I BOSQICH BAKALAVRLARI UCHUN KUZGI Oʻ QUV MAVSUMI

MUSTAQIL ISH TOPSHIRIQLARI

I topshiriq.

Ushbu chiziqli tenglamalar sistemasini Kramer, Gauss hamda matritsalar usulida yeching:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

Izoh: Sistemadagi a_{ij} koeffitsient va b_i ozod hadlardan iborat parametrlar variant boʻyicha jadvaldan olinadi.

Variant №	Sistema tenglamalarining parametrlari											
	a_{11}	a_{12}	a_{13}	b_1	a_{21}	a_{22}	a_{23}	b_2	a_{31}	a_{32}	a_{33}	b_3
1	1	−3	3	–2	2	1	−3	1	1	−1	4	3
2	2	1	6	2	3	−1	−3	10	−2	4	−1	3
3	−2	3	2	0	4	−4	−4	4	1	6	1	13
4	5	−3	2	−3	−5	2	6	2	0	−1	−4	−1
5	1	1	1	0	1	0	2	−2	0	2	3	−1
6	3	−2	3	4	0	2	1	–4	2	−4	0	2
7	0	2	−1	3	1	3	0	9	5	−2	1	12
8	1	1	2	−3	2	1	1	−4	1	2	3	−7
9	2	−5	7	−1	1	1	−1	1	−3	2	−3	0
10	1	1	−1	1	1	−1	1	5	1	1	1	9
11	3	2	−1	5	0	2	−2	6	–3	7	–3	2
12	10	3	4	7	2	3	−4	–1	7	−5	−4	−9
13	3	2	−3	5	0	1	−1	–1	4	−2	8	4
14	8	1	−4	1	3	−3	1	–4	4	9	−1	1
15	9	−3	7	–7	−8	−2	1	1	1	−1	1	–3
16	8	6	−1	−6	6	1	−2	0	2	4	2	−2
17	1	−6	−6	4	2	–1	2	5	1	3	6	1
18	1	−2	3	−1	2	1	−2	2	4	3	−3	10
19	5	3	4	−1	4	4	1	9	4	2	3	–1
20	1	0	−1	3	5	−1	7	–10	4	9	5	3
21	2	−3	6	–7	3	4	−1	–6	1	−5	2	10
22	1	4	−2	8	1	−5	2	−3	5	6	1	−1
23	2	−2	1	−6	4	3	−1	1	1	−4	2	−9
24	1	3	1	−2	1	4	2	−4	1	−5	−3	10
25	3	0	5	−1	0	2	1	−1	1	−3	1	2
26	3	2	1	9	2	3	1	5	2	1	3	11
27	4	−3	2	12	2	5	−3	−3	5	6	−2	0
28	1	1	−3	6	2	−1	1	−1	3	1	2	3
29	7	2	4	1	1	−3	−2	6	1	−4	−1	6
30	2	−3	−2	3	3	−2	1	1	3	−4	−1	5

II topshiriq

Fazoda uchlari $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ va $D(x_4, y_4, z_4)$ nuqtalarda joylashgan piramida berilgan. Bu piramida bo’yicha quyidagilarni bajaring:

1. $\overrightarrow{AB}$ vektor koordinatalarini toping va undan foydalanib *AB* qirra uzunligini hisoblang;

2. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib *AB* va *AD* qirralar orasidagi φ burchak kosinusini toping;

3. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib piramidaning ABD tomoni yuzasini toping ;

4. $\overrightarrow{AB}$, $\overrightarrow{AC}$ va $\overrightarrow{AD}$ vektorlar yordamida *ABCD* piramidaning hajmini aniqlang;

5. *AD* qirra yotgan toʻgʻri chiziqning kanonik va parametrik tenglamalarini yozing;

6. *ABC* yoq yotgan tekislikning umumiy, kesmalardagi va normal tenglamalarni yozing;

7. Piramidaning *ABC* va *ABD* yoqlari orasigi ikki yoqli α burchak kosinusini toping;

8. Piramidaning *D* uchidan tushirilgan *DH* balandligi yotuvchi *L* to'g'ri chiziqning kanonik tenglamasini aniqlang;

9. Piramidaning *D* uchidan tushirilgan *DH* balandligining uzunligini toping.

Izoh: $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ va $D(x_4, y_4, z_4)$ nuqtalarning koordinatalari variantga asosan jadvaldan olinadi.

Variant №	x_1	y_1	z_1	x_2	y_2	z_2	x_3	y_3	z_3	x_4	y_4	z_4
1	2	4	8	–3	5	1	6	–4	3	5	8	–1
2	–1	–3	–7	2	–4	0	–5	3	–2	–4	–7	2
3	3	5	9	0	6	–2	7	1	4	6	9	0
4	0	–2	7	–3	–3	5	–4	4	–1	–3	–6	3
5	–4	6	–3	7	7	–1	8	0	5	7	–3	1
6	1	–2	3	–4	5	–6	7	–8	9	–3	6	–5
7	2	–3	4	–5	6	–7	8	–9	3	–4	7	1
8	3	–4	–5	6	7	–8	9	–3	7	–4	–1	–2
9	4	5	–6	–7	8	–9	3	–5	7	1	2	–3
10	–5	6	7	–8	9	–2	–1	3	–4	–2	3	4
11	–6	7	–8	–9	0	3	2	1	–3	–3	–4	–5
12	7	–8	9	0	–1	2	1	–2	3	4	5	6
13	8	–9	1	–1	2	–3	–4	–5	–6	–7	0	4
14	9	–1	1	–2	1	–2	3	4	5	6	7	8
15	0	–1	2	1	–2	–3	–4	5	–6	7	–8	–9
16	1	–2	–1	–2	3	4	5	6	7	–5	0	8
17	2	1	2	3	–4	–5	–6	7	8	–9	0	–3
18	–3	4	–5	1	–8	7	–4	–2	1	2	–1	0
19	2	–5	3	–2	7	–8	3	–1	2	–3	1	5
20	–4	3	–5	0	–9	6	5	–3	0	1	–3	2
21	2	−3	6	17	3	4	−1	3	1	−5	2	10
22	1	4	−2	8	1	−5	−3	1	−4	6	1	4
23	2	−2	1	−6	4	3	−1	3	1	−4	2	−9
24	1	3	1	−2	1	4	2	−36	1	−5	−3	10
25	3	0	5	−1	0	2	1	−1	1	−3	1	2
26	3	2	1	5	2	3	1	1	2	1	3	11
27	4	−3	2	9	2	5	−3	4	5	6	−2	18
28	1	1	−3	6	2	−1	1	5	3	1	2	7
29	7	2	4	1	1	−3	−2	2	1	−4	−1	8
30	2	−3	−2	4	3	−2	1	11	3	−4	−1	7

III topshiriq

III.1-masala

Berilgan a), b), c) va d) hollardagi $y=f(x)$ funksiyalarning hosilalarini toping.

№	a) b)	$y = f(x)$	c) d)	$y = f(x)$
1	a)	$y = \dfrac{x-1}{x+1}$	c)	$y = \operatorname{arctg}(1+\ln x);$
	b)	$y = (x+1)\ln(x+1);$	d)	$x = \ln t\,,\ y = t^2$
2	a)	$y = \dfrac{x-2x^2}{1-\sin x};$	c)	$y = \arcsin(1-\ln x);$
	b)	$y = (x^2+1)\sin x;$	d)	$x = \sin t\,,\ y = t^2 - t$

3	a)	$y=\dfrac{10^x+x^{10}}{\sin x}$	c)	$y=\arccos\sqrt{1-\ln x};$
	b)	$y=(x^2+1)\mathrm{arcctg}x$	d)	$x=\cos t,\ y=t+t^2$
4	a)	$y=\dfrac{\mathrm{tg}x+\sin x}{x^2}$	c)	$y=e^{1-\cos 5x}$
	b)	$y=(1-x^2)\arcsin x;$	d)	$x=2t+1,\ y=\cos t^2$
5	a)	$y=\dfrac{\cos x+\sin x}{1+x}$	c)	$y=\arcsin(1-x^3)$
	b)	$y=x^2\ln(1+x^2);$	d)	$x=\ln(t^2+1),\ y=t^3$
6	a)	$y=\dfrac{\ln x}{1+x^2}$	c)	$y=\ln(1-\sqrt{x-1});$
	b)	$y=x\mathrm{tg}(1+x^2);$	d)	$x=e^{2t},\ y=t^2$
7	a)	$y=\dfrac{x}{x^2-1};$	c)	$y=\mathrm{arctg}\sqrt{1+x^2};$
	b)	$y=(x+\sin x)(x-\cos x);$	d)	$x=t^2,\ y=t^3+t^2+1$
8	a)	$y=\dfrac{1+\sin x}{1-\cos x};$	c)	$y=\sqrt{1-\sin(x^2+1)};$
	b)	$y=(x-\mathrm{tg}x)(x-\mathrm{ctg}x)$	d)	$x=t^2+t,\ y=t^3+1$
9	a)	$y=\dfrac{1-\mathrm{tg}x}{1+\mathrm{ctg}x}$	c)	$y=\sin(e^x+\cos x)$
	b)	$y=(x-1)\mathrm{arctg}\sqrt{x-2}$	d)	$x=t^2-4t,\ y=t^3+t$
10	a)	$y=\dfrac{\sqrt{x}}{1-\sqrt{x}};$	c)	$y=\ln(x+\ln x)$
	b)	$y=(x-1)\arcsin\sqrt{2-x};$	d)	$x=t^2-4t,\ y=(t+1)^3$
11	a)	$y=\dfrac{x-1}{5x-2}$	c)	$\left(\sqrt{x+1}\right)\left(\dfrac{1}{\sqrt{x}}-1\right)$
	b)	$y=\ln x\cdot\sin\sqrt{\ln x}$	d)	$x=(t-2)^2,\ y=t^3+t$
12	a)	$y=\dfrac{2x+3}{3x+7}$	c)	$y=5\mathrm{arctg}e^{\sqrt{5x}}$
	b)	$y=\left(x^2-3x+3\right)\left(x^2+2x-1\right)$	d)	$x=\sin(t-4),\ y=\cos(t+3)$
13	a)	$y=\dfrac{5x^2}{x-3}$	c)	$\left(\dfrac{2}{\sqrt{x}}-\sqrt{3}\right)\left(4\sqrt[3]{x}+\dfrac{\sqrt[3]{x^2}}{3x}\right)$
	b)	$y=\ln\left(e^{5x}+1\right)$	d)	$x=\sin(2t-1),\ y=\cos 2(2t-1)$
14	a)	$y=\dfrac{x^2+2x}{3-4x}$	c)	$y=\mathrm{tg}(2^x+x+1)$

	b)	$y=\left(1-x^2\right)\left(1-2x^3\right)$	d)	$x=2^t$, $y=t^2$
15	a)	$y=\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$	c)	$y=\sin 3^x\cdot\cos^2 3^x$
	b)	$y=\left(x^2+x+1\right)\left(x^2-x+1\right)$	d)	$x=\sin(2t+1)$, $y=\cos 2(2t+1)$
16	a)	$y=\dfrac{x^2}{x+1}$	c)	$y=(x-1)(x-2)(x-3)$
	b)	$y=2\ln\operatorname{tg}(x/8)$	d)	$x=(2t-1)^2$, $y=\ln(2t+1)$
17	a)	$y=\dfrac{\sqrt{x}-2}{\sqrt{x}+2}$	c)	$y=\dfrac{1}{2}\cdot\left(\operatorname{tg}2x+\ln\cos^2 2x\right)$
	b)	$y=\left(\sqrt[3]{x}+1\right)(x-1)$	d)	$x=\ln(2t-1)$, $y=\ln(2t+1)$
18	a)	$y=\dfrac{\sqrt{x^3}-x}{x+\sqrt[3]{x^2}}$	c)	$y=\operatorname{ctg}^2\operatorname{ctg}x$
	b)	$y=\left(x^2-4\right)\left(x^2-9\right)$	d)	$x=\operatorname{tg}(2t-1)$, $y=(2t+1)^2$
19	a)	$y=\dfrac{x^2+7x+5}{x^2-3x}$	c)	$y=\arcsin\sqrt{1-e^x}$
	b)	$y=\left(1+\sqrt{2x}\right)\left(1+\sqrt{3x}\right)$	d)	$x=(2t-1)^2$, $y=(2t+1)^3$
20	a)	$y=\dfrac{-x^2+2x+3}{x^3-2}$	c)	$y=\ln\dfrac{1-\sin 3x}{1+\sin 3x}$
	b)	$y=\left(x^2+x-1\right)\left(x^3+1\right)$	d)	$x=10^{2t-1}$, $y=\lg(2t-1)$
21	a)	$y=\dfrac{x^2-1}{x^2+1}$;	c)	$y=\operatorname{tg}(1+\ln x)$
	b)	$y=(x+2)^2\ln(x+2)$;	d)	$y=(x^2-1)\operatorname{tg}x$;
22	a)	$y=\dfrac{2x-x^2}{1-\cos x}$;	c)	$y=\arccos(1+\ln x)$
	b)	$y=(x^2-1)\operatorname{tg}x$;	d)	$x=10^{\sin t}$, $y=t^2-2t$
23	a)	$y=\dfrac{1+e^{3x}}{\ln x}$	c)	$y=\cos\sqrt{1-\ln x}$
	b)	$y=(x^2-1)\operatorname{arcctg}x$	d)	$x=\arccos t$, $y=\arcsin t$
24	a)	$y=\dfrac{x+\ln x}{x^3}$	c)	$y=e^{\sin x}+e^{-\cos x}$
	b)	$y=(1+x^2)\operatorname{arctg}x$	d)	$x=(2t-3)^2$, $y=\sin t^2$
25	a)	$y=\dfrac{\sin x}{1+\cos x}$	c)	$y=\arccos(1+x^2)$
	b)	$y=e^x\ln(1+x^2)$	d)	$x=(t^2+1)^3$, $y=t^3$

26	a)	$y=\frac{x-1}{1+x^2}$	c)	$y=\ln(1+\sqrt{x+1})$
	b)	$y=x^3\sin(1+x^2)$	d)	$x=e^{-4t}$, $y=t^2+2t$
27	a)	$y=\frac{1+\sqrt{x}}{1-\sqrt{x}}$	c)	$y=\ln(x-\ln x)$
	b)	$y=(x-1)\arccos\sqrt{2-x}$	d)	$x=t^2+2t$, $y=(t+2)^3$
28	a)	$y=\frac{2x-1}{4x+3}$	c)	$y=\ln x\cdot\cos\sqrt{\ln x}$
	b)	$y=\left(\sqrt{x-1}\right)\left(1-\sqrt{x}\right)$	d)	$x=(t+2)^3$, $y=t^3+3t$
29	a)	$y=\frac{2x^2+1}{3x^2+5}$	c)	$y=\mathrm{ctg}(2^x-x^2+3)$
	b)	$y=\left(x^2+5x-3\right)\left(x^2-4x+5\right)$	d)	$x=\ln(t^2-4)$, $y=\lg(t+2)$
30	a)	$y=\frac{5x^2+3}{x^2-1}$	c)	$y=\ln\left(e^{5x}+1\right)$
	b)	$y=\left(1-x^2\right)\left(1-2x^3\right)$	d)	$x=\sin(2t+1)$, $y=\cos 2(2t+1)$

III.2-masala

$y=f(x)$ tenglama bilan berilgan egri chiziqning absissasi $x=x_0$ bo'lgan nuqtasiga o'tkazilgan urinma va normal tenglamalarini yozing.

№	$y=f(x)$	x_0	№	$y=f(x)$	x_0
1	$y=x^2+2x$	2	16	$y=3tg2x+1$	π/2
2	$y=80x-x^2$	−1	17	$y=1-4x+e^{3x}$	0
3	$y=1+2\cos x$	π/2	18	$y=6tg5x$	π/20
4	$y=\frac{1}{4}x^4+\frac{1}{3}x^3$	1	19	$y=4\sin 6x$	π/18
5	$y=\frac{1}{3}x^3+4x+3$	4	20	$y=\frac{x^2}{3}-\frac{x^2}{2}-7x+9$	1
6	$y=x+\sin 2x$	π/4	21	$y=x^2-3x+1$	−1
7	$y=xe^x$	0	22	$y=8x^3-x^2+1$	3
8	$y=13+tgx$	π/3	23	$y=1-2\cos x$	−π/2
9	$y=1-x^2$	1	24	$y=4tg3x$	π/9
10	$y=1+3x+e^{2x}$	0	25	$y=x^3-3x+5$	−2
11	$y=x^3-5x^2+7x-2$	−1	26	$y=x-\cos 2x$	π/4
12	$y=x^2-6x+2$	2	27	$y=e^x\cos x$	0

13	$y=\frac{x^2}{4}-x+5$	4	28	$y=\mathrm{ctg}x+\mathrm{tg}x$	π/4
14	$y=\frac{x^4}{4}-27x+60$	−2	29	$y=\sin(1-x^2)$	−1
15	$y=-\frac{x^2}{2}+7x-\frac{15}{2}$	3	30	$y=1-5x+e^{3x}$	0

III.3-masala

Moddiy nuqta $s=s(t)$ tenglama bo'yicha harakatlanmoqda. Bu moddiy nuqtaning berilgan $t=t_0$ vaqtdagi $v(t_0)$ tezligini va $a(t_0)$ tezlanishini aniqlang.

№	$s=s(t)$	t_0	№	$s=s(t)$	t_0
1	$s=e^{\sin 2t}$	π/2	16	$s=2^{\ln t}$	*E*
2	$s=te^t$	0	17	$s=e^t\cos t$	0
3	$s=\ln(t^2-9)$	5	18	$s=\ln^2(t-1)$	2
4	$s=t^2\ln t$	1	19	$s=\frac{\ln t}{t}$	*E*
5	$s=\frac{t^2}{t+2}$	4	20	$s=\frac{1}{1-e^t}$	ln2
6	$s=\frac{4t}{4-t^2}$	$\sqrt{2}$	21	$s=\ln(t^2+1)$	0
7	$s=\frac{t^2+1}{t^2-1}$	3	22	$s=\frac{4t}{4+\sin t}$	π/2
8	$s=\frac{t^2}{t-2}$	5	23	$s=t\sqrt{5+t}$	4
9	$s=\ln(4-t^2)$	1	24	$s=\sqrt{t^2-t}$	2
10	$s=\frac{t^2+1}{t-1}$	0	25	$s=\frac{t^2}{t-1}$	3
11	$s=e^{2\cos t}$	π/2	26	$s=\sqrt{t}e^t$	1
12	$s=t\sin t$	π/4	27	$s=t^3\ln t$	1
13	$s=\ln(t^2-1)$	3	28	$s=\frac{t}{t^2+1}$	2
14	$s=(2+t^2)\ln t$	*e*	29	$s=\ln^2(t+1)$	0
15	$s=e^t\ln(t+1)$	0	30	$s=\frac{t^2}{t+4}$	0

III.4-masala

Berilgan $y=f(x)$ funksiyani ekstremumga tekshiring va uning monotonlik oraliqlarini toping.

№	$y = f(x)$	№	$y = f(x)$	№	$y = f(x)$
1	$y = e^{2x-x^2}$	11	$y = 2^{1/x}$	21	$y = e^{2x+x^2}$
2	$y = xe^{x^2}$	12	$y = x \cdot e^{-x}$	22	$y = xe^{x}$
3	$y = \ln(x^2 - 1)$	13	$y = e^{x} - x$	23	$y = \ln(x^2 - 9)$
4	$y = (2 + x^2)e^{-x^2}$	14	$y = \frac{\ln x}{x}$	24	$y = x^2 + 2\ln x$
5	$y = x^2 - 2\ln x$	15	$y = \ln(x^2 - 1)$	25	$y = \frac{x^2}{x+2}$
6	$y = \frac{x^2}{x-1}$	16	$y = \frac{1}{1-e^{x}}$	26	$y = \frac{4x}{4-x^2}$
7	$y = \frac{4x}{4+x^2}$	17	$y = x - \ln x$	27	$y = \frac{x^2+1}{x^2-1}$
8	$y = \frac{x^2-1}{x^2+1}$	18	$y = x\sqrt{x+5}$	28	$y = \frac{x^2}{x-2}$
9	$y = \ln(9 - x^2)$	19	$y = \sqrt{x^2 - x}$	29	$y = \ln(4 - x^2)$
10	$y = \frac{x^2}{x+4}$	20	$y = \sqrt{x - x^2}$	30	$y = \frac{x^2+1}{x-1}$

I BOSQICH BAKALAVRLARI UCHUN KUZGI O'QUV MAVSUMI

MUSTAQIL ISH TOPSHIRIQLARIDAGI MASALALARNING NAMUNAVIY YECHIMLARI.

I topshiriq

Berilgan uch noma'lumli chiziqli tenglamalar sistemasini Kramer, Gauss va matritsalar usullarida yeching:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 5x_1 - 3x_2 + 2x_3 = -3 \\ 2x_1 - x_2 + 3x_3 = 1 \end{cases}$$

Yechish: Berilgan sistemani Kramer usulida yechish uchun dastlab uning asosiy Δ va yordamchi Δ_1, Δ_2, Δ_3 aniqlovchilarini hisoblaymiz. Asosiy Δ aniqlovchi sistemaning koeffitsientlaridan tuziladi:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 5 & -3 & 2 \\ 2 & -1 & 3 \end{vmatrix} = 1\cdot(-3)\cdot 3 + (-1)\cdot 1\cdot 5 + 1\cdot 2\cdot 2 - 2\cdot 1\cdot(-3) - 2\cdot 1\cdot(-1) - 5\cdot 1\cdot 3 =$$

$$= -9 - 5 + 4 + 6 + 2 - 15 = -17,$$

Yordamchi Δ_1, Δ_2, Δ_3 aniqlovchilar asosiy Δ aniqlovchining mos ravishda birinchi, ikkinchi, uchinchi ustunlarini ozod hadlar bilan almashtirishdan hosil qilinadi:

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ -3 & -3 & 2 \\ 1 & -1 & 3 \end{vmatrix} = 0 + 3 + 2 + 3 - 0 + 9 = 17,$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ 5 & -3 & 2 \\ 2 & 1 & 3 \end{vmatrix} = -9 + 5 + 0 - (-6) - 2 - 0 = 0,$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ 5 & -3 & -3 \\ 2 & -1 & 1 \end{vmatrix} = -3 + 0 - 6 - 0 - 3 - 5 = -17.$$

Bu aniqlovchilar yordamida berilgan chiziqli tenglamalar sistemasining ildizlarini Kramer formulalari orqali quyidagicha topamiz:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{17}{-17} = -1, \quad x_2 = \frac{\Delta_2}{\Delta} = \frac{0}{-17} = 0, \quad x_3 = \frac{\Delta_3}{\Delta} = \frac{-17}{-17} = 1.$$

Demak, berilgan sistemaning ildizlari $x_1 = -1$, $x_2=0$, $x_3=1$ boʻladi.

Yechim toʻgʻriligini tekshirish uchun bu ildizlar qiymatlarini berilgan sistemaga qoʻyamiz:

$$\begin{cases} x_1 + x_2 + x_3 = -1 + 0 + 1 \equiv 0 \\ 5x_1 - 3x_2 + 2x_3 = 5 \cdot (-1) - 3 \cdot 0 + 2 \cdot 1 \equiv -3 \\ 2x_1 - x_2 + 3x_3 = 2 \cdot (-1) - 0 + 3 \cdot 1 \equiv 1 \end{cases}.$$

Bu yerdan koʻrinadiki $x_1 = -1$, $x_2=0$, $x_3=1$ boʻlganda berilgan sistemaning uchala tenglamasi ham ayniyat boʻldi. Demak, sistema toʻgʻri yechilgan va $x_1=-1$, $x_2=0$, $x_3=1$ berilgan sistema ildizlari boʻladi.

Bu sistemani Gauss usulida yechish uchun dastlab uni «toʻrtburchakli» shakldan «uchburchakli» shaklga keltiramiz. Buning uchun dastlab sistemaning ikkinchi va uchinchi tenglamalaridan x_1 noma'lumni yoʻqotamiz. Bunga erishish uchun sistemaning birinchi tenglamasini 5ga (yoki 2ga) koʻpaytirib, uning ikkinchi (yoki uchinchi) tenglamasidan ayiramiz. Natijada quyidagi sistemaga kelamiz:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ -8x_2 - 3x_3 = -3 \\ -3x_2 + x_3 = 1 \end{cases}$$

Endi bu sistemaning uchinchi tenglamasidan x_2 noma'lumni yoʻqotamiz. Buning uchun oxirgi sistemaning ikkinchi tenglamasini 3 ga, uchinchi tenglamasini esa 8 ga koʻpaytirib, hosil boʻlgan uchinchi tenglamadan ikkinchi tenglamani ayiramiz. Natijada ushbu «uchburchak» shaklidagi sistemaga kelamiz:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ -8x_2 - 3x_3 = -3 \\ -17x_3 = -17 \end{cases}.$$

Oxirgi uchburchakli sistemaning uchinchi tenglamasidan x_3 noma'lumni topamiz:

$$-17x_3 = -17 \Rightarrow x_3 = \frac{-17}{-17} = 1.$$

$x_3=1$ natijani uchburchakli sistemaning ikkinchi tenglamasiga qoʻyib, x_2 noma'lumni topamiz:

$$-8x_2 - 3 \cdot 1 = -3 \Rightarrow -8x_2 - 3 = -3 \Rightarrow -8x_2 = 0 \Rightarrow x_2 = 0.$$

Topilgan $x_3=1$ va $x_2=0$ natijalarni uchburchakli sistemaning birinchi tenglamasiga qoʻyib, x_1 noma'lum qiymatini topamiz:

$$x_1 + 0 + 1 = 0 \Rightarrow x_1 + 1 = 0 \Rightarrow x_1 = -1.$$

Demak, berilgan sistemaning ildizlari $x_1 = -1$, $x_2=0$, $x_3=1$ boʻladi va Kramer usulida topilgan natijalar bilan ustma−ust tushadi.

Endi bu sistemani matritsalar usulida yechamiz. Buning uchun berilgan sistema boʻyicha quyidagi matritsalarni kiritamiz:

$$A=\begin{pmatrix}1 & 1 & 1\\ 5 & -3 & 2\\ 2 & -1 & 3\end{pmatrix},\quad B=\begin{pmatrix}0\\ -3\\ 1\end{pmatrix},\quad X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}.$$

Bu holda berilgan chiziqli tenglamalar sistemasi $AX=B$ koʻrinishga keladi va uning ildizlaridan iborat X matritsa $X=A^{-1}\cdot B$ formula bilan topiladi. Bu yerda A^{-1} yuqoridagi A matritsaga teskari matritsa boʻlib, u

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}$$

formula orqali topiladi. Shu sababli dastlab Δ=detA aniqlovchini va A_{ij} algebraik toʻldiruvchilarni hisoblaymiz. Kramer usuli koʻrilayotganda

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 5 & -3 & 2\\ 2 & -1 & 3\end{vmatrix}=-17$$

ekanligi topilgan edi. Algebraik toʻldiruvchi ta’rifiga asosan

$$A_{11}=\begin{vmatrix}-3 & 2\\ -1 & 3\end{vmatrix}=-7,\quad A_{12}=-\begin{vmatrix}5 & 2\\ 2 & 3\end{vmatrix}=-11,\quad A_{13}=\begin{vmatrix}5 & -3\\ 2 & -1\end{vmatrix}=1$$

$$A_{21}=-\begin{vmatrix}1 & 1\\ -1 & 3\end{vmatrix}=-4,\quad A_{22}=\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}=1,\quad A_{23}=-\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}=3$$

$$A_{31}=\begin{vmatrix}1 & 1\\ -3 & 2\end{vmatrix}=5,\quad A_{32}=-\begin{vmatrix}1 & 1\\ 5 & 2\end{vmatrix}=3,\quad A_{33}=\begin{vmatrix}1 & 1\\ 5 & -3\end{vmatrix}=-8$$

ekanligini topamiz.

Demak,

$$A^{-1}=\frac{1}{-17}\begin{pmatrix}-7 & -4 & 5\\ -11 & 1 & 3\\ 1 & 3 & -8\end{pmatrix}=\begin{pmatrix}\frac{7}{17} & \frac{4}{17} & -\frac{5}{17}\\ \frac{11}{17} & -\frac{1}{17} & -\frac{3}{17}\\ -\frac{1}{17} & -\frac{3}{17} & \frac{8}{17}\end{pmatrix}$$

va matritsalarni koʻpaytirish ta’rifiga asosan

$$X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=A^{-1}B=\begin{pmatrix}\frac{7}{17} & \frac{4}{17} & -\frac{5}{17}\\ \frac{11}{17} & -\frac{1}{17} & -\frac{3}{17}\\ -\frac{1}{17} & -\frac{3}{17} & \frac{8}{17}\end{pmatrix}\cdot\begin{pmatrix}0\\ -3\\ 1\end{pmatrix}=\frac{1}{17}\begin{pmatrix}-17\\ 0\\ 17\end{pmatrix}=\begin{pmatrix}-1\\ 0\\ 1\end{pmatrix}.$$

Bu yerdan yana bir marta berilgan sistemaning yechimi $x_1=-1$, $x_2=0$ va $x_3=1$ ekanligini koʻramiz.

II topshiriq

Fazoda uchlari $A(8,6,4)$, $B(10,5,5)$, $C(5,6,8)$ va $D(9,10,7)$ nuqtalarda joylashgan piramida berilgan. Bu piramida bo’yicha quyidagilarni bajaring:

1. $\overrightarrow{AB}$ vektor koordinatalarini toping va undan foydalanib AB qirra uzunligini hisoblang;

2. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib *AB* va *AD* qirralar orasidagi φ burchak kosinusini toping;

3. $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlardan foydalanib piramidaning ABD tomoni yuzasini toping ;

4. $\overrightarrow{AB}$, $\overrightarrow{AC}$ va $\overrightarrow{AD}$ vektorlar yordamida *ABCD* piramidaning hajmini aniqlang;

5. *AD* qirra yotgan toʻgʻri chiziqning kanonik va parametrik tenglamalarini yozing;

6. *ABC* yoq yotgan tekislikning umumiy, kesmalardagi va normal tenglamalarni yozing;

7. Piramidaning *ABC* va *ABD* yoqlari orasigi ikki yoqli *α* burchak kosinusini toping;

8. Piramidaning *D* uchidan tushirilgan *DH* balandligi yotuvchi *L* to'g'ri chiziqning kanonik tenglamasini aniqlang;

9. Piramidaning *D* uchidan tushirilgan *DH* balandligining uzunligini toping.

Yechish: **1.** $\overrightarrow{AB}=(x,\ y,\ z)$ vektorning *x*, *y* va *z* koordinatalari uning B(10,5,5) uchi va A(8,6,4) boshi mos koordinatalarining ayirmasiga teng , ya'ni

$$\overrightarrow{AB}=(x, y, z)=(x_2-x_1, y_2-y_1, z_2-z_1)=(10-8, 5-6, 5-4)=(2, -1, 1).$$

AB qirraning |AB| uzunligi topilgan $\overrightarrow{AB}$ vektor moduliga teng bo'ladi va | $\overrightarrow{AB}$ | modul formulasiga asosan

$$|AB|=\left|\overrightarrow{AB}\right|=\sqrt{x^2+y^2+z^2}=\sqrt{2^2+(-1)^2+1^2}=\sqrt{6}\,.$$

2. Dastlab yuqoridagi singari A(8,6,4) va D(9,10,7) nuqtalar bo'yicha $\overrightarrow{AD}$ vektor koordinatalarini topamiz:

$$\overrightarrow{AD}=(9-8, 10-6, 7-4)=(1, 4, 3).$$

AB va *AD* qirralar orasidagi φ burchak kosinusini $\overrightarrow{AB}=(2, -1, 1)$ va $\overrightarrow{AD}=(1, 4, 3)$ vektorlar orasidagi burchak formulasi, vektorlar skalyar ko'paytmasi va modullarini koordinatalar orqali ifodasidan foydalanib topamiz:

$$\cos\varphi=\frac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{AD}\right|}=\frac{2\cdot1+(-1)\cdot4+1\cdot3}{\sqrt{2^2+(-1)^2+1^2}\cdot\sqrt{1^2+4^2+3^2}}=\frac{1}{\sqrt{6}\cdot\sqrt{26}}=\frac{1}{\sqrt{156}}\,.$$

3. Piramidaning ABD yog'ining *S* yuzasini topish uchun $\overrightarrow{AB}=(2, -1, 1)$ va $\overrightarrow{AD}=(1, 4, 3)$ vektorlarning vektorial ko'paytmasidan foydalanamiz. Vektorial ko'paytmaning koordinatalardagi ifodasi va III tartibli aniqlovchini hisoblash formulasiga asosan

$$\overrightarrow{AB}\times\overrightarrow{AD}=\begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 4 & 3 \end{vmatrix}=-3i+8k+j+k-4i-6j=-7i-5j+9k=(-7,-5,9).$$

Bu yerdan, vektorial ko'paytma modulining geometrik ma'nosiga asosan,

$$S=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AD}\right|=\frac{1}{2}\sqrt{(-7)^2+(-5)^2+9^2}=\frac{\sqrt{155}}{2}\ \text{kv.birlik}$$

javobga ega bo'lamiz.

4. Dastlab A(8,6,4) va C(5,6,8) nuqtalar bo'yicha $\overrightarrow{AC}$ vektor koordinatalarini topamiz:

$$\overrightarrow{AC}=(5-8, 6-6, 8-4)=(-3, 0, 4).$$

ABCD piramidaning *V* hajmini

$$\overrightarrow{AB}=(2, -1, 1),\quad \overrightarrow{AC}=(-3, 0, 4),\quad \overrightarrow{AD}=(1, 4, 3)$$

vektorlarning aralash ko'paytmasi yordamida topamiz. Aralash ko'paytmaning koordinatalar orqali ifodasi formulasidan foydalanib

$$V=\pm\frac{1}{6}\overrightarrow{AB}\cdot\overrightarrow{AC}\cdot\overrightarrow{AD}=\pm\frac{1}{6}\begin{vmatrix} 2 & -1 & 1 \\ -3 & 0 & 4 \\ 1 & 4 & 3 \end{vmatrix}=$$

$$=\pm\frac{1}{6}(0+(-4)+(-12)-0-32-9)=\pm\frac{1}{6}\cdot(-57)=\frac{57}{6}=9\frac{1}{2}\text{ kub birlik}$$

natijani olamiz.

5. *AD* qirra yotgan toʻgʻri chiziqning kanonik tenglamasini ikkita A(8,6,4) va D(9,10,7) nuqtalardan oʻtuvchi toʻgʻri chiziq tenglamasi formulasidan foydalanib topamiz:

$$AD:\quad \frac{x-8}{9-8}=\frac{y-6}{10-6}=\frac{z-4}{7-4}\Rightarrow\frac{x-8}{1}=\frac{y-6}{4}=\frac{z-4}{3}.$$

Endi AD qirraning kanonik tenglamasidagi kasrlarni *t* parametrga tenglashtirib, uning parametrik tenglamasini hosil qilamiz :

$$\frac{x-8}{1}=\frac{y-6}{4}=\frac{z-4}{3}=t\Rightarrow x-8=t,\quad y-6=4t,\quad z-4=3t\Rightarrow$$
$$x=t+8,\quad y=4t+6,\quad z=3t+4.$$

6. *ABC* yoq yotgan tekislikning $Ax+By+Cz+D=0$ koʻrinishdagi umumiy tenglamasini uchta *A*(8,6,4) , *B*(10,5,5) va *C*(5,6,8) nuqtalardan oʻtuvchi tekislik tenglamasining ifodasi yordamida topamiz:

$$\begin{vmatrix} x-8 & y-6 & z-4\\ 10-8 & 5-6 & 5-4\\ 5-8 & 6-6 & 8-4\end{vmatrix}=0\Rightarrow\begin{vmatrix} x-8 & y-6 & z-4\\ 2 & -1 & 1\\ -3 & 0 & 4\end{vmatrix}=0$$

$$\Rightarrow -4(x-8)-3(y-6)-3(z-4)-8(y-6)=0\Rightarrow$$
$$-4x+32-3y+18-3z+12-8y+48=0$$
$$-4x-11y-3z+110=0\Rightarrow 4x+11y+3z-110=0$$

Endi *ABC* yoqning kesmalarga nisbatan $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ tenglamasini topish uchun uning umumiy tenglamasini $-D=110$ ga boʻlamiz:

$$\frac{4x}{110}+\frac{11y}{110}+\frac{3z}{110}-\frac{110}{110}=0\Rightarrow\frac{x}{5/2}+\frac{y}{10}+\frac{z}{110/3}=1.$$

Bu yerdan izlangan kesmalardagi tenglamada $a=5/2$, $b=10$ va $c=110/3$ ekanligini koʻramiz.

ABC yoqning normal $x\cos\alpha+y\cos\beta+z\cos\gamma-p=0$ tenglamasini topish uchun normallashtiruvchi *M* koʻpaytuvchini topib, *ABC* yoqning umumiy tenglamasining ikkala tomonini *M* ga koʻpaytiramiz. Umumiy tenglamada ozod had $D=-110<0$ boʻlgani uchun

$$M=\frac{1}{\sqrt{A^2+B^2+C^2}}=\frac{1}{\sqrt{4^2+11^2+3^2}}=\frac{1}{\sqrt{16+121+9}}=\frac{1}{\sqrt{146}}\quad\Rightarrow$$
$$\frac{4}{\sqrt{146}}x+\frac{11}{\sqrt{146}}y+\frac{3}{\sqrt{146}}z-\frac{110}{\sqrt{146}}=0.$$

Demak, $\cos\alpha=4/\sqrt{146}$, $\cos\beta=11/\sqrt{146}$, $\cos\gamma=3/\sqrt{146}$ va $p=110/\sqrt{146}$.

7. Dastlab *ABD* yoq yotgan tekislikning umumiy tenglamasini topamiz:

$$\begin{vmatrix} x-8 & y-6 & z-4\\ 10-8 & 5-6 & 5-4\\ 9-8 & 10-6 & 7-4\end{vmatrix}=0\Rightarrow\begin{vmatrix} x-8 & y-6 & z-4\\ 2 & -1 & 1\\ 1 & 4 & 3\end{vmatrix}=0$$

$$\Rightarrow -3(x-8)+(y-6)+8(z-4)+(z-4)-6(y-6)+4(x-8)=0\Rightarrow$$
$$\Rightarrow x-5y+9z-14=0.$$

Umumiy tenglamalari $4x+11y+3z-110=0$ va $x-5y+9z-14=0$ bo'lgan *ABC* va *ABD* tekisliklar orasidagi burchak formulasiga asosan cosα qiymatini topamiz:

$$\cos\alpha = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\cdot\sqrt{A_2^2 + B_2^2 + C_2^2}} =$$

$$= \frac{4\cdot 1 + 11\cdot(-5) + 3\cdot 9}{\sqrt{4^2 + 11^2 + 3^2}\cdot\sqrt{1^2 + (-5)^2 + 9^2}} = -\frac{24}{\sqrt{146}\cdot\sqrt{107}} = -\frac{24}{\sqrt{15622}} .$$

8. Piramidaning D(9,10,7) uchidan tushirilgan *DH* balandlik yotgan *L* to'g'ri chiziq tenglamasini topish uchun dastlab bu nuqtadan o'tuvchi to'g'ri chiziqlar dastasi tenglamasidan foydalanamiz:

$$L: \quad \frac{x-9}{m} = \frac{y-10}{n} = \frac{z-7}{p} .$$

Bu to'g'ri chiziq *ABC* yoq yotgan va $4x+11y+3z-110=0$ umumiy tenglama bilan aniqlangan tekislikka perpendikulyar joylshgan. Shu sababli, fazodagi to'g'ri chiziq va tekislikning perpendikulyarlik shartiga asosan, $m=4$, $n=11$ va $p=3$ deb olish mumkin. Demak, *DH* balandlik yotgan *L* to'g'ri chiziqning kanonik tenglamasi quyidagicha bo'ladi:

$$L: \quad \frac{x-9}{4} = \frac{y-10}{11} = \frac{z-7}{3} .$$

9. Piramidaning D(9,10,7) uchidan tushirilgan *DH* balandlikning *h* uzunligini bu nuqtadan umumiy tenglamasi $4x+11y+3z-110=0$ bo'lgan *ABC* yoq yotgan tekislikkacha bo'lgan *d* masofa formulasidan foydalanib topamiz:

$$h = d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} = \frac{|4\cdot 9 + 11\cdot 10 + 3\cdot 7 - 110|}{\sqrt{4^2 + 11^2 + 3^2}} =$$

$$= \frac{|36 + 110 + 21 - 110|}{\sqrt{146}} = \frac{57}{\sqrt{146}} .$$

III topshiriq

III.1-masala

Quyidagi berilgan funksiyalarning hosilalarini toping:

$a)\ y = \dfrac{x}{\sqrt{a^2 - x^2}}$, b) $y = (3x^2 + 5x - 4)\sin x$,

c) $y = \ln(3tgx + e^x)$ d) $x = t(\cos t - \sin t),\ y = t(\cos t + \sin t)$.

Yechish: *a*) Bo'linmaning hosilasi

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

formulasida $u = x, \quad v = \sqrt{a^2 - x^2}$ deb olib va hosilalar jadvalidan foydalanib, ushbu natijani olamiz:

$$y' = \left(\frac{x}{\sqrt{a^2 - x^2}}\right)' = \frac{x'\sqrt{a^2 - x^2} - x\left(\sqrt{a^2 - x^2}\right)'}{(\sqrt{a^2 - x^2})^2} = \frac{\sqrt{a^2 - x^2} - \dfrac{1}{2\sqrt{a^2 - x^2}}(a^2 - x^2)'}{a^2 - x^2} =$$

$$=\frac{\sqrt{a^2-x^2}+\frac{x^2}{\sqrt{a^2-x^2}}}{a^2-x^2}=\frac{(\sqrt{a^2-x^2})^2+x^2}{\sqrt{\left(a^2-x^2\right)^3}}=\frac{a^2-x^2+x^2}{\sqrt{\left(a^2-x^2\right)^3}}=\frac{a^2}{\sqrt{\left(a^2-x^2\right)^3}}.$$

b) Ko'paytmaning hosilasi $(uv)'=u'v+uv'$ formulasida

$$u=3x^2+5x-4,\quad v=\sin x$$

deb olib va hosilalar jadvalidan foydalanib, ushbu javobga kelamiz:

$$y'=\left((3x^2+5x-4)\sin x\right)'=(3x^2+5x-4)'\sin x+(3x^2+5x-4)(\sin x)'=$$
$$=(6x+5)\sin x+(3x^2+5x-4)\cos x.$$

c) Murakkab funksiyaning hosilasi $[f(u)]'=f'(u)\cdot u'$ formulasida $f(u)=\ln u,\quad u=3tgx+e^x$ deb olib va hosilalar jadvaliga asosan

$$y'=[\ln(3tgx+e^x)]'=(u=3tgx+e^x)=(\ln u)'=\frac{1}{u}u'=$$
$$=\frac{1}{3tgx+e^x}(3tgx+e^x)'=\frac{1}{3tgx+e^x}(3\cdot\frac{1}{\cos^2 x}+e^x)=\frac{3+e^x\cos^2 x}{(3tgx+e^x)\cos^2 x}$$

natijaga erishamiz.

d) Parametrik $x=\varphi(t)$, $y=\psi(t)$ ko'rinishda berilgan funksiyaning hosilasini topish

$$y'=\frac{\varphi'(t)}{\psi'(t)}$$

formulasida $x=\varphi(t)=t(\cos t-\sin t)$, $y=\psi(t)=t(\cos t+\sin t)$ deb, izlanayotgan y' hosilaning parametrik ko'rinishdagi ifodasini topamiz:

$$y'=\frac{[t(\cos t-\sin t)]'}{[t(\cos t+\sin t)]'}=\frac{t'(\cos t-\sin t)+t(\cos t-\sin t)'}{t'(\cos t+\sin t)+t(\cos t+\sin t)'}=$$
$$=\frac{(\cos t-\sin t)+t(-\sin t-\cos t)}{(\cos t+\sin t)+t(-\sin t+\cos t)}=\frac{\cos t-\sin t-t(\sin t+\cos t)}{\cos t+\sin t-t(\sin t-\cos t)}.$$

III.2-masala

Ushbu $y=\sqrt[3]{x^2}-2x-2$ funksiya grafigiga absissasi $x_0=1$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.

Yechish: Ma'lumki, differentsiallanuvchi $y=f(x)$ funksiya grafigining $(x_0, y_0)=(x_0, f(x_0))$ nuqtasiga o'tkazilgan urinma tenglamasi

$$y-y_0=f'(x_0)(x-x_0),$$

normal tenglamasi esa

$$y-y_0=-\frac{1}{f'(x_0)}(x-x_0)$$

formulalar bilan topiladi. Bizning masalada $f(x)=\sqrt[3]{x^2}-2x-2$, $x_0=1$ va

$$y_0=f(x_0)=f(1)=\sqrt[3]{1^2}-2\cdot1-2=1-4=-3,$$

$$y'(x)=f'(x)=\frac{2}{3\sqrt[3]{x}}-2\Rightarrow f'(x_0)=\frac{2}{3\sqrt[3]{x_0}}-2=\frac{2}{3\sqrt[3]{1}}-2=\frac{2}{3}-2=-\frac{4}{3}$$

bo'ladi. Bu yerdan urinma tenglamasi

$$y+3=-\frac{4}{3}(x-1) \Rightarrow 3y+9=-4x+4 \Rightarrow 4x+3y+5=0,$$

normal tenglamasi esa

$$y+3=\frac{3}{4}(x-1) \Rightarrow 4y+12=3x-3 \Rightarrow 3x-4y-15=0$$

ko'rinishda ekanligi kelib chiqadi.

III.3-masala

Moddiy nuqta $s=t\sin^2 t$ tenglama bo'yicha harakatlanmoqda. Bu moddiy nuqtaning berilgan $t=\pi/4$ vaqtdagi $v(\pi/4)$ tezligini va $a(\pi/4)$ tezlanishini aniqlang.

Yechish: Harakat tenglamasi $s=s(t)$ bo'lgan moddiy nuqtaning $t=t_0$ vaqtdagi tezligi $v(t_0)=s'(t_0)$ va tezlanishi $a(t_0)=s''(t_0)$ hosilalar orqali topiladi. Shu sababli dastlab I tartibli $s'(t)$ va II tartibli $s''(t)$ hosilalarni hisoblaymiz:

$$s'(t)=(t\sin^2 t)'=t'\sin^2 t+t(\sin^2 t)'=\sin^2 t+t\cdot 2\sin t\cos t=\sin^2 t+t\sin 2t,$$

$$s''(t)=[s'(t)]'=[\sin^2 t+t\sin 2t]'=\sin 2t+\sin 2t+2t\cos 2t=2(\sin 2t+t\cos 2t)$$ Bu yerdan, yuqoridagi formulalarga asosan,

$$v(\frac{\pi}{4})=s'(\frac{\pi}{4})=\sin^2\frac{\pi}{4}+\frac{\pi}{4}\sin\frac{\pi}{2}=(\frac{\sqrt{2}}{2})^2+\frac{\pi}{4}\cdot 1=\frac{2+\pi}{4},$$

$$a(\frac{\pi}{4})=s''(\frac{\pi}{4})=2(\sin\frac{\pi}{2}+\frac{\pi}{4}\cos\frac{\pi}{2})=2(1+\frac{\pi}{4}\cdot 0)=2.$$

III.4-masala

Berilgan $f(x)=x^3+4,5x^2-12x+1$ funksiyani ekstremumga tekshiring va uning monotonlik oraliqlarini toping.

Yechish: Berilgan $f(x)=x^3+4,5x^2-12x+1$ funksiyani ekstremumga tekshirish uchun dastlab $f'(x)=0$ tenglamadan uning kritik nuqtalarini topamiz:

$$f'(x)=(x^3+4,5x^2-12x+1)'=3x^2+9x-12=0 \Rightarrow 3x^2+9x-12=0 \Rightarrow x^2+3x-4=0,$$

$$x_{1,2}=\frac{-3\pm\sqrt{9+16}}{2}=\frac{-3\pm 5}{2} \Rightarrow x_1=-4,\ x_2=1.$$

Dastlab funksiyaning $x_1=-4$ kritik nuqtadagi xarakterini aniqlaymiz. Bunda $x<-4$ holda $f'(x)>0$ va $x>-4$ holda $f'(x)<0$ bo'ladi. Demak, $x_1=-4$ kritik nuqtada funksiya lokal maksimumga ega bo'ladi va

$$f_{max}=f(-4)=(-4)^3+4.5\cdot(-4)^2-12\cdot(-4)+1=57.$$

Endi funksiyaning $x_2=1$ kritik nuqtadagi xarakterini aniqlaymiz. Bunda $x<1$ holda $f'(x)<0$ va $x>1$ holda $f'(x)>0$ bo'ladi. Demak, $x_2=1$ kritik nuqtada funksiya lokal minimumga ega bo'ladi va

$$f_{min}=f(1)=1^3+4.5\cdot 1^2-12\cdot 1+1=-5.5\ .$$

Funksiyaning monotonlk oraliqlari, ya'ni o'sish va kamayish sohalari, $f'(x)>0$ va $f'(x)<0$ tengsizliklarning yechimlari kabi topiladi. Bunda

$$f'(x)>0 \Rightarrow 3x^2+9x-12>0 \Rightarrow x<-4,\ x>1$$

bo'lgani uchun funksiyaning o'sish sohasi $(-\infty,-4)\cup(1,\infty)$ ekanligi kelib chiqadi.

Xuddi shunday tarzda

$$f'(x)<0 \Rightarrow 3x^2+9x-12<0 \Rightarrow -4<x<1$$

bo'lgani uchun funksiyaning kamayish sohasi $(-4,1)$ oraliqdan iborat ekanligi kelib chiqadi.