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MUNDARIJA

KO‘P TIPLI GALTON – VATSON TARMOQLANUVCHI TASODIFIY JARAYONLARI

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Ushbu maqolada biologiya, gen muhandisligi, kimyo, atom fizikasi, virusologiya va boshqa ko‘plab sohalarda o‘zining amaliy hamda nazariy tadbiqlariga ega bo‘lgan tarmoqlanuvchi tasodifiy jarayonlar ayrim tiplari yoritilgan. Ikki tipli galton-vatson tarmoqlanuvchi tasodifiy jarayonlari uchun yuqori vaqt momentlaridagi holatlari tavsiflangan.

Kalit so‘zlar: *Tarmoqlanuvchi jarayonlar, ikki tipli Galton Vatson tarmoqlanuvchi jarayoni, limit teoremalar.*

This article describes some types of branching random processes that have practical and theoretical applications in biology, genetic engineering, chemistry, nuclear physics, virology and many other fields. Two types of Galton-Watson branching random processes at high time moments are described.

Key words: *Branching processes, Two types of Galton-Watson branching processes, limit theorems.*

В данной статье описаны некоторые типы ветвящихся случайных процессов, которые имеют практическое и теоретическое применение в биологии, генной инженерии, химии, ядерной физике, вирусологии и многих других областях. Описаны два типа ветвящихся случайных процессов Гальтона - Ватсона в большие моменты времени.

Ключевые слова: *Ветвящиеся процессы, два типа ветвящихся случайных процессов Гальтона-Ватсона, предельные теоремы.*

Ko'p tipli Galton–Vatson jarayonlari XIX asr o'rtalarida Kolmogorov, Sevastyanov va boshqa bir qator olimlar tomonidan o'rganilgan [1]-[3].

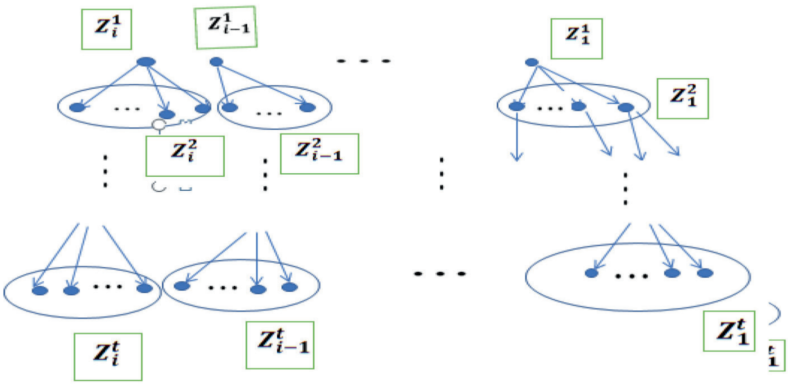
Faraz qilaylik, Z_t Sanoqli tipdagi Galton – Vatson jarayonlari bo'lsin. Z_t holatlari sanoqlita bo'lgan Markov zanjiridir.

$$Z_t = (Z_{1t}, Z_{2t}, \ldots) \qquad (1)$$

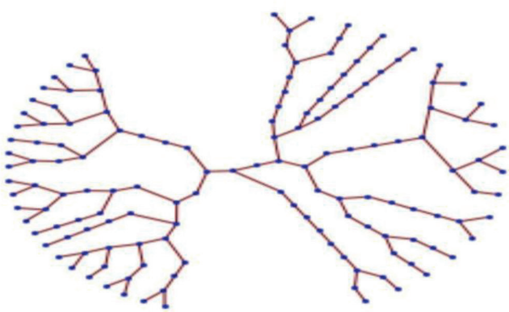
bu yerda Z_t cheksiz o'lchovli Markov zanjirini hosil qiladi.

Z_{it} i - tipdagi t-vaqt momentidagi zarralar sonini anglatadi.

Vatson jarayoni bo'lsin.



1- chizma



2 - chizma

Bu jarayon ikta holatdan iborat bo‘lib 1- holatda jarayon birinchi vaqt momentida quyidagi qonuniyatga ega bo‘lsin.

$$f_1(s) = h_{01} + \frac{h_{11}s}{1 + r_1 - r_1 s}$$

bu yerda $h_{01} + h_{11} = 1, r_1 > 0$ h_{01} 1-holat uchun avlodlar bo‘lmaslik ehtimoli, h_{11} 1-holatda jarayon kamida bitta avlodga ega bo‘lish ehtimoli.

2- holatda jarayon birinchi vaqt momentida quyidagi qonuniyatga ega bo‘lsin:

$$f_2(s) = h_{02} + \frac{h_{12}s}{1 + r_2 - r_2 s}$$

bu yerda $h_{02} + h_{12} = 1, r_1 > 0, h_{02}$ 2-holat uchun avlodlar bo‘lmaslik ehtimoli, h_{11} 2-holatda jarayon kamida bitta avlodga ega bo‘lish ehtimoli.

Birinchi va ikkinchi holatda jarayon uchun birinchi vaqt momentidagi matematik kutilmalarni aniqlaymiz.

$$f_1'(1) = M_1 = (1 + r_1)h_{11}, \quad f_2'(1) = M_2 = (1 + r_2)h_{12}.$$

1-holat n_1 avlod, 2-holat n_2 avlodga ega bo‘lsin. $n = n_1 + n_2$ va

$$f_1^{\{n_1\}}(s) == h_{01}^{\{n_1\}} + \frac{h_{11}^{\{n_1\}}s}{1 + r_1^{\{n_1\}} - r_1^{\{n_1\}}s},$$

$$f_2^{\{n_2\}}(s) == h_{02}^{\{n_2\}} + \frac{h_{12}^{\{n_2\}}s}{1 + r_2^{\{n_2\}} - r_2^{\{n_2\}}s},$$

$$f_1^{\{n_1\}}(f_2^{\{n_2\}}(s)) = h_0^{\{n\}} + \frac{h_1^{\{n\}}s}{1 + r^{\{n\}} - r^{\{n\}}s}$$

bu yerda $r^{\{n\}} = r_2^{\{n_2\}} + r_1^{\{n_1\}} M_2^{\;n_2}$

$$h_1^{\{n\}} = \frac{M_1^{n_1} M_2^{n_2}}{1 + r_2^{\{n_2\}} - r_1^{\{n_1\}} M_2^{n_2}} \tag{2}$$

Yuqoridagilardan

$$\frac{1}{P(Z_n \neq \theta)} = \frac{1}{M_1^{n_1} M_2^{n_2}} + \frac{r_2^{\{n_2\}}}{M_1^{n_1} M_2^{n_2}} + \frac{r_1^{\{n_1\}}}{M_1^{n_1}} \tag{3}$$

bulardan

$$r_2^{\{n_2\}} + r_1^{\{n_1\}} M_2^{n_2} = r_1 M_2^{n_2} \sum_{j=1}^{n_1} M_1^{j-1} + r_2 \sum_{j=1}^{n_2} M_2^{j-1} \tag{4}$$

u holda (3) (3,2,8) va (4) (3,2,9) munosabatdan quyidagi munosabat o‘rinli [3]:

$$P(Z_n \neq \theta) = \frac{M_1^{n_1} M_2^{n_2}}{1 + r_1 M_2^{n_2} \sum_{j=1}^{n_1} M_1^{j-1} + r_2 \sum_{j=1}^{n_2} M_2^{j-1}} \tag{5}$$

Z_n uchun maxsus holatlarni qaraylik .

1-hol: $M_1 > 1$, $M_2 > 1$, $n_1 \to \infty, n_2 \to \infty$

$$P(Z_n \neq \theta) = \frac{M_1^{n_1} M_2^{n_2}}{1 + r_1 M_2^{n_2} \sum_{j=1}^{n_1} M_1^{j-1} + r_2 \sum_{j=1}^{n_2} M_2^{j-1}} \to \frac{M_1 - 1}{r_1}$$

bundan $P(Z_n = \theta) \to \dfrac{r_1 - M_1 + 1}{r_1}$

2-hol: $M_1 < 1$, $M_2 < 1$, $n_1 \to \infty, n_2 \to \infty$

$$P(Z_n \neq \theta) = \frac{M_1^{n_1} M_2^{n-n_1}}{1 + r_1 M_2^{n-n_1} \sum_{j=1}^{n_1} M_1^{j-1} + r_2 \sum_{j=1}^{n-n_1} M_2^{j-1}} = \frac{M_1^{n_1} M_2^{n_2}}{1 + r_1 M_2^{n_2} \frac{1}{1-M_1} + r_2 \frac{1}{1-M_2}}$$

$$\frac{M_1^{n_1} M_2^{n_2}}{1 + r_1 M_2^{n_2} \frac{1}{1-M_1} + r_2 \frac{1}{1-M_2}} \sim \frac{M_1^{n_1} M_2^{n_2}(1-M_2)}{r_2}$$

u holda $$P(Z_n = \theta) \sim 1 - \frac{M_1^{n_1} M_2^{n_2}(1-M_2)}{r_2},$$

$n_1 \to \infty$, $n_2 \to \infty$ $\quad P(Z_n = \theta) = 1$

3-hol: $M_1 < 1,\ M_2 > 1,\ n_1 = const,\ n_2 \to \infty$

$$P(Z_n \neq \theta) = \frac{M_1^{n_1} M_2^{n_2}}{1 + r_1 M_2^{n_2} \sum_{j=1}^{n_1} M_1^{j-1} + r_2 \sum_{j=1}^{n_2} M_2^{j-1}} =$$

$$= \frac{M_1^{n_1} M_2^{n_2}}{1 + r_1 M_2^{n_2} \frac{1-M_1^{n_1}}{1-M_1} + r_2 \frac{M_2^{n_2}-1}{M_2-1}} \to \frac{M_1^{n_1}}{r_1 \frac{1-M_1^{n_1}}{1-M_1} + r_2 \frac{1}{M_2-1}}$$

u holda $$P(Z_n = \theta) \to 1 - \frac{M_1^{n_1}}{r_1 \frac{1-M_1^{n_1}}{1-M_1} + r_2 \frac{1}{M_2-1}},$$

$n_1 \to \infty, n_2 \to \infty$ $\quad P(Z_n = \theta) = 1.$

4-hol: $M_1 = 1, M_2 = 1,\ n_1 \to \infty, n_2 \to \infty$

$$P(Z_n \neq \theta) = \frac{M_1^{\,n_1} M_2^{\,n_2}}{1 + r_1 M_2^{\,n_2} \sum_{j=1}^{n_1} M_1^{\,j-1} + r_2 \sum_{j=1}^{n_2} M_2^{\,j-1}} = \frac{1}{1 + r_1 n_1 + r_2 n_2}.$$

Agar $\frac{n_1}{n} \to \alpha,\ 0 < \alpha < 1$ $\quad \frac{1}{1 + r_1 n_1 + r_2 n_2} \to \frac{k_\alpha}{n}$

$$k_\alpha = \frac{1}{\alpha r_1 + (1-\alpha) r_2}$$

$$P(Z_n = \theta) \to 1 - \frac{1}{1 + r_1 n_1 + r_2 n_2}$$

$n_1 \to \infty,\ n_2 \to \infty, n \to \infty$ $\qquad P(Z_n = \theta) = 1.$

Demak bu keltirilgan tasdiqlardan quyidagi teoremani keltirishimiz mumkin.

Teorema: Faraz qilaylik, tasodifiy jarayon bir jinsli bo‘lmagan 2 tipli kasr – chiziqli jarayon bo‘lib boshlang‘ich vaqtda Z_1 va Z_2 holatlardan iborat bo‘lsin Z_1 holat n_1 avlodga va Z_2 holat n_2 avlodga ega bo‘lib boshlang‘ich vaqtda har bir holat turli parametrli geometrik taqsimot bilan taqsimlangan bo‘lib ushbu kasr – chiziqli hosil qiluvchi funksiyalarga ega bo‘lsa

$$f_{Z_1^1} = \frac{(1-q_{11})s}{1-q_{11}s}, \quad f_{Z_2^1} = \frac{(1-q_{21})s}{1-q_{21}s}, \qquad n = n_1 + n_2.$$

Bunday tasodifiy jarayon yetarlicha yuqori vaqt momentida kamida bitta avlodga ega bo‘ladi.

Isbot: O‘z navbatida Z_1 holat va Z_2 holatning n_1 va n_2 avlodlardagi hosil qiluvchi funksiyasini quyidagi ko‘rinishda belgilab olamiz:

$$f_{Z_1^{n_1}}(s) = f_1^{\{n_1\}} = \frac{q_{1n_1}^{\{1\}} - q_{1n_1}^{\{2\}} s}{q_{1n_1}^{\{3\}} - q_{1n_1}^{\{4\}} s},$$

$$f_{Z_2^{n_2}}(s) = f_2^{\{n_2\}} = \frac{q_{2n_2}^{\{1\}} - q_{2n_2}^{\{2\}} s}{q_{2n_2}^{\{3\}} - q_{2n_2}^{\{4\}} s}$$

u holda $f_1^{\{n_1\}}(f_2^{\{n_2\}}(s)) = \dfrac{q_{1n}^{\{1\}} + q_{1n}^{\{2\}} s}{q_{1n}^{\{3\}} + q_{1n}^{\{4\}} s}$.

Shrederning funksiyalar itteratsiyasiga doir ishlarida keltirilgan kasr chiziqli funksiyalar t-tartibli iteratsiyalaridan foydalanilanib $q_{1n}^{\{1\}}, q_{1n}^{\{2\}}, q_{1n}^{\{3\}}, q_{1n}^{\{4\}}$ larni topish mumkin va bu jarayonning tugash ehtimolligini topaylik. Yuqoridagi (5) munosabatga ko‘ra boshlang‘ich vaqtda 2 tipli bir jinsli bo‘lmagan Galton – Vatson jarayoni Z_1 va Z_2 holatlari avlodlar hosil qilishi geometrik taqsimot qununiga bo‘ysunsa jarayonning tugash ehtimoli quyidagiga teng bo‘ladi:

$$P(Z_n \neq \theta) = \frac{\left(\dfrac{1}{1-q_{11}}\right)^{n_1}\left(\dfrac{1}{1-q_{21}}\right)^{n_2}}{1+\left(\dfrac{q_{11}}{1-q_{11}}\right)\left(\dfrac{1}{1-q_{21}}\right)^{n_2}\sum\limits_{j=1}^{n_1}\left(\dfrac{1}{1-q_{11}}\right)^{j-1}+\left(\dfrac{q_{21}}{1-q_{21}}\right)\sum\limits_{j=1}^{n_2}\left(\dfrac{1}{1-q_{21}}\right)^{j-1}}$$

keltirilgandan tenglikdan $\dfrac{1}{1-q_{11}} \geq 1, \dfrac{1}{1-q_{21}} \geq 1$ bo‘lganligi uchun

$$n_1 \to \infty, n_2 \to \infty \qquad P(Z_n \neq \theta) \to \frac{\frac{1}{1-q_{11}} - 1}{\frac{q_{11}}{1-q_{11}}} = 1$$

$n_1 \to \infty, n_2 \to \infty$, $P(Z_n = \theta) = 0$.

Demak, bunday jarayonlar bir ehtimollik bilan tugamas ekan ya’ni bu tasodifiy jarayon yetarlicha yuqori vaqt momentida kamida bitta avlodga ega bo‘lar ekan.

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