4-MAVZU : FUNKSIYA HOSILASI

REJA

1. Hosila tushunchasiga olib keladigan masalalar.

2. Fuksiya hosilasi.

3. Differensiallash, uning asosiy qoidalari va formulalari.

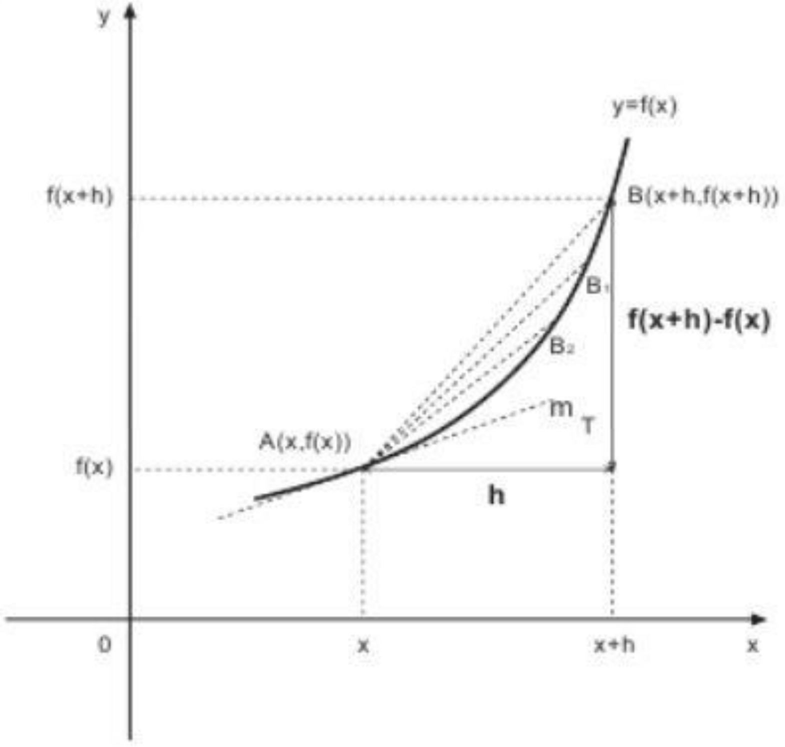
1. Hosila tushunchasiga olib keladigan masalalar.

Hosila tushunchasiga olib keladigan masalalar jumlasiga qattiq jismni to‘g‘ri chiziqli harakatini, yuqoriga vertikal holda otilgan jismning harakatini yoki dvigatel silindridagi porshen harakatini tekshirish kabi masalalarni kiritish mumkin. Bunday harakatlarni tekshirganda jismning konkret o‘lchamlarini va shaklini e‘tiborga olmay, uni harakat qiluvchi moddiy nuqta shaklida tasavvur qilamiz. Biz bitta masalani olib qaraymiz.

2. Fuksiya hosilasi.

Hosila ta'rifi.

Faraz qilaylik biz $y = f(x)$ chiziqning $A(x, f(x))$ nuqtasidagi urinmasini topmoqchimiz. m_T - A nuqtada chiziqqa o'tkazilgan urinmaning burchak koeffisienti bo'lsin. A nuqtaga o'tkazilgan urinmaning ikkinchi $B(x+h, f(x+h))$ nuqtasini olaylik.



Hamda AB vatarning gradientini m_{AB} deb qaraylik. Yetalicha kichik h uchun

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{f(x+h) - f(x)}{h}$$

Agar biz B nuqtani A ga yaqinlashtirsak $B_1, B_2, B_3 \ldots$ nuqtalar ketma-ketligi hosil bo'ladi. Bu nuqtalarga mos $AB_1, AB_2, AB_3 \ldots$vatarlarni chiziqning A nuqtasidagi urinmasiga qadar yaqinlashtiraylik.

$$f'(x) = \lim_{B_n \to A} m_{AB_n} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (*)$$

(*) tenglikka funksiyaning x nuqtadagi hosilasi deyiladi.

Namunaviy misollar.

1. $f(x)=x^2$ funksiya limitini hisoblang.

   Yechish.

Agar $f(x)=x^2$ bo'lsa u holda $f(x+h)=(x+h)^2$ bo'ladi. Bundan

$$f'(x)==\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}=\lim_{h\to 0}\frac{h(2x+h)}{h}=2x$$

Misollar:

Quyidagi funksiyalarning hosilalarini (*) formulasidan foydalanib toping.

1. $f(x)=x^2$
2. $f(x)=3x^2$
3. $f(x)=\sqrt{x}$

An estimation of the gradient at A can be found by taking a second point

B (x + h, f (x + h)) and calculating m_{AB}, the gradient of the chord AB. Consider h to be small then

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{f(x + h) - f(x)}{h}$$

If we move B closer and closer to A, say to points B_1, B_2 and B_3 then the gradient of the chords AB_1, AB_2 and AB_3 will give better and better approximations for the gradient of the curve at A.

If m_{AB_n} tends to a limit value as B_n approaches A then this value is denoted by $f'(x)$ and we write

$$f'(x) = \lim_{B_n \to A} m_{AB_n} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

where $\lim_{h \to 0}$ means the limit value as h approaches 0.

Note that when $f(x) = x^2$ then $f(x + h) = (x + h)^2$

hence

hence $$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x$$

Therefore when $f(x) = x^2$ then $f'(x) = 2x$ (as you knew already).

Bibliography:

4.Jane S Paterson, Dorothy A Watson "SQA Advanced Higher Mathematics" Unit1 pp 43-44

y=f(x) funksiya (a,b) intervalda aniqlangan bo'lsin (a,b) intervalga tegishli x_0 va $x_0+\Delta x$ nuqtalarni olamiz.

Argument biror (musbat yoki manfiy - bari bir) Δx orttirmasini olsin, u vaqtda y funksiya biror Δy orttirmani oladi. Shunday qilib argumentning x_0 qiymatida $y_0=f(x_0)$ ga, argumentning $x_0+\Delta x$ qiymatda $y_0+\Delta y=f(x_0+\Delta x)$ ga ega bo'lamiz. Funksiya orttirmasi Δy ni topamiz.

$$\Delta y=f(x_0+\Delta x)-f(x_0) \qquad (1)$$

Funksiya orttirmasini argument orttirmasiga nisbatini tuzamiz.

$$\frac{\Delta y}{\Delta x}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \qquad (2)$$

Bu – nisbatning $\Delta x\to 0$ dagi limitini topamiz.

Bibliography:

4.Jane S Paterson, Dorothy A Watson "SQA Advanced Higher Mathematics" Unit1 pp 43-44

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Bu – nisbatning $\Delta x \to 0$ dagi limitini topamiz.

Agar bu limit mavjud bo‘lsa, u berilgan f(x) funksiyaning x_0 nuqtadagi hosilasi deyiladi va $f'(x_0)$ bilan belgilanadi. Shunday qilib, ta'rifga ko‘ra

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} \quad \text{yoki} \quad f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \qquad (3)$$

Demak, berilgan y=f(x) funksiyaning argument x bo‘yicha hosilasi deb, argument orttirmasi Δx ixtiyoriy ravishda nolga intilganda funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining limitiga aytiladi.

Umumiy holda x ning har bir qiymati uchun $f'(x)$ hosila ma'lum qiymatga ega, ya’ni hosila ham x ning funksiyasi bo‘lishini qayd qilamiz. Hosilada $f'(x)$ belgi bilan birga boshqacha belgilar ham ishlatiladi. $y'; y'_x, \frac{dy}{dx}$

Hosilaning x=a dagi konkret qiymati $f'(a)$ yoki $y'\Big|_{x=a}$ bilan belgilanadi.

Funksiya hosilasini hosila ta'rifiga ko‘ra hisoblashni ko`ramiz.

Misol: $y = x^2$ funksiya berilgan: uning:

1) ixtiyoriy x nuqtadagi va 2) x=5 nuqtadagi hosilasi y' topilsin.

Yechish:

1) argumentning x ga teng qiymatida $y = x^2$ ga teng. Argument $x + \Delta x$ qiymatida $y + \Delta y = (x + \Delta x)^2$ ga ega bo‘lamiz.

$\Delta y = (x + \Delta x)^2 - x^2 = 2x(\Delta x) + (x)^2,$ $\frac{\Delta y}{\Delta x}$ nisbatni tuzamiz.

$\frac{\Delta y}{\Delta x} = \frac{2x + \Delta x(\Delta x)^2}{\Delta x} = 2x + \Delta x$ Limitga o‘tib, berilgan funksiyadan hosila topamiz. $y' = \lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} (2x + \Delta x) = 2x$

Demak, $y = x^2$ funksiyaning ixtiyoriy nuqtadagi hosilasi $y' = 2x$

2) x=5 da $y'\Big|_{x=5} = 2 \cdot 5 = 10$

3. Differensiallash, uning asosiy qoidalari va formulalari.

Berilgan f(x) funksiyadan hosila topish amali shu funksiyani differensiallash deyiladi.

Differensiallashning asosiy qoidalari.

1. O‘zgarmas miqdorning hosilasi nolga teng, ya‘ni agar y=c bo‘lsa (c=const) y'=0 bo‘ladi.

2. O‘zgarmas ko‘paytuvchini hosila ishorasidan tashqariga chiqarish mumkin: y=cu(x) bo‘lsa y'=cu'(x) bo‘ladi.

3. Chekli sondagi differensiallanuvchi funksiyalar yig‘indisining hosilasi shu funksiyalar hosilalarining yig‘indisiga teng:

$$y = U(x) + V(x) + W(x); \quad y' = U'(x) + V'(x) + W'(x)$$

4. Ikkita differensiallanuvchi funksiyalar ko‘paytmasining hosilasi birinchi funksiya hosilasining ikkinchi funksiya bilan ko‘paytmasi hamda birinchi funksiyaning ikkinchi funksiya hosilasi bilan ko‘paytmasining yig‘indisiga teng:

$$y = u\vartheta \quad \text{bo‘lsa} \quad y' = u'\vartheta + u\vartheta'.$$

5. Ikkita differensiallanuvchi funksiyalar bo‘linmasining hosilasi (kasrda ifodalanib) bo‘linuvchi funksiya hosilasini bo‘luvchi funksiya bilan ko‘paytmasi hamda bo‘linuvchi funksiyani bo‘luvchi funksiya hosilasi bilan ko‘paytmasining ayirmasini bo‘luvchi(maxrajdagi) funksiya kvadratining nisbatiga teng:

$$y = \frac{u}{\vartheta} \quad \text{bo‘lsa} \quad y' = \frac{u'\vartheta - u\vartheta'}{\vartheta^2}$$

Theorem 6.4 (Algebraic operations) *Let $f(x), g(x)$ be differentiable maps at $x_0 \in \mathbb{R}$. Then the maps $f(x) \pm g(x)$, $f(x)g(x)$ and, if $g(x_0) \neq 0$, $\frac{f(x)}{g(x)}$ are differentiable at x_0. To be precise,*

$$(f \pm g)'(x_0) = f'(x_0) \pm g'(x_0), \tag{6.3}$$

$$(f\,g)'(x_0) = f'(x_0)g(x_0) + f(x_0)g'(x_0), \tag{6.4}$$

$$\left(\frac{f}{g}\right)'(x_0) = \frac{f'(x_0)g(x_0) - f(x_0)g'(x_0)}{[g(x_0)]^2}. \tag{6.5}$$

2.Canuto, C., Tabacco, A. Mathematical Analysis I,172p.

6. Aytaylik, y=F(u) murakkab funksiya bo'lsin ya'ni y=F(u), $u=\varphi(x)$ yoki $y=F[\varphi(x)]$, u - o'zgaruvchi, oraliq argumenti deyiladi. y=F(u) va $u=\varphi(x)$ differensiallanuvchi funksiyalar bo'lsin.

Murakkab funksiyaning differensiallash qoidasini keltirib chiqaramiz.

Teorema: Murakkab F(u) funksiyaning erkli o'zgaruvchi x bo'yicha hosilasi bu funksiya oraliq argumenti bo'yicha hosilasini oraliq argumentining erkli o'zgaruvchi x bo'yicha hosilasining ko'paytmasiga teng, ya'ni

$$y'_x = F'_u(u)\cdot u'_x(x)........(1)$$

Misol: $y=\left(x^5+4x^4+3x^2+2\right)^5$ funksiyaning hosilasini toping.

Yechish: berilgan funksiyani murakkab funksiya deb qaraymiz ya'ni $y=u^5;\quad u=x^5+4x^4+3x^2+2$ (1) formulaga asosan

$$y'_x = y'_u\cdot u'_x = \left((x^5+4x^4+3x^2+2)^5\right)' = 5\left(x^5+4x^4+3x^2+2\right)^4\cdot\left(5x^4+16x^3+6x\right);$$

Differensiallashning asosiy formulalari jadvali:

1) y=const ; $y' = 0$

2) $y = x^{\alpha}$; $y = \alpha x^{\alpha-1}$

3) $y = \sqrt{x}$; $y' = \frac{1}{2\sqrt{x}}$

4) $y = \frac{1}{x}$; $y = -\frac{1}{x^2}$

5) $y = a^x$; $y' = a^x \ln a$

6) $y = e^x$; $y' = e^x$

7) $y = \log_a x$; $y' = \frac{1}{x}\log_a e$

8) $y = \ln x$; $y' = \frac{1}{x}$

9) $y = \sin x$; $y' = \cos x$

10) $y = \cos x$; $y' = -\sin x$

11) $y = tgx$; $y' = \frac{1}{\cos^2 x}$

12) $y = ctgx$; $y' = -\frac{1}{\sin^2 x}$

Misollar.

1) $f(x)=(x^3+4x+7)^4$ funksiyaning hosilasini toping.

Yechish: Bu yerda $y(u)=u^4$ va $u(x)=x^3+4x+7$ U holda

$$f(x)=(u^4)'\cdot(x^3+4x+7)'=4u^3(3x^2+4)=4(x^3+4x+7)^3(3x^2+4)$$

2) $(x^2+x)'=(x^2)'+(x)'=2x+1$

3) $(2x\sin x)'=(2x)'\sin x+2x(\sin x)'=2(x)'\sin x+2x\cos x=$
$2\sin x+2x\cos x=2(\sin x+x\cos x)$

4) $y=\sin 3x$ $\quad y'-?$ $\quad y'=(\sin 3x)'=3\cos 3x$