12-ma'ruza. Qutb koordinatalar sistemasi.

Adabiyot

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Jane S Paterson Heriot-Watt (University Dorothy) A Watson Balerno (High School) SQA Advanced Higher Mathematics. Unit 1. This edition published in 2009 by Heriot-Watt University SCHOLAR. Copyright © 2009 Heriot-Watt University.

Ma’ruza rejasi:

* Qutb koordinatalar sistemasi.
* Nuqtaning qutb va dekart koordinatalari orasidagi bog’lanish.
* Qutb koordinatalari bilan berilgan ikkita nuqta orasidagi bog’lanish.
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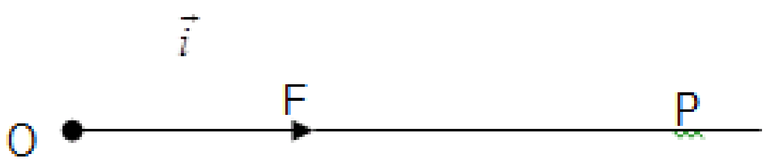
1.Qutb koordinatalar sistemasi.

Geometriyada affin, to'g'ri burchakli dekart koordinatalar bilan bir qatorda qutb koordinatalari ham qaraladi. Ko'plab tadqiqotlarda va egri chiziqning muhim sinflarini o'rganishda qutb koordinatalar sistemasi qo'l kelmoqda.

Shu sistema bilan tanishaylik. Yo'nalish tekislikda 0 nuqta va bu nuqtadan chiquvchi OP nur va OP nurda yotuvchi $\overline{OE} = \vec{i}$ birlik vektor olamiz (1-chizma).



1 - chizma

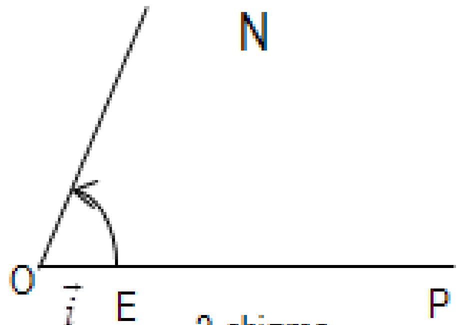
Hosil bo'lgan geometrik obraz qutb koordinatalar sistemasi deyiladi va (0, i) ko'rinishda belgilanadi.

O nuqtani qutb boshi, OP nur esa qutb o'qi deyiladi.

Tekislikda (0, i) qutb koordinatalar sistemasi va ixtiyoriy N nuqta berilgan bo'lsin, bu nuqtaning tekislikdagi vaziyatini ma'lum tartibda olingan ikkita son:

1) OE birlik kesmada o'lchangan $\rho = |ON|$ masofa (2 - chizma).
2) OR nur ON nurning ustiga tushishi uchun burilishi kerak bo'lgan yo'nalishli $\varphi = (i\wedge ON)$ burchak bilan to'liq aniqlanadi.

ρ, N nuqtaning qutb radius φ ni N nuqtaning qutb burchagi deyiladi, ularni birgalikda N nuqtaning qutb koordinatalari deyiladi va (ρ, φ) ko'rinishda yoziladi. O nuqta uchun $\rho = 0$, φ - aniqlanmagan.

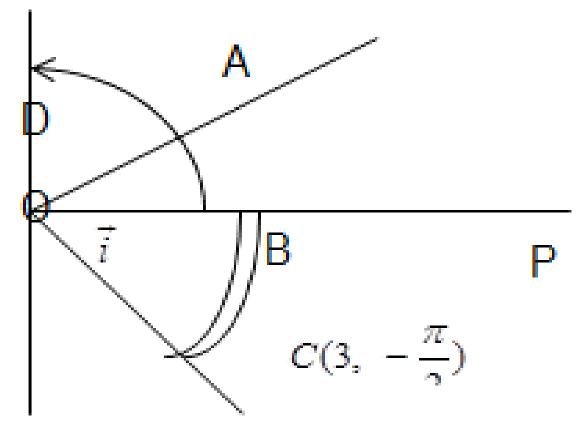


2-chizma

Agar $0 \le \rho < \infty$, $0 \le \varphi < 2\pi$ o'zgarsa, tekislikni har bir nuqtasi qutb koordinatalar bilan ta'minlanadi.

1-misol. $A(2;\ \frac{\pi}{3})$, $B(1;\ 0)$, $C(3;\ \frac{\pi}{4})$, $D(1;\ \frac{\pi}{2})$. 3- chizmada berilgan nuqtalar tasvirlangan.

Ravshanki, har qanday (ρ, φ) juft haqiqiy sonlar uchun tekislikning bitta nuqtasi mavjud bo'lib, bu sonlar shu nuqtaning koordinatalari bo'ladi. Ammo bir nuqtaning o'ziga cheksiz ko'p sonlar mos keladi. Chunki, N nuqtaning koordinatalari $\rho = a > 0,\ \ \varphi = \alpha$ bo'lsa, $\rho = a,\ \varphi = \alpha + 2\pi k$ (bu yerda k=0, 1…). Juftlari ham shu N nuqtaning koordinatalari bo'ladi, chunki ON



3 - chizma

nur OR qutb o'qini α burchakka qadar burishdan hosil bo'ladi deb faraz qilinsin, u holda OR nurni $\varphi = \alpha \pm 2\pi k$ qadar burishdan ham o'sha nurning o'zini hosil qilish mumkin.

N nuqtaning qutb burchagi qabul qilishi mumkin bo'lgan qiymatlar orasidan $-\pi \le \varphi < \pi$ tengsizlikni qanoatlantiradigan aniq bir qiymati OP nurni ON nurni ustiga tushishi uchun burish kerak bo'lgan burchak olinadi. ON nur OP nurga qarama-qarshi yo'nalgan bo'lsa, 180^0 ga ikki yo'nalishda burish mumkin, bu vaqtda qutb burchagining bosh qiymati uchun $\varphi = \pi$ qabul qilinadi.

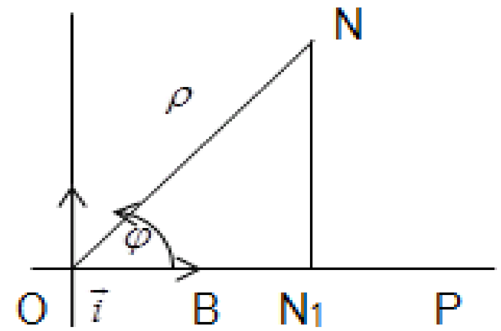
2. Nuqtaning qutb va dekart koordinatalari orasidagi bog'lanish.

Tekislikda $(0, \vec{i})$ qutb koordinatalar sistemasi berilgan. Koordinatalar boshi qutb boshi bilan, absissalar o'qining musbat qismi qutb o'qi bilan ustma-ust tushadigan musbat yo'nalishli $(0, \vec{i}, \vec{j})$ dekart reperini kiritamiz (4-chizma).

Tekislikdagi N nuqtaning qutb koordinatalar ρ, φ dekart koordinatalari x, y bo'lsin.

To'g'ri burchakli ONN_1 uchburchakdan

$$x = \rho\cos\varphi \qquad (12.1)$$
$$y = \rho\sin\varphi$$



4-chizma

Nuqtaning qutb koordinatalari ma'lum bo'lsa, uning dekart koordinatalari (12.1) formuladan topiladi.

Agar N nuqtaning dekart koordinatalari ma'lum bo'lsa, uning qutb koordinatalarini ushbu

$$\rho = \sqrt{x^2 + y^2}, \quad tg\varphi = \frac{y}{x} \Rightarrow \varphi = arctg\frac{y}{x};$$
$$\cos\varphi = \frac{x}{\sqrt{x^2 + y^2}}; \quad \sin\varphi = \frac{y}{\sqrt{x^2 + y^2}}. \qquad (12.2)$$

formuladan topiladi.

Eslatma. N nuqtaning dekart koordinatalaridan qutb koordinatalariga o'tishda $tg\varphi = \frac{y}{x}$ formula qutb burchagini qiymatini to'liq aniqlaydi, chunki buning uchun yana φ ning miqdori musbat yoki manfiy ekanligini ham bilish kerak. Odatda bu N nuqtaning qaysi chorakda joylashishiga qarab aniqlanadi. Masalan, (12.2) formulada x=3, u=3 bo'lsa, $tg\varphi = 1$ bo'lib, $\varphi = 45^0$. Lekin, x=-3, u=-3 bo'lganda ham $tg\varphi = 1$ bo'lib, 45^0 emas, 135^0 bo'lishi kerak, chunki (-3; -3) nuqta uchinchi chorakda joylashgan φ burchakning qiymati va ishorasini $\cos\varphi$, $\sin\varphi$ ga qarab aniqlash qulayroq.

3.Ikki nuqta orasidagi masofa.

Qutb koordinatalari bilan $N_1(\rho_1, \varphi_1)$ va $N_2(\rho_2, \varphi_2)$ nuqtalar orasidagi masofani hisoblash formulasini chiqaraylik.

Tekislikdagi N_1 va N_2 nuqtalarning dekart koordinatalari $N_1(x_1, y_1)$ va $N_2(x_2, y_2)$ bo'lsin. (12.1) formulaga ko'ra

$$\begin{matrix} x_1 = \rho_1 \cos\varphi_1 \\ y_1 = \rho_1 \sin\varphi_1 \end{matrix} \quad \text{va} \quad \begin{matrix} x_2 = \rho_2 \cos\varphi_2 \\ y_2 = \rho_2 \sin\varphi_2 \end{matrix}$$

U holda

$$N_1N_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(\rho_2 \cos\varphi_2 - \rho_1 \cos\varphi_1)^2 + (\rho_2 \sin\varphi_2 - \rho_1 \sin\varphi_1)^2} = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos(\varphi_2 - \varphi_1)} \qquad (12.3)$$

(12.3) qutb koordinatalar bilan ikki nuqta orasidagi masofani hisoblash formulasi.

1-masala. Dekart koordinatalar sistemasida A(7, -7), N(-5, 12), P(3, 0) nuqtalar berilgan. Ularning qutb koordinatalarini toping?

Yechish Bu masalani yechishda (12.2) formuladan foydalanamiz.

$$A(7,\ -7),\ \ \rho = \sqrt{7^2 + (-7)^2} = 7\sqrt{2} \qquad tg\varphi = \frac{-7}{7} = -1,\quad \varphi = \frac{3\pi}{4}$$

$$N(-5;\ 12),\ \ \rho = \sqrt{(-5)^2 + 12^2} = 13 \qquad tg\varphi = -\frac{12}{5},\quad \varphi = arctg(-\frac{12}{5})$$

$$m(3;\ 0),\quad \rho = \sqrt{3^2} = 3 \qquad tg\varphi = \frac{0}{3} = 0,\quad \varphi = 0$$

2-masala. Uchlarini $A(5;\ \frac{\pi}{2})$, $B(8;\ \frac{5\pi}{6})$ va $C(3;\ \frac{7\pi}{6})$ nuqtalarda joylashgan uchburchakning muntazam ekanligini isbotlang.

Yechish Uchburchakning muntazam ekanligini isbotlash uchun AB=BC=AC ni isbotlash etarli. Buning uchun (12.3) formuladan

$$AB = \sqrt{5^2 + 8^2 - 2\cdot 5\cdot 8\cdot \cos(\frac{5\pi}{6} - \frac{\pi}{2})} = \sqrt{25 + 64 - 80\cdot \cos\frac{\pi}{3}} = \sqrt{89 - 80\frac{1}{2}} = \sqrt{49} = 7$$

$$AC = \sqrt{5^2 + 3^2 - 2\cdot 5\cdot 3\cdot \cos(\frac{7\pi}{6} - \frac{\pi}{2})} = \sqrt{25 + 9 - 30\cdot \cos(-\frac{2\pi}{3})} = \sqrt{25 - 30(-\frac{1}{2})} = \sqrt{49} = 7$$

$$BC = \sqrt{64 + 9 - 2\cdot 8\cdot 3\cdot \cos\frac{\pi}{3}} = \sqrt{73 - 24} = \sqrt{49} = 7$$

Demak, AB=AC=BC ekan, ABC uchburchak muntazam.