8-MAVZU. ANIQMAS INTEGRAL VA UNING XOSSALARI.

REJA

1. **Boshlang‘ich funksiya**.
2. **Aniqmas integral va uning xossalari.**
3. **Asosiy integrallash jadvali:**

1. Boshlang‘ich funksiya. Differensial hisobning asosiy vazifasi berilgan *F(x)* funksiyaga ko‘ra uning hosilasi $f(x)=F'(x)$ ni yoki differensialini topishdan iborat edi.

Integral hisobning asosiy vazifasi buning teskarisi bo‘lib, *F(x)* funksiyani uning ma’lum *f(x)* hosilasiga yoki $f(x)dx$ differensialiga ko‘ra topishdan iborat. Demak, *f(x)* funksiya berilgan, shunday *F(x)* funksiyani topish kerakki, uning hosilasi *f(x)* ga teng bo`lsin, ya’ni

$$F`(x) = f(x) \qquad (1)$$

bo`lsin.

Ta‘rif. Agar *[a,b]* kesmada aniqlangan *f(x)* funksiya uchun bu kesmaning barcha nuqtalarida $F^{l}(x)=f(x)$ tenglik bajarilsa, *F(x)* funksiya shu kesmada *f(x)* funksiyaga nisbatan boshlang`ich funksiya deb ataladi.

Masalan: Boshlang`ich funksiya ta'rifiga asosan, $F(x)=\frac{x^4}{4}$ funksiya $f(x)=x^3$ funksiyasi uchun boshlang`ich ekani kelib chiqadi, chunki $\left(\frac{x^4}{4}\right)'=x^3$

Agar $f(x)$ funksiya uchun boshlang`ich funksiya mavjud bo`lsa, u boshlang`ich yagona bo`lmasligini ko`rish oson. $F(x)=\frac{x^4}{4}+6; F(x)=\frac{x^4}{4}+7$. Umuman $F(x)=\frac{x^4}{4}+c$.

Agar $F_1(x)$ va $F_2(x)$ funksiyalar $f(x)$ funksiyadan $[a,b]$ kesmada boshlang`ich funksiyalari bo`lsa, ular orasida ayirma o`zgarmas songa teng bo`ladi. Agar berilgan $f(x)$ funksiya uchun qanday bo`lmasin birgina $F(x)$ boshlang`ich funksiya topilgan bo`lsa, $F(x)$ funksiya uchun har qanday boshlang`ich funksiya $F(x)+C$ ko`rinishga ega bo`ladi.

2. Aniqmas integral va uning xossalari.

Ta'rif. Agar $F(x)$ funksiya biror oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, u holda $F(x)+C$ (bu yerda C – ihtiyoriy doimiy) funksiyalar to'plami shu kesmada $f(x)$ funksiyaning aniqmas integrali deyiladi va $\int f(x)dx = F(x) + C$ kabi belgilanadi.

Bu yerda $f(x)$ – integral ostidagi funksiya, $f(x)dx$ integral ostidagi ifoda,

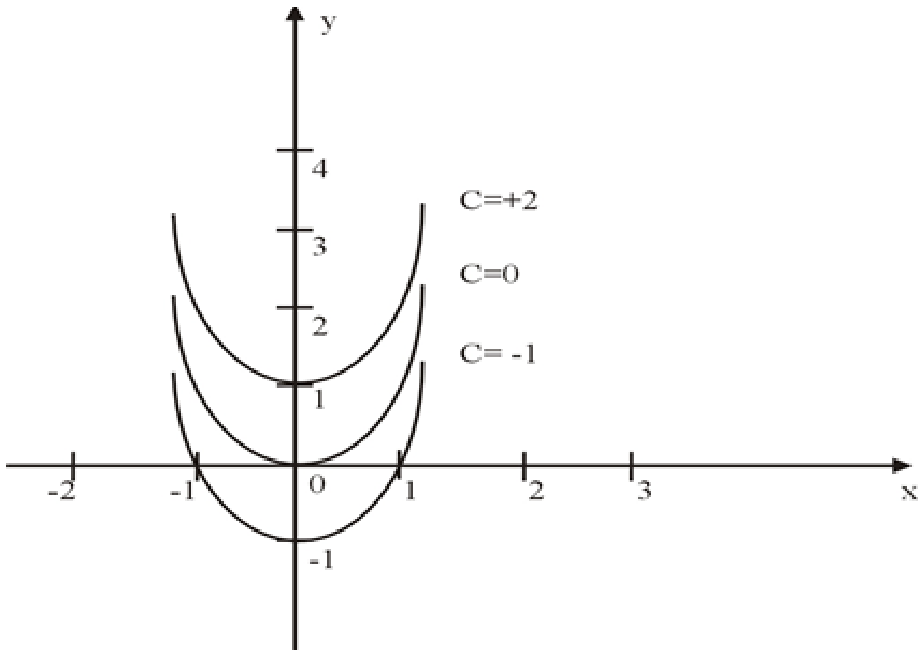
$\int$ – integral belgisi deyiladi.

Aniqmas integralni topish jarayoni yoki berilgan funksiyaning boshlang'ich funksiyasini topish jarayoni **integrallash** deyiladi.

1-misol: $\int \cos x dx = \sin \tilde{o} + C$, chunki $(\sin x)' = \cos x$

2-misol: $\int 3x^2 dx = x^3 + C$, chunki $(x^3)' = 3x^2$.

Boshlang‘ich funksiyalarning grafigi integral egri chizig‘i deyiladi, shuning uchun aniqmas integral geometrik jihatdan ihtiyoriy C o‘zgarmasga bog‘liq bo‘lgan hamma egri chiziqlar to‘plamini ifodalaydi.



Aniqmas integralning xossalari:

1) Aniqmas integralning hosilasi integral ostidagi funksiyaga teng, ya'ni

$$\left(\int f(x)dx\right)' = f(x)$$

2) Aniqmas integralning differensiali integral belgisi ostidagi ifodaga teng, ya'ni

$$d\left(f(x)dx\right) = f(x)dx$$

3) Biror funksiyaning hosilasidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o'zgarmasning yig'indisiga teng, ya'ni

$$\int F'(x)dx = F(x) + C$$

4) Biror funksiyaning differentsialidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o'zgarmasning yig'indisiga teng, ya'ni

$$\int dF(x) = F(x) + C$$

5) If $\int f(x)dx = F(x) + C$, then $\int \alpha f(x)dx = \alpha \int f(x)dx = \alpha[F(x) + C] = \alpha F(x) + K$

for all constants α. Here $k = \alpha C$ is just some new constant of integration. This property is read, "The integral of a constant times a function equals the constant times the integral of the function."

Adabiyot: J.H.Heinbockel. Introduction to Calculus Volume 1, p.181 prop.of int.

Agar $\int f(x)dx = F(x) + C$ b'lsa, u holda barcha o'zgarmas α lar uchun

$\int \alpha f(x)dx = \alpha \int f(x)dx = \alpha[F(x) + C] = \alpha F(x) + K$ bo'ladi. Bu yerda $k = \alpha C$ - integraldagi yangi o'zgarmas sondir. Bu xossa quyidagichadir: "funktsiyani o'zgarmas songa ko'paytmasining integrali o'zgarmas sonni shu funktsiya integraliga ko'paytmasiga teng".

6) Chekli sondagi funksiyalarning algerbaik yig'indisidan olingan aniqmas integral shu funksiyalarning har biridan olingan aniqmas integrallarning algebraik yig'indisiga teng, ya'ni

$$\int (f_1(x) + f_2(x) + f_3(x))dx = \int f_1(x)dx + \int f_2(x)dx + \int f_3(x)dx$$

7) Agar F(x) funksiya f(x) uchun boshlang'ich funksiya bo'lsa, ya'ni

$$\int f(x)dx = F(x) + C \text{ bo'lsa u holda } \int f(u)du = F(u) + C$$

tenglik to'g'ri bo'ladi, bu yerda $u = u(x)$ x ning differensiallanuvchi funksiyasi. Bu xossa integrallash formulalarining invariantligi deyiladi.

3. Asosiy integrallash jadvali:

1) $\int 0 \cdot dx = C$

2) $\int 1 \cdot dx = x + C$

3) $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha + 1} + C \qquad (\alpha \neq -1)$

4) $\int \frac{1}{x} dx = \ln |x| + C$

5) $\int \frac{1}{1 + x^2} dx = \operatorname{arctg} x + C$

6) $\int \frac{1}{\sqrt{1 - x^2}} dx = \arcsin x + C$

7) $\int a^x dx = \frac{a^x}{\ln a} + C$

8) $\int \sin x dx = -\cos x + C$

9) $\int \cos x dx = \sin x + C$

10) $\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C$

11) $\int \frac{1}{\sin^2 x} dx = -\operatorname{ctg} x + C$

The following integrals occur quite often and should be memorized.

If $\frac{d}{dx}x = 1$, then $\int 1dx = x + C$ or $\int dx = x + C$.

If $\frac{d}{dx}x^2 = 2x$, then $\int 2xdx = x^2 + C$ or $\int d(x^2) = x^2 + C$.

If $\frac{d}{dx}x^3 = 3x^2$, then $\int 3x^2 dx = x^3 + C$ or $\int d(x^3) = x^3 + C$.

If $\frac{d}{dx}x^n = nx^{n-1}$, then $\int nx^{n-1}dx = x^n + C$ or $\int d(x^n) = x^n + C$.

If $\frac{d}{dx}(\frac{u^{m+1}}{m+1}) = u^m$, then $\int u^m du = \frac{u^{m+1}}{m+1} + C$ or $\int d(\frac{u^{m+1}}{m+1}) = \frac{u^{m+1}}{m+1} + C$.

If $\frac{d}{dt}\sin t = \cos t$, then $\int \cos t dx = \sin t + C$ or $\int d(\sin t) = \sin t + C$.

If $\frac{d}{dt}\cos t = -\sin t$, then $\int \sin t dx = -\cos t + C$ or $-\int d(\cos t) = -\cos t + C$.

Adabiyot: J.H.Heinbockel. Introduction to Calculus Volume 1, p.181

Quyidagi integrallar ko'p qo'llanilgani uchun eslab qolish lozim:

Agar $\frac{d}{dx}x = 1$ bo'lsa, u holda $\int 1dx = x + C$ yoki $\int dx = x + C$ bo'ladi.

Agar $\frac{d}{dx}x^2 = 2x$ bo'lsa, u holda $\int 2xdx = x^2 + C$ yoki $\int d(x^2) = x^2 + C$ bo'ladi.

Agar $\frac{d}{dx}x^3 = 3x^2$ bo'lsa, u holda $\int 3x^2dx = x^3 + C$ yoki $\int d(x^3) = x^3 + C$ bo'ladi.

Agar $\frac{d}{dx}x^n = nx^{n-1}$ bo'lsa, u holda $\int nx^{n-1}dx = x^n + C$ yoki $\int d(x^n) = x^n + C$ bo'ladi.

Agar $\frac{d}{dx}(\frac{u^{m+1}}{m+1}) = u^m$ bo'lsa, u holda $\int u^m du = \frac{u^{m+1}}{m+1} + C$ yoki $\int d(\frac{u^{m+1}}{m+1}) = \frac{u^{m+1}}{m+1} + C$ bo'ladi.

Agar $\frac{d}{dt}\sin t = \cos t$ bo'lsa, u holda $\int \cos t dx = \sin t + C$ yoki $\int d(\sin t) = \sin t + C$ bo'ladi.

Agar $\frac{d}{dt}\cos t = -\sin t$ bo'lsa, u holda $\int \sin t dx = -\cos t + C$ yoki $-\int d(\cos t) = -\cos t + C$ bo'ladi.

1-misol. $\int 5dx.$

Yechilishi: 1-xossaga asosan o'zgarmas ko'paytuvchi 5 ni integral ishorasi tashqarisiga chiqaramiz va formulani qo'llab quyidagini hosil qilamiz:

$$\int 5dx = 5\int dx = 5x + C.$$

Tekshirish. $d(5x + C) = 5dx.$ Integral ostidagi ifodani hosil qildik, demak, integral to'g'ri olingan.

2-misol. $\int x^3 dx = \frac{x^{3+1}}{3+1} + C = \frac{1}{4}x^4 + C.$ Tekshirish: $d\left(\frac{1}{4}x^4 + C\right) = \frac{1}{4}\cdot 4x^2 dx = x^3 dx.$

Tekshirish: $d\left(\frac{4}{3}x^3 - 2x^2 + 12x + C\right) = (4x^2 - 4x + 12)dx = 4(x^2 - x + 3)dx.$

3-misol. To find the integral given by $I=\int(3x+7)^2\,dx$ you would make a substitution $u=3x+7$ with $du=3\,dx$ and then perform the necessary scaling to write

$$I=\frac{1}{3}\int(3x+7)^2\,3dx=\frac{1}{3}\int u^2\,du=\frac{1}{3}\int u^2\,du=\frac{1}{3}\frac{u^3}{3}+C=\frac{1}{3}\frac{(3x+7)^3}{3}+C=\frac{1}{9}(3x+7)^3+C$$

Adabiyot: J.H.Heinbockel. Introduction to Calculus Volume 1, p.184, example 3-3

Quyidagi $I=\int(3x+7)^2\,dx$ integralni hisoblash uchun $u=3x+7$ ni $du=3\,dx$ ga almashtirishingiz va o'rniga qo'yib yozishingiz kerak:

$$I=\frac{1}{3}\int(3x+7)^2\,3dx=\frac{1}{3}\int u^2\,du=\frac{1}{3}\int u^2\,du=\frac{1}{3}\frac{u^3}{3}+C=\frac{1}{3}\frac{(3x+7)^3}{3}+C=\frac{1}{9}(3x+7)^3+C$$