O’ZBEKISTON ALOQA VA AXBOROTLASHTIRISH AGENTLIGI

TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI
SAMARQAND FILIALI

Yaxshiboyev M.U., Narzullayev U.X., Muxiddinov S. R., Xasanov X.A.

OLIY MATEMATIKADAN
MISOL VA MASALALAR TO’PLAMI

ALGEBRA VA ANALITIK GEOMETRIYA, LIMIT, UZLUKSIZLIK, HOSILA, INTEGRAL

1 - QISM

S A M A R Q A N D – 2012

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5521900 – «Informatika va axborot texnologiyalari», 5522200 – “Telekommunikatsiya”, 5140900 – “Kasb ta’limi (Informatika va axborot texnologiyalari)”, 5811100 - «Korxonalar servisi» (edektron va komp’yuter texnikasi) bakalavriat yo’nalishlari uchun o’quv–uslubiy qo’llanma

TATU Samarqand filiali o’quv–uslubiy
Kengashining 2011 yil 31 may qarori
bilan (9- bayonnoma) nashrga tavsiya etilgan

SAMARQAND – 2012

M.U.Yaxshiboyev, U.X.Narzullayev, S.R.Muxiddinov, X.A.Xasanov *Oliy matematikadan misol va masalalar to'plami:* algebra va analitik geometriya, limit, uzluksizlik, hosila, integral. 1 - qism Samarqand: SamDU nashri, 2012, –116 bet.

O'quv–uslubiy qo'llanmada ikkinchi, uchinchi va yuqori tartibli determinatlar, matritsa, chiziqli tenglamalar sistemasi, vektorlar, tekislik va fazodagi to'g'ri burchakli Dekart koordinatalar sistemasi, vektorlarning vektor va aralash ko`paytmalari, tekislikda to'g'ri chiziq, ikkinchi tartibli chiziqlar, fazoda tekislik va to'g'ri chiziq, sirtlar, kompleks sonlar, to'plam, to'plamlar ustida amallar, haqiqiy sonlar, sonlar ketma-ketligi va uning limiti, funksiyaning limit, funksiyaning uzluksizligi, funksiyaning hosilasi va differensiali, funksiyaning yuqori tartibli hosilasi va differensiali, Lopital qoidalari va Teylor formulasi, funksiyani to'liq tekshirish, aniqmas, aniq va xosmas integrallar hamda ularning tadbiqlari.

Taqrizchilar: A. G'aziev, SamDU matematik analiz kafedrasi professori,
B. Tursunqulov, TATU Samarqand filiali dotsenti

TATU Samarqand filiali, 2012

MUNDARIJA

SO’Z BOSHI

Ilm-fan jadal taraqqiy etayotgan, zamonaviy axborot-kommunikatsiya tizimlari vositalari keng joriy etilgan jamiyatda turli fan sohalarida bilimlarning tez yangilanib borishi, ta’lim oluvchilar oldiga ularni jadal egallash bilan bir qatorda, muntazam va mustaqil ravishda bilim izlash vazifasini qo’ymoqda.

Hozirgi vaqtda oliy matematika fani, texnika va iqtisodning turli –tuman masalalarini hal qilishda keng qo’llanilmoqda. Talabalar o’quv adabiyotni mustaqil o’rganish va undan foydalana bilish malakalarini hosil qilish, mantiqiy fikrlashni o’stirish va matematikaviy madaniyatning umumiy saviyasini ko’tarish; tatbiqiy masalalarni matematikaviy tomondan tekshirish malakalarini hosil qilish kabilar talab qilinadi.

Uslubiy qo‘llanma bakalavriatning 5521900 – «Informatika va axborot texnologiyasi», 5522200 – “Telekommunikasiya”, 5140900 – “Kasb ta’lim (Informatika va axborot texnologiyalari)”, 5811100 - «Korxonalar servisi» (edektron va komp’yuter texnikasi) ta’lim yo‘nalishlari uchun mo‘ljallangan bo‘lib, u amaldagi davlat ta’lim standartlari va «Oliy matematika» fani namunaviy dasturiga asosan tuzildi.

O’quv–uslubiy ko’llanma ikkinchi, uchinchi va yuqori tartibli determinantlar, matritsa, chiziqli tenglamalar sistemasi, vektorlar, chiziqli fazo, fundamental yechimlar sistemasi, tekislik va fazodagi to’g’ri burchkli Dekart koordinatalar sistemasi, vektorlarning skalyar, vektor va aralash ko`paytmalari, tekislikda to’g’ri chiziq, ikkinchi tartibli chiziqlar, fazoda tekislik va to’g’ri chiziq, sirtlar, kompleks sonlar, to’plam, to’plamlar ustida amallar, haqiqiy sonlar, sonlar ketma-ketligi va uning limiti, funksiyaning limit, funksiyaning uzluksizligi, funksiyaning hosilasi va differensiali, funksiyaning yuqori tartibli hosilasi va differensiali, Lopital qoidalari va Teylor formulasi, funksiyani to’liq tekshirish, aniqmas, aniq va xosmas integrallar hamda ularning tadbiqlari mavzularini o’z ichiga olgan.

1-amaliy mashg'ulot.

2-, 3-, $n-$ TARTIBLI DETERMINANTLAR. HISOBLASH USULLARI

1. Determinantlarni hisoblang:

$$a)\begin{vmatrix}3 & 5\\ 5 & 8\end{vmatrix};\ b)\begin{vmatrix}ab & ac\\ bd & cd\end{vmatrix};\ c)\begin{vmatrix}\sin\alpha & \sin\beta\\ \cos\alpha & \cos\beta\end{vmatrix};\ d)\begin{vmatrix}\ell og_b a & 1\\ 1 & \ell og_a b\end{vmatrix};$$

$$e)\begin{vmatrix}\cos\alpha + i\sin\alpha & 1\\ 1 & \cos\alpha - i\sin\alpha\end{vmatrix};\qquad f)\begin{vmatrix}a+bi & c+di\\ -c+di & a-bi\end{vmatrix}.$$

2. Determinantlarni hisoblang:

$$a)\begin{vmatrix}-1 & 5 & 4\\ 3 & -2 & 0\\ -1 & 3 & 6\end{vmatrix};\qquad b)\begin{vmatrix}0 & 2 & 2\\ 2 & 0 & 2\\ 2 & 2 & 0\end{vmatrix};\qquad c)\begin{vmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{vmatrix};$$

$$d)\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix};\qquad e)\begin{vmatrix}0 & a & 0\\ b & c & d\\ 0 & e & 0\end{vmatrix};\quad f)\begin{vmatrix}\sin\alpha & \cos\alpha & 1\\ \sin\beta & \cos\beta & 1\\ \sin\gamma & \cos\gamma & 1\end{vmatrix};\ g)\begin{vmatrix}1 & 0 & 1+i\\ 0 & 1 & i\\ 1-i & -i & 1\end{vmatrix};$$

$$h)\begin{vmatrix}1 & \varepsilon & \varepsilon^2\\ \varepsilon^2 & 1 & \varepsilon\\ \varepsilon & \varepsilon^2 & 1\end{vmatrix}\left(\varepsilon = -\frac{1}{2} + i\frac{\sqrt{3}}{2}\right);\ i)\begin{vmatrix}1 & 1 & 1\\ 1 & \varepsilon & \varepsilon^2\\ 1 & \varepsilon^2 & \varepsilon\end{vmatrix}\left(\varepsilon = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right).$$

3. Determinantlarni yoymasdan turib, quyidagi ayniyatlarni isbotlang:

$$a)\begin{vmatrix}1 & a & bc\\ 1 & b & ca\\ 1 & c & ab\end{vmatrix} = (b-a)(c-a)(c-b);$$

$$b)\begin{vmatrix}1 & 1 & 1\\ a^2 & b^2 & c^2\\ a^3 & b^3 & c^3\end{vmatrix} = (ab+ac+bc)(b-a)(c-a)(c-b);$$

$c)\begin{vmatrix}1 & a & bc\\ 1 & b & ca\\ 1 & c & ab\end{vmatrix}=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$; $d)\begin{vmatrix}1 & a & a^3\\ 1 & b & b^3\\ 1 & c & c^3\end{vmatrix}=(a+b+c)\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$.

4. *a*) $\begin{vmatrix}5x & 1 & 2 & 3\\ x & x & 1 & 2\\ 1 & 2 & x & 3\\ x & 1 & 2 & 2x\end{vmatrix}$ determinantning x^4 va x^3 ni saqlovchi hadlarni toping.

b) $\begin{vmatrix}x & 1 & 2 & 3\\ 1 & x & 3 & 2\\ 3 & 1 & x & 2\\ 5 & 3 & 1 & x\end{vmatrix}$, determinantning x^4, x^3 va x^2 ni saqlovchi hadlarni toping.

5. *a*) $\begin{vmatrix}1 & 1 & 1 & a\\ 2 & 2 & 1 & b\\ 3 & 2 & 1 & c\\ 1 & 2 & 3 & d\end{vmatrix}$ determinantni 4-ustun elementlari bo'yicha yoying;

b) $\begin{vmatrix}a & 1 & 1 & 1\\ b & 0 & 1 & 1\\ c & 1 & 0 & 1\\ d & 1 & 1 & 0\end{vmatrix}$ determinantni 1-ustun elementlari bo'yicha yoying;

c) $\begin{vmatrix}1 & 2 & -1 & -2\\ 2 & -1 & -2 & 1\\ a & b & c & d\\ -2 & -1 & 1 & 2\end{vmatrix}$ determinantni 3-satr elementlari bo'yicha yoying.

6. Determinantni uni yoymasdan turib hisoblang:

$$\begin{vmatrix} a & b & c & 1 \\ b & c & a & 1 \\ c & a & b & 1 \\ \frac{b+c}{2} & \frac{c+a}{2} & \frac{a+b}{2} & 1 \end{vmatrix}.$$

7. Determinantning xossalaridan foydalanib (satr va ustun bo'yicha yoyishni ham hisobga olganda) ayniyatlarni isbotlang:

$$a)\begin{vmatrix} (a+b)^2 & c^2 & c^2 \\ a^2 & (b+c)^2 & a^2 \\ b^2 & b^2 & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3;$$

$$b)\begin{vmatrix} \frac{1}{a+x} & \frac{1}{a+y} & 1 \\ \frac{1}{b+x} & \frac{1}{b+y} & 1 \\ \frac{1}{c+x} & \frac{1}{c+y} & 1 \end{vmatrix} = \frac{(a-b)(a-c)(b-c)(x-y)}{(a+x)(b+x)(c+x)(a+y)(b+y)(c+y)};$$

$$c)\begin{vmatrix} a^2+(1-a^2)\cos\varphi & ab(1-\cos\varphi) & ac(1-\cos\varphi) \\ ba(1-\cos\varphi) & b^2+(1-b^2)\cos\varphi & bc(1-\cos\varphi) \\ ca(1-\cos\varphi) & cb(1-\cos\varphi) & c^2+(1-c^2)\cos\varphi \end{vmatrix} = \cos^2\varphi;$$

$$d)\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2+b^2+c^2+d^2)^2;$$

$$e)\begin{vmatrix} 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \\ 1 & 1 & 0 & c \\ a & b & c & d \end{vmatrix} = a^2+b^2+c^2-2ab-2bc-2ac+2d.$$

8. Determinantlarni hisoblang:

$$a)\begin{vmatrix} -1 & 2 & 7 & 5 \\ 1 & 3 & -1 & 2 \\ 2 & 1 & 2 & 3 \\ -5 & 2 & -1 & 3 \end{vmatrix};\quad b)\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix};\quad c)\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix};\quad d)\begin{vmatrix} 3 & -1 & 2 & 5 \\ 2 & 1 & 0 & 3 \\ 4 & -2 & 1 & 6 \\ -1 & 3 & -2 & 4 \end{vmatrix};$$

$$e)\begin{vmatrix} 10 & 6 & 8 & 3 \\ 1 & 1 & 3 & -4 \\ 4 & -2 & -1 & 11 \\ 9 & 7 & 6 & -13 \end{vmatrix};\quad f)\begin{vmatrix} 1 & 3 & 3 & 1 \\ 3 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 2 & 4 & 2 & 3 \end{vmatrix};\quad g)\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & -4 & -1 & 2 \\ 4 & 3 & -2 & -1 \end{vmatrix};\quad h)\begin{vmatrix} 4 & 7 & 9 & 2 \\ 2 & 3 & 4 & -6 \\ 5 & 9 & 6 & 9 \\ 3 & 6 & 8 & 5 \end{vmatrix};$$

$$i)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 4 \\ 3 & 1 & 3 & 3 \\ 4 & 4 & 3 & 4 \end{vmatrix};\quad j)\begin{vmatrix} 2 & 3 & 0 & 0 & 0 \\ 4 & 7 & 0 & 0 & 0 \\ -8 & 2 & 1 & 0 & 0 \\ 25 & 17 & 0 & 1 & 0 \\ 17 & 8 & 0 & 0 & 1 \end{vmatrix};\quad k)\begin{vmatrix} 5 & -1 & 3 & -3 & 1 \\ 8 & 4 & 1 & 0 & 1 \\ 13 & 3 & 4 & -3 & 2 \\ 3 & 2 & -2 & 4 & -5 \\ 7 & -6 & 0 & 8 & 1 \end{vmatrix};\quad e)\begin{vmatrix} 6 & 1 & 1 & 1 & 1 \\ 1 & 5 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{vmatrix};$$

9. Laplas teoremasidan foydalanib, quyidagi determinantlarni hisoblang:

$$\text{a)}\begin{vmatrix} 1 & 0 & 0 & -1 \\ 2 & 3 & 4 & 7 \\ -3 & 4 & 5 & 9 \\ -4 & -5 & 6 & 1 \end{vmatrix};\quad \text{b)}\begin{vmatrix} 5 & 62 & -79 & 4 \\ 0 & 2 & 3 & 0 \\ 6 & 183 & 201 & 5 \\ 0 & 3 & 4 & 0 \end{vmatrix};\quad \text{c)}\begin{vmatrix} 3 & -1 & 5 & 2 \\ 2 & 0 & 7 & 0 \\ -3 & 1 & 2 & 0 \\ 5 & -4 & 1 & 2 \end{vmatrix};$$

$$\text{d)}\begin{vmatrix} 9 & 7 & 6 & 8 & 5 \\ 3 & 0 & 0 & 2 & 0 \\ 5 & 3 & 0 & 4 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 7 & 5 & 4 & 6 & 0 \end{vmatrix};\quad \text{e)}\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 2 & 3 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{vmatrix};\quad \text{f)}\begin{vmatrix} 1 & 0 & 2 & 0 & 3 \\ 0 & 2 & 0 & 3 & 0 \\ 2 & 0 & 3 & 0 & 4 \\ 0 & 3 & 0 & 4 & 0 \\ 3 & 0 & 4 & 0 & 3 \end{vmatrix};$$

10. Quyidagi determinantlarni avval almashtirishlar bajarib soddalashtiring, so'ng Laplas teoremasidan foydalanib hisoblang:

a) $\begin{vmatrix} 3 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ -8 & 5 & 9 & 5 \\ -11 & 7 & 7 & 4 \end{vmatrix}$; b) $\begin{vmatrix} 9 & 7 & 9 & 7 \\ 8 & 6 & 8 & 6 \\ -9 & -7 & 9 & 7 \\ -8 & -6 & 8 & 6 \end{vmatrix}$; c) $\begin{vmatrix} 6 & 8 & -9 & -12 \\ 4 & 6 & -6 & -9 \\ -3 & -4 & 6 & 8 \\ -2 & -3 & 4 & 6 \end{vmatrix}$;

d) $\begin{vmatrix} 213 & 186 & 162 & 137 \\ 344 & 157 & 295 & 106 \\ 419 & 418 & 419 & 418 \\ 417 & 416 & 417 & 416 \end{vmatrix}$; e) $\begin{vmatrix} 8 & 10 & 3 & 1 & 4 \\ 7 & 9 & 4 & 1 & 6 \\ 1 & -2 & 2 & 1 & 3 \\ 2 & 5 & -4 & -2 & -6 \\ -1 & 2 & 6 & 3 & 9 \end{vmatrix}$;

Mustaqil yechish uchun misollar va masalalarning javoblari

1. a) -1; b) 0; s) $sin(\alpha - \beta)$; d) 0; e) 0; f) $a^2 + b^2 + c^2 + d^2$. **2.** a) -50; b) 16; c) 0; в) $3abc - a^3 - b^3 - c^3$; e) 0; f) $sin(\beta - \delta) + sin(\delta - \alpha) + sin(\alpha - \beta)$; g) –2; h) 0; i) $3i\sqrt{3}$. **4**. a) $10x^4, -2x^2, -3x^3$; b) $x^4, -x^3, -32x^2$**;5.** a) $4a - c - d$; b) $2a + b - c - d$; c) $-5a - 5b - 5c - 5d$. **6.** 0. **8.** a) –252; b) –3; c) –3; d) –65; e) –1455; f) 8; g) 900; h)-74; i) 54; j) 2: k) 0; l) 394; **9.** a) 216; b) 1; c) –106; d) 120; e) –11; f) –2. **10.** a) –12; b)16; c) 1; d) –400: e) –36.

2-amaliy mashg'ulot.

MATRITSALAR VA ULAR USTIDA AMALLAR. TESKARI MATRITSA, MATRITSA RANGI.

1. Matritsalarning chiziqli kombinatsiyasi topilsin:

a) $3\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} - 4\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$; b) $2\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - 3\begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix}$;

c) $2\begin{pmatrix} 1 & 8 & 7 & -15 \\ 1 & -5 & -6 & 11 \end{pmatrix} - \begin{pmatrix} 5 & 24 & -7 & -1 \\ -1 & 2 & 7 & 3 \end{pmatrix}$;

d) $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}+\begin{pmatrix}4 & -2\\ -3 & 1\end{pmatrix}$; e) $\begin{pmatrix}1 & 5\\ 5 & 1\end{pmatrix}-\begin{pmatrix}-1 & -1\\ -1 & -1\end{pmatrix}$;

f) $2\begin{pmatrix}0 & 0 & 1 & 1 & 1\\ 0 & 1 & 0 & 0 & 1\end{pmatrix}-\begin{pmatrix}1 & 0 & 1 & 1 & 1\\ 0 & 1 & 1 & 1 & 1\end{pmatrix}$.

2. Qanday shartlarda quyidagi ayniyatlar o'rinli bo'ladi:

a) $A+B=B+A$; b) $A+(B+C)=(A+B)+C$;

c) $\alpha(\beta A)=(\alpha\beta)A$; d) $\alpha(A+B)=\alpha A+\alpha B$;

e) $(\alpha+\beta)A=\alpha A+\beta A$.

3. Matritsalarning ko'paytmasini hisoblang:

a) $\begin{pmatrix}2 & -3 & 0\end{pmatrix}\begin{pmatrix}4\\ 3\\ 1\end{pmatrix}$; b) $\begin{pmatrix}4\\ 3\\ 1\end{pmatrix}\begin{pmatrix}2 & -3 & 0\end{pmatrix}$; c) $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}3 & 5\\ 5 & 9\end{pmatrix}$;

d) $\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$; g) $\begin{pmatrix}3 & 1\\ 2 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$; h) $\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 5\end{pmatrix}$;

e) $\begin{pmatrix}0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 4 & 3\\ 0 & 3 & 2\\ 0 & 1 & 0\\ 0 & 2 & 1\end{pmatrix}$; f) $\begin{pmatrix}3 & 3 & -4 & -3\\ 0 & 6 & 1 & 1\\ 5 & 4 & 2 & 1\\ 2 & 3 & 3 & 2\end{pmatrix}\begin{pmatrix}0\\ 1\\ 0\\ 0\end{pmatrix}$; i) $\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & 0 & 1 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 1\end{pmatrix}\begin{pmatrix}1\\ 1\\ 1\\ 1\end{pmatrix}$;

4. Ko'paytmaning mavjudligini tekshiring va mavjud bo'lganda hisoblang:

a) $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}$; b) $\begin{pmatrix}2\\ 4\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}$; c) $\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}\begin{pmatrix}2 & 4\end{pmatrix}$;

d) $\begin{pmatrix}-12 & 13\end{pmatrix}\begin{pmatrix}13547 & 13647\\ 28423 & 28523\end{pmatrix}\begin{pmatrix}-1\\ 1\end{pmatrix}\begin{pmatrix}-12 & 13\end{pmatrix}$.

5. Hisoblang:

a) $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}^n$; b) $\begin{pmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}^n$; c) $\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{pmatrix}^3$;

d) $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}^n$; e) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n$; f) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}^n$.

6. Ayniyatlarning to'g'riligini tekshiring:

a) $(\alpha A)^T = \alpha A^T$; b) $(AB)^T = B^T A^T$;

c) $(ABC)^T = C^T B^T A^T$; d) $(A+B)^T = A^T + B^T$.

7. *f(A)* ni hisoblang, agar

a) $f(x) = x^2 - 2x + 1,\ A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$; b) $f(x) = x^2 - 2x + 1,\quad A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$;

c) $f(x) = x^2 - 3x + 2,\quad A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$; d) $f(x) = (x-\varepsilon)^2,\quad A = \begin{pmatrix} \varepsilon & 1 \\ -1 & \varepsilon \end{pmatrix}$;

e) $f(x) = x^2 + x + 1,\quad A = \begin{pmatrix} -1 & 1 & 1 \\ -5 & 21 & 17 \\ 6 & -26 & -21 \end{pmatrix}$;

8. Agar $AB=BA$ shart bajarilsa, quyidagi tenglamalarning to'g'riligini isbotlang:

a) $(A+B)^2 = A^2 + 2AB + B^2$; b) $(A+B)(A-B) = A^2 - B^2$;

c) $A^n - B^n = (A-B)(A^{n-1} + A^{n-2}B + \ldots + AB^{n-2} + B^{n-1})$;

d) $(A+B)^n = A^n + nA^{n-1} + \dfrac{n(n-1)}{2!}A^{n-2}B^2 + \ldots + B^n$.

Agar $AB \neq BA$ bo'lsa, yuqoridagi tengliklar to'g'ri bo'ladimi?

9. *A* matritsa bilan o'rin almashinuvchi bo'lgan hamma matritsalar topilsin, agar:

a) $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$; b) $A = \begin{pmatrix} 2 & -1 \\ 3 & -1 \end{pmatrix}$; c) $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$;

d) $A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$; e) $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$; bunda $i \neq j$ bo'lsa, $a_i \neq a_j$.

10. $\begin{pmatrix}17 & -6\\ 35 & -12\end{pmatrix}=\begin{pmatrix}2 & 3\\ 5 & 7\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}-7 & 3\\ 5 & -2\end{pmatrix}$ tenglikdan foydalanib,

$\begin{pmatrix}17 & -6\\ 35 & -12\end{pmatrix}^5$ ni hisoblang.

11. $\begin{pmatrix}4 & 3 & -3\\ 2 & 3 & -2\\ 4 & 4 & -3\end{pmatrix}=\begin{pmatrix}1 & 3 & 1\\ 2 & 2 & 1\\ 3 & 4 & 2\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}0 & 2 & -1\\ 1 & 1 & -1\\ -2 & -5 & 4\end{pmatrix}$ tenglikdan foydalanib,

$\begin{pmatrix}4 & 3 & -3\\ 2 & 3 & -2\\ 4 & 4 & -3\end{pmatrix}^6$ ni hisoblang.

12. $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ matritsa $x^2-(a+d)x+ad-bc=0$ tenglamani qanoatlantirishini isbotlang.

13. Teskari matritsani topish formulasidan foydalanib quyidagi matritsalar uchun teskari matritsani toping:

a) $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$; b) $\begin{pmatrix}3 & 4\\ 5 & 7\end{pmatrix}$; c) $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$; d) $\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$;

e) $\begin{pmatrix}2 & 7 & 3\\ 3 & 9 & 4\\ 1 & 5 & 3\end{pmatrix}$; f) $\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{pmatrix}$; g) $\begin{pmatrix}2 & -2 & 1\\ 2 & 1 & -2\\ 1 & 2 & 2\end{pmatrix}$;

h) $\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}$; i) $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 1 & 2\\ 1 & 1 & 1 & -1\\ 1 & 0 & -2 & -6\end{pmatrix}$.

14. Quyidagi ayniyatlar o'rinli bo'ladimi?

a) $\left(A^T\right)^{-1}=\left(A^{-1}\right)^T$; b) $(\alpha A)^{-1}=\alpha^{-1}A^{-1}$;

c) $(AB)^{-1}=B^{-1}A^{-1}$; d) $(ABC)^{-1}=C^{-1}B^{-1}A^{-1}$;

15. Berilgan matritsani elementar matritsalar ko'paytmasiga yoying:

a) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$; b) $\begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$; c) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}$.

16. Matritsaviy tenglamalar sistemasini yeching:

a) $\begin{cases} X+Y=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \\ 2X+3Y=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{cases}$; b) $\begin{cases} 2X-Y=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ -4X-2Y=\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \end{cases}$.

17. Elementar almashtirishlar yordamida berilgan matritsa uchun teskari matritsani toping:

a) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$; b) $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$; c) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$;

d) $\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$; e) $\begin{pmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{pmatrix}$; f) $\begin{pmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{pmatrix}$;

18. Quyidagi tenglamalardan X matritsani toping:

a) $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} X = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$; b) $X \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$;

c) $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -2 & 5 \end{pmatrix} X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$; d) $X \begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 2 \\ 5 & 8 & -1 \end{pmatrix}$;

19. Xoshiyalovchi minor usulidan foydalanib matritsa rangini toping:

a) $\begin{pmatrix} 2 & -4 & 3 & -3 & 5 \\ 1 & -2 & 1 & 5 & 3 \\ 1 & -2 & 4 & -34 & 0 \end{pmatrix}$; **b)** $\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$;

c) $\begin{pmatrix} 2 & 3 & 1 & -1 \\ 3 & 1 & 4 & 2 \\ 1 & 2 & 3 & -1 \\ 1 & -4 & -7 & 5 \end{pmatrix}$; **d)** $\begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -3 & 1 & -4 \\ 1 & 9 & 8 & -2 \\ 1 & -12 & -7 & -2 \end{pmatrix}$;

e) $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$; **f)** $\begin{pmatrix} 3 & -1 & 3 & 5 & 2 \\ 5 & -3 & 2 & 4 & 3 \\ 1 & -3 & -5 & -7 & 0 \\ 7 & -5 & 1 & 1 & 4 \end{pmatrix}$;

g) $\begin{pmatrix} 3 & 4 & -1 & 5 & -2 \\ 1 & 5 & -2 & 3 & 4 \\ 2 & -1 & 1 & 2 & 3 \\ 3 & -7 & 4 & 1 & -7 \\ 0 & 11 & -5 & 4 & -4 \end{pmatrix}$; h) $\begin{pmatrix} 4 & 2 & 3 & -1 & 5 \\ 2 & 1 & -1 & 3 & -2 \\ 2 & 1 & -6 & 10 & -11 \\ 2 & 1 & 9 & -11 & 16 \\ 10 & 5 & 10 & -6 & 17 \end{pmatrix}$.

20. Elementar almashtirishlar yordami bilan quyidagi matritsalarning rangini toping:

a) $\begin{pmatrix} 17 & 51 & 27 & 31 \\ 93 & 25 & 14 & 121 \\ 94 & 27 & 15 & 120 \\ 18 & 53 & 28 & 30 \end{pmatrix}$; **b)** $\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix}$;

c) $\begin{pmatrix} 12 & -51 & 23 & 48 & 94 \\ 25 & 31 & 48 & 73 & -103 \\ 11 & -186 & 21 & 71 & 385 \\ 19 & 203 & 27 & 39 & 111 \end{pmatrix}$; **d)** $\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}$;

e) $\begin{pmatrix} 3 & 18 & 23 & 48 \\ 17 & 15 & 31 & -47 \\ 23 & 27 & 42 & 33 \\ 4 & 6 & 27 & -110 \\ 26 & 21 & 30 & 67 \end{pmatrix}$; f) $\begin{pmatrix} 8 & 5 & 3 & 7 & 1 & 2 \\ 4 & 6 & 7 & 9 & 11 & 12 \\ 3 & 0 & -1 & 2 & 7 & 1 \\ 15 & 11 & 9 & 13 & 19 & 15 \\ 17 & 10 & 2 & -5 & 6 & 3 \end{pmatrix}$.

Mustaqil yechish uchun misollar va masalalarning javoblari

1. a) 0; b) $\begin{pmatrix} 4 \\ -11 \\ -16 \end{pmatrix}$; c) $\begin{pmatrix} -3 & -8 & 21 & -29 \\ 3 & -8 & -19 & 19 \end{pmatrix}$; d) $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$; e) $\begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}$; f) $\begin{pmatrix} -1 & 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 & 1 \end{pmatrix}$. **2**. a), b), d) lar o'rinlidir, agar matritsalar bir xil o'lchovli bo'lsa; c), e) lar doimo o'rinli. **3.** a) (-1); b) $\begin{pmatrix} 8 & -12 & 0 \\ 6 & -9 & 0 \\ 2 & 3 & 0 \end{pmatrix}$;

c) $\begin{pmatrix} 8 & 14 \\ 8 & 14 \end{pmatrix}$; d) (1 1); e) $\begin{pmatrix} 4 & 4 \\ 3 & 3 \end{pmatrix}$; f)(6, 9, 12); g) $\begin{pmatrix} 0 & 3 & 2 \end{pmatrix}$; h) $\begin{pmatrix} 3 \\ 6 \\ 4 \\ 3 \end{pmatrix}$; i) $\begin{pmatrix} 4 \\ 3 \\ 1 \\ 2 \end{pmatrix}$.

4. a) mavjud emas; b)$\begin{pmatrix} 8 \\ 16 \end{pmatrix}$; c) $\begin{pmatrix} 8 & 16 \end{pmatrix}$; d) $\begin{pmatrix} -1200 & 1300 \end{pmatrix}$.

5. a) $2^{n-1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$; b) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; c) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$; d) 0 agar $n > 1$ bo'lsa;

e) $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$; f) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$. **6**. b), c), d) lar o'rinli, agar ularda foydalanilgan amallar bajarilsa; a) doimo o'rinlidir. 7**.** a) 0; b) 0; c) 0; d) -*E*; e)$\begin{pmatrix} 2 & -5 & -4 \\ -3 & 16 & 12 \\ 4 & -20 & -15 \end{pmatrix}$. **8.** Yo'q.

9. a) $\left\{ \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} \mid \alpha, \beta, \gamma \in \boldsymbol{C} \right\}$;

b) $\left\{\begin{pmatrix} \alpha & \beta \\ -3\beta & \alpha+3\beta \end{pmatrix} \mid \alpha,\beta \in \mathbf{C}\right\}$; c) $\left\{\begin{pmatrix} \alpha & \beta \\ 0 & \alpha \end{pmatrix} \mid \alpha,\beta \in \mathbf{C}\right\}$; d), e) $\left\{\begin{pmatrix} \alpha & \beta & \gamma & \delta \\ 0 & \alpha & \beta & \gamma \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \alpha \end{pmatrix} \mid \alpha,\beta,\gamma,\delta \in \mathbf{C}\right\}$.**10.** $\begin{pmatrix} 3197 & -1266 \\ 7385 & -922 \end{pmatrix}$. **11.** $\begin{pmatrix} 190 & 189 & -189 \\ 126 & 127 & -126 \\ 252 & 252 & -251 \end{pmatrix}$.

13. a) $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$; b) $\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$; c) $\frac{1}{ad\ \ bc}\begin{pmatrix} d & b \\ c & a \end{pmatrix}$;

d) $\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$; e) $\begin{pmatrix} -\frac{7}{3} & 2 & -\frac{1}{3} \\ \frac{5}{3} & -1 & -\frac{1}{3} \\ -2 & 1 & 1 \end{pmatrix}$; f) $\frac{1}{9}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$; g) $\frac{1}{9}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}$;

h) $\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$; i) $\begin{pmatrix} 22 & -6 & -26 & 17 \\ -17 & 5 & 20 & -13 \\ -1 & 0 & 2 & -1 \\ 4 & -1 & -5 & 3 \end{pmatrix}$; **14**. f) tasdiq umumiy holda noto'g'ridir. **15.** Mumkin bo'lgan javoblarning bittasini keltiramiz

a) $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$; b) $\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$;

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$. **16.** a) $X = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}, Y = \begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix}$;

b) $Y = 2X + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, bunda X – ikkinchi tartibli ixtiyoriy matritsa.

17. a) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$; b) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$; c) $\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$;

d) $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$;e) $\frac{1}{7}\begin{pmatrix} -5 & 2 & 2 & 2 \\ 2 & -5 & 2 & 2 \\ 2 & 2 & -5 & 2 \\ 2 & 2 & 2 & -5 \end{pmatrix}$; f) $\begin{pmatrix} 0 & -1 & 1 & -1 \\ 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{pmatrix}$;

18. a) $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$; b) $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$; c) $\begin{pmatrix} 21 & -14 & -10 \\ -10 & 7 & 5 \\ -4 & 3 & 2 \end{pmatrix}$; d) $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{pmatrix}$;

e) yechimlar mavjud emas; f) $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$ bunda a va b ixtiyoriy sonlar;

g) $\begin{pmatrix} a & b & c \\ 1-a & 2-b & 4-c \end{pmatrix}$, a, b, c – lar ixtiyoriy sonlar; h) $\begin{pmatrix} 0 & 0 \\ -1 & 0 \\ -2 & -1 \end{pmatrix}$;

i) $\begin{pmatrix} -1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$; j) yeichmlar mavjud emas;

19. a) $r = 2$; b) $r = 2$; c) r=4; d)r=2;e) r=3;
f) r=3;g) r=3; h) r=2. **20.** a) r=3; b) r=3; c) r=4;
d)r=3; e) r=4; f) r=5.

3-amaliy mashg'ulot.

CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMANI YECHISHDA KRAMER, MATRISA VA GAUSS USULLARI

1. Tenglamalar sistemasini Kramer qoidasi bo'yicha yeching:

a) $\begin{cases} x + 4y = -10, \\ 3x - y = 9; \end{cases}$ b) $\begin{cases} 2x - 5y = 1, \\ a\,x - 5y = -2a - 5; \end{cases}$

c) $\begin{cases} x+2y-z=12, \\ 3x-y+4z=-13, \\ -x+5y-z=27; \end{cases}$ d) $\begin{cases} 2x+4y+3z=14, \\ 3x-y+4z=-13, \\ -x+5y-z=27; \end{cases}$

e) $\begin{cases} 2x_1+2x_2-x_3+x_4=4, \\ 4x_1+3x_2-x_3+2x_4=6, \\ 8x_1+5x_2-3x_3+4x_4=12, \\ 3x_1+3x_2-2x_3+2x_4=6; \end{cases}$ f) $\begin{cases} 2x_1+3x_2+11x_3+5x_4=2, \\ x_1+x_2+5x_3+2x_4=1, \\ 2x_1+x_2+3x_3+2x_4=-3, \\ x_1+x_2+3x_3+4x_4=-3; \end{cases}$

g) $\begin{cases} 2x_1+5x_2+4x_3+x_4=20, \\ x_1+3x_2+2x_3+x_4=11, \\ 2x_1+10x_2+9x_3+7x_4=40, \\ 3x_1+8x_2+9x_3+2x_4=37; \end{cases}$ h) $\begin{cases} 3x_1+4x_2+x_3+2x_4+3=0, \\ 3x_1+5x_2+3x_3+5x_4+6=0, \\ 6x_1+8x_2+x_3+5x_4+8=0, \\ 3x_1+5x_2+3x_3+7x_4+8=0; \end{cases}$

i) $\begin{cases} 2x-y-6z+3t+1=0, \\ 7x-4y+2z-15t+32=0, \\ x-2y-4z+9t-5=0, \\ x-y+2z-6t+8=0; \end{cases}$ j) $\begin{cases} 2x+y+4z+8t=-1, \\ x+3y-6z+2t=3, \\ 3x-2y+2z-2t=8, \\ 2x-y+2z=4; \end{cases}$

2. Tenglamalar sestemasini Gauss usulida yeching:

a) $\begin{cases} x-2y=3, \\ 2x-4y=6; \end{cases}$ b) $\begin{cases} x-2y=3, \\ 2x-4y=5; \end{cases}$

c) $\begin{cases} x-2y=1-i, \\ 2x-4y=2-2i; \end{cases}$ d) $\begin{cases} x-2y=1-i, \\ 2x-4y=2+i; \end{cases}$

e) $\begin{cases} x+2y-z=5, \\ 3x-y+z=-4; \end{cases}$ f) $\begin{cases} 2z_1-(1+i)z_2-3iz_3=b_1, \\ -2z_1+(1+i)z_2+(1+3i)z_3=b_2, \\ z_3=b_3; \end{cases}$

g) $\begin{cases} 2x+3y-z=0, \\ x-2y+4z=9, \\ y+z=2; \end{cases}$ h) $\begin{cases} 2x+3y-z=-1, \\ x+2y-4z=9, \\ -x-12y+14z=1; \end{cases}$

i) $\begin{cases} 3x_1 - 2x_2 - 5x_3 + x_4 = 3, \\ 2x_1 - 3x_2 + x_3 + 5x_4 = -3, \\ x_1 + 2x_2 - 4x_4 = -3, \\ x_1 - x_2 - 4x_3 + 9x_4 = 22; \end{cases}$ j) $\begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 5, \\ x_1 + 3x_2 + 5x_3 - 2x_4 = 3, \\ x_1 + 5x_2 - 9x_3 + 8x_4 = 1, \\ 5x_1 + 18x_2 + 4x_3 + 5x_4 = 12; \end{cases}$

k) $\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 1, \\ 8x_1 + 12x_2 - 9x_3 + 8x_4 = 3, \\ 4x_1 + 6x_2 + 3x_3 - 2x_4 = 3, \\ 2x_1 + 3x_2 + 9x_3 - 7x_4 = 3; \end{cases}$ l) $\begin{cases} x_1 + 2x_2 - x_3 - 3x_4 + 4x_5 = 3, \\ 2x_1 - x_2 + 3x_3 + 2x_4 - x_5 = 4, \\ x_1 + 4x_2 + 2x_3 - 5x_4 + 3x_5 = 6, \\ x_1 + 15x_2 + 6x_3 - 19x_4 + 9x_5 = 2; \end{cases}$

m) $\begin{cases} 3x_1 + x_2 + x_3 - 2x_4 + 3x_5 = 4, \\ 2x_1 + 3x_2 - 2x_3 + x_4 - 4x_5 = 5, \\ x_1 + 2x_2 + 3x_3 + 4x_4 + x_5 = 3, \\ x_1 - 2x_2 + 3x_3 - 3x_4 + 7x_5 = -1; \end{cases}$ n) $\begin{cases} \alpha x_1 + x_2 + x_3 + x_4 = 1, \\ x_1 + \alpha x_2 + x_3 + x_4 = 1, \\ x_1 + x_2 + \alpha x_3 + x_4 = 1, \\ x_1 + x_2 + x_3 + \alpha x_4 = 1. \end{cases}$

3. Agar $f(1) = -1,\ f(-1) = 9,\ f(2) = -3$ bo'lsa, $f(x)$ kvadrat ko'phadni toping.

4. Agar $f(-1) = 0,\ f(1) = 4,\ f(2) = 3,\ f(3) = 16$ bo'lsa, uchinchi darajali ko'phadni toping.

5. k ning qanday haqiqiy qiymatlarida sistema nol yechimga ega bo'lmaydi:

a) $\begin{cases} k x_1 + x_2 + x_3 + x_4 = 0, \\ x_1 + (1+k)x_2 + x_3 + x_4 = 0, \\ x_1 + x_2 + (2+k)x_3 + x_4 = 0, \\ 2x_1 + 2x_2 + (3+k)x_3 + 2x_4 = 0 \end{cases}$ b) $\begin{cases} k x_1 + x_2 + x_3 = 0, \\ x_1 + k x_2 + x_3 = 0, \\ x_1 + x_2 + k x_3 = 0 \end{cases}$?

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. a) $\{(2,-3)\}$; b) $\{(-2,-1) \mid a \neq -2\}$; c) $\{(0,5,-2)\}$;

d) Крамер qoidasi bo'yicha sistemani yechish mumkin emas;

e) $\{(1,1,-1,-1)\}$; f) $\{(-2,0,1,-1)\}$; g) $\{(1,2,2,0)\}$;

h) $\{(2,-2,1,-1)\}$; i) $\left\{\left(-3,0,-\frac{1}{2},\frac{2}{3}\right)\right\}$; j) $\left\{\left(2,-3,-\frac{3}{2},\frac{1}{2}\right)\right\}$;

k) $\{(-0,4;\ -1,2;\ 3,4;\ 1)\}$; l) $\left\{\left(\frac{2}{3},-1,\frac{3}{2},0\right)\right\}$. **2.**a) $\{(3+2\alpha,\alpha)\,|\,\alpha\in\mathbf{R}\}$;

b) Yechimi yo'q; c) $\{(1-i+2\alpha,\alpha)\,|\,\alpha\in\mathbf{C}\}$; d) Yechimi yo'q;

e) $\{(\alpha,1-4\alpha,-3-7\alpha)\,|\,\alpha\in\mathbf{R}\}$; f) Agar $b_3\neq b_1+b_2$, u holda yechimi yo'q;

$\left\{\left(\frac{1}{2}(1+i)\alpha+b_1+3ib_3),\alpha,\ b_3\right)\,|\,\alpha\in\mathrm{C},b_3=b_1+b_2\right\}$; g) $\{(1,0,2)\}$;

h) $\left(\frac{13}{3},-\frac{13}{3},-\frac{10}{3}\right)$; i) $\{(-1,3,-2,2)\}$;

j) $\{(6-28\alpha+17\beta,\ -1+7\alpha-5\beta,\ \alpha,\beta)\,|\,\alpha,\beta\in\mathbf{R}\}$;

k) $\left\{\left(\frac{1}{10}(6-15\alpha-\beta),\ \alpha,\frac{1}{5}(1+4\beta),\beta\right)\,|\,\alpha,\beta\in\mathbf{R}\right\}$; l) Yechimi yo'q ;

m) $\left\{\left(1+\frac{1}{32}(47\alpha-38\beta),1+\frac{1}{4}(-7\alpha+6\beta),\frac{1}{32}(-21\alpha-30\beta),\alpha,\beta\right)\,|\,\alpha,\beta\in\mathbf{R}\right\}$;

n) Agar $\alpha=-3$, u holda yechimi yo'q;

$\left\{\left(\frac{1}{3+\alpha},\frac{1}{3+\alpha},\frac{1}{3+\alpha},\frac{1}{3+\alpha}\right)\,|\,\alpha\neq 1,\alpha\neq -3\right\}$;

$\{(1-\alpha-\beta-\gamma,\alpha,\beta,\gamma)\,|\,\alpha,\beta,\gamma\in\mathbf{R},\ \alpha=1\}$. **3.** $f(x)=x^4-5x+3$.

4. $f(x)=2x^3-5x^2+7$. **5.** a) $-1;\ 0;1$; b) $-2;1$.

4-amaliy mashg'ulot.

VEKTORLAR VA ULAR USTIDA CHIZIQLI AMALLAR. VEKTORLARNING SKALYAR, VEKTOR VA ARALASH KO'PAYTMALARI

1. Quyida berilgan vektorlarning uzunligi va vektor yo'nalishidagi birlik vektorni toping.

1) $\vec{a}$ = {2;-6;3}; 2) $\vec{b}$ = {4;-5;2}; 3) $\vec{c}$ = {6;10;0}.

2. Ushbu $\vec{a}$ = {-2;11;*z*} vektorning uzunligi 15 ga teng bo'lsa, *z* ni toping.

3. *A*(-2; 5; -4), *B*(3; -7; 8), *C*(2; 4; 0) nuqtalar berilgan. $\overrightarrow{AB}$, $\overrightarrow{BA}$, $\overrightarrow{AC}$, $\overrightarrow{BC}$ va $\overrightarrow{CA}$ vektorlarni toping.

4. Agar $\vec{a}$ = {-1; 3; 7} vektor va *M*(4; -3; 0) nuqta berilgan bo'lib, $a = MN$ bo'lsa, *N* nuqtaning koordinatalarini toping.

5. Agar $\left|\vec{a}\right| = 4$ bo'lib, $\vec{a}$ vektorning *Ox*, *Oy*, *Oz* o'qlari bilan mos ravishda $\alpha = 60^0, \beta = 45^0, \gamma = 60^0$ tashkil etsa, $\vec{a}$ vektorining koordinata o'qlaridagi proeksiyalarini toping.

6. Quyida berilgan vektorlarning yo'naltiruvchi kosinuslarini toping.

1) $\vec{a}$ = {-3;12;-4}; 2) $\vec{b}$ = {3;-4;5}.

7. Vektor koordinata o'qlari bilan quyidagi burchaklarni tashkil etishi mumkinmi?

1)$\alpha = 225^0, \beta = 240^0, \gamma = 120^0$; 2)$\alpha = 60^0, \beta = 45^0, \gamma = 60^0$;

3)$\alpha = 60^0, \beta = 90^0, \gamma = 120^0$.

8. $\vec{a}$ vektor *Ox* va *Oy* o'qlari bilan $\alpha = 120^0$ va $\beta = 60^0$ tashkil etadi. Agar $|\vec{a}| = 5$ bo'lsa, uning koordinatalarini toping.

9. Agar *M*(x,y) nuqtani aniqlovchi radius vektori koordinata o'qlari bilan bir xil burchakni tashkil qilib, uning uzunligi $5\sqrt{3}$ ga teng bo'lsa, *M* nuqtaning koordinatalarini toping.

10. $\vec{a}$ va $\vec{b}$ vektorlar berilgan. Quyidagi berilgan:

1) $2\vec{a}$**;** 2) -0,5$\vec{b}$; 3**)** – $\vec{a}$ + $\vec{b}$; 4) 0,5$\vec{a}$ - 3$\vec{b}$ vektorlarni yasang.

11. $|\vec{a}| = 15$, $|\vec{b}| = 25$ va $|\vec{a} + \vec{b}| = 32$ bo'lsa, $|\vec{a} - \vec{b}|$ ni toping.

12. $\vec{a}$ va $\vec{b}$ ***b*** vektorlar 60^0 burchakni tashkil etadi. $|\vec{a}| = 6$, $|\vec{b}| = 3$ ekanligini bilgan holda $(2\vec{a} + \vec{b})(2\vec{a} - 3\vec{b})$ vektorni hisoblang.

13. ABC uchburchakda $\overrightarrow{AB}$ vektor $\vec{m}$ ga va $\overrightarrow{AC}$ vektor $\vec{n}$ ga teng bo'lsa, quyidagi vektorlarni yasang:

$$1)\ \vec{m}+\frac{\vec{n}}{2};\qquad 2)\ \vec{n}-\frac{\vec{m}}{2};\qquad 3)\ \frac{\vec{m}-\vec{n}}{2}.$$

14. $\vec{a}$ = {5; -3;7} va $\vec{b}$={3; -1; -2} vektorlar berilgan. Quyida berilgan vektorlarning koordinata o'qlaridagi proeksiyalarini toping:

1) $\vec{a}$ + $\vec{b}$; 2) $\vec{a}$ - $\vec{b}$; **3) -3** $\vec{a}$; 4) $\frac{1}{3}\vec{b}$; 5) **2** $\vec{a}$ **- 3** $\vec{b}$; 6) $\frac{1}{3}\vec{a}+\vec{b}$

15. α va β ning qanday qiymatlarida $\vec{a}$ = 3***i*** - 2***j*** + α ***k*** va $\vec{b}$ = β***i*** + 3***j*** - 6***k*** vektorlar kolleniar bo'ladi?

16. $\vec{c}=11\vec{i}+1\vec{j}-10\vec{k}$ vektor berilgan. $\vec{c}$ vektorga parallel, unga qarama-qarshi yo'nalgan va uzunligi 45 ga teng bo'lgan $\vec{d}$ vektorni toping.

17. $\vec{AB}$ = {4; 6; 2} va $\vec{AC}$={-8; 10; -12} vektorlar ABC uchburchakning tomonlari bilan ustma- ust tushadi. Boshlari uchburchakning uchlarida va medianalar bilan ustma- ust tushgan vektorlarning koordinatalarini toping.

18. $\vec{a}$= {1; -2}, $\vec{b}$ = {-2; 3}, $\vec{c}$ = {-4; 7} vektorlar berilgan. Har bir vektorni, qolgan ikki vektorlarni bazis deb olganda, yoyilmalarini aniqlang.

19. Tekislikda A(2; -1), B(-1; -2), C(-2; -3) , D(-3; 2) nuqtalar berilgan. $\vec{AB}$ va $\vec{AC}$ vektorlarni bazis vektorlari deb, quyidagi vektorlarning yoyilmalarini toping:

1) $\vec{AD}$; 2) $\vec{BD}$; 3) $\vec{CD}$; 4) $\vec{BD}+\vec{CD}$; 5) $\vec{AB}+\vec{BD}+\vec{CD}$.

20. $\vec{a}$ = {1; -3; 2}, $\vec{b}$ = {-2; 1; 3}, $\vec{c}$ ={1; -2; -1} vektorlar berilgan.

$\vec{d}$ = {-6; 5; 11} vektorning $\vec{a}$, $\vec{b}$, $\vec{c}$ bazis vektorlari bo'yicha yoyilmasini toping.

21. $\vec{a}$ = {2; -1; 2}, $\vec{b}$ = {1; 0; 2}, $\vec{c}$ = {7; -7; 3}, $\vec{d}$ = {-1; 2; 1} vektorlar berilgan. Bu vektorlarning har birining yo'yilmasini qolgan uchta vektorlarni bazis vektori deb qabul qilgan holda toping.

22. $\vec{a}$ va $\vec{b}$ vektorlar $\frac{\pi}{3}$ burchakni tashkil qiladi. $\left|\vec{a}\right|=5$, $\left|\vec{b}\right|=6$ ekanligini bilgan holda quyidagilarni hisoblang.

1) $(\vec{a},\vec{b})$; 2) $\vec{a}^2$; 3) $\vec{b}^2$; 4) $(\vec{a}+\vec{b})^2$; 5) $(\vec{a}-\vec{b})^2$;

6) $(2\vec{a}-3\vec{b},\ 2\vec{a}+3\vec{b})$; 7) $(2\vec{a}-3\vec{b})^2$.

23. $\vec{a}$ va $\vec{b}$ vektorlar o'zaro ortogonal bo'lib, $\vec{c}$ vektor bilan esa $\frac{2\pi}{3}$ burchakni tashkil etadi. $\left|\vec{a}\right|=2, \left|\vec{b}\right|=4, \left|\vec{c}\right|=8$ ekanini bilgan holda, quyidagilarni hisoblang.

1) $(2\vec{a}+3\vec{b}, 2\vec{b}-3\vec{c})$; 2) $(\vec{a}+\vec{b}-\vec{c})^2\vec{a}$;

3) $(2\vec{a}-3\vec{b}+4\vec{c})^2$; 4) $(\vec{a}-\vec{b}-\vec{c})^2$

24. Ushbu $(\vec{a}+\vec{b})^2+(\vec{a}-\vec{b})^2=2(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2)$ ayniyatni isbotlang va uning geometrik ma'nosini aniqlang.

25. Ushbu $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ shartni qanoatlantiruvchi birlik $\vec{a},\vec{b}$ va $\vec{c}$ vektorlar berilgan. $(\vec{a},\vec{b})+(\vec{b},\vec{c})+(\vec{c},\vec{a})$ ni hisoblang.

26. $\left|\vec{a}\right|=7$, $\left|\vec{b}\right|=7$ ekanini bilgan holda, α ning qanday qiymatlarida $\vec{a}+\alpha\cdot\vec{b}$ va $\vec{a}-\alpha\cdot\vec{b}$ vektorlar perpendikulyar bo'ladi.

27. ABC uchburchakning tomonlari bilan ustma-ust tushgan $\vec{b}=\vec{AB}$, $\vec{c}=\vec{AC}$ vektorlar berilgan. Boshi C uchida va CD balandlik bilan ustma-ust tushgan vektorning $\vec{b},\vec{c}$ bazis vektorlari bo'yicha yoyilmasini toping.

28. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\frac{2\pi}{3}$ burchakni tashkil etadi. $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=4$ ekanini bilgan holda, $\vec{a}-\vec{b}$ va $\vec{a}+\vec{b}$ vektorlar orasidagi burchakni toping.

29. $\vec{a}=\{2;-4;-2\}, \vec{b}=\{4;-3;2\}$ vektorlar berilgan. Quyidagilarni hisoblang:

1) $(\vec{a},\vec{b})$; 2) $\sqrt{\vec{a}^2}$; 3) $\sqrt{\vec{b}^2}$;

4) $(3\vec{a}-\vec{b},2\vec{a}+3\vec{b})$; 5) $(\vec{a}+\vec{b})^2$; 6) $(\vec{a}-\vec{b})^2$

30. To'rtburchakning $A(3; -1; 2)$, $B(2; 1; -3)$, $C(5; -4; 7)$ va $D(6; 7; -1)$ uchlari berilgan. Uning AC va BD diagonallari perpendikulyar ekanligini isbotlang.

31. ABC uchburchakning $A(2; -3; 1)$, $B(5; -3; 5)$, $C(9; -3; 2)$ uchlari be rilgan. A uchidagi ichki burchakni toping.

32. ABC uchburchakning $A(4; -2; 7)$, $B(2; 1; 1)$ va $C(-1; 7; 3)$ uchlari berilgan. ABC uchburchakning ichki burchaklarini hisoblab, uning teng yonli uchburchak ekanligini isbotlang.

33. $\vec{a} = \{2;-6;3\}$ va $\vec{x}$ vektorlar o'zaro kollenear bo'lib, Oz o'qi bilan o'tkir burchakni tashkil qiladi. $\left|\vec{x}\right| = 42$ ekanini bilgan holda $\vec{x}$ vektorning koordinatalarini toping.

34. $\vec{a} = \{2;-3;4\}, \vec{b} = \{-1;2;3\}$ vektorlar berilgan. $(\vec{x},\vec{a}) = -7, (\vec{x},\vec{b}) = 2$ ekanligini bilgan holda Oz o'qqa perpendikulyar bo'lgan $\vec{x}$ vektorni toping.

35. $\vec{c} = \{3;6;3\}$ vektorning koordinata o'qlari bilan teng o'tkir burchaklarni tashkil etuvchi o'qdagi proeksiyalarini toping.

36. $\vec{a} = \{4; -2; -5\}$, $\vec{b} = \{6; 1; -3\}$, $\vec{c} = \{4; -12; 3\}$ vektorlar berilgan**.**

$\vec{a} + \vec{b}$ vektorning $\vec{c}$ vektordagi proeksiyalarini toping.

37. $\vec{a} = \{4; -2; -4\}$, $\vec{b} = \{-1; -4; 1\}$ va $\vec{c} = \{5; -8; -4\}$ vektorlar berilgan**.** $\vec{a}$ vektorning $\vec{b} + \vec{c}$ vektorlardagi proeksiyalni toping.

39. $\vec{a}$ va $\vec{b}$ vektorlar $\frac{5\pi}{6}$ burchakni tashkil etadi. $|\vec{a}| = 3$ va $|\vec{b}| = 4$ ekanini bilgan holda $|[\vec{a},\vec{b}]|$ ni hisoblang.

40. $\left|\vec{a}\right|=6,\left|\vec{b}\right|=12$ va ular 36^0 burchak tashkil etishini bilgan holda $\left|[\vec{a},\vec{b}]\right|$ ni hisoblang.

41. O'zaro ortogonal bo'lgan $\vec{a}$ va $\vec{b}$ vektorlarning uzunliklari $\left|\vec{a}\right|=2,\left|\vec{b}\right|=3$ ekanini bilgan holda quyidagilarni hisoblang:

1) $\left|[2\vec{a}-\vec{b},\vec{a}-2\vec{b}]\right|$; 2) $[\vec{a}-3\vec{b},3\vec{a}+\vec{b}]^2$.

42. Ushbu $[\vec{a},\vec{b}]^2+(\vec{a},\vec{b})^2=a^2b^2$ ayniyatni isbotlang.

43. $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ shartni qanoatlantiradi. Ushbu $[\vec{a},\vec{b}]=[\vec{b},\vec{c}]=[\vec{c},\vec{a}]$ o'rinli ekanligini isbotlang.

44. $\vec{a}=\{2;1;-3\}$ va $\vec{b}=\{1;-2;-1\}$ vektorlar berilgan. Quyidagi vektor ko'paytmalarining koordinatalarini toping.

1) $[\vec{a}-\vec{b}]$; 2) $[\vec{a},\vec{a}-2\vec{b}]$; 3) $[\vec{a}-2\vec{b},\vec{a}+2\vec{b}]$; 4) $[3\vec{a}-2\vec{b},2\vec{a}+3\vec{b}]$.

45. ABC uchburchakning $A(1; 2; -1)$, $B(3; -1; 2)$ va $C(-2; 3; 5)$ uchlari berilgan. ABC uchburchakning yuzini hisoblang.

46. ABC uchburchakning $A(1;-2;-1)$, $B(6;-2;-5)$, $C(-3;1;-1)$ uchlari berilgan. B uchudan AC tomoniga tushirilgan balandlikning uzunligini toping.

47. $\vec{a}$ vektor $\vec{b}$ va $\vec{c}$ vektorlar bilan o'zaro perpendikulyar bo'lib, $\vec{b}$ va $\vec{c}$ vektorlar $\frac{5\pi}{6}$ burchakni tashkil qiladi. $\left|\vec{a}\right|=5,\left|\vec{b}\right|=8,\left|\vec{c}\right|=3$ ekanligini bilgan holda $(\vec{a},[\vec{b},\vec{c}])$ aralash ko'paytmani hisoblang.

48. $\vec{a}=\{2;-3;1\}$, $=\{-1;2;4\ \vec{b}\}$, $\vec{c}=\{3;-5;2\}$ vektorlar berilgan.

$([\vec{a},\vec{b}],\vec{c})$aralash ko'paytmani hisoblang.

49. Ushbu $A(1; 2; 3)$, $B(1; 0; 5)$, $C(2; 1; -1)$ va $D(2; -1; 1)$ nuqtalarning bitta tekislikda yotishini isbotlang.

50. Uchlari $A(2; -1; 3)$, $B(1; 3; 4)$, $C(-1; 1; 2)$, $D(5; 4; 5)$ nuqtalarda joylashgan tetraedrning hajmini toping.

51. Piramidaning $A(3;\ 7;\ 6)$, $B(3;\ 1;\ 2)$, $C(-4\ ;\ 8;\ -5)$, $D(1;\ -2;\ 4)$ uchlari berilgan. C uchudan tushirilgan piramidaning balandligini toping.

52. $ABCD$ tetraedrning uchta $A(1;\ -3;\ -2)$, $B(3;\ -1;\ 4)$, $C(2;\ -3;\ 4)$ uchlari va uning hajmi 3 ga teng. Tetraedrning D uchi Ox o'qiga tegishli ekanini bilgan holda, uning koordinatalarini toping.

Mustaqil yechish uchun misol va masalalarning javoblari

1. 1)7; $\{\frac{2}{7},-\frac{6}{7},\frac{3}{7}\}$; 2)13; $\{\frac{4}{13},-\frac{3}{13},-\frac{12}{13}\}$; 3)15; $\{\frac{2}{15},\frac{2}{3},\frac{11}{15}\}$. **2.** ± 10.

3. a) {5, -12, 12}; b) {-5, 12, -12}; c) {4, -1,4}; d) (-1, 11, -8);e) {-4,1,-4}.

4.$N(3, 0, 7)$. **5**. $\{2;2\sqrt{2};2\}$. **6**. 1) $-\frac{3}{13};\frac{12}{13};-\frac{4}{13}$; 2) $\frac{3}{5\sqrt{2}};-\frac{4}{5\sqrt{2}};\frac{1}{\sqrt{2}}$;

7.1) Ha; 2) Ha; 3) Yo'q. **8**. $(-\frac{5}{2};\frac{5}{2};\pm\frac{5}{\sqrt{2}})$. **9**. $\pm\{5;5;5\}$. **11**. 26.

12 81. **14**. 1) {8; -4; 5};2) {2; -2; 9}; 3) {-15; 9; -21}; 4) $\{1;-\frac{1}{3};-\frac{2}{3}\}$;

5) {1; -3; 20};6) $\{\frac{14}{3};-2;\frac{1}{3}\}$.**15**. 4; -4,5. **16**. {-33; -6; 30}.**17**.{-2; 2; -5};

{-8; 11; - 8}; {10; -13; 13}. **18**. $\vec{a}=0{,}5\vec{b}-0{,}5\vec{c}$; $\vec{b}=2\vec{a}+\vec{c}$; $\vec{c}=-2\vec{a}+\vec{b}$. **19**. 1) {11; -7};2) {10; -7}; 3) {11; -8}; 4) {21; -15};5) {32; -22}.

20. $\vec{c}=\vec{p}+2\vec{q}-3\vec{r}$.**21.** $\vec{a}=-2\vec{b}+\vec{c}+3\vec{d}$, $\vec{b}=-\frac{1}{2}\vec{a}+\frac{1}{2}\vec{c}+\frac{3}{2}\vec{d}$,

$\vec{c}=\vec{a}+2\vec{b}-3\vec{d}$, $\vec{b}=\frac{1}{3}\vec{a}+\frac{2}{3}\vec{b}-\frac{1}{3}\vec{d}$.**22**. 1) 15; 2) 25; 3) 36; 4) 91; 5) 31;6) -118;

7) 244 **23**.1) 240; 2) 132; 3) 1440;4) 68 . **25**. -1,5.**26**. $\alpha=\pm 1$.

27. $\frac{(\vec{b},\vec{c})}{|b|^2}\vec{b}-\vec{c}$. **28**. $\arccos(-\frac{7}{\sqrt{481}})$. **29**. 1) 16; 2) $\sqrt{24}$; 3) $\sqrt{29}$;

4) 169; 5) 85; 6) 21. **31**. $\frac{\pi}{4}$. **33**.{12;-36;18}. **34**.{-8;-3;0}. **35**. $\{3\sqrt{2};3\sqrt{2};3\sqrt{2}$. **37**. 4. **39**. 6. **40**. $36\sqrt{3}$. **41**.1) 18; 2) 5184. **44**.1) {7; 1; 5}; 2) {14; 2; 10}; 3) {-28; -4; -20}; 4) {-98; -13; -69}. **46**. h = 5. **47**. 60. **48**. 5. **50**. 3. **51**. $\frac{22}{3}$. **52**. $\frac{13}{3}$.

5-amaliy mashg'ulot.
TEKISLIKDA TO'G'RI CHIZIQLAR

1. Ushbu 1) P(4,0) va Q(3,1), 2) C(-1,1) va D(2,7), 3) A(2,-4) va B(-3,11) nuqtalardan o'tgan to'g'ri chiziqning burchak koeffitsienti va ordinatalar o'qidan ajratgan kesmasini toping.

2. To'g'ri burchakli dekart koordinatalar sistemasining boshidan o'tuvchi va x o'qiga: 1) 45^0, 2) 60^0, 3) 135^0, 4) 180^0 og'ma bo'lgan to'g'ri chiziq tenglamasini yozing.

3. To'g'ri burchakli koordinatalar sistemasiga nisbatan, koordinatalar boshidan o'tuvchi va

1) $y = \frac{1}{4}x + 1$ $y = 3x + 5$ to'g'ri chiziqqa parallel bo'lgan;

2) to'g'ri chiziqqa perpendikulyar bo'lgan;

3) $y = 2x + 5$ to'g'ri chiziq bilan 45^0 burchak tashkil qilgan;

4) $y = x - 1$ to'g'ri chiziqqa 60^0 li burchak ostida o'gma bo'lgan to'g'ri chiziqning tenglamasini yozing.

4. Uchburchakning uchlari berilgan: A(2,3), B(-2,-1) va C(4,-2).

1) Uning uchala tomonining;

2) *C* uchidan o'tkazilgan medianasining;

3) *A* uchidan BC tomoniga tushirilgan balandligining tenglamasini tuzing.

5. Berilgan uchta nuqtaning bir to'g'ri chiziqda yotishi yoki yotmasligini tekshiring:

1) $(1,3),(5,7)$ va $(10,12)$ 2) $(2,4),(4,-1)$ va $(0,3)$

6. 1) A(-2,-3) nuqtadan o'tuvchi va burchak koeffitsienti $k=1$ bo'lgan to'g'ri chiziq tenglamasini tuzing; 2) (-2,0) nuqtadan o'tuvchi va burchak koeffitsienti $k=-2$ ga teng bo'lgan to'g'ri chiziq tenglamasini tuzing.

7. (-3,-2) nuqtadan o'tuvchi va *Ox* o'qi bilan arctg2 burchak tashkil etuvchi to'g'ri chiziq tenglamasini tuzing.

8. 1) C(3,1) va $D(4,-2)$, 2) A(2,3) va B(-3,1) nuqtalardan o'tuvchi to'g'ri chiziqning Ox o'qqa o'g'ish burchagini toping.

9. A(6,2) va (-3,8) nuqtalardan o'tuvchi to'g'ri chiziqning koordinata o'qlarida ajratuvchi kesmalarini toping.

10. Quyidagi to'g'ri chiziqlarning kesishish nuqtalarini toping:

1) $y=5x$ va $x+y-12=0$, 2) $x-4y-7=0$ va $x+2y-4=0$

11. Ushbu to'g'ri chiziqlar orasidagi o'tkir burchakni toping:

1) $y=3x$ va $y=-x$ 2) $2x-3y+6=0$ va $3x-y-3=0$

3) $\frac{x}{5}+\frac{y}{2}=1$ va $\frac{x}{3}+\frac{y}{4}=1$

12. $5x-12y-16=0$ va $3x+4y-12=0$ to'g'ri chiziqlar orasidagi o'tkir burchakni toping:

13. Uchlari A(-6,-1), B(4,6) va C(2,1) bo'lgan uchburchak berilgan. Bu uchburchakning ichki burchaklarini toping.

14. Uchburchakning A(2,-1), B(-7,3) va C(-1,-5) uchlari berilgan. C burchak bissektrisasining tenglamasini tuzing:

15. 1) A(-7,3) nuqtadan $5x-7y+21=0$ to'g'ri chiziqqa parallel holda o'tuvchi to'g'ri chiziq tenglamasini tuzing; 2) $A(-1,-4)$ nuqtadan $\frac{x}{4}+\frac{y}{3}=1$ to'g'ri chiziqqa parallel holda o'tuvchi to'g'ri chiziq tenglamasini tuzing.

16. 1) $B(5;-2)$ nuqtadan $6x-12y+5=0$ to'g'ri chiziqqa perpendikulyar holda o'tuvchi to'g'ri chiziq tenglamasini tuzing; 2) $M(-4;1)$ nuqtadan $\frac{x}{5}-\frac{y}{6}=1$ to'g'ri chiziqqa perpendikulyar holda o'tuvchi to'g'ri chiziq tenglamasini tuzing.

17. 1) $M(6;8)$ nuqtadan $4x+3y+2=0$ to'g'ri chiziqqacha bo'lgan masofani toping;

2) $N(4;6)$ nuqtadan $3x+4y+14=0$ to'g'ri chiziqqacha bo'lgan masofani toping;

3) Ikkita parallel $4x+3y-8=0$ va $4x+3y-33=0$ to'g'ri chiziqlar orasidagi masofani toping.

18. To'g'ri burchakli dekart koordinatalar sistemasida berilgan to'g'ri chiziqlarning tenglamalari normal shaklga keltiring:

1) $4x-3y+10=0$, 2) $6x+8y-15=0$

3) $y-x\sqrt{3}=4$ 4) $x\cos 10^0+y\sin 10^0+4=0$

19. $7x-y+3=0$ va $3x+5y-4=09$ to'g'ri chiziqlarning kesishish nuqtasidan va $A(2,-1)$ nuqtadan o'tuvchi to'g'ri chiziqning tenglamasini yozing.

20. *m* va *n* ning qanday qiymatlarida $mx+8y+n=0$ va $2x+my-1=0$ to'g'ri chiziqlar: 1) parallel; 2) ustma-ust; 3) perpendikulyar bo'ladi?

21. Ushbu $(x+2y-7)+\lambda(3x-y+5)=0$ dastaga tegishli va dastaning asosiy to'g'ri chiziqlaridan har biriga perpendikulyar bo'lgan to'g'ri chiziqlarning tenglamasini toping.

22. Teng tomonli to'g'ri burchakli uchburchak gipotenuzasi tenglamasi $y=7x-4$ va uning to'g'ri burchak uchi $C(3,4)$ nuqtada bo'lganda uchburchak katetlarining tenglamasini tuzing.

23. Quyidagi to'g'ri chiziqlarning parametrik tenglamasini yozing:

1) $y=2x-3$, 2) $y=0,5x+1$, 3) $6x+11y+9=0$,

4) $\frac{x}{3}-\frac{y}{4}=1$; 5) $\frac{x-1}{2}=\frac{y}{3}$; 6) $4y+5=0$.

24. μ va λ koeffitsientlar qanday shartni qanoatlantirganda $\lambda x + \mu y + 2 = 0, 3x - 2y + 3 = 0, y - 1 = 0$ to'g'ri chiziqlar bir nuqtada kesishadi?

25. Agar $A_1 x + B_1 y + C_1 = 0, A_2 x + B_2 y + C_2 = 0, A_3 x + B_3 y + C_3 = 0,$ to'g'ri chiziqlar bir nuqtada kesishsa, $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0$ bo'lishini isbotlang.

26. M nuqtaning $5x - 12y - 13 = 0$ va $3x - 4y - 19 = 0$ to'g'ri chiziqlardan chetlanishi mos ravishda -3 va -5 ga teng, M nuqtaning koordinatalarini toping.

Mustaqil yechish uchun misollar va masalalarning javoblari

1.1) $k = -1,\ b = 4$; **2)** $k = 2,\ b = 3$; **3)** $k = -3,\ b = 2$. **2. 1)** $y = x$; **2)** $y = \sqrt{3}x$; **3)** $y = -x$; **4)** $y = 0$. **3. 1)** $y = 3x$; 2). $y = -4x$; 3). $y = -3x$ *yoki* $y = \frac{1}{3}x$.

4) $y = -(2 + \sqrt{3})x$ yoki $y = -(2 - \sqrt{3})x$.**4.**1). $AB: y = x + 1;\ AC:\ y = -\frac{5}{2}x + 8$.

$BC: y = -\frac{1}{6}x - \frac{4}{3}$. **2)** $y = -\frac{3}{4}x + 1$,**3)** $y = 6x - 9$. **5.**1). $y = x - 1$; 2).$y = -2x - 4$.

7. $2x - y + 4 = 0$. **8.**1)$\alpha = arctg0{,}4 = 21^0 48'$; 2)$180^0 - arctg3 \approx 108^0 26'$ **.9.** $x = 9, y = 6$ **10. 1)** (2;10) 2)(5;-0,5**).** **11.**

1)$\alpha = arctg2 \approx 63^0 26'$; 2)$\alpha = arctg\frac{7}{9} \approx 37^0 52'$; 3)$\alpha = arctg\frac{14}{23} \approx 31^0 20'$.

12. $\varphi = \arccos\frac{33}{65} \approx 59^0 29'$. **13.** $tgA = 0{,}383; A = 20^0 57'$;

$tgB = 0{,}6545; B = 33^0 12'$; $tgC = 1{,}3846, C = 125^0 50'$.

14. $x + 1 = 0$. **15.1)** $5x - 7y + 56 = 0$, **2)** $3x + 4y + 19 = 0$. **16.1)** $2x + y - 89 = 0$,

2) $5x + 6y + 14 = 0$. **17. 1)**10, **2)** 10, **3)**5. **18.** 1)$\frac{4x - 3y + 10}{-5} = 0$;

2)$0{,}6x + 0{,}8y - 1{,}5 = 0$ 3)$\frac{y}{2} - \frac{\sqrt{3}}{2}x - 2 = 0$; **4)** $x\cos 100 - y\sin 100 = 0$.

19. $25x + 29y - 21 = 0.$

20. 1) $m = -4,\ n \neq 2$ *yoki* $m = 4,\ n \neq -2;$

2) $m = -4,\ n = 2,\ m = 4,\ n = -2;$ 3) $m = 0,\ n - ixtiyoriy$

21. $14x - 7y + 32 = 0,\ 7x + 21y - 75 = 0.$ **22.** $y = \frac{3}{4}x + \frac{7}{4}; y = -\frac{4}{3}x + 8.$

23. 1) $x = 2 + t, y = 1 + 2t,$ **2)** $x = 2 + 2t, y = 2 + t;$

3) $x = -7 + 11t, y = 3 - 6t,$ **4)** $x = 3t; y = -4 + 4t$ **5)** $x = 2 - 2t, y = 3t;$ **6)** $x = t, y = -1{,}25$ **24.** $-\lambda + 3\mu + 6 = 0$. **26.** M(2;3).

6-amaliy mashg'ulot.

IKKINCHI TARTIBLI CHIZIQLAR

1.Quyidagi ma'lumotlarga ko'ra fokuslari abssissa o'qida, koordinata boshiga nisbatan simmetrik bo'lgan ellipsning eng sodda tenglamasini tuzing: 1) Yarim o'qlari $a = 16$ va $b = 8$ ga teng; 2) Fokuslari orasidagi masofa $2c = 12$ va katta o'qi $2a = 20$ ga teng; 3) Katta yarim o'qi $a = 10$ ga, ekssentrisiteti esa $\varepsilon = 0{,}4$ ga teng; 4) kichik o'qi $2b = 36$, fokuslari orasidagi masofa esa $2c = 20$ ga teng; 5) Uning katta o'qi $2a = 24$, direktrisalar orasidagi masofa esa, $D = 32$ ga teng; 6) uning kichik yarim o'qi $b = 6$, direktrisalar orasidagi masofa esa 26 ga teng; 7) direktrisalar orasidagi masofa $D = 36$, ekssentrisiteti esa $\varepsilon = 1/3$ ga teng:

2. Ellips tenglamasi berilgan: $16x^2 + 25y^2 = 400.$ 1) O'qlarining uzunlik-lari; 2) fokuslarining koordinatalari; 3) ekssentrisitetini hisoblang.

3. $\frac{x^2}{30} + \frac{y^2}{24} = 1$ ellipsda uning kichik o'qidan 5 birlik masofadagi nuqtani toping.

4. Ellips $A(\sqrt{5};-3)$ va $B(-2\sqrt{5};2)$ nuqtalardan o'tadi. Ellipsning tenglamasini tuzing.

5. $\frac{x^2}{12}+\frac{y^2}{6}=1$ ellipsning $x - y - 3 = 0$ to'g'ri chiziq bilan kesishish nuqtalarini toping.

6. $\frac{x^2}{49}+\frac{y^2}{24}=1$ ellipsga ichki to'g'ri to'rtburchak chizilgan, uning ikkita qarama-qarshi tomoni fokuslaridan o'tadi. Shu to'g'ri to'rtburchakning yuzini toping.

7. Quyida tenglamasi bilan berilgan chiziqlarni aniqlang va chizing.

1) $y=\frac{3}{4}\sqrt{16-x^2}$; 2) $y=-\frac{5}{4}\sqrt{16-x^2}$

3) $y=\frac{9}{7}\sqrt{49-x^2}$; 4) $y=-\frac{4}{3}\sqrt{9-x^2}$

8. $\frac{x^2}{100}+\frac{y^2}{25}=1$ ellipsning $x + 2y - 14 = 0$ to'g'ri chiziq bilan kesishish nuqtalarining koordinatalarini toping.

9. Agar fokuslari Ox o'qida yotuvchi ellips $A(\sqrt{3};\sqrt{6})$ va $B(3;\sqrt{2})$ nuqtadan o'tsa, shu ellipsning tenglamasini tuzing.

10. $\frac{x^2}{16}+\frac{y^2}{12}=1$ ellipsga (2;-3) nuqtada urinuvchi to'g'ri chiziqning tenglamasini tuzing.

11. $x=\pm 8$ to'g'ri chiziqlar kichik o'qi 8 ga teng bo'lgan ellipsning direktrisalaridir. Shu ellipsning tenglamasini va ekssentrisitetini toping.

12. Ekssentrisiteti $\varepsilon=\frac{4}{5}$ bo'lgan ellips koordinata o'qlariga simmetrik bo'lib, M(4;-2,8) nuqtadan o'tadi. M nuqtaning fokal radiuslarini aniqlang.

13. $\frac{x^2}{30}+\frac{y^2}{24}=1$ ellipsning $2x - y + 17 = 0$ to'g'ri chiziqqa parallel bo'lgan urinmalarini toping.

Giperbola

14. Quyidagilarni bilgan holda fokuslari abssissa o'qida koordinata boshiga nisbatan simmetrik joylashgan giperbolaning eng sodda tenglamasini tuzing:

1) haqiqiy o'qi $2a$ = 20 va mavhum o'qi esa $2b$ =16 ga teng; 2) fokuslar orasidagi masofa $2c$ = 20, mavhum o'qi esa $2b$ = 12 ga teng; 3) fokuslar orasidagi masofa $2c = 10$, ekssentrisiteti esa $\varepsilon = \frac{5}{4}$ teng; 4) haqiqiy o'qi 2a = 8, ekssentrisiteti esa $\varepsilon = \frac{3}{2}$ ga teng; 5) asimptotalari $y = \pm\frac{4}{3}x$ tenglamalar bilan berilgan fokuslari orasidagi masofa esa $2c$ = 10 teng; 6) direktrisalar orasidagi masofa $\frac{225}{16}$, fokuslar orasidagi masofa esa $2c$ = 32 teng; 7) direktrisalar orasidagi masofa $\frac{32}{5}$, mavhum o'qi esa 2 b = 16 ga teng; 8) direktrisalar orasidagi masofa $\frac{24}{5}$, ekssentrisiteti esa $e = \frac{5}{2}$ ga teng; 9) asimptota tenglamalari $y = \pm\frac{3}{4}x$, direktrisalari orasidagi masofa $\frac{64}{5}$ ga teng.

15. $\frac{x^2}{81} - \frac{y^2}{144} = 1$ giperbolaning uchlari, fokuslari va asimptotalarini toping.

16. $16x^2 - 25y^2 = 400$ giperbola berilgan. 1) a va b; 2) fokuslari;

3) ekssentrisiteti; 4) asimptota tenglamalari; 5) direktrisalarini toping.

17. Fokuslarining koordinatalari F_1(-20;0) va F_2(20;0), $e = \frac{5}{3}$ ekssentrisiteti bo'yicha giperbola tenglamasini tuzing.

18. Haqiqiy va mavhum o'qlarining yig'indisi 14 ga, fokuslari orasidagi masofa esa 20 ga teng bo'lib, fokuslari Ox o'qida yotgan giperbolaning tenglamasini tuzing:

19. $\frac{x^2}{36} - \frac{y^2}{9} = 1$ giperbolaga $M_1(5; \frac{9}{4})$ nuqta tegishli. M_1 nuqtaning fokal radiuslarini toping.

20. Quyidagi shartda giperbolaning ekssentrisitetini hisoblang:

1) asimptotalar orasidagi burchak 60^0 ga teng;

2) asimptotalar orasidagi burchak 90^0 ga teng;

21. Quyidagi tenglamasi bilan berilgan chiziqlarni aniqlang va chizing:

1) $y = \frac{4}{5}\sqrt{x^2 - 25}$ 3) $y = \frac{4}{15}\sqrt{x^2 + 225}$

2) $y = \frac{4}{3}\sqrt{x^2 - 9}$ 4) $y = 4\sqrt{x^2 + 1}$

22. Agar giperbolaning asimptotalari $y = \pm\frac{\sqrt{6}}{3}x$ tenglamalar bilan berilgan bo'lsa, y (6;-4) nuqtadan o'tsa, shu giperbolaning tenglamasini tuzing.

23. $9x + 2y-24=0$ to'g'ri chiziq va $\frac{x^2}{4} - \frac{y^2}{9} = 1$ giperbolaning asimptotalari bilan chegaralangan uchburchakning yuzini hisoblang:

24. $\frac{x^2}{5} - \frac{y^2}{4} = 1$ giperbolaga (5;4) nuqtada urinuvchi to'g'ri chiziq tenglamasini tuzing.

25. Quyida berilganlarga ko'ra koordinata boshiga nisbatan simmetrik, fokuslari abssissa o'qida yotgan giperbola tenglamasini tuzing:

1) $M_1(5;\frac{9}{4}), M_2(\ 8;3\sqrt{3})$ giperbola nuqtalari;

2) $M_1(\ 5;3)$ giperbola nuqtasi, $e = \sqrt{2}$ esa uning ekssentrisiteti;

3) $M(4,5;-1)$ giperbola nuqtasi, $y = \pm\frac{2}{3}x$ to'g'ri chiziqlar esa uning asimptotalari;

4) $M(-3;2,5)$ giperbola nuqtasi, $x = \pm\frac{4}{3}$ esa uning direktrisa tenglamalari.

26. $\frac{x^2}{15} - \frac{y^2}{6} = 1$ giperbolaga 1) $x + y - 7=0$ to'g'ri chiziqqa parallel;

2) $x - 2y = 0$ to'g'ri chiziqqa perpendikulyar bo'lgan urinmalarni o'tkazing.

27. $\frac{x^2}{49}+\frac{y^2}{24}=1$ ellips bilan umumiy fokuslarga ega va ekssentrisiteti $e=1{,}25$ bo'lgan giperbolaning tenglamasini tuzing.

Parabola

28. Quyida berilganlarga ko'ra parabolaning eng sodda tenglamasini tuzing:

1) fokusi $F(6;0)$ nuqtada, uchi koordinatalar boshida; 2) direktrisasi x = -5 to'g'ri chiziqdan iborat va uchi koordinatalar boshida; 3) direktrisasi y=-4 to'g'ri chiziqdan iborat va uchi koordinatalar boshida; 4) parabola y o'qiga nisbatan simmetrik bo'lib, fokusi (0;6) nuqtada va uchi koordinatalar boshida;

29. $y^2=16x$ parabolada fokal radius vektori 29 ga teng bo'lgan nuqta topilsin.

30. Uchi koordinatalar boshida bo'lib, Ox o'qiga nisbatan simmetrik bo'lgan va quyidagi nuqtalardan o'tuvchi parabolaning tenglamasini tuzing:

1) (10;-3); 2) (-8;6); 3) (-4;4).

31. Parabolaning tenglamasi berilgan: $y^2=6x$. Uning direktrisasi tenglamasini tuzing.

32. Parabolaning berilgan tenglamasiga ko'ra uning fokusi koordinatalrini hisoblang: 1) $y^2=6x$; 2) $y^2=-4x$; 3) $x^2=14y$; 4) $x^2=-5y$.

33. $y^2=16x$ parabolaning $4x-3y+8=0$ to'g'ri chiziq bilan kesishish nuqtalarini toping.

34.Uchi $A(2;3)$ nuqtada, fokusi $F(6;3)$ nuqtada bo'lgan parabola tenglamasini tuzing.

35. $y^2+4y-24x+76=0$ parabola fokusining koordinatalarini toping:

36. $y^2=12x$ parabolaning $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellips bilan kesishish nuqtalarini toping.

37. $y^2=18x$ parabola bilan $(x+6)^2+y^2=100$ aylana umumiy vatarining tenglamasini tuzing.

38. $y^2=3x$ parabolaning $\frac{x^2}{20}-\frac{y^2}{5}=1$ giperbola bilan kesishish nuqtalarini toping.

39. $y^2 = 2px$ parabolaga muntazam uchburchak ichki chizilgan. Uchburchak uchlarining koordinatalarini aniqlang.

Ellips

1. 1) $\frac{x^2}{256}+\frac{y^2}{64}=1$**; 2)** $\frac{x^2}{100}+\frac{y^2}{64}=1$**;** **3)** $\frac{x^2}{100}+\frac{y^2}{84}=1$**;**

4) $\frac{x^2}{424}+\frac{y^2}{324}=1$**; 5)** $\frac{x^2}{144}+\frac{y^2}{63}=1$**;** **6)** $\frac{x^2}{52}+\frac{y^2}{36}=1, \frac{x^2}{234}+\frac{y^2}{36}=1$**;** **7)**

$\frac{x^2}{36}+\frac{y^2}{32}=1$. **2.** $1)2a=10,\ 2b=8;\ \ 2)F(-3,0);\ \ 3)\varepsilon=\frac{3}{5}$**.3.** $(\pm5;\pm2)$**. 4.** $\frac{x^2}{32}+\frac{3y^2}{32}=1$.

5. $(2\pm\sqrt{3};-1\pm\sqrt{3})$**. 6.** $S=64\frac{4}{7}$*кв.бир.* **8.** (8;3), (6;4)**. 9.** $\frac{x^2}{12}+\frac{y^2}{8}=1$**. 10.** x-2y-8=0**.**

11. $\frac{x^2}{32}+\frac{y^2}{16}=1, \varepsilon=\frac{\sqrt{2}}{2}$**.** **12.** $r_1=\frac{9}{5}, r_2=8\frac{1}{5}$**.**

13. 2x-y+12=0 va 2x-y-12=0

Giperbola

14. 1) $\frac{x^2}{100}-\frac{y^2}{64}=1;$ 2) $\frac{x^2}{64}-\frac{y^2}{36}=1;$ 3) $\frac{x^2}{16}-\frac{y^2}{9}=1;$

4) $\frac{x^2}{16}-\frac{y^2}{20}=1;$ 5) $\frac{x^2}{9}-\frac{y^2}{16}=1;$ 6) $\frac{x^2}{225}-\frac{y^2}{31}=1;$ 7) $\frac{x^2}{16}-\frac{y^2}{9}=1;$ 8) $\frac{x^2}{36}-\frac{y^2}{185}=1;$

9) $\frac{x^2}{64}-\frac{y^2}{36}=1.$ **15**. (-9;0),(9;0), $F_1(-15;0), F_2(15;0), y=\pm\frac{4}{3}x$

16. $1)a=5,b=4;\ 2)F_1(-\sqrt{41};0), F_2(\sqrt{41};0);$ $3)\varepsilon=\frac{\sqrt{41}}{5};\ 4)y=\pm\frac{4}{5}x;$

$5)x=\pm\frac{a}{\varepsilon}=\pm\frac{25}{\sqrt{41}}$**.17.** $\frac{x^2}{144}-\frac{y^2}{256}=1.$ **18.** $\frac{x^2}{36}-\frac{y^2}{64}=1, \frac{x^2}{64}-\frac{y^2}{36}=1.$ **20**.

$1)\varepsilon=\frac{2}{3}\sqrt{3}\ ;2)\varepsilon=\sqrt{2}$. **22.** $\frac{x^2}{12}\quad\frac{y^2}{8}=1$**.23.** $S_\Delta=12$*кв.бир***.**

24. $x + y = 1$. **25.** 1) $\frac{x^2}{16} - \frac{y^2}{9} = 1$, 2) $x^2 - y^2 = 16$; 3) $\frac{x^2}{18} - \frac{y^2}{8} = 1$;

4) $\frac{x^2}{4} - \frac{y^2}{5} = 1 yoki \frac{9x^2}{61} - \frac{16y^2}{305} = 1$. **26.** 1) $x + y + 3 = 0, x + y - 3 = 0$;

2) $2x + y + \sqrt{54} = 0, \quad 2x + y - \sqrt{54} = 0$. **27.** $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Parabola

28. 1) $y^2 = 24x$; 2) $y^2 = 10x$; 3) $x^2 = 16y$; 4) $x^2 = 24y$. **29.** $A(25;-20)$; $B(25;20)$. **30.** 1) $y^2 = 0{,}9x$; 2) $y^2 = -4{,}5x$. **31**. $x = -1{,}5$. **32**. 1) (1,5;0); 2) (-1;0);3) (0;3,5); 4) (0;-1,25). **33.** (4;8) yoki (1;4). **34.** $(y-3)^2 = 16(x-2)$. **35**. F(9;-2). **36.** $(\frac{5}{4};\sqrt{15}),(\frac{5}{4};-\sqrt{15})$. **37.** x-2=0. **38.** $(\frac{5}{4};\sqrt{15}),(\frac{5}{4};-\sqrt{15})$.

39. $O(0;0), A(6;2\sqrt{3}), B(6;-2\sqrt{3})$.

7-amaliy mashg'ulot.

TEKISLIK VA FOZADA TO'GRI CHIZIQ

Fazoda tekislik

1. Ushbu $A(3;2;-2)$, $B(-2;0;0)$, $C(-3;1;0)$, $D(-4;-2;2{,}5)$ nuqtalar berilgan. Bu nuqtalardan qaysilari $2x - 3y + 2z + 4 = 0$ tekislikka tegishli bo'lishini ko'rsating.

2. 1) $M(-3,0,2)$ nuqtadan o'tuvchi va $\boldsymbol{n}=(1,3,4)$ vektorga perpendikulyar tekislikning tenglamasini tuzing.

2) $M(6,4,5)$ nuqtadan o'tuvchi va $\boldsymbol{n}=(-1,-3,2)$ vektorga perpendikulyar tekislikning tenglamasini tuzing.

3) $A(4;-2;3)$ va $B(1;4;2)$ nuqtalar berilgan. A nuqtadan o'tuvchi va AB vektorga perpendikulyar bo'lgan tekislikning tenglamasini tuzing.

3. 1) Ox o'qdan va $M(3,2,4)$ nuqtadan o'tuvchi;

2) Oy o'qdan va $M(-2,-3,-4)$ nuqtadan o'tuvchi;

3) Oz o'qdan va $M(1,1,1)$ nuqtadan o'tuvchi tekislik tenglamasini tuzing.

4. $M(2,-1,3)$ nuqtadan o'tuvchi va $\boldsymbol{a} = (3,0,-1)$ hamda $\boldsymbol{b} = (-3,2,2)$ vektorlarga parallel ravishda o'tuvchi tekislikning tenglamasini tuzing.

5. 1) $M(-2,3,4)$ nuqtadan o'tuvchi va $x + 2y - 3z + 4=0$ tekislikka parallel bo'lgan tekislikning tenglamasini tuzing.

2) $M_1(-2, -3, 1)$ va $M_2(1, 4, -2)$ nuqtalardan o'tuvchi va $2x - 3y - z + 4 = 0$ tekislikka perpendikulyar bo'lgan tekislikning tenglamasini tuzing.

6. Quyidagi tekisliklarning koordinata o'qlaridan ajratgan kesmalarini hisoblang:

1) $4x - 3y - z + 12=0$; 2) $5x + y - 4z - 20=0$;3) $x - 8z - 16 = 0$;4) $y - 7 = 0$.

7. Quyidagi berilgan tekislik tenglamalarini normal shaklga keltiring.

1) $2x - 9y + 6z - 22=0$; 2) $5x + \sqrt{8}y - 4z + 5 = 0$;

3) $4x + 3y + 12z + 6 = 0$.

8. 1) $A(2,3,4)$ nuqtadan $4x + 3y + 12z - 5 = 0$ tekislikkacha

2) $B(3, 1, -1)$ nuqtadan $3x - y + 2z + 1 = 0$ tekislikkacha

3) $C(2, 0, -1/2)$ nuqtadan $4x - 4y + 2z + 17 = 0$ tekislikkacha bo'lgan masofani toping.

9. Quyida berilgan tekisliklar orasidagi o'tkir burchaklarni toping.

1) $2x - 3y + 4z - 1 = 0$ va $3x - 4y - z + 3 = 0$;

2) $x - y + z + 1 = 0$ va $2x + 3y + z - 3 = 0$;

3) $4x - 5y + 3z - 1 = 0$ va $x - 4y - z + 9 = 0$.

10. Quyidagi 1) $11x - 2y - 10z + 75 = 0$ va $11x - 2y - 10z - 45 = 0$;

2) $2x - 3y + 6z + 28 = 0$ va $2x - 3y + 6z - 14 = 0$ parallel tekisliklar orasidagi masofani toping.

11. Quyida berilgan uchta tekislikning kesishish nuqtasini toping.

1) $3x - 5y + 3z - 1 = 0$, $x + 2y + z - 4 = 0$, $2x + 7y - z - 8 = 0$;

2) $2x - 4y + 9z - 28 =0$, $7x + 9y - 9z - 5 = 0$, $7x + 3y - 6z + 1 = 0$;

3) $2x + y - 5 = 0$, $x + 3z - 16 = 0$, $5y - z - 10 = 0$.

12. Kubning ikkita yog'i $2x - 2y + z - 1 = 0$ va $2x - 2y + z + 5 = 0$ tekisliklarda yotadi. Bu kubning hajmini hisoblang.

13. $M_1(3, 4, -5)$ nuqtadan o'tgan, $\boldsymbol{a_1} = \{3, 1, -1\}$ va $\boldsymbol{a_2} = \{1, -2, 1\}$ vektorlarga parallel bo'lgan tekislik tenglamasini tuzing.

14. $M_1(3, -1, 2)$, $M_2(4, -1, -1)$ va $M_3(2, 0, 2)$ nuqtalar orqali o'tgan tekislik tenglamasini tuzing.

15. $M_1(2, -1, 3)$ va $M_2(3, 1, 2)$ nuqtalar orqali o'tgan $\boldsymbol{a} = \{3, -1, 4\}$ vektorga parallel bo'lgan tekislik tenglamasini tuzing.

Fazoda to'g'ri chiziq

16. Ozod had D ning qanday qiymatlarida quyidagi $\begin{cases} 3x - y + 2z - 6 = 0 \\ 2x + 3y - z + D = 0 \end{cases}$ to'g'ri chiziq: 1) Ox 2) Oy 3) Oz o'qini kesadi.

17. $6x - 17y + 12z - 13 = 0$ tekislik bilan koordinatalar tekisligining kesishishidan hosil bo'lgan to'g'ri chiziq tenglamalarini tuzing.

18. $M_1(3,5,0)$ nuqtadan o'tgan: 1) $a = (8, -3, 2)$ vektorga;

2) $\frac{x+1}{6} = \frac{y+2}{5} = \frac{z-6}{-3}$ to'g'ri chiziqqa; 3) Ox o'qiga; 4) Oy o'qiga; 5) Oz o'qiga parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.

19. Quyida berilgan ikki nuqta orqali o'tgan to'g'ri chiziq tenglamasini tuzing:

1) (2; -2; 3), (3; 4; -1); 2) (-8;-1;6), (4;0;-5);

3) (6; -2;5), (8;-3;4); 4) (7;-2;-4), (5;2;6).

20. Quyida berilgan to'g'ri chiziqlarning kanonik tenglamasini tuzing.

1) $2x + 2y + 3z - 4 = 0$, $x + 2y - 3z + 4 = 0$;

2) $5x + y + z = 0$, $2x + 3y - 2z + 5 = 0$;

3) $2x - y + 3z - 2 = 0$, $3x + y - 4z - 8 = 0$.

21. Quyida berilgan to'g'ri chiziqlarning parametrik tenglamasini tuzing.

1) $2x + 3y - z - 4 = 0$, $3x - 5y + 2z - 1 = 0$;

2) $x + 2y - z - 6 = 0$, $2x - y + z + 1 = 0$.

22. Quyida berilgan to'g'ri chiziqlarning parallelligini isbotlang.

1) $\frac{x+2}{3} = \frac{y-1}{-2} = \frac{z}{1}$ va $\begin{cases} x + y - z = 0 \\ x - y - 3z - 2 = 0 \end{cases}$

2) $\begin{cases} x = 2t + 5 \\ y = -t + 2 \\ z = t - 7 \end{cases}$ va $\begin{cases} x + 3y + z + 2 = 0 \\ x - y - 3z - 2 = 0 \end{cases}$

3) $\begin{cases} x + y - 3z + 1 = 0 \\ x - y + z + 3 = 0 \end{cases}$ va $\begin{cases} x + 2y - 5z - 1 = 0 \\ x - 2y + 3z - 9 = 0 \end{cases}$

23. Quyidagi berilgan to'g'ri chiziqlarning perpendikulyarligini isbotlang. 1) $\frac{x}{1} = \frac{y-1}{-2} = \frac{z}{3}$ va $\begin{cases} 3x + y - 5z + 1 = 0 \\ 2x + 3y - 8z + 3 = 0 \end{cases}$

2) $\begin{cases} x = 2t + 1 \\ y = 3t - 2 \\ z = -6t + 1 \end{cases}$ va $\begin{cases} 2x + y - 4z + 2 = 0 \\ 4x - y - 5z + 4 = 0 \end{cases}$

3) $\begin{cases} x + y - 3z - 1 = 0 \\ 2x - y - 9z - 2 = 0 \end{cases}$ va $\begin{cases} 2x + y + 2z + 5 = 0 \\ 2x - 2y - z + 2 = 0 \end{cases}$

24. Quyida berilgan ikki to'g'ri chiziq orasidagi o'tkir burchakni toping.

1) $\frac{x-2}{2} = \frac{y-1}{1} = \frac{z+4}{2}$ va $\frac{x+1}{12} = \frac{y+3}{3} = \frac{z-2}{4}$.

2) $\frac{x-3}{1} = \frac{y+2}{-1} = \frac{z}{\sqrt{2}}$ va $\frac{x+2}{1} = \frac{y-3}{1} = \frac{z+5}{\sqrt{2}}$.

3) $\frac{x-1}{3} = \frac{y+4}{-2} = \frac{z-2}{4}$ va $\frac{x+3}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$

25. Uchburchakning A(3,6,-7), B(-5,2,3) va C(4,-7,-2) uchlari berilgan. Uning S uchidan tushirilgan medianasining parametrik tenglamasini tuzing.

Tekislik va to'g'ri chiziq.

26. $\frac{x+2}{4} = \frac{y-1}{4} = \frac{z+3}{2}$ to'g'ri chiziq bilan $12x + 3y - 4z + 4 = 0$ tekislik orasidagi burchakni hisoblang.

27. $\frac{x+4}{3} = \frac{y-1}{2} = \frac{z-3}{4}$ to'g'ri chiziq bilan $2x - 3y - 2z + 5 = 0$ tekislik orasidagi burchakni hisoblang.

28. M(2,-3,4) nuqtadan $\frac{x-3}{3} = \frac{y+1}{2} = \frac{z+4}{4}$ to'g'ri chiziqqa perpendikulyar holda o'tuvchi tekislik tenglamasini tuzing.

29. N(-1,2,-3) nuqtadan $\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+3}{2}$ to'g'ri chiziqqa perpendikulyar holda o'tuvchi tekislik tenglamasini tuzing.

30. Quyida berilgan to'g'ri chiziq bilan tekislik kesishish nuqtasini toping.

1) $\frac{x-2}{4}=\frac{y-3}{2}=\frac{z+1}{5}$, $x + 2y - 3z - 4 = 0.$

2) $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z}{6}$, $2x + 3y + z - 1 = 0.$

3) $\frac{x+2}{-2}=\frac{y-1}{3}=\frac{z-3}{2}$, $x + 2y - 2z + 6 = 0.$

31. $M(1,3,2)$ nuqtadan o'tib, $x - 2y + 2z - 3 = 0$ tekislikka perpendikulyar to'g'ri chiziq tenglamasini tuzing.

32. $M(-1,1,-2)$ nuqtadan o'tib, $4x - 5y - z - 3 = 0$ tekislikka perpendikulyar ravishda o'tuvchi to'g'ri chiziq tenglamasini tuzing.

33. Quyida berilgan to'g'ri chiziqning tekislikda yotishini tekshiring.

1) $\frac{x+3}{4}=\frac{y-1}{2}=\frac{z+2}{3}$, $2x-y-2z-9=0$.

2) $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-4}{5}$, $3x-4y-2z-7=0$.

3) $\frac{x+3}{3}=\frac{y-2}{-1}=\frac{z+1}{-5}$, $x-2y+z-15=0$.

34. m ning qanday qiymatida $\frac{x+1}{3}=\frac{y-2}{m}=\frac{z+3}{-2}$ to'g'ri chiziq $x - 3y + 6z + 7 = 0$ tekislikka parallel bo'ladi?

35. C ning qanday qiymatida $\begin{cases}3x-2y+z+3=0\\4x-3y+4z+1=0\end{cases}$ to'g'ri chiziq $2x - y + Cz - 2=0$ tekislikka parallel bo'ladi?

36. Ushbu $\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{5}$ to'g'ri chiziqqa nisbatan $P(4;3;10)$ nuqtaga simmetrik bo'lgan nuqtani toping.

37. $P(7;9;7)$ nuqtadan $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z}{2}$ to'g'ri chiziqqacha bo'lgan masofani toping.

38. $P(1,-1,-2)$ nuqtadan $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z}{2}$ to'g'ri chiziqqacha bo'lgan masofani toping.

39. Quyidagi ikki parallel to'g'ri chiziq orasidagi masofani toping.

$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{2}$ va $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$

40. $x = 3t - 2$, $y = -4t + 1$, $z = 4t - 5$ to'g'ri chiziq bilan $4x - 3y - 6z - 5 = 0$ tekislikning parallel ekanligini isbotlang.

Mustaqil yechish uchun misol va masalalarning javoblari

Fazoda tekislik

1. A va B nuqtalar. **2**. 1) $x + 3y + 4z - 5 = 0$; 2) $x + 3y - 2z - 8 = 0$;
3) $3x - 6y + z - 27 = 0$. **3**.1) $2y + z = 0$; 2) $2x - z = 0$; 3) $x - y = 0$. **4**.$2x - 3y + 6z - 25 = 0$. **5**. 1) $x + 2y - 3z + 8 = 0$; 2) $2x - 3y - 5z = 0$. **6**.1) -3; 4; 12; 2) 4;20;-5; 3) 16; 0; -2; 4) 0; 7; 0. **7**.

1) $\frac{2}{11}x-\frac{9}{11}y+\frac{6}{11}z-2=0$; 2) $\frac{5}{7}x+\frac{\sqrt{8}}{7}y-\frac{4}{7}z+\frac{5}{7}=0$;

3) $\frac{4}{13}x+\frac{3}{13}y+\frac{12}{13}z+\frac{6}{13}=0$. **8**.1) $d=\frac{60}{13}$; 2) $d=\frac{4}{\sqrt{35}}$; 3) $d=4$. **9**.

1) $\varphi=\arccos\frac{14}{\sqrt{29\quad 26}}=\arccos 0{,}5098$; 2) $\varphi=90^0$; 3) $\varphi=\arccos 0{,}7$.

10. 1) $d = 8$; 2) $d = 6$. **11**. 1) (1,1,1); 2) (2,3,4); 3) (1,3,5). **12**. V = 8. **13**.$x + 4y + 7z + 16 = 0$. **14**.$3x + 3y + z - 8 = 0$. **15**. $x - y - z = 0$.

Fazoda to'g'ri chiziq

16.1) $D = -4$; 2) $D = 9$; 3) $D = 3$. **17.** $6x - 17y + -13 = 0$, $z = 0$;
$6x + 12z - 13 = 0$, y = 0; $-17y + 12z - 13 = 0$, $x = 0$. **18**.

1) $\frac{x-3}{8}=\frac{y}{-3}=\frac{z-4}{2}$; 2) $\frac{x-3}{6}=\frac{y}{5}=\frac{z-4}{-3}$; 3) $\frac{x-3}{1}=\frac{y}{0}=\frac{z-4}{0}$; 4) $\frac{x-3}{0}=\frac{y}{0}=\frac{z-4}{1}$.

19. 1) $\frac{x-1}{1}=\frac{y+2}{6}=\frac{z-3}{-4}$; 2) $\frac{x+4}{12}=\frac{y+1}{1}=\frac{z-6}{-11}$;

3) $\frac{x-6}{2}=\frac{y+2}{-1}=\frac{z-5}{-1}$; 4) $\frac{x-7}{-2}=\frac{y+2}{4}=\frac{z+4}{10}$.

20. 1) $\frac{x}{-12}=\frac{y}{9}=\frac{z-\frac{4}{3}}{2}$; 2) $\frac{x}{-5}=\frac{y+1}{12}=\frac{z-1}{13}$; 3) $\frac{x-2}{1}=\frac{y-2}{17}=\frac{z}{5}$;

21. 1) $x = t + 1$, $y = -7t$, $z = -19t - 3$; 2) $x = -t + 1$, $y = 3t + 2$, $z = 5t - 1$.

24. 1) $= \arccos 0{,}8974 = 26^0 71'$; 2) $\varphi = 60^0$; 3) $\varphi = 68^0 53'$. **25.** $x = 5t + 4$, $y = -11t - 7$, $z = -2$.

Tekislik va to'g'ri chiziq

26. $\varphi = \arcsin\frac{2}{3}$. **27.** $\varphi = \arcsin 0{,}3604$. **28.** 3x+2y+4z-16=0.

29. $4x + 3y + 2z + 4 = 0$. **30.1)** (6,5,4); **2)** (2,-3,6); **3)** To'g'ri chiziq tekislikda yotadi.

31. $\frac{x-1}{1}=\frac{y-3}{-2}=\frac{z-2}{2}$. **32.** $\frac{x+1}{4}=\frac{y-1}{-5}=\frac{z+2}{-1}$. **33.** 1) Yotadi;

2) Yotadi; 3) Yotmaydi. **34.** $m = -3$. **35.** $c = -2$. **36.**(2,9,6). **37.** $d = \sqrt{22}$. **38.** $d = 7$. **39**. $d = 3$.

8-amaliy mashg'ulot.

IKKINCHI TARTIBLI SIRTLAR

Quyida berilgan tenglamalar qanday sirtlarni aniqlaydi. Kesimlar usulida bu sirtlarni tekshiring va ularni chizing.

1. $x^2 + 2y^2 - 6z^2 = 0$.

2. $3x^2 + 2y^2 - 4z^2 = 12$.

3. $3x^2 - 4y^2 + 24z = 0$.

4. $x^2 + 14y = 0$.

5. $4y^2 + z^2 + 8x = 0$.

6. $x^2 - 9y^2 - 4z^2 + 1 = 0$.

7. $z = 4 - x^2 - y^2$.

9. $x^2 + 3y^2 - 9z = 0$.

10. $4x^2 + y^2 = 9$.

11. $6x^2 - y^2 + 4z^2 = 0$.

12. $10x^2 + 5y^2 - 2z^2 - 50 = 0$.

13. $4z^2 - 5y^2 + 40 = 0$.

14. $z^2 - 6y^2 = 12x$.

15. $4x^2 - 12y^2 - 6z^2 = 12$.

8. $x^2+4y^2+8z^2-16=0$.

16. $\frac{x^2}{16}+\frac{y^2}{9}-\frac{z^2}{1}=1$ sirtning $\frac{x-4}{4}=\frac{y+3}{0}=\frac{z-1}{1}$ to'g'ri chiziq bilan kesishish nuqtalarini toping.

17. $\frac{x^2}{9}-\frac{y^2}{6}+\frac{z^2}{4}=1$ sirtning $\frac{x}{3}=\frac{y-2}{4}=\frac{z-3}{2}$ to'g'ri chiziq bilan kesishish nuqtalarini toping.

Mustaqil yechish uchun misol va masalalarning javoblari

1. Konus. 2.Bir pallali giperboloid. 3. Konis. 4.Parabolik silindir.

5.Elliptik paraboloida. 6. Ikki pallali giperboloid. 7. Elliptik paraboloida.8. Ellipsoida.9. Elliptik paraboloida.10. Elliptik silindir. 11. Konus.12. Bir pallali giperboloid.13. Giperbolik silindir.14. Giperbolik paraboloid. 15. Ikki pallali giperboloid.16. Kesishmaydi.

9-amaliy mashg'ulot.

KOMPLEKS SONLAR

1. Berilgan z_1 va z_2 kompleks sonlarning yig'indisi va ko'paytmasini toping:

a) $z_1=5+4i$, $z_2=-2+3i$; b) $z_1=-8-7i$, $z_2=-3i$;

c) $z_1=5+\sqrt{3}\,i$, $z_2=5-\sqrt{3}\,i$.

2. z_2-z_1 ayirmani va $\frac{z_2}{z_1}$ bo'linmani toping:

$z_1=1+2i$, $z_2=5$; b) $z_1=-1+\sqrt{3}i$, $z_2=-\sqrt{2}+\sqrt{6}i$;

c) $z_1=a-\sqrt{b}i$, $z_2=a+\sqrt{b}i$.

3. Hisoblang:

a) $(4+i)(5+3i)-(3+i)(3-i)$; b) $\dfrac{(5+i)(7-6i)}{3+i}$; *c)* $\dfrac{(5+i)(3+5i)}{2i}$;

d) $\dfrac{(1+3i)(8-i)}{(2+i)^2}$; *e)* $\dfrac{(2+i)(4+i)}{1+i}$; *f)* $\dfrac{(3\text{-}i)(1\text{-}4i)}{z\text{-}i}$; *g)* $(2+i)^3+(2-i)^3$;

h) $(3+i)^3-(3-i)^3$; *i)* $\dfrac{(1+i)^5}{(1-i)^3}$; *j)* $\left(-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i\right)^3$.

4. Kompleks sonning haqiqiy qismini toping:

$z=\dfrac{(1+2i)^3}{i}+i^{19}$; b) $z=\dfrac{5+2i}{2-5i}-\dfrac{3-4i}{4+3i}+\dfrac{1}{i}$.

5. Kompleks sonning mavhum qismini toping:

$z=(2-i)^3(2+11i)$ b) $z=\dfrac{2-3i}{1+4i}+i^6$.

6. Tenglikni isbotlang:

a) $(1+i)^{8n}=2^{4n}$ $(n\in\boldsymbol{Z})$; *b)* $(1+i)^{4n}=(-1)^n 2^{2n}$ $(n\in\boldsymbol{Z})$.

7. Tenglamalar sistemasini yeching:

a) $\begin{cases} iz_1+(1+i)z_2=2+2i \\ 2iz_1+(3+2i)z_2=5+3i \end{cases}$; b) $\begin{cases} (1-i)z_1-3z_2=-i \\ 2z_2-(3+3i)z_2=3-i \end{cases}$;

c) $\begin{cases} 2z_1-(2+i)z_2=-i \\ (4-2i)z_1-5z_2=-1-2i \end{cases}$

8. Hisoblang:

a) $i^4+i^{14}+i^{24}+i^{34}+i^{44}$; b) $i+i^2+i^3+\ldots i^n$, $n>4$; c) $i\cdot i^2\cdot i^3\cdot i^4\ldots i^{50}$.

9. $\omega=-\dfrac{1}{2}+\dfrac{i\sqrt{3}}{2}$ bo'lganda quyidagilarni hisoblang:

a) $(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$; b) $(a+b)(a+b\omega)(a+b\omega^2)$;

c) $(a+b\omega+c\omega^2)^3+(a+b\omega^2+c\omega)^3$.

10. Tenglamani yeching:

$(i-z)(1+2i)+(1-iz)(3-4i)=1+7i$; b) $z^2+\bar{z}=0$;

c) $(1-i)\bar{z}-3iz=2-i$; d) $z\bar{z}+3(z-\bar{z})=4+3i$;

e) $z\bar{z}+3(z+\bar{z})=7$; f) $z\bar{z}+3(z+\bar{z})=3i$.

11. Hisoblang:

$\sqrt{2i}$; b) $\sqrt{-8i}$; c) $\sqrt{3-4i}$; d) $\sqrt{-15+8i}$; e) $\sqrt{-11+60i}$;

f) $\sqrt{-8-6i}$; q) $\sqrt{2-3i}$; h) $\sqrt{1-i\sqrt{3}}$; i) $\sqrt[4]{2-i\sqrt{12}}$; j) $\sqrt[4]{-1}$.

12. Tenglamani yeching:

$x^2-(2+i)x+(-1+7i)=0$; b) $x^2-(3-2i)x+(5-5i)=0$;

c) $(2+i)x^2-(5-i)x+(2-2i)=0$; d) $x^4-6x^2+25=0$;

e) $x^4+34x^2+289=0$; f) $x^2-(4+3i)x+1+5i=0$;

g) $x^2+5x+9=0$; h) $x^2+x+1+i=0$.

13. Quyidagi kompleks sonlarni ifodalovchi nuqtalarni yasang:

1; –1; i; - i; –1 + i; 2 – 3i; – 6 + 3i; cos 30° – i sin30°; cos 150° + i sin150°.

14. Kompleks tekislikda berilgan z_1, z_2, z_3 nuqtalar parallelogramning ketma-ket uchlaridan iborat. Bu parallelogramning to'rtinchi uchini toping.

15. Kompleks tekislikda $z_1 = 6 + 8i, z_2 = 4 - 3i$ nuqtalar berilgan. z_1 va z_2 vektorlar hosil qilgan burchak bissektrisasining nuqtalariga mos keluvchi kompleks sonlarni toping.

16. Tenglamani yeching:

a) $|z|-iz=1-2i$; b) $z^2+3|z|=0$; c) $z^2+|z|^2=0$.

17. Tenglamalar sistemasini yeching: $|z+1-i|=|3+2i-z|=|z+i|$.

18. Tenglamalar sistemasini yeching: $\begin{cases}|z+1|=|z+2|\\|3z+9|=|5z+10i|.\end{cases}$

19. Quyidagi nuqtalarga mos kompleks sonlarni toping:

a) markazi koordinatalar boshida, tomonlari koordinata o'qlariga parallel va tomonlarining uzunligi 1 ga teng bo'lgan kvadratning uchlariga;

b) markazi koordinatalar boshida, bir tomoni ordinata o'qiga parallel, bitta uchi manfiy haqiqiy yarim o'qda joylashgan va tashqi chizilgan aylana radiusi 1 ga teng bo'lgan muntazam uchburchakning uchlariga;

c) markazi $2+i\sqrt{3}$ nuqtaga joylashgan, tomonlaridan biri abssissa o'qiga parallel va tashqi chizilgan aylana radiusi 2 ga teng bo'lgan muntazam oltiburchakning uchlariga.

20. Tekislikda quyidagi shartlarni qanoatlanturvchi z kompleks sonlarga mos keladigan nuqtalar to'plamini tasvirlang:

$|z|=1$; b) $arg\, z=\frac{\pi}{3}$; c) $|z|\le 2$; d) $|z-1-i|<1$; e) $|z+3+4i|\le 5$;

f) $3<|z|<5$; g)$1\le|z-2i|<2$; h) $|argz|<\frac{\pi}{6}$; i) $|\mathrm{Re}\, z|\le 1$;

j) $-1<Re\, iz<0$; k) $|\mathrm{Im}\, z|=1$; l) $|Re\, z+Im\, z|<1$; m) $|z-1|+|z+1|=3$;

n) $|z+2|-|z-2|=3$; o) $|z-2|=Re\, z+2$; p) $|z+1|<|1-z|$.

21. $|z+1-i|\le 1$ shartni qanoatlantiruvchi z kompleks sonlar ichidan eng kichik musbat argumentga ega bo'lgan sonni toping.

22. $|z-5i|\le 3$ shartni qanoatlantiruvchi z kompleks sonlar ichidan eng kichik musbat argumentga ega bo'lgan sonni toping.

23. *Oxy* tekislikdagi qanday *M(x,y)* nuqtalar uchun quyidagi tengliklar o'rinli:

a) $\left|\sqrt{2x+y}+i\sqrt{x+2y}\right|=\sqrt{3}$. b) $\left|\sqrt{x^2+4}+i\sqrt{y-4}\right|=\sqrt{10}$?

24. Kompleks son moduli va argumentini unga qo'shma bo'lgan son moduli va argumenti orqali ifodalang.

25. *A* va *B* nuqtalar *Oxy* tekislikda mos ravishda $a = 6 + 8i$ va $b = 4 - 3i$ sonlarni ifodalaydi. Hech bo'lmaganda bitta shunday c soni topingki, unga mos keluvchi C nuqta AOB burchakning bissektrisasida yotsin.

26. Qanday shartlar bajarilganda:

a) $|z_1+z_2|=|z_1|+|z_2|$; b) $|z_1+z_2|=|z_1|-|z_2|$?

27*. (-1) dan farqli va moduli 1 ga teng bo'lgan har qanday z kompleks sonni $z=\frac{1+ti}{1-ti}$, bunda $t\in \boldsymbol{R}$, shaklda tasvirlash mumkinligini isbotlang.

28. Kompleks sonlarni trigonometrik shaklga keltiring:

a) 7; b) i; c) –3; d) $-5i$; e) $1+i\sqrt{3}$; f) $-1+i\sqrt{3}$ g)$1-i\sqrt{3}$; h) $\sqrt{3}+i$;

i) $-\sqrt{3}+i$; j) $-\sqrt{3}-i$; k) $\sqrt{3}-i$; l) $1+i\frac{\sqrt{3}}{3}$;

29. Kompleks sonlarni algebraik va trigonometrik shaklga keltiring:

a) $\frac{i(cos\frac{5}{3}p+isin\frac{5}{3}p)}{cos\frac{p}{6}+isin\frac{p}{6}}$; b) $\frac{1}{cos\frac{4}{3}\pi-isin\frac{4}{3}\pi}$; c) $\frac{i}{(1+i)^2}$;

d) $\frac{-cos\frac{5}{12}\pi+isin\frac{5}{12}\pi}{cos\frac{13\pi}{12}-isin\frac{13\pi}{12}}$; e) $\frac{(cos\frac{p}{3}-isin\frac{p}{3})(\frac{1}{2}+i\frac{\sqrt{3}}{3})}{i}$.

30. Kompleks sonlarni trigonometrik shaklga keltiring:

a) $\frac{5(cos100^0+isin100^0)i}{3(cos40^0-isin40^0)}$; b) $\frac{sin\frac{2}{5}p+i(1-cos\frac{2}{5}p)}{i-1}$.

31. Ayniyatni isbotlang: $|x+y|^2+|x-y|^2=2(|x|^2+|y|^2)$. Bu ayniyat qanday geometrik ma'noga ega?

32. Hisoblang**: a)** $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$**; b)** $\left(1-\frac{\sqrt{3}-i}{2}\right)^{24}$**;**

c) $\frac{\left(-1+i\sqrt{3}\right)^{15}}{(1-i)^{20}}+\frac{\left(-1-i\sqrt{3}\right)^{15}}{(1+i)^{20}}$**; d)** $\frac{(1+i)^{2n+1}}{(1-i)^{2n-1}}$, $n\in N$**; e)** $z=(tg1-i)^4$**;**

33. Isbotlang: $\left(\frac{1+itg\alpha}{1-itg\alpha}\right)^n=\frac{1+itg\alpha n}{1-itg\alpha n}$.

34. Agar $z+\frac{1}{z}=2cos\alpha$ bo'lsa, $z^m+\frac{1}{z^m}=2cos\,m\alpha$ bo'lishini isbotlang.

35. $(1+\omega)^n$ ifodani soddalashtiring, bu yerda $\omega=cos\frac{2}{3}\pi+isin\frac{2}{3}\pi$.

36. Ildizning qiymatlarini trigonometrik shaklda yozing:

a) $\sqrt[6]{i}$; b) $\sqrt[10]{512\left(1-i\sqrt{3}\right)}$; c) $\sqrt[8]{8\sqrt{2}(1-i)}$.

37. Ildizning qiymatlarini algebraik shaklda yozing:

a) $\sqrt[3]{1}$; b) $\sqrt[4]{1}$; c) $\sqrt[6]{1}$; d) $\sqrt[3]{i}$; e) $\sqrt[4]{-4}$; f) $\sqrt[6]{64}$; g) $\sqrt[8]{16}$; h) $\sqrt[6]{-27}$;

i) $\sqrt[4]{8\sqrt{3}i-8}$; j) $\sqrt[4]{-72\left(1-i\sqrt{3}\right)}$; k) $\sqrt[3]{1+i}$; l) $\sqrt[3]{2-2i}$; m) $\sqrt[3]{\dfrac{8+24i}{3-i}}$;

n) $\sqrt[3]{\dfrac{27-54i}{2+i}}$; o) $\sqrt[4]{-\dfrac{18}{1+i\sqrt{3}}}$; p) $\sqrt[4]{\dfrac{-32}{9\left(1-i\sqrt{3}\right)}}$.

38. Tenglamani yeching: a) $z^5-1-i\sqrt{3}=0$; b) $z^6+64=0$.

39. $\sqrt{5+12i}$ va $\sqrt{5-12i}$ sonlarning haqiqiy qismlari manfiy bo'lgan holda $z=\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$ sonning algebraik shaklini yozing.

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. a) $3+7i$; $-22+7i$; b) $-8-10i$; $21-24i$; c) 10; 28. **2.** a) $4-2i$; $1-2i$; b) $\left(1-\sqrt{2}\right)+\left(\sqrt{6}-\sqrt{3}\right)i$; $\sqrt{2}$; c) $2\sqrt{b}i$, $\dfrac{a^2-b}{a^2+b}+\dfrac{2a\sqrt{b}}{a^2+b}i$;

3. a) $7+17i$; b) $10-11i$; c) $14-5i$; d) $5+i$; e) $\dfrac{13}{2}-\dfrac{1}{2}i$; f) $\dfrac{11}{5}-\dfrac{27}{5}i$;

d) 4; h) 52 i; i) 2; j) 1.**4.** a) –2; b) 0. **5.** a) 0; b) $-\dfrac{11}{17}$.

7. a) $z_1=2,\quad z_2=1-i$; b) $\varnothing$; c) $z_1=\dfrac{(2+i)z_2-i}{2}$. **8.** a) 1; b) 0, agar $n=4\kappa$; i, agar $n=4\kappa+1$; i-1, agar $n=4\kappa+2$; -1, agar $n=4\kappa+3$; c) $-i$.

9. a) $a^2+b^2+c^2$ - $(ab+bc+ac)$; b) a^3+b^3;

c) $2\left(a^3+b^3+c^3\right)-3\left(a^2b+a^2c+b^2a+b^2c+c^2a+c^2b\right)+12abc$.

10. a) $-1-i$; b) 0,–1, $\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$; c) i; d) $z_1=\dfrac{\sqrt{15}}{2}+\dfrac{1}{2}i, z_2=\dfrac{\sqrt{15}}{2}+\dfrac{1}{2}i$;

e) $\left\{x+yi\,|-7\le x\le 1,\ y=\pm\sqrt{7-6x-x^2}\right\}$; f) $\varnothing$, **11.** a) $\pm(1+i)$;

b) $\pm(2-2i)$; c) $\pm(2-i)$; d) $\pm(1+4i)$; e) $\pm(5+6i)$; f) $\pm(1-3i)$;

g) $\pm\left(\sqrt{\dfrac{\sqrt{13}+2}{2}}-i\sqrt{\dfrac{\sqrt{13}-2}{2}}\right)$; h) $\pm\left(\sqrt{\dfrac{3}{2}}-i\sqrt{\dfrac{1}{2}}\right)$;

i) $i^{\alpha}\left(\frac{1+\sqrt{3}}{2}+\frac{1-\sqrt{3}}{2}i\right),\ \alpha=0,1,2,3;$ j) $\frac{\sqrt{2}(\pm1\pm i)}{2}$.

12. a) $x_1 = 3 - i;\ x_2 = -1 + 2i$; b) $x_1 = 2 + i,\ x_2 = 1 - 3i;$ c) $x_1 = 1 - i;$;

d) $x_1 = 2-i, x_2 = -2+i, x_3 = 2+i,\ x_4 = -2 - i$; e) $x_1 = x_2 = i\sqrt{17}$, $x_3 = x_4 = -i\sqrt{17}$;

f) $x_1 = 3+2i, x_2 = 1+i$; g) $x_1 = -\frac{5}{2}+\frac{\sqrt{11}}{2}i$, $x_2 = -\frac{5}{2}-\frac{\sqrt{11}}{2}i$; h) $x_1 = -i,\ x_2 = -1+i$. **14.** z_4 $= z_1+z_3 - 2z_2$. **15.** $t(7+i)$, t –ixtiyoriy musbat son. **16.** a) $2-\frac{3}{2}i$; 0, $3i$, $-3i$; c) bi, $b \in \boldsymbol{R}$. **17.** $\frac{7}{6}+\frac{5}{6}i$.**18.** $-\frac{3}{2}-\frac{17}{4}i$; $-\frac{3}{2}-2i$.**19.** a) $\pm\frac{1}{2}\pm\frac{1}{2}i$; b) -1, $\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$; c) $4+i\sqrt{3}$, $2+2i\sqrt{3}$, $1+2i\sqrt{3}$, $i\sqrt{3}$, 1, 3. **21.** i. **22.** $\frac{12}{5}+\frac{16}{5}i$. **24.** $|z|=|\bar{z}|$, $argz = -arg\bar{z}$.**25.** c = 7+i, $C(7,1)$.**26.** a) $argz_1 = argz_2$; b) $argz_1 = -argz_2$, $|z_1| \geq |z_2|$. **28.** a) $7(cos0 + isin0)$; b) $\left(cos\frac{\pi}{2}+isin\frac{\pi}{2}\right)$; c) $3(cos\pi + isin\pi)$;

d) $5\left(cos\frac{3\pi}{2}+isin\frac{3\pi}{2}\right)$; e) $2\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)$; f) $2\left(cos\frac{2\pi}{3}+isin\frac{2\pi}{3}\right)$;

g) $2\left(cos\left(-\frac{\pi}{3}\right)+isin\left(-\frac{\pi}{3}\right)\right)$; h) $2\left(cos\frac{\pi}{6}+isin\frac{\pi}{6}\right)$;

i) $2\left(cos\frac{5}{6}\pi+isin\frac{5}{6}\pi\right)$;j) $2\left(cos\left(-\frac{5}{6}\pi\right)+isin\left(-\frac{5}{6}\pi\right)\right)$;

k) $2\left(cos\left(-\frac{\pi}{6}\right)+isin\left(-\frac{\pi}{6}\right)\right)$; l) $\frac{2}{\sqrt{3}}\left(cos\frac{\pi}{6}+isin\frac{\pi}{6}\right)$;

29.a) $1 = cos0 + isin0$; b) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i = cos\frac{4}{3}\pi + sin\frac{4}{3}\pi$;c) $\frac{1}{2} = \frac{1}{2}(cos0 + sin0)$;

d) $\frac{1}{2}-\frac{\sqrt{3}}{2}i = cos\frac{5}{3}\pi + isin\frac{5}{3}\pi$; e) $-i = cos\frac{3}{2}\pi + sin\frac{3}{2}\pi$.

30. a) $\frac{5}{3}\left(cos230^0 + isin230^0\right)$; b) $\sqrt{2}\,sin\frac{\pi}{5}\left(cos\frac{29}{20}\pi + isin\frac{29}{20}\pi\right)$.

32. a) $2^9\left(1-i\sqrt{3}\right)$; b) $\left(2-\sqrt{3}\right)^{12}$; c) -64; d) 2, agar n – juft, –2, agar n –toq; e) $\frac{1}{cos^4 1}\left(cos4+isin4\right)$; f) $\frac{1}{cos^4 2}\left(cos2+isin2\right)$; g) $-32icos^5\frac{3\pi}{5}$.

36. a) $cos\frac{(4k+1)\pi}{12}+isin\frac{(4k+1)\pi}{12}(0\le k\le 5)$;

b) $(cos\frac{(6k-1)\pi}{30}+isin\frac{(6k-1)\pi}{30}(0\le k\le 9)$;

c) $\sqrt{2}\left(cos\frac{(8k-1)\pi}{32}+isin\frac{(8k-1)\pi}{32}\right)(0\le k\le 7)$.

37. a) $\left\{1,-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}\right\}$; b) $\{\pm 1,\pm i\}$; c) $\left\{\pm 1,\pm\frac{1+i\sqrt{3}}{2};\pm\frac{1-i\sqrt{3}}{2}\right\}$;

d) $\left\{\frac{\sqrt{3}}{2}+\frac{1}{2}i,-\frac{\sqrt{3}}{2}+\frac{1}{2}i,-i\right\}$; e) $\{1\pm i;-1\pm i\}$; f) $2\sqrt[6]{1}(см(c))$;

g) $\left\{\pm\sqrt{2},\pm\sqrt{2}i,\pm\sqrt{2}(1+i),\pm\sqrt{2}(1-i)\right\}$; h) $\left\{\pm i\sqrt{3},\pm\frac{\sqrt{3}}{2}\left(\sqrt{3}+i\right),\pm\frac{\sqrt{3}}{2}\left(\sqrt{3}-i\right)\right\}$;

i) $\left\{\sqrt{3}+i,-1+i\sqrt{3},-\sqrt{3}-i,1-i\sqrt{3}\right\}$; j) $\left\{3-i\sqrt{3},\sqrt{3}+3i,-3+i\sqrt{3},-\sqrt{3}-3i\right\}$;

k) $\left\{\frac{1}{2}\sqrt[6]{2}\left(\sqrt{2\sqrt{3}}+i\sqrt{2\sqrt{3}}\right),\ \frac{1}{2}\sqrt[3]{4}(i-1),\frac{1}{6}\sqrt[6]{2}\left(\sqrt{2-\sqrt{3}}+i\sqrt{2+\sqrt{3}}\right)\right\}$;

l) $\left\{\frac{1}{2}\sqrt{2}\left(\sqrt{2+\sqrt{3}}-i\sqrt{2-\sqrt{3}}\right),-\frac{1}{2}\sqrt{2}\left(\sqrt{2-\sqrt{3}}-i\sqrt{2+\sqrt{3}}\right),1-i\right\}$;

m) $\left\{\pm\sqrt{3}+i,-2i\right\}$; n) $\left\{\frac{3}{2}\left(\pm\sqrt{3}+i\right),-3i\right\}$; o) $\left\{\pm\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right),\pm\left(\frac{\sqrt{3}}{2}-i\frac{3}{2}\right)\right\}$;

p) $\left\{\pm\left(1-i\frac{\sqrt{3}}{3}\right),\pm\left(\frac{\sqrt{3}}{3}+i\right)\right\}$. **38.** a)

$\sqrt[5]{2}\left(cos\frac{\pi}{15}+isin\frac{\pi}{15}\right);\sqrt[5]{2}\left(cos\frac{7\pi}{15}+isin\frac{7\pi}{15}\right);\sqrt[5]{2}\left(cos\frac{13\pi}{15}+isin\frac{13\pi}{15}\right)$;

$\sqrt[5]{2}\left(cos\frac{19\pi}{5}+isin\frac{19\pi}{5}\right);\sqrt[5]{2}\left(cos\frac{5\pi}{3}+isin\frac{5\pi}{3}\right)$ b)

$\sqrt{3}+i,\ 2\text{i},\ -\sqrt{3}+i,\ -\sqrt{3}-i,-2\text{i},\sqrt{3}-i$. **39.** $-\frac{3}{2}i$;

10- amaliy mashg'ulot.

SONLAR KETMA-KETLIGI VA UNING LIMITI

Quyidagi ketma-ketliklarning dastlabki beshta hadini yozing:

1. $x_n=2+(-1)^n\frac{2}{n+1}$. **2.** $x_n=n(3-3(-1)^n)$. **3.** $x_n=\frac{4n+3}{3n-2}$.

4. $x_n=(-1)^n\arcsin\frac{\sqrt{3}}{2}+\pi m$. **5.** $x_n=\cos\frac{n\pi}{2}$. **6.** $x_n=2^{(-1)^n}$.

Quyidagi ketma-ketliklarning umumiy hadini yozing.

7. $-\frac{1}{2},\ \frac{1}{3},\ -\frac{1}{4},\ \frac{1}{5},\ldots$. **8.** $0,\ 4,\ \ 0,\ 4,\ldots$. **9** $1,\ 0,-3,\ 0,\ 5,\ 0,-7,\ 0,\ldots$.

10. $2,\frac{4}{3},\frac{5}{6},\frac{8}{7},\ldots$. **11.** $-3,\ \frac{5}{3},\ -\frac{7}{5},\ \frac{9}{7},\ -\frac{11}{9},\ldots$.

Quyida berilgan $\{x_n\}$ ketma-ketliklarni chegaralanganlikka tekshiring.

12. $\left\{\frac{3n+40}{n}\right\}$. **13.** $\left\{\frac{5n}{6n-7}\right\}$. **14.** $\left\{\frac{4n^2-3}{4n^2}\right\}$. **15.** $\left\{\frac{n^2-5}{2+3n^2}\right\}$.

16. $\left\{(-1)^n(n^2+1)\right\}$. **17.** $\left\{(-1)^{n+1}[1+(-1)^n]\cdot n\right\}$. **18.** $\left\{5n^2\right\}$.

Quyidagi ketma-ketliklarning chegaralanganligini isbotlang.

19. $\left\{\frac{4n^2-3}{3+n^2}\right\}$. **20.** $\left\{\frac{2n+(-1)^n}{3n+2}\right\}$. **21.** $\left\{\frac{n^2+5n-6}{(n+2)^2}\right\}$.

22. $\left\{\frac{4n^5+3}{(n^3+1)(n+1)^2}\right\}$. **23.** $\left\{\frac{2-n}{\sqrt{n^2+3}}\right\}$. **24.** $\left\{\frac{\sin^2 n}{n^2+3}\right\}$.

Quyida berilgan $\{x_n\}$ ketma-ketliklarning cheksiz kichik ketma-ketlik ekanligini ta'rif bo'yicha ko'rsating.

25. $x_n=\frac{3}{n}$. **26.** $x_n=\frac{(-1)^{n+1}}{n}$. **27.** $x_n=\frac{1+(-1)^n}{3^n}$.

28. $x_n = \frac{1}{n}\cos\frac{n\pi}{2}$. **29.** $x_n = \frac{1}{\sqrt{n}}$. **30.** $x_n = \frac{2}{\sqrt{2n-1}}$.

Quyida berilgan $\{x_n\}$ ketma-ketliklarning cheksiz katta ketma-ketlik ekanligini ta'rif bo'yicha ko'rsating:

31. $x_n = n.$ **32.** $x_n = 4 - 3 \cdot n$. **33.** $x_n = 2^n.$

34. $x_n = \ln n$. **35.** $x_n = n^{1/p} (p \in N)$

Ketma-ketlik limiti ta'rifidan foydalanib, quyidagi tengliklarni isbotlang.

36. $\lim\limits_{n\to\infty} \frac{4n-3}{4n+5} = 1.$ **37.** $\lim\limits_{n\to\infty} \frac{5n-2}{3n+4} = \frac{5}{3}$.

38. $\lim\limits_{n\to\infty} \frac{n^2+3}{n^2+2n+1} = 1.$ **39.** $\lim\limits_{n\to\infty} \frac{5n^2+4}{8n^2+7} = \frac{5}{8}$.

40. $\lim\limits_{n\to\infty} \frac{\sqrt{n^2+n}}{n} = 1$. **41.** $\lim\limits_{n\to\infty} \frac{5n+1}{7-9n} = \frac{-5}{9}.$

42. Quyida berilgan $\{x_n\}$ ketma-ketliklarning yaqinlashuvchi ekanligini isbotlang:

1) $x_n = \frac{3+(-1)^n}{n}$; 2) $x_n = \frac{n+1}{n}$;

3) $x_n = \sqrt[n]{2}$ $(a > 0)$; 4) $x_n = \frac{1}{\sqrt[n]{n!}}$;

43. Quyida berilgan $\{x_n\}$ ketma-ketliklarning uzoqlashuvchi ekanligini isbotlang.

1) $x_n = (-1)^n \cdot 2^n.$ 2) $x_n = n^{(-1)^n}$. 3) $x_n = \frac{n^2-10}{n}.$

4) $x_n = (-1)^n + \frac{1}{n}.$

44. a soni $\{x_n\}$ ketma-ketliklarning limiti emasligini ta'rif yordamida ko'rsating.

1) $x_n = (-1)^n \cdot 2 + 2, \quad a = 0.$ 2) $x_n = \frac{n^2-1}{n^2}$ $a = 1.$

3) $x_n = \frac{(-1)^n}{n}, \quad a = -1.$ 4) $x_n = \cos\frac{\pi n}{3}, \quad a = \frac{1}{2}.$

Limitlarni toping

45. $\lim\limits_{n\to\infty}\left(\frac{1}{6}+\frac{1}{12}+....+\frac{1}{n^2+3n+2}\right)$.

46. $\lim\limits_{n\to\infty}\left(\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+....+\frac{1}{(3n-2)(3n+1)}\right)$.

47. $\lim\limits_{n\to\infty}\left(\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+....+\frac{1}{(4n-3)(4n+1)}\right)$.

48. $\lim\limits_{n\to\infty}\left(\frac{1}{1\cdot 3\cdot 5}+\frac{1}{3\cdot 5\cdot 7}+....+\frac{1}{(2n-1)(2n+1)(2n+3)}\right)$.

$\{a_n\}$- ketma-ketlik arifmetik progressiya bo'lib, uning ayirmasi $d\neq 0$ va barcha xadlari $a_n\neq 0 \quad (n\in N)$ bo'lsin. Limitlarni toping.

49. $\lim\limits_{n\to\infty}\left(\frac{1}{a_1\cdot a_2}+\frac{1}{a_2\cdot a_3}+....+\frac{1}{a_n\cdot a_{n+1}}\right)$.

50. $\lim\limits_{n\to\infty}\left(\frac{1}{a_1a_2\,a_3}+\frac{1}{a_2a_3a_4}+....+\frac{1}{a_na_{n+1}a_{n+2}}\right)$.

51. $\lim\limits_{n\to\infty}\left(\frac{1}{a_1a_2\,a_3\,a_4}+\frac{1}{a_2a_3a_4\,a_5}+....+\frac{1}{a_na_{n+1}a_{n+2}\,a_{n+3}}\right)$.

52. $\lim\limits_{n\to\infty}\frac{\sqrt{4n^2+2}-2n}{\sqrt{n+2}-\sqrt{n}}$.

53. $\lim\limits_{n\to\infty}\frac{\sqrt{n^2+7}-\sqrt{n^2-7}}{\sqrt{n^2+1}-n}$.

54. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n}-\sqrt[3]{n+1}}{\sqrt[4]{n+1}-\sqrt[4]{n}}$.

55. $\lim\limits_{n\to\infty}\frac{\sqrt[4]{n^3+n}-\sqrt{n}}{n+2+\sqrt{n+1}}$.

56. $\lim\limits_{n\to\infty}\frac{\sqrt{2n}}{\sqrt{3n+\sqrt{3n+\sqrt{3n}}}}$.

57. $\lim\limits_{n\to\infty}\frac{n^2-\sqrt{n}}{\left(\sqrt{n^3}+n+\sqrt{n}\right)(n-1)}$.

Quyidagi limitlarni toping:

58. $\lim\limits_{n\to\infty}\left(1+\frac{p}{m}\right)^{n\cdot q}$, $p,q\in N$.

59. $\lim\limits_{n\to\infty}\left(\frac{3^n+1}{3^n}\right)^{3^n}$.

60. $\lim\limits_{n\to\infty}\left(\frac{n+2}{n+5}\right)^{n}$.

61. $\lim\limits_{n\to\infty}\left(\frac{n^2}{n^2+1}\right)^{n^2}$.

62. $\lim\limits_{n\to\infty}\left(1+\frac{1}{k+n}\right)^{n}$, $k\in N$.

63. $\lim\limits_{n\to\infty} n\ln\left(1+\frac{1}{n}\right)$.

64. Kuyida berilgan $\{x_n\}$ ketma-ketliklarni Koshi kriteriysidan foydalanib, yaqinlashuvchi ekanligini isbotlang.

1) $x_n = \frac{\cos a}{3} + \frac{\cos 2a}{3^2} + \frac{\cos 3a}{3^3} + ... + \frac{\cos na}{2^n}$, $a \in R$.

2) $x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}$.

3) $x_n = a_1 q + a_2 q^2 + ... + a_n q^n$, bunda $|q| < 1, \forall n \in N$ uchun $|a_n| \le C$, $C = const$.

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. 1, $\frac{8}{3}, \frac{3}{2}, \frac{12}{5}, \frac{5}{3}, ...$. **2.** 6, 0, 18, 0, 30,... . **3.** 7, $\frac{11}{4}, \frac{15}{7}, \frac{19}{10}, ...$.

4. $\frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}, ...$. **5.** 0; -1; 0; 1; 0;... . **6.** $\frac{1}{2}$; 2, $\frac{1}{2}$, 2, $\frac{1}{2}, ...$.

7. $x_n = \frac{(-1)^n}{n+1}$. **8.** $x_n = 2(1 + (-1)^n)$. **9.** $x_n = n \cos\frac{\pi(n-1)}{2}$. **10.** $x_n = \frac{2n}{2n-1}$. **11.** $x_n = (-1)^n \frac{2n+1}{2n-1}$. **12.** Chegaralangan. 13. Chegaralangan. **14.** Chegaralangan. **15.** Chegaralangan. 16. Chegaralanmagan. **17.** Yuqoridan chegaralangan, quyidan chegaralanmagan. **18.** Quyidan chegaralangan, yuqoridan chegaralanmagan. **45.** $\frac{1}{2}$. **46.** $\frac{1}{3}$. **47.** $\frac{1}{4}$. **48.** $\frac{1}{12}$. **49.** $\frac{1}{d \cdot a_1}$. **50.** $\frac{1}{2d \cdot a_1 \cdot a_2}$. **51.** $\frac{1}{3d \cdot a_1 \cdot a_2 \cdot a_3}$. **52.** 0. **53.** 14. **54.** $-\infty$. **55.** 0. **56.** $\sqrt{\frac{2}{3}}$. **57.** 0. **58.** e^{pq}. **59.** e. **60.** e^{-3}. **61.** e^{-1}. **62.** e. **63.** 1.

11- amaliy mashg'ulot.

FUNKSIYaNING LIMITI

δ ning qanday qiymatlarida $0 < |x - x_0| < \delta$ ekanligidan $|f(x) - b| < \varepsilon$ tengsizlikning o'rinliligi kelib chiqadi?

1. $f(x) = 3x - 2;\ \ x_0 = 1;\ b = 1;\ \varepsilon = 0{,}001$.

2. $f(x) = x^2;\ \ x_0 = 2;\ b = 4;\ \varepsilon = 0{,}001$.

3. $f(x) = \sqrt{x};\ \ x_0 = a;\ b = \sqrt{a};\ \varepsilon = 0{,}01$.

4. $f(x) = 3x^2 - 2;\ \ x_0 = 2;\ b = 10;\ \varepsilon = 0{,}01$.

$x \to x_0$ da $f(x)$ funksiyaning cheksiz katta ekanligi ma'lum. $|f(x)| > E$ tengsizlik o'rinli bo'lishi uchun x qanday bo'lishi lozim.

5. $f(x) = \dfrac{2+3x}{x};\ \ x_0 = 0;\ \ E = 10^3$. **6** $f(x) = \dfrac{x+2}{x-4};\ \ x_0 = 4;\ \ E = 1000.$

7. $f(x) = \dfrac{1}{e^x - 1};\ \ x_0 = 0;\ \ E = 1000.$ **8.** $f(x) = \dfrac{1}{x-2};\ \ x_0 = 2;\ \ E = 100.$

Funksiya limitining Geyne ta'rifidan foydalanib, quyidagi limitlarni toping.

9. $\lim\limits_{x\to 1} \dfrac{2x+1}{5x+3}.$ **10.** $\lim\limits_{x\to 0} x\cos\dfrac{1}{x}.$

11. $\lim\limits_{x\to 0} x^2 \sin\dfrac{1}{x}.$ **12.** $\lim\limits_{x\to 0} x\,\mathrm{arcctg}\dfrac{1}{x}.$

Funksiya limitining Geyne ta'rifidan foydalanib, quyidagi limitlarning mavjud emasligini isbotlang:

13. $\lim\limits_{x\to 2} \sin\dfrac{1}{x-2}.$ **14.** $\lim\limits_{x\to\infty} \cos x.$ **15.** $\lim\limits_{x\to 0} \cos\dfrac{1}{x}.$

Quyidagi munosabatlarni ta'rif yordamida yozing va tegishli misollar keltiring.

16 $\lim\limits_{x\to a} f(x) = b.$ **17.** $\lim\limits_{x\to a-0} f(x) = b.$

18. $\lim\limits_{x\to a+0} f(x) = b.$ **19.** $\lim\limits_{x\to\infty} f(x) = b.$

20. Ushbu $\lim\limits_{x\to c} f(x) = 3,\ \lim\limits_{x\to c} g(x) = -2,\ \lim\limits_{x\to c} h(x) = 0$ limitlar berilganda, quyidagi mavjud limitlarni hisoblang, agar limit mavjud bo'lmasa, nima uchun mavjud emasligini izohlang.

1) $\lim\limits_{x\to c}[f(x) - g(x)].$ 2) $\lim\limits_{x\to c}[f(x)]^2.$ 3) $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)}.$

4) $\lim\limits_{x\to c} \dfrac{h(x)}{g(x)}.$ 5) $\lim\limits_{x\to c} \dfrac{f(x)}{h(x)}.$ 6) $\lim\limits_{x\to c}[f(x)\cdot h(x)].$

7) $\lim\limits_{x\to c} \dfrac{g(x)}{f(x) - h(x)}.$ 8) $\lim\limits_{x\to c} \dfrac{1}{f(x) - g(x)}.$

21. Ushbu $\lim\limits_{x\to c} f(x) = 5,\ \lim\limits_{x\to c} g(x) = 0,\ \lim\limits_{x\to c} h(x) = -5$ limitlar berilganda,

quyidagi limitlarni hisoblang, agar limit mavjud bo'lmasa, nima uchun mavjud emasligini izohlang.

1) $\lim\limits_{x\to c}[2f(x)-3h(x)]$. 2) $\lim\limits_{x\to c}\dfrac{f(x)}{x-c}$. 3) $\lim\limits_{x\to c}[h(x)]^2$.

4) $\lim\limits_{x\to c}\dfrac{3}{f(x)+h(x)}$. 5) $\lim\limits_{x\to c}[3+g(x)]^3$.

Quyidagi limitlarni hisoblang.

22. $\lim\limits_{t\to 0} t(2-\dfrac{3}{t})$. **23.** $\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3}$. **24.** $\lim\limits_{x\to -2}\dfrac{(x^2-x-6)^2}{x+2}$.

25 $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{x-4}$. **26** $\lim\limits_{t\to 0}\dfrac{1-1/t^2}{1-\dfrac{1}{t}}$. **27.** $\lim\limits_{t\to 0}\dfrac{1-\dfrac{1}{t}}{1+\dfrac{1}{t}}$.

28 $f(x)=x^2-3x$ funksiya berilganda, quyidagi limitlarni hisoblang:

1) $\lim\limits_{x\to 3}\dfrac{f(x)-f(3)}{x-3}$. 2) $\lim\limits_{x\to 1}\dfrac{f(x)-f(1)}{x-1}$.

Quyida berilgan funksiyalarning bir tomonli limitlarini toping.

29. $f(x)=2^{\frac{1}{x-1}}$, $x\to 1\pm 0$. **30.** $f(x)=\dfrac{4}{(x-2)^3}$, $x\to 2\pm 0$.

31. $f(x)=\dfrac{x^2-1}{|x-1|}$, $x\to 1\pm 0$. **32.** $f(x)=\dfrac{\sqrt{1-\cos 2x}}{x}$, $x\to 0\pm 0$.

33. $f(x)=\begin{cases}3x+1, & x\le 1,\\ -2x+3, & x>1;\end{cases}$ $x\to 1\pm 0$. **34.** $f(x)=\begin{cases}9x+2\\ 4x^2-3\end{cases}, x\to 0\pm 0$

Quyidagi limitlarni hisoblang.

35. $\lim\limits_{x\to 4}\dfrac{x^2-16}{x^2+x-20}$. **36.** $\lim\limits_{x\to -1}\dfrac{7x^2+4x-3}{2x^2+3x+1}$.

37. $\lim\limits_{x\to -5}\dfrac{x^2-2x-35}{2x^2+11x+5}$. **38.** $\lim\limits_{x\to 0}\dfrac{3x^2+x}{4x^2-5x+1}$.

39. $\lim\limits_{x\to -8}\dfrac{2x^2+15x-8}{3x^2+25x+8}$. **40.** $\lim\limits_{x\to 1}\dfrac{2x^2-3x+1}{4x-3x^2-1}$.

41. $\lim\limits_{x\to 7}\dfrac{3x^2-17x-28}{x^2-9x+14}$.

42. $\lim\limits_{x\to 3}\dfrac{2x^2-3x-9}{3x^2-5x-10}$.

43. $\lim\limits_{x\to -\infty}\dfrac{4x^2+7x-10}{2x^4-3x+7}$.

44. $\lim\limits_{x\to -\infty}\dfrac{5x^4-2x^3+3}{2x^2+5x-7}$.

45. $\lim\limits_{x\to 0}\dfrac{3-\sqrt{x^2+9}}{5x^2}$.

46. $\lim\limits_{x\to 9}\dfrac{\sqrt{2x+7}-5}{3-\sqrt{x}}$.

47. $\lim\limits_{x\to -1}\dfrac{\sqrt{5+x}-2}{\sqrt{8-x}-3}$.

48. $\lim\limits_{x\to 10}\dfrac{\sqrt{x-1}-3}{\sqrt{x+6}-4}$.

49. $\lim\limits_{x\to -4}\dfrac{\sqrt{x+12}-\sqrt{4-x}}{x^2+2x-8}$.

50. $\lim\limits_{x\to 5}\dfrac{x^2-3x-10}{\sqrt{x-1}-\sqrt{9-x}}$.

51. $\lim\limits_{x\to 4}\dfrac{x-4}{\sqrt{5x+5}-5}$.

52. $\lim\limits_{x\to 2}\dfrac{x^3-8}{\sqrt{4x+1}-3}$.

53. $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{4x^2}$.

54. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^3 x}{3x^2}$.

55. $\lim\limits_{x\to 0}\dfrac{\sin 7x+\sin 3x}{x\cos 2x}$.

56. $\lim\limits_{x\to 0}\dfrac{\sin 4x-\sin 2x}{3x}$.

57. $\lim\limits_{x\to 0}\dfrac{\cos^2 x-\cos^2 2x}{x^2}$.

58. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}$.

59. $\lim\limits_{x\to 0}\sin 3x\operatorname{ctg} 2x$.

60. $\lim\limits_{x\to 0}\dfrac{\arcsin^2 3x}{2x\sin 5x}$.

61. $\lim\limits_{x\to 0}\dfrac{\arcsin 8x}{\sin 5x}$.

62. $\lim\limits_{x\to 0}\dfrac{\operatorname{tg} 2x-\sin 2x}{x^2}$.

63. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+10}\right)^{-4x}$

64. $\lim\limits_{x\to\infty}\left(\dfrac{2x}{2x-3}\right)^{5x}$

65. $\lim\limits_{x\to\infty}\left(\dfrac{4x}{3+4x}\right)^{-3x}$.

66. $\lim\limits_{x\to\infty}\left(\dfrac{3x-4}{3x+2}\right)^{5x}$.

67. $\lim\limits_{x\to\infty}\left(\dfrac{1-x}{2-x}\right)^{3x}$.

68. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+8}\right)^{-2x}$.

69. $\lim\limits_{x\to-\infty}\left(\dfrac{2x-1}{4x+1}\right)^{3x-1}$. **70.** $\lim\limits_{x\to-\infty}\left(\dfrac{x+3}{4x-5}\right)^{2x}$.

Birinchi ajoyib limitga doir misollarni yeching.

71. $\lim\limits_{x\to0}\dfrac{\sin 20x}{x}$. **72.** $\lim\limits_{x\to0}\dfrac{\sin 9x}{\sin 6x}$. **73.** $\lim\limits_{x\to0}\dfrac{1-\cos 4x}{2x^2}$.

74. $\lim\limits_{x\to0}\dfrac{\sin ax}{\sin bx}$. **75.** $\lim\limits_{x\to0}\dfrac{tg8x}{3x}$. **76.** $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$.

77. $\lim\limits_{x\to0}\dfrac{\sin^2 x-\sin^2 a}{x-a}$. **78.** $\lim\limits_{x\to0}\dfrac{1-\cos^3 x}{x\cdot\sin 4x}$.

Quyidagi funksiyalarning qaysi biri cheksiz kichik bo'ladi:

79. $f(x)=\dfrac{x^2-6x+9}{x^3-27},\ x\to3.$

80. $f(x)=\dfrac{1}{1+4^x};\ a)\ x\to+\infty;\ b)\ x\to-\infty.$

81. $f(x)=\sqrt{x^4+4}-x^2;\ a)\ x\to+\infty,\ b)\ x\to-\infty.$

Quyidagi funksiyalardan qaysi biri cheksiz katta bo'ladi?

82. $f(x)=\dfrac{4x^3-8x+4}{x^3-3x^2+3x-1},\ x\to1.$

83. $f(x)=\dfrac{1}{x^3-6x^2+9x}-\dfrac{1}{x^2-2x-3},\ x\to3.$

84. $f(x)=x\left(\sqrt{x^2+1}-x\right),\ a)\ x\to+\infty,\ b)\ x\to-\infty.$

Quyidagi tasdiqlardan qaysilari $x\to+\infty$ da to'g'ri ekanligini ko'rsating.

85. $20x^2+x\sin x=O(x^2)$ **86.** $x^2=O(20x^2+x\sin x)$

87. $e^x+x^2=O(e^x)$ **88.** $e^x=O(e^x+x^2)$

$x\to0+0$ da quyidagi tengliklarni isbotlang.

89. $4x^2-x^3=\mathrm{O}(x^2).$ **90.** $x\sin\sqrt[5]{x}=\mathrm{O}(x^{\frac{6}{5}}).$

91. $x\sin\dfrac{1}{x}=\mathrm{O}(|x|).$ **92.** $\ln x=o(\dfrac{1}{x^{\varepsilon}}),\ \varepsilon>0.$

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. $\delta \le \frac{0{,}001}{3} = \frac{1}{3000}$. **2** $\delta \le \sqrt{4{,}001} - 2 \approx 0{,}00025$. **3.** $\delta = \min(a; 0{,}01\sqrt{a})$ **4.** $\delta = \frac{1}{1500}$.

5. $\left(-\frac{2}{1003};0\right)\cup\left(0;\frac{2}{9997}\right)$ **6.** $\left(\frac{3996}{1001};4\right)\cup\left(4;\frac{4003}{999}\right)$. **7.** $\left(\ln\frac{999}{1000};0\right)\cup\left(0;\ln\frac{1001}{1000}\right)$.**8.** $\left(\frac{199}{100};2\right)\cup\left(2;\frac{201}{100}\right)$. **9.** $\frac{3}{8}$. **10.** 0. **11.** 0.

19. 0.**20.** 1) 5. 2) 9. 3) $-\frac{3}{2}$. 4) 0. 5) Mavjud emas, chunki mahrajning limiti nolga teng. 6) 0. 7) $-\frac{2}{3}$. 8) $\frac{1}{5}$. **21.** 1) 25. 2) Mavjud emas, chunki maxraj nolga aylanadi. 3) 25. 4) Mavjud emas, maxrajning limiti nolga aylanadi. 5) 27. **22.** -3. **23.** 27. **24.** 0. **25.** $\frac{1}{4}$. **26.** Mavjud emas. **27.** -1. 28.1) 3. 2) Mavjud emas. **29.** $f(1-0)=0,\ f(1+0)=+\infty$. **30.** $f(0-0)=-\infty,\ f(0+0)=+\infty$. **31.** $f(1-0)=-2,\ f(1+0)=2$. **32.** $f(0-0)=-\sqrt{2},\ f(0+0)=\sqrt{2}$. **7.82.** $f(1-0)=4$, $f(1+0)=1$. **34.** $f(0-0)=2,\ f(0+0)=-3$ **35.** $\frac{8}{9}$. **36.**10. **37.** $\frac{4}{3}$.**38.**0. **39.** $\frac{17}{23}$. **40.** -0.5.**41.** 5.**42.** $\frac{8}{13}$.**43.** 0. **44.** ∞. **45.** $\frac{1}{30}$. **46.** $-\frac{6}{5}$. **47.** $-\frac{3}{2}$. **48.** $\frac{4}{3}$. **49.** $-\frac{1}{12\sqrt{2}}$. **50.**14. **51.** 2. **52.** 18. **53.** 3.125. **54.** $\frac{1}{3}$. **55.**10. **56.** $-\frac{2}{3}$. **57.** 3. **58.** $\frac{1}{2}$. **59.** $\frac{3}{2}$. **60.**0.9.**61.** $\frac{8}{3}$. **62.** 0. **63.** e^{20}. **64.** $e^{7,5}$. **65.** $e^{\frac{9}{4}}$. **66.** e^{-10}. **67.** e^{3}. **68.** $e^{1,5}$. **69.** $+\infty$. **70.** $+\infty$. **71.** 20. **72.** $\frac{9}{8}$. **73.** 4. **74.** $\frac{a}{b}$. **75.** $\frac{8}{3}$. **76.** $\cos a$. **77.** $\cos a \sin a = \sin 2a$. **78.** $\frac{3}{8}$. **79.** Ha. **80.** *a*) Ha. *b*) Yo'q. **81.** *a*) Ha. *b*) Ha. **82.** Ha. **83.** Ha. **84.** a) yo'q *b*) yo'q.

12- amaliy mashg'ulot.

FUNKSIYANING UZLUKSIZLIGI

1. Ushbu 1) $f(x)=3x-2,$ 2) $f(x)=x^3$ funksiyalar uchun uzluksizlikning "$\varepsilon-\delta$" ta'rifiga ko'ra, $a=1$ nuqta uchun quyidagi jadvalni to'ldiring.

1)

ε	2	0,5	0,01	0,001	0,0001
δ					

2)

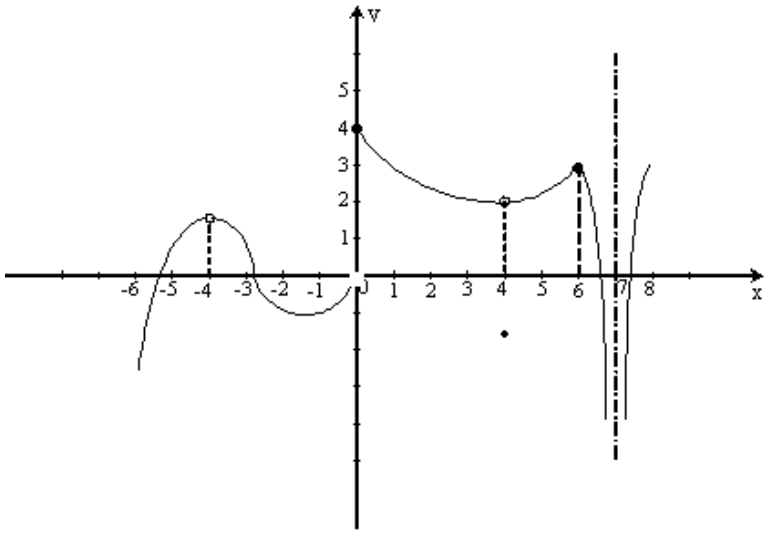
ε	2	0,5	0,01	0,001	0,0001
δ					

Koshi ta'rifdan foydalanib quyidagi funksiyalarning uzluksizligini ko'rsating.

2. $f(x)=x^2$**.** **3.** $f(x)=\sqrt{x}$**.** **4.** $f(x)=|x|$**.**

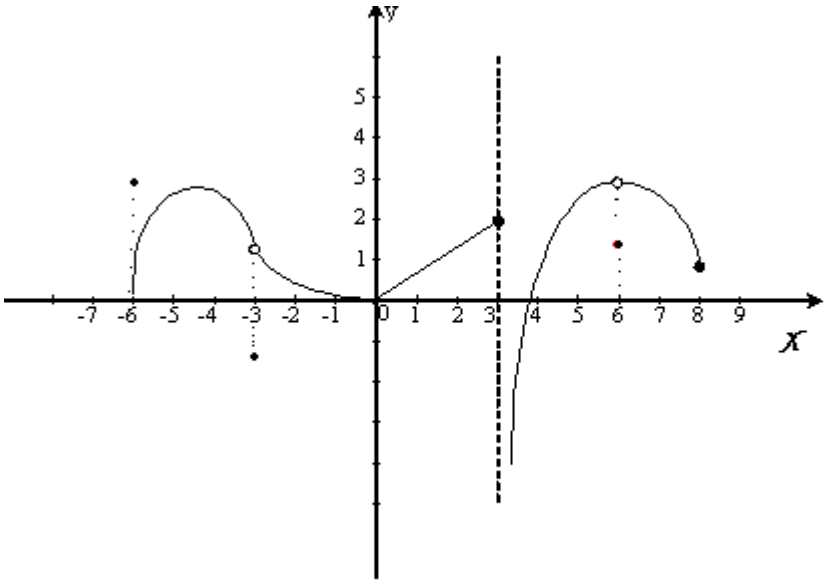
5. $f(x)=arctgx.$ **6.** $f(x)=\begin{cases} x^2, & x-\textit{рационал сон бўлганда,} \\ -x^2, & x-\textit{иррационал сон бўлганда.} \end{cases}$

7. 1- chizmada f funksiyaning grafigi berilgan. 1) Qaysi nuqtalarda f funksiya uzilishga ega? 2) Har bir uzilish nuqtasida f funksiya chapdan uzluksizmi, o'ngdanmi yoki har ikala tomondan uzulishga egami, shuni aniqlang. 3) Agar uzluksizga ega bo'lsa, f funksiya qaysi quqtalarda yo'qotish mumkin bo'lgan uzluksizga ega, qaysi nuqtalarda birinchi tur uzilishiga.



1-chizma

8. 2- chizmada f funksiyaninggrafigi berilgan. Funksiyaning uzluksizlik integvallarini toping.



2-chizma.

Berilgan funksiya ko'rsatilgan nuqtada uzluksizmi, yo'qmi, shuni aniqlang. Agar uzluksiz bo'lmasa, uzulishining turini aniqlang.

9. $f(x)=x^2-3x+1, \quad x_0=3.$

10. $f(x)=\sqrt{x^2+4}, \quad x_0=2.$

11. $f(x)=\begin{cases} x^2+4, & x<2, \\ x^3, & x\geq 2, \end{cases} \quad x_0=2.$

12. $f(x)=\begin{cases} x^2+9, & x<2, \\ 7, & x=2, \\ x^3, & x>3, \end{cases} \quad x_0=3.$

13. $f(x)=x^2 signx$ funksiyaning grafigini chizing. Agar funksiya grafigi uzilishga ega bo'lsa, uzilish nuqtalarda uzilishning turini aniqlang.

14. $f(x)=\dfrac{x^2-9}{x-3}$.

15. $f(x)=|x-1|$.

16. $f(x)=\begin{cases} x-1, & x<1, \\ 0, & x=1, \\ x^2, & x>1. \end{cases}$

17. $f(x)=\begin{cases} -1, & x<-1, \\ x^3, & -1\le x\le 1, \\ 1, & x>1. \end{cases}$

18-19- misollarda $f(x)$ funksiya $x=1$ nuqtadan tashqari R ning barcha nyqtalarida aniqlangan va uzluksiz. Agar mumkin bo'lsa, $f(1)$ qiymatni shunday tanlanki,natijada $f(x)$ funksiya butun R da uzluksiz bo'lsin.

18. $f(x)=\dfrac{x^2-1}{x-1}$.

19. $f(x)=\dfrac{x-1}{|x-1|}$.

Quyidagi funksiyalarning uzilish nuqtalarini toping, turlarini aniqlang, 1-tur uzilish nuqtalarida funksiyaning sakrashini hisoblang hamda grafigini chizing.

20. $f(x)=\begin{cases} x^2+5, & x<2, \\ x^3, & x\ge 2. \end{cases}$

21. $f(x)=\begin{cases} x^2+5, & x<2, \\ 10, & x=2, \\ x^3+1, & x<2. \end{cases}$

22. $f(x)=(signx)^2$.

23. $f(x)=\dfrac{|x|-x}{x^2}$.

Quyidagi funksiyalarning uzilish nuqtalarini toping, ularning turlarini aniqlang va grafiklarini chizing.

24. $f(x)=\dfrac{|x+3|}{x+3}$.

25. $f(x)=\dfrac{1+x}{1+x^3}$.

26. $f(x)=\dfrac{|x-1|}{x^2-x^3}$.

27. $f(x)=\dfrac{x^2-4}{x^2-5x+6}$.

28. $f(x)=\begin{cases} x^2+2, & x\le 0, \\ x-1, & x>0. \end{cases}$

29. $f(x)=\dfrac{1}{\ln x}$.

Ko'rsatilgan nuqtalarda berilgan funksiyalarni uzluksizlikka tekshiring:

30. $f(x)=2^{\frac{1}{x-5}}+1;\ x_1=5,\ x_2=6$. **31**. $f(x)=\dfrac{(x-1)^2}{|x-1|},\ \ x_1=1,\ \ x_2=2.$

32. $f(x)=6^{\frac{1}{x-1}}-3;\ x_1=1, x_2=2$. **33.** $f(x)=\dfrac{x-3}{x^2-9},\ \ x_1=3,\ \ x_2=-3.$

34. $f(x)=\dfrac{x+4}{x-3};\ x_1=3, x_2=4$. **35.** $f(x)=\dfrac{x^2-4}{x-2},\ \ x_1=2,\ \ x_2=-2.$

Quyidagi funksiyalarning uzluksizlikka tekshiring va grafigini chizing:

36. $y=\lim\limits_{n\to\infty}(1-x)^{2n},\ |x|\le 1$. **37** $y=\lim\limits_{n\to\infty}\dfrac{x^n-x^{-n}}{x^n+x^{-n}},\ x\ne 0.$

Quyidagi funksiyalar a va b ning qanday qiymatlarida uzluksiz bo'ladi?

39. $f(x)=\begin{cases}(x-2)^3, & x\le 0,\\ ax^2+b, & 0<x<1,\\ 4\sqrt{x}, & x\ge 1.\end{cases}$ **40.** $f(x)=\begin{cases}ax^2+1, & x>0,\\ -x, & x\le 0.\end{cases}$

41. $f(x)=\begin{cases}\dfrac{(x-1)^2}{x^2-1}, & |x|\ne 1,\\ a, & x=-1,\\ b, & x=1.\end{cases}$ **42.** $f(x)=\begin{cases}\dfrac{2^x-1}{x}, & x\ne 0,\\ a, & x=0.\end{cases}$

Quyidagi funksiyalarning berilgan kesmada chegaralanganligini ko'rsating:

43. $f(x)=\sin x\cos^2 x-\sqrt{x+6},\ x\in[0;\ 10].$

44. $f(x)=arctg\dfrac{x^2+1}{2x}+2^{\sin x}-x^2,\ x\in[1;\ 4].$

45. $f(x)=x(x-2)^2\ln(4-x),\ x\in[0;3].$ **46.** $f(x)=\dfrac{x^2+1}{x^2+4},\ x\in[-4;\ 4].$

Quyidagi funksiyalarning berilgan kesmada eng katta va eng kichik qiymatlari mavjud bo'ladimi?

47. $f(x)=\sin x+3^x, x\in[-1;2].$ **48.** $f(x)=x^2-2,\ x\in[-1;2].$

Quyidagi tenglamalarning ko'rsatilgan kesmada yechimga ega ekanligini ko'rsating:

49. $x^3+2x+1=0,\ x\in[-1;\ 0].$ **50.** $x^3-x^5-x+2=0,\ x\in[0{,}5;\ 2].$

51. $\sin^3 x-5\sin x+1=0,\ x\in\left[0;\dfrac{\pi}{2}\right].$ **52.** $\sin x-x+1=0,\ x\in[0;\ \pi].$

53. $2^x=1,\ x\in[-0{,}2\ 3].$ **54.** $2^x=4x,\ x\in[2;\ 5].$

Quyidagi funksiyalarga teskari bo'lgan funksiyalar mavjudmi?

55. $y = 3x - 6,\ x \in R.$ **56.** $y = 3^{x-1},\ x \in R.$ **57.** $y = (x-2)^2,\ x \in (-\infty; 2]$.

58. $y = \cos 2x,\quad x \in \left[-\frac{\pi}{2}; 0\right]$. **59.** $y = tgx,\ x \in \left[0; \frac{\pi}{4}\right]$.

60. $y = \sin x,\ x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$. **61.** $y = \ln(x-2),\ x \in (2; +\infty)$.

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. 1)

ε	2	0,5	0,01	0,001	0,0001
δ	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{300}$	$\frac{1}{3000}$	$\frac{1}{30000}$

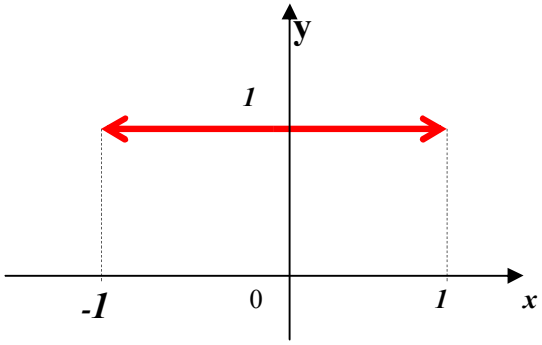
2)

ε	2	0,5	0,01	0,001	0,0001
δ	$\frac{2}{7}$	$\frac{1}{14}$	$\frac{1}{700}$	$\frac{1}{7000}$	$\frac{1}{70000}$

7. 1) $x = -4,\ x = 0,\ x = 4\ x = 7$ uzilish; 2) $x = -4$ ikki tomnlama uzilish, $x = 0$ da o'ngdan uzluksiz, $x = 4$ da har ikki tomondan uzilish, $x = 7$ har ikki tomondan uzilish; 3) $x = 4$ yo'qotilishi mumkin bo'lgan uzilish, $x = 0$ birinchi tur tuzilish. **8.** $(-6; -3), (-3; 3], (3; 6), (6; 8]$. **9.** Uzluksiz. 10**.** Uzluksiz. **11.** Uzluksiz. **12.** Yo'qotilishi mumkin bo'lgan uzulish. 14. $x = 3$ da yo'qotilishi mumkin bo'lgan uzilish. **15**. Uzluksiz. **16.** Birinchi tur uzulish. **17**. Uzluksiz.

18. $f(1) = 2$. **19**. Mumkin emas. **20**. $x = 2$ birinchi tur uzilish, $\Delta f(2) = -1$. **21**. $x = 2$ birinchi tur uzilish, $\Delta f(2) = 1$. **22**. $x = 0$ yo'qotilishi mumkin bo'lgan nuqta. **23.** $x = 0$ ikkinchi tur uzilish. **24**. $x = -3$ birinchi tur uzilish nuqtasi. **25.** $x = -1$ da yo'qotilishi mumkin bo'lgan uzilish. **26.** $x = 0$ ikkinchi tur uzilish, $x = 1$ birinchi tur uzilish. **27.** $x = 2$ da yo'qotilishi mumkin bo'lgan uzilish, $x = 3$ ikkinchi tur uzilish. **28.** $x = 0$ birinchi tur uzilish. **29.** $x = 0$ da yo'qotilishi mumkin bo'lgan nuqta, $x = 1$ ikkinchi tur uzilish. **30**. $x_1 = 5$ ikkinchi tur uzilish , $x_2 = 6$ nuqtada uzluksiz. **31**.

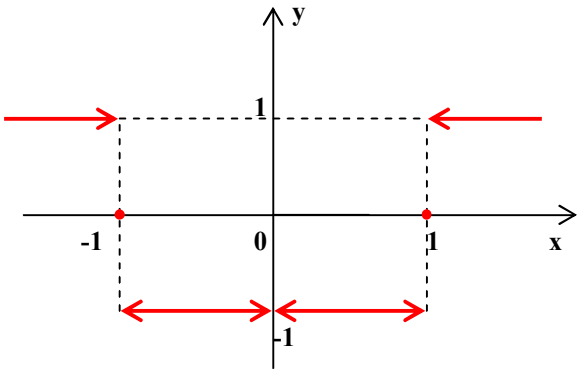
$x_1 = 1$ yo'qotilishi mumkin bo'lgan nuqta, $x_2 = 2$ nuqtada uzluksiz. **32.** $x_1 = 1$ ikkinchi tur uzilish , $x_2 = 2$ nuqtada uzluksiz. **33.** $x_1 = 3$ yo'qotilishi mumkin bo'lgan nuqta, $x_2 = -3$ ikkinchi tur uzilish. **34**. $x_1 = 3$ ikkinchi tur uzilish , $x_2 = 4$ nuqtada uzluksiz. **35**. $x_1 = 2$ yo'qotilishi mumkin bo'lgan nuqta, $x_2 = -2$ nuqtada uzluksiz.



1-чизма.

36. $x = -1,\ x = 1$ yo'qotilishi mumkin bo'lgan uzilish nuktalari. Bu funksiyaning grafigi 1-chizmada tasvirlangan

37. $x = -1,\ x = 1$ da birinchi tur uzilish, $x = 0$ da esa, yo'qotilishi mumkin bo'lgan uzilish. Bu funksiyaning grafigi 2-chizmada tasvirlangan



2-чизма.

39. $a = 12,\ b = 8.$ **40**. Shunday a son mavjud emas. **41.** Shunday a va b sonlar mavjud emas. **47.** Mavjud. **48.** Mavjud. 55**.** Mavjud. **56.** Mavjud . **57.** Mavjud. **58.** Mavjud. **59.** Mavjud emas. **60.** Mavjud emas.

13- amaliy mashg'ulot.

FUNKSIYANING HOSILASI VA DIFFERENSIALI

1. Hosila ta'rifidan foydalanib, quyidagi funksiyalarning $f'(x)$ hosilalarini toping.

1) $f(x)=x^3-2x$. 2) $f(x)=2^x\cos x$. 3) $f(x)=\cos 5x$.

4) $f(x)=\operatorname{tg}x+2x$. 5) $f(x)=\sqrt[3]{x}$. 6) $f(x)=\ln(4x-1)$.

2. Hosila ta'rifidan foydalanib, quyidagi funksiyalarning $f'(x_0)$ hosilalarini toping:

1) $f(x)=(x-4)^3(x+3),\ f'(4)$. 2) $f(x)=\dfrac{(x-2)^3\ln x}{\sin x},\ f'(2)$.

3) $f(x)=\operatorname{ctg}x+2x,\ f'\left(\dfrac{\pi}{4}\right)$. 4) $f(x)=\sqrt[5]{x-4},\ f'(4)$.

3. Hosila ta'rifidan foydalanib, quyidagi funksiyalarning hosilalari mavjudligini tekshiring.

1) $f(x)=|\ln x|,\ x_0=1$. 2) $f(x)=|(x-1)(x-2)|,\ x_0=1,\ x_0=2$.

3) $f(x)=|\sin x|,\ x_0=\pi$. 4) $f(x)=|\cos x|$. 5) $f(x)=|\pi^2-x^2|\cdot\sin^2 x$.

6) $f(x)=\begin{cases}x^2, & x>0,\\ x^4, & x\le 0\end{cases}\quad x_0=0$. 7) $f(x)=\begin{cases}x\cdot\cos\dfrac{1}{x}, & x\neq 0,\\ 0, & x=0.\end{cases}$

Hosilalar jadvali va sodda qoidalar yordamida quyidagi funksiyalarning hosilalarini toping.

Darajali funksiyalar

4. $y=3x^2+6x+3$. **5.** $y=2\sqrt{x}-\dfrac{1}{x^3}+\sqrt{3}$. **6.** $y=\dfrac{1-x^3}{1+x^3}$.

7. $y=(1+2x)^{30}$. **8.** $y=\left(7x^2-\dfrac{4}{x}+6\right)^8$. **9.** $y=\left(t^3-\dfrac{1}{t^3}+3\right)^4$.

10. $y=\sqrt{1-x^2}$. **11.** $y=\dfrac{1+x}{\sqrt{1-x}}$. **12.** $y=\dfrac{t^3}{(1-t)^2}$.

Trigonometrik funksiyalar

13. $y = x^2 \cdot \sin x + 2x\cos x - 2\sin x.$ **14.** $y = \sin^3 x + \sin^{-2} x$.

15. $y = \cos^{-4} x.$ **16.** $y = \dfrac{\operatorname{tg} x}{x}.$ **17.** $y = \cos x - \dfrac{1}{3}\cos^3 x$ **.**

Logarifmik funksiyalar.

18. $y = x^3 \log_4 x.$ **19.** $y = x \ln x$. **20.** $y = \sqrt{\ln x}$.

21. $y = \ln\left(1 - 2x^2\right).$ **22.** $y = \log_2\left(x^2 - 4x\right).$ **23.** $y = \ln^5\left(\sin x\right).$

Ko'rsatkichli funksiyalar.

24. $y = 3^x$ **.** **25.** $y = 2^{\sin 2x}$. **26.** $y = 5^{\cos 4x}.$

27. $y = \dfrac{x^2}{4^x}$ **.** **28.** $y = e^x \cos x.$ **29.** $y = x^4 - 5^{2x}$.

Giperbolik funksiyalar.

30. $y = \operatorname{sh}^2 x$ **.** **31.** $y = \ln\left(\operatorname{ch} x\right).$ **32.** $\mathrm{y} = \operatorname{sh}^2 x + \operatorname{ch}^2 x.$

33. $\mathrm{y} = \operatorname{th}^3 x$. **34.** $y = x^2 \operatorname{sh} x$. **35.** $\mathrm{y} = \dfrac{1 + \operatorname{th} x}{1 - \operatorname{th} x}.$

Teskari trigonometrik funksiyalar.

36. $y = x \arcsin x$. **37.** $y = \left(\arccos x\right)^2$. **38.** $\mathrm{y} = \sqrt{x}\operatorname{arctg} x.$

39. $y = \arcsin \dfrac{3}{x}.$ **40.** $y = \dfrac{x^2}{\operatorname{arctg} x}.$ **41.** $y = \dfrac{\arccos x}{x}$.

Quyidagi funksiyalarning hosilalarini toping.

42. $y = \sqrt{x} + \ln x - \dfrac{1}{\sqrt{x}}$ **.** **43.** $y = \dfrac{x^4}{4}\left[(\ln x)^2 - \dfrac{1}{2}\ln x + \dfrac{1}{8}\right]$

44. $y = \operatorname{ctg} \pi x + \dfrac{\cos \pi x}{2\sin^3 \pi x}$**.** **45.** $y = e^{\sqrt[3]{x}}\left(\sqrt[3]{x^2} - 2\sqrt[3]{x^2} + 2\right).$

46. $y = \dfrac{1}{\sqrt{2}}\ln\left(\sqrt{2} \cdot \operatorname{tg} x + \sqrt{1 + 2\operatorname{tg}^2 x}\right).$ **47.** $y = \dfrac{1}{2a}\left[\ln \dfrac{\sqrt{a^2 + x^2}}{a + x} - \dfrac{a}{a + x}\right]$ **48.**

$y = + \dfrac{\sin x}{2\cos^2 x} - \dfrac{1}{2}\ln \operatorname{tg}\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right).$ **49.** $y = \ln \operatorname{tg} \dfrac{x}{2} - \operatorname{ctg} x \cdot \ln(1 + \sin x) - x.$

50. Quyidagi funksiyalarning ko'rsatilgan nuqtalardagi o'ng va chap

hosilalarini toping.

1) $f(x)=|x+3|$, $f'(-3+0)$, $f'(-3-0)$.

2) $y=f(x)=|x^2-5x+6|$, $x=2$, $x=3$. 3) $f(x)=|2^x-2|$, $x=1$.

51. Quyidagi funksiyalarning $x=0$ nuqtadagi o'ng va chap hosilalarini toping:

1) $f(x)=\begin{cases} x, & x\le 0, \\ \sqrt[3]{x^4}\ln x, & x>0; \end{cases}$ 2) $f(x)=\begin{cases} 1+e^{1/x}, & x<0, \\ \sqrt{1+\sqrt[3]{x^4}}, & x\ge 0. \end{cases}$

52. Funksiyalarning uzilish nuqtalaridagi o'ng va chap hosilalarini toping.

1) $f(x)=\sqrt{\dfrac{x^2+x^3}{x}}$; 2) $f(x)=\begin{cases} \dfrac{1}{1+e^{1/x}}, & x\ne 0, \\ 0, & x=0; \end{cases}$

3) $f(x)=\operatorname{arc\,tg}\dfrac{1+x}{1-x}$; 4) $f(x)=(1-x^2)\operatorname{signx}$.

53. $f(x)=|x-x_0|\cdot\varphi(x)$ funksiya uchun $f'_+(x_0)$ va $f'_-(x_0)$ larni toping, bunda $\varphi(x)$ -berilgan x_0 nuqtada uzluksiz funksiya.

54. $y=\dfrac{1}{2}x^2-\ln x$ funksiya grafigiga abssissasi $x_0=2$ bo'lgan nuqtada o'tkazilgan urinmaning burchak koeffisiyentini toping.

55. $y=x^2-3x+2$ parabolaga abssissasi $x_0=2$ bo'lgan nuqtada o'tkazilgan urinmaning burchak koeffisiyentini toping.

56. $y=4\sin\dfrac{x}{3}$ funksiya grafigining $M(\dfrac{3\pi}{2},4)$ nuqtasidan o'tkazilgan urinma tenglamasini yozing.

57. $\mathrm{y}=\mathrm{x}^2+1$ egri chiziqqa o'tkazilgan urinma $y=2x+3$ to'g'ri chiziqqa parallel. Urinish nuqtasining ordinatasini toping.

58. $y=x^2-2x+1$ egri chiziqdagi qanday nuqtada unga o'tkazilgan urinma $y=-4(x+1)$ to'g'ri chiziqqa parallel bo'ladi?

59. $y=\dfrac{x}{1-x}$ funksiya grafigiga abssissasi $x_0=3$ bo'lgan nuqtadan o'tkazilgan urinmaning Ox o'qi bilan tashkil etgan burchagi α bo'lsa, $\operatorname{tg}2\alpha$ ni toping.

60. $y=\dfrac{x+2}{x-2}$ funksiya grafigiga qanday nuqtalarda o'tkazilgan urinma, Ox o'qining musbat yo'nalishi bilan 135^0 li burchak tashkil etadi?

61. $y=\sqrt[3]{x}$ funksiyaning grafigi qanday nuqtada abssissa o'qiga 30^0 li burchak ostida joylashgan bo'ladi?

62. $y=x^3+2x-1$ funksiya grafigiga qanday nuqtada o'tkazilgan urinma, $x+y=0$ to'g'ri chiziqqa perpendikulyar bo'ladi?

63. $y=x^4$ va $y^4=x$ funksiyalarning grafiklari qaysi nuqtalarda, qanday burchak ostida kesishishlarini aniqlang.

64. $y=\ln x$ chiziq Ox o'qni qanday burchak ostida kesadi?

65. $y=\sin x$ chiziq sinusoida Ox o'qni qanday burchak ostida kesadi?

66. a ning qanday qiymatida $y=a^x$ chiziq Oy o'qni 45^0 li burchak ostida kesadi.

67.Ushbu

1) $y=\sin x\sqrt{3}$; 2) $y=\dfrac{x}{1+x^2}$; 3) $y=\dfrac{x}{\sqrt{3+x^2}}$

funksiyalar Oy o'qni qanday burchak ostida kesadi?

68. $x=a\cos t,\ y=b\sin t$ ellipsga o'tkazilgan urinmaning Ox o'q bilan tashkil qilgan burchagini toping.

69. $(2;1)$ nuqtada $x=t^2\text{-}3t+4,\ y=t^2\text{-}4t+4$ chiziqqa o'tkazilgan urinmani toping.

70. $x=2t^3\text{-}9t^2+12t\text{-}1,\ y=t^2+t+1$ chiziqqa qanday nuqtada o'tkazilgan urinma Oy o'qqa parallel bo'lali.

71. $x=2t-t^2,\ y=3t+t^3$ chiziqqa: 1) t = -1; 2) t = 1; 3) $t=\sqrt{2}$ nuqtalarda o'tkazilgan urinma to'g'ri chiziqlar tenglamasini yozing.

72. Quyidagi funksiyalarning grafiklari qaysi nuqtalarda qanday burchak ostida kesishishlarini aniqlang:

1) $f_1(x)=x\text{-}x^3$, $f_2(x)=5x$.

2) $f_1(x)=\sqrt{2}\sin x,\quad f_2(x)=\sqrt{2}\cos x$. . **3)** $f_1(x)=\dfrac{1}{x},\qquad f_2(x)=\sqrt{x}.$

4) $f_1(x)=\ln x,\ f_2(x)=\dfrac{x^2}{2e}.$ **5)** $f_1(x)=x^2-4x+4,\quad f_2(x)=-x^2+6x-4.$

6) $f_1(x)=x^3,\qquad f_2(x)=\dfrac{1}{x^2}.$ **7)** $f_1(x)=4x^2+2x-8,\quad f_2(x)=-x^3-x+10.$

73. Qanday nuqtalarda quyidagi $y=y(x)$ funksiyalar grafigiga o'tkazilgan urinmalar berilgan to'g'ri chiziqlarga parallel bo'ladi?

1) $y=x^2-7x+3,\quad 5x+y-3=0.$ **2)** $y=\dfrac{1}{3}\sin 3x+\dfrac{\sqrt{3}}{3}\cos 3x,\quad y=-x.$

3) $y=x^3-3x+5,\quad y=-2x.$ **4)** $y=\ln(4x-1),\ y=x.$ **5)** $y=x^2,\ y=2x+5.$

6) $y=\left(3-x^2\right)e^x\quad x=0.$ 7) $y=|x-5|\cdot(x-3)^3,\quad x=0.$

74. Qanday nuqtalarda quyidagi $y=y(x)$ unksiyalar grafigiga o'tkazilgan urinmalar berilgan to'g'ri chiziqlarga perpendikulyar bo'ladi?

1) $y=\ln x,\qquad 2y+x+1=0.$ 2) $y = x^3 - 3x + 5,\ \ y = -x/9.$

3) $y=-\sqrt{2x^3},\quad 4x-3y+2=0$. **4)** $y=\sin x,\ x-10=0.$

5) $y= tgx,\ \ x+y=0$. 6) $y = x^2,\ y = 2x+5$

75. $y=f(x)$ funksiya grafigiga berilgan nuqtada o'tkazilgan urinma tenglamasini yozing:

1) $y=\operatorname{arctg}2x,\ x=0.$ 2) $y=e^x,\ x=1.$

3) $y=|x-1|\sqrt[3]{x+2}\ ,\ x=6.$ 4) $y=\sqrt{5-x^2}\ ,\ x=1.$

5) $y=4\operatorname{ctg}x-\dfrac{\cos x}{\sin^2 x},\qquad x=\dfrac{\pi}{2}.$ 6) $x^2+y^2-2x+6y=0\ ,\quad y>-3,\ \ x=0.$

7) $x=a(t-\sin t),\ y=a(1-\cos t)\ ,\ t=t_0\neq 2n\pi\ ,\ n\in Z.$

8) $x=t\,e^t\ ,\ y=t\,e^{-t}\ ,\ \ t>-1, t=t_0>-1.$

76. $s=2\sin 3t$ qonuniyat bo'yicha to'g'ri chiziqli harakat qilayotgan nuqtaning $t=\dfrac{\pi}{9}$ paytdagi tezligini toping.

77. $s=\sin^2 t$ qonuniyat bo'yicha to'g'ri chiziqli harakat qilayotgan nuqtaning $t=\dfrac{\pi}{6}$ paytdagi tezligini toping.

78. $s=e^t+\cos t+5t$ qonuniyat bo'yicha harakatlanayotgan moddiy nuqtaning

$t = 0$ dagi tezligini toping.

79. Ikki moddiy nuqta, mos ravishda, $s_1 = 2{,}5t^2 - 6t + 1$ va $s_2 = 0{,}5t^2 + 2t - 3$ qonuniyatlar bo'yicha harakatlanmoqda. Qaysi vaqtda birinchi nuqtaning tezligi ikkinchisinikidan 3 marta katta bo'ladi?

80. Moddiy nuqta $s = \ln t + \frac{1}{16}t$ qonuniyat bo'yicha to'g'ri chiziqli harakat qilmoqda. Harakat boshlangandan qancha vaqt o'tgach, nuqtaning tezligi $\frac{1}{8}$ $м/с$ bo'ladi?

81. Massasi $m = 1{,}5$ bo'lgan jism $s(t) = t^2 + t + 1$ qonuniyat bo'yicha to'g'ri chiziqli harakat qilmoqda. Jismning harakati boshlangandan 5 sekund vaqt o'tgandagi kinetik energiyasini toping (m massa kilogrammlarda, s yo'l – metrlarda berilgan).

82. Absissalar o'qi bo'ylab ikkita nuqta, mos ravishda, $x = 100 + 5t$ va $x = \frac{t^2}{2}$ qonuniyatlar bo'yicha harakat qilmoqda. Bu nuqtalar uchrashish paytida (momentida) bir-biridan qanday tezlikda uzoqlashadi? (x metrlar bilan o'lchanadi, t – sekundlar bilan).

83. G'ildirak shunday aylanadiki, uning burilish burchagi vaqtning kvadratiga proporsionaldir. Birinchi aylanish 8 sekund vaqt davomida amalga oshirildi. Harakat boshlangandan 64 sekund vaqt o'tgandagi burchak tezligini toping.

84. Ko'rsatilgan nuqtalarda quyidagi funksiyalarga teskari bo'lgan funksiyalarning hosilalarini toping:

1) $y = 2x - \frac{\cos x}{2}$, $y = -\frac{1}{2}$. 2) $y = 2x^2 - x^4$, $0 < x < 1$, $y = \frac{3}{4}$.

85. Parametrik shaklda berilgan quiydagi $y = y(x)$ funksiyalarning y'_x hosilalarini toping:

1) $x = e^{-t}$, $y = t^3$, $-\infty < t < +\infty$. 2) $x = a\cos t$, $y = b\sin t$, $0 < t < \pi$.

3) $x = \ln(1 + t^2)$, $y = t - \operatorname{arctg} t$. 4) $x = a(t - \sin t)$, $y = a(1 - \cos t)$.

5) $x = \frac{a\sin t}{1 + b\cos t}$, $y = \frac{c\cos t}{1 + b\cos t}$. 6) $x = a\cos^3 t, y = a\sin^3 t$.

7) $x = a\,\mathrm{ch}\,t,\ y = b\,\mathrm{sh}\,t.$ 8) $x = \arcsin\dfrac{t}{\sqrt{1+t^2}},\quad y = \arccos\dfrac{1}{\sqrt{1+t^2}}.$

86. Oshkormas shaklda berilgan quiydagi $y = y(x)$ funksiyalarning y'_x hosilalarini toping:

1) $x + \sqrt{xy} + y = a.$ 2) $e^x \cdot \sin y - e^{-y}\cos x = 0\ .$ 3) $y^2 = 2px,\ y > 0\ .$

4) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}},\ \ y > 0\ .$ 5) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1,\quad y > 0\ .$ 6) $e^y + xy = e,\ x = 0,\ y = 1\ .$

Quyidagi funksiyalarning differensialini toping.

87. $y = \ln x + x^2.$ **88.** $y = e^{3x} + \sqrt{x}.$ **89.** $\mathrm{y} = \cos^2 x + 3.$

90. $y = \mathrm{tg}\,4x + \dfrac{2}{x}.$ **91.** $y = \log_3 x + \sin 5x.$

92. Quyidagi funksiyalarning differensialini toping.

1) $y = \ln\ln\left(\dfrac{x}{2}\right).$ 2) $y = \cos\dfrac{1}{\log_2 x}\ .$ 3) $y = e^{\frac{\sqrt{1-x}}{1+x}}\ .$ 4) $y = x^{x^2}\ .$

5) $y = \mathrm{arc\,tg}\dfrac{\ln x}{x},\ x_0 = \dfrac{1}{e},\ x_0 = e.$

6) $y = \dfrac{x^2 \cdot 2x}{x^x},\quad x_0 = 1,\quad x_0 = 2.$ 7) $y = \arcsin^2 x + \mathrm{arctg}^3\, x.$

93. Funksiyaning orttirmasini uning differensiali bilan almashtirib, quyidagi $y = f(x)$ funksiyalarning ko'rsatilgan nuqtadagi taqribiy qiymatlarini toping:

1) $y = \sqrt[3]{x},\quad a)\, x = 65;\qquad b)\, x = 125{,}1324\ .$ 2) $y = \sqrt[4]{x},\quad a)\ x = 90;\ b)\ x = 15{,}8.$

3) $y = \mathbf{tg}x,\ x = \mathbf{44^0\ 50'}\ .$ 4) $y = \sqrt{\dfrac{2-x}{2+x}}\ ,\ x = 0{,}15\ .$

94. Funksiyaning orttirmasini uning differensiali bilan almashtirib, quyidagi ifodalarning taqribiy qiymatlarini toping:

1) $\sqrt[3]{1{,}02}.$ 2) $\sin 29^0.$ 3) $\cos 151^0.$ 4) $\lg 11.$

96. Quyidagi $y = f(x)$ funksiyalarni berilgan nuqtalarda differensiallanuvchilikka tekshiring.

1) $y = \sqrt{x^3},\ x_0 = 0.$ 2) $y = \sqrt{1-x},\ x_0 = \dfrac{1}{2}.$ 3) $y = 3x^3 + 4x^2 + 5x - 2\ ,\qquad \forall x_0 \in R.$

4) $y = x\cos\frac{1}{x}$, $x_0 = 0$. 5) $y = \begin{cases} x \cdot \sin\frac{1}{x^2}, & x \neq 0, \\ 0, & x = 0, \end{cases}$ $x_0 = 0$.

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. 1) $3x^2 - 2x$. 2) $2^x(\ln 2\cos x - \sin x)$. 3) $-5\sin 5x$. 4) $\frac{1}{\cos^2 x} + 2$. 5) $\frac{1}{3\sqrt[3]{x^2}}$.

6) $\frac{4}{4x-1}$. **2.** 1) 0. 2) 0. 3) 0. 4) ∞. **3.** 1) Mavjud emas. 2) Mavjud emas. 3) Mavjud emas. 4) $x = \frac{2k-1}{2}\pi$, $k \in Z$ nuqtada hosilaga ega emas. 5) . Hamma joyda hosilaga ega. 6) $f'(0) = 0$. 7) Mavjud emas. **4.** $6(x+1)$ **5.** $\frac{1}{\sqrt{x}} + \frac{3}{x^4}$. **6.** $-\frac{6x^2}{(1+x^3)^2}$. **7.** $60(1+2x)^{29}$. **8.** $16\left(7x^2 - \frac{4}{x} + 6\right)^7\left(7x + \frac{2}{x^2}\right)$. **9.** $12\left(t^3 - \frac{1}{t^3} + 3\right)^3\left(t^2 + \frac{1}{t^4}\right)$. **10.** $-\frac{x}{\sqrt{1-x^2}}$. **11.** $\frac{3-x}{2(1-x)^{3/2}}$. **12.** $\frac{t^2(3-t)}{(1-t)^3}$.

13. $x^2\cos x$. **14.** $\cos x\left(3\sin^2 x - 2\cos ec^3 x\right)$. **15.** $4\sin x \cdot \sec^5 x$. **16.** $\frac{x - 0{,}5\sin 2x}{x^2\cos^2 x}$. **17.** $-\sin^3 x$. **18.** $3x^2\log_4 x + \frac{x^2}{2\ln 2}$. **19.** $\ln x + 1$.

20. $\frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$. **21.** $-\frac{4x}{1-2x^2}$. **22.** $\frac{2(x-2)}{(x^2-4x)\ln 2}$. **23.** $5\ln^4(\sin x)\operatorname{ctg} x$. **24.** $3^x \ln 3$. **25.** $2^{1+\sin 2x} \cdot \cos 2x \cdot \ln 2$. **26.** $-4 \cdot 5^{\cos 4x} \cdot \sin 4x \cdot \ln 5$. **27.** $\frac{2(x - x^2\ln 2)}{4^x}$. **28.** $e^x(\cos x - \sin x)$.

29. $4x^3 - 2 \cdot 5^{2x}\ln 5$. **30.** $\frac{1}{2}\operatorname{sh} 2x$. **31.** $\operatorname{th} x$. **32.** $2\operatorname{sh} 2x$. **33.** $3\operatorname{th}^2 x \cdot \frac{1}{ch^2 x}$. **34.** $x(2\operatorname{sh} x + x\operatorname{ch} x)$. **35.** $\frac{2}{\operatorname{ch}^2 x \cdot (1 - \operatorname{th} x)^2}$. **36.** $\arcsin x + \frac{x}{\sqrt{1-x^2}}$. **37.** $-\frac{2\arccos x}{\sqrt{1-x^2}}$. **38.** $\frac{1}{2\sqrt{x}}\operatorname{arctg} x + \frac{\sqrt{x}}{1+x^2}$. **39.** $-\frac{3x^3}{\sqrt{x^2-9}}\operatorname{sign} x$. **40.**

$\frac{2x\operatorname{arctg}x+2x^3\operatorname{arctg}x-x^2}{(1+x^2)\operatorname{arctg}^2 x}$. **41.** $-\frac{x+\sqrt{1-x^2}\arccos x}{x^2\sqrt{1-x^2}}$. **42.** $\frac{x+2\sqrt{x}+1}{2x\sqrt{x}}$. **43.** $x^3\ln^2 x$. **44.** $-\frac{3\pi}{2\sin^4\pi x}$. **45.** $\frac{1}{3}e^{\sqrt[3]{x}}$. **46.** $\frac{1}{\sqrt{1-\sin^4 x}}$.

47. $\frac{x^2}{(x^2+a^2)(x+a)^2}$. **48.** $\frac{1}{\cos^3 x}$. **49.** $\frac{\ln(1+\sin x)}{\sin^2 x}$. **50.**

1) $f'_+(-3+0)=1,\ f'_-(-3-0)=-1$. 2) $f'_-(2)=f'_-(3)=-1,\quad f'_+(2)=f'_+(3)=1$.

3) $f'_+(1)=\ln 4,\ f'_-(1)=-\ln 4$. 4) $f'(2k)=+\infty,\quad f'(2k-1)=-\infty,\ k\in Z$.

51. 1) $f'_-(0)=1,\quad f'_+(0)=0$. 2) $f'(0)=0$. **52.** 1) $f'_-(0)=-\frac{1}{2},\quad f'_+(0)=\frac{1}{2}$.

2) $f'_-(0)=-\infty,\quad f'_+(0)=0$. 3) $f'_-(1)=f'_+(1)=\frac{1}{2}$. 4) $f'_+(0)=+\infty,\ f'_-(0)=+\infty$. **53.**

$f'_+(x_0)=\varphi(x_0),\ f'_-(x_0)=-\varphi(x_0)$. **54**. k=3/2. **55.** k = 1. **56.** $y=4$. **57**. $y=2$. **58.** (-1; 4). 59. $\frac{8}{15}$. **60.** (4;3), (0;-1). **61.** $\left(\frac{1}{\sqrt[4]{27}};\frac{1}{\sqrt[4]{3}}\right)$. **62.** (-1;0), (1,-2). **63.** $\varphi=\operatorname{arctg}\frac{3}{8}$.

64. 45^0. 65. $x=2\pi n$ da 45^0, $x=(2n+1)\pi$ da -45^0 $(n\in Z)$. **66**. $a=e$. 67. 1) 30^0. 2) 45^0. 3) 60^0. **68**. $\operatorname{tg}\delta=-\frac{b}{a}\operatorname{ctg}t$. **69.** $y=2x-3$. **70**. (4;3) *ва* (3;7). **71**. 1) va 3) hollarda, mos ravishda, $y+2=0$ va $y-\sqrt{2}=\frac{3}{2}(1+\sqrt{2})(x-2\sqrt{2}+2)$; 2) holda $t=1$ nuqtada $\frac{3(1-t^2)}{2(1-t)}$ funksiya aniqlanmagan. 72. 1) $(0;0),\ \varphi=\operatorname{arctg}(2/3)$.

2) $\left(\frac{\pi}{4}+\pi k;\ (-1)^k\right),(k\in Z),\varphi=\frac{\pi}{2}$.

3) $(1;1),\ \varphi=\operatorname{arctg}3$. 4) $(\sqrt{e};1/2)$, $\varphi=0$. 5) (1;1) **ва** (4; 4), $\varphi=\operatorname{arctg}\frac{6}{7}$.

6) (1; 1), $\varphi=\frac{5\pi}{4}$. 7) (3; 34), $\varphi=0$. **73**. 1) $(1;-3)$. 2) $\left(\frac{1}{3}(\frac{\pi}{3}-2k\pi);\ \frac{1}{\sqrt{3}}\right)$,

$\left(\frac{1}{3}(-\frac{2\pi}{3}-2m\pi);-\frac{1}{\sqrt{3}}\right),\ k,m\in Z$. 3) $M_1\left(-\frac{1}{\sqrt{3}};\ 5+8\frac{\sqrt{3}}{9}\right),\quad M_2\left(\frac{1}{\sqrt{3}};\ 5+8\frac{\sqrt{3}}{9}\right)$.. 4) $\left(\frac{5}{4};\ \ln 4\right)$.

5) $(1;1)$. 6) $(1;2e)$. 7) $(3;0);\ \left(\frac{9}{2};\ \frac{27}{16}\right)$. **74**. 1) $\left(\frac{1}{2};-\ln 2\right)$. 2) $M_1(-2;3),\ M_2(2;7)$.

3) $\left(\frac{1}{8};\ -1/16\right)$. 4) $\left(\frac{\pi}{2}+k\pi;1\right)$, $(k\in Z)$. 5) $(k\pi;\ 0)$, $(k\in Z)$. 6) $\left(-\frac{1}{4};\ \frac{1}{16}\right)$. **75.**

1) $2x-y=0$. 2) $ex-y=0$. 3) $29x-12y-54=0$. 4) $x+2y-5=0$.

5) $y=-3x+\frac{3\pi}{2}$. 6) $x-3y=0$.

7) $y=\left(\operatorname{ctg}\left(\frac{t_0}{2}\right)\right)x+2a-at_0\operatorname{ctg}(t_0/2)$. 8) $(1-t_0)e^{-t0}\,x-(1+t_0)e^{t0}\,y+2t_0^2=0$. **76.** $3\sqrt{3}$ (m/c). **77.**

$\frac{\sqrt{3}}{2}(m/c)$. **78.** 6 (m/s). **79.** 6 (s). **80.** 16 (s). **81.** 90,75 Jaul . **82.**

15 $\frac{\textbf{м}}{\textbf{сек}}$. **83.** $4\pi\cdot$ **рад/сек.** **84.** 1) $x'\left(-\frac{1}{2}\right)=\frac{1}{2}$. 2) $x'\left(\frac{3}{4}\right)=\frac{\sqrt{2}}{2}$. **85.** 1) $y'_x=-3t^2e^t$.

2) $y'_x=-\frac{b}{a}\operatorname{ctg}t$. 3) $y'_x=\frac{t}{2}$. 4) $y'_x=\operatorname{ctg}\frac{t}{2}\ (t\neq 2k\pi, k\in Z)$. 5) $y'_x=\frac{-c\sin t}{a(b+\cos t)}$.

6) $y'_x=-\operatorname{tg}t\ \left(t\neq\frac{2k+1}{2}\pi,\ k\in Z\right)$. 7) $y'_x=\frac{b}{a}\operatorname{cth}t\ (|t|>0)$. 8) $y'_x=\operatorname{sign}t\ (0<|t|<+\infty)$.

86. 1) $y'_x=\frac{2a-2x-y}{x+2y-2a}$. 2) $y'_x=\frac{e^x\sin y+e^{-y}\sin x}{e^x\cos y+e^{-y}\cos x}$. 3) $y'_x=\frac{p}{y}$.

4) $y'_x=-\sqrt[3]{\frac{y}{x}}$, $|x|<a$. 5) $y'_x=\frac{b^2x}{a^2y}$, $|x|>a$. 6) $y'_x(0)=-\frac{1}{e}$. 7) $y'_x=-\frac{b^2x}{a^2y}$. 8) $y'_x=\frac{x+y}{x-y}$.

9) $y'_x(0)=\frac{1}{\sqrt{3}}$. 10) $y'_x=\frac{4y-2x-4}{8y-4x-3}$. **87.** $\left(\frac{1}{x}+2x\right)dx$. **88.** $\left(3e^{3x}+\frac{1}{2\sqrt{x}}\right)dx$. **89.**

$-\sin 2x\cdot dx$. **90.** $(\frac{4}{\cos^2 4x}-\frac{2}{x^2})dx$. **91.** $\left(\frac{1}{x\ln 3}+5\cos 5x\right)dx$.

92. 1) $\frac{dx}{x\ln\frac{x}{2}}$, $x>2$. 2) $\frac{\sin\left(\frac{1}{\log_2 x}\right)}{\left(x\log\frac{2}{2}x\right)\ln 2}dx$. 3) $-\frac{1}{\sqrt{1+x}\sqrt{1-x^2}}e^{\sqrt{\frac{1-x}{1+x}}}\,dx$.

4) $x^{1+x^2}(1+2\ln x)dx$. 5) $\frac{2e^2dx}{e^2+1}$; 0. 6) $(2+\ln 4)dx$; 0. 7) $\left(\frac{\arcsin x}{\sqrt{1-x^2}}+\frac{3\ arctg^2 x}{1+x^2}\right)dx$. **93.**

1) *a*) 4,0208 ; *b*) 5,00177. 2) *a*) 3,083 ; *b*) 1,9938. 3) 0,9942 . 4) 0,925.

94. 1) 1,007. 2) 0,4849. 3) $-0{,}8748$. 4) 1,043. **95.** 1) $\frac{12}{11}dx$. 2) $\frac{y_0^2-4x_0^3}{5y_0^4-2x_0y_0}dx$. 3) $-\frac{11}{20}dx$.

4) 0 . 5) $\frac{1}{2}dx$. 6) $\frac{1}{8}dx$. **96**. 1) Differensiallanuvchi; 2) Differensiallanuvchi; 3) Differensiallanuvchi; 4)Differensiallanuvchi emas; 5) Differensiallanuvchi emas.

14- amaliy mashg'ulot.

FUNKSIYANING YUQORI TARTIBLI HOSILASI VA DIFFERENSIALI

Quyidagi misollarning ko'rsatilgan tartibdagi hosilalarini toping.

1. $y = x(2x-1)^2(x+3)^3, \quad y^{(7)} = ?$

2. $y = \frac{1}{1+x^3}, \quad y'' = ?$

3. $y = 1 + 10x + \frac{1}{x^{98}}, \quad y'' = ?$

4. $y = \cos^2 x, \quad y'' = ?$

5. $y = (1+x^2)\operatorname{arctg} x, \quad y'' = ?$

6. $y = \operatorname{arctg}(x + \sqrt{x^2+1}), \quad y'' = ?$

7. $y = \sqrt{1-x^2}\arcsin x, \quad y'' = ?$

8. $y = \ln(x + \sqrt{x^2+1}), \quad y'' = ?$

9. $y = \sqrt[5]{x^3}, \quad y''' = ?$

10. $y = x^5 \ln x, \quad y''' = ?$

Quyidagi funksiyalarning berilgan nuqtalardagi ko'rsatilgan tartibdagi hosilalarini toping.

11. $y = x^6 - 4x^3 + 4, \quad y^{(IV)}(1) = ?$

12. $y = \frac{x^5}{(x-1)^4}, \quad y^{(/)}(5) = ?$

13. $y = \frac{\arcsin x}{\sqrt{1+x^2}}, \quad y^{(/)}(0) = ?$

14. $y = \operatorname{arctg} x, \quad y^{(/)}(1) = ?$

15. $y = e^{\sqrt{x}}, \quad y^{(/)}(4) = ?$

16. $y = \frac{1+x}{\sqrt{1-x}}, \quad y^{(100)}(x) = ?$

Quyidagi funksiyalarning ko'rsatilgan tenglamalarni qanoatlantirishini isbotlang.

17. $y = e^x \sin x, \quad y'' - 2y' + 2y = 0;$

18. $y = e^{-x} \sin x, \quad y'' + 2y' + 2y = 0;$

19. $y = c_1 \cos x + c_2 \sin x, \quad y'' + y = 0$ (c_1 va c_2 - ixtiyoriy o'zgarmas sonlar);

20. $y = c_1 chx + c_2 shx, \quad y'' - y = 0$ (c_1 va c_2 - ixtiyoriy o'zgarmas sonlar);

Quyidagi parametrik shaklda berilgan $y = y(x)$ funksiyalarning ko'rsatilgan tartibdagi hosilalarini toping:

21. $x = at^2, \quad y = bt^3; \quad x''_{yy} = ?$

22. $x = t^3 + 3t + 1, \quad y = t^3 - 3t + 1; \quad y''_{xx} = ?$

23. $x = e^{\alpha t} \cos \beta t, \quad y = e^{\alpha t} \sin \beta t; \quad y''_{xx} = ?$

24. $x = a(\cos t - \ln ctg \frac{t}{2}), \quad y = a \sin t; \quad y'''_{xxx} = ?$

Quyidagi parametrik shaklda berilgan $y = y(x)$ funksiyalarning berilgan nuqtada ko'rsatilgan tartibdagi hosilalarini toping:

25. $x = \ln(1 + \sin \varphi), \quad y = \ln(1 - \cos 2\varphi); \quad (\ln(\frac{3}{2}); \ln(\frac{1}{2})); \quad y''_{xx} = ?$

26. $x = \mathrm{ch}\, t \sin t + \mathrm{sh}\, t \cos t, \quad y = \mathrm{ch}\, t \cos t - \mathrm{sh}\, t \sin t; \quad (0;1); \quad y''_{xx} = ?$

Parametrik shaklda berilgan $y = y(x)$ funksiyalarning berilgan tenglamalarni qanoatlantirishini isbotlang:

27. $x = e^t \sin t, \quad y = e^t \cos t; \quad y''(t + y)^2 = 2(xy' - y).$

28. $x = \sin t, \quad y = \sin kt; \quad (1 - x^2)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} + k^2 y = 0.$

29. $x = \sin t, \quad y = Ae^{\sqrt{2}t} + Be^{\sqrt{2}t}; \quad (1 - x^2)y'' - xy' - 2y = 0, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$, A va B - ixtiyoriy o'zgarmas sonlar;

Quyidagi oshkormas shaklda berilgan $y = y(x)$ funksiyalarning x bo'yicha ko'rsatilgan tartibdagi hosilalarini toping:

30. $y^2 = 2px, \quad y''_{xx} = ?$ **31.** $e^{x-y} = x + y, \quad y''_{xx} = ?$

32. $\mathrm{arctg}\, y - y + x = 0, \quad y''_{xx} = ?$ **33.** $e^x - e^y = y - x, \quad y''_{xx} = ?$

34. $x^2 + 5xy + y^2 - 2x + y - 6 = 0, \quad (1;1)$ nuqtadagi $y''_{xx} = ?$

Quyidagi funksiyalarning $x = 0$ nuqtada nechanchi tartibli hosilalarga ega ekanligini aniqlang va mavjud hosilalarning bu nuqtadagi qiymatini hisoblang:

35. $y=\begin{cases}1-\cos x, & x<0 \text{ bo'lganda},\\ \ln(1+x)-x, & x\geq 0 \text{ bo'lganda}.\end{cases}$

36. $y=\begin{cases}shx-x, & x<0 \text{ bo'lganda},\\ x-\sin x, & x\geq 0 \text{ bo'lganda}.\end{cases}$

37. $y=\begin{cases}shx, & x<0 \text{ bo'lganda},\\ \sin xchx, & x\geq 0 \text{ bo'lganda}.\end{cases}$

x ni erkli o'zgaruvchi deb, quyidagi $y=y(x)$ funksiyalarning ko'rsatilgan tartibdagi differensiallarini toping:

38. $y=(x+1)^3(x-1)^2,\ d^2y=?$ **39.** $y=(x^3+2x^2+x+3)e^{-2x},\ d^2y=?$

40. $y=\sin^2 x,\ d^4y=?$ **41.** $y=x\cos 2x,\ d^{12}y=?$

42. $y=\operatorname{arctg}\left(\frac{b}{a}tgx\right);\ d^2y=?$ **43.** $y=\cos xchx;\ d^8y=?$

Agar du, d^2u, dv, dv^2 lar mavjud bo'lsa, quyidagi $y=y(x)$ funksiyalar uchun d^2y ni toping.

44. $y=\sqrt{u^2+v^2};\ d^2y=?$ **45.** $y=u^v;\ d^2y=?$

46. $y=\frac{2u+v}{u};\ d^2y=?$ **47.** $y=u\ln v;\ d^2y=?$

Quyidagi $y=y(x)$ funksiyalarning berilgan nuqtadagi ko'rsatilgan tartibdagi differensiallarini toping:

48. $y=xe^{x^2};\ d^2y\big|_{x=1}=?$ **49.** $y=\cos^2 x;\ d^3y\Big|_{x=\frac{\pi}{4}}=?$

50. $y=x\sqrt[3]{(x-5)^2};\ d^2y\big|_{x=-3}=?$ **51.** $y=\frac{1}{ax+b};\ d^ny\big|_{x=0}=?$

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. 0 . **2.** $\frac{6x(2x^3-1)}{(x^3+1)^3}$. **3.** $9702/x^{100}$. **4.** $-2\cos 2x$. **5.** $\frac{2x}{1+x^2}+2arctgx$.

6. $-x(1+x^2)^{-2}$. **7.** $-\dfrac{\arcsin x + x\sqrt{1-x^2}}{\sqrt{(1-x^2)^3}}$. **8.** $-x(x^2+1)^{-3/2}$. **9.** $\dfrac{42}{125}x^{-\frac{12}{5}}$. **10.** $x^2(60\ln x + 47)$. **11.** 360. **12.** $\dfrac{625}{1024}$. **13.** 0. **14.** $-\dfrac{1}{2}$. **15.** $e^{2/32}$.

16. $\dfrac{1\cdot 3\cdot 5\cdot \ldots \cdot 197(399-x)}{2^{100}(1-x)^{201/2}}$. **21.** $-\dfrac{2a}{9b^2t^4}$. **22.** $\dfrac{4t}{3(t^2+1)^3}$. **23.** $\dfrac{(\alpha^2+\beta^2)\beta e^{-\alpha t}}{(\alpha\cos\beta t - \beta\sin\beta t)^3}$. **24.** $\dfrac{\sin t(1+3\sin^2 t)}{a^2\cos^7 t}$. **25.** -12. **26.** $-\dfrac{1}{2}$. **30.** $-p^2/y^3$. **31.** $4(x+y)/(x+y+1)^3$.

32. $-\dfrac{2(1+y^2)}{y^5}$. **33.** $(e^x - e^y)(1-e^{x+y})/(1+e^y)^3$. **34.** $\dfrac{111}{256}$. **35.** $y'(0)=0,\ y''(0)$ mavjud emas. **36.** $y'(0)=0,\ y''(0)=0,\ y'''(0)=1,\ y^{(IV)}(0)=0,\ y^{(V)}(0)$ mavjud emas. **37.** $y'(0)=1,\ y''(0)=0,\ y'''(0)$ mavjud emas. **38.** $4(x+1)(5x^2-2x-1)dx^2$. **39.** $2(2x^3-2x^2-3x+6)e^{-2x}dx^2$. **40.** $-8\cos 2x dx^4$. **41.** $4096(6\sin 2x + x\cos 2x)dx^{12}$.

42. $\dfrac{ab(a^2-b^2)\sin 2x}{(a^2\cos^2 x + b^2\sin^2 x)}dx^2$. **43.** $17\cos x chx dx^8$. **44.** $\dfrac{(u^2-v^2)(ud^2u+vd^2v)+(vdu-udv)^2}{(u^2-v^2)^{3/2}}$. **45.** $u^v\left(\dfrac{v}{u}d^2u + \ln u d^2v + \dfrac{v(v-1)}{u^2}du^2 + \dfrac{2(v\ln u+1)}{u}dudv + \ln^2 u dv^2\right)$.

46. $\dfrac{1}{u^3}\left(u^2d^2v - uvd^2u - 2ududv + 2vudu^2\right)$. **47.** $\ln v d^2u + \dfrac{2}{v}dudv + \dfrac{u}{v}d^2v - \dfrac{u}{v^2}dv^2$.

48. $10edx^2$. **49.** $4dx^3$. **50.** $-\dfrac{5}{8}dx^2$. **51.** $\dfrac{(-1)^n a^n n!}{b^{n+1}}dx^n$.

15- amaliy mashg'ulot.

FUNKSIYANI HOSILA YORDAMIDA TEKSHIRISH

Quyidagi funksiyalarni monotonlikka tekshiring.

1. $y=3x-x^2$**.** **2.** $y=\dfrac{\sqrt{x}}{x+100}\ (x\ge 0)$**.** **3.** $y=x+\sin x$**.**

4. $y=x^2-\ln x^2$**.** **5.** $y=x^2e^{-x}$**.** **6.** $y=\dfrac{1-x+x^2}{1+x+x^2}$.

7. Quyidagi funksiyalarning o'suvchi va kamayuvchi bo'lish oraliqlarini toping:

1) $y=\dfrac{\sin x+\cos x}{1+|\cos x|}$. 2) $y=(x-2)^5(2x+1)^4$.

3) $y=\sqrt[3]{(2x-a)(a-x)^2}$. 4) $y=\dfrac{2x}{1+2x}$.

5) $y=x-e^x$. 6) $y=x-2\sin x\ (0\le x\le 2\pi)$.

8. Ushbu 1) $y=\dfrac{a^2-1}{3}x^3+(a-1)x^2+2x$; 2) $y=ax+3\sin x+4\cos x$ funksiyalar a ning qanday qiymatlarida o'suvchi bo'ladi.

Quyidagi funksiyalarni ekstremumga tekshiring.

9. $y=2+x-x^2$. **10.** $y=(x-1)^3$.

11. $y=\dfrac{3}{4}x^4+x^3-9x^2+7$. **12.** $y=x^4e^{-x^2}$.

13. $y=2\sin x+\cos 2x$. **14.** $y=\dfrac{3x^2+4x+4}{x^2+x+1}$ **.**

15. $y=\sin x+\dfrac{1}{2}\sin 2x$. **16.** $y=(x^2-2x)\ln x-\dfrac{3}{2}x^2+4x$.

Quyidagi funksiyalarning ko'rsatilgan oraliqlarda eng katta va eng kichik qiymatlarini toping.

17. $y = 2x^3 - 3x^{25} - 12x + 1, \quad x \in [-2;2,5]$. **18.** $y = x + \sqrt{x}, \quad x \in [0;4]$.

19. $y = x^3 - 3x^2 + 1, \quad x \in [-1;4]$. **20.** $y = \operatorname{arctg} x - \frac{1}{2}\ln x, \; x \in \left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$.

21. $y = 2\sin x + \sin 2x, \; x \in \left[0; \frac{3}{2}\pi\right]$. **22.** $y = x - 2\ln 2, \; x \in [1; e]$.

23. $y = -\frac{1}{3}x^3 - \frac{1}{6}x$ funksiyaning $x \in [-1; 1]$ kesmadagi eng katta va eng kichik qiymatlari yig'indisini hisoblang.

Quyidagi funksiyalar grafigining qavariqlik va botiqlik oraliqlarini toping.

24. $y = x^4 + x^3 - 18x^2 + 24x - 12$. 25. $y = x + x^{5/3}$.

26. $y = x + \sin x$. **27.** $y = 2 - \left|x^5 - 1\right|$

28. $y = 3x^4 - 4x^3 + 1$. **29** $y = x^\alpha, \alpha > 1, x > 0$.

Quyidagi funksiyalar grafigining egilish nuqtalarini toping.

30. $y = x + 36x^2 - 2x^3 - x^4$. **31.** $y = 1 + x^2 - \frac{x^4}{2}$.

32. $y = 3x^4 - 8x^3 + 6x^2 - 12$. **33.** $y = \frac{x+1}{x^2+1}$.

34. a parametrning qanday qiymatlarida $f(x) = ax^3 + e^x$ funksiya egilish nuqtasiga ega bo'ladi.

Quyidagi funksiyalarni to'liq tekshiring va ularning grafigini chizing.

35. $y = \frac{x^4}{(1+x)^3}$. **36.** $y = \sqrt[3]{x^2} - \sqrt[3]{x^2 - 4}$. **37.** $y = x^2 \ln(x+2)$.

38. $y = x^3 e^{-4x}$. **39.** $y = \frac{1}{3}\sqrt{(2x+1)^3} + 4\sqrt{x}$. **40.** $y = \frac{x^2\sqrt{x^2-1}}{2x^2-1}$.

41. $y = |x|\sqrt{1-x^2}$. **42.** $y = (x^2-2)e^{-2x}$.

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. $\left(-\infty; \frac{1}{2}\right)$ da funksiya o'suvchi, $\left(\frac{1}{2}; \infty\right)$ da esa funksiya kamayuvchi.

2. $(0;\ 100)$ da funksiya o'suvchi, $(100;\infty)$ da esa, funksiya kamayuvchi. **3.** R da funksiya o'suvchi. 4. $(-\infty;-1)\cup(0;1)$ da funksiya kamayuvchi, $(-1;0)\cup(1;\infty)$ da esa, funksiya o'suvchi. **5**. $(-\infty;0)\cup(2;\infty)$ da funksiya kamayuvchi, $(0;2)$ da funksiya o'suvchi. **6**. $(-\infty;\ -1)\cup(1;\ +\infty)$ da funksiya o'suvchi, $(-1;\ 1)$ da esa, funksiya kamayuvchi. **7.** 1) $\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2\pi k\right), k\in Z$ da funksiya o'suvchi, $\left(\frac{\pi}{2}+2k\pi;\frac{3\pi}{2}+2k\pi\right), k\in Z$ da esa funksiya kamayuvchi. 2) $\left(-\infty;-\frac{1}{2}\right)$ da o'suvchi; $\left(-\frac{1}{2},\frac{11}{18}\right)$ da kamayuvchi, $\left(\frac{11}{18};\infty\right)$ da o'suvchi. 3) $\left(-\infty;-\frac{2}{3}\right)$da o'suvchi; $\left(\frac{2}{3}a, a\right)$ da kamayuvchi, $(a;\infty)$ da o'suvchi. 4) $(-\infty;-1)$da kamayuvchi; $(-1,1)$ da o'suvchi; $(1;\infty)$ da kamayuvchi. 5) $(-\infty;0)$ da o'suvchi; $(0,\infty)$ da kamayuvchi. 6) $\left(-\infty;\frac{\pi}{3}\right)$ kamayuvchi; $\left(\frac{\pi}{3},\frac{5\pi}{3}\right)$ da o'suvchi ; $\left(\frac{5\pi}{3},2\pi\right)$da kamayuvchi. **8**. 1) $a\le 3,\ a\ge 1$; 2) $a\ge 5$.**9**. $y_{\max}\left(\frac{1}{2}\right) = 2\frac{1}{4}$. **10**. Ekstremumga ega emas. **11.** $y_{\min}(-2) = -9,\ y_{\min}(3) = -40{,}5,\ y_{\max}(0) = 7$. **12.** $y_{\max}\left(\pm\sqrt{2}\right) = 4e^{-2},\ y_{\min}(0) = 0$.

13. $y_{\max}\left(\frac{\pi}{6}\right) = \frac{3}{2},\quad y_{\max}\left(\frac{5\pi}{6}\right) = \frac{3}{2},\quad y_{\min}\left(\frac{\pi}{2}\right) = 1,\ y_{\max}\left(\frac{\pi}{2}\right) = 1,\ y_{\min}\left(\frac{3\pi}{2}\right) = -3.$

14. $y_{\max}(0)=4,\ \ y_{\max}(-2)=\frac{2}{3}$. **15.** $y_{\min}\left(2\pi k-\frac{\pi}{3}\right)=-\frac{3\sqrt{3}}{4}, k\in Z$,

$y_{\max}\left(2\pi k+\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{4}, k\in Z$. **16.** $y_{\max}(1)=2\frac{1}{2},\ \ y_{\min}(e)=\frac{e(4-e)}{2}$.

17. $y_{eng\ katta}(-1)=8,\ y_{eng\ kichik}(2)=19.$

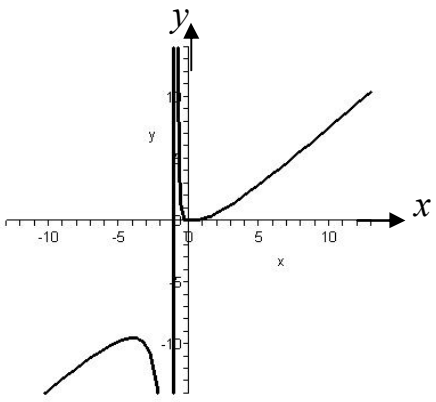
18. $y_{eng\ katta}(4)=6,\ y_{eng\ kichik}(0)=0.$ **19.** $y_{eng\ katta}(4)=17,\ y_{eng\ kichik}(2)=y_{eng\ kichik}(-1)=-3.$

20. $y_{eng\ katta}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}+0{,}25\cdot\ln 3,\ \ y_{eng\ kichik}(\sqrt{3})=\frac{\pi}{6}-0{,}25\cdot\ln 3.$ **21.**

$y_{eng\ katta}\left(\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{2},\ \ y_{eng\ kichik}\left(\frac{3\pi}{2}\right)=-2.$ **22.** $y_{eng\ katta}(1)=1,\ y_{eng\ kichik}(2)=2(1-\ln 2).$ **16.37**.Eng kattasi yo'q, $y_{eng\ kichik}(0)=1.$ **23**. 0 . **24**. $(-\infty;-2)\cup\left(\frac{3}{2};\infty\right)$ da qavariq; $\left(-2;\frac{3}{2}\right)$da botiq. **25**. $(-\infty;0)$da botiq; $(0;\infty)$ da qavariq. **26**. $(2\pi k,(2k+1)\pi), k\in Z$ da botiq; $((2k+1)\pi,(2k+2)\pi), k\in Z$ da qavariq. **27**. $(-\infty;0)\cup(1;\infty)$ da botiq; $(0;1)$ da qavariq. **28**. $(-\infty;0)\cup\left(\frac{2}{3};+\infty\right)$ da qavariq; $\left(0;\frac{2}{3}\right)$ da botiq**. 29.** Qavariq. **30**. $(-3;\ 294)$. **31**. $\left(\frac{1}{\sqrt{3}};\ \frac{23}{18}\right),\left(-\frac{1}{\sqrt{3}};\ \frac{23}{18}\right)$. **32.** $\left(\frac{1}{3};\ 12\frac{11}{27}\right),\ (1;\ 13)$ **. 33.** $\left(-2-\sqrt{3};\frac{-\sqrt{3}-1}{4}\right)$**,** $\left(-2+\sqrt{3};\frac{\sqrt{3}+1}{4}\right),\ (1;1)$**.** **34**. $a\in\left(-\infty;-\frac{e}{6}\right),\ a\in(0;+\infty)$. **35.** Funksiyaning aniqlanish sohasi: $(-\infty;-1)\cup(-1;+\infty)$. x = -1- vertikal asimptota, $y=x-3$ og'ma asimptota. $y_{\min}(0)=0,\ y_{\max}(-4)=-\frac{256}{27}$. $\left(-6;-\frac{3296}{125}\right)$ va $\left(2;\frac{16}{27}\right)$nuqtalar egilish nuqtalari (1-chizma). **36**. R da aniqlangan, juft funksiya. Grafik Oy o'qiga nisbatan simmetrik, $y=0$ - gorizontal asimptota. $y_{\min}(0)=\sqrt[3]{4},\ y_{\max}(\pm\sqrt{2})=2\sqrt[3]{2}.$ $\left(2;\sqrt[3]{4}\right),\ \left(-2;\sqrt[3]{4}\right)$- egilish nuqtalari (2-chizma). **37.** Funksiya $(-2;+\infty)$ oraliqda aniqlangan. x = -2 vertikal asimptota. $y_{\min}(0)=0,\ \ y_{\max}(-0{,}73)\approx 0{,}12.$ $(-0{,}37;0{,}075)$ – egilish nuqtasi (3-chizma).**38.** Funksiya R

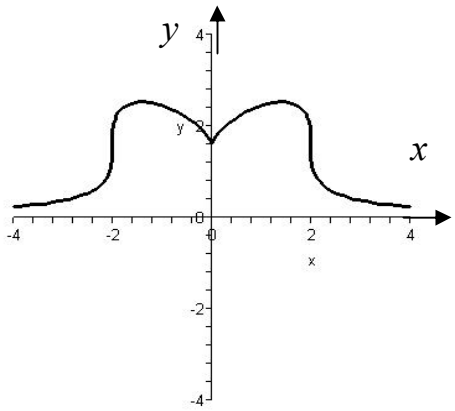
da aniqlangan, $x \to +\infty$ da $y = 0$- gorizontal asimptota. $y_{\max}\left(\frac{3}{4}\right) = \left(\frac{3}{4e}\right)^3$. Egilish nuqtalari: $(0;\ 0)$, $\left(\frac{3-\sqrt{3}}{4}; \left(\frac{3-\sqrt{3}}{4}\right)^3 e^{\sqrt{3}-3}\right)$, $\left(\frac{3+\sqrt{3}}{4}; \left(\frac{3+\sqrt{3}}{4}\right)^3 e^{-\sqrt{3}-3}\right)$. (4-chizma). **39.** Funksiya $x \geq 0$ da aniqlangan, ordinata o'qi bilan esa $\left(0; \frac{1}{3}\right)$ nuqtada kesishadi; funksiya qa'tiy o'suvchi; $\left(\frac{\sqrt{5}+1}{2}; \approx 8\right)$- egilish nuqtasi (5-chizma). **40**. Funksiya $|x| \geq 1$ da aniqlangan; ordinata o'qiga nisbatan simmetrik; ordinata o'qi bilan kesishish nuqtalari: $(1;\ 0), (-1;\ 0)$; $x \to +\infty$ da $y = \frac{x}{2}$ va $x \to -\infty$ da $y = -\frac{x}{2}$ asimptotalari; $(-\infty; -1)$ da kamayuvchi $(1; +\infty)$ da o'suvchi (6-chizma). **41.** Funksiya $|x| \leq 1$ da aniqlangan; ordinata o'qiga nisbatan simmetrik; o'qlar bilan kesishish nuqtalari: $(-1;\ 0); (0; 0), (1;\ 0)$; $y_{\min}(0) = 0,\ y_{\max}\left(\pm\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$ (7-chizma). **42.** Funksiya R da aniqlangan, koordinata o'qlari bilan kesishish nuqtalari: $\left(-\sqrt{2};\ 0\right), \left(\sqrt{2};\ 0\right), (0;\ -2)$; $x \to +\infty$ da $y = 0$ asimptota; $y_{\min}(-1) \approx -7{,}4$, $y_{\max}(2) \approx 0{,}04$; funksiyaning egilish nuqtalarining abssissalari: $x = 1 - \sqrt{10}/2 \approx -0{,}6$, $x = 1 + \sqrt{10}/2 \approx 2{,}6$. (8-chizma).

35.



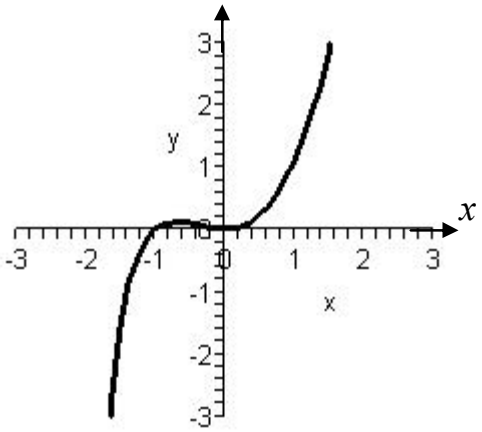
1- chizma.

36.



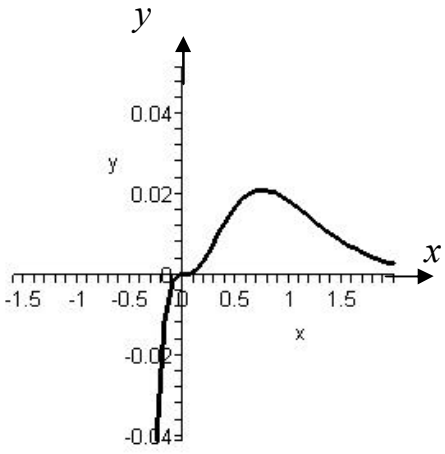
2- chizma.

37.



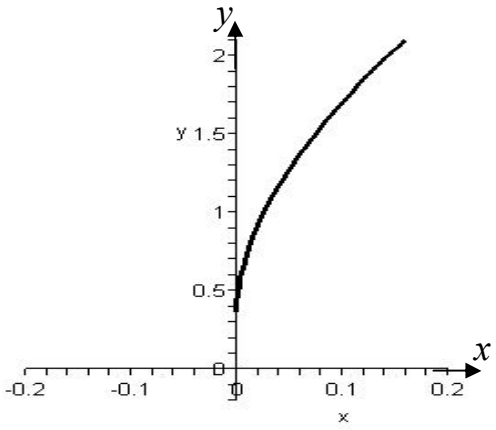
3- chizma .

38.



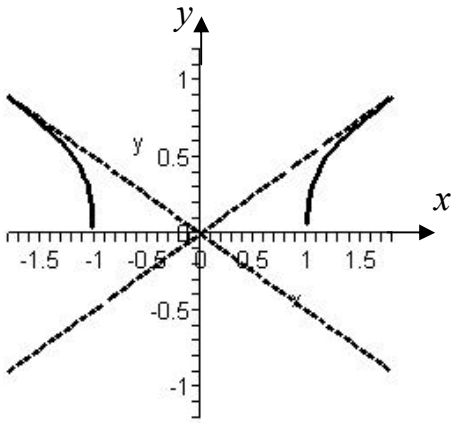
4- chizma.

39.



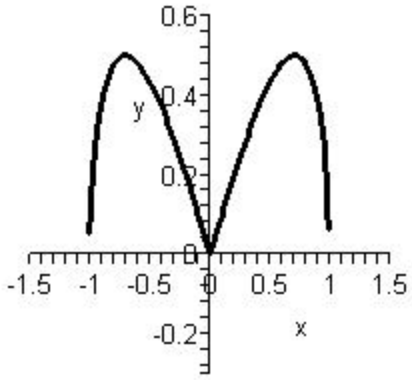
5- chizma.

40.



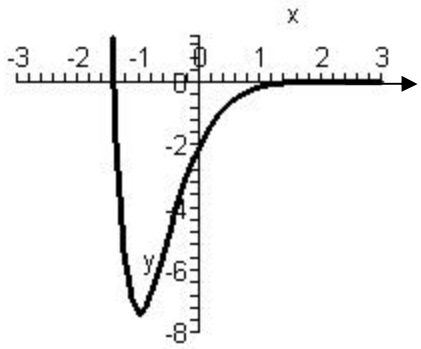
6- chizma.

41.



7- chizma.

42.



8- chizma.

16- amaliy mashg'ulot.

LOPITAL QOIDALARI. TEYLOR FORMULASI

Lopital qoidalaridan foydalanib, quyidagi funksiyalarning limitini hisoblang

1. $\lim\limits_{x\to 1}\dfrac{3x^2+5x-8}{4x^2+3x-7}$.

2. $\lim\limits_{x\to 4}\dfrac{\ln(x^2-15)}{3x^2-10x-8}$.

3. $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{-2ax}}{\ln(1+x)}$.

4. $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{1+2x}+1}{\sqrt{2+x}+x}$.

5. $\lim\limits_{x\to 0}\dfrac{\ln\sin 2x}{\ln\sin x}$.

6. $\lim\limits_{x\to 0}\dfrac{\ln x}{\ln\sin x}$.

7. $\lim\limits_{x\to 0}\dfrac{e^{1/x^2}-1}{\operatorname{arctg} x^2-\pi}$.

8. $\lim\limits_{x\to 0}\dfrac{\ln\cos 2x}{\ln\cos 5x}$.

9. $\lim\limits_{x\to 1}\dfrac{a^{\ln x}-x}{\ln x}$.

10. $\lim\limits_{x\to 1}\dfrac{\operatorname{tg} x-x}{x-\sin x}$.

11. $\lim\limits_{x\to 0}\dfrac{x\cdot\operatorname{ctg} x-1}{x^2}$.

12. $\lim\limits_{x\to 0}\dfrac{\sin ax-\sin bx}{\operatorname{sh} ax-\operatorname{sh} bx}$.

13. $\lim\limits_{x\to 0}\dfrac{3\operatorname{tg}4x-12\operatorname{tg}x}{3\sin 4x-12\sin x}$.

14. $\lim\limits_{x\to 1}\dfrac{x^{\alpha}-1}{x^{\beta}-1}$, $\beta\neq 0$.

15. $\lim\limits_{x\to+\infty}\dfrac{x^3}{e^x}$.

16. Quyidagi limitlarni Lopital qoidasi bo'yicha hisoblash mumkin emasligini ko'rsating va ularning limitini hisoblang:

1). $\lim\limits_{x\to 0}\dfrac{x^3\sin(1/x)}{\sin^2 x}$; 2). $\lim\limits_{x\to\infty}\dfrac{x+\cos x}{x-\cos x}$.

17. $\lim\limits_{x\to\infty}\dfrac{2+2x+\sin 2x}{(2x+\sin 2x)e^{\sin x}}$ limitni hisoblashga Lopital qoidasini qo'llash mumkinmi, agar limit mavjud bo'lsa, uni hisoblang.

Quyidagi ko'phadlarni $x-x_0$ ning manfiy bo'lmagan darajalari bo'yicha Teylor formulasiga yoying:

18. $P(x)=1+3x+5x^2-2x^3,\quad x_0=-1.$

19. $P(x)=x^4-4x^3+7x^2-5x+3,\quad x_0=2.$

20. $P(x)=x^9+3x^4+2x-2,\quad x_0=1.$

Quyidagi funksiyalarni x ning manfiy bo'lmagan darajalari bo'yicha ko'rsatilgan tartibgacha Makloren formulasiga yoying:

21. $f(x)=\dfrac{(1+x)^{100}}{(1-2x)^{40}\cdot(1+2x)^{60}}$, $o(x^2)$ hadgacha.

22. $f(x)=e^{\sqrt{1+2x}}$, $o(x^2)$ hadgacha .

23. $f(x)=xe^x$, $o(x^3)$ hadgacha.

24. $f(x)=\dfrac{e^x+e^{-x}}{2}$, $o(x^4)$ hadgacha.

Teylor formulasidan foydalanib, quyidagi limitlarni toping.

25. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+3x}-\sqrt{1+2x}}{x^2}$.

26. $\lim\limits_{x\to 0}\dfrac{e^x-(1+x)}{x}$.

27. $\lim\limits_{x\to 0}\dfrac{\cos x-1+\dfrac{x^2}{2}}{x^4}$.

28. $\lim\limits_{x\to\infty}x^2\left(e^{-\frac{1}{x^2}}-1\right)$.

29. $\lim\limits_{y\to 0}\dfrac{1-\cos y-\dfrac{y^2}{2}}{y^4}$.

30. $\lim\limits_{x\to 0}\dfrac{\ln(1+x)-x}{x^2}$.

31. $\lim\limits_{t\to\infty}(t+1)\cdot\sin\dfrac{1}{t+1}$.

32. $\lim\limits_{x\to 0}\dfrac{\cos x-e^{-x^2/2}}{x^4}$.

Mustaqil yechish uchun berilgan misol va masalalarning javoblari

1. 1. **2.** $\frac{4}{7}$. **3.** $3a$. **4.** $\frac{4}{9}$. **5.** 1. **6.** 1. **7.** 0. **8.** $\frac{4}{25}$. **9.** $\ln a - 1$. **10.** 2. **11.** $-\frac{1}{3}$. **12.** 1.

13. -2. **14.** $\frac{\alpha}{\beta}$. **15.** 0. **17.** Lopital qoidasini qo'llash mumkin emas, limit mavjud emas. **18.** $5-13(x+1)+11(x+1)^2-2(x+1)^3$. **19.** $5+7(x-2)+7(x-2)^2+4(x-2)^3+(x-2)^4$. **20.** $2-3(x-1)+(x-1)^2+15(x-1)^3+25(x-1)^4+\frac{7}{3}(x-1)^5+7(x-1)^6+(x-1)^7$.

21. $1+60x+195x^2+o(x^2)$. **22.** $e+ex+o(x^2)$. **23.** $x+\frac{x^2}{1!}+\frac{x^3}{2!}+o(x^3)$. **24.** $1+\frac{x^2}{2!}+\frac{x^4}{4!}+o(x^4)$. **25.** $-\frac{1}{2}$ **26.** $\frac{1}{2}$ **27.** $\frac{1}{24}$ **28.** -1 **29.** $\frac{1}{4}$.

30. $-\frac{1}{2}$. **31.** 0. **32.** $-\frac{1}{12}$.

17- amaliy mashg'ulot.

BOSHLANG'ICH FUNKSIYA VA ANIQMAS INTEGRAL

Quyidagi funksiyalarning bitta boshlang'ich funksiyasini toping.

1. $f(x)=5x^4$. **2.** $f(x)=\cos 2x$. **3.** $f(x)=e^{1-3x}$.

4. 4) $f(x)=ctg3x$. **5.** $f(x)=\frac{1}{\cos^2 3x}+2^{3x}$. **6.** $f(x)=\sqrt{x}+\sqrt[3]{x}$.

Berilgan oraliqda $F(x)$ funksiya $f(x)$ funksiyaning boshlang'ich funksiyasi ekanligini ko'rsating va aniqmas integralini yozing:

7. $f(x)=2-\frac{3}{\cos^2 3x}$. **8.** $f(x)=ctg^2 5x$.

9. $f(x)=3(3x+5)^4$. **10.** $f(x)=\sin 2x\cdot\cos x$.

11. $f(x)=\left(\dfrac{\sin 2x-2\sin^2 x}{1-tgx}\right)^2$. **12.** $f(x)=\ln^{-1}x-\ln^{-2}x$.

$f(x)$ funksiyaning, grafigi berilgan $A(x_0,y_0)$ nuqta orqali o'tadigan boshlang'ich funksiyasini toping.

13. $f(x)=x^2,\ A(3;2).$ **14.** $f(x)=\cos x+2x\ ,\ A\left(\dfrac{\pi}{2};3\right).$

15. $f(x)=e^x+\sin 2x,\ A\left(0;\dfrac{7}{2}\right).$ **16.** $f(x)=\dfrac{1}{x}+2x,\ A(e,1).$

Agar *F(x)* funksiya $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, berilgan funksiyaning boshlang'ich funksiyalarini toping.

17. $5f(5x)$. **18.** $3f\left(\dfrac{x}{2}\right)$. **19.** $-4f(-4x+3)$.

20. $3f(-3x+2)$. **21.** $\dfrac{4}{5}f\left(\dfrac{2}{3}x+7\right)$. **22.** $cf(ax+b)$.

Quyidagi integrallarni hisoblang:

23. $\int 4x^7dx.$ **24.** $\int\dfrac{dx}{x^5}.$

25. $\int(5x^3-2x^2+3x-8)dx.$ **26.** $\int\dfrac{3x^4+5x^3-6x\sqrt[4]{x}+4}{x}dx.$

27. $\int\dfrac{2x^4-5x\sqrt[3]{x}+7\sqrt{x}}{x\sqrt{x}}dx.$ **28.** $\int\left(x^{3/2}-\dfrac{2}{x^{3/2}}\right)^2dx.$

Quyidagi integrallarni hisoblang:

29. $\int(x-1)(x+2)\,dx.$ **30.** $\int x^2(x+1)(5x-3)\,dx.$

31. $\int\sqrt[3]{x^2}(8\sqrt[3]{x}-1)\,dx.$ **32.** $\int\dfrac{\left(\sqrt{x}-1\right)^3}{x\sqrt{x}}dx.$

Kuyidagi integrallarni hisoblang:

33. $\int 8^x\,dx.$ **34.** $\int 5^{3x}\cdot e^x\,dx.$

35. $\int 5^{x-2}\,dx.$ **36.** $\int\dfrac{32^x-2^x}{4^x}dx.$

Quyidagi integrallarni hisoblang:

37. $\int 8\cos x\,dx.$

38. $\int \frac{\sin x}{9}dx.$

39. $\int \frac{1}{5\sin^2 x}dx.$

40. $\int\left(\frac{2}{\cos^2 x}-\frac{5}{\sin^2 x}\right)dx.$

Quyidagi integrallarni hisoblang:

41. $\int \frac{dx}{\cos^2 x\sin^2 x}.$

42. $\int \frac{1-\cos 2x}{6\sin x}dx.$

43. $\int \frac{dx}{\sin^2 x+\cos 2x}.$

44. $\int \frac{1-4ctg^2 x}{\cos^2 x}dx.$

Quyidagi integrallarni hisoblang:

45. $\int \frac{1+\sqrt{4-x^2}}{\sqrt{4-x^2}}dx.$

46. $\int \sqrt{\frac{4+x^2}{16-x4}}dx.$

47. $\int \frac{\sqrt{4+x^2}-\sqrt{4-x^2}}{\sqrt{16-x^4}}dx.$

48. $\int \sqrt{\frac{3+x^2}{x^4-9}}dx.$

Quyidagi integrallarni hisoblang:

49. $\int \frac{3+x^2}{1+x^2}dx.$

50. $\int \frac{1-2x^2}{x^2(1-x^2)}dx.$

51. $\int \frac{(2x^2+5)\,dx}{x^2(x^2+5)}.$

52. $\int \frac{x^2-7}{9-x^2}dx.$

Quyidagi integrallarni hisoblang:

53. $\int \frac{dx}{ch^2 x\ sh^2 x}.$

54. $\int \frac{ch2x}{ch^2 x\ sh^2 x}.$

55. $\int sh^2 x\,dx\ .$

56. $\int ch^2 x\,dx.$

Quyidagi integrallarni hisoblang $(a\neq 0)$

57. $\int e^{ax}\,dx.$

58. $\int \sin(ax+b)\,dx.$

59. $\int \cos(ax+b)\,dx.$

60. $\int b^{ax}\,dx,\quad b\neq 1,\ b>0.$

61. $\int \frac{dx}{\cos^2 ax}.$

62. $\int \frac{dx}{\sin^2 ax}.$

Quyidagi integrallarni hisoblang:

63. $\int e^{3x+4}dx.$

64. $\int (12x-5)^7\,dx.$

65. $\int \sqrt{9x+7}\,dx.$

66. $\int \cos 5x\,dx.$

67. $\int \sin\left(\frac{\pi}{4}-3x\right)dx.$

68. $\int \frac{dx}{(6-5x)^4}.$

69. $\int \frac{dx}{\sqrt[3]{9x-7}}$. **70.** $\int \frac{dx}{6x+5}$. **71.** $\int \frac{dx}{7-5x}$.

72. $\int 6^{3x+2}dx$. **73.** $\int \frac{dx}{9+25x^2}$. **74.** $\int \frac{dx}{25-16x^2}$.

Mustaqil yechish uchun misollarning javoblari

1. $F(x)=x^5$ **.2.** $F(x)=\frac{1}{2}\sin 2x$. **3.** $F(x)=-\frac{1}{3}e^{1-3x}$ **.4.** $F(x)=\frac{1}{3}\ln|\sin 3x|$.

5. $F(x)=\frac{1}{3}tg3x+\frac{1}{3}\frac{8^x}{\ln 2}$. **6.** $F(x)=\frac{2}{3}x\sqrt{x}+\frac{3}{4}x\sqrt[3]{x}$. **7.** $2x-tg3x+C$. **8.** $-\frac{1}{5}ctg5x-x+C$.

9. $\frac{1}{5}(3x+5)^5+C$. **10.** $-\frac{2}{3}\cos x-\frac{1}{6}\cos 3x+C$. **11.**

$\frac{1}{2}x-\frac{1}{8}\sin 4x+C$. **12.** $\frac{x}{\ln x}+C$. **13.** $\frac{x^3}{3}-7$ **.** **14.** $\sin x+x^2+2-\frac{\pi^2}{4}$. **15.**

$e^x-\frac{1}{2}\cos 2x+3$. **.16.** $\ln|x|+x^2-e^2$. **19.** $F(5x)$. **20.** $6F\left(\frac{x}{2}\right)$. **21.** $F(-4x+3)$. **22.** $-F(-3x+2)$.

23. $\frac{6}{5}F\left(\frac{2}{3}x+7\right)$. **24.** $\frac{c}{a}F(ax+b)$. **25.** $\frac{1}{2}x^8+C$. **26.** $-\frac{1}{4x^4}+C$ **.**

27. $\frac{5}{4}x^3-\frac{2}{3}x^3+\frac{3}{2}x^2-8x+C$. **28.** $\frac{3}{4}x^4+\frac{5}{3}x^3-\frac{24}{5}x^{\frac{5}{4}}+4\ln|x|+C$. **29.** $\frac{x^3}{3}+\frac{x^2}{2}-2x+C$.

30. $x^5+\frac{x^4}{2}-x^3+C$. **31.** $4x^2-\frac{3}{5}x^{5/3}+C$. **32.**

$x-6\sqrt{x}+3\ln|x|+\frac{2}{\sqrt{x}}+C$. **33.** $\frac{8^x}{\ln 8}+C$. **34.** $\frac{125^x\cdot e^x}{3\ln 5+1}+C$. **35.** $\frac{5^{x-2}}{\ln 5}+C$. **36.** $\frac{16^x+3}{2^x\cdot\ln 2}+C$. **37.**

$8\sin x+C$. **38.** $-\frac{1}{9}\cos x+C$. **39.** $-\frac{1}{5}ctgx+C$. **40.** $2tgx+5tgx+C$. **41.** $tgx-ctgx+C$.

42. $-\frac{1}{3}\cos x+C$. **43.** $tgx+C$. **44.** $tgx+4ctgx+C$. **45.** $\arcsin\frac{x}{2}+x+C$. **46.** $\arcsin\frac{x}{4}+C$. **47.**

$\arcsin\frac{x}{2}-\ln\left|x+\sqrt{x^2+4}\right|+C$. **48.** $\ln\left|x+\sqrt{x^2-3}\right|+C$. **49.** $x+2arctgx+C$. **50.**

$\frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|-\frac{1}{x}+C$. **51.** $\frac{1}{\sqrt{5}}arctg\frac{x}{\sqrt{5}}-\frac{1}{x}+C$. **52.** $\frac{1}{3}\ln\left|\frac{3+x}{3-x}\right|-x+C$ **.** **53.** $-cthx-thx+C$.

54. $thx-cthx+C$. **55.** $\frac{1}{4}sh2x-\frac{1}{2}x+C$. **56.** $\frac{1}{4}sh2x+\frac{1}{2}x+C$. **57.** $\frac{1}{a}e^{ax}+C$.

58. $-\frac{1}{a}\cos(ax+b)+C.$ **59.** $\frac{1}{a}\sin(ax+b)+C.$ **60.**

$\frac{1}{a}\frac{b^{ax}}{\ln b}+C,\ \ b\neq 1,\ b>0.$ **61.** $\frac{1}{a}tgax+C.$ **62.** $-\frac{1}{a}ctgax+C.$ **63.**

$\frac{1}{3}e^{3x+4}+C.$ **64.** $\frac{1}{96}(12x-5)^{8}+C.$ **65.** $\frac{2}{27}\sqrt{(9x+7)^{3}}+C.$

66. $\frac{1}{5}\sin 5x+C.$ **67.** $\frac{1}{3}\cos\left(\frac{\pi}{4}-3x\right)+C.$ **68.** $\frac{1}{15}\frac{1}{(6-5x)^{3}}+C.$ **69.**

$\frac{1}{6}\sqrt[3]{(9x-7)^{2}}+C.$ **70.** $\frac{1}{6}\ln|6x+5|+C.$ **71.** $-\frac{1}{5}\ln(7-5x)+C.$ **72.**

$\frac{6^{3x+2}}{3\ln 6}+C.$ **73.** $\frac{1}{15}arctg\frac{5x}{3}+C.$ **74.** $\frac{1}{40}\ln\left|\frac{5+4x}{5-4x}\right|+C.$

18-amaliy mashg'ulot.

INTEGRALLARNI HISOBLASHDA O'ZGARUVCHILARNI ALMASHTIRISH VA BO'LAKLAB INTEGRALLASH

Quyidagi integrallarni hisoblang:

1. $\int x\sqrt{x^{2}-7}dx.$ **2.** $\int\frac{x^{2}}{\sqrt[5]{3-x^{3}}}dx.$ **3.** $\int\frac{e^{3x}\,dx}{(e^{3x}-4)^{3}}.$

4. $\int\frac{xdx}{x^{2}+8}.$ **5.** $\int\frac{2x+1}{x^{2}+x-5}dx.$ **6.** $\int\frac{2x-3}{8+3x-x^{2}}dx.$

Quyidagi integrallarni hisoblang:

7. $\int\frac{e^{x}dx}{2e^{x}+7}.$ **8.** $\int\frac{\sqrt[5]{e^{x}}dx}{x^{2}}dx.$ **9.** $\int\frac{1}{x\ln x}dx.$

10. $\int\frac{\sqrt[7]{\ln^{5}x}}{x}dx.$ **11.** $\int x^{2}e^{1-x^{3}}dx.$ **12.** $\int\frac{5^{x}dx}{9+5^{x}}.$

Quyidagi integrallarni hisoblang:

13. $\int tgxdx.$ **14.** $\int ctgxdx.$ **15.** $\int\sin x\cos^{5}x\,dx.$

16. $\int\frac{\cos x}{\sqrt[5]{\sin x}}dx.$ **17.** $\int\frac{\sin x}{\cos^{2}x\sqrt{\cos x}}dx.$ **18.** $\int\frac{\sqrt[7]{128\,tgx}}{\cos^{2}x}dx.$

Quyidagi integrallarni hisoblang:

19. $\int \frac{1-4\arcsin x}{\sqrt{1-x^2}}dx.$ **20.** $\int \frac{\sqrt[4]{arctgx}}{1+x^2}dx.$ **21.** $\int \frac{dx}{(x^2+1)arctgx}.$

22. $\int e^{5\cos x-1}\sin xdx.$ **23.** $\int \frac{\sin xdx}{\cos^2 x-5}.$ **24.** $\int \frac{\cos\frac{2}{x^3}}{x^4}dx.$

Quyidagi integrallarni hisoblang:

25. $\int ch^3 x\, shxdx.$ **26.** $\int \frac{shx}{ch^2 x}dx.$ **27.** $\int sh^7 x\, chx\, dx.$

28. $\int \frac{chx}{sh^2 x}dx.$ **29.** $\int \frac{cthx}{sh^2 x}dx.$ **30.** $\int \frac{thx}{ch^2 x}dx.$

Integrallarni hisoblang:

31. $\int x\sin x\,dx$ **32.** $\int (x+3)\cos xdx.$

33. $\int (1-4x)\sin xdx.$ **34.** $\int (2x+4)e^x\,dx.$

35. $\int (x+5)6^x\,dx.$ **36.** $\int x^2\cos x\,dx.$

37. $\int \left(x^2+3\right)\sin x\,dx.$ **38.** $\int (2x-x^2)e^{-x}\,dx.$

Integrallarni hisoblang:

39. $\int \ln x\,dx.$ **40.** $\int \ln\left(1+x^2\right)dx.$

41. $\int x^2\ln(x+4)dx.$ **42.** $\int xarctgx\,dx.$

43. $\int \frac{\arcsin x}{x^2}dx.$ **44.** $\int \ln\left(x+\sqrt{1+x^2}\right)dx.$

Quyidagi integlarni hisoblang:

45. $\int e^x\cos xdx.$ **46.** $\int e^x\sin xdx.$

47. $\int 2^x\cos xdx.$ **48.** $\int e^{ax}\sin bx\,dx$, $a^2+b^2\neq 0.$

Quyidagi integrallarni hisoblang:

49. $\int x^5\sin x\,dx.$ **50.** $\int x^6e^{-x}dx.$

51. $\int x^4\cos 3x\,dx.$ **52.** $\int x^3e^{-x^2}\,dx.$

Quyidagi integrallarni hisoblang:

53. $\int \sqrt{x}\sin\sqrt{x}\,dx.$ **54.** $\int \frac{x\ln x}{\sqrt{1+x}}dx.$ **55.** $\int \cos^2\ln x\,dx.$

56. $\int \frac{\ln tgx}{\cos^2 x}dx.$ **57.** $\int x\,arctg\,x^2\,dx.$

Mustakil yechish uchun misollarning javoblari

1. $\frac{1}{3}\sqrt{(x^2-7)^3}+C.$ **2.** $-\frac{5}{12}\sqrt[5]{(3-3^3)^4}+C.$ **3.** $-\frac{1}{6}\frac{1}{(e^{3x}-4)^2}+C.$

4. $\frac{1}{2}\ln\left|x^2+8\right|+C.$ **5.** $\ln\left|x^2+x-5\right|+C.$ **6.** $-\ln\left|8+3x-x^2\right|+C.$ **7.**

$\frac{1}{2}\ln\left|2e^x+7\right|+C.$ **8.** $-\frac{1}{5}e^{5/x}+C.$ **9.** $\ln\left|\ln x\right|+C.$ **10.** $\frac{7}{12}(\ln x)^{12/7}+C.$ **11.** $-\frac{1}{3}e^{1-x^3}+C.$

12. $\frac{\ln\left|9+5^x\right|}{\ln 5}+C.$ **13.** $-\ln\left|\cos x\right|+C.$ **14.** $\ln\left|\sin x\right|+C.$ **15.** $-\frac{\cos^6 x}{6}+C.$ **16.**

$\frac{5}{6}\sqrt[5]{\sin^6 x}+C.$ **17.** $\frac{2}{3\cos x\sqrt{\cos x}}+C.$ **18.** $\frac{7}{4}tgx\sqrt[7]{tgx}+C.$ **19.**

$\arcsin x-2\arcsin^2 x+C.$ **20.** $\frac{4}{5}\sqrt[4]{arctg^5 x}+C.$ **21.** $\ln\left|arc\,tgx\right|+C.$ **22.** $-\frac{1}{5}e^{5\cos x-1}+C.$ **23.**

$\frac{1}{2\sqrt{5}}\ln\left|\frac{\cos x+\sqrt{5}}{\cos x-\sqrt{5}}\right|+C.$ **24.** $-\frac{1}{6}\sin\frac{2}{x^3}+C.$ **25.** $\frac{1}{4}ch^4 x+C.$

26. $-\frac{1}{chx}+C.$ **27.** $\frac{sh^8 x}{8}+C.$ **28.** $-\frac{1}{shx}+C.$ **29.** $-0{,}5cgh^2 x+C.$

30. $0{,}5th^2 x+C.$ **31.** $-x\cos x+\sin x+C.$ **32.** $\sin x-(x+3)\cos x+C.$

33. $(4x-1)\cos x-4\sin x+C.$ **34.** $2e^x(1+x)+C.$ **35.** $\frac{x\ln 6+5\ln 6-1}{\ln^2 6}6^x+C.$

36. $x^2\sin x-2\sin x+2x\cos x+C.$ **37.** $2x\sin x-\left(x^2+1\right)\cos x+C.$ **38.** $e^{-x}x^2+C.$ **39.**

$x\ln x-x+C.$ **40.** $x\ln\left(1+x^2\right)-2x+2arctgx+C.$ **41.** $\frac{1}{3}(4+x)^3\ln(x+4)-$

$\frac{352}{9}-\frac{16}{3}x+\frac{2}{3}x^2-\frac{1}{9}x^3-4\ln(x+4)(x+4)^2+16(4+x)+16(4+x)\ln(x+4)+C.$

42. $\frac{x^2}{2}arctgx-\frac{1}{2}x+\frac{1}{2}arctgx+C.$ **43.** $-\frac{\arcsin x}{x}-arctg\frac{1}{\sqrt{1-x^2}}+C.$ **44.**

$x\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}+C.$ **45.** $\frac{1}{2}e^x\cos x+\frac{1}{2}e^x\sin x+C.$ **46.**

$\frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x + C.$ **47.** $\left(2^{1+x} tg\frac{x}{2} - 2^x tg^2\frac{x}{2}\ln 2 + 2^x \ln 2\right)\frac{\cos^2\frac{x}{2}}{1+\ln^2 2} + C.$

48. $\frac{a\sin bx - b\cos bx}{a^2+b^2}e^{ax} + C.$ **49.**

$-x^5\cos x + 5x^4 \sin x + 20x^3 - 60x^2 \sin x + 120\sin x - 120x\cos x + C.$

50. $-\left(x^6 + 6x^5 + 30x^4 + 120x^3 + 360x^2 + 720x + 720\right)e^{-x} + C.$ **51.**

$\frac{1}{3}x^4 \sin 3x + \frac{4}{9}x^3\cos 3x - \frac{4}{9}x^2 \sin 3x + \frac{8}{81}\sin 3x - \frac{8}{27}x\cos 3x + C.$ **52.** $-\frac{1}{2}\left(1+x^2\right)e^{-x^2} + C.$ **53.**

$-2x\cos\sqrt{x} + 4\cos\sqrt{x} + 4\sqrt{x}\sin\sqrt{x} + C.$ **54.** $\left(-\frac{4}{9}\sqrt{1+x} + \frac{2}{3}\ln x\sqrt{1+x}\right)x -$

$-\frac{20}{9} + \frac{20}{9}\sqrt{1+x} + \frac{1}{2}\ln x\left(\frac{8}{3} - \frac{8}{3}\sqrt{1+x}\right) - \frac{8}{3}\ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{1+x}\right) + C.$ **55.**

$\frac{3}{5}x tg^4\left(\frac{\ln x}{2}\right) - \frac{4}{5}x tg^3\left(\frac{\ln x}{2}\right) + \frac{2}{5}x tg^2\left(\frac{\ln x}{2}\right) + \frac{4}{5}x tg\left(\frac{\ln x}{2}\right) + \frac{3}{5}x + C.$ **56.** $tgx\ln tgx - tgx + C.$

57. $\frac{1}{2}x^2 arctgx^2 - \frac{1}{4}\ln\left(1+x^4\right) + C.$

19-amaliy mashg'ulot.

RASIONAL FUNKSIYaLARNI INTEGRALLASh

Integrallarni hisoblang:

1. $\int\frac{dx}{x(2+x)}.$ **2.** $\int\frac{dx}{x(5+2x)}.$ **3.** $\int\frac{dx}{x\left(\frac{3}{2}+\frac{5}{7}x\right)}.$

4. $\int\frac{dx}{x(a+bx)}.$ **5.** $\int\frac{dx}{(x+1)(x-3)}.$ **6.** $\int\frac{dx}{(2x+3)(x+4)}.$

Integrallarni hisoblang:

7. $\int\frac{dx}{x(4+x^2)}.$ **8.** $\int\frac{dx}{x(3+4x)^2}.$ **9.** $\int\frac{dx}{x(a+bx)^2}.$

10. $\int\frac{dx}{(2+x)(x-3)^2}.$ **11.** $\int\frac{dx}{(3-4x)(5-2x)^2}.$ **12.** $\int\frac{dx}{(a+bx)(c+fx)^2}.$

Integrallarni hisoblang:

13. $\int \frac{dx}{x(a+bx)(c+fx)}$. **14.** $\int \frac{dx}{x(x^2+3x+5)}$. **15.** $\int \frac{dx}{(x-3)(3x^2+4x+2)}$.

16. $\int \frac{dx}{(2x+5)(x^2-4x+7)}$. **17.** $\int \frac{4x+8}{4x^2+6x-13}dx$. **18.** $\int \frac{x+2}{3x^2-x+5}dx$.

Integrallarni hisoblang:

19. $\int \frac{x^6-2x^4-3x^3-9x^2+4}{x^5-5x^3+4x}dx$. **20.** $\int \frac{x^5-2x^2+3}{x^2-4x+4}dx$.

21. $\int \frac{2x^4-3x^3+4x^2-5x+6}{x^2-3x+1}dx$. **22.** $\int \frac{x^3-x+1}{(x-2)^5}dx$.

Integrallarni hisoblang:

23. $\int \frac{x^2+1}{x(x-1)^3}dx$. **24.** $\int \frac{dx}{(x-2)^2(x+3)^3}$.

25. $\int \frac{dx}{(x+1)(x+2)^2(x+3)^3}$. **26.** $\int \frac{x^2}{(1-x^2)^3}dx$.

Integrallarni hisoblang:

27. $\int \frac{x^3+x+1}{x^4-1}dx$. **28.** $\int \frac{dx}{(x^2-4x+5)(x^2-4x+3)}$.

29. $\int \frac{x^3-6}{x^4+6x^2+8}dx$. **30.** $\int \frac{(3x^2-2)dx}{9x^4-13x^2+4}$.

31. $\int \frac{(3x^2-2)x\,dx}{(x+2)^2(3x^2-2x+4)}$. **32.** $\int \frac{18-11x}{(x^2-9x+20)(x^2+2x+2)}dx$.

Mustaqil yechish uchun misollarning javoblari

1. $-\frac{1}{2}\ln\left|\frac{2}{x}+1\right|+C$. **2.** $-\frac{1}{5}\ln\left|\frac{5}{x}+2\right|+C$. **3.** $-\frac{2}{3}\ln\left|\frac{3}{2x}+\frac{5}{7}\right|+C$. **4.** $-\frac{1}{a}\ln\left|\frac{a}{x}+b\right|+C$.

5. $-\frac{1}{4}\ln\left|\frac{x+1}{x-3}\right|+C$. **6.** $\frac{1}{5}\ln\left|\frac{2x+3}{x+4}\right|+C$. **7.** $\frac{1}{4}\left(\frac{1}{4+x}-\frac{1}{4}\ln\left|\frac{4}{x}+1\right|\right)+C$. **8.** $\frac{1}{3}[\frac{1}{3+4x}-\frac{1}{3}\ln\left|\frac{3}{x}+4\right|]+C$.

9. $\frac{1}{a}[\frac{1}{a+bx}-\frac{1}{a}\ln\left|\frac{a}{x}+b\right|]+C$. **10.** $\frac{-1}{5(x-3)}+\frac{1}{25}\ln\left|\frac{x+2}{x-3}\right|+C$. **11.** $\frac{1}{14(2x-5)}-\frac{1}{49}\ln\left|\frac{3-4x}{5-2x}\right|+C$.

12. $\frac{1}{(bc-af)(c+fx)}+\frac{b}{(af-bc)^2}\ln\left|\frac{a+bx}{c+fx}\right|+C$.

13. $\frac{1}{ac}\ln|x|+\frac{b}{a(af-bc)}\ln|a+bx|-\frac{c}{c(af-bc)}\ln|c+fx|+C.$

14. $\frac{1}{10}\ln\frac{x^2}{x^2+3x+5}-\frac{3}{5\sqrt{11}}arctg\frac{2x+3}{\sqrt{11}}+C.$ **15.** $\frac{1}{82}\ln\left|\frac{(x-3)^2}{3x^2+4x+2}\right|-\frac{11}{41\sqrt{2}}arctg\frac{3x+2}{\sqrt{2}}+C.$

16. $\frac{1}{93}\ln\left|\frac{(2x+5)^2}{x^2-4x+7}\right|+\frac{\sqrt{3}}{31}arctg\frac{2x-2}{\sqrt{3}}+C.$ **17.** $\frac{1}{2}\ln|4x^2+6x-13|+\frac{5}{2\sqrt{61}}\ln\left|\frac{4x+3-\sqrt{61}}{4x+3+\sqrt{61}}\right|+C.$

18. $\frac{1}{6}\ln|3x^2-x+5|+\frac{13}{3\sqrt{59}}arctg\frac{6x-1}{\sqrt{59}}+C.$ **19.** $\frac{x^2}{2}+\ln\left|\frac{x(x-2)(x+1)\sqrt{|x^2-1|}}{x+2}\right|+C.$

20. $\frac{x^4}{4}+\frac{4}{3}x^3+6x^2+30x-\frac{27}{x-2}+72\ln|x-2|+C.$

21. $\frac{2}{3}x^3+\frac{3}{2}x^2+11x+\frac{5}{2}\ln|x^2-3x+1|+\frac{13}{2\sqrt{5}}\ln\left|\frac{2x-3-\sqrt{5}}{2x-3+\sqrt{5}}\right|+C.$

22. $-\frac{1}{x-2}-\frac{3}{(x-2)^2}-\frac{11}{3(x-2)^3}-\frac{7}{4(x-2)^4}+C.$ **23.** $-\frac{1}{(x-2)^2}+\ln\left|\frac{x-1}{x}\right|+C.$

24. $\frac{16-21x-6x^2}{250(x-2)(x+3)^2}-\frac{3}{625}\ln\left|\frac{x-2}{x+3}\right|+C.$ **25.** $\frac{9x^2+50x+68}{4(x+2)(x+3)^2}+\frac{1}{8}\ln\left|\frac{(x+1)(x+2)^{16}}{(x+3)^{17}}\right|+C.$

26. $\frac{x^3+x}{8(1-x^2)^2}-\frac{1}{16}\ln\left|\frac{1+x}{1-x}\right|+C.$ **27.** $\frac{3}{4}\ln|x-1|+\frac{1}{4}\ln|x+1|-\frac{1}{2}arctg\,x+C.$ **28.**

$\frac{1}{52}\ln|x-3|-\frac{1}{20}\ln|x-1|+\frac{1}{65}\ln(x^2+4x+5)+\frac{7}{130}arctg(x+2)+C.$

29. $\ln\frac{x^2+4}{\sqrt{x^2+2}}+\frac{3}{2}arctg\frac{x}{2}-\frac{3}{\sqrt{2}}arctg\frac{x}{\sqrt{2}}+C.$ **30.** $\frac{1}{10}\ln\frac{3x^2-5x+2}{3x^2+5x+2}+C.$

31. $\frac{1}{x+2}+\ln|x+2|-\frac{1}{\sqrt{11}}arctg\frac{3x-1}{\sqrt{11}}+C.$ **32.** $\ln\left|\frac{x-4}{x-5}\right|+arctg(x+1)+C.$

20-amaliy mashg'ulot.

TARKIBIDA TRIGONOMETRIK FUNKSIYALAR QATNASHGAN IFODALARNI INTEGRALLASH

Integrallarni hisoblang.

1. $\int \cos^2 x\,dx.$ **2.** $\int \cos^3 px dx.$ **3.** $\int \sin^3 px dx.$

4. $\int \cos^4 px dx.$ **5.** $\int \sin^5 x dx.$ **6.** $\int tg^3 dx.$

7. $\int ct^3 x dx.$ **8.** $\int \cos^5 x dx.$ **9.** $\int \sin^4 px\,dx.$

$\int \sin px \cos qx\,dx,\ \int \sin px \sin qx\,dx,\ \int \cos px \cos qx\,dx$ ko'rinishdagi integrallarni hisoblang.

10. $\int \cos 5x \cos 9x\,dx.$ **11.** $\int \sin 5x \sin 3x\,dx.$

12. $\int \cos 4x \cos x\,dx.$ **13.** $\int \sin 7x \cos 3x\,dx.$

14. $\int \sin 3x \cos 5x\,dx.$ **15.** $\int \cos px \cos qx\,dx.$

16. $\int \sin px \sin qx\,dx.$ **17.** $\int \sin px \cos qx\,dx.$

18. $\int \sin x \sin 2x \sin 3x\,dx.$ **19.** $\int \cos x \cos \cos 5x\,dx.$

$\int \frac{dx}{\sin^m x}, \int \frac{dx}{\cos^m x}$ ko'rinishdagi integrallarni hisoblang.

20. $\int \frac{dx}{\sin x}.$ **21.** $\int \frac{dx}{\sin^3 x}.$ **22.** $\int \frac{dx}{\sin^4 x}.$

23. $\int \frac{dx}{\sin^5 x}.$ **24.** $\int \frac{dx}{\sin^6 x}.$ **25** $\int \frac{dx}{\cos x}.$

26. $\int \frac{dx}{\cos^6 x}.$ **27.** $\int \frac{dx}{sh^3 x}.$ **28.** $\int \frac{dx}{sh^4 x}.$

29. $\int \frac{dx}{ch^3 x}.$ **30.** $\int \frac{dx}{ch^4 x}.$ **31.** $\int \frac{sh^3 x}{chx} dx.$

$\int \frac{\cos^p x}{\sin^q x} dx, \int \frac{\sin^p x}{\cos^q x} dx$ ko'rinishdagi integrallarni hisoblang.

32. $\int \frac{\sin^2 x}{\cos x} dx.$ **33.** $\int \frac{\sin^3 x}{\cos x} dx.$ **34.** $\int \frac{\sin^4 x}{\cos x} dx.$

35. $\int \frac{\sin x}{\cos^2 x} dx.$ **36.** $\int \frac{\sin^2 x}{\cos^2 x} dx.$ **37.** $\int \frac{\sin^3 x}{\cos^2 x} dx.$

$\int \frac{ch^p x}{sh^q x} dx, \int \frac{sh^p x}{ch^q x} dx$ ko'rinishdagi integrallarni hisoblang.

38. $\int \frac{sh^2 x}{chx} dx.$ **39.** $\int \frac{ch^3 x}{chx} dx.$ **40.** $\int \frac{sh^4 x}{chx} dx.$

41. $\int \frac{sh^2 x}{ch^2 x} dx.$ **42.** $\int \frac{sh^3 x}{ch^2 x} dx.$ **43.** $\int \frac{sh^2 x}{ch^3 x} dx.$

44. $\int \frac{sh^3 x}{ch^4 x} dx$ **45.** $\int \frac{sh^4 x}{ch^4 x} dx.$ **46.** $\int \frac{sh^2 x}{ch^4 x} dx.$

47. $\int \frac{ch^2 x}{shx} dx.$ **48.** $\int \frac{ch^3 x}{shx} dx.$ **49.** $\int \frac{ch^4 x}{shx} dx.$

$\int \sin^p x \cos^q x dx$ ko'rinishdagi integrallarni hisoblang.

50. $\int \sin^2 x \cos^2 dx.$ **51.** $\int \sin^2 x \cos^3 x dx.$

52. $\int \sin^2 x \cos^4 x dx.$ **53.** $\int \sin^3 x \cos x dx.$

54. $\int \sin^3 x \cos^2 x dx.$ **55.** $\int \sin^3 x \cos x dx.$

$\int sh^p x ch^q x dx$ ko'rinishdagi integrallarni hisoblang.

56. $\int shx chx \, dx.$ **57.** $\int sh^2 x ch^2 x \, dx.$

58. $\int sh^2 x ch^4 x dx.$ **59.** $\int sh^3 x \, ch^2 x.$

60. $\int sh^3 x \, ch^3 x dx.$ **61.** $\int sh^3 x ch^2 x dx.$

$\int \frac{dx}{\sin^p x \cos^q x}$ ko'rinishdagi integrallarni hisoblang.

62. $\int \frac{dx}{\sin x \cos x}.$ **63.** $\int \frac{dx}{\sin \cos^2 x}.$ **64.** $\int \frac{dx}{\sin x \cos^3 x}.$

65. $\int \frac{dx}{\sin \cos^4 x}.$ **66.** $\int \frac{dx}{\sin^2 x \cos x}.$ **67.** $\int \frac{dx}{\sin^2 x \cos^2 x}.$

$\int \frac{dx}{sh^p x ch^q x}$ ko'rinishdagi integrallarni hisoblang.

68. $\int \frac{dx}{shx\,chx}$. **69.** $\int \frac{dx}{shx\ ch^2x}$. **70.** $\int \frac{dx}{shx\ ch^3x}$.

$\int R(\sin x, \cos)dx,$ (bunda $R-$ rasional funksiya) ko'rinishdagi integrallarni hisoblang.

71. $\int \frac{dx}{3+5\cos x}$. **72.** $\int \frac{dx}{10+8\cos x}$.

73. $\int \frac{dx}{5+4\cos x}$. **74.** $\int \frac{dx}{13+5\sin x}$.

75. $\int \frac{dx}{3+5\sin x}$. **76.** $\int \frac{dx}{5+3\sin x}$.

77. $\int \frac{dx}{a+b\cos x}$. **78.** $\int \frac{dx}{a+b\sin x}$.

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5.1. $\frac{1}{2}x+\frac{1}{4}\sin 2x+C$. **5.2.** $\frac{1}{p}\sin px-\frac{1}{3p}\sin^3 px+C$. **5.3.** $\frac{1}{3p}\cos^3 px-\frac{1}{p}\cos px+C$.

5.4. $\frac{3}{8}x+\frac{1}{4p}\sin 4px+C$. **5.5.** $-\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+C$. **5.6.** 6) $\frac{1}{2}tg^2x+\ln|\cos x|+C$.

5.7. $-\frac{1}{2}ctg^2-\ln|\sin x|+C$. **5.8.** $\sin x-\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+C$. **9.**

$\frac{3x}{8}-\frac{1}{np}\sin 2px+\frac{1}{32p}\sin 4px+C$. **10.** $\frac{1}{8}\sin 4x+\frac{1}{28}\sin 4x+C$. **11.** $\frac{1}{4}\sin 2x-\frac{1}{16}\sin 8x+C$.

12. $\frac{1}{6}\sin 3x+\frac{1}{10}\sin 5x+C$. **13.** $-\frac{1}{8}\cos 4x-\frac{1}{20}\cos 10x+C$. **14.** $-\frac{1}{16}\cos 8x+\frac{1}{4}\cos 2x+C$.

15. $\frac{\sin(p+q)x}{2(p+q)}+\frac{\sin(p-q)x}{2(p-q)}+C,\ (p^2\neq q^2)$.

16. $-\frac{\sin(p+q)x}{2(p+q)}+\frac{\sin(q-p)x}{2(q-p)}+C,\ (p^2\neq q^2)$.

17. $-\frac{\cos(p+q)x}{2(p+q)}-\frac{\cos(p-q)x}{2(p-q)}+C,\ (p^2\neq q^2)$. **18.** $\frac{\cos 6x}{24}-\frac{\cos 4x}{16}-\frac{\cos 2x}{8}+C$.

19. $\frac{\sin 9x}{36}+\frac{\sin 7x}{28}+\frac{\sin 3x}{12}+\frac{\sin x}{4}+C$. **20.** $\ln\left|tg\frac{x}{2}\right|+C$. **21.** $-\frac{\cos x}{2\sin^2 x}+\frac{1}{2}\ln\left|tg\frac{x}{2}\right|+C$.

22. $-\frac{\cos x}{3\sin^3 x}-\frac{3}{2}ctgx+C=-\frac{1}{3ctg^3 x}-ctgx+C.$ **23.** $-\frac{\cos x}{4\sin^4 x}-\frac{3}{8}\frac{\cos x}{\sin^2 x}+\frac{3}{8}\ln\left|tg\frac{x}{2}\right|+C.$

24. $-\frac{1}{5}ctg^5 x-\frac{2}{3}ctg^3 x-ctgx+C.$ **25.** $\ln\left|tg\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+C.$

26. $\frac{\sin x}{5\cos^5 x}+\frac{4}{5}tg^3 x+\frac{4}{5}tgx=\frac{1}{5}tg^5 x+\frac{2}{3}tg^3 x+tgx+C.$ **27.**

$-\frac{chx}{2sh^2 x}-\frac{1}{2}\ln\left|tg\frac{x}{2}\right|+C.$ **28.** $-\frac{1}{3}cth^3 x+cthx+C.$ **29.** $\frac{shx}{2ch^2 x}+\frac{1}{2}arctg(shx)+C.$

30. $-\frac{1}{3}th^3 x+thx+C.$ **31.** $\frac{1}{2}ch^2 x-\ln chx+C.$

32. $-\sin x+\ln\left|tg\left(\frac{\pi}{2}+\frac{x}{2}\right)\right|+C.$ **33.** $\frac{1}{2}\cos^2 x-\ln|\cos x|+C.$

34. $\frac{1}{3}\sin^3 x-\sin x+\ln\left|tg\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+C.$ **35.** $\frac{1}{\cos x}+C.$ **36.** $tgx-x+C.$ **37.** $\cos x+\frac{1}{\cos x}+C.$

38. $shx-arctg(shx)+C.$

39. $\frac{1}{2}ch^2 x-\ln chx+C.$ **40.** $\frac{1}{3}sh^3 x-shx+arctg(shx)+C.$ **41.** $x-thx+C.$ **42.**

$-\frac{3}{2}x+\frac{1}{4}sh2x+thx+C.$ **43.** $-\frac{shx}{2ch^2 x}+\frac{1}{2}arctg(shx)+C.$ **44.** $-\frac{1}{chx}+\frac{1}{3ch^3 x}+C.$

45. $-\frac{1}{3}th^3 x-thx+x+C.$ **46.** $\frac{1}{3}th^3 x+C.$ **47.** $chx+\ln\left|th\frac{x}{2}\right|+C.$ **48.** $\frac{1}{2}ch^2 x+\ln|shx|+C.$

49. $\frac{1}{3}ch^3 x+chx+\ln\left|th\frac{x}{2}\right|+C.$

50. $\frac{1}{8}x-\frac{1}{32}\sin 4x+C.$ **51.** $\frac{1}{3}\sin^3 x-\frac{1}{5}\sin^5 x+C.$ **52.** $\frac{x}{16}+\frac{1}{64}\sin 2x-\frac{1}{64}\sin 4x-\frac{1}{192}\sin 6x+C.$

53. $\frac{1}{4}\sin^4 x+C.$ **54.** $\frac{1}{5}\cos^5 x-\frac{1}{3}\cos^3 x+C.$

55. $\frac{1}{64}\left(\frac{1}{3}\cos 5x-3\cos 2x\right)+C.$

56. $\frac{sh^2 x}{2}+C.$ **57.** $-\frac{x}{8}+\frac{1}{32}sh4x+C.$ **58.** $\frac{1}{5}sh^3 x\,ch^2 x+\frac{2}{15}sh^3 x+C.$

59. $-\frac{1}{16}x-\frac{1}{64}sh2x+\frac{1}{64}sh4x+\frac{1}{192}sh\,6x+C.$ **60.** $\frac{1}{5}sh^2 x\ ch^3 x-\frac{2}{15}ch^3 x+C.$ **5.105.**

$\frac{1}{6}sh^6x+\frac{1}{4}sh^2x=\frac{1}{6}ch^6x-\frac{1}{4}ch^4x+C.$ **61.** $\ln\left|tgx\right|+C.$ **62.** $\frac{1}{\cos x}+\ln\left|tg\frac{x}{2}\right|+C.$

63. $\frac{1}{2\cos^2 x}+\ln\left|tgx\right|+C.$ **64.** $\frac{1}{\cos x}+\frac{1}{3\cos^3 x}+\ln\left|tg\frac{x}{2}\right|+C.$

65. $\ln\left|tg\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|-\frac{1}{\sin x}+C.$ **66.** $-2ctg2x+C.$ **67.** $-\frac{1}{2\sin^2 x}+\ln\left|tgx\right|+C.$

68. $\ln\left|thx\right|+C.$ **69.** $\frac{1}{chx}+\ln\left|tg\frac{x}{2}\right|+C.$ **70.** $-\frac{1}{2}th^2x+\ln\left|thx\right|+C.$ **71.** $\frac{1}{4}\ln\left|\frac{tg\frac{x}{2}+2}{tg\frac{x}{2}-2}\right|+C.$ **72.**

$\frac{1}{3}arctg\left(\frac{tg\frac{x}{2}}{3}\right)+C.$ **73.** $\frac{2}{3}arctg\left(\frac{tg\frac{x}{2}}{3}\right)+C.$ **74.** $\frac{1}{6}arctg\left(\frac{13tg\frac{x}{2}+5}{12}\right)+C.$ **75.**

$\frac{1}{4}\ln\left|\frac{3tg\frac{x}{2}+1}{3tg\frac{x}{2}+9}\right|+C.$ **76.** $\frac{1}{2}arctg\left(\frac{5+tg\frac{x}{2}+3}{4}\right)+C.$ **77.**

$$\begin{cases}\frac{2}{\sqrt{a^2-b^2}}arctg\frac{\sqrt{a^2-b^2}tg\frac{x}{2}}{a+b}+C,\ a^2>b^2,\\ \frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{\sqrt{b^2-a^2}tg\frac{x}{2}+a+b}{\sqrt{b^2-a^2}tg\frac{x}{2}-a-b}\right|+C, a^2<b^2\end{cases}$$ **.78.**

$$\begin{cases}\frac{2}{\sqrt{a^2-b^2}}arctg\frac{atg\frac{x}{2}+b}{\sqrt{a^2-b^2}}+C,\ a^2>b^2,\\ \frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{atg\frac{x}{2}+b-\sqrt{b^2-a^2}}{atg\frac{x}{2}+b+\sqrt{b^2-a^2}}\right|+C, a^2>b^2.\end{cases}$$

21-amaliy mashg'ulot.

BA'ZI IRRASIONAL IFODALARNI INTEGRALLASH

Quyidagi integrallarni hisoblang.

1. $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}.$ **2.** $\int \frac{dx}{\sqrt[3]{x}+\sqrt{x}}.$

3. $\int \frac{dx}{1+\sqrt{x+1}}.$ **4.** $\int \frac{1+\sqrt{x+1}}{2+\sqrt{x+1}}dx.$

5. $\int \frac{dx}{\sqrt[3]{x}+2\sqrt[4]{x}+\sqrt{x}}.$ **6.** $\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}dx..$

7. $\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx.$ **8.** $\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}dx.$

Quyidagi integrallarni (Eyler almashtirishlaridan foydalanib) hisoblang.

9. $\int \frac{dx}{1+\sqrt{x^2+2x+2}}.$ **10.** $\int \frac{dx}{x\sqrt{4x^2+4x+3}}.$ **11.** $\int \frac{dx}{(1+x)\sqrt{1+x+x^2}}.$

12. $\int \frac{dx}{x+\sqrt{x^2-x+1}}.$ **13.** $\int \frac{dx}{x\sqrt{x^2-x-5}}.$ **14.** $\int \frac{dx}{x+\sqrt{x^2-x+4}}.$

15. $\int \frac{dx}{x\sqrt{2+x-x^2}}.$ **16.** $\int \frac{x\,dx}{\sqrt{\left(6x-8-x^2\right)^3}}.$

Quyidagi integrallarni, binomial differensiallarni integrallash usulidan foydalanib,hisoblang.

17. $\int \sqrt[3]{x}\left(1+\sqrt{x}\right)^2 dx.$ **18.** $\int \frac{1}{\sqrt[6]{x}}\left(\sqrt[3]{x}-1\right)^2 dx.$

19. $\int \frac{dx}{\sqrt[4]{x^3}\left(\sqrt[8]{x}-1\right)^5}.$ **20.** $\int \frac{dx}{x\left(1+\sqrt[3]{x}\right)^2}.$

21. $\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx.$ **22.** $\int \frac{\sqrt[3]{1+x^3}}{x^2}dx.$

23. $\int \sqrt{x}\sqrt[3]{1+\sqrt[4]{x^3}}\,dx.$ **24.** $\int \frac{dx}{x\sqrt{1+x^3}}.$

Quyidagi integrallarni, $\int R\left(x,\sqrt{x^2+a^2}\right)dx,\ \int R\left(x,\sqrt{a^2-x^2}\right)dx$ ifodalarni

integrallash usulidan foydalanib, hisoblang.

25. $\int \sqrt{9-x^2}\,dx.$ **26.** $\int \sqrt{9-16x^2}\,dx.$

27. $\int \sqrt{a^2-b^2x^2}\,dx.$ **28.** $\int \frac{dx}{\sqrt{\left(4-x^2\right)^3}}.$

29. $\int \frac{dx}{\sqrt{\left(a^2-b^2x^2\right)^3}}.$ **30.** $\int \frac{dx}{x\sqrt{a^2-b^2x^2}}.$

31. $\int x\sqrt{\left(a^2-b^2x^2\right)^m}\,dx.$ **32.** $\int x^2\sqrt{a^2-b^2x^2}\,dx.$

Quyidagi integrallarni hisoblang.

33. $\int \frac{dx}{\sqrt{\left(9-x^2\right)^3}}.$ **34.** $\int \frac{x^2dx}{\sqrt{25-x^2}}.$

35. $\int \frac{x^2dx}{\sqrt{a^2-b^2x^2}}.$ **36.** $\int \frac{x^3}{\sqrt{4-x^2}}dx.$

37. $\int \frac{x^3}{\sqrt{a^2-b^2x^2}}dx.$ **38.** $\int \frac{x^4}{\sqrt{a^2-b^2x^2}}dx.$

39. $\int \frac{dx}{x^2\sqrt{a^2-b^2x^2}}.$ **40.** $\int \frac{dx}{x^4\sqrt{a^2-b^2x^2}}.$

Mustaqil yechish uchun misollarning javoblari

1. $2\sqrt{x}-4\sqrt[4]{x}+4\ln\left|\sqrt[4]{x}+1\right|+C.$ **2.** $6[\frac{1}{3}.\sqrt{x}-\frac{1}{2}\sqrt[3]{x}+\sqrt[6]{x}-\ln\left|1+\sqrt[6]{x}\right|]+C.$

3. $2\sqrt{x+1}-2\ln\left|\sqrt{x+1}+1\right|+C.$ **4.** $x+1-2\sqrt{x+1}+2\ln\left|\sqrt{x+1}+2\right|+C.$ **5.**

$2\sqrt{x}-3\sqrt[3]{x}-8\sqrt[4]{x}+6\sqrt[6]{x}+48\sqrt[12]{x}++3\ln\left(1+\sqrt[12]{x}\right)+\frac{33}{2}\ln\left(\sqrt[6]{x}-\sqrt[12]{x}+2\right)--\frac{171}{\sqrt{7}}arctg\frac{2\sqrt[12]{x}-1}{\sqrt{7}}+C$

.6. $6t-3t^2+\frac{3}{2}t^4+\frac{6}{5}t^5-\frac{6}{7}t^7+3\ln\left(1+t^2\right)-6arctgt+C,\ t=\sqrt[6]{x+1}.$

7. $x+4\sqrt{x+1}+4\ln\left|\sqrt{x+1}-1\right|+C.$ **8.** $\frac{1}{2}x^2-\frac{1}{2}x\sqrt{x^2-1}+\frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right|+C.$ **9.**

$\ln\left(x+1+\sqrt{x^2+2x+2}\right)+\frac{2}{x+2+\sqrt{x^2+2x+2}}+C.$ **10.** $\frac{1}{\sqrt{3}}\ln\frac{2x+\sqrt{4x^2+4x+3}-\sqrt{3}}{2x+\sqrt{4x^2+4x+3}+\sqrt{3}}+C.$

11. $-2arctg\frac{1+x+\sqrt{1+x-x^2}}{x}+C.$ **12.** $2\ln|t|-\frac{1}{2}\ln|t-1|+\frac{3}{t+1}-\frac{3}{2}\ln|t+1|+C,$ **bunda**

$t=\dfrac{\sqrt{x^2-x+1}+1}{x}$. **13.** $\dfrac{1}{\sqrt{5}}\ln\left|\dfrac{\sqrt{x^2-x+5}+x-\sqrt{5}}{\sqrt{x^2x+5}+x+\sqrt{5}}\right|+C.$

14. $8\ln|2t+1|-\dfrac{1}{2}\ln|t-1|-\dfrac{15}{2}\ln|t+1|+\dfrac{5}{t+1}+C,$ bunda $t=\dfrac{\sqrt{x^2-x+4}+2}{x}.$

15. $-\dfrac{1}{\sqrt{2}}\ln\left|\dfrac{\sqrt{2+x-x^2}+\sqrt{2}}{x}+\dfrac{1}{2\sqrt{2}}\right|+C.$ **16.** $\dfrac{2(x-2)}{\sqrt{6x-8-x^2}}-\dfrac{\sqrt{6x-8-x^2}}{x-2}+C.$

17. $\dfrac{3}{7}x^{7/3}+\dfrac{24}{11}x^{11/6}+3x^{4/3}+C.$

18. $6\sqrt[6]{x}+\dfrac{9}{2}\ln\left|\dfrac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right|-\dfrac{3\sqrt[6]{x}}{\sqrt[3]{x-1}}+C.$ **19.** $\dfrac{-8}{3\left(\sqrt[8]{x}-1\right)^3}-\dfrac{2}{\left(\sqrt[8]{x}-1\right)^4}+C.$

20. $3\left(\ln\left|\dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}\right|+\dfrac{1}{1+\sqrt[3]{x}}\right)+C.$ **21.** $2\left(1+\sqrt[3]{x}\right)^{3/2}+C.$

22. $\dfrac{1}{6}\ln\left(\dfrac{\sqrt[3]{\left(1+x^3\right)^2}}{x^2}+\dfrac{\sqrt[3]{1+x^3}}{x}+1\right)+\dfrac{1}{\sqrt{3}}arctg\,\dfrac{2\sqrt[3]{1+x^3}+x}{\sqrt{3}x}-\dfrac{1}{3}\ln\left|1-\dfrac{\sqrt[3]{1+x^3}}{x}\right|-\dfrac{\sqrt[3]{1+x^3}}{x}+C.$

23. $\dfrac{4}{7}\left(1+\sqrt[4]{x^3}\right)^{7/3}-\left(1+\sqrt[4]{x^3}\right)^{4/3}+C.$ **24.** $\dfrac{1}{3}\ln\left|\dfrac{\sqrt{1+x^3}-1}{\sqrt{1+x^3}+1}\right|+C.$ **25.** $\dfrac{x\sqrt{9-x^2}}{2}+\dfrac{9}{2}arc\sin\dfrac{x}{3}+C.$

26. $x\dfrac{\sqrt{9-16x^2}}{2}+\dfrac{9}{8}arc\sin\dfrac{3x}{4}+C.$ **27.** $\dfrac{x\sqrt{a^2-b^2x^2}}{2}+\dfrac{a^2}{2^b}arc\sin\dfrac{bx}{a}+C.$

28. $\dfrac{x}{4\sqrt{4-x^2}}+C.$ **29.** $\dfrac{x}{a^2\sqrt{a^2-b^2x^2}}+C.$ **30.** $-\dfrac{1}{a}\ln\left|\dfrac{a+\sqrt{a^2-b^2x^2}}{bx}\right|+C.$

31. $-\dfrac{\sqrt{\left(a^2-b^2x^2\right)^{m+2}}}{(m+2)b^2}+C.$ **32.** $\dfrac{2b^2x^3-a^2x}{8b^2}\sqrt{a^2-b^2x^2}+\dfrac{a^4}{8b^3}\arcsin\dfrac{bx}{a}+C.$ **33.** $\dfrac{x}{9\sqrt{9-x^2}}+C.$

34. $-\dfrac{x\sqrt{25-x^2}}{2}+\dfrac{25}{2}arc\sin\dfrac{x}{5}+C.$ **35.** $-\dfrac{\sqrt{a^2-b^2x^2}}{2b^2}+\dfrac{a^2}{2b^3}arc\sin\dfrac{bx}{a}+C.$

36. $-\dfrac{8+x^2}{3}\sqrt{4-x^2}+C.$ **37.** $-\dfrac{2a^2+b^2x^2}{3b^4}\sqrt{a^2-b^2x^2}+C.$

38. $-\dfrac{2b^2x^3+3a^2x}{8b^4}\sqrt{a^2-b^2x^2}+\dfrac{3a^4}{8b^5}arc\sin\dfrac{bx}{a}+C.$

39. $-\dfrac{\sqrt{a^2-b^2x^2}}{a^2x}+C.$ **40.** $-\dfrac{a^2+2b^2x^2}{3a^4x^3}\sqrt{a^2-b^2x^2}+C.$

22-amaliy mashg'ulot.

ANIQ INTEGRALNI HISOBLASH. NYUTON-LEYBNIS FORMULASI

Quyidagi integrallarni, Nyuton – Leybnis formulasiga asosan, hisoblang.

1. $\int_0^1 (2x-3)dx$. **2.** $\int_1^0 5x^4 dx.$ **3.** $\int_1^4 2\sqrt{x}dx$.

4. $\int_1^5 2\sqrt{x-1}dx.$ **5.** $\int_{-2}^0 (x+1)(x-2)dx.$ **6.** $\int_1^2 \left(3t+\frac{4}{t^2}\right)dt.$

7. $\int_0^\pi (1+\cos)dx.$ **8.** $\int_0^1 \left(x^2+\sqrt{x}\right)dx.$ **9.** $\int_0^1 \left(x^{\frac{3}{2}}-x^{\frac{1}{2}}\right)dx$

Quyidagi integrallarni, Nyuton – Leybnis formulasiga asosan, hisoblang.

10. $\int_{-1}^1 \left(3x^2-4x+7\right)dx.$ **11.** $\int_1^2 \frac{4}{x^2}dx.$ **15.** $\int_1^4 \frac{dt}{t\sqrt{t}}.$

16. $\int_0^1 \frac{36}{(2x+1)^3}dx.$ **17.** $\int_{1/8}^1 x^{-1/3}\left(1-x^{2/3}\right)^{3/2}dx.$ **18.** $\int_0^\pi \sin^2 5xdx.$

19. $\int_0^{\pi/3} \sec^2 tdt.$ **20.** $\int_\pi^{3\pi} ctg^2\frac{x}{2}dx.$ **21.** $\int_{-\frac{\pi}{3}}^0 \sec xtgxdx.$.

Quyidagi integrallarga, Nyuton–Leybnis formulasini formal ravishda qo'llaganda, noto'g'ri natijaga kelinishini izohlang.

22. $\int_0^{dx} \frac{dx}{\left(2+tg^2x\right)\cos^2 x}.$ **23.** $\int_{-1}^1 \frac{d}{dx}\left(arctg\frac{1}{x}\right)dx.$ **24**. $\int_{-1}^1 \frac{dx}{x}.$

Quyidagi integrallarni hisoblang.

25. $\int_{-2}^0 (2x+5)dx.$ **26.** $\int_0^1 \left(x^2+\sqrt{x}\right)dx.$ **27.** $\int_{-1}^1 \sqrt[3]{x}dx.$

28. $\int_{-1}^2 \sqrt[3]{x}dx.$ **28.** $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{dx}{1+x^2}.$ **29.** $\int_2^9 \sqrt[3]{x-1}dx.$

30. $\int_0^\pi (1+\cos x)dx.$ **31.** $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos ec\,x\,dx.$ **32.** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(8y^2+\sin y\right)dy.$

33. $\int_1^{\sqrt{2}}\left(\frac{u^2}{2}-\frac{1}{4^5}\right)du.$ **34.** $\int_{-4}^{4}|x|dx.$ **35.** $\int_{-6}^{6}x|x|dx.$

Quyidagi integrallarni hisoblang.

36. $\int_0^2 3^x dx.$ **37.** $\int_0^1 \frac{x^2dx}{1+x^6}.$ **38.** $\int_{e^2}^{e^3}\frac{dx}{x\ln x}.$

39. $\int_{-\pi}^{\pi}\sin^2 xdx.$ **40.** $\int_0^1 \frac{dx}{4x^2+4x+5}.$ **41.** $\int_2^3 \frac{dx}{x^2-2x-8}.$

42. $\int_1^e \frac{\cos(\ln x)dx}{x}.$ **43.** $\int_1^e \frac{dx}{x(1+\ln^2 x)}.$ **44.** $\int_0^1 \sqrt{4-x^2}\,dx.$

Quyidagi aniq integrallarni, o'zgartiruvchilarni almashtirish yordamida, hisoblang.

45. $\int_0^3 \sqrt{y+1}dy.$ **46.** $\int_{-1}^0 \sqrt{y+1}\,dx.$ **47.** $\int_0^{\pi} 3\cos^2 x\sin dx.$

48. $\int_{2\pi}^{3\pi}\cos^2 x\sin xdx.$ **49.** $\int_{-1}^1 \frac{5x}{(4+x^2)^2}dx.$ **50.** $\int_0^1 \frac{5x}{(4+x^2)^2}dx.$

51. $\int_0^{\frac{\pi}{6}}(1-\cos 3t)\sin 3tdt.$ **52.** $\int_{\frac{\pi}{6}}^{\pi/3}(1-\cos 3t)\sin 3tdt$. **53.** $\int_0^{2\pi}\frac{\cos t}{\sqrt{4+3\sin t}}dt.$

54. $\int_{-\pi}^{\pi}\frac{\cos t}{\sqrt{4+3\sin t}}dt.$ **55.** $\int_0^1 \sqrt{x^5+2x(5x^4+2)}dx.$ **56.** $\int_0^{\frac{\pi}{6}}\cos^{-3}2x\sin 2xdx.$

Quyidagi integrallarni, o'zgaruvchilarni almashtirish usulidan foydalanib, hisoblang.

57. $\int_1^2 \frac{dx}{x\sqrt{1+x^2}}.$ **58.** $\int_0^1 \frac{\sqrt{e^x}dx}{\sqrt{e^x+e^{-x}}}.$ **59.** $\int_3^6 \frac{\sqrt{x^2-9}}{x^4}dx.$

60. $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}.$ **61.** $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x+\cos x}.$ **62.** $\int_0^1 \frac{arc\sin\sqrt{x}}{\sqrt{x(1-x)}}dx.$

63. Ushbu $\int_1^7 (x^2-6x+13)dx$ integralda, $x^2-6x+13=t$ almashtirishni olish mumkinmi? Javobingizni sharhlang.

64. Ushbu $\int_0^1 \sqrt{1-x^2}\,dx$ integralda, $x=\sin t$ almashtirishni olish mumkinmi? Javobingizni sharhlang.

65. Ushbu $\int_0^{\pi} \frac{dx}{1+\sin^2 x}$ integralda, $tgx=t$ almashtirishni olish mumkinmi? Javobingizni sharhlang.

Bo'laklab integrallash formulasi yordamida, aniq integrallarni hisoblang.

66. $\int_0^4 xe^{-x}dx.$ **67.** $\int_1^2 x\ln x dx.$ **68.** $\int_0^{\pi/2} t^2 \sin 2t dt.$

69. $\int_0^1 x\,arctg\left(x^2\right)dx.$ **70.** $\int_0^{\pi/3} x\,tg^2 x dx.$ **71.** $\int_1^2 x^3 \ln x dx.$

72. $\int_0^1 x^2 e^{-x}dx.$ **73.** $\int_1^{e^2} x\ln\sqrt{x}dx.$ **74.** $\int_1^{e^2} \ln x\,dx.$

75. $\int_0^{1/2} x\cdot\cos\pi x dx..$ **76.** $\int_0^1 \ln\left(1+x^2\right)dx.$ **77.** $\int_0^{1/4} \arcsin 2x dx.$

Quyidagi integrallarni, bo'laklab integrallash formulasidan foydalanib, hisoblang.

78. $\int_0^{\frac{\pi}{2}} x\cos x dx.$ **79.** $\int_1^2 (3x+2)\ln x dx..$ **80.** $\int_0^{\pi} e^x \sin 2x dx.$

81. $\int_0^{\pi} x^3 \sin x dx.$ **82.** $\int_0^{\sqrt{3}} x\,arc\,tgx\,dx.$ **83.** $\int_1^2 \frac{\ln x}{x^5}dx.$

84. $\int_0^{\frac{\pi}{2}} \sin^4 x\cos^2 x dx.$ **85.** $\int_0^{\pi} e^x \cos^2 x dx.$ **86.** $\int_{-a}^{a} \frac{\ln(2a-x)}{\ln\left(4a^2-x^2\right)}dx\ \left(a>\frac{1}{\sqrt{3}}\right).$

Mustaqil yechish uchun misollarning javoblari

1. $-2.$ **2.** -1. **3.** $7/3.$ **4.** $8/3.$ **5.** $\frac{2}{3}.$ **6.** $13/2.$

7. $\pi.$ **8.** 1. **9.** $-\frac{4}{15}.$ **13.** 16. **14.** 2. **15.** 2. **16.** 8. **17.**

$\frac{27\sqrt{3}}{160}$. **18.** $\frac{\pi}{2}$. **19.** $\sqrt{3}$. **20.** -2π. **21.** $1/2$. **22.** $\frac{1}{\sqrt{2}}arctg\left(\frac{tgx}{\sqrt{2}}\right)$ funksiya, integral ostidagi funksiya uchun boshlang'ich funksiya bo'lib, u, $0 \le x \le 2\pi$ da uzilishga ega. **23.** $arctg\frac{1}{x}$ funksiya, $x = 0$ nuqtada uzilishga ega. **24.** Integral ostidagi $\frac{1}{x}$ funksiya va uning $\ln|x|$ boshlang'ich funksiyasi, $[-1;1]$ kesmada uzilishga ega. **25.** 6. **26.** 1. 27. 1.5. **28.** $\pi/6$. **29.** $\frac{45}{7}$. **30.** π. **31.** 0. **32.** $\frac{2\pi^3}{3}$. **33.** $\frac{16\sqrt{2}-19}{48}$. **34.** 16. **35.** 0. **36.** $\frac{8}{\ln 3}$. **37.** $\frac{\pi}{12}$. **38.** $\ln 1{,}5$. **39.** π. **40.** $\frac{1}{4}arctg(4/7)$. **41.** $\frac{1}{6}\ln(2/5)$. **42.** $\sin 1$. **43.** $\frac{\pi}{4}$. **44.** $\frac{\pi}{3}+\frac{\sqrt{3}}{2}$. **45.** $14/3$. **46.** $2/3$. **47.** 3) 2. **48.** 2. **49.** 0. **50.** $1/8$. **51.** $1/6$. **52.** $1/2$. **53.** 0. **54.** 0. **55.** $2\sqrt{3}$. **56.** $\frac{3}{4}$. **57.** $\ln\frac{e+\sqrt{1-e^2}}{1+\sqrt{2}}$. **58.** $\frac{\sqrt{3}}{72}$. **59.** $\frac{\pi}{4}$. **60.** $\ln 2$. **61.** $\frac{\pi}{12}$. **63.** Yo'q. **64.** Mumkin. **65.** Yo'q. **66.** $1-5e^{-4}$. **67.** $\ln 4-\frac{3}{4}$. **68.** $\frac{\pi^2-4}{8}$. **69.** $\frac{\pi}{8}-\frac{1}{4}\ln 2$. **70.** $\frac{\pi\sqrt{3}}{3}-\ln 2-\frac{\pi^2}{18}$. **71.** $4\ln 2-\frac{15}{16}$.. **72.** $2-5e^{-1}$. **73.** $\frac{3}{8}e^4+\frac{1}{8}$. **74.** e^2+1. **75.** $\frac{1}{2\pi}-\frac{1}{\pi^2}$. **76.** $\ln 2+\frac{\pi}{2}-2$. 77. $\frac{\pi}{24}+\frac{\sqrt{3}-2}{4}$. **78.** $\frac{\pi}{2}-1$. **79.** $10\ln 2-\frac{17}{4}$. **80.** 0. **81.** $\pi^3-6\pi$. **82.** $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$. **83.** $\frac{15}{256}-\frac{\ln 2}{64}$. **84.** $\frac{\pi}{32}$. **85.** $\frac{3}{5}(e^{\pi}-1)$. **86.** *a*.

23-amaliy mashg'ulot.

CHEGARALARI CHEKSIZ XOSMAS INTEGRALLAR

Quyidagi xosmas integrallarning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping.

1. $\int_1^{+\infty}\frac{dx}{\sqrt[3]{x^5}}$. 2. $\int_0^{+\infty}e^{-5x}dx$. 3. $\int_{-\infty}^{+\infty}\frac{dx}{x^2+2x+2}$.

4. $\int_0^{+\infty} xe^{-x^2}dx$. 5 $\int_1^{+\infty} \frac{arctgx}{1+x^2}dx$.. 6. $\int_1^{+\infty} \frac{dx}{(x+2)\ln^2(x+2)}$.

7. $\int_0^{+\infty} \frac{2xdx}{(x^2+1)^3}$. 8. $\int_{-\infty}^{0} \frac{dx}{(x+1)^3}$. 9. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+x+1)^2}$.

Quyidagi xosmas integrallarning uzoqlashuvchi ekanligini isbotlang.

10. $\int_1^{+\infty} \frac{dx}{\sqrt[3]{x}}$. 11. $\int_0^{+\infty} \frac{xdx}{x^2+5}$. 12. $\int_0^{+\infty} \cos x dx$.

13. $\int_1^{+\infty} \frac{dx}{\sqrt{16+x^2}}$. 14. $\int_e^{+\infty} \frac{dx}{(x+1)\ln(x+1)}$. 15. $\int_1^{+\infty} 4^x dx$.

Quyidagi xosmas integrallarni hisoblang.

16. $\int_{\sqrt{2}}^{+\infty} \frac{xdx}{(x^2+1)^3}$. **17.** $\int_1^{+\infty} \frac{dx}{(1+x)\sqrt{x}}$. **18.** $\int_0^{+\infty} \frac{dx}{e^x+\sqrt{e^x}}$. **19.** $\int_{\sqrt{2}}^{+\infty} \frac{dx}{(x-1)\sqrt{x^2-2}}$.

Quyidagi funksiyalarning grafiklari va abssissalar o'qi bilan chegaralangan shakllarning yuzini hisoblang.

20. $f(x)=\frac{1}{4+x^2}$, $-\infty < x < +\infty$. **21.** $f(x)=x^2e^{-x^3}$, $0 \le x < +\infty$.

22. $f(x)=\frac{\sqrt{x}}{(1+x)^2}$, $1 \le x < +\infty$. **23.** $f(x)=\frac{1}{\sqrt{1+e^x}}$, $0 \le x < +\infty$.

Quyidagi integrallarning yaqinlashuvchiligini isbotlang.

24. $\int_0^{+\infty} \frac{x^3}{x^5+1}dx$. **25.** $\int_0^{+\infty} \frac{x}{\sqrt[3]{1+x^7}}dx$. **26.** $\int_2^{+\infty} (\cos\frac{x}{2}-1)dx$.

27. $\int_0^{+\infty} \left(e^{-\frac{1}{x^2}}-e^{-\frac{4}{x^2}}\right)dx$. **28.** $\int_e^{+\infty} \frac{dx}{x\cdot\ln^5 x}$. **29.** $\int_0^{+\infty} \frac{\ln(1+x+x\sqrt{x})}{\sqrt{x^3}}dx$.

Quyidagi integrallarning uzoqlashuvchiligini isbotlang.

30. $\int_0^{+\infty} \frac{x^3+1}{x^4}dx$. **31.** $\int_0^{+\infty} \frac{xdx}{\sqrt[3]{x^5+2}}$. **32.** $\int_0^{+\infty} \frac{\sin^2 x}{x}dx$.

33. $\int_0^{+\infty} \frac{\sin\frac{1}{x}}{\left(x-\cos\frac{\pi}{x}\right)^2}dx$. **34.** $\int_0^{+\infty} \frac{xdx}{1+x^2\sin^2 x}$. **35.** $\int_0^{+\infty} \frac{1}{\sqrt{x}} arctg\frac{x}{2+x}dx$.

Quyidagi integrallarni absolyut va shartli yaqinlashuvchilikka tekshiring.

36. $\int_0^{+\infty} x\cos x^4 dx$. **37.** $\int_0^{+\infty} \frac{\sin(\ln x)}{\sqrt{x}}dx$. **38.** $\int_0^{+\infty} \frac{x\cos 7x}{x^2+2x+2}dx$.

Mustaqil yechish uchun misollarning javoblari

1. $\frac{3}{2}$. **2.** $\frac{1}{5}$. **3.** π. **4.** $\frac{1}{2}$. **5.** $\frac{3\pi^2}{32}$. **6.** $\frac{1}{\ln 3}$. **7.** $\frac{1}{2}$. **8.** $-\frac{1}{2}$. **9.** $\frac{4\pi}{3\sqrt{3}}$. **16.** $\frac{1}{36}$. **17.** $\frac{\pi}{2}$. **18.** $2(1-\ln 2)$. **19.** $\frac{3\pi}{4}$. **20.** $\frac{\pi}{2}$. **21.** $\frac{1}{3}$. **22.** $\frac{1}{2}+\frac{\pi}{4}$. **23.** $2\ln(1+\sqrt{2})$. **36.** $2\sqrt{3}$. **37.** $\frac{3}{4}$. **38.** $\frac{1}{5}$.

24-amaliy mashg'ulot.

CHEGARALANMAGAN FUNKSIYANING XOSMAS INTEGRALLARI

Quyidagi xosmas integrallarning yaqinlashuvchiligini ko'rsating va qiymatini toping.

1. $\int_0^1 \frac{dx}{\sqrt[3]{x}}$. **2.** $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$. **3.** $\int_1^e \frac{dx}{x\sqrt{\ln x}}$. **4.** $\int_0^4 \frac{dx}{x+\sqrt{x}}$.

5. $\int_1^2 \frac{xdx}{\sqrt{x-1}}$. **6.** $\int_{-1}^1 \frac{x+1}{\sqrt[5]{x^3}}dx$. **7.** $\int_0^1 \frac{\arcsin x}{\sqrt{1-x^2}}dx$. **8.** $\int_0^{\frac{1}{2}} \frac{dx}{x\ln^2 x}$.

9. $\int_0^3 \frac{x^2dx}{\sqrt{9-x^2}}$. **10.** $\int_{-1}^1 \frac{\arccos x}{\sqrt{1-x^2}}dx$. **11.** $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3}dx$. **12.** $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}dx$.

Quyidagi xosmas integrallarning uzoqlashuvchi ekanligini isbotlang.

13. $\int_{-1}^{3}\frac{dx}{x}$. **14.** $\int_{0}^{e}\frac{dx}{e^x-1}$. **15.** $\int_{-3}^{3}\frac{xdx}{x^2-1}$. **16.** $\int_{0}^{\frac{\pi}{2}}\frac{\cos x}{\sqrt{\sin^3 x}}dx$.

17. $\int_{0}^{\frac{1}{3}}\frac{dx}{x\ln x}$. **18.** $\int_{-1}^{1}\frac{e^{\frac{1}{x}}}{x^3}dx$. **19.** $\int_{0}^{1}\frac{e^{\frac{1}{x}}}{x^3}dx$. **20.** $\int_{0}^{\frac{\pi}{2}}tg\,x\,dx$.

Quyidagi xosmas integrallarni ќisoblang.

21. $\int_{0}^{1}\frac{2-\sqrt[3]{x}-x^3}{\sqrt[5]{x^3}}dx$. **22.** $\int_{\sqrt{2}}^{2}\frac{dx}{(x-1)\sqrt{x^2-2}}$. **23** $\int_{0}^{1}\frac{dx}{\sqrt{x\sqrt{x}}}$.

24. $\int_{0}^{\frac{\pi}{2}}\sqrt{tgx}dx$. **25.** $\int_{0}^{\frac{\pi}{2}}\ln\cos x dx$. **26.** $\int_{0}^{\pi}x\ln\sin x dx$. **27.** $\int_{0}^{\frac{\pi}{4}}\sqrt{ctgx}dx$.

28. $\int_{-1}^{1}\frac{dx}{(16-x^2)\sqrt{1-x^2}}$. **29.** $\int_{a}^{b}\frac{xdx}{\sqrt{(x-a)(b-x)}}$. **30.** $\int_{0}^{1}\frac{x^3\arcsin x}{\sqrt{1-x^2}}dx$.

Quyidagi limitlarni ќisoblang

31. $\lim\limits_{x\to+\infty}\frac{\int_{0}^{x}\sqrt{1+t^{10}}dt}{x^6}$. **32.** $\lim\limits_{x\to+\infty}\frac{\int_{0}^{x}t^{-1}e^{-t}dt}{\ln\frac{1}{x}}$.

Quyidagi funksiyaning grafigi va abssissalar o'qi bilan chegaralangan shaklning yuzini toping.

33. $y=\frac{-x}{\sqrt{x+1}},\quad x\in(-1;0]$. **34.** $y=\frac{1}{\sqrt{2-5x}},\quad x\in[0;0,4)$.

35. $y=\frac{x}{\sqrt{(x-2)(5-x)}},\quad x\in(2;5)$. **36.** $y=\frac{1}{\sqrt{1-x}},\quad x\in[0;1)$.

Quyida berilgan chiziq va uning asimptotalari bilan chegaralangan shaklning yuzini toping.

37. $y^2=\frac{8-4x}{x}$. **38.** $y^2(x+1)=x^2,\quad x<0$.

39. $(1-x^2)y^2=x^2,\quad x>0$. **40.** $x=\cos t, y=\cos 2t\,tgt,\quad t\in\left[\frac{\pi}{4};\ \frac{3\pi}{4}\right]$.

Quyidagi integrallarning yaqinlashuvchiligini isbotlang.

41. $\int_{0}^{8}\frac{dx}{x^2+\sqrt[3]{x}}$. **42.** $\int_{0}^{2\pi}\frac{dx}{\sqrt[3]{x}}$. **43.** $\int_{0}^{8}\frac{\sqrt{x}\,dx}{e^{\sin x}-1}$. **44.** $\int_{0}^{1}\frac{dx}{\sqrt{x}+arctgx}$.

Quyidagi integrallarning uzoqlashuvchiligini isbotlang.

45. $\int_2^4 \frac{dx}{\ln(x-1)}$. **46.** $\int_{-1}^1 \frac{dx}{\ln(1+x)}$. **47.** $\int_0^1 \frac{dx}{e^x-\cos x}$.

Misollarning javoblari

1. $\frac{3}{2}$. **2.** $\frac{\pi}{2}$. **3.** 2. **4.** $2\ln 3$. **5.** $\frac{8}{3}$. **6.** $\frac{10}{7}$.

7. $\frac{\pi^2}{8}$. **8.** $\frac{1}{\ln 2}$. **9.** $\frac{9\pi}{4}$. **10.** $\frac{\pi^2}{2}$. **11.** $-2e^{-1}$. **12.** 2. **21.** $\frac{625}{187}$. **22.** $\frac{\pi}{2}$. **23.** $\frac{4}{3}$.

24. $\frac{\pi}{\sqrt{2}}$. **25.** $-\frac{\pi\ln 2}{2}$. **26.** $-\frac{\pi^2\ln 2}{2}$. **27.** $\frac{1}{2\sqrt{2}}\left(\pi+\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$. **28.** $\frac{\pi}{4\sqrt{15}}$. **29.** $\frac{\pi(a+b)}{2}$. **30.** $\frac{7}{9}$. **31.** $\frac{1}{6}$. **32.** 1. **33.** 0. **33.** $\frac{4}{3}$. **34.** $\frac{2\sqrt{2}}{5}$. **35.** $\frac{7\pi}{2}$.

36. 2. **37.** 4π. **38.** $\frac{8}{3}$. **39.** 2. **40.** $2+\frac{\pi}{2}$.

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OLIY MATEMATIKADAN MISOL VA MASALALAR TO'PLAMI

ALGEBRA VA ANALITIK GEOMETRIYA, LIMIT, UZLUKSIZLIK, HOSILA, INTEGRAL

1 - QISM

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