\\ 18 & -76 & 148 & 14 & 39\\ 13 & -72 & 146 & 15 & 44\end{pmatrix}$

26. a. $\alpha A^2+\beta BC=\begin{pmatrix}-7 & 7 & -12\\ -23 & 1 & -14\\ -4 & 1 & 10\end{pmatrix}$

b. $B\times D=\begin{pmatrix}9 & 25 & -18 & 3 & 13\\ 21 & 41 & -30 & 6 & 24\\ 26 & 26 & -2 & 10 & 14\end{pmatrix}$

27. a. $\alpha A^2+\beta BC=\begin{pmatrix}5 & -2 & 52\\ 54 & 29 & 40\\ -10 & 20 & 3\end{pmatrix}$

b. $B\times D=\begin{pmatrix}21 & 0 & 2 & 8 & 22\\ 16 & 14 & -17 & 16 & 9\\ 16 & -4 & 7 & 4 & 21\end{pmatrix}$

28. a. $\alpha A^2+\beta BC=\begin{pmatrix}-25 & -20 & 2\\ -27 & -17 & -4\\ -20 & 40 & -1\end{pmatrix}$

b. $B\times D=\begin{pmatrix}9 & 0 & -11 & 18 & 16\\ 17 & 4 & -13 & 24 & 23\\ 16 & 11 & 3 & 3 & -3\end{pmatrix}$

29. a. $\alpha A^2+\beta BC=\begin{pmatrix}54 & -23 & -62\\ 7 & 5 & 16\\ -28 & -38 & 36\end{pmatrix}$

b. $B\times D=\begin{pmatrix}12 & -15 & 63 & 45 & -19\\ 1 & 1 & -15 & -12 & 1\\ -8 & -2 & 66 & 54 & 4\end{pmatrix}$

30. a. $\alpha A^2+\beta BC=\begin{pmatrix}-15 & -16 & 11\\ 3 & 23 & 47\\ -25 & 26 & -21\end{pmatrix}$

b. $B\times D=\begin{pmatrix}14 & 8 & 19 & -5 & 11\\ -6 & 4 & 18 & -2 & 1\\ 21 & 6 & 17 & 12 & -25\end{pmatrix}$

2- topshiriq.

$x=A^{-1}B,\quad x=BA^{-1},\quad x=A^{-1}BC^{-1}$

1. a. $\begin{pmatrix}\frac{2}{14} & \frac{-6}{14}\\ \frac{8}{14} & \frac{-10}{14}\end{pmatrix}$ b. $\begin{pmatrix}\frac{1}{7} & \frac{2}{7}\\ -\frac{6}{7} & -\frac{5}{7}\end{pmatrix}$ c. $\begin{pmatrix}\frac{2}{70} & -\frac{16}{70}\\ -\frac{6}{70} & -\frac{22}{70}\end{pmatrix}$

2. a. $\begin{pmatrix}-\frac{3}{7} & \frac{1}{7}\\ \frac{5}{7} & -\frac{4}{7}\end{pmatrix}$ b. $\begin{pmatrix}\frac{5}{7} & \frac{1}{7}\\ \frac{3}{7} & -\frac{2}{7}\end{pmatrix}$ c. $\begin{pmatrix}-\frac{10}{14} & \frac{8}{14}\\ \frac{12}{14} & -\frac{11}{14}\end{pmatrix}$

3. a. $\begin{pmatrix}\frac{3}{5} & -\frac{3}{5}\\ \frac{4}{5} & \frac{11}{5}\end{pmatrix}$ b. $\begin{pmatrix}\frac{9}{5} & 3\\ 0 & 1\end{pmatrix}$ c. $\begin{pmatrix}-\frac{3}{30} & \frac{9}{30}\\ -\frac{19}{30} & \frac{27}{30}\end{pmatrix}$

4. a. $\begin{pmatrix}\frac{2}{17} & \frac{6}{17}\\ \frac{22}{17} & -\frac{19}{17}\end{pmatrix}$ b. $\begin{pmatrix}-\frac{22}{17} & -\frac{15}{17}\\ -\frac{4}{17} & -\frac{5}{17}\end{pmatrix}$ c. $\begin{pmatrix}-\frac{16}{85} & \frac{14}{85}\\ \frac{79}{85} & -\frac{16}{85}\end{pmatrix}$

5. a. $\begin{pmatrix}\frac{21}{6} & -\frac{3}{6}\\ \frac{3}{6} & \frac{1}{6}\end{pmatrix}$ b. $\begin{pmatrix}\frac{20}{6} & -\frac{2}{6}\\ -\frac{5}{6} & \frac{2}{6}\end{pmatrix}$ c. $\begin{pmatrix}-\frac{66}{60} & -\frac{54}{60}\\ -\frac{8}{60} & -\frac{2}{60}\end{pmatrix}$

b. $B \times D = \begin{pmatrix} 16 & 11 & -9 & 6 & 3 \\ 0 & 24 & -66 & 12 & 3 \\ 6 & -7 & 31 & -4 & 3 \end{pmatrix}$

23. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 22 & -14 & -18 \\ -51 & -17 & 23 \\ 58 & -33 & -70 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 11 & -4 & 23 & 26 & 18 \\ -18 & 10 & -12 & 9 & -1 \\ 34 & -13 & 59 & 11 & 5 \end{pmatrix}$

24. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -11 & 19 & -5 \\ 10 & -26 & 2 \\ 4 & 83 & -23 \end{pmatrix}$

25. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -17 & 10 & -5 \\ -23 & 6 & 21 \\ 38 & -39 & 23 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} -18 & 27 & -36 & 0 & 5 \\ 18 & -76 & 148 & 14 & 39 \\ 13 & -72 & 146 & 15 & 44 \end{pmatrix}$

26. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -7 & 7 & -12 \\ -23 & 1 & -14 \\ -4 & 1 & 10 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 9 & 25 & -18 & 3 & 13 \\ 21 & 41 & -30 & 6 & 24 \\ 26 & 26 & -2 & 10 & 14 \end{pmatrix}$

27. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 5 & -2 & 52 \\ 54 & 29 & 40 \\ -10 & 20 & 3 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 21 & 0 & 2 & 8 & 22 \\ 16 & 14 & -17 & 16 & 9 \\ 16 & -4 & 7 & 4 & 21 \end{pmatrix}$

28. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -25 & -20 & 2 \\ -27 & -17 & -4 \\ -20 & 40 & -1 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 9 & 0 & -11 & 18 & 16 \\ 17 & 4 & -13 & 24 & 23 \\ 16 & 11 & 3 & 3 & -3 \end{pmatrix}$

29. a. $\alpha A^2 + \beta BC = \begin{pmatrix} 54 & -23 & -62 \\ 7 & 5 & 16 \\ -28 & -38 & 36 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 12 & -15 & 63 & 45 & -19 \\ 1 & 1 & -15 & -12 & 1 \\ -8 & -2 & 66 & 54 & 4 \end{pmatrix}$

30. a. $\alpha A^2 + \beta BC = \begin{pmatrix} -15 & -16 & 11 \\ 3 & 23 & 47 \\ -25 & 26 & -21 \end{pmatrix}$

b. $B \times D = \begin{pmatrix} 14 & 8 & 19 & -5 & 11 \\ -6 & 4 & 18 & -2 & 1 \\ 21 & 6 & 17 & 12 & -25 \end{pmatrix}$

2- topshiriq.

$$x = A^{-1}B, \quad x = BA^{-1}, \quad x = A^{-1}BC^{-1}$$

1.a. $\begin{pmatrix} \frac{2}{14} & \frac{-6}{14} \\ \frac{8}{14} & \frac{-10}{14} \end{pmatrix}$ b. $\begin{pmatrix} \frac{1}{7} & \frac{2}{7} \\ -\frac{6}{7} & -\frac{5}{7} \end{pmatrix}$ c. $\begin{pmatrix} \frac{2}{70} & -\frac{16}{70} \\ -\frac{6}{70} & -\frac{22}{70} \end{pmatrix}$

2.a. $\begin{pmatrix} -\frac{3}{7} & \frac{1}{7} \\ \frac{5}{7} & -\frac{4}{7} \end{pmatrix}$ b. $\begin{pmatrix} \frac{5}{7} & \frac{1}{7} \\ \frac{3}{7} & -\frac{2}{7} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{10}{14} & \frac{8}{14} \\ \frac{12}{14} & -\frac{11}{14} \end{pmatrix}$

3.a. $\begin{pmatrix} \frac{3}{5} & -\frac{3}{5} \\ \frac{4}{5} & \frac{11}{5} \end{pmatrix}$ b. $\begin{pmatrix} \frac{9}{5} & 3 \\ 0 & 1 \end{pmatrix}$ c. $\begin{pmatrix} -\frac{3}{30} & \frac{9}{30} \\ -\frac{19}{30} & \frac{27}{30} \end{pmatrix}$

4.a. $\begin{pmatrix} \frac{2}{17} & \frac{6}{17} \\ \frac{22}{17} & -\frac{19}{17} \end{pmatrix}$ b. $\begin{pmatrix} -\frac{22}{17} & -\frac{15}{17} \\ -\frac{4}{17} & -\frac{5}{17} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{16}{85} & \frac{14}{85} \\ \frac{79}{85} & -\frac{16}{85} \end{pmatrix}$

5.a. $\begin{pmatrix} \frac{21}{6} & -\frac{3}{6} \\ \frac{3}{6} & \frac{1}{6} \end{pmatrix}$ b. $\begin{pmatrix} \frac{20}{6} & -\frac{2}{6} \\ -\frac{5}{6} & \frac{2}{6} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{66}{60} & -\frac{54}{60} \\ -\frac{8}{60} & -\frac{2}{60} \end{pmatrix}$

6. a. $\begin{pmatrix} -7 & -6 \\ -30 & -31 \end{pmatrix}$ b. $\begin{pmatrix} 29 & 66 \\ -30 & -67 \end{pmatrix}$ c. $\begin{pmatrix} \frac{5}{9} & -\frac{34}{9} \\ \frac{32}{9} & -\frac{151}{9} \end{pmatrix}$

7. a. $\begin{pmatrix} \frac{17}{2} & \frac{4}{2} \\ -\frac{7}{2} & -\frac{6}{2} \end{pmatrix}$ b. $\begin{pmatrix} \frac{17}{2} & 14 \\ -\frac{1}{2} & -3 \end{pmatrix}$ c. $\begin{pmatrix} \frac{97}{4} & \frac{76}{4} \\ -\frac{53}{4} & -\frac{40}{4} \end{pmatrix}$

8. a. $\begin{pmatrix} 5 & \frac{4}{3} \\ 3 & \frac{2}{3} \end{pmatrix}$ b. $\begin{pmatrix} \frac{10}{3} & \frac{38}{3} \\ \frac{2}{3} & \frac{7}{3} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{19}{3} & -14 \\ -\frac{11}{3} & -7 \end{pmatrix}$

9. a. $\begin{pmatrix} -\frac{11}{2} & \frac{7}{2} \\ \frac{48}{2} & -\frac{30}{2} \end{pmatrix}$ b. $\begin{pmatrix} -\frac{27}{2} & 32 \\ 3 & -7 \end{pmatrix}$ c. $\begin{pmatrix} -\frac{47}{2} & -\frac{29}{2} \\ 102 & 63 \end{pmatrix}$

10. a. $\begin{pmatrix} -1 & 15 \\ 0 & 6 \end{pmatrix}$ b. $\begin{pmatrix} -6 & -10 \\ 6 & 11 \end{pmatrix}$ c. $\begin{pmatrix} -\frac{34}{6} & \frac{14}{6} \\ -2 & 1 \end{pmatrix}$

11. a. $\begin{pmatrix} -\frac{12}{6} & -\frac{16}{6} \\ \frac{24}{6} & \frac{32}{6} \end{pmatrix}$ b. $\begin{pmatrix} \frac{7}{6} & \frac{22}{6} \\ \frac{5}{6} & \frac{14}{67} \end{pmatrix}$ c. $\begin{pmatrix} 3 & -\frac{88}{12} \\ -\frac{75}{12} & \frac{150}{12} \end{pmatrix}$

12. a. $\begin{pmatrix} 21 & -18 \\ 2 & -2 \end{pmatrix}$ b. $\begin{pmatrix} 30 & 27 \\ -12 & -11 \end{pmatrix}$ c. $\begin{pmatrix} \frac{81}{2} & -60 \\ 4 & -6 \end{pmatrix}$

13. a. $\begin{pmatrix} \frac{18}{22} & -\frac{8}{22} \\ -\frac{8}{22} & \frac{17}{22} \end{pmatrix}$ b. $\begin{pmatrix} \frac{23}{22} & -\frac{17}{22} \\ -\frac{2}{22} & \frac{12}{22} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{10}{22} & \frac{38}{22} \\ -\frac{9}{22} & \frac{10}{22} \end{pmatrix}$

14. a. $\begin{pmatrix} -\frac{4}{5} & \frac{11}{5} \\ -\frac{7}{5} & \frac{28}{5} \end{pmatrix}$ b. $\begin{pmatrix} -\frac{6}{5} & \frac{29}{5} \\ -\frac{5}{5} & 6 \end{pmatrix}$ c. $\begin{pmatrix} -\frac{52}{50} & \frac{29}{50} \\ -\frac{126}{50} & \frac{77}{50} \end{pmatrix}$

15. a. $\begin{pmatrix} 8 & 3 \\ -3 & -\frac{1}{2} \end{pmatrix}$ b. $\begin{pmatrix} \frac{17}{2} & 6 \\ -\frac{5}{2} & -1 \end{pmatrix}$ c. $\begin{pmatrix} \frac{13}{5} & \frac{14}{5} \\ -\frac{11}{10} & -\frac{8}{10} \end{pmatrix}$

16. a. $\begin{pmatrix} -1 & 7 \\ 6 & -32 \end{pmatrix}$ b. $\begin{pmatrix} -8 & 15 \\ 14 & -25 \end{pmatrix}$ c. $\begin{pmatrix} -15 & -23 \\ 70 & 108 \end{pmatrix}$

17. a. $\begin{pmatrix} \frac{7}{2} & \frac{1}{2} \\ -\frac{32}{2} & -\frac{3}{2} \end{pmatrix}$ b. $\begin{pmatrix} 5 & 9 \\ -2 & -3 \end{pmatrix}$ c. $\begin{pmatrix} -\frac{19}{14} & -\frac{4}{14} \\ \frac{93}{14} & \frac{24}{14} \end{pmatrix}$

18. a. $\begin{pmatrix} 0 & \frac{1}{13} \\ -1 & -\frac{3}{13} \end{pmatrix}$ b. $\begin{pmatrix} \frac{10}{13} & \frac{11}{13} \\ -1 & -1 \end{pmatrix}$ c. $\begin{pmatrix} \frac{1}{55} & \frac{1}{55} \\ \frac{10}{55} & \frac{62}{52} \end{pmatrix}$

19. a. $\begin{pmatrix} -3 & -3 \\ 5 & 4 \end{pmatrix}$ b. $\begin{pmatrix} -\frac{8}{3} & -\frac{23}{3} \\ \frac{5}{3} & \frac{11}{3} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{6}{2} & -\frac{12}{2} \\ \frac{9}{2} & \frac{19}{2} \end{pmatrix}$

20. a. $\begin{pmatrix} 14 & -8 \\ 37 & -22 \end{pmatrix}$ b. $\begin{pmatrix} -20 & -12 \\ 19 & 12 \end{pmatrix}$ c. $\begin{pmatrix} \frac{52}{5} & -\frac{46}{5} \\ \frac{140}{5} & -\frac{125}{5} \end{pmatrix}$

21. a. $\begin{pmatrix} 22 & -6 \\ 9 & -2 \end{pmatrix}$ b. $\begin{pmatrix} 13 & -9 \\ -9 & 7 \end{pmatrix}$ c. $\begin{pmatrix} 6 & -10 \\ \frac{5}{2} & -4 \end{pmatrix}$

22. a. $\begin{pmatrix} -\frac{21}{8} & 1 \\ \frac{11}{8} & 0 \end{pmatrix}$ b. $\begin{pmatrix} -\frac{23}{8} & -\frac{3}{8} \\ -\frac{14}{8} & \frac{2}{8} \end{pmatrix}$ c. $\begin{pmatrix} \frac{29}{48} & -\frac{10}{48} \\ -\frac{11}{48} & \frac{22}{48} \end{pmatrix}$

23. a. $\begin{pmatrix} -4 & \frac{34}{7} \\ -2 & \frac{19}{7} \end{pmatrix}$ b. $\begin{pmatrix} -\frac{9}{7} & -\frac{8}{7} \\ -1 & 0 \end{pmatrix}$ c. $\begin{pmatrix} \frac{22}{14} & \frac{10}{14} \\ \frac{9}{14} & -\frac{49}{14} \end{pmatrix}$

24. a. $\begin{pmatrix} \frac{1}{4} & -\frac{9}{4} \\ \frac{3}{4} & \frac{61}{4} \end{pmatrix}$ b. $\begin{pmatrix} 7 & 12 \\ \frac{9}{2} & \frac{17}{2} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{12}{28} & -\frac{17}{28} \\ \frac{52}{28} & \frac{125}{28} \end{pmatrix}$

25. a. $\begin{pmatrix} -\frac{26}{10} & -\frac{5}{10} \\ \frac{2}{10} & 0 \end{pmatrix}$ b. $\begin{pmatrix} -\frac{7}{10} & -\frac{41}{10} \\ -\frac{3}{10} & -\frac{19}{10} \end{pmatrix}$ c. $\begin{pmatrix} \frac{31}{10} & -\frac{88}{10} \\ -\frac{2}{10} & \frac{6}{10} \end{pmatrix}$

26. a. $\begin{pmatrix} -\frac{20}{3} & -\frac{22}{3} \\ \frac{78}{3} & \frac{93}{3} \end{pmatrix}$ b. $\begin{pmatrix} -\frac{17}{3} & 22 \\ -7 & 30 \end{pmatrix}$ c. $\begin{pmatrix} \frac{42}{3} & \frac{62}{3} \\ -\frac{171}{3} & -\frac{249}{3} \end{pmatrix}$

27. a. $\begin{pmatrix} \frac{23}{5} & \frac{13}{5} \\ 16 & 10 \end{pmatrix}$ b. $\begin{pmatrix} \frac{22}{5} & \frac{46}{5} \\ \frac{22}{5} & \frac{51}{5} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{82}{50} & \frac{66}{50} \\ -\frac{58}{10} & \frac{44}{10} \end{pmatrix}$

28. a. $\begin{pmatrix} \frac{13}{7} & \frac{25}{7} \\ \frac{5}{7} & \frac{22}{7} \end{pmatrix}$ b. $\begin{pmatrix} \frac{3}{7} & -\frac{5}{7} \\ \frac{13}{7} & \frac{32}{7} \end{pmatrix}$ c. $\begin{pmatrix} -\frac{62}{49} & \frac{51}{49} \\ -\frac{61}{49} & \frac{32}{49} \end{pmatrix}$

29. a. $\begin{pmatrix} 2 & \frac{5}{2} \\ 3 & 3 \end{pmatrix}$ b. $\begin{pmatrix} -3 & -\frac{5}{2} \\ 3 & 8 \end{pmatrix}$ c. $\begin{pmatrix} -\frac{2}{6} & \frac{5}{6} \\ -1 & 1 \end{pmatrix}$

30. a. $\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} \\ \frac{13}{3} & -\frac{16}{3} \end{pmatrix}$ b. $\begin{pmatrix} 1 & 3 \\ \frac{5}{3} & -\frac{17}{3} \end{pmatrix}$ c. $\begin{pmatrix} \frac{14}{6} & \frac{10}{6} \\ \frac{109}{6} & \frac{77}{6} \end{pmatrix}$

3 – Topshiriq.

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27) 4
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29) 3
30) 4

3.7. Paragrafdagi misollarning javoblari.

4 – Topshiriq.

1. $\lim\limits_{x\to\infty} \dfrac{\sqrt[4]{x^{16}+3x}+1}{\sqrt[8]{x^{32}+x^2+x}+x^4}$ J: 1

2. $\lim\limits_{x\to\infty} \dfrac{\sqrt[4]{x^{20}+x^5+x}+3}{\sqrt[3]{x^{15}+3x+2}}$ J: 1

3. $\lim\limits_{x\to\infty} \dfrac{\sqrt[5]{x^{10}+4x^2+1}}{\sqrt[5]{x^5+7x}+5x^2}$ J: $\frac{1}{5}$

4. $\lim\limits_{x\to\infty} \dfrac{\sqrt[7]{x^7+x^6+5x}+2x^3}{\sqrt[9]{x^{27}+6x^{20}}+7}$ J: 2

5. $\lim\limits_{x\to\infty} \dfrac{\sqrt[3]{x^6+2x^3}+3x^2}{\sqrt[7]{x^{21}+5x^2+x}}$ J: 0

6. $\lim\limits_{x\to\infty} \dfrac{\sqrt[15]{x^{30}+5x^{10}}+10x}{\sqrt[10]{x^{20}+7x^6+9}+x^2}$ J: $\frac{1}{2}$

7. $\lim\limits_{x\to\infty} \dfrac{\sqrt[3]{x^{12}+4x+7}+4x^2}{\sqrt[5]{x^{20}+5x^7}+9x^4}$ J: $\frac{1}{10}$

8. $\lim\limits_{x\to\infty} \dfrac{\sqrt[30]{x^{60}+5x^{10}+x}}{\sqrt[8]{x^{81}+5x^7}+3x^2}$ J: $\frac{1}{3}$

9. $\lim\limits_{x\to\infty} \dfrac{\sqrt[18]{x^{36}+x^{10}}+7x^6}{\sqrt[5]{x^{40}+x^{20}+10x}}$ J: 0

10. $\lim\limits_{x\to\infty} \dfrac{\sqrt[8]{x^{72}+x^{15}+5x-15}}{\sqrt[4]{x^{16}+5}+3x^9}$ J: $\frac{1}{3}$

11. $\lim\limits_{x\to\infty} \dfrac{\sqrt[7]{x^{14}+5x^{10}+9}}{\sqrt[6]{x^{12}+x^5+3}+88}$ J: 1

12. $\lim\limits_{x\to\infty} \dfrac{\sqrt[7]{x^{14}+x^{13}+5}-7x^5}{\sqrt{x^{10}+5x^5+x}+2x}$ J: -7

13. $\lim\limits_{x\to\infty} \dfrac{\sqrt[5]{x^{30}+7x^{20}+x^3}}{\sqrt[10]{x^{10}+5x^6+10}+8x^6}$ J: $\frac{1}{8}$

14. $\lim\limits_{x\to\infty} \dfrac{\sqrt[3]{x^{30}+10x^{15}+3}}{\sqrt{5x^{20}+10x}-12}$ J: $\frac{1}{5}$

15. $\lim\limits_{x\to\infty} \dfrac{\sqrt[7]{x^{24}+7x^2+x}+2x^3}{\sqrt[8]{x^{24}+5x^{10}+3}}$ J: ∞

16. $\lim\limits_{x\to\infty} \dfrac{\sqrt[20]{x^{40}+4x^{30}-3}}{\sqrt[3]{x^3+x^2-3x}+5x^2}$ J: $\frac{1}{5}$

17. $\lim\limits_{x\to\infty} \dfrac{\sqrt[10]{x^{10}+x^9+7}+3}{\sqrt[5]{x^5+x^4+x}+2x}$ J: $\frac{1}{3}$

29. a. $\begin{pmatrix} 2 & \frac{5}{2} \\ 3 & 3 \end{pmatrix}$ b. $\begin{pmatrix} -3 & -\frac{5}{2} \\ 3 & 8 \end{pmatrix}$ c. $\begin{pmatrix} -\frac{2}{6} & \frac{5}{6} \\ -1 & 1 \end{pmatrix}$

30. a. $\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} \\ \frac{13}{3} & -\frac{16}{3} \end{pmatrix}$ b. $\begin{pmatrix} 1 & 3 \\ \frac{5}{3} & -\frac{17}{3} \end{pmatrix}$ c. $\begin{pmatrix} \frac{14}{6} & \frac{10}{6} \\ \frac{109}{6} & \frac{77}{6} \end{pmatrix}$

3 – Topshiriq.

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30) 4

3.7. Paragrafdagi misollarning javoblari.

4 – Topshiriq.

1. $\lim\limits_{x\to\infty} \frac{\sqrt[4]{x^{16}+3x}+1}{\sqrt[8]{x^{32}+x^2+x}+x^4}$ J: 1
2. $\lim\limits_{x\to\infty} \frac{\sqrt[4]{x^{20}+x^5+x}+3}{\sqrt[3]{x^{15}+3x+2}}$ J: 1
3. $\lim\limits_{x\to\infty} \frac{\sqrt[5]{x^{10}+4x^2+1}}{\sqrt[5]{x^5+7x}+5x^2}$ J: $\frac{1}{5}$
4. $\lim\limits_{x\to\infty} \frac{\sqrt[7]{x^7+x^6+5x}+2x^3}{\sqrt[9]{x^{27}+6x^{20}}+7}$ J: 2
5. $\lim\limits_{x\to\infty} \frac{\sqrt[3]{x^6+2x^3}+3x^2}{\sqrt[7]{x^{21}+5x^2+x}}$ J: 0
6. $\lim\limits_{x\to\infty} \frac{\sqrt[15]{x^{30}+5x^{10}}+10x}{\sqrt[10]{x^{20}+7x^6+9}+x^2}$ J: $\frac{1}{2}$
7. $\lim\limits_{x\to\infty} \frac{\sqrt[3]{x^{12}+4x+7}+4x^2}{\sqrt[5]{x^{20}+5x^7}+9x^4}$ J: $\frac{1}{10}$
8. $\lim\limits_{x\to\infty} \frac{\sqrt[30]{x^{60}+5x^{10}+x}}{\sqrt[8]{x^{81}+5x^7}+3x^2}$ J: $\frac{1}{3}$
9. $\lim\limits_{x\to\infty} \frac{\sqrt[18]{x^{36}+x^{10}}+7x^6}{\sqrt[5]{x^{40}+x^{20}+10x}}$ J: 0
10. $\lim\limits_{x\to\infty} \frac{\sqrt[8]{x^{72}+x^{15}+5x}-15}{\sqrt[4]{x^{16}+5}+3x^9}$ J: $\frac{1}{3}$
11. $\lim\limits_{x\to\infty} \frac{\sqrt[7]{x^{14}+5x^{10}+9}}{\sqrt[6]{x^{12}+x^5+3}+88}$ J: 1
12. $\lim\limits_{x\to\infty} \frac{\sqrt[7]{x^{14}+x^{13}+5}-7x^5}{\sqrt{x^{10}+5x^5+x}+2x}$ J: -7
13. $\lim\limits_{x\to\infty} \frac{\sqrt[5]{x^{30}+7x^{20}+x^3}}{\sqrt[10]{x^{10}+5x^6+10}+8x^6}$ J: $\frac{1}{8}$
14. $\lim\limits_{x\to\infty} \frac{\sqrt[3]{x^{30}+10x^{15}+3}}{\sqrt{5x^{20}+10x}-12}$ J: $\frac{1}{5}$
15. $\lim\limits_{x\to\infty} \frac{\sqrt[7]{x^{24}+7x^2+x}+2x^3}{\sqrt[8]{x^{24}+5x^{10}+3}}$ J: ∞
16. $\lim\limits_{x\to\infty} \frac{\sqrt[20]{x^{40}+4x^{30}-3}}{\sqrt[3]{x^3+x^2-3x}+5x^2}$ J: $\frac{1}{5}$
17. $\lim\limits_{x\to\infty} \frac{\sqrt[10]{x^{10}+x^9+7}+3}{\sqrt[5]{x^5+x^4+x}+2x}$ J: $\frac{1}{3}$

18. $\lim\limits_{x\to\infty}\frac{\sqrt{x^8+7x^6+x}-10}{\sqrt[30]{x^{10}+2x^7+5}+3x^2}$ J: $\frac{1}{3}$

19. $\lim\limits_{x\to\infty}\frac{\sqrt[8]{x^{40}+x^{10}+10}}{\sqrt{x^{10}+x^9+x}+15}$ J: 1

20. $\lim\limits_{x\to\infty}\frac{\sqrt[5]{x^{20}+4x^3+7}}{\sqrt[8]{x^{32}+x}-9x^2}$ J: 1

21. $\lim\limits_{x\to\infty}\frac{\sqrt[7]{x^{21}+x^{20}+5x}+8x^6}{\sqrt{x^{40}+10^{10}+x^3}}$ J: 0

22. $\lim\limits_{x\to\infty}\frac{\sqrt[7]{x^{28}+5x^{20}+x}+7}{\sqrt[5]{x^{40}+x^{25}+3}}$ J: 0

23. $\lim\limits_{x\to\infty}\frac{\sqrt[6]{x^{12}+3x-4}+x^2}{\sqrt[5]{x^{10}+x^2+6}+7x}$ J: 2

24. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x^{30}+2x-5}+2x^{10}}{\sqrt{x^{20}+x^{10}+x}+4x^5}$ J: 3

25. $\lim\limits_{x\to\infty}\frac{\sqrt[7]{x^{49}+x^3+x}+2x}{2x^7+\sqrt{x^6+3x^2+9}}$ J: $\frac{1}{2}$

26. $\lim\limits_{x\to\infty}\frac{\sqrt[10]{x^{10}+5x^9+4}}{\sqrt[15]{x^{15}+x^{10}+x}+9x}$ J: $\frac{1}{10}$

27. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x^3+5x^2+x}+2x}{\sqrt[3]{x^{18}+4x^6+3}-7}$ J: ∞

28. $\lim\limits_{x\to\infty}\frac{\sqrt[11]{x^{33}+5x}-7}{\sqrt[5]{x^{10}+x^9+4}+3x^3}$ J: $\frac{1}{3}$

29. $\lim\limits_{x\to\infty}\frac{\sqrt[16]{x^{16}+x^5+5}+2x}{\sqrt{3x^2+2x+5}}$ J: 3

30. $\lim\limits_{x\to\infty}\frac{\sqrt[12]{x^{24}+x^{20}+x}}{\sqrt[10]{x^{20}+x^8+4}+20x^2}$ J: $\frac{1}{21}$

5 – Topshiriq. Limitlarni hisoblash.

1. $-\frac{2}{5}$; 2. 0; 3. $\frac{5}{2}$; 4. 6; 5. $-\frac{1}{2}$; 6. $\frac{1}{2}$; 7. 0; 8. 0;

3.9. Paragrafdagi misollarning javoblari.

2 – topshiriq.

1. $1-\frac{e^x\left(\sqrt{e^{2x}+e^x+1}\right)+2e^x+e^x}{\left(\sqrt{e^{2x}+e^x+1}\right)\left(2+e^x+2\sqrt{e^{2x}+e^x+1}\right)}$

2. $y'=\frac{1}{2}e^{2x}(1-cos2x)$

3. $y'=\frac{e^x}{e^{2x}-6e^x+13}$

4. $y'=\frac{2^x(2^x-1)}{(2^x+1)^3}$

5. $y'=\frac{e^x+\sqrt{e^x+1}}{\sqrt{e^x+1}}$

6. $y'=\frac{\sqrt{arctg\, e^x}\cdot e^x}{1+e^{2x}}$

7. $y'=\frac{e^x(e^x-1)}{e^{2x}+1}$

8. $y'=\frac{e^x}{e^x+1}+\frac{(36e^{2x}+27e^x)(e^x+1)^3-3e^x(18e^{2x}+27e^x+11)(e^x+1)^2}{6(e^x+1)^6}$

9. $y'=\sqrt{e^x-1}$

10. $y'=2\sqrt{1+e^x}+\frac{e^x-4e^x-2\sqrt{1+e^x}}{\sqrt{1+e^x}}$

11. $y'=e^{\alpha x}\cdot\sin\beta x$

12. $y'=e^{\alpha x}\cdot\frac{(\alpha^2-\beta^2)}{\alpha^2+\beta^2}\sin\beta x$

13. $y'=e^{\alpha x}\cdot cos^2\beta x$

14. $y'=\frac{1-e^x\cdot x}{(1+e^x)^2}$

16. $y'=1-\frac{2e^{\frac{x}{4}}}{\left(1+e^{\frac{x}{4}}\right)^2}$

17. $y'=\sqrt{\frac{e^x-1}{e^x+1}}$

18. $y'=1-\left(-e^x\cdot arcsine^x+\frac{1}{\sqrt{1-e^{2x}}}+\frac{e^{2x}}{\left(\sqrt{1-e^{2x}}\right)\cdot\left(1+\sqrt{1-e^{2x}}\right)}\right)$

19. $y'=\frac{1}{1+e^x}+e^{-\frac{x}{2}}\cdot arctg\, e^{\frac{x}{2}}+\frac{1}{1+e^{\frac{x^2}{4}}}-\frac{4arctg\, e^{\frac{x}{2}}\cdot e^{\frac{x}{2}}}{1+e^{\frac{x^2}{4}}}$

3 – topshiriq

1. $y'=\frac{tg\,6x}{\cos 6x}$
2. $y'=\frac{tg\,30x}{\cos 30x}$
3. $y'=\frac{1}{4\cdot sin^2 3x}$
4. $y'=-\frac{1}{4\cdot sin^2 16x}$
5. $y'=\frac{tg\,8x}{\cos 8x}$
6. $y'=\frac{tg\,34x}{\cos 34x}$
7. $y'=\frac{1}{4\cdot sin^2 4x}$
8. $y'=-\frac{1}{4\cdot sin^2 18x}$

18. $\lim\limits_{x\to\infty}\frac{\sqrt{x^8+7x^6+x}-10}{\sqrt[30]{x^{10}+2x^7+5}+3x^2}$ $\qquad J:\ \frac{1}{3}$

19. $\lim\limits_{x\to\infty}\frac{\sqrt[8]{x^{40}+x^{10}+10}}{\sqrt{x^{10}+x^9}+x+15}$ $\qquad J:\ 1$

20. $\lim\limits_{x\to\infty}\frac{\sqrt[5]{x^{20}+4x^3+7}}{\sqrt[8]{x^{32}+x}-9x^2}$ $\qquad J:\ 1$

21. $\lim\limits_{x\to\infty}\frac{\sqrt[7]{x^{21}+x^{20}+5x}+8x^6}{\sqrt{x^{40}+10^{10}+x^3}}$ $\qquad J:\ 0$

22. $\lim\limits_{x\to\infty}\frac{\sqrt[7]{x^{28}+5x^{20}+x}+7}{\sqrt[5]{x^{40}+x^{25}+3}}$ $\qquad J:\ 0$

23. $\lim\limits_{x\to\infty}\frac{\sqrt[6]{x^{12}+3x-4}+x^2}{\sqrt[5]{x^{10}+x^2}+6+7x}$ $\qquad J:\ 2$

24. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x^{30}+2x-5}+2x^{10}}{\sqrt{x^{20}+x^{10}+x}+4x^5}$ $\qquad J:\ 3$

25. $\lim\limits_{x\to\infty}\frac{\sqrt[7]{x^{49}+x^3+x}+2x}{2x^7+\sqrt{x^6+3x^2+9}}$ $\qquad J:\ \frac{1}{2}$

26. $\lim\limits_{x\to\infty}\frac{\sqrt[10]{x^{10}+5x^9+4}}{\sqrt[15]{x^{15}+x^{10}+x}+9x}$ $\qquad J:\ \frac{1}{10}$

27. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x^3+5x^2+x}+2x}{\sqrt[3]{x^{18}+4x^6+3}-7}$ $\qquad J:\ \infty$

28. $\lim\limits_{x\to\infty}\frac{\sqrt[11]{x^{33}+5x}-7}{\sqrt[5]{x^{10}+x^9+4}+3x^3}$ $\qquad J:\ \frac{1}{3}$

29. $\lim\limits_{x\to\infty}\frac{\sqrt[16]{x^{16}+x^5+5}+2x}{\sqrt{3x^2+2x}+5}$ $\qquad J:\ 3$

30. $\lim\limits_{x\to\infty}\frac{\sqrt[12]{x^{24}+x^{20}+x}}{\sqrt[10]{x^{20}+x^8+4}+20x^2}$ $\qquad J:\ \frac{1}{21}$

5 – Topshiriq. Limitlarni hisoblash.

1. $-\frac{2}{5}$; 2. 0; 3. $\frac{5}{2}$; 4. 6; 5. $-\frac{1}{2}$; 6. $\frac{1}{2}$; 7. 0; 8. 0;

3.9. Paragrafdagi misollarning javoblari.

2 – topshiriq.

1. $1-\frac{e^x(\sqrt{e^{2x}+e^x+1})+2e^x+e^x}{(\sqrt{e^{2x}+e^x+1})(2+e^x+2\sqrt{e^{2x}+e^x+1})}$

2. $y'=\frac{1}{2}e^{2x}(1-cos2x)$

3. $y'=\frac{e^x}{e^{2x}-6e^x+13}$

4. $y'=\frac{2^x(2^x-1)}{(2^x+1)^3}$

5. $y'=\frac{e^x+\sqrt{e^x+1}}{\sqrt{e^x+1}}$

6. $y'=\frac{\sqrt{arctg\,e^x}\cdot e^x}{1+e^{2x}}$

7. $y'=\frac{e^x(e^x-1)}{e^{2x}+1}$

8. $y'=\frac{e^x}{e^x+1}+\frac{(36e^{2x}+27e^x)(e^x+1)^3-3e^x(18e^{2x}+27e^x+11)(e^x+1)^2}{6(e^x+1)^6}$

9. $y'=\sqrt{e^x-1}$

10. $y'=2\sqrt{1+e^x}+\frac{e^x-4e^x-2\sqrt{1+e^x}}{\sqrt{1+e^x}}$

11. $y'=e^{\alpha x}\cdot\sin\beta x$

12. $y'=e^{\alpha x}\cdot\frac{(\alpha^2-\beta^2)}{\alpha^2+\beta^2}\sin\beta x$

13. $y'=e^{\alpha x}\cdot cos^2\beta x$

14. $y'=\frac{1-e^x\cdot x}{(1+e^x)^2}$

16. $y'=1-\frac{2e^{\frac{x}{4}}}{\left(1+e^{\frac{x}{4}}\right)^2}$

17. $y'=\sqrt{\frac{e^x-1}{e^x+1}}$

18. $y'=1-\left(-e^x\cdot arcsine^x+\frac{1}{\sqrt{1-e^{2x}}}+\frac{e^{2x}}{(\sqrt{1-e^{2x}})\cdot(1+\sqrt{1-e^{2x}})}\right)$

19. $y'=\frac{1}{1+e^x}+e^{-\frac{x}{2}}\cdot arctg\,e^{\frac{x}{2}}+\frac{1}{1+e^{\frac{x^2}{4}}}-\frac{4arctg\,e^{\frac{x}{2}}\cdot e^{\frac{x}{2}}}{1+e^{\frac{x^2}{4}}}$

3 – topshiriq

1. $y'=\frac{tg\,6x}{\cos 6x}$

2. $y'=\frac{tg\,30x}{\cos 30x}$

3. $y'=\frac{1}{4\cdot sin^2 3x}$

4. $y'=-\frac{1}{4\cdot sin^2 16x}$

5. $y'=\frac{tg\,8x}{\cos 8x}$

6. $y'=\frac{tg\,34x}{\cos 34x}$

7. $y'=\frac{1}{4\cdot sin^2 4x}$

8. $y'=-\frac{1}{4\cdot sin^2 18x}$

9. $y' = \frac{tg\,4x}{\cos 4x}$

10. $y' = \frac{tg\,38x}{\cos 38x}$

11. $y' = -\frac{1}{4 \cdot sin^2 2x}$

12. $y' = \frac{1}{4 \cdot sin^2 20x}$

13. $y' = \frac{tg\,14x}{\cos 14x}$

14. $y' = \frac{tg\,42x}{\cos 42x}$

15. $y' = \frac{1}{4 \cdot sin^2 8x}$

16. $y' = \frac{1}{4 \cdot sin^2 22x}$

17. $y' = \frac{tg\,12x}{\cos 12x}$

18. $y' = \frac{tg 46x}{\cos 46x}$

19. $y' = \frac{1}{4 \cdot sin^2 10x}$

20. $y' = \frac{1}{4 \cdot sin^2 24x}$

21. $y' = \frac{tg\,20x}{\cos 20x}$

22. $y' = \frac{tg\,50x}{\cos 50x}$

23. $y' = \frac{1}{4 \cdot sin^2 12x}$

24. $y' = \frac{1}{4 \cdot sin^2 26x}$

25. $y' = \frac{tg\,10x}{\cos 10x}$

26. $y' = \frac{tg\,54x}{\cos 54x}$

27. $y' = -\frac{1}{4 \cdot sin^2 14x}$

28. $y' = -\frac{1}{4 \cdot sin^2 28x}$

29. $y' = \frac{tg\,58x}{\cos 58x}$

30. $y' = -\frac{1}{4 \cdot sin^2 30x}$

4 – topshiriq.

1. $y' = -\frac{\sqrt{x^2-1}}{x} + \arcsin\frac{a}{x} + \frac{1}{\sqrt{x^2-1}}$

2. $y' = \frac{4\sqrt{2}x(8x-1-8x^3)}{(1-4x^2)^2}$

3. $y' = \frac{x-1}{(\sqrt{2x+1})(x+\sqrt{1+2x})}$

4. $y' = 2x \cdot arctg\,\sqrt{x^2-1}$

5. $y' = \frac{-\sqrt{2}}{\sqrt{1+2x^2}}$

6. $y' = \ln\left|x + \sqrt{x^2+3}\right|$

8. $y' = \frac{\sqrt{2}}{x^3\sqrt{1-\left(\frac{x^2-1}{x^2\sqrt{2}}\right)^2}}$

9. $y' = -\frac{\sin 2x\,(2\sqrt{1+cos^4 x}+cos^2 x)}{cos^2 x+\sqrt{1+cos^4 x}}$

10. $y' = -\frac{x\,arctg\,x}{\sqrt{1+x^2}}$

11. $y' = \frac{-2\ln x}{(1+x^2)^2}$

12. $y' = \frac{e^x\sqrt{e^{2x}-1}+e^{2x}}{e^x+\sqrt{e^{2x}-1}}$

13. $y' = \frac{-4x(x^3+x^2+8)}{\sqrt{4-x^2}}$

14. $y' = \frac{x \cdot \cos x}{sin^2 x}$

15. $y' = \frac{5\cos x}{|\sin x + 2\cos x|}$

16. $y' = \frac{4\sqrt{tg\,x} \cdot ctg\,2x}{\sin 2x}$

17. $y' = \frac{x-\sqrt{x^2+1}}{x}$

18. $y' = -\frac{4}{3(x-2)^2} \cdot \sqrt[3]{\left(\frac{x+2}{x-2}\right)^2}$

19. $y' = \frac{x^2}{x^4-x^2+1}$

20. $y' = \frac{2x^3}{|x^2-1|}$

21. $y' = \frac{1}{2}$

22. $y' = \frac{1}{\sqrt{x^2+x}}$

23. $y' = \frac{1}{2cos^2\sqrt{x}}$

24. $y' = 5 \cdot e^x \cdot \cos 2x$

25. $y' = 2\sin\ln x$

"Matematika" fanidan glossariy

Atamaning o'zbek tilida nomlanishi	Atamaning ingliz tilida nomlanishi	Atamaning rus tilida nomlanishi	Atamaning ma'nosi
Ajoyib limitlar	He canonical form of the quadratic form	Замечательные пределы	1- ajoyib limit: $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$; 2- ajoyib limit: $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e$.
Algebraik to'ldiruvchi	The algebraic addition	Алгебраическое дополнение	a_{ij} minorning (elementning) algebraik to'ldiruvchisi $A_{ij}=(-1)^{i+j}M$ formula bilan aniqlanadi.
Aniq integral	Definite integral	Определенный интеграл	$\sigma=\sum\limits_{i=1}^{n}f(\overline{x}_i)\Delta x_i$ integral yig'indining eng katta qismiy kesma uzunligi nolga intilgandagi limitiga $f(x)$ funksiyaning $[a;b]$ kesmadagi aniq integrali deb aytiladi va $\int\limits_a^b f(x)dx$ kabi belgilanadi. Bu yerda, a va b sonlar integralning **quyi va yuqori chegaralari** deyiladi.
Aniq integralda o'zgaruvchini almashtirish va bo'laklab integrallash	Integration by parts and replacement of the variable in a definite integral.	Интегрирование по частям и замена переменной в определенном интеграле.	$f(x)$ funksiya $[a;b]$ kesmada uzluksiz va $\int\limits_a^b f(x)dx$ integral berilgan bo'lsin. $x=\varphi(t)$ almashtirish bilan ifoda integrallash o'zgaruvchisi t bo'lgan

			yangi aniq integralga keladi. Bunda $\varphi(t)$, $\varphi'(t)$ funksiyalar $[\alpha;\beta]$ kesmada, $\varphi(\alpha)=a$, $\varphi(\beta)=b$, uzluksiz bo'lishi kerak. Bu shartlar bajarilganda, $\int\limits_a^b f(x)dx=\int\limits_\alpha^\beta f(\varphi(t))\varphi'(t)dt$ formula o'rinli bo'ladi.
Aniq sistema	Certain system	Определенная система	Birgalikda bo'lgan sistema yagona yechimga ega bo'lsa aniqsistema deyiladi.
Aniqmas integral	Indefinite integral	Неопределенный интеграл	(a,b) intervalda berilgan $f(x)$ funksiya boshlang'ich funksiyalarning umumiy ifodasi $F(x)+C$, bu yerda C=const, shu $f(x)$ funksiyaning aniqmas integrali deb ataladi va u $\int f(x)dx$ kabi belgilanadi. Bunda $\int$ - integral belgisi, $f(x)$ integral ostidagi funksiya, $f(x)dx$ - integral ostidagi ifoda, x – integrallash o'zgaruvchisi deb ataladi.
Aniqmas sistema	Uncertain system	Неопределенная система	Birgalikda bo'lgan sistema cheksiz ko'p yechimga ega bo'lsa aniqmassistema deyiladi.
Arifmetik vektor fazo	Arithmetic vector space	Арифметическое векторное пространство	n o'lchovli vektorlar to'plamiga chiziqli (vektorlarni qo'shish va vektorlarni songa ko'paytirish) amallar bilan birgalikda n o'lchovli arifmetik vektor fazo deyiladi.

"Matematika" fanidan glossariy

Atamaning o'zbek tilida nomlanishi	Atamaning ingliz tilida nomlanishi	Atamaning rus tilida nomlanishi	Atamaning ma'nosi
Ajoyib limitlar	He canonical form of the quadratic form	Замечательные пределы	1- ajoyib limit: $\lim_{x\to 0}\frac{\sin x}{x}=1$; 2- ajoyib limit: $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$.
Algebraik to'ldiruvchi	The algebraic addition	Алгебраическое дополнение	a_{ij} minorning (elementning) algebraik to'ldiruvchisi $A_{ij}=(-1)^{i+j}M$ formula bilan aniqlanadi.
Aniq integral	Definite integral	Определенный интеграл	$\sigma=\sum_{i=1}^{n}f(\bar{x}_i)\Delta x_i$ integral yig'indining eng katta qismiy kesma uzunligi nolga intilgandagi limitiga $f(x)$ funksiyaning $[a;b]$ kesmadagi aniq integrali deb aytiladi va $\int_a^b f(x)dx$ kabi belgilanadi. Bu yerda, a va b sonlar integralning **quyi va yuqori chegaralari** deyiladi.
Aniq integralda o'zgaruvchini almashtirish va bo'laklab integrallash	Integration by parts and replacement of the variable in a definite integral.	Интегрирование по частям и замена переменной в определенном интеграле.	$f(x)$ funksiya $[a;b]$ kesmada uzluksiz va $\int_a^b f(x)dx$ integral berilgan bo'lsin. $x=\varphi(t)$ almashtirish bilan ifoda integrallash o'zgaruvchisi t bo'lgan

			yangi aniq integralga keladi. Bunda $\varphi(t)$, $\varphi'(t)$ funksiyalar $[\alpha;\beta]$ kesmada, $\varphi(\alpha)=a$, $\varphi(\beta)=b$, uzluksiz bo'lishi kerak. Bu shartlar bajarilganda, $\int_a^b f(x)dx=\int_\alpha^\beta f(\varphi(t))\varphi'(t)dt$ formula o'rinli bo'ladi.
Aniq sistema	Certain system	Определенная система	Birgalikda bo'lgan sistema yagona yechimga ega bo'lsa aniqsistema deyiladi.
Aniqmas integral	Indefinite integral	Неопределенный интеграл	(a,b) intervalda berilgan $f(x)$ funksiya boshlang'ich funksiyalarning umumiy ifodasi $F(x)+C$, bu yerda C=const, shu $f(x)$ funksiyaning aniqmas integrali deb ataladi va u $\int f(x)dx$ kabi belgilanadi. Bunda $\int$ - integral belgisi, $f(x)$ integral ostidagi funksiya, $f(x)dx$ - integral ostidagi ifoda, x – integrallash o'zgaruvchisi deb ataladi.
Aniqmas sistema	Uncertain system	Неопределенная система	Birgalikda bo'lgan sistema cheksiz ko'p yechimga ega bo'lsa aniqmassistema deyiladi.
Arifmetik vektor fazo	Arithmetic vector space	Арифметическое векторное пространство	n o'lchovli vektorlar to'plamiga chiziqli (vektorlarni qo'shish va vektorlarni songa ko'paytirish) amallar bilan birgalikda n o'lchovli arifmetik vektor fazo deyiladi.

Aylana	Definitely negative quadratic form	Окружность	Fiksirlangan $M_0(a,b)$ nuqtadan bir xil R-masofada yotgan nuqtalarning geometrik o'rniga aylana deyiladi. $\sqrt{(x-a)^2+(y-b)^2}=R$
Bir tomonlama chekli limitlar	Unilateral finite limits	Односторонние конечные пределы	Agar ixtiyoriy $\varepsilon>0$ son uchun $\exists\delta>0$ sonni ko`rsatish mumkin bo`lsaki va $x_0-\delta<x<x_0$ $(x_0<x<x_0+\delta)$ shartni qanoatlantiruvchi barcha x lar uchun $\lvert f(x)-b\rvert<\varepsilon$ tengsizlik bajarilsa, $b=f(x_0-0)$ $(b=f(x_0+0))$ son $f(x)$ funksiyaning $x\to x_0$ da chapdan (o`ngdan) limiti deyiladi.
Bir tomonlama va cheksiz hosila	Unilateral and endless derivatives	Односторонние и бесконечные производные	Agar $f(x)$ funksiya x_0 nuqtada uzluksiz bo'lib, $\lim\limits_{\Delta x\to 0}\dfrac{f(x_0+\Delta x)-f(x_0)}{\Delta x}\to\infty$ bo'lsa, u holda $f(x)$ funksiyaning x_0 nuqtadagi hosilasi chegaralanmagan deyiladi. $\lim\limits_{\Delta x\to 0-0}\dfrac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$, $\lim\limits_{\Delta x\to 0+0}\dfrac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ limitlarga, mos ravishda, $f(x)$ funksiyaning x_0 nuqtada chap va o'ng hosilalari deyiladi. Bu hosilalarni mos ravishda

			$f'_-(x_0)$, $f'_+(x_0)$ ko'rinishda belgilash mumkin.
Birgalikda bo‘lgan sistema	Co (permissible) system	Совместная (разрешимая) система	Chiziqli tenglamalar sistemasi kamida bitta yechimga ega bo'lsa, u holda bunday sistema birgalikda deyiladi.
Birgalikda bo'lmagan sistema	Incompatibility (insoluble) system	Несовместная (неразрешимая) система	Bitta ham yechimga ega bo'lmagan chiziqli tenglamalar sistemasi birgalikda bo'lmagan sistema deyiladi.
Birlik matritsa	The identity matrix	Единичная матрица	$A=(a_{ij})$ kvadrat matritsada $i\neq j$ bo'lganda $a_{ij}=0$ va $i=j$ bo'lganda esa $a_{ii}=1$ bo'lsa,u holdabunday matritsaga birlik matritsadeyiladi.
Bo‘laklab integrallash formulasi	The formula for integration by parts	Формула интегрирования по частям	Bo‘laklab integrallash usuli ikki funksiya ko‘paytmasining differensiali formulasidan kelib chiqadi. Ma'lumki, agar *u(x)* va *v(x)* funksiyalar differensiallanuvchi funksiyalar bo‘lsa, u holda *d(uv)=udv+vdu* yoki *udv=d(uv)-vdu* bo‘ladi. Bu tenglikni ikkala tomonini integrallasak, $\int udv=\int d(uv)-\int vdu$, yoki $\int udv=uv-\int vdu$ formula hosil bo‘ladi. Bu formula bo‘laklab integrallash formulasi deyiladi.
Boshlang‘ich funksiya	The primitive	Первообразная	Agar *(a,b)* da *f(x)* funksiya biror *F(x)* funksiyaning

			hosilasiga teng, ya'ni (a,b) intervaldan olingan ixtiyoriy x uchun $F'(x)=f(x)$ bo'lsa, u holda $F(x)$ funksiya (a,b) intervalda $f(x)$ funksiyaning boshlang'ich funksiyasi deyiladi.
Burchak koeffitsiyenti	The quadratic form	Угловой коэффи-циент	$k=tg\varphi$ to'g'ri chiziqning burchak koeffitsienti
Burilish nuqtasi	Inflection point	Точка перегиба	x_0 nuqta atrofida berilgan y=f(x) funksiya uchun, x_0 nuqta burilish nuqtasi deyiladi, agarda x_0 nuqta y=f(x) funksiyaning botiqlik va qavariqlik intervallarini ajratib turuvchi chegaraviy nuqta bo'lsa.
Chegaralangan ketma – ketlik	Limited sequence	Ограни-ченная последова-тельность	Agar R^n fazoda nuqtalar ketma – ketligi chekli limitga ega bo`lsa u chegaralangan bo`ladi.
Chiziqli bog'liq va chiziqli erkli vektorlar	Linearly dependent and linearly independe nt vectors	Линейно зависимые и линейно незави-симые вектора	Agar $\lambda_1,\lambda_2,...,\lambda_n$ koeffitsentlardan aqqali bittasi noldan farqli bo'lganda $\lambda_1 X_1+\lambda_2 X_2+\cdots+\lambda_n X_n=\Theta$ (1) tenglik o'rinli bo'lsa, u holda $X_1,X_2,...,X_n$ vektorlar chiziqli bog'liq deyiladi. Bunda, Θ - nol vektor. Aks holda $X_1,X_2,...,X_n$ vektorlar chiziqli erkli deyiladi.
Chiziqli fazo	The linear space	Линейное про-странство	Agar elementlari ixtiyoriy tabiatli bo'lgan L to`plam berilgan va bu toplam elementlari orasida qo'shish va songa ko'paytirish

			amallari kiritilgan bo'lsa u holda L to'plam *chiziqli* (yoki *vertor*) *fazo* deyiladi.
Chiziqli tenglamalar sistemasi	The system of linear equations	Система линейных уравнений	Quyidagi $\begin{cases} a_{11}x_1+a_{12}x_2+...+a_{1n}x_n=b_1, \\ a_{21}x_1+a_{22}x_2+...+a_{2n}x_n=b_2, \\ ... \quad ... \quad ... \quad ... \quad ... \\ a_{m1}x_1+a_{m2}x_2+...+a_{mn}x_n=b_m \end{cases}$ sistemaga n noma'lumli m ta chiziqli algebraik tenglamalar sistemasi (yoki soddalik uchun chiziqli tenglamalar sistemasi) deyiladi.
Chegaralangan funksiya	Limited function	Ограничен-ные функция	Agar $y=f(x)$ funksiya $V_1\in D(f)$ nuqtalar to'plamida ham quyidan, va ham yuqoridan chegaralangan bo'lsa, u holda $y=f(x)$ funksiya V_1 to'plamda chegaralangan funksiya deb ataladi.
Cheksiz kichik va cheksiz katta ketma – ketlik	Infinitely small and infinitely large	Бесконечно малые и бесконечно большие последова-тельности	Oldindan tayinlanadigan har qanday $\varepsilon>0$ son uchun $\{a_k\}$ sonli ketma – ketlikning shunday bir N (ε ga bog'liq) tartib raqamini ko'rsatish mumkin bo'lsaki, barcha $k>N$ tartib raqamli hadlari uchun $\|a_k\|<\varepsilon$ tengsizlik qanoatlantirilsa, $\{a_k\}$ sonli ketma – ketlikka cheksiz kichik sonli ketma – ketlik deyiladi. Limiti nolga teng har qanday sonli ketma – ketlikkacheksiz kichik sonli ketma – ketlik deyiladi.

			Oldindan tayinlanadigan har qanday $A>0$ son uchun $\{\gamma_k\}$ sonli ketma – ketlikning shunday bir N (A ga bog‘liq) tartib raqamini tanlash mumkin bo‘lsaki, barcha $k>N$ tartib raqamli hadlari uchun $\lvert\gamma_k\rvert>A$ tengsizlik o‘rinli bo‘lsa, $\{\gamma_k\}$ sonli ketma – ketlik cheksiz katta sonli ketma – ketlik deyiladi.
Determinant elementlari	The elements of the determinant	Элементы определителя	a_{ij} – determinantning i–satr j–ustunda joylashgan elementini ifodalaydi.
Diagonal matritsa	Diagonal matrix	Диагональная матрица	$A=(a_{ij})$ kvadrat matritsada $i\neq j$ bo’lganda, $a_{ij}=0$ bo’lsa, u holda A matritsaga *diagonal matritsa* deyiladi.
Differensiallash	Differentiation	Дифференцирование	Berilgan $f(x)$ funksiyaning hosilasini topish amali ko’p hollarda $f(x)$ funksiyani differensiallash deb yuritiladi.
Davriy funksiya	Periodic function	Периодическая функция	Agar $y=f(x)$ funksiya uchun shunday bir musbat t son mavjud bo‘lsaki, funksiyaning aniqlanish sohasiga tegishli har qanday x va $x+t$ nuqtalar uchun $f(x+t)=f(x)$ tenglik bajarilsa, $y=f(x)$ funksiya davriy funksiya deyiladi.
Ekstremumning yetarli sharti	Sufficient optimality	Достаточные	Teorema (etarli shart). $f(x)$ funksiya x_0 kritik nuqtaning

	conditions	условия экстремума	biror δ atrofida differensiallanuvchi, x_0 nuqtaning o’zida esa uzluksiz bo’lib, differensiallanuvchi bo’lishi shart bo’lmasin. Agar $(x_0-\delta;x_0)$ va $(x_0;x_0+\delta)$ intervallarda $f'(x)$ hosila qarama–qarshi ishorali qiymatlarga erishsa, x_0 nuqta $f(x)$ funksiyaning ekstremum nuqtasi bo’ladi.
Ekstremumning zaruriy sharti	Necessary optimality conditions	Необходимые условия экстремума	Teorema. (funksiya ekstremumining zaruriylik sharti). Agar x_0 nuqta $f(x)$ funksiyaning ekstremum nuqtasi bo’lib, funksiya uning biror bir atrofida aniqlangan bo’lsa, u holda $f'(x_0)=0$ yoki $f'(x_0)$ – mavjud emas.
Ekvivalent sistemalar	Equivalent (tantamount to) system	Эквивалентные (равносильные) системы	Agar ikkita sistemaning yechimlari bir xil sonlar to’plamidan iborat bo’lsa, bunday sistemalar teng kuchliyoki ekvivalent deyiladi.
Ellips	Positive matrix	Эллипс	Fiksirlangan F_1 va F_2 nuqtalargacha bo’lgan masofalar yig’indisi o’zgarmas $2a$ kattalikka teng bo’lgan nuqtalarning geometrik o’rniga ellips deyiladi. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Fazoda to’g’ri chiziqning kanonik	Canonical equations of a	Канонические уравнения	$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ ko‘rinishidagi tenglama

			Oldindan tayinlanadigan har qanday $A>0$ son uchun $\{\gamma_k\}$ sonli ketma – ketlikning shunday bir N (A ga bog'liq) tartib raqamini tanlash mumkin bo'lsaki, barcha $k>N$ tartib raqamli hadlari uchun $\lvert\gamma_k\rvert>A$ tengsizlik o'rinli bo'lsa, $\{\gamma_k\}$ sonli ketma – ketlik cheksiz katta sonli ketma – ketlik deyiladi.
Determinant elementlari	The elements of the determinant	Элементы определителя	a_{ij} – determinantning i–satr j–ustunda joylashgan elementini ifodalaydi.
Diagonal matritsa	Diagonal matrix	Диагональная матрица	$A=(a_{ij})$ kvadrat matritsada $i\neq j$ bo'lganda, $a_{ij}=0$ bo'lsa, u holda A matritsaga *diagonal matritsa* deyiladi.
Differensiallash	Differentiation	Дифференцирование	Berilgan $f(x)$ funksiyaning hosilasini topish amali ko'p hollarda $f(x)$ funksiyani differensiallash deb yuritiladi.
Davriy funksiya	Periodic function	Периодическая функция	Agar $y=f(x)$ funksiya uchun shunday bir musbat t son mavjud bo'lsaki, funksiyaning aniqlanish sohasiga tegishli har qanday x va $x+t$ nuqtalar uchun $f(x+t)=f(x)$ tenglik bajarilsa, $y=f(x)$ funksiya davriy funksiya deyiladi.
Ekstremumning yetarli sharti	Sufficient optimality	Достаточные	Teorema (etarli shart). $f(x)$ funksiya x_0 kritik nuqtaning

	conditions	условия экстремума	biror δ atrofida differensiallanuvchi, x_0 nuqtaning o'zida esa uzluksiz bo'lib, differensiallanuvchi bo'lishi shart bo'lmasin. Agar $(x_0-\delta; x_0)$ va $(x_0; x_0+\delta)$ intervallarda $f'(x)$ hosila qarama–qarshi ishorali qiymatlarga erishsa, x_0 nuqta $f(x)$ funksiyaning ekstremum nuqtasi bo'ladi.
Ekstremumning zaruriy sharti	Necessary optimality conditions	Необходимые условия экстремума	Teorema. (funksiya ekstremumining zaruriylik sharti). Agar x_0 nuqta $f(x)$ funksiyaning ekstremum nuqtasi bo'lib, funksiya uning biror bir atrofida aniqlangan bo'lsa, u holda $f'(x_0)=0$ yoki $f'(x_0)$ – mavjud emas.
Ekvivalent sistemalar	Equivalent (tantamount to) system	Эквивалентные (равносильные) системы	Agar ikkita sistemaning yechimlari bir xil sonlar to'plamidan iborat bo'lsa, bunday sistemalar teng kuchliyoki ekvivalent deyiladi.
Ellips	Positive matrix	Эллипс	Fiksirlangan F_1 va F_2 nuqtalargacha bo'lgan masofalar yig'indisi o'zgarmas $2a$ kattalikka teng bo'lgan nuqtalarning geometrik o'rniga ellips deyiladi. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Fazoda to'g'ri chiziqning kanonik	Canonical equations of a	Канонические уравнения	$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ ko'rinishidagi tenglama

tenglamasi	straight line in space	прямой в пространстве	fazoda to'g'ri chiziqning kanonik tenglamasi deyiladi.
Ferma teoremasi	Fermat's last theorem	Теорема Ферма	**Teorema.** (Ferma). Agar x_0 nuqtada $f(x)$ funksiya differensiallanuvchi va lokal ekstremumga erishsa, u holda shu nuqtada $f'(x)=0$ bo'ladi.
Fueksiya grafigiga o'tkazilgan normal	Normal to the graph of the function	Нормаль к графику функции	$y-f(x_0)=-\left(\frac{1}{f'(x_0)}\right)(x-x_0)$ (normal tenglamasi).
Funksiya differensiali	Differential function	Диффе-ренциал функции	Agar $y=f(x)$ x_0 nuqtaning δ atrofida aniqlangan bo`lib, uning $\Delta y$ orttirmasini $\Delta y = A\Delta x + \Delta x\,\varepsilon(\Delta x)$ ko`rinishda tasvirlash mumkin bo`lsa, u holda $y=f(x)$ funksiya $x_0$ nuqtada differensiallanuvchi $A\Delta x$ esa uning differensiali deb ataladi. Bu yerda $A(x_0)$ $\Delta x$ ga bog`liq emas, $\lim_{\Delta\varepsilon\to 0}\varepsilon(\Delta x)\to 0$. Funksiya differensiali quyidagicha yoziladi: $dy=df=Adx,\ A=f'(x)$.
Funksiya grafigiga urinma	The tangent to the graph of the function	Касатель-ная к графику функции	$f'(x_0)$ qiymat $M(x_0;f(x_0))$ nuqtada $f(x)$ funksiyaga o`tkazilgan urinmaning $tg\varphi=f'(x_0)$ – burchak koeffitsientini bildiradi. $M(x_0;f(x_0))$ nuqtada

			$f(x)$ funksiyaga o`tkazilgan urinma tenglamasi quyidagi ko`rinishga ega bo`ladi: $y-f(x_0)=f'(x_0)(x-x_0)$.
Funksiya hosilasi	Derivative function	Производ-ная функции	$y=f(x)$ funksiya $x=x_0$ nuqtaning biror bir atrofida aniqlangan va $\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ mavjud bo`lsin. U holda bu limit $f(x)$ funksiyaning x_0 nuqtadagi hosilasi deb ataladi va quyidagicha belgilanadi: $f'(x_0)$, $f'_x(x_0)$, $y'(x_0)$, $f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$
Funksiya qavariqligi	Function bulge	Выпуклость функции	(a;b) intervalda hosilaga ega bo'lgan y=f(x) funksiya, bu oraliqda qavariq (botiq) bo'lishi uchun, uning f '(x) hosilasi (a;b) intervalda kamayuvchi (o'suvchi) bo'lishi zarur va yetarlidir.
Funksiyaning nuqtadagi limiti	The limit function at	Предел функции в точке	Agar har bir hadi V to'plamga tegishli va M_0 quyuqlanish nuqtasidan har qanday farqli $M_1, M_2, ..., M_k, ...$ nuqtalar ketma – ketligi M_0 nuqtaga intilganda, bu nuqtalarga mos funksiya qiymatlarining $f(M_1), f(M_2), ..., f(M_k), ...$ sonli ketma – ketligi b songa intilsa, uholda b soni $f(M)$ funksiyaning $M\to M_0$ dagi

tenglamasi	straight line in space	прямой в пространстве	fazoda to'g'ri chiziqning kanonik tenglamasi deyiladi.
Ferma teoremasi	Fermat's last theorem	Теорема Ферма	**Teorema.** (Ferma). Agar x_0 nuqtada $f(x)$ funksiya differensiallanuvchi va lokal ekstremumga erishsa, u holda shu nuqtada $f(x)=0$ bo'ladi.
Fueksiya grafigiga o'tkazilgan normal	Normal to the graph of the function	Нормаль к графику функции	$y-f(x_0)=-\left(\frac{1}{f'(x_0)}\right)(x-x_0)$ (normal tenglamasi).
Funksiya differensiali	Differential function	Дифференциал функции	Agar $y=f(x)$ x_0 nuqtaning δ atrofida aniqlangan bo'lib, uning Δy orttirmasini $\Delta y=A\Delta x+\Delta x\varepsilon(\Delta x)$ ko'rinishda tasvirlash mumkin bo'lsa, u holda $y=f(x)$ funksiya x_0 nuqtada differensiallanuvchi $A\Delta x$ esa uning differensiali deb ataladi. Bu yerda $A(x_0)$ Δx ga bog'liq emas, $\lim_{\Delta x\to 0}\varepsilon(\Delta x)\to 0$. Funksiya differensiali quyidagicha yoziladi: $dy=df=Adx,\ A=f'(x)$.
Funksiya grafigiga urinma	The tangent to the graph of the function	Касательная к графику функции	$f'(x_0)$ qiymat $M(x_0;f(x_0))$ nuqtada $f(x)$ funksiyaga o'tkazilgan urinmaning $tg\varphi=f'(x_0)$ –burchak koeffitsientini bildiradi. $M(x_0;f(x_0))$ nuqtada

			$f(x)$ funksiyaga o'tkazilgan urinma tenglamasi quyidagi ko'rinishga ega bo'ladi: $y-f(x_0)=f'(x_0)(x-x_0)$.
Funksiya hosilasi	Derivative function	Производная функции	$y=f(x)$ funksiya $x=x_0$ nuqtaning biror bir atrofida aniqlangan va $\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ mavjud bo'lsin. U holda bu limit $f(x)$ funksiyaning x_0 nuqtadagi hosilasi deb ataladi va quyidagicha belgilanadi: $f'(x_0)$, $f'_x(x_0)$, $y'(x_0)$, $f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$
Funksiya qavariqligi	Function bulge	Выпуклость функции	(a;b) intervalda hosilaga ega bo'lgan y=f(x) funksiya, bu oraliqda qavariq (botiq) bo'lishi uchun, uning f '(x) hosilasi (a;b) intervalda kamayuvchi (o'suvchi) bo'lishi zarur va yetarlidir.
Funksiyaning nuqtadagi limiti	The limit function at	Предел функции в точке	Agar har bir hadi V to'plamga tegishli va M_0 quyuqlanish nuqtasidan farqli har qanday $M_1, M_2, ..., M_k, ...$nuqtalar ketma – ketligi M_0 nuqtaga intilganda, bu nuqtalarga mos funksiya qiymatlarining $f(M_1), f(M_2), ..., f(M_k), ...$ sonli ketma – ketligi b songa intilsa, uholda b soni $f(M)$ funksiyaning $M\to M_0$ dagi

			limiti deyiladi.
Funksiyaning aniqlanish sohasi	The domain of the function	Область определения функции	Agar $X \subset R^n$, $Y \subset R^n$ bo'isa, u holda f qonuniyat funksya deb ataladi. Bu yerda X aniqlanish sohasi $(D(f))$.
Funksiyaning eng katta va eng kchik qiymatlari	Maximum and minimum values of the function	Наибольшее и наименьшее значения функции	Funksiyaning kesmada eng katta va eng kichik qiymatlarini topish uchun: a) funksiyaning kesmaga tegishli kritik nuqtalari aniqlaniladi; b) funksiyaning topilgan kritik nuqtalaridagi va kesmaning chetki nuqtalaridagi qiymatlari hisoblanadi; c) ushbu qiymatlar o'zaro solishtirilib uning eng katta, eng kichigi tanlanadi.
Funksiyaning o'sishi va kamayishi	Increase and decrease of function	Возрастание и убывание функции	V oraliqda differensiallanuvchi $f(x)$ funksiya shu oraliqda o'suvchi (kamayuvchi) bo'lishi uchun, oraliqning har bir ichki nuqtasida $f'(x) \geq 0$ $(f'(x) \leq 0)$ bo'lishi zarur va yetarli.
Funksiyaning qiymatlar to'plami	The set values of the function	Множество значений функции	Agar $X \subset R^n$, $Y \subset R^n$ bo'lsa, u holda f qonuniyat funktsya deb ataladi. Y esa qiymatlar to'plami deyiladi $(E(f))$.
Giperbola	A non-negative matrix	Гипербола	Fiksirlangan F_1 va F_2 nuqtalargacha bo'lgan masofalar ayirmasining absolyut qiymati o'zgarmas $2a$ kattalikka teng bo'lgan

			nuqtalarning geometrik o'rniga giperbola deyiladi. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
Gradiyent	Gradient	Градиент	$y=f(M)$ funksiyaning $\nabla y = grad\ y(M)$ gradiyenti deb $(y'_{x_1}, y'_{x_2}, \ldots, y'_{x_n})$ koordinatali vektorga aytiladi.
Ichki nuqta	Inner point	Внутренняя точка	$\varepsilon > 0$ son mavjud bo'lsin. Agar $M_0(x_1^0, x_2^0, \ldots, x_n^0) \in V$ nuqtaning ε atrofi $S_\varepsilon(M_0)$ V to'plamga tegishli bo'lsa, u holda M_0 nuqta V to'plamning ichki nuqtasi deyiladi.
Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi	The equation of a straight line passing through two points	Уравнение прямой проходящей через две точки	$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ ikkita nuqtadan o'tuvchi to'g'ri chiziq tenglamasi.
Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi	The equation of a straight line passing through two points	Уравнение прямой проходящей через две точки	$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$ tenglamaga fazoda ikkita nuqtadan o'tuvchi to'g'ri chiziq tenglamasi deyiladi.
Ikki to'g'ri chiziq orasidagi burchak	The angle between the straight lines	Угол между прямыми	$tg\theta = \left\|\frac{k_2-k_1}{1+k_1k_2}\right\|$ ikki to'g'ri chiziq orasidagi burchakni topish formulasi.

			limiti deyiladi.
Funksiyaning aniqlanish sohasi	The domain of the function	Область определения функции	Agar $X \subset R^n$, $Y \subset R^n$ bo'lsa, u holda f qonuniyat funksya deb ataladi. Bu yerda X aniqlanish sohasi $(D(f))$.
Funksiyaning eng katta va eng kchik qiymatlari	Maximum and minimum values of the function	Наибольшее и наименьшее значения функции	Funksiyaning kesmada eng katta va eng kichik qiymatlarini topish uchun: a) funksiyaning kesmaga tegishli kritik nuqtalari aniqlaniladi; b) funksiyaning topilgan kritik nuqtalaridagi va kesmaning chetki nuqtalaridagi qiymatlari hisoblanadi; c) ushbu qiymatlar o'zaro solishtirilib uning eng katta, eng kichigi tanlanadi.
Funksiyaning o'sishi va kamayishi	Increase and decrease of function	Возрастание и убывание функции	V oraliqda differensiallanuvchi $f(x)$ funksiya shu oraliqda o'suvchi (kamayuvchi) bo'lishi uchun, oraliqning har bir ichki nuqtasida $f'(x) \geq 0$ $(f'(x) \leq 0)$ bo'lishi zarur va yetarli.
Funksiyaning qiymatlar to'plami	The set values of the function	Множество значений функции	Agar $X \subset R^n$, $Y \subset R^n$ bo'lsa, u holda f qonuniyat funktsya deb ataladi. Y esa qiymatlar to'plami deyiladi $(E(f))$.
Giperbola	A non-negative matrix	Гипербола	Fiksirlangan F_1 va F_2 nuqtalargacha bo'lgan masofalar ayirmasining absolyut qiymati o'zgarmas $2a$ kattalikka teng bo'lgan

			nuqtalarning geometrik o'rniga giperbola deyiladi. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
Gradiyent	Gradient	Градиент	$y=f(M)$ funksiyaning $\nabla y = grad\ y(M)$ gradiyenti deb $(y'_{x_1}, y'_{x_2}, ..., y'_{x_n})$ koordinatali vektorga aytiladi.
Ichki nuqta	Inner point	Внутренняя точка	$\varepsilon > 0$ son mavjud bo'lsin. Agar $M_0(x_1^0, x_2^0, ..., x_n^0) \in V$ nuqtaning ε atrofi $S_\varepsilon(M_0)$ V to'plamga tegishli bo'lsa, u holda M_0 nuqta V to'plamning ichki nuqtasi deyiladi.
Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi	The equation of a straight line passing through two points	Уравнение прямой проходящей через две точки	$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ ikkita nuqtadan o'tuvchi to'g'ri chiziq tenglamasi.
Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi	The equation of a straight line passing through two points	Уравнение прямой проходящей через две точки	$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$ tenglamaga fazoda ikkita nuqtadan o'tuvchi to'g'ri chiziq tenglamasi deyiladi.
Ikki to'g'ri chiziq orasidagi burchak	The angle between the straight lines	Угол между прямыми	$tg\theta=\left\|\frac{k_2-k_1}{1+k_1k_2}\right\|$ ikki to'g'ri chiziq orasidagi burchakni topish formulasi.

Ikkinchi tartibli determinant	The determinant of order 2	Определитель 2-го порядка	$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ ifoda ikkinchi tartibli determinant deyiladi.
Ikkinchi tur uzilish nuqtasi	The second kind of break point	Точка разрыва второго рода	Agar $y=f(x)$ funksiyaning x_0 nuqtada chapdan yoki o‘ngdan limitlarining hech bo’lmaganda bittasi mavjud bo‘lmasa yoki cheksiz bo‘lsa u holda x_0 nuqta $y=f(x)$ funksiyaning ikkinchi tur uzilish nuqtasi deyiladi.
Ixtiyoriy ikki nuqta orasidagi masofa	The distance between any two points	Расстояние между любыми двумя точками	R^n fazoda $M(x_1,x_2,...,x_n)$ va $N(y_1,y_2,...,y_n)$ nuqtalar berilgan bo‘lsin. Bu nuqtalar orasidagi masofa real fazoda qo‘llanilgan formulani umumlashtirish asosida aniqlanadi. Berilgan n o‘lchovli M va N nuqtalar orasidagi masofa $\rho(M,N)$ ko’rinishda belgilanib, u $\rho(M,N)=\sqrt{\sum_{i=1}^{n}(x_i-y_i)^2}$ formula asosida hisoblanadi.
Juft va toq funksiya	Even and odd function	Четная и нечетная функция	Agar har qanday $x \in V$ uchun $f(-x)=f(x)$ $(f(-x)=-f(x))$ tenglik o‘rinli bo‘lsa, $y=f(x)$ funksiya V to‘plamda juft (toq) funksiya deyiladi.
Keltirilgan sistema	Present system	Приведенная система	m noma’lumli n ta chiziqli bir jinsli bo’lmagan tenglamalar sistemasi vektor shaklda berilgan bo’lsin: $a_1x_1+a_2x_2+...+a_mx_m=b$

			Sistemaning ozod xadlari ustuni nol ustun bilan almashtirilgan $a_1x_1+a_2x_2+...+a_mx_m=\theta$ ko’rinishiga dastlabki bir jinslimas sistemaning keltirilgan sistemasi deyiladi.
Kroneker-kapelli teoremasi	Theorem of kronecker - capelli	Теорема Кронекера – Капелли	Chiziqli tenglamalar sistemasi birgalikda bo’lishi uchun uning asosiy va kengaytirilgan matritsalarining ranglari teng bo’lishi zarur va yetarli, ya’ni $r(A)=r(A\|B)$
Kvadrat matritsa	A square matrix	Квадратная матрица	Ham satrlar soni,ham ustunlar soni n ga teng bo’lgan, ya’ni(n x n) o’lchamli matritsaga n - tartibli kvadrat matritsa deyiladi.
Kvadratik shakl	The quadratic form	Квадратичная форма	n ta $x_1,x_2,...,x_n$ noma’lumlarning $f(x)$ kvadratik shakli deb har bir hadi bu noma’lumlarning ikkita kvadrati yoki ikkita noma’lumning ko’paytmasidan iborat bo’lgan $f=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}x_ix_j$ yig’ndiga aytiladi.
Kvadratik shaklning kanonik shakli	He canonical form of the quadratic form	Канонический вид квадратичной формы	Agar kvadratik shaklda turli noma’lumlar ko’paytmalari oldidagi barchak koeffitsiyentlar nolga teng bo’lsa, u holda bu shakl kanonik shakl deb ataladi. $f=b_1y_1^2+b_2y_2^2+....+b_ny_n^2$

Laplas teoremasi	Laplace theorem	Теорема Лапласа	Laplas teoremasi. Determinantning qiymati uning ixtiyoriy satr (ustun) elementlari bilan, shu elementlarga mos algebraik to'ldiruvchilar ko'paytmalari yig'indisiga teng.
Lokal ekstremum	Local extremum	Локальный экстремум	Funksiyaning lokal maksimum valokal minimum nuqtalariga, uning local ekstremum nuqtalari deyiladi.
Lokal maksimum	Local maximum	Локальный максимум	Agar barcha $x \in (x_0-\delta;x_0)\cup(x_0;x_0+\delta)$ nuqtalar uchun $f(x)<f(x_0)$ tengsizlik o'rinli bo'lsa, u holda x_0 nuqta $f(x)$ funksiyaning qat'iy lokal maksimum nuqtasi deyiladi.
Lokal minimum	Local minimum	Локальный минимум	Agar barcha $x \in (x_0-\delta;x_0)\cup(x_0;x_0+\delta)$ nuqtalar uchun $f(x)>f(x_0)$ tengsizlik o'rinli bo'lsa, u holda x_0 nuqta $f(x)$ funksiyaning qat'iy lokal minimum nuqtasi deyiladi.
Manfiy aniqlangan kvadratik shakl	Definitely negative quadratic form	Определенно отрицательная квадратичная форма	Agar n ta noma'lumning haqiqiy koeffitsientli f kvadratik shakli n ta manfiy kvadratdan iborat normal ko'rinishga keltirilsa bu shakl manfiy aniqlangan deyiladi.
Matritsa	Matrix	Матрица	Matritsa deb m ta satr va n ta ustunga ega bo'lgan to'rtburchakli sonlar jadvaliga aytiladi

Matritsaning rangi	The rank of the matrix	Ранг матрицы	A matritsaningrangi deb, noldan farqli matritsa osti minorlarining eng katta tartibiga aytiladi va $rang(A)=r(A)$ ko'rinishida ifodalanadi.
Minor	Minor	Минор	n-tartibli d determinantning $1\le k\le n-1$ shartni qanoatlantiruvchi ixtiyoriy k ta satrlari va k ta ustunlari kesishgan joyda turgan, ya'ni bu satrlardan biriga hamda ustunlardan biriga tegishli bo'lgan elementlardan tashkil topgan k-tartibli matritsa d determinantning k-tartibli minori deb ataladi.
Murakkab funksiyani differensiallash	Differentiation of a composite function	Дифференцирование сложной функции	Agar $u=g(x)$ funksiya x_0 nuqtada differensiallanuvchi, o'z navbatida $y=f(u)$ funksiya ham $u_0=g(x_0)$ nuqtada differensiallanuvchi bo'lsa, u holda $y=f[g(x)]$ murakkab funksiya ham x_0 nuqtada differensiallanuvchi va bo'ladi $dy/dx=(dy/du)(du/dx)$ ya'ni $y'(x_0)=f'(u_0)g'(x_0)$bo'ladi.
Musbat aniqlangan kvadratik shakl	Definitely a positive quadratic form	Определенно положительная квадратичная форма	Agar n ta noma'lumning haqiqiy koeffitsientli f kvadratik shakli n ta musbat kvadratdan iborat normal ko'rinishga keltirilsa bu shakl musbat aniqlangan deyiladi.
Musbat matritsa	Positive matrix	Положительная	Har bir koordinatasi musbat vektorga musbat vektor

		матрица	deyilsa, har bir elementi musbat sonlardan iborat matritsaga esa musbat matritsa deyiladi.
Monoton ketma – ketliklar	Monotonous sequence	Монотонная последовательность	O'suvchi yoki kamayuvchi sonli ketma – ketliklar monoton ketma – ketliklar deb yuritiladi.
Nol matritsa	Zero matrix	Нулевая матрица	Har bir elementi nolga teng bo'lgan, ixtiyoriy o'lchamli matritsaga nol matritsa deyiladi.
Nuqtadan to'g'ri chiziqqacha masofa	Definitely a positive quadratic form	Расстояния от точки до прямой	$d=\left\|\frac{Ax_0+By_0+C}{\sqrt{A^2+B^2}}\right\|$ formulaga berilgan nuqtadan berilgan to'g'ri chiziqqacha masofani topish formulasi deyiladi.
Nuqtalar ketma – ketligining limiti	Unilateral finite limits	Предел последовательности точек	Agar ixtiyoriy $\varepsilon>0$ son uchun shunday K natural sonni (ε ga bog'liq ravishda) ko'rsatish mumkin bo'lsaki, barcha $k>K$ tartib raqamli hadlar uchun $M_k\in S_\varepsilon(M_0)$ bo'lsa, $M_0(x_1^0,x_2^0,...,x_n^0)$ nuqtaga $\{M_k\}$ nuqtalar ketma – ketligining limiti deyiladi.
Og'ma va gorizontal asimptota	Inclined and vertical asymptote	Наклонная и горизонтальная асимптоты.	$y=f(x)$ funksiya grafigi $y=kx+b$ og'ma asimptotaga ega bo'lishi uchun chekli limitlarning mavjud bo'lishi zarur va yetarli.
Ortogonal vektorlar	Orthogonal vectors	Ортогональные вектора	Agar ikkita vektorning skalyar ko'paytmasi nolga teng bo'lsa, u holda bunday vektorlar ortogonal vektorlar

			deyiladi.
Ortogonal vektorlar	Orthogonal vectors	Ортогональные векторы	n o'lchovli vektorlardan tarkib topgan vektorlar sistemasi berilgan bo'lib, sistema vektorlarining har qanday ikki jufti o'zaro ortogonal bo'lsa, u holda sistemaga ortogonal vektorlar sistemasi deyiladi.
Parabola	Negative matrix	Парабола	Berilgan F nuqtadan berilgan va berilgan to'g'ri chizig'idan bir xil uzoqlikda yotuvchi nuqtalarning geometrik o'rniga parabola deyiladi. $y^2=2px$
Parametrik va oshkormas funksiyalarni differensiallash	Parametrically defined and differentiation of implicit functions	Дифференцирование параметрически заданных и неявных функций	$y=f(x)$ funksiya parametrik ko'rinishda berilgan bo'lsin: $x=\varphi(t),\ y=\psi(t),\ t\in[\alpha,\beta]$. Agar $x=\varphi(t),\ y=\psi(t)$ funksiyalar differensiallanuvchi va $\varphi'(t)\neq 0$ bo'lsa, u holda y'_x mavjud bo'lib quyidagicha aniqlanadi: $y'_x=\frac{y'_t}{x'_t}=\frac{\psi'(t)}{\phi'(t)}$.
Qiya simmetrik matritsa	Skew-symmetric matrix	Кососимметрическая матрица	Agar A kvadrat matritsada $A=-A^T$ munosabat o'rinli bo'lsa, bunday matritsaga qiya simmetrik matritsa deb ataladi.
Qavariq to'plam	A convex set	Выпуклое множество	Agar V to'plamga tegishli ixtiyoriy M_1 va M_2

			nuqtalarini tutashtiruvchi $[M_1M_2]$ kesma ham V to'plamga tegishli bo'lsa, uholda V nuqtalarto'plamiga R^n fazoda qavariqto'plamdeyiladi.
R^n Fazoda ochiq to'plam	An open set in the space	Открытое мно-жество в пространстве R^n,	Agar V to`plamning barcha nuqtalari ichki nuqtalardan tashkil topgan bo'lsa, $V\subset R^n$ to`plam ochiq to`plam deyiladi.
R^n Fazoda yopiq to'plam	A closed set in the space	Замкнутое множество в простран-стве R^n	Agar V to`plamning barcha quyuqlanish nuqtalari o`ziga tegishli bo`lsa, $V\subset R^n$ to`plam yopiq to`plam deyiladi.
R^n Fazoda ochiq to'plam	An open set in the space	Открытое множество в простран-стве R^n,	Agar V to`plamning barcha nuqtalari ichki nuqtalardan tashkil topgan bo'lsa, $V\subset R^n$ to`plam ochiq to`plam deyiladi.
Satr matritsa	Matrix row	Матрица строка	(1x n) o'lchamli matritsaga satr matritsa deyiladi.
Silvestr mezoni	Criteria sylvester	Критерии сильвестра	Kvadratik shakl matritsasi bosh minorlari har birining musbat bo'lishi, uning musbat aniqlanishi uchun zarur va yetarli. Toq tartibli bosh *minorlarning har biri manfiy* bo'lib, juft tartibli bosh minorlar har birining musbat bo'lishi, kvadratik shaklning manfiy aniqlanishi uchun zarur va yetarli.
Simmetrik matritsa	The symmetric	Симмет-рическая	Agar A kvadrat matritsada $A=A^T$ munosabat o'rinli

	matrix	матрица	bo'lsa, u holda bunday matritsaga simmetrik matritsa deb ataladi.
Skalyar matritsa	Scalar matrix	Скалярная матрица	Agar diagonal matritsaning barcha a_{ii} elementlari o'zaro teng bo'lsa, u holda bunday matritsaga skalyar matritsa deyiladi.
Sonli ketma-ketlik	The limit points of the sequence	Числовая последова-тельность	Natural sonlar to'plamida aniqlangan funksiya sonli-ketma-ketlik deyiladi. $y=f(n),\ n\in N$.
n – tartibli determinant	The determina nt of order n –	Определи-тель n – го порядка	n – tartibli determinant deb $n!$ hadning quyidagi tartibda tuzilgan algebraik yig'indisiga aytiladi: hadlari matritsaning har qaysi satridan va har qaysi ustunidan bittadan olingan n ta elementdan tuzilgan bo'lib, mumkin bo'lgan barcha ko'paytmalar hizmat qiladi; shu bilan birga hadning indekslari juft o'rniga qo'yishni tashkil etsa, musbat ishora bilan, aks holda esa manfiy ishora bilan olinadi.
Teskari matritsa	Inverse matrix	Обратная матрица	Agar A kvadrat matritsa uchun $AA^{-1}=A^{-1}A=E$ tenglik bajarilsa, A^{-1} matritsa A matritsaga teskari matritsa deyiladi.
To'g'ri chiziqning	He canonical	Уравнение прямой с	$y=kx+b$

			nuqtalarini tutashtiruvchi $[M_1M_2]$ kesma ham V to'plamga tegishli bo'lsa, uholda V nuqtalarto'plamiga R^n fazoda qavariqto'plamdeyiladi.
R^n Fazoda ochiq to'plam	An open set in the space	Открытое мно-жество в пространстве R^n,	Agar V to`plamning barcha nuqtalari ichki nuqtalardan tashkil topgan bo'lsa, $V\subset R^n$ to`plam ochiq to`plam deyiladi.
R^n Fazoda yopiq to'plam	A closed set in the space	Замкнутое множество в простран-стве R^n	Agar V to`plamning barcha quyuqlanish nuqtalari o`ziga tegishli bo`lsa, $V\subset R^n$ to`plam yopiq to`plam deyiladi.
R^n Fazoda ochiq to'plam	An open set in the space	Открытое множество в простран-стве R^n,	Agar V to`plamning barcha nuqtalari ichki nuqtalardan tashkil topgan bo'lsa, $V\subset R^n$ to`plam ochiq to`plam deyiladi.
Satr matritsa	Matrix row	Матрица строка	(1x n) o'lchamli matritsaga satr matritsa deyiladi.
Silvestr mezoni	Criteria sylvester	Критерии сильвестра	Kvadratik shakl matritsasi bosh minorlari har birining musbat bo'lishi, uning musbat aniqlanishi uchun zarur va yetarli. Toq tartibli bosh *minorlarning har biri manfiy* bo'lib, juft tartibli bosh minorlar har birining musbat bo'lishi, kvadratik shaklning manfiy aniqlanishi uchun zarur va yetarli.
Simmetrik matritsa	The symmetric	Симмет-рическая	Agar A kvadrat matritsada $A=A^T$ munosabat o'rinli

	matrix	матрица	bo'lsa, u holda bunday matritsaga simmetrik matritsa deb ataladi.
Skalyar matritsa	Scalar matrix	Скалярная матрица	Agar diagonal matritsaning barcha a_{ii} elementlari o'zaro teng bo'lsa, u holda bunday matritsaga skalyar matritsa deyiladi.
Sonli ketma-ketlik	The limit points of the sequence	Числовая последова-тельность	Natural sonlar to'plamida aniqlangan funksiya sonli-ketma-ketlik deyiladi. $y=f(n),\ n\in N.$
$n-$tartibli determinant	The determina nt of order $n-$	Определи-тель $n-$го порядка	$n-$tartibli determinant deb $n!$ hadning quyidagi tartibda tuzilgan algebraik yig'indisiga aytiladi: hadlari matritsaning har qaysi satridan va har qaysi ustunidan bittadan olingan n ta elementdan tuzilgan bo'lib, mumkin bo'lgan barcha ko'paytmalar hizmat qiladi; shu bilan birga hadning indekslari juft o'rniga qo'yishni tashkil etsa, musbat ishora bilan, aks holda esa manfiy ishora bilan olinadi.
Teskari matritsa	Inverse matrix	Обратная матрица	Agar A kvadrat matritsa uchun $AA^{-1}=A^{-1}A=E$ tenglik bajarilsa, A^{-1} matritsa A matritsaga teskari matritsa deyiladi.
To'g'ri chiziqning	He canonical	Уравнение прямой с	$y=kx+b$

burchak koeffitsientli tenglamasi	form of the quadratic form	угловым коэффи-циентом	to'g'ri chiziqning burchak koeffitsientli tenglamasi.
To'g'ri chiziqning kanonik tenglamasi	Canonical equations of a straight line in space	Канони-ческоеурав нение-прямой	To'g'ri chiziqning kanonik tenglamasi $\frac{x-x_0}{m}=\frac{y-y_0}{n}$.
To'g'ri chiziqning umumiy tenglamasi	The general equation of a straight line	Общее уравнение прямой	$Ax+By+C=0$, $(A^2+B^2\neq 0)$ tenglamaga to'g'ri chiziqning umumiy tenglamasi deyiladi.
Transponirlangan matritsa	The transposed matrix	Транспонир ованная матрица	Agar A matritsada barcha satrlar mos ustunlar bilan almashtirilsa, u holda hosil bo'lgan A^T matritsaga A matritsaga transponirlangan matritsa deyiladi.
Uchinchi tartibli determinant	The determinant of order	Определи-тель 3-го порядка	$a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-$ $-a_{13}a_{22}a_{31}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}$ ifoda uchinchi tartibli determinant deyiladi.
Ustun matritsa	Column matrix	Матрица столбец	(m x1) o'lchamli matritsaga esa ustun matritsa deyiladi.
Vertikal asimptota	Vertical asymptote	Вертикаль-ная асимптота	Faraz qilaylik, a nuqtadagi bir tomonli limitlarning kamida biri cheksizga teng bo'lsin. U holda $y=f(x)$ egri chiziqdagi M(x,y) nuqta $x\to a$ da koordinatalar boshidan cheksiz uzoqlashadi, shu

			nuqtadan $x=a$ to'g'ri chiziqqacha bo'lgan masofa MN=$\|x-a\|$ nolga intiladi. Demak, ta'rifga ko'ra $x=a$ to'g'ri chiziq $y=f(x)$ egri chiziqning (funksiya grafigining) vertikal asimptotasi bo'ladi.
Xos matritsa	En matrix	Собствен-ная матрица	Agar matritsa determinanti nolga teng bo'lsa, bu matritsa xos yoki maxsus matritsa deyiladi.
Xosmas matritsa	Improper matrix	Несобст-венная матрица	Kvadrat matritsa elementlaridan tuzilgan determinant noldan farqli bo'lsa, u holda bunday matritsa xosmas yoki maxsusmas matritsa deyiladi.
Xusustiy hosila	Partial derivatives	Частные производ-ные	Agar $\lim\limits_{\Delta x_i\to 0}(f(M_i)-f(M_0))/\Delta x_i$ mavjud bo'lib bu limit chekli bo'lsa, u holda bu limitga $y=f(M)$ funksiyaning M_0 nuqtadagi x_i o'zgaruvchi bo'yicha xususiy hosilasi deyiladi.
Yevklid fazosi	Euclidean space	Евклидово про-странство	Agar n o'lchovli haqiqiy chiziqli fazoda skalyar ko'paytma aniqlangan bo'lsa, bu fazo n o'lchovli Evklid fazosi deyiladi va E^n ko'rinishda belgilanadi.
Yuqori tartibli	Derivative	Произ-	$y=f(x)$ funksiyaning yuqori

hosla va differensiallar	s and differentials of higher orders	водные и дифферен-циалы высших порядков	tartibli differensiallari ham ketma – ket ravishda, mos hosilalari kabi aniqlanadi: $d^3y=d(d^2y)$ –uchinchi tartibli differensial; $d^ny=d\left(d^{n-1}y\right)$ – n –tartibli differensial.
Matrisalar usuli	Matrix method of system solutions	Матричный способ решения системы	$X=A^{-1}B$ ifoda chiziqli tenglamalar sistemasining matritsalar usuli bilan yechish formulasi.
Tekislikning umumiy tenglamasi	n-dimensional coordinate space R^n	Общее уравнение плоскости	$Ax+By+Cz+D=0$ ko'rinishidagi tenglama tekislikning umumiy tenglamasi deyiladi.

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MATEMATIKA

informatika o'qitish metodikasi yo'nalishi uchun amaliy mashg'ulotlar

(1-qism)

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2019 yil 14 dekabrda tahririy-nashriyot bo'limiga qabul qilindi.
2019 yil 15 dekabrda original-maketdan bosishga ruxsat etildi.
Qog'oz bichimi 60x84 1/8. "Times new roman" garniturasi. Offset qog'ozi.
Shartli bosma tabog'i – 10,0.
Adadi 100 nusxa. Buyurtma № 12/6.

ISBN-978-9943-6179-3-3

SamDU tahririy-nashriyot bo'limida chop etildi.
140104, Samarqand sh., Universitet xiyoboni, 15.

978-9943-6179-3-3
9 789943 617933