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OLIV MATEMATIKADAN QO'LLANMA

O'ZBEKISTON RESPUBLIKASI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

MIRZO ULUG'BEK NOMIDAGI
O'ZBEKISTON MILLIY UNIVERSITETI

OLIY MATEMATIKADAN QO'LLANMA

Toshkent
"Universitet"
2008

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I-qism. Analitik geometriya va oliy algebra

1-Bob. Tekislikda analitik geometriya.

§1. Analitik geometriyaning sodda masalalari.

Abtsissalar o'qidagi $A(x_1)$ va $B(x_2)$ nuqtalar orasidagi masofa

$$|AB| = |x_2 - x_1|$$

formula yordamida topiladi. AB kesmadagi

$|AC|:|CB| = \lambda$ shartni qanoatlantiruvchi C nuqtaning koordinatasi

$$x = \frac{x_1 + \lambda \cdot x_2}{1 + \lambda}$$

formula yordamida topiladi. Xususan, $\lambda = 1$ bo'lganda, AB kesma markazi koordinatasi

$$x = \frac{x_1 + x_2}{2}$$

ko'rinishda topiladi.

Tekislikdagi ikki $A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalar orasidagi masofa

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

koordinatalar boshidan $A(x_1; y_1)$ nuqttagacha bo'lgan masofa

$$|OA| = \sqrt{x_1^2 + y_1^2}$$

formuladan topiladi. AB kesmani

$|AC|:|CB| = \lambda$ nisbatda bo'luvchi $C(x, y)$ nuqta koordinatalari

$$x = \frac{x_1 + \lambda \cdot x_2}{1 + \lambda}, \quad y = \frac{y_1 + \lambda \cdot y_2}{1 + \lambda}$$

formulalardan, o'rtasining koordinatalari esa :

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

formulalardan topiladi.

Uchlari $A(x_1; y_1)$, $B(x_2; y_2)$, $C(x_3; y_3)$
nuqtalarda bo'lgan uchburchak yuzi

$$S = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

formula yordamida topiladi, og'irlik markazi,
ya'ni medianalar kesishish nuqtasi koordinatalari

$$x = \frac{1}{3}(x_1 + x_2 + x_3), \quad y = \frac{1}{3}(y_1 + y_2 + y_3) \quad \text{dan}$$

topiladi.

1.1. $A(3)$ va $B(-5)$ orasidagi masofani toping.

1.2. $C(2)$ nuqtaga nisbatan $A(-3)$ nuqtaga simmetrik
bo'lgan nuqtani toping.

1.3. AB kesma ikki nuqta yordamida teng uch qismga
bo'lingan. $A(-1)$, $B(5)$ bo'lsa, bo'linish nuqtalari
koordinatalarini toping.

1.4. $A(3; 8)$ va $B(-5; 14)$ nuqtalar orasidagi
masofani toping.

1.5. Uchlari $A(-3; -2)$, $B(0; -1)$, $C(-2; 5)$
nuqtalarda bo'lgan uchburchak to'g'ri burchakli
ekanligini isbotlang.

1.6. Ordinatalar o'qida $A(4; -1)$ nuqtadan 5 birlik
uzoqlikdagi nuqtani toping.

1.7. Uchlari $A(2; 0)$, $B(5; 3)$, $C(2; 6)$ nuqtalarda
bo'lgan uchburchak yuzini toping.

1.8. Uchlari $A(3; 1)$, $B(4; 6)$, $C(6; 3)$, $D(5; -2)$
nuqtalarda bo'lgan to'rtburchak yuzini toping.

1.9. $A(1; 2)$ va $B(4; 4)$ nuqtalar berilgan. Abtissalar
o'qida shunday S nuqta topingki, ABC uchburchak
yuzi 5 kv.b. ga teng bo'lsin.

1.10. Kvadratning ikki yonma-yon uchlari
 $A(3; -7)$, $B(-1; 4)$ nuqtalarda bo'lsa, uning yuzini
toping.

1.11. $E(3; 5)$ va $F(1; -3)$ nuqtalar kvadrat qarama-
qarshi uchlari bo'lsa, uning yuzini toping.

1.12. $A(-3; 2)$ va $B(1; 6)$ nuqtalar muntazam uchburchak uchlari bo'lsa, bu uchburchak perimetri va yuzini toping.

1.13. ABCD parallelogramm uchta uchi $A(3; -7)$, $B(5; -7)$, $C(-2; 5)$ nuqtalarda. D nuqta B ga qarama-qarshi uchi bo'lsa, bu parallelogramm dioganallari uzunligini toping.

1.14. Agar $A(3; 0)$, $C(-4; 1)$ nuqtalar kvadrat qarama-qarshi uchlari bo'lsa, qolgan ikki uchini toping.

1.15. Uchta uchi $A(-2; 3)$, $B(4; -5)$, $C(-3; 1)$ nuqtalarda bo'lgan parallelogramm yuzini hisoblang.

1.16. Yuzi 3 ga teng, ikki uchi $A(3; 1)$, $B(1; -3)$ nuqtalarda, uchinchi uchi O_y o'qida yotuvchi uchburchak berilgan. Uchinchi uchi koordinatalarini toping.

1.17. Parallelogramm ikki uchi $A(-1; 3)$, $B(-2; 4)$ nuqtalarda, yuzi 12 kv.b. bo'lsa, va dioganallari abtsissalar o'qida kesishsa, qolgan ikki uchini toping.

§2. To'g'ri chiziq tenglamalari.

a, b, c — o'zgarmas sonlar, $a^2 + b^2 \neq 0$ shart

bajarilganda $ax + by + c = 0$ tenglama to'g'ri chiziqning umumiy tenglamasi deyiladi.

1) $a \neq 0$, $b \neq 0$ bo'lsa, $ax + by = 0$ ko'rinishga kelib, koordinatalar boshidan o'tuvchi to'g'ri chiziqlar hosil bo'ladi.

2) $a = 0$, $b \neq 0$, $c \neq 0$ bo'lsa, tenglama

$y = -\frac{c}{b}$ ko'rinish olib, OX o'qiga parallel

to'g'ri chiziqni ifodalaydi.

$$3) \quad e = 0, \quad a \neq 0, \quad c \neq 0 \text{ bo'lsa,}$$

tenglama $x = -\frac{c}{a}$ ko'rinish oladi va OY

o'qiga parallel to'g'ri chiziqni ifodalaydi.

$$4) \quad e = c = 0, \quad a \neq 0 \text{ bo'lsa, } ax = 0 \text{ yoki}$$

$x = 0$ ko'rinish oladi va Oy o'qini ifodalaydi.

$$5) \quad a = c = 0, \quad e \neq 0 \text{ bo'lsa, } ey = 0 \text{ yoki}$$

$y = 0$ ko'rinish oladi va Ox o'qini anglatadi.

To'g'ri chiziq Ox o'qi musbat yo'nalishi bilan α burchak hosil qilsa va $tg\alpha = k$ deyilsa,

$y = kx + b$ tenglama to'g'ri chiziqning burchak koeffitsientli tenglamasi deyiladi. Bunda b —soni to'g'ri chiziqning Oy o'qi bilan kesishish nuqtasi ordinatasi. Umumiy tenglama tomonlari b ga bo'linib, y topilsa, burchak koeffitsientli tenglama hosil bo'ladi. Agar to'g'ri chiziq Ox o'qini a —abtsissali, Oy o'qini b —ordinatali nuqtalarda kesib o'tsa, to'g'ri chiziq tenglamasi

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{ko'rinishda yoziladi va to'g'ri}$$

chiziqning kesmalar bo'yicha tenglamasi deyiladi.

To'g'ri chiziqqa koordinatalar boshidan tushirilgan perpendikulyar uzunligi p bo'lib, bu perpendikulyar Ox o'qining musbat yo'nalishi bilan α burchak hosil qilsa, to'g'ri chiziq tenglamasi

$$x \cos \alpha + y \sin \alpha - p = 0$$

ko'rinishda yoziladi va to'g'ri chiziqning normal tenglamasi deyiladi. Bu tenglamani umumiy tenglama

tomonlarini $\mu = \pm \frac{1}{\sqrt{a^2 + b^2}}$ ga ko'paytirib hosil

qilish mumkin.

2.1. Quyidagi to'g'ri chiziqlarni yasang: 1) $2y + 7 = 0$

2) $5x - 2 = 0$ 3) $4x - 3y = 0$ 4) $x - 3y - 3 = 0$

2.2. Koordinata boshidan o'tuvchi va Ox o'qi musbat yo'nalishi bilan 1) 45° 2) 60° 3) 90° 4) 120° burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

2.3. To'g'ri chiziq $y = kx + b$ $A(2; 3)$ nuqtadan o'tadi va Ox o'qi musbat yo'nalishi bilan 45° burchak hosil qiladi. k va b ni aniqlang.

2.4. To'g'ri chiziq umumiy tenglamasi $12x - 5y - 65 = 0$. Bu to'g'ri chiziqning 1) burchak koeffitsientli; 2) kesmalar bo'yicha; 3) normal tenglamalarini yozing.

2.5. $2x - 5y = 0$ to'g'ri chiziqning kesmalar bo'yicha tenglamasini yozish mumkinmi?

2.6. Agar to'g'ri chiziq $A(2; 5)$ nuqtadan o'tsa va ordinatalar o'qidan $b = 7$ kesma ajratsa, uning tenglamasini yozing.

2.7. To'g'ri chiziq son o'qlaridan bir xil kesma ajratadi, son o'qlari orasidagi kesmasining uzunligi $5\sqrt{2}$ bo'lsa, tenglamasini yozing.

2.8. $B(-4; 6)$ nuqtadan o'tib, son o'qlari bilan yuzasi 6 kv.b. uchburchak hosil qiluvchi to'g'ri chiziq tenglamasini toping.

2.9. Rombning diagonalari 10 va 6 sm bo'lib, mos ravishda Ox va Oy o'qlarida joylashsa, tomonlari tenglamalarini yozing.

§3. To'g'ri chiziqqa doir masalalar.

$y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri

chiziqlar orasidagi o'tkir burchak $\text{tg}\varphi = \left| \frac{k_2 - k_1}{1 + k_1k_2} \right|$

formuladan topiladi. Ikki to'g'ri chiziqlarning parallelizm sharti $k_2 = k_1$, perpendikulyarlik sharti esa

$$k_1 = -\frac{1}{k_2} \text{ ko'rinishda bo'ladi.}$$

Agar to'g'ri chiziqlar $a_1x + b_1y + c_1 = 0$,
 $a_2x + b_2y + c_2 = 0$ umumiy tenglamalar bilan berilsa

$$\operatorname{tg} \varphi = \left| \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2} \right| \text{ ko'rinish oladi. Parallelizm,}$$

perpendikulyarlik shartlari mos ravishda $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

va $a_1a_2 + b_1b_2 = 0$ bo'ladi

k - burchak koeffitsientli, $A(x_0, y_0)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi

$y - y_0 = k(x - x_0)$ ko'rinishda bo'ladi.

$A(x_0, y_0), B(x_1, y_1)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} \text{ ko'rinishda}$$

bo'ladi.

$A(x_0, y_0)$ nuqtadan $x \cos \alpha + y \sin \alpha - p = 0$ to'g'ri chiziqgacha bo'lgan masofa

$$d = |x_0 \cos \alpha + y_0 \sin \alpha - p| \text{ formuladan,}$$

$ax + by + c = 0$ to'g'ri chiziqgacha masofa esa

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} \text{ formula yordamida topiladi.}$$

3.1. $y = -3x + 4$ va $y = 2x - 1$ to'g'ri chiziqlar orasidagi o'tkir burchakni aniqlang.

3.2. $3x - 2y + 7 = 0$, $6x - 4y - 9 = 0$,
 $6x + 4y - 5 = 0$, $2x + 3y - 6 = 0$ to'g'ri chiziqlardan
o'zaro parallellari va perpendikulyarlarini ko'rsating.

3.3. $A(2; -1)$, $B(4; 3)$, $C(12; -3)$ nuqtalar
uchburchak uchlari bo'lsa, uning tomonlari
tenglamalarini yozing.

3.4. $A(0; 7)$, $B(6; -6)$, $O(0; 0)$ uchlarga ega
uchburchak tomonlari tenglamalarini va ichki
burchaklarini toping.

3.5. $A(-2; 1)$ nuqtadan o'tuvchi,
 $3x + 4y - 1 = 0$ to'g'ri chiziqda parallel va
perpendikulyar to'g'ri chiziqlar tenglamalarini yozing.

3.6. Uchburchak tomonlari tenglamalari
berilgan: $x + 2y = 0$, $x + 4y - 6 = 0$,
 $x - 4y - 6 = 0$. Ichki burchaklarini aniqlang.

3.7. $A(3; 1)$ nuqtadan o'tib, $x - 3y + 1 = 0$
to'g'ri chiziq bilan 45° li burchak hosil qiluvchi to'g'ri
chiziq tenglamasini toping.

3.8. Uchlari $A(-4; 2)$, $B(2; -5)$, $C(5; 0)$
nuqtalarda bo'lgan uchburchak medinalari va
balandliklari kesishadigan nuqtalarni toping.

3.9. Normalining uzunligi 5 bo'lib, Ox o'qi
musbat yo'nalishi bilan 45° burchak hosil qiluvchi
to'g'ri chiziq tenglamasini yozing.

3.10. Koordinata boshidan $12x - 5y + 39 = 0$
to'g'ri chiziqqacha bo'lgan masofani toping.

3.11. $A(5; 2)$, $B(1; 2)$ nuqtalardan
 $x - 2y - 1 = 0$ to'g'ri chiziqqacha masofani hisoblang.

3.12. O'zaro parallel $2x + y - 7 = 0$,
 $2x + y + 1 = 0$ to'g'ri chiziqlar orasidagi masofani
toping.

3.13. Agar $y = kx + 5$ to'g'ri chiziqdan koordinata boshigacha masofa $\sqrt{5}$ bo'lsa k ni aniqlang.

3.14. $x + y - 5 = 0$, $7x - y - 19 = 0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalari tenglamalarini yozing.

§4. Ikkinchi tartibli chiziqlar

Ikkinchi tartibli chiziqlar

$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$ umumiy tenglama bilan beriladi. Bu paragrafda aylana, ellips, giperbola, parabolalar, ularning xossalari doir masalalar o'rganiladi.

1. Markazi $C(a, b)$ nuqtada, radiusi R bo'lgan aylana tenglamasi

$$(x - a)^2 + (y - b)^2 = R^2.$$

Koordinata boshi $O(0; 0)$ nuqtada bo'lsa, aylana tenglamasi

$$x^2 + y^2 = R^2$$

ko'rinish oladi

2. Tekislikda fokuslar deb ataluvchi F_1 va F_2 nuqtalargacha bo'lgan masofalar yig'indisi $2 \cdot a$ songa teng bo'lgan nuqtalarning geometrik o'rni ellips deyiladi.

Fokuslar orasidagi masofani $2c$, $a^2 - c^2 = b^2$ desak, ellips

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

kanonik tenglamaga ega bo'ladi $e = \frac{c}{a} < 1$ miqdor ellips ekstsentrishiteti deyiladi.

Ellips $A(x, y)$ nuqtasidan fokuslarga masofa (fokal radiuslari) $r = a - ex$, $r = a + ex$ formulalardan topiladi.

3. Tekislikda fokuslar deb ataluvchi F_1 va F_2 nuqtalarga bo'lgan masofalar ayirmasi $2a$ songa teng bo'lgan nuqtalar geometrik o'rni giperbola deyiladi.

Fokuslar orasidagi masofa $2c$, $c^2 - a^2 = b^2$ bo'lsa, giperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

kanonik ko'rinishiga keladi.

$$e = \frac{c}{a} > 1 \quad \text{nisbat} \quad \text{giperbola}$$

ekstsentsritetidir.

Giperbola $A(x, y)$ nuqtasidan fokuslarga masofa (fokal radiuslari)

$$r_1 = |ex - a|, \quad r_2 = |ex + a|$$

formulalardan topiladi.

4. Tekislikda fokus deb ataluvchi

$F(\frac{p}{2}; 0)$ nuqtadan va direktrisa deb ataluvchi $x = -\frac{p}{2}$ to'g'ri chiziqdan bir xil uzoqlashgan nuqtalar geometrik o'rni parabola deyib,

$$y^2 = 2px$$

kanonik tenglamaga ega bo'ladi.

Parabola $A(x, y)$ nuqtasidan fokusgacha

masofa $r = x + \frac{p}{2}$ formuladan topiladi

4.1. $A(1;2), B(0;1), C(-3;0)$ nuqtalardan o'tuvchi aylana tenglamasini yozing.

4.2. $A(-4;8)$ nuqta berilgan Diametri OA bo'lgan aylana tenglamasini yozing.

4.3. Aylanalar markazlari va radiuslarini toping.

1) $x^2 + y^2 - 6x + 4y - 23 = 0$

2) $x^2 + y^2 + 5x - 7y + 2,5 = 0$

3) $x^2 + y^2 + 7y = 0$ r

4.4. $x^2 + y^2 - 8x - 4y + 16 = 0$ aylanaga koordinata boshidan o'tkazilgan urinmalar tenglamalarini yozing.

4.5. $x^2 + 4y^2 = 16$ ellips fokusi va ekstsentrisitetini toping

4.6. Er shari biror fokusda Quyosh joylashgan ellips bo'yicha harakatlanadi. Yerdan quyoshgacha eng qisqa masofa 147,5 mln.km, eng uzun masofa 152,5 mln.km bo'lsa, Yer orbitasi katta yarim o'qini va ekstsentrisitetini toping.

4.7. $x^2 + y^2 = 36$ aylana ordinatalari ikki marta qisqartirilsa qanday chiziq hosil bo'ladi?

4.8. $A(\sqrt{3}; \sqrt{2})$ nuqtadan o'tuvchi, ekstsentrisiteti $\sqrt{2}$ ga teng bo'lgan giperbola tenglamasini yozing.

4.9. $\frac{x^2}{64} - \frac{y^2}{36} = 1$ giperbolaning chap shoxida shunday nuqta topingki, o'ng fokal radiusi 18 ga teng bo'lsin.

4.10. $x^2 + 4x + y^2 = 0$ aylanadan va $M(2;0)$ nuqtadan bir xil masofada yotuvchi nuqtalar tenglamasini yozing.

4.11. Fokus $4x - 3y - 4 = 0$ to'g'ri chiziq va OX o'qi kesishgan nuqtada bo'lgan parabola tenglamasini yozing.

4.12. $y^2 = 2x$ parabola koordinatalar boshidan o'tuvchi to'g'ri chiziqdan $\frac{3}{4}$ uzunlikdagi vatar ajratadi. To'g'ri chiziq tenglamasini toping.

§5. Koordinatalarni almashtirish.

Qutb koordinatalar boshidan qutb o'qi OX o'qi musbat yo'nalishi bilan ustma-ust qo'yilsa, tekislikdagi nuqta qutb va dekart koordinatalari quyidagicha bog'lanadi:

$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \\ r = \sqrt{x^2 + y^2} \\ \operatorname{tg} \varphi = \frac{y}{x} \end{cases}$$

XOY koordinatalar sistemasidan koordinata boshi $O_1(a, b)$ bo'lgan $x'Oy'$ koordinatalariga o'tish — parallel ko'chirishda tekislikning biror A nuqtasi eski va yangi koordinatalari quyidagicha bog'lanadi.

$$\begin{cases} x = x' + a \\ y = y' + b \end{cases} \quad \begin{cases} x = x' - a \\ y' = y - b \end{cases}$$

Agar koordinata o'qlari musbat α burchakka burilsa nuqtaning eski x, y koordinatalari va yangi x', y' koordinatalari quyidagicha bog'lanadi:

$$\begin{cases} x = x' \cos \alpha - y' \sin \alpha \\ y = x' \sin \alpha + y' \cos \alpha \\ x' = x \cos \alpha + y \sin \alpha \\ y' = -x \sin \alpha + y \cos \alpha \end{cases}$$

Agar ikkinchi tartibli chiziq (ITCh)

$$Ax^2 + Cy^2 + 2Dx + 2Ey + F = 0$$

ko'rinishida berilsa, parallel ko'chirish yordamida kanonik ko'rinish oladi.

Agar ITCh

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0 \text{ tenglama}$$

bilan aniqlansa, koordinata o'qlarini

$$\operatorname{ctg} 2\alpha = \frac{A-C}{2B}$$

shart bajarilganda α - burchakka burish yordamida oldingi holga keltirish mumkin, bunda xy had yo'qoladi.

$$5.1. \quad A(2\sqrt{3}; 2), B(0; -3), \quad C(-4; 4),$$

$D(\sqrt{2}; -\sqrt{2})$ nuqtalar qutb koordinatalarini toping.

$$5.2. \quad A\left(10; \frac{\pi}{2}\right), B\left(2; \frac{5\pi}{4}\right), C\left(0; \frac{\pi}{10}\right), D\left(1; -\frac{\pi}{4}\right)$$

nuqtalar dekart koordinatalarini toping.

$$5.3. \quad A\left(6; -\frac{\pi}{4}\right) \text{ va } B\left(8; \frac{\pi}{4}\right), \quad C\left(4; \frac{\pi}{6}\right) \text{ va}$$

$D\left(4; \frac{\pi}{2}\right), E\left(5; \frac{\pi}{4}\right) \text{ va } F(12; \pi)$ nuqtalar orasidagi masofalarni toping.

5.4. Agar parallel ko'chirishda $O(0; 0)$ nuqta

$O_1(3; -4)$ nuqtaga o'tsa, $A(5; 6)$ nuqta qanday nuqtaga o'tadi?

5.5. Agar parallel ko'chirishda $A(4;3)$ nuqta $A_1(-3;-4)$ nuqtaga o'tsa, koordinata boshi qanday nuqtaga o'tadi?

5.6. Koordinatalar sistemasi $\alpha = \frac{\pi}{4}$

burchakka burilsa,

$A(\sqrt{3};3)$ nuqtaning koordinatalari qanday o'zgaradi?

5.7. Dastlab koordinatalar boshi $O_1(3;4)$ nuqtaga ko'chirildi, so'ngra bu sistema son o'qlari $\alpha = \frac{\pi}{6}$ ga burildi, bunda $A(2;1)$ nuqta koordinatalari qanday o'zgaradi?

5.8. Parallel ko'chirish yordamida $y = Ax^2 + Bx + C$ ko'rinishidagi parabolaning $y' = Ax'^2$ ko'rinishga keltiring:

a) $y = 9x^2 - 6x + 2$

b) $y = 4x - 2x^2$

5.9. Parallel ko'chirish yordamida $y = \frac{kx+l}{px+q}$

ko'rinishidagi giperbolani $y' = \frac{m}{x'}$ ko'rinishda yozing.

a) $y = \frac{4x+5}{2x-1}$

b) $y = \frac{2x}{4x-1}$

5.10. $Ax^2 + Cy^2 + 2Dx + 2Ey + F = 0$ ko'rinishdagi ITCh kanonik ko'rinishga keltirilsin.

a) $4x^2 + 9y^2 - 8x - 36y + 4 = 0$

b) $x^2 - 9y^2 + 2x + 36y - 44 = 0$

$$v) 16x^2 + 25y^2 - 32x + 50y - 359 = 0$$

$$g) x^2 + 4y^2 - 4x - 8y + 8 = 0$$

$$d) x^2 - y^2 - 6x + 10 = 0$$

$$e) x^2 + 2x + 5 = 0$$

5.11. Kanonik ko'rinishga keltiring.

$$a) 5x^2 + 4xy + 8y^2 + 8x + 14y + 5 = 0$$

$$b) x^2 - 2xy + y^2 - 10x - 6y + 25 = 0$$

$$v) 4xy + 4\sqrt{3}y^2 + 16x + 12y - 36 = 0$$

5.12. Quyidagi chiziqlarni yasang:

$$a) r = a\varphi \quad (\text{arximed spirali})$$

$$b) r = a(1 - \cos \varphi) \quad (\text{kardoida})$$

$$v) r^2 = a^2 \cos 2\varphi \quad (\text{lemniskata})$$

$$g) r = \frac{a}{\varphi} \quad (\text{Giperbolik spirali})$$

$$d) r = a \sin 3\varphi \quad (\text{uch yaproqli gul})$$

$$e) r = a \sin 2\varphi \quad (\text{to'rt yaproqli gul})$$

5.13. Qutb koordinatalar sistemasiga o'tkazing:

$$a) x^2 - y^2 = a^2$$

$$b) x^2 + y^2 = a^2$$

$$v) (x^2 + y^2)^2 = a^2 (x^2 - y^2)$$

$$g) y = x$$

5.14. Dekart koordinatalar sistemasiga o'tkazing.

$$a) r \cos \varphi = a$$

$$b) r^2 \sin 2\varphi = 2a^2$$

$$v) r \sin(\varphi + \frac{\pi}{4}) = a\sqrt{2}$$

$$g) r = a(1 + \cos \varphi)$$

5.15. Kanonik tenglamasini yozing.

$$a) r = \frac{9}{5 - 4 \cos \varphi}$$

$$b) r = \frac{3}{1 - \cos \varphi}$$

$$c) r = \frac{1}{2 - \sqrt{3} \sin \varphi}$$

Bobga doir misollar echish namunalari

1. Kesma bir uchi $A(-7)$, o'rtasi $C(2)$ bo'lsa ikkinchi uchi koordinatasini toping.

Ikkinchi uchi $B(x_2)$ nuqtada bo'lsa,

$2 = \frac{-7 + x_2}{2}$ kelib chiqadi. Bundan $B(11)$ ekanligini topamiz.

2. Uchlari $A(-4; 2)$, $B(0; -1)$, $C(3; 3)$ nuqtalarda bo'lgan uchburchak yuzi, perimetri va ichki burchaklarini toping.

$$S = \frac{1}{2} |-4(-1-3) + 0(3-2) + 3(2+1)| = \frac{1}{2} \cdot |16+9| = 12,5$$

$$|AB| = \sqrt{(0+4)^2 + (-1-2)^2} = \sqrt{4^2 + 3^2} = 5.$$

$$|BC| = \sqrt{(3-0)^2 + (3+1)^2} = \sqrt{3^2 + 4^2} = 5.$$

$$|AC| = \sqrt{(3+4)^2 + (3-2)^2} = \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2}$$

$$\text{Demak, } P = 5 + 5 + 5\sqrt{2} = 5(2 + \sqrt{2}).$$

Kosinuslar teoremasidan

$$\cos < A = \frac{|AC|^2 + |AB|^2 - |BC|^2}{2 \cdot |AC| \cdot |AB|} = \frac{50 + 25 - 25}{2 \cdot 2\sqrt{5} \cdot 5} = \frac{1}{\sqrt{2}},$$

ya'ni $< A = 45^\circ$ ekanligi kelib chiqadi.

Uchburchakning teng yonli ekanligidan
 $\angle C = 45^\circ$, $\angle B = 90^\circ$.

Bu uchburchak to'g'ri burchakli ekanligi
 $|AB|^2 + |BC|^2 = |AC|^2$ tenglikdan kelib chiqadi.

3. Uchburchakning ikkita uchi $A(3; 8)$, $B(10; 2)$
 nuqtalarda bo'lib, medianalari $O(1; 1)$ nuqtada
 kesishsa, uchi $C(x, y)$ koordinatalarini toping.

A nuqtadan chiqqan mediana AD bo'lsa,
 medianalar xossasidan $|AO| : |OD| = 2 : 1 = \lambda$ bo'lib,
 D nuqta koordinatalari x_1, y_1 uchun quyidagi
 tengliklar o'rinli:

$$1 = \frac{3 + 2 \cdot x_1}{1 + 2}; \quad 1 = \frac{8 + 2 \cdot y_1}{1 + 2}$$

Bundan $D(0; -2,5)$. O'z navbatida, bu nuqta CB
 kesma markazi ekanligidan

$$\frac{x + 10}{2} = 0; \quad \frac{y + 2}{2} = -2,5$$

ya'ni $C(-10; -7)$ kelib chiqadi.

4. Agar $O(0; 0)$, $E(3; 0)$, $F(0; 4)$ uchburchak
 tomonlari o'rtalari bo'lsa, bu uchburchak yuzini
 toping.

$|OE| = 3$; $|OF| = 4$; $|EF| = 5$ kesmalar berilgan
 uchburchak o'rta chiziqlari ekanligidan, uning
 tomonlari 6, 8, 10 uzunlikka egaligi kelib chiqadi.
 Pifagor teoremasi o'rinniligidan bu uchburchak to'g'ri
 burchakli va

$$S = \frac{6 \cdot 8}{2} = 24 \quad (\text{kv.b})$$

5. Uchlari $O(0; 0)$, $A(8; 0)$, $B(0; 6)$ nuqtalarda
 bo'lgan uchburchakda OS — mediana, OD —
 bissektisa, OE — balandlik uzunliklarini hisoblang.
 $a = |OA| = 8$, $b = |OB| = 6$, $c = |AB| = 10$
 ekanligidan, uchburchak to'g'ri burchakli.

$$|OC| = \frac{|AB|}{2} = 5;$$

$$L_c = |OD| = \frac{1}{a+b} \sqrt{ab(a+b+c)(a+b-c)}$$

$$\text{formuladan } |OD| = \frac{24\sqrt{2}}{7}; \quad S = \frac{ab}{2} = \frac{c \cdot h}{2}$$

$$\text{formuladan } h = |OE| = 4,8.$$

6. $2x - 3y - 12 = 0$ to'g'ri chiziqning son o'qlari bilan kesishish nuqtalarini aniqlang.

To'g'ri chiziq Ox o'qi bilan kesishish nuqtasida $y = 0$ bo'ladi. Demak, $2x - 12 = 0$. Bundan $x = 6$, ya'ni to'g'ri chiziq Ox o'qini $(6; 0)$ nuqtada kesib o'tadi.

Aksincha, $x = 0$ bo'lsa, $y = -4$ kelib chiqadi. To'g'ri chiziq Oy o'qini $(0; -4)$ nuqtada kesib o'tadi.

7. $A(0; 4)$ nuqtadan o'tuvchi va Ox o'qi musbat

yo'nalishi bilan $\alpha = \frac{2\pi}{3}$ burchak hosil qiluvchi to'g'ri chiziq tenglamasini yozing.

$$\operatorname{tg} \frac{2\pi}{3} = -\sqrt{3} \text{ ekanligidan } k = -\sqrt{3}. \text{ Demak,}$$

to'g'ri chiziq burchak koeffitsientli tenglamasi $y = -\sqrt{3}x + 4$ bo'ladi.

8. To'g'ri chiziq koordinata o'qlarida teng musbat kesmalar ajratadi. Bu to'g'ri chiziq va son o'qlari bilan chegaralangan uchburchak yuzi 8 kv.b. bo'lsa, to'g'ri chiziq tenglamasini yozing.

Bu uchburchak to'g'ri burchaklidir. Agar uning katetlarini a , b deb belgilasak, ularning tengligi va $a^2 = 16$, $a = 4$ ekanligi kelib chiqadi. Demak, to'g'ri chiziqning kesmalar bo'yicha tenglamasi

$$\frac{x}{4} + \frac{y}{4} = 1 \text{ ko'rinishda bo'ladi.}$$

9. To'g'ri chiziq $\frac{(x+2\sqrt{5})}{4} + \frac{y-2\sqrt{5}}{2} = 0$ tenglama

bilan berilgan. Bu to'g'ri chiziqning umumiy, burchak koeffitsientli, kesmalar bo'yicha va normal tenglamalarini yozing.

1) Berilgan tenglamani umumiy maxrajga keltiramiz:

$$x + 2\sqrt{5} + 2(y - 2\sqrt{5}) = 0$$

Bundan $x + 2y - 2\sqrt{5} = 0$ umumiy tenglamasi kelib chiqadi.

2) Umumiy tenglamani y ga nisbatan echamiz:

$$2y = -x + 2\sqrt{5} \quad \text{ya'ni} \quad y = -\frac{1}{2}x + \sqrt{5}$$

burchak koeffitsientli tenglamasidir.

3) Umumiy tenglama tomonlarini $2\sqrt{5}$ ga bo'lamiz:

$$\frac{x}{2\sqrt{5}} + \frac{y}{\sqrt{5}} = 1$$

Kesmalar bo'yicha tenglama hosil bo'ldi.

4) Umumiy tenglama tomonlarini

$$\mu = \pm \frac{1}{\sqrt{a^2 + b^2}} = \pm \frac{1}{\sqrt{1^2 + 2^2}} \quad \text{ya'ni} \quad \mu = \frac{1}{\sqrt{5}} \quad \text{ga}$$

ko'paytiramiz.

$$\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y - 2 = 0 \quad \text{bu to'g'ri chiziqning}$$

normal tenglamasi bo'lib, $\sin \alpha = \frac{2}{\sqrt{5}}$, $\cos \alpha = \frac{1}{\sqrt{5}}$,

$$p = 2.$$

10. Ixtiyoriy nuqtasidan $x = -3$ to'g'ri chiziqqacha masofa Ox o'qigacha bo'lgan masofadan ikki marta kichik bo'ladigan chiziq tenglamasini toping.

Bu chiziqning ixtiyoriy $M(x, y)$ nuqtasini olamiz. Bu nuqtadan $x = -3$ to'g'ri chiziqqacha masofa $|x + 3|$ ga, Ox o'qigacha masofa esa u ga teng. Shartga ko'ra, $y = 2 \cdot |x + 3|$ yoki $y = \pm 2(x + 3)$.

11. $y = -2x$ va $y = 3x + 4$ to'g'ri chiziqlar orasidagi o'tkir burchakni toping. $k_1 = -2$; $k_2 = 3$ ekanligidan

$$\operatorname{tg} \varphi = \left| \frac{3 + 2}{1 - 3 \cdot 2} \right| = \left| \frac{5}{-5} \right| = 1. \quad \text{Demak: } \varphi = 45^\circ.$$

12. $A(-2; 0), B(2; 6)$ va $C(1; 2)$ nuqtalar uchburchak uchlari bo'lsa, uchburchak AS tomoni, BE medianasi, BD — balandligi tenglamalarini yozing.

A va S nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi $\frac{x - (-2)}{4 - (-2)} = \frac{y - 0}{2 - 0}$ yoki $y = \frac{x}{3} + \frac{2}{3}$ dir.

E nuqta A va S nuqtalarni tutashtiruvchi kesma o'rtasi, demak uning koordinatalari

$$x = \frac{-2 + 4}{2} = 1, \quad y = \frac{0 + 2}{2} = 1$$

ekanligi kelib chiqadi. Mediana B va E nuqtalardan o'tganligi uchun uning tenglamasi $\frac{x - 1}{2 - 1} = \frac{y - 1}{6 - 1}$, ya'ni $y = 5x - 4$ ko'rinishda bo'ladi.

BD balandlik AC tomonga perpendikulyarligidan, tenglamasi $y = -3x + b$ ko'rinishda bo'ladi. Uning $B(2; 6)$ nuqtadan o'tishidan foydalanib $6 = -3 \cdot 2 + b$, ya'ni $b = 12$ ekanligini topamiz.

Demak, BD balandlik $y = -3x + 12$ tenglama bilan aniqlanadi.

13. m va n ning qanday qiymatlarida $mx + 8y + n = 0$ va $2x + my - 1 = 0$ to'g'ri chiziqlar

1) parallel 2) ustma-ust 3) perpendikulyar bo'ladi?

1) bu to'g'ri chiziqlar $\frac{m}{2} = \frac{8}{m}$ shart bajarilsa,

ya'ni $m = \pm 4$ bo'lsa parallel bo'ladi.

2) Ular ustma-ust tushishi uchun

$\frac{m}{2} = \frac{8}{m} = \frac{n}{-1}$ shartlar bajarilishi zarur. Bundan

$m = \pm 4$; $n = \mp 2$ kelib chiqadi.

3) Perpendikulyarlik shartidan $2m + 8m = 0$,

ya'ni $m = 0$ kelib chiqadi.

14. Ikki to'g'ri chiziq $x + 3y = 0$ va

$x - 2y + 3 = 0$ larning kesishish nuqtasini toping.

Noparallel bu to'g'ri chiziqlar kesishish nuqtasini topish uchun, ularning tenglamasini birgalikda echish kerak.

$$\Rightarrow \begin{cases} y = -\frac{x}{3} \\ y = \frac{x+3}{2} \end{cases} \Rightarrow -\frac{x}{3} = \frac{x+3}{2} \Rightarrow x = -\frac{9}{5}; y = -\frac{3}{5}$$

Demak, bu to'g'ri chiziqlar $A(-\frac{9}{5}; -\frac{3}{5})$

nuqtada kesishadi.

15. $A(-1; 1)$ nuqtadan o'tib $y = \frac{6-2x}{3}$ to'g'ri chiziq

bilan 45° burchak hosil qiluvchi to'g'ri chiziqlar tenglamasini yozing.

Berilgan to'g'ri chiziq burchak koeffitsienti $k_1 = -\frac{2}{3}$,
 qidirilayotgan to'g'ri chiziqlardan birinikini k_2 deb
 olamiz.

$$\operatorname{tg} 45^\circ = \left| \frac{k_2 + \frac{2}{3}}{1 - \frac{2}{3} \cdot k_2} \right| = 1. \text{ dan } |3k_2 + 2| = |3 - 2k_2|, \text{ ya'ni}$$

$3k_2 + 2 = \pm(3 - 2k_2)$. Bundan $k_2 = \frac{1}{5}$ yoki $k_2 = -5$
 kelib chiqadi. Qidirilayotgan to'g'ri chiziqlar
 $y - 1 = -5(x + 1)$ ko'rinishdadir. Soddalashtirib
 $y = \frac{x}{5} + \frac{6}{5}$ va $y = -5x - 4$ tenglamalarga ega
 bo'lamiz.

16. $A(3; 0), B(5; -3)$ nuqtalardan $2x - 3y - 6 = 0$
 to'g'ri chiziqqacha bo'lgan masofani toping.
 A nuqtadan to'g'ri chiziqqacha masofa

$$d_1 = \frac{|2 \cdot 3 - 3 \cdot 0 - 6|}{\sqrt{2^2 + (-3)^2}} = 0$$

Demak, bu nuqta chiziqda yotadi. B
 nuqtadan bu to'g'ri chiziqqacha bo'lgan masofa

$$d_2 = \frac{|2 \cdot 5 - 3 \cdot 5 - 6|}{\sqrt{2^2 + (-3)^2}} = \frac{13}{13} = 1$$

17. Ikki parallel $2x - 3y - 6 = 0$ va $4x - 6y - 25 = 0$
 to'g'ri chiziqlar orasidagi masofani toping.

Birinchi to'g'ri chiziqdan biror nuqta tanlab
 olamiz, buning uchun $y = 0$ desak, $x = 3$ chiqadi.

$A(3; 0)$ nuqta birinchi chiziqda yotadi. Shu nuqtadan
 ikkinchi to'g'ri chiziqqacha masofa

$$d = \frac{|4 \cdot 3 - 6 \cdot 0 - 25|}{\sqrt{4^2 + (-6)^2}} = \frac{13}{\sqrt{52}} = \frac{13}{2\sqrt{13}} = \frac{\sqrt{13}}{2}$$

bo'lib, bu izlanayotgan masofadir.

18. $2x + 3y = 10$ va $3x + 2y = 10$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalari tenglamalarini yozing.

$ax + by + c = 0$ va $a_1x + b_1y + c_1 = 0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalari

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm \frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}$$

formula yordamida topiladi. Bundan

$$\frac{2x + 3y - 10}{\sqrt{2^2 + 3^2}} = \pm \frac{3x + 2y - 10}{\sqrt{3^2 + 2^2}} \quad \text{ya'ni}$$

$$y = x \text{ va } y = 4 - x.$$

19. Parallelogramm ikki tomoni tenglamalari $y = x - 2$ va $5y = x + 6$ bo'lib, dioganallari koordinata boshida kesishsa, qolgan ikki tomon va dioganallar tenglamalarini yozing.

Berilgan ikki tomon kesishish nuqtasini topamiz:

$$x - 2 = \frac{x + 6}{5}, \text{ ya'ni } x = 4, y = 2.$$

Parallelogramm $A(4;2)$ uchiga qarama-qarshi

uchini $C(x, y)$ desak, $\frac{4+x}{2} = 0, \quad \frac{2+y}{2} = 0$

ekanligidan $C(-4; -2)$ bo'ladi.

$C(-4; -2)$ dan o'tib, berilgan tomonlarga parallel bo'lgan to'g'ri chiziqlar izlanayotgan tomonlar bo'ladi:

$$y+2=1 \cdot (x+4) \quad \text{va} \quad y+2=\frac{1}{5} \cdot (x+4) \quad \text{dan}$$

$$y=x+2, \quad y=\frac{x}{5}-\frac{6}{5} \quad \text{kelib chiqadi.}$$

Bu tomonlarning berilgan tomonlar bilan kesishadigan nuqtalari

$$\begin{cases} y=x+2 \\ y=\frac{x}{5}-\frac{6}{5} \end{cases} \quad \text{va} \quad \begin{cases} y=x-2 \\ y=\frac{x}{5}-\frac{6}{5} \end{cases} \quad \text{sistemalar}$$

echimlari, ya'ni $B(-1;1), D(1;-1)$ dir.

$$AC \text{ diagonal} \quad \frac{x-4}{-4-4} = \frac{y-2}{-2-2} \quad \text{dan} \quad y = \frac{x}{2}$$

va

$$BD \text{ diagonal} \quad \frac{x+1}{1+1} = \frac{y-1}{-1-1} \quad \text{dan} \quad y = -x$$

ekanligi kelib chiqadi.

20. $x^2 + 6x + y^2 - 8y + 21 = 0$ aylana markazi koordinatalari va radiusini toping.

$(x+3)^2 - 9 + (y-4)^2 - 16 + 21 = 0$ ko'rinishda to'la kvadratlar ajratsak,

$$(x+3)^2 + (y-4)^2 = 2^2$$

Aylana markazi $A(-3;4)$ nuqtada va radius $R=2$

21. $A(5;0), B(1;4)$ nuqtalardan o'tuvchi, markazi $y=-x+3$ to'g'ri chiziqda yotuvchi aylana tenglamasini tuzing.

AB kesma o'rtasining koordinatalari

$$x = \frac{5+1}{2} = 3 \quad ; \quad y = \frac{0+4}{2} = 2 \quad C(3;2)$$

AB chiziq tenglamasi

$$\frac{x-5}{1-5} = \frac{y-0}{4-0}, \quad \text{ya'ni } y = x+5.$$

Aylana markazi $C(3;2)$ dan o'tuvchi,

$y = -x + 5$ ga

perpendikulyar to'g'ri chiziqda yotadi, ya'ni

$$y-2 = 1(x-3), \quad y = x-1$$

Demak, aylana markazi $y = -x+3$ va $y = x-1$ to'g'ri chiziqlar kesishish nuqtasi $O_1(2;1)$ dir

Aylana radiusi esa.

$$R = |OA| = \sqrt{(5-2)^2 + (0-1)^2} = \sqrt{10}$$

Demak,

$$(x-2)^2 + (y-1)^2 = 10$$

$$22. \quad \frac{x^2}{25} + \frac{y^2}{9} = 1 \quad \text{ellipsda fokal radiuslari ayirmasi}$$

6,4 ga teng bo'lgan nuqtani aniqlang.

$$a = 5, b = 3 \text{ ekanligidan} \quad s = \sqrt{5^2 - 3^2} = 4.$$

$$\text{Demak, } \varepsilon = \frac{4}{5}$$

$$6,4 = |r_2 - r_1| = 2\varepsilon x \text{ dan } x = \pm 4.$$

Topilgan x ni ellips tenglamasiga qo'yib $y = \pm 1,8$.

Demak: $(4; 1,8); (4; -1,8); (-4; 1,8), (-4; -1,8)$ nuqtalar shartni qanoatlantiradi.

$$23. \quad \frac{x^2}{16} - \frac{y^2}{9} = 1 \quad \text{giperbola o'ng shoxida shunday}$$

nuqta topingki, undan o'ng fokusgacha masofa chap fokusgacha bo'lgan masofadan ikki marta qisqa bo'lsin.

$\varepsilon x + a = 2(\varepsilon x - a)$ shartdan $X = \frac{3a}{\varepsilon}$ kelib chiqadi.

$a = 4$; $\varepsilon = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{16+9}}{4} = \frac{5}{4}$
 ekanligidan $x = 9,6$ demak,

$$y = \pm \frac{3}{4} \sqrt{x^2 - 16} = \pm \frac{3}{4} \sqrt{\left(\frac{48}{5}\right)^2 - 16} = \pm \frac{3}{4} \sqrt{119}.$$

Shartni $A_1(9,6; \frac{3}{4}\sqrt{119})$ va

$A_2(9,6; -\frac{3}{4}\sqrt{119})$ nuqtalari qanoatlantirar ekan.

24. $y^2 = 8x$ parabolada direktrisadan 4 birlik uzoqlikdagi nuqtani toping.

Bunday nuqtadan direktrisagacha masofa

$$d = \frac{p}{2} + x \text{ ga teng demak, } \frac{p}{2} + x = 4$$

$p = 4$ ekanligini hisobga olsak, $x = 2$ kelib chiqadi.

Uni tenglamaga qo'yib $y = \pm 4$ ekanligini topamiz.

Demak, $A_1(2; 4), A_2(2; -4)$ izlanayotgan nuqtalardir.

25. $\frac{x^2}{100} + \frac{y^2}{225} = 1$ ellips va $y^2 = 24x$ parabola kesishish nuqtalarini toping.

$$\frac{x^2}{100} + \frac{24x}{25 \cdot 9} = 1 \text{ dan } 3x^2 + 32x - 300 = 0$$

tenglamaga ega bo'lamiz. Bundan $x_1 = -\frac{50}{3}, x_2 = 6$

kelib chiqib, $x = 6$ shartni qanoatlantiradi, $y^2 = 144$.

Demak, $A_1(6;12), A_2(6;-12)$.

26. $M(1;\sqrt{3})$ nuqtaning qutb koordinatalarini
 $A(2;\frac{5\pi}{4})$ nuqta dekart koordinatalarini toping.

$$r = \sqrt{1^2 + (\sqrt{3})^2} = 2, \quad \operatorname{tg} \varphi = \frac{\sqrt{3}}{1} \quad \text{dan} \quad \varphi = \frac{\pi}{3}$$

Demak, $M(2;\frac{\pi}{3})$

$$x = 2 \cos \frac{5\pi}{4} = 2 \cos(\pi + \frac{\pi}{4}) = -2 \cos \frac{\pi}{4} = -\sqrt{2}$$

$$y = 2 \sin \frac{5\pi}{4} = 2 \sin(\pi + \frac{\pi}{4}) = -2 \sin \frac{\pi}{4} = -\sqrt{2}$$

Demak, $A(-\sqrt{2}; -\sqrt{2})$

27. $E(3;\frac{\pi}{4})$ va $F(4;\frac{3\pi}{4})$ nuqtalar orasidagi
 masofani toping

$$\angle EOF = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}.$$

Kosinuslar (xususan, Pifagor) teoremasidan foydalanib:

$$|EF|^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos \frac{\pi}{2}, \quad \text{ya'ni} \quad |EF| = 5$$

5.3. $x^2 + y^2 = ax$ ni qutb koordinatalarida,
 $r = 2a \sin \varphi$ ni dekart koordinatalarda yozing.

a) $x^2 + y^2 = r^2, x = r \cos \varphi, y = r \sin \varphi$ ekanligidan
 $r^2 = ar \cos \varphi$, ya'ni $r = a \cos \varphi$;

b) $\sqrt{x^2 + y^2} = 2a \cdot \frac{y}{\sqrt{x^2 + y^2}} \quad \text{dan}$

$$x^2 + y^2 = 2ay \quad \text{yoki} \quad x^2 + (y - a)^2 = a^2.$$

28. a) $2x^2 + 5y^2 - 12x + 10y + 13 = 0$ tenglamani kanonik ko'rinishga keltiring.

$2x^2 - 12x + 5y^2 + 10y + 13 = 0$ tenglamada to'la kvadratlar ajratamiz:

$$2[x^2 - 6x + 9 - 9] + 5[y^2 + 2y + 1] + 8 = 0. \text{ Bundan,}$$

$$2(x-3)^2 - 18 + 5(y-1)^2 - 5 + 13 = 0, \text{ ya'ni}$$

$$2(x-3)^2 + 5(y-1)^2 = 10$$

yoki $x' = x - 3$, $y' = y + 1$ parallel ko'chirish natijasida

$$2x'^2 + 5y'^2 = 10 \quad \text{yoki} \quad \frac{x'^2}{5} + \frac{y'^2}{2} = 1$$

Ellips tenglamasiga ega bo'lamiz.

b) $5x^2 - 4xy + 2y^2 - 24 = 0$

29. a) $2x^2 + 6\sqrt{3}xy - 4y^2 + 20x + 10y + \frac{480\sqrt{3} - 85}{7} = 0$

kanonik tenglamasini yozing.

$$\operatorname{ctg} 2\alpha = \frac{2+4}{6\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \text{dan} \quad 2\alpha = 60^\circ, \text{ ya'ni}$$

$$\alpha = 30^\circ$$

$$x = x' \cos 30^\circ - y' \sin 30^\circ = \frac{\sqrt{3}x' - x'}{2}$$

$$y = x' \sin 30^\circ + y' \cos 30^\circ = \frac{x' + \sqrt{3}y'}{2}$$

almashtirish yordamida tenglama quyidagi

ko'rinishga keladi:

$$5\left\{x' + 2(\sqrt{3} + 1)\right\}^2 - 4(\sqrt{3} + 1)^2 - 7\left\{\left[y' - \frac{10}{7}(\sqrt{3} - 1)\right]^2 - \frac{100}{49}(\sqrt{3} - 1)^2\right\} + \frac{480\sqrt{3} - 85}{7} = 0$$

$$\begin{cases} x'' = x' + (\sqrt{3} + 1) \\ y'' = y' - \frac{10}{7}(\sqrt{3} - 1) \end{cases}$$

almashtirishdan so'ng
ko'rinishga keladi, ya'ni

$$5x'^2 - 7y'^2 = 35$$

$$\frac{x'^2}{7} - \frac{y'^2}{5} = 1$$

giperbola tenglamasiga ega bo'lamiz.

b) $r = \frac{1}{2 - \sqrt{3} \cos \varphi}$ dekart kordinatalari

sistemasiga o'tkazing va kanonik ko'rinishga keltiring.

$$\sqrt{x^2 + y^2} = \frac{1}{2 - \sqrt{3} \cdot \frac{x}{\sqrt{x^2 + y^2}}} \quad \text{dan}$$

$$2\sqrt{x^2 + y^2} - \sqrt{3}x = 1 \quad \text{yoki} \quad 4(x^2 + y^2) = 1 + 2\sqrt{3}x + 3x^2$$

$$\text{kelib chiqadi.} \quad x^2 - 2\sqrt{3}x + 4y^2 = 1 \quad \text{da}$$

$$(x - \sqrt{3})^2 - 3 + 4y^2 = 1 \quad \text{uchun} \quad x' = x - \sqrt{3}; \quad y' = y$$

deb almashtirsak:

$$x'^2 + 4y'^2 = 4 \quad \text{ellips tenglamasiga ega bo'lamiz.}$$

I-bob bo'yicha uy vazifalari.

1. Tekislikda

uchta

$A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ nuqtalar berilgan.

Quyidagilar topilsin.

1^o. ABC uchburchak perimetri;

2^o. ABC uchburchak medinalari kesishishi nuqtasi;

3^o. ABC uchburchak yuzi;

4^o. C nuqtadan o'tuvchi to'g'ri chiziqlar dastasi;

5^o. A va B nuqtalardan o'tuvchi to'g'ri chiziq 4 xil tenglamasi;

6^o. C nuqtadan o'tib, AB chiziqqa parallel va perpendikulyar

bo'lgan to'g'ri chiziqlar;

7^o. C nuqtadan AB to'g'ri chiziqqacha bo'lgan masofa;

8^o. ABC uchburchak ichki burchaklari.

9^o. C nuqtadan o'tkazilgan mediana va bissektrisa

tenglamalari.

- 1). $A(-4;2), B(-1;4), C(1;2)$
- 2). $A(-5;0), B(1;4), C(1;-4)$
- 3). $A(1;4), B(3;-1), C(4;3)$
- 4). $A(-4;1), B(1;4), C(4;-1)$
- 5). $A(-2;-2), B(1;1), C(+2;6)$
- 6). $A(-4;-3), B(-1;-2), C(1;0)$
- 7). $A(-1;-1), B(1;4), C(4;-2)$
- 8). $A(-3;2), B(0;-4), C(2;4)$
- 9). $A(-2;3), B(-1;4), C(1;-3)$
- 10). $A(-1;-4), B(1;2), C(4;-2)$
- 11). $A(5;1), B(-2;1), C(0;5)$
- 12). $A(3;4), B(0;-3), C(-2;4)$
- 13). $A(0;3), B(1;1), C(3;1)$
- 14). $A(-2;4), B(1;-4), C(3;5)$
- 15). $A(1;1), B(5;3), C(2;-3)$

2. Quyida beriladigan shartni qanoatlantiruvchi $M(x, y)$ nuqtalar tenglamasini tuzing va grafigini chizing.

1) Koordinatalar boshigacha va $A(5; 0)$ nuqtagacha masofalar 2:1 nisbatda;

2) $A(-1; 0)$ gacha masofa $x = -4$ gacha masofadan ikki marta kichik;

3) $A(2; 0)$ gacha va $5x + 8 = 0$ gacha masofalar 5:4 nisbatda

4) $B(1; 0)$ gacha masofa $A(4; 0)$ gacha masofadan 2 marta kichik;

5) $A(2; 0)$ gacha va $2x + 5 = 0$ gacha masofalar 4:5 nisbatda;

6) $A(3; 0)$ gacha masofa $B(26; 0)$ gacha masofadan 2 marta kichik;

7) $A(0; 2)$ nuqtadan va $y - 4 = 0$ to'g'ri chiziqdan bir xil uzoqlikda;

8) Ordinatalar o'qidan va $x^2 + y^2 = 4x$ aylanadan bir xil uzoqlikda;

9) $A(2; 6)$ nuqtadan va $y + 2 = 0$ to'g'ri chiziqdan bir xil uzoqlikda;

10) $A(-4; 0)$ gacha masofa $O(0; 0)$ gacha masofadan 3 marta katta;

11) $A(-4; 2)$ nuqtadan va $x = 1$ to'g'ri chiziqdan bir xil uzoqlikda;

12) $B(2; 0)$ gacha masofa $A(6; 0)$ gacha masofadan 3 marta kichik;

13) $A(4; 0)$ gacha va $x + 3 = 0$ gacha masofalar 2:3 nisbatda;

14) $A(2;2)$ gacha masofa $B(16;0)$ gacha masofadan 3 marta katta;

15) $A(1;1)$ dan va $B(6;4)$ dan bir xil uzoqlikda;

3.Qutb koordinatalar sistemasida ($r = r(\varphi)$) tenglama bilan chiziq berilgan.

a) φ ga $[0,2\pi]$ oraliqdagi qiymatlarni berib, nuqtalar bo'yicha chiziqni yasang.

b) Dekart koordinatalariga o'tkazing va qanday chiziqligini aniqlang

v)Kanonik ko'rinishiga keltiring.

$$1) r = \frac{1}{(1 + \cos \varphi)} \quad 2) r = \frac{1}{(2 + \cos \varphi)}$$

$$3) r = \frac{4}{(2 - 3 \cos \varphi)} \quad 4) r = \frac{8}{(3 - \cos \varphi)}$$

$$5) r = \frac{1}{(2 + 2 \cos \varphi)} \quad 6) r = \frac{5}{(3 - 4 \cos \varphi)}$$

$$7) r = \frac{10}{(2 + \cos \varphi)} \quad 8) r = \frac{3}{(1 - 2 \cos \varphi)}$$

$$9) r = \frac{1}{(3 - 3 \cos \varphi)} \quad 10) r = \frac{5}{(6 + 3 \cos \varphi)}$$

$$11) r = \frac{9}{(5 - 4 \cos \varphi)} \quad 12) r = \frac{9}{(4 - 5 \cos \varphi)}$$

$$13) r = \frac{3}{(1 - \cos \varphi)} \quad 14) r = \frac{1}{(4 - \sqrt{3} \cos \varphi)}$$

$$15) r = \frac{1}{(2 - \sqrt{5} \cos \varphi)}$$

4. Ikkinchi tartibli chiziqli

$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$ tenglama bilan berilgan. Kanonik ko'rinishga keltiring va dastlabki koordinatalar sistemasida chizing.

- 1) $x^2 - 2xy + y^2 + 6x - 14y + 29 = 0$
- 2) $x^2 - 2xy + y^2 - 12x + 12y - 14 = 0$
- 3) $3x^2 - 2xy + 3y^2 - 4x - 4y - 12 = 0$
- 4) $x^2 - 6xy + y^2 - 4x - 4y + 12 = 0$
- 5) $x^2 - xy + y^2 - 2x - 2y - 2 = 0$
- 6) $3x^2 + 10xy + 3y^2 - 12x - 12y + 4 = 0$
- 7) $x^2 - 2xy + y^2 - 10x - 6y + 25 = 0$
- 8) $x^2 + 2xy + y^2 - 4y + 3 = 0$
- 9) $2x^2 + 6\sqrt{3}xy - 4y^2 - 9 = 0$
- 10) $x^2 - 3y^2 + 16x + 12y - 36 = 0$
- 11) $4xy + 3y^2 + 16x + 12y - 36 = 0$
- 12) $25x^2 + 10xy + y^2 - 1 = 0$
- 13) $8x^2 - 18xy + 9y^2 + 2x - 1 = 0$
- 14) $14x^2 + 24xy + 21y^2 - 4x + 18y - 139 = 0$
- 15) $9x^2 - 24xy + 16y^2 - 20x + 110y - 50 = 0$

II-bob. Oliy algebra elementlari.
§ 6. Determinantlar, xossalari. Kramer qoidasi.

$n \times n$ ta elementdan tuzilgan, quyidagicha

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdot & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & a_{n3} & \cdot & \cdot & \cdot & a_{nn} \end{vmatrix}$$

yoziuvchi son n – tartibli determinant deyiladi. Bunda a_{ij} – son i – yo'l (satri), j – ustunda turadi.

i – yo'l, j – ustun o'chirilishidan hosil bo'ladigan $(n - 1)$ – tartibli determinant a_{ij} – element minori deyiladi va M_{ij} ko'rinishda belgilanadi.

$A_{ij} = (-1)^{i+j} \cdot M_{ij}$ esa a_{ij} element algebraik to'ldiruvchisi deyiladi.

Ikkinchi tartibli determinant.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

tenglik yordamida aniqlanadi.

Agar n – tartibli determinant Δ – songa teng bo'lsa,

$$\Delta = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}$$

$$\Delta = a_{1j}A_{1j} + a_{2j}A_{2j} + \dots + a_{nj}A_{nj}$$

tengliklar o'rinni, ya'ni determinant istalgan satri (yoki ustuni) elementlari bilan shu elementlar algebraik to'ldiruvchilari ko'paytmalarining yig'indisiga teng (bu determinant hisoblashning asosiy teoremasidir)

Determinantlar quyidagi xossalarga ega:

$$1) \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} \quad 2) \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \quad 3) \begin{vmatrix} a^2 & ae \\ ae & e^2 \end{vmatrix} \quad 4) \begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix}$$

$$5) \begin{vmatrix} a^2 + ae + e^2 & a^2 - ae + e^2 \\ a + e & a - e \end{vmatrix}$$

$$6) \begin{vmatrix} \sin \alpha & \cos \alpha \\ \sin \beta & \cos \beta \end{vmatrix} \quad 7) \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \beta & \cos \beta \end{vmatrix}$$

$$8) \begin{vmatrix} \sin \alpha + \sin \beta & \cos \beta + \cos \alpha \\ \cos \beta - \cos \alpha & \sin \alpha - \sin \beta \end{vmatrix}$$

6.2. Uchburchak qoidasi yordamida quyidagi uchinchi tartibli determinantlarni hisoblang:

$$1) \begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$$

$$2) \begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}$$

$$3) \begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$$

$$4) \begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}$$

6.3. Qulay qator bo'yicha yoyib hisoblang:

$$1) \begin{vmatrix} 5 & 6 & 3 \\ 0 & 1 & 0 \\ 7 & 4 & 5 \end{vmatrix}$$

$$2) \begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}$$

$$3) \begin{vmatrix} 1 & e & 1 \\ 0 & e & 0 \\ e & 0 & -e \end{vmatrix}$$

$$4) \begin{vmatrix} 0 & a & -e \\ -a & 0 & c \\ e & -c & 0 \end{vmatrix}$$

6.4. Determinant xossalaridan foydalanib

hisoblang:

$$1) \begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$$

$$2) \begin{vmatrix} a^2 & a & 1 \\ a^2 & a & 1 \\ c^2 & c & 1 \end{vmatrix}$$

$$3) \begin{vmatrix} a & a & c \\ a & c & a \\ c & a & a \end{vmatrix}$$

$$4) \begin{vmatrix} a & x & x \\ x & a & x \\ x & x & c \end{vmatrix}$$

$$5) \begin{vmatrix} a+x & x & x \\ x & a+x & x \\ x & x & c+x \end{vmatrix}$$

$$6) \begin{vmatrix} 1+\cos\alpha & 1+\sin\alpha & 1 \\ 1-\sin\alpha & 1+\cos\alpha & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$7) \begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$$

$$8) \begin{vmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ 3 & -4 & -1 & -2 \\ 4 & 3 & 2 & -1 \end{vmatrix}$$

$$9) \begin{vmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \end{vmatrix}$$

$$10) \begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$$

6.5. Tenglamalar sistemasini a) Kramer formulasi yordamida, b) Gauss usulida eching.

$$1) \begin{cases} 2x - y - z = 4 \\ 3x + 4e - 2z = 11 \\ 3x - 2y + 4z = 11, \end{cases} \quad 2) \begin{cases} x + y + 2z = -1 \\ 2x - e + 2z = -4 \\ 4x + e + 4z = -2 \end{cases}$$

$$3) \begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases} \quad 4) \begin{cases} x_1 + y_2 + 2z + 3t = 1 \\ 3x - y - z - 2t = -4 \\ 2x + 3y - z - t = -6 \\ x + 2y + 3z - t = -4 \end{cases}$$

$$5) \begin{cases} x + 2y + 3z + 4t = 5 \\ 2x + y + 2z + 3t = 1 \\ 3x + 2y + z + 2t = 1 \\ 4x + 3y + 2z + t = -5 \end{cases} \quad 6) \begin{cases} y - 3z + 4t = -5 \\ x - 2z + 3t = -4 \\ 3x + 2y - 5t = 12 \\ 4x + 3y - 5z = 5 \end{cases}$$

$$7) \begin{cases} x - 3y + 5z - 7t = 12 \\ 3x - 5y + 7z - t = 0 \\ 5x - 7y + z - 3t = 4 \\ 7x - y + 3z - 5t = 16 \end{cases} \quad 8) \begin{cases} x + 2y = 5 \\ 3y + 4z = 18 \\ 7n + 8v = 68 \\ 9v + 10x = 55 \end{cases}$$

6.6. $A(x_1; y_1), B(x_2; y_2)$ nuqtalardan o'tuvchi to'g'ri chiziqli tenglamasini 3-tartibli determinant yordamida yozing.

6.7. Uchlari $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ nuqtalarda bo'lgan uchburchak kozi formulasini 3-tartibli determinant yordamida yozing.

6.8. Tenglamalarni eching:

$$1) \begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0 \quad 2) \begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0$$

Yuqori tartibli determinantlarni hisoblash

Yuqori tartibli determinantlar, asosan, xossalari yordamida diagonalling bir tomonidagi elementlarni nolga aylantirish yordamida hisoblanadi.

$$\begin{vmatrix} d_1 & & & \\ & d_2 & & * \\ & & \ddots & \\ & 0 & & \\ & & & d_n \end{vmatrix} = d_1 d_2 \dots d_n = \prod_{k=1}^n d_k$$

Misol.

$$1) \Delta = \begin{vmatrix} 1 & a_1 & a_2 & \dots & a_n \\ 1 & a+b_1 & a_2 & \dots & a_n \\ 1 & a_1 & a_2+b_2 & \dots & a_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ 1 & a_1 & a_2 & \dots & a_n+b_n \end{vmatrix} =$$

$$q = \begin{vmatrix} 1 & a_1 & a_2 & \dots & a_n \\ 1 & a_1 & a_2 & \dots & a_n \\ 1 & a_1 & a_2 & \dots & a_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ 1 & a_1 & a_2 & \dots & a_n \end{vmatrix} + \begin{vmatrix} 1 & a_1 & a_2 & \dots & a_n \\ 1 & b_1 & a_2 & \dots & a_n \\ 1 & a_1 & b_2 & \dots & a_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ 1 & a_1 & a_2 & \dots & b_n \end{vmatrix}$$

1-determinant nolga teng, chunki $\overline{2, n+1}$ ustunlardan a_i ($i = \overline{1, n}$) ko'paytuvchilar determinant belgisidan tashqariga chiqarilsa, hosil bo'lgan determinat barcha elementlari bir xil bo'lib qoladi.

2-determinat 1-yo'lini (-1) ga ko'paytirib qolgan barcha yo'llariga qo'shsak, u quyidagi ko'rinishga keladi.

$$\begin{vmatrix} 1 & a_1 & a_2 & \dots & a_n \\ 0 & b_1 - a_1 & 0 & \dots & 0 \\ 0 & 0 & b_2 - a_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & b_n - a_n \end{vmatrix}$$

Demak, $\prod_{k=1}^n (b_k - a_k)$.

Ba'zi hollarda determinat bir echim bir necha o'zgaruvchi ko'phadi deb qaralib, bu ko'phad chiziqli bo'luvchilarini topish mumkin bo'ladi. Chiziqli bo'luvchilar ko'paytmasi tartibi ko'phad tartibiga teng bo'lsa, ular ishorasi farqli bo'lishi mumkin, xolos.

Misol. 1) $f(x) = \begin{vmatrix} 1 & 2 & 3 & \dots & n \\ 1 & x+1 & 3 & \dots & n \\ 1 & 2 & x+1 & \dots & n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 2 & 3 & \dots & x+1 \end{vmatrix}$

Agar x o'rniga mos ravishda $1, 2, \dots, n-1$ qo'yilsa determinant nolga teng bo'ladi, ya'ni $f(x) = \pm(x-1)(x-2)\dots(x-n+1)$ bo'lishi mumkin.

Determinat yoyilmasida x^{n-1} had +1 koeffitsient bilan qatnashishini hisobga olsak,

$$f(x) = (x-1)(x-2)\dots(x-n+1) = \prod_{k=1}^{n-1} (x-k) \text{ bo'lishi}$$

kelib chiqadi.

$$2) \begin{vmatrix} 1 & x_1 & x_2 & \dots & x_n \\ 1 & x & x_2 & \dots & x_n \\ 1 & x_1 & x_2 & \dots & x_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1 & x_2 & \dots & x_n \\ 1 & x_1 & x_2 & \dots & x \end{vmatrix} = 0 \text{ tenglamani eching.}$$

Echish.

$$\begin{vmatrix} 1 & x_1 & x_2 & \dots & x_n \\ 0 & x-x_1 & 0 & \dots & 0 \\ 0 & 0 & x-x_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & x-x_n \end{vmatrix} = (x-x_1)(x-x_2)\dots(x-x_n) =$$

$$= \prod_{k=1}^n (x-x_k)$$

$$\prod_{k=1}^n (x-x_k) = 0 \quad \text{tenglama echimlari } x_1, x_2, \dots, x_n$$

bo'ladi.

1. Determinantlar xossalariidan foydalanib hisoblang:

$$\text{a)} \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$$\text{b)} \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix}$$

$$\text{v)} \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & a & b \\ 1 & a & 0 & c \\ 1 & b & c & 0 \end{vmatrix}$$

$$\text{g)} \begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$$

$$\text{v)} \begin{vmatrix} 0 & a & b & c & d \\ -a & 0 & e & f & g \\ -b & -e & 0 & h & k \\ -c & -f & -h & 0 & e \\ -d & -g & -k & -l & 0 \end{vmatrix}$$

$$\text{e)} \begin{vmatrix} 1 & 2 & 3 & . & . & n \\ -1 & 0 & 3 & . & . & n \\ -1 & -2 & 0 & . & . & n \\ . & . & . & . & . & . \\ -1 & -2 & -3 & . & . & 0 \end{vmatrix} \quad \text{f)} \begin{vmatrix} 1 & 2 & 2 & . & . & 2 \\ 2 & 2 & 2 & . & . & 2 \\ 2 & 2 & 3 & . & . & 2 \\ . & . & . & . & . & . \\ 2 & 2 & 2 & . & . & n \end{vmatrix}$$

$$g) \begin{vmatrix} 0 & 1 & 1 & . & . & 1 \\ 1 & 0 & 1 & . & . & 1 \\ 1 & 1 & 0 & . & . & 1 \\ . & . & . & . & . & . \\ 1 & 1 & 1 & . & . & 0 \end{vmatrix}$$

$$h) \begin{vmatrix} n & 1 & 1 & . & . & 1 \\ 1 & n & 1 & . & . & 1 \\ 1 & 1 & n & . & . & 1 \\ . & . & . & . & . & . \\ 1 & 1 & 1 & . & . & n \end{vmatrix}$$

$$z) \begin{vmatrix} 1 & 2 & 0 & 0 & . & . & 0 \\ 1 & 3 & 2 & 0 & . & . & 0 \\ 0 & 1 & 3 & 2 & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{vmatrix}$$

2. Chiziqli ko'paytuvchilarni ajratish yordamida hisoblang.

$$a) \begin{vmatrix} 1 & x_1 & x_2 & . & . & x_{n-1} & x_n \\ 1 & x & x_2 & . & . & x_{n-1} & x_n \\ 1 & x_1 & x & . & . & x_{n-1} & x_n \\ . & . & . & . & . & . & . \\ 1 & x_1 & x_2 & . & . & x_{n-1} & x \end{vmatrix} \quad b) \begin{vmatrix} -x & a & b & c \\ a & -x & c & b \\ b & c & -x & a \\ c & b & a & -x \end{vmatrix}$$

$$i) \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{vmatrix}$$

3. $f(x, \xi, \dots) = 0$ tenglamani eching.

$$a) f(x) = \begin{vmatrix} x & a_1 & a_2 & . & . & a_{n-1} & 1 \\ a_1 & x & a_2 & . & . & a_{n-1} & 1 \\ a_1 & a_2 & x & . & . & a_{n-1} & 1 \\ . & . & . & . & . & . & . \\ a_1 & a_2 & a_3 & . & . & x & 1 \\ a_1 & a_2 & a_3 & . & . & a_n & 1 \end{vmatrix}$$

$$b) f(x) = \begin{vmatrix} 1 & x_1 & x_1^2 & . & . & x_1^{n-1} \\ 1 & x_2 & x_2^2 & . & . & x_2^{n-1} \\ 1 & x_3 & x_3^2 & . & . & x_3^{n-1} \\ . & . & . & . & . & . \\ 1 & x_n & x_n^2 & . & . & x_n^{n-1} \end{vmatrix}$$

§7. Matritsalar. Chiziqli tenglamalar sistemasini teskari matritsa yordamida echish.

Turli tabiatli a_{ij} ($i = \overline{1, m}, j = \overline{1, n}$) sonlardan tuzilgan

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2n} \\ . & . & . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & . & . & . & a_{nn} \end{pmatrix}$$

jadval $n \times m$ o'lchamli matritsa deyiladi.

$n=m$ bo'lgan holda matritsa kvadrat matritsa deyiladi.

Kvadrat matritsa elementlari nolardan iborat bo'lsa, nol matritsa, $i=j$ da 1, $j=1$ da 0 bo'lsa,

$$\begin{pmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & \cdot & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{pmatrix},$$

ya'ni $A \cdot X = B$

ko'rinishda yozish mumkin. Demak, $X = A^{-1} \cdot B$.

$$7.1. A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

bo'lsa, $A^2 + 2B - 5C$ ni hisoblang.

$$7.2. A = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \text{ bo'lsa, } A^4 - B^4 \text{ ni hisoblang.}$$

$$7.3. \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}^n \text{ ni hisoblang.}$$

$$7.4. A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix} \text{ bo'lsa,}$$

$A \cdot B - B \cdot A$ ni hisoblang.

7.5. Berilgan matritsalariga teskari matritsalarini toping.

$$1) A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix},$$

$$2) A = \begin{pmatrix} a & e \\ c & d \end{pmatrix},$$

$$3) A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$4) A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 3 \end{pmatrix}.$$

$$5) A = \begin{pmatrix} 1 & 3 & -5 & 7 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad 6) A = \begin{pmatrix} 0 & 1 & . & 1 \\ 1 & 0 & 1 & 1 \\ . & . & . & . \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

7.6. Noma'lum matritsani toping

$$1) \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$$

$$2) \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$$

$$3) \begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} \cdot X \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix},$$

$$4) \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}.$$

7.7. Quyidagi sistemalarni matritsaviy usulda eching.

$$1) \begin{cases} 3x + 4y = 11 \\ 5y + 6z = 28, \\ x + 2z = 7 \end{cases} \quad 2) \begin{cases} x + 3y + 5z + 7t = 12 \\ 3x + 5y + 7z + t = 0 \\ 5x + 7y + z + 3t = 4 \\ 7x + y + 3z + 5t = 16 \end{cases}$$

$$3) \begin{cases} x+2z=0 \\ y+2n=0 \\ x+y+v=0 \\ z-n=2 \\ n+v=-1 \end{cases}$$

§ 8. Kompleks sonlar, formalari. Muavr formulalari.

Haqiqiy x, y sonlar yordamida tuzilgan $z = x + iy$ son kompleks son $i = \sqrt{-1}$ esa mavhum birlik deyiladi. Bunda x kompleks sonning haqiqiy qismi, y esa mavhum qismi deyiladi.

$z = x + iy$ yozuv kompleks sonning algebraik formasi deyiladi, amallar bu formada quyidagicha kiritiladi:

$$z_1 \pm z_2 = (x_1 + iy_1) \pm (x_2 + iy_2) = (x_1 \pm x_2) + (y_1 \pm y_2)i$$

$$z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i \frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}.$$

$z = x + iy$ uchun $x - iy$ son qo'shma kompleks son deyiladi va $\bar{z}$ tarzida belgilanadi. $z = x + iy$

songa tekislikdagi $A(x, y)$ nuqtani mos qo'yish

mumkin, $|OA| = r = \sqrt{x^2 + y^2}$ kompleks son moduli,

OX o'qi bilan hosil qilgan φ burchagi kompleks son

argumenti deyiladi va $\operatorname{tgy} = \frac{y}{x}$ ko'rinishda yoziladi.

$x = r \cos \varphi$, $y = r \sin \varphi$ ekanligidan

$z = x + yi = r(\cos \varphi + i \sin \varphi)$ kelib chiqadi. Oxirgi yozuv kompleks son trigonometrik formasi deyiladi, amallar quyidagicha kiritiladi:

$$Z_1 \cdot Z_2 = r_1(\cos \varphi_1 + i \sin \varphi_1) \cdot r_2(\cos \varphi_2 + i \sin \varphi_2) = \\ = r_1 r_2 [\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2)],$$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} [\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)],$$

$$Z^n = [r(\cos \varphi + i \sin \varphi)]^n = r^n (\cos n\varphi + i \sin n\varphi),$$

$$\sqrt[n]{r(\cos \varphi + i \sin \varphi)} = \sqrt[n]{r} \left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n} \right), \quad k = 0, 1, \dots, (n-1)$$

Oxirgi ikkita formula Muavr formulalari deyiladi.

$e^{i\varphi} = \cos \varphi + i \sin \varphi$ tenglik o'rinli bo'lib Eyler formulasi deyiladi.

8.1. Amallarni bajaring.

- | | |
|-------------------------|-------------------------|
| 1) $(2 + 3i)(3 - 2i)$, | 2) $(a + bi)(a - bi)$, |
| 3) $(3 - 2i)^2$ | 4) $(1 + i)^3$, |
| 5) $\frac{1+i}{1-i}$, | 6) $\frac{2i}{1+i}$ |

8.2. Tenglamalarni eching.

- | | |
|------------------------|-------------------------|
| 1) $x^2 + 4 = 0$, | 2) $x^2 - 2x + 5 = 0$, |
| 3) $x^2 + 4x + 13 = 0$ | 4) $x^8 - 1 = 0$, |
| 5) $x^6 + 1 = 0$, | 6) $x^4 + 1 = 0$ |

8.3. Berilgan kompleks sonni trigonometrik formada tasvirlang.

- | | |
|--------------|---------------|
| 1) $z = 3$, | 2) $z = 2i$, |
|--------------|---------------|

- 3) $z = 2 - 2i$, 4) $z = \sqrt{3} + i$
 5) $z = -\sqrt{3} - i$, 6) $z = \sqrt{2} - \sqrt{2}i$,
 7) $z = \sin \alpha + i(1 - \cos \alpha)$

8.4. Muavr formulasidan foydalanmay, hisoblang:

- 1) $\frac{(1-i)^5 - 1}{(1+i)^5 + 1}$, 2) $\frac{(1+i)^5}{(1-i)^7}$, 3) $\sqrt{2}i$, 4) $\sqrt{1-i\sqrt{3}}$.

8.5. Muavr formulalari bo'yicha hisoblang.

- 1) $(1+i)^{25}$, 2) $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$, 3) $\sqrt[3]{i}$,
 4) $\sqrt[3]{2-2i}$, 5) $\sqrt[6]{-27}$, 6) $\sqrt[6]{1}$
 7) $\sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}$

§9. IOqori darajali tenglamalar.

Algebraning asosiy teoremasi.

$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$ tenglama
 n -darajali tenglama deyiladi.

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

ko'phadni $(x-a)$ ga bo'lganda hosil bo'ladigan qoldiq berilgan ko'phadning $x=a$ dagi qiymati $f(a)$ ga teng bo'ladi.

Teorema (Bezu): a son $f(x)$ ko'phadning ildizi bo'lishi uchun $f(x)$ ning $(x-a)$ ga bo'linishi zarur va etarli.

Teorema (algebraning asosiy teoremasi): Darajasi birdan kichik bo'lmagan ixtiyoriy ko'phad kamida bitta ildizga ega.

Natija: ixtiyoriy n -darajali ($n \geq 1$) ko'phad n ta ildizga ega va

$$(x - \alpha_1)^{l_1} \cdot (x - \alpha_2)^{l_2} \dots (x - \alpha_m)^{l_m} \cdot (x^2 + p_1x + q_1)^{r_1} \cdot (x^2 + p_2x + q_2)^{r_2} \dots (x^2 + p_kx + q_k)^{r_k}$$

ko'rinishda yoziladi, bunda

$$l_1 + l_2 + \dots + l_m + 2(r_1 + r_2 + \dots + r_k) = k$$

$$x^3 + a_1x^2 + a_2x + a_3 = 0 \quad \text{tenglama} \quad x = z - \frac{a_1}{3}$$

almashtirishda

$z^3 + pz + q = 0$ ko'rinish oladi, ildizlari esa Kardanof formulasidan topiladi:

$$z = u + v = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

$$\Delta = \frac{q^2}{4} + \frac{p^3}{27}$$

diskriminant deyiladi. $u_1 = \operatorname{Re} u, v_1 = \operatorname{Re} v$.

1) $\Delta > 0$ bo'lsa,

$$z_1 = u_1 + v_1, z_{2,3} = -\frac{u_1 + v_1}{2} \pm \frac{u_1 - v_1}{2} \cdot i\sqrt{3}.$$

2) $\Delta = 0$ bo'lsa, $z_1 = \frac{3q}{p}, z_2 = z_3 = -\frac{z_1}{2}$.

3) $\Delta < 0$ bo'lsa, $z_1 = 2\sqrt{-\frac{p}{3}} \cos \frac{\varphi}{3},$

$$z_{2,3} = 2\sqrt{-\frac{p}{3}} \cos(\frac{\varphi}{3} \pm 120^\circ), \quad \text{bunda}$$

$$\cos \varphi = -\frac{q}{2} \div \sqrt{-\frac{p^3}{27}}.$$

Agar $f(a)$ va $f(b)$ turli ishorali bo'lsa (a, b) intervalda $f(x) = 0$

Tenglamani kamida bitta ildizi bor.

9.1. Biror ildizni tanlab tenglamani eching:

a) $x^3 - 4x^2 + x + 6 = 0$

b) $x^3 - 4x^2 - 4x - 5 = 0$

v) $x^4 + x^3 + 2x - 4 = 0$

g) $4x^3 - 4x^2 + x - 1 = 0$

9.2. Kardano formulasi bo'yicha eching:

a) $z^3 - 6z - 9 = 0$

b) $z^3 - 12z - 8 = 0$

v) $z^3 - 6z - 7 = 0$

g) $x^3 + 9x^2 + 18x + 9 = 0$

9.3. Ildizlari berilgan sonlar bo'lgan tenglamani tuzing:

a) 2; 1; -2; 3

b) -1 uch karrali, 1 va $2+i$

9.4. Ko'paytuvchilarga ajrating:

a) $x^8 - 1$

b) $x^5 + 1$

v) $x^5 - 1$

g) $x^4 + 4$

Yuqori tartibli tenglamalar ratsional ildizlari.

1. Agar qisqarmas $\frac{p}{q}$ - ratsional son ($p \in Z, q \in N$)

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$$

tenglama ildizi bo'lsa, $q \in N$ soni a_0 ning, $p \in Z$ soni esa a_n ning bo'luvchilari bo'ladi. Chunki

$$a_0 p^n + a_1 p^{n-1} q + a_2 p^{n-2} q^2 + \dots + a_{n-1} p q^{n-1} + a_n q^n = 0$$

dan

$$\begin{cases} a_0 p^n = -q(a_1 p^{n-1} + a_2 p^{n-2} q + \dots + a_{n-1} p q^{n-2} + a_n q^{n-1}) \\ a_n q^n = -p(a_0 p^{n-1} + a_1 p^{n-2} q + a_2 p^{n-3} q^2 + \dots + a_{n-1} q^{n-1}) \end{cases}$$

tengliklar kelib chiqib, yuqoridagilarni tasdiqlaydi.

Misollar: 1) $x^4 + 2x^2 - 13x^2 - 38x - 24 = 0$

tenglamada $a_0 = 1, a_n = -24$. Demak, $q = 1$:

$p = \pm 1; \pm 2; \pm 3; \pm 4; \pm 6; \pm 8; \pm 12; \pm 24$ bo'lishi mumkin.

O'rniga qo'yib tekshirishlar

$x_1 = -1; x_2 = -2; x_3 = -3; x_4 = 4$ ekanligini, ya'ni

$$\text{tenglama } (x+1)(x+2)(x+3)(x-4) = 0$$

Ko'rinishda yozilishi mumkinligini ko'rsatadi.

2) $24x^5 + 10x^4 - x^3 - 19x^2 - 5x + 6 = 0$ tenglama

uchun $p = \pm 1; \pm 2; \pm 3; \pm 6$, $q = 1; 2; 3; 4; 6; 8; 12; 24$

bo'lishi mumkin.

$$\left\{ \frac{p}{q} \right\} = \left\{ \frac{1}{2}; -\frac{2}{3}; \frac{3}{4} \right\} \text{ ekanligini topish mumkin, xolos.}$$

Qolgan ikki ildiz irratsional yoki kompleks ekanligi kelib chiqdi. Berilgan tenglama chap tomonidagi

ko'phadni $\left(x - \frac{1}{2}\right)\left(x + \frac{2}{3}\right)\left(x - \frac{3}{4}\right)$ ga qisqartirib

(bo'lib), bo'linma ko'phadni nolga tenglab qolgan ikki ildizi topiladi.

1. Yuqori darajali tenglamalar uchun Viet teoremasi.

Keltirilgan $x^2 + px + q = 0$ tenglama uchun x_1, x_2

ildizlar bo'lsa, $x_1 + x_2 = -p$, $x_1 \cdot x_2 = q$ (Viet teoremasi)

kelib chiqishi elementar matematikadan ma'lum.

$x^3 + a_0 x^2 + a_1 x + a_2 = 0$ tenglama ildizlari x_1, x_2, x_3

bo'lsa, tenglama $(x-x_1)(x-x_2)(x-x_3)=0$ ga
ekvivalent bo'ladi va

$$x^3 - (x_1 + x_2 + x_3)x^2 + (x_1x_2 + x_2x_3 + x_1x_3)x - x_1x_2x_3 = 0$$

kubik tenglama uchun Viet tengliklari

$$\begin{cases} x_1 + x_2 + x_3 = -a_0 \\ x_1x_2 + x_1x_3 + x_2x_3 = a_1 \\ x_1x_2x_3 = -a_2 \end{cases}$$

ko'rinishda bo'lishi kelib chiqadi.

$$x^4 + a_0x^3 + a_1x^2 + a_2x + a_3 = 0$$

Tenglama uchun Viet tengliklari

$$\begin{cases} a_0 = -(x_1 + x_2 + x_3 + x_4) \\ a_1 = x_3x_4 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_1x_2 \\ a_2 = -(x_1x_3x_4 + x_2x_3x_4 + x_1x_2x_3 + x_1x_2x_4) \\ a_3 = x_1x_2x_3x_4 \end{cases}$$

ko'rinishda bo'ladi.

Misollar:

1. Agar $x^3 + px + q = 0$ tenglama ildizlari x_1, x_2, x_3

bo'lsa, $\frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_1} + \frac{x_2}{x_1} + \frac{x_3}{x_2} + \frac{x_1}{x_3}$ ni hisoblang.

$x_1 + x_2 + x_3 = 0$ ekanligi ma'lum, undan

$x_1 + x_2 = -x_3$ deyish mumkin. U holda

$$\begin{aligned} & \frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_1} + \frac{x_2}{x_1} + \frac{x_3}{x_2} + \frac{x_1}{x_3} = \\ &= \frac{1}{x_1x_2x_3} \cdot [x_1^2x_3 + x_2^2x_1 + x_2x_3^2 + x_2^2x_3 + x_1x_3^2 + x_1^2x_2] = \\ &= \frac{1}{x_1x_2x_3} \cdot [x_1^2(x_2 + x_3) + x_2x_3(x_3 + x_2) + x_1(x_2^2 + x_3^2)] = \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{x_1 x_2 x_3} \cdot [-x_1^3 - x_1 x_2 x_3 + x_1(x_2^2 + x_3^2)] = \frac{1}{x_2 x_3} [-x_1^2 - x_2 x_3 + x_2^2 + x_3^2] = \\
 &= \frac{1}{x_2} [x_1 - x_2 - x_2 + x_3] = \frac{1}{x_2} [x_1 + x_2 x_3 - 3x_2] = \frac{1}{x_2} (-3x_2) = -3
 \end{aligned}$$

2. $x^3 + a_0 x^2 + a_1 x + a_2 = 0$ tenglama ildizlari x_1, x_2, x_3 bo'lsa, ildizlari $x_1 + x_2, x_2 + x_3, x_3 + 1$ bo'lgan tenglama tuzing.

3. $x^3 + 2x - 3 = 0$ uchun $x_1 + x_2 + x_3, x_1 \cdot x_2 \cdot x_3$ va $x_1^2 + x_2^2 + x_3^2$ larni toping.

Bob bo'yicha misollar echish namunalari

1. $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$ va $\begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix}$ larni hisoblang

$$\begin{aligned}
 \text{a) } \begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix} &= (a+b)(a+b) - (a-b)(a-b) = \\
 &= (a+b)^2 - (a-b)^2 = (a+b+a-b)(a+b-a+b) = \\
 &= 2a2b = 4ab
 \end{aligned}$$

b)

$$\begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix} = \cos \alpha \cos \alpha - \sin \alpha \sin \alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$2. \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \text{ determinantni hisoblang.}$$

$$\begin{aligned} \text{Teorema ga ko'ra } \Delta &= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = \\ &= a_{11}(-1)^{1+1}M_{11} + a_{12}(-1)^{1+2}M_{12} + a_{13}(-1)^{1+3}M_{13} = \\ &= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \\ &= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - \\ &- a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} \end{aligned}$$

Oxirgi oltita qo'shiluvchilar uchburchak qoidasi deb ataluvchi quyidagi chiziqlardan topiladi:



3. Determinant xossalaridan foydalanib hisoblang.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 4 & 9 & 16 & 25 \\ 8 & 27 & 64 & 125 \end{vmatrix}$$

i -yo'lni 2 ga ko'paytirib $(i+1)$ - yo'ldan ayiramiz:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 4 & 9 & 16 & 25 \\ 8 & 27 & 64 & 125 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 8 & 15 \\ 0 & 9 & 32 & 75 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 8 & 15 \\ 9 & 32 & 75 \end{vmatrix}$$

i-yo'lni 3 ga ko'paytirib $(i+1)$ -yo'ldan ayiramiz:

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 8 & 30 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 8 & 30 \end{vmatrix} = 60 - 48 = 12$$

$$4. \begin{cases} x + 2y + 3z = 14 \\ y + 2z + 3t = 20 \\ 3x + z + 2t = 14 \\ 2x + 3y + t = 12 \end{cases}$$

tenglamalar sistemasini Kramer qoidasi va Gaussning noma'lumlarni ketma-ket yo'qotish usuli yordamida eching.

a) Asosiy determinant va yordamchi determinantlar quyidagicha ko'rinishda bo'ladi:

$$\Delta = \begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \end{vmatrix}, \quad \Delta_x = \begin{vmatrix} 14 & 2 & 3 & 0 \\ 20 & 1 & 2 & 3 \\ 14 & 0 & 1 & 2 \\ 12 & 3 & 0 & 1 \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} 1 & 14 & 3 & 0 \\ 0 & 20 & 2 & 3 \\ 3 & 14 & 1 & 2 \\ 2 & 12 & 0 & 1 \end{vmatrix}.$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 14 & 0 \\ 0 & 1 & 20 & 3 \\ 3 & 0 & 14 & 2 \\ 2 & 3 & 12 & 1 \end{vmatrix}.$$

$$\Delta_t = \begin{vmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 2 & 20 \\ 3 & 0 & 1 & 14 \\ 2 & 3 & 0 & 12 \end{vmatrix};$$

Determinant xossalaridan foydalanib $\Delta=96$, $\Delta_x=96$

$\Delta_y = 192, \Delta_z = 288, \Delta_t = 384$ ekanligini topamiz,

Demak, $x = 1; y = 2; t = 4$

b) tenglamalar sistemasi koeffitsientlaridan tuzilgan quyidagi jadvalni tuzamiz:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 3 & 0 & 1 & 2 & 14 \\ 2 & 3 & 0 & 1 & 12 \end{array} \right)$$

Tenglamalar sistemasidagi biror tenglamani songa ko'paytirib, boshqa tenglamaga qo'shsak, ekvivalent tenglama hosil bo'ladi. Demak, 1-tenglamani mos ravishda (-3) va (-2) ga ko'paytirib, 3- va 4-tenglamalarga qo'shamiz.

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 0 & -6 & -8 & 2 & -28 \\ 0 & -1 & -6 & 1 & -16 \end{array} \right)$$

Endi 3-va 4-tenglamalarni mos ravishda 6 va 1 ga ko'paytirilgan 2-tenglamaga qo'shamiz:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 0 & 0 & 4 & 20 & 92 \\ 0 & 0 & -4 & 4 & 4 \end{array} \right)$$

Oxirgi tenglamalarni qo'shib quyidagi natijaga kelamiz:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 14 \\ 0 & 1 & 2 & 3 & 20 \\ 0 & 0 & 4 & 20 & 92 \\ 0 & 0 & 0 & 24 & 96 \end{array} \right)$$

Oxirgi jadvalga mos sistema quyidagi ko'rinishda bo'ladi:

$$\left. \begin{array}{l} x + 2y + 3z = 14 \\ y + 2z + 3t = 20 \\ 4z + 20t = 92 \\ 24t = 96 \end{array} \right\}$$

Bundan $t = 4, z = 3, y = 2, x = 1$ ekanligini ketma-ket topishimiz qiyin emas.

Qarab chiqilgan usul Gaussning noma'lumlarni ketma-ket yo'qotish usuli bo'lib, talabalarga o'rta maktab kursidan tanish.

5. $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ bo'lsa, $A^2 - 2B$ ni hisoblang.

$$A^2 = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix} \quad \text{va}$$

$$2 \cdot B = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \text{ ekanligidan,}$$

$$A^2 - 2B = \begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -11 & 0 \end{pmatrix} \quad \text{kelib}$$

chiqadi.

6. $A = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{pmatrix}$ ga teskari A^{-1} matritsani toping.

$$|A| = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{vmatrix} = -6.$$

$$A_{11} = \begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix} = 14, \quad A_{21} = -\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 5,$$

$$A_{31} = \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = 13, \quad A_{12} = -\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} = -10,$$

$$A_{22} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -4, \quad A_{32} = -\begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix} = 8,$$

$$A_{13} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2, \quad A_{23} = -\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 1,$$

$$A_{33} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

Demak:

$$A^{-1} = \frac{-1}{6} \cdot \begin{pmatrix} 14 & 5 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{pmatrix}$$

$$7. \begin{cases} 3x + 2y + 2z = 13 \\ x + 3y + z = 10 \\ 5x + 3y + 4z = 23 \end{cases}$$

tenglamalar sistemasini matritsaviy usulda eching.

Sistemani $A \cdot X = B$ ko'rinishda yozib olamiz, bunda

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B = \begin{pmatrix} 13 \\ 10 \\ 23 \end{pmatrix}.$$

A matritsa determinanti va elementlar to'ldiruvchilarini hisoblaymiz:

$$|A| = \begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} = 27 + 2 - 24 = 5.$$

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 3 & 4 \end{vmatrix} = 9, \quad A_{21} = -\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2,$$

$$A_{31} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -4,$$

$$A_{12} = -\begin{vmatrix} 1 & 1 \\ 5 & 4 \end{vmatrix} = 1, \quad A_{22} = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 2,$$

$$A_{32} = -\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -1, \quad A_{13} = \begin{vmatrix} 1 & 3 \\ 5 & 3 \end{vmatrix} = -12,$$

$$A_{23} = -\begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} = 1, \quad A_{33} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7.$$

$$\text{Demak, } A^{-1} = \frac{1}{5} \cdot \begin{pmatrix} 9 & -2 & -4 \\ 1 & 2 & -1 \\ -12 & 1 & 7 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \frac{1}{5} \begin{pmatrix} 9 & -2 & -4 \\ 1 & 2 & -1 \\ -12 & 1 & 7 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 10 \\ 23 \end{pmatrix} =$$

$$= \frac{1}{5} \cdot \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Ya'ni $x = 1$, $y = 2$, $z = 3$.

8. Amallarni bajaring:

a) $(3-i) + (4+2i) = (4+3) + (2-1)i = 7+i$

b) $(3-i) \cdot (4+2i) = (12+2) + i(6-4) = 14+2i$

v) $\frac{3-i}{2(2+i)} = \frac{3-i}{2(2+i)} \cdot \frac{2-i}{2-i} = \frac{7-5i}{2 \cdot 5} = 0,7 - 0,5i$

r) $i^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = -i$.

9. $\sqrt{3-4i}$ ni hisoblang.

$\sqrt{3-4i} = \alpha + \beta i$ chiqadi deb faraz qilamiz, demak,

$$(\alpha + \beta i)^2 = \alpha^2 - \beta^2 + 2\alpha\beta i = 3 - 4i.$$

Bundan: $\begin{cases} \alpha^2 - \beta^2 = 3 \\ 2\alpha\beta = -4 \end{cases}$ sistema hosil bo'lib,

$(2; -1)$, $(-2; 1)$ echimlar ekanligini topish mumkin.

Demak: $\sqrt{3-4i} = \begin{cases} 2-2i \\ -2+2i \end{cases}$

10. $(1+i)^{10}$ ni hisoblang.

$r = \sqrt{2}$, $\operatorname{tg} \varphi = \frac{y}{x} = \frac{1}{1} = 1$, ya'ni $\varphi = \frac{\pi}{4}$

ekanligidan foydalanib:

$$\begin{aligned}
 (1+i)^{10} &= (\sqrt{2})^{10} \cdot \left(\cos 10 \cdot \frac{\pi}{4} + i \sin 10 \cdot \frac{\pi}{4} \right) = \\
 &= 2^5 \left(\cos \frac{5\pi}{2} + i \sin \frac{5\pi}{2} \right) = \\
 &= 32 \left[\cos \left(2\pi + \frac{\pi}{2} \right) + i \sin \left(2\pi + \frac{\pi}{2} \right) \right] = \\
 &= 32(0+i) = 32i
 \end{aligned}$$

11. $\sqrt[3]{-2+2i}$ ni hisoblang.

$$r = \sqrt{4+4} = 2\sqrt{2}; \quad \operatorname{tg} \varphi = \frac{2}{-2} = -1, \quad \varphi = \frac{3\pi}{4}$$

bo'lgani uchun

$$\sqrt[3]{-2+2i} = \sqrt[3]{2\sqrt{2}} \left(\cos \frac{\frac{3\pi}{4} + 2k\pi}{3} + i \sin \frac{\frac{3\pi}{4} + 2k\pi}{3} \right),$$

$$z_0 = \sqrt[3]{\sqrt{2^3}} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) = \sqrt[3]{2^3} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = 1+i;$$

$$z_1 = \sqrt{2} \left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right) = \sqrt{2} \left(-\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$$

$$z_2 = \sqrt{2} \left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12} \right) = \sqrt{2} \left(+\cos \frac{5\pi}{12} - i \sin \frac{5\pi}{12} \right)$$

12. Biror $x = \alpha_1$ ildizini tanlab topib, so'ngra qolgan ildizlarini toping.

$$a) x^5 - x^4 - 16x + 16 = 0 \quad \text{tenglama ildizi,}$$

$$x_0 = 1$$

shuning uchun $(x-1)$ ni ajratamiz:

$$x^4(x-1) - 16(x-1) = 0 \quad \text{yoki}$$

$$(x-1)(x^2+4)(x-2)(x+2) = 0.$$

bundan ildizlar $1, \pm 2, \pm 2i$ ekanligi kelib chiqadi.

$$b) x^4 + x^3 + 2x - 28 = 0; \quad x_1 = 2$$

$$x^4 - 2x^3 + 3x^3 - 6x^2 + 6x^2 - 12x + 14x - 28 = 0$$

$$x^3(x-2) + 3x^2(x-2) + 6x(x-2) + 14(x-2) = 0$$

$$(x-2)(x^3 + 3x^2 + 6x + 14) = 0$$

Ikkinchi qavsda $x = z - 1$ almashtirish bajaramiz va

$$z^3 + 3z + 10 = 0 \quad \text{ko'rinishga kelimiz;}$$

Demak, $\Delta > 0$ ekanligidan, Kardano formulasiga ko'ra:

$$x_2 = \sqrt[3]{-5 + \sqrt{25+1}} + \sqrt[3]{-5 - \sqrt{26}}$$

va x_3, x_4 ni ham topish mumkin.

13. Ko'phad ildizlari berilgan: ikki karrali 1, 2, 3 va $1+i$.

Bu ko'phadni yozing:

$(1-i)$ ham ko'phad ildizi bo'ladi, chunki $1 \pm i$

$x^2 - 2x + 2 = 0$ ning ildizlardir. Demak, izlangan ko'phad quyidagicha bo'ladi;

$$(x-1)^2(x-2)(x-3)(x^2-2x+2)$$

II-Bob bo'yicha uy vazifalari.

$$1. \begin{cases} ax + by + cz = d \\ bx - cy + az = e \\ cx + ay - bz = f \end{cases} \quad \text{tenglamalar sistemasi}$$

berilgan. Uch usulda eching: a) Kramer qoidasi, b) Gauss usuli, v) Matritsaviy.

- 1) $a = 1; b = 2; c = 3; d = 14; e = -1; f = -1.$
- 2) $a = 2; b = 1; c = 4; d = 5; e = -2; f = 10.$
- 3) $a = 3; b = 2; c = -2; d = 1; e = 7; f = -2$
- 4) $a = 2; b = -1; c = 1; d = -1; e = -5; f = -4$
- 5) $a = 1; b = 3; c = 0; d = 17; e = 18; f = -5$
- 6) $a = 1; b = 0; c = 2; d = 8; e = -3; f = 5$
- 7) $a = 2; b = -1; c = 3; d = 15; e = 9; f = 8$
- 8) $a = 3; b = -2; c = -1; d = 14; e = 2; f = -11$
- 9) $a = 2; b = -2; c = 3; d = 1; e = -3; f = 9$
- 10) $a = 4; b = 1; c = 1; d = 4; e = -6; f = 4$
- 11) $a = 5; b = 1; c = -1; d = 12; e = -2; f = 4$
- 12) $a = 5; b = -1; c = 12; d = 12; e = 26; f = 10$
- 13) $a = 2; b = -1; c = 1; d = 5; e = -17; f = 24$
- 14) $a = 4; b = 1; c = 2; d = 5; e = -7; f = 24$
- 15) $a = 8; b = 2; c = 3; d = -1; e = -22; f = 9.$

$$2). z = \frac{a}{b + ci} \quad \text{berilgan.}$$

a) Algebraik formada yozing va z^2 ni hisoblang.

b) Trigonometrik formada yozing va z^{20} , $\sqrt[3]{z}$ larni hisoblang.

- 1) $a = 1; b = \sqrt{2}; c = -\sqrt{2}.$
- 2) $a = 2; b = \sqrt{2}; c = \sqrt{2}.$
- 3) $a = 3; b = 1; c = \sqrt{3}.$

- 4) $a = 4; b = 1; c = -\sqrt{3}$.
- 5) $a = 5; b = \sqrt{3}; c = 1$.
- 6) $a = -1; b = -\sqrt{3}; c = 1$.
- 7) $a = -2; b = -1; c = 1$.
- 8) $a = -3; b = 3; c = \sqrt{3}$.
- 9) $a = -4; b = 1; c = -1$.
- 10) $a = -5; b = -3; c = \sqrt{3}$.
- 11) $a = 1; b = \sqrt{3}; c = -3$.
- 12) $a = 2; b = 1; c = -\sqrt{3}$.
- 13) $a = 3; b = 1; c = \sqrt{3}$.
- 14) $a = 4; b = -\sqrt{3}; c = 1$.
- 15) $a = 5; b = 2; c = 2\sqrt{3}$.

3. $x^3 + ax^2 + bx + c = 0$ tenglamani eching.

- 1) $a = 2; b = -3; c = 0$.
- 2) $a = -4; b = 2; c = 1$.
- 3) $a = 3; b = -5; c = 2$.
- 4) $a = 1; b = -2; c = -5$.
- 5) $a = 2; b = 3; c = -1$.
- 6) $a = 4; b = 1; c = 2$.
- 7) $a = 3; b = 2; c = 3$.
- 8) $a = 2; b = 4; c = 1$.
- 9) $a = -2; b = 2; c = -4$.
- 10) $a = 10; b = -10; c = 1$.
- 11) $a = -3; b = 2; c = -6$.
- 12) $a = 1; b = -1; c = 2$.
- 13) $a = -4; b = 3; c = 2$.

$$14) \quad a = 4; b = -7; c = 2.$$

$$15) \quad a = 3; b = 2; c = 6.$$

3-bob. FAZODA ANALITIK GEOMETRIYA.

§10. Vektorlar nazariyasi va tatbiqlari.

Fazoda M nuqta berilib, to'g'ri burchakli dekart koordinatalar sistemasi $OXYZ$ aniqlansa, nuqta uchta koordinata x (abtsissa) y (ordinata), z (applikata) ga ega bo'ladi va $M(x, y, z)$ tarzida yoziladi.

$O(0; 0; 0)$ dan $M(x, y, z)$ gacha masofa

$$|OM| = \sqrt{x^2 + y^2 + z^2}, \quad A(x_1, y_1, z_1) \quad \text{va}$$

$B(x_2; y_2; z_2)$ nuqtalar orasidagi masofa esa

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

formula yordamida topiladi.

AB kesmani $|AB| : |CB| = \lambda$ nisbatda bo'luvchi

$C(x, y, z)$ nuqta koordinatalari tekislikdagiga o'xshash topiladi, ya'ni

$$x = \frac{x_1 + \lambda x_2}{1 + \lambda}; y = \frac{y_1 + \lambda y_2}{1 + \lambda}; z = \frac{z_1 + \lambda z_2}{1 + \lambda}.$$

Yo'naltirilgan kesma vektor bo'lib, $OXYZ$ koordinatalari fazosida o'qlardagi birlik $\vec{i}, \vec{j}, \vec{k}$ vektorlar - ortlar orqali quyidagicha yoyiladi:

$$\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}.$$

Bu holda $\vec{a}$ vektor x, y, z — koordinatalarga ega bo'ladi va

$\vec{a}(x, y, z)$ tarzida yoziladi. Bu vektor uzunligi

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2} \text{ formuladan topiladi.}$$

$\vec{a}$ vektorning ox, oy, oz o'qlari bilan hosil qilgan burchaklari mos ravishda α, β, γ bo'lsa, bu burchaklar kosinuslari

$$\cos \alpha = \frac{x}{|\vec{a}|}, \quad \cos \beta = \frac{y}{|\vec{a}|}, \quad \cos \gamma = \frac{z}{|\vec{a}|}$$

yo'naltiruvchi kosinuslar deyiladi. Ular o'zaro quyidagicha bog'langan: $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

koordinatalari bilan berilgan $\vec{b}(x_1, y_1, z_1), \vec{c}(x_2, y_2, z_2)$ vektorlar ustida amallar quyidagicha aniqlanadi:

$$\vec{b} \pm \vec{c} = (x_1 \pm x_2; y_1 \pm y_2; z_1 \pm z_2)$$

$$\lambda \vec{b} = (\lambda x_1; \lambda y_1; \lambda z_1)$$

Boshi $A(x_0, y_0, z_0)$ oxiri $B(x_1, y_1, z_1)$

nuqtada bo'lgan $\overrightarrow{AB}$ vektor koordinatalari

$$\overrightarrow{AB}(x_1 - x_0, y_1 - y_0, z_1 - z_0) \quad \text{tarzida}$$

aniqlanadi.

1. Skalyar ko'paytma xossalari.

Ikki $\vec{a}$ va $\vec{e}$ vektorlar skalyar ko'paytmasi deb ular uzunliklari va ular orasidagi burchak kosinusi ko'paytmasiga aytiladi, $\vec{a} \bullet \vec{e}$ yoki $(\vec{a}, \vec{e})$ tarzida belgilanadi.

$$\vec{a} \bullet \vec{e} = |\vec{a}| \cdot |\vec{e}| \cdot \cos \varphi$$

$$\cos \varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \text{ formuladan } \vec{a}, \vec{b} \text{ vektorlar}$$

$$\text{parallellik sharti } \frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2},$$

perpendikulyarlik sharti $x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2 = 0$.
quyidagi xossalar o'rinni:

$$1) \vec{a} \cdot \vec{a} = |\vec{a}|^2$$

2) $\vec{a} \cdot \vec{e} = 0 \Leftrightarrow \vec{a} \perp \vec{e}$ yoki $\vec{a}, \vec{e}$ lardan kamida bittasi nol vektor.

$$3) \vec{a} \cdot \vec{e} = \vec{e} \cdot \vec{a} \quad 4) \vec{a}(\vec{e} + \vec{c}) = \vec{a}\vec{e} + \vec{a}\vec{c}$$

$$5) \vec{i}^2 = \vec{j}^2 = \vec{k}^2 = 1, \quad \vec{i}\vec{j} = \vec{j}\vec{k} = \vec{k}\vec{i} = 0$$

Koordinatalari bilan berilgan
 $\vec{a}(x_1, y_1, z_1), \vec{b}(x_2, y_2, z_2)$ vektorlar skalyar
ko'paytmasi quyidagicha topiladi:

$$\vec{a} \cdot \vec{e} = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2$$

Ikki vektor orasidagi burchak esa

$$\cos \varphi = \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_2^2}}$$

formuladan topiladi.

2. Vektor ko'paytma.

Ikki $\vec{a}$ va $\vec{e}$ vektorlar vektor ko'paytmasi deb
shunday $\vec{c}$ vektorga aytiladiki:

1) $\vec{c}$ vektori $\vec{a}$ va $\vec{e}$ vektorlarga perpendikulyar;

2) $\vec{c}$ vektor uzunligi $\vec{a}$ va $\vec{e}$ vektorlarga qurilgan
parallelogramm yuziga teng, ya'ni

$$|\vec{c}| = |\vec{a}| \cdot |\vec{e}| \cdot \sin \varphi,$$

3) $\vec{c}$ vektor uchidan qaralganda, $\vec{a}$ dan $\vec{e}$ ga yo'nalish soat mili yo'nalishiga teskari bo'lishi kerak.

Vektor ko'paytma $\vec{a} \times \vec{b}$ yoki $[\vec{a}, \vec{b}]$ tarzida belgilanadi,

Quyidagi xossalar o'rinli:

$$1) \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$2) \vec{a} = 0 \text{ yoki } \vec{b} = 0 \text{ dan tashqari } \vec{a} \parallel \vec{b} \text{ bo'lsa ham } \vec{a} \times \vec{b} = 0$$

$$3) (k\vec{a}) \times \vec{b} = \vec{a} \times (k\vec{b}) = k(\vec{a} \times \vec{b})$$

$$4) \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$5) \vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0, \quad \vec{i} \times \vec{j} = -\vec{j} \times \vec{i} = \vec{k}, \\ \vec{j} \times \vec{k} = -\vec{k} \times \vec{j} = \vec{i}, \quad \vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j}$$

Koordinatalari bilan berilgan $\vec{a}(x_1, y_1, z_1)$,

$\vec{b}(x_2, y_2, z_2)$ vektorlar vektor ko'paytamasi quyidagicha topiladi:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Demak, $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogramm va uchburchak yuzi uchun quyidagi formulalar o'rinli:

$$S = |\vec{a} \times \vec{b}|, S_{\Delta} = \frac{1}{2} |\vec{a} \times \vec{b}|$$

3. Aralash ko'paytma.

Uchta $\vec{a}, \vec{b}, \vec{c}$ vektorlar aralash ko'paytmasi deb, $\vec{a}\vec{x}\vec{b}$ vektor ko'paytmaning $\vec{c}$ vektor bilan skalyar ko'paytmasiga teng songa aytiladi va $\vec{a}\vec{b}\vec{c}$ ko'rinishda belgilanadi.

Bu ko'paytma moduli berilgan vektorlar^{ga} qurilgan parallelepiped hajmiga teng, ya'ni

$$V_{nap} = |\vec{a}\vec{b}\vec{c}|$$

Yasovchilari $\vec{a}, \vec{b}, \vec{c}$ bo'lgan piramida hajmi esa

$$V_{nap} = \frac{1}{6} |\vec{a}\vec{b}\vec{c}|$$

Aralash ko'paytma quyidagi xossalarga ega:

1) Ko'paytuvchilardan kamida bittasi nol vektor, kamida ikkitasi parallel, uchalasi bir tekislikda yotadigan hollarda aralash ko'paytma nolga teng.

$$2) (\vec{a}\vec{x}\vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \cdot \vec{c})$$

Koordinatalari bilan berilgan $\vec{a}(x_1, y_1, z_1),$

$\vec{b}(x_2, y_2, z_2), \vec{c}(x_3, y_3, z_3)$

vektorlar aralash ko'paytmasi

$$(\vec{a}\vec{x}\vec{b})\vec{c} = \vec{a}\vec{b}\vec{c} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

formula yordamida topiladi.

10.1. Ox o'qida $A(2; -4; 5)$ va $B(-3; 2; 7)$ nuqtalardan bir xil uzoqlikda joylashgan nuqtani toping.

10.2. Uchlari $A(2; 3; 4), B(3; 1; 2), C(4; -1; 3)$ nuqtalarda bo'lgan uchburchak og'irlik markazi koordinatalarini toping.

- 10.3. XOY tekislikda
 $A(1; -1; 5)$, $B(3; 4; 4)$, $C(4; 6; 1)$ nuqtalardan
 bir xil uzoqlikda joylashgan nuqtani toping.
- 10.4. $\vec{a}(4; -12; z)$ vektor berilgan. $|\vec{a}| = 13$ bo'lsa,
 z ni toping.
- 10.5. $A(3; -1; 2)$, $B(-1; 2; 1)$ bo'lsa, $\overline{AB}$ va $\overline{BA}$
 vektorlar koordinatalarini toping.
- 10.6. Vektor OX va OZ o'qlari bilan $\alpha = 120^\circ$, $\beta = 45^\circ$
 burchaklar hosil qilsa, OY o'qi bilan
 qanday burchak hosil qiladi?
- 10.7. $|\vec{a}| = 13$, $|\vec{b}| = 19$ va $|\vec{a} + \vec{b}| = 24$ bo'lsa, $|\vec{a} - \vec{b}|$
 ni hisoblang.
- 10.8. $|\vec{a}| = 11$, $|\vec{b}| = 23$ va $|\vec{a} - \vec{b}| = 30$ bo'lsa, $|\vec{a} + \vec{b}|$ ni
 hisoblang.
- 10.9. $\vec{a}$ va $\vec{b}$ vektorlar $\varphi = 120^\circ$ burchak hosil qiladi.
 $|\vec{a}| = 3$, $|\vec{b}| = 5$ bo'lsa, $|\vec{a} + \vec{b}|$, $|\vec{a} - \vec{b}|$ larni
 hisoblang.
- 10.10. $\vec{a}(-2; 3; \beta)$, $\vec{b}(\alpha; -6; 2)$ vektorlar α, β larning
 qanday qiymatlarida kollinear bo'ladi?
- 10.11. $\vec{a}(\alpha; 2; \beta)$, $\vec{b}(\alpha; -2; 3)$ vektorlar α ning qanday
 qiymatlarida perpendikulyar bo'ladi?
- 10.12. ABC uchburchak uchlari
 $A(2; -1; 3)$, $B(1; 1; 1)$, $C(0; 0; 5)$ nuqtalarda
 bo'lsa, bu uchburchak ichki burchaklarini
 aniqlang.
- 10.13. Qavslarni oching:
 $(2\vec{i} - \vec{j}) \cdot \vec{j} + (\vec{j} - 2\vec{k}) \cdot \vec{k} + (\vec{i} - 2\vec{k})^2$
- 10.14. $(\vec{a} + \vec{b})^2 + (\vec{a} - \vec{b})^2 = 2(a^2 + b^2)$ tenglikni
 isbotlang, geometrik ma'nosini
 tushuntiring.

10.15. Oldingi masala yordamida medianalar xossasini isbotlang.

10.16. $|\vec{a}| = 3$, $|\vec{b}| = 26$, $|\vec{a} \times \vec{b}| = 72$ bo'lsa, $\vec{a} \cdot \vec{b}$ ni hisoblang.

10.17. Uchlari $A(1; 2; 0)$, $B(3; 0; -3)$, $C(5; 2; 6)$ nuqtalarda bo'lgan uchburchak yuzini hisoblang.

10.18. $\vec{a}(2; -3; 1)$, $\vec{b}(-3; 1; 2)$, $\vec{c}(1; 2; 3)$ bo'lsa, $(\vec{a} \times \vec{b}) \cdot \vec{c}$ va $\vec{a} \cdot (\vec{b} \times \vec{c})$ larni hisoblang.

10.19. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak 30° , $|\vec{a}| = 6$, $|\vec{b}| = 3$. Bu vektorlarga perpendikulyar $\vec{c}$ vektor berilgan va $|\vec{c}| = 3$. $(\vec{a} \times \vec{b}) \cdot \vec{c}$ ni hisoblang.

10.20. $\vec{a}(1; -1; 3)$, $\vec{b}(-2; 2; 1)$, $\vec{c}(3; -2; 5)$ bo'lsa $(\vec{a} \times \vec{b}) \cdot \vec{c}$ ni hisoblang.

10.21. $A(1; 2; -1)$, $B(0; 1; 5)$, $C(-1; 2; 1)$, $D(2; 1; 3)$ nuqtalarning bitta tekislikda yotishini isbotlang.

10.22. Uchlari $A(2; -1; 1)$, $B(5; 5; 4)$.

$C(3; 2; -1)$, $D(4; 1; 3)$ nuqtalarda bo'lgan tetraedr hajmini hisoblang.

§11 Fazoda tekislik tenglamalari.

$\vec{N}\{A, B, C\}$ vektorga perpendikulyar,

$F(x_0, y_0, z_0)$ nuqtadan o'tuvchi tekislik tenglamasi

$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ ko'rinishda

bo'lib, undan $Ax + By + Cz + D = 0$ umumiy

tenglamasini keltirib chiqarish mumkin.

I. $D = 0$ bo'lsa, $Ax + By + Cz = 0$ tekislik koordinatalar boshidan o'tadi.

II. $C = 0$ bo'lsa, $Ax + By + D = 0$ tenglama Oz o'qiga parallel bo'ladi.

III. $C = D = 0$ bo'lsa $Ax + By = 0$ tekislik Oz o'qidan o'tadi.

IV. $B = C = 0$ bo'lsa $Ax + D = 0$ tekislik YOZ tekisligiga parallel.

V. Koordinata tekisliklari tenglamalari:
 $x = 0, y = 0, z = 0$

Tekislikning o'qlardagi kesmalar bo'yicha

$$\text{tenglamasi: } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

ko'rinishda bo'ladi

$A_1x + B_1y + C_1z + D_1 = 0$ va $A_2x + B_2y + C_2z + D_2 = 0$
 tekisliklar orasidagi burchak

$$\cos \varphi = \pm \frac{|\vec{N}_1 \vec{N}_2|}{|\vec{N}_1| |\vec{N}_2|} = \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}}$$

formuladan topiladi.

$$\text{Bu tekisliklar parallel bo'lsa } \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2},$$

perpendikulyar bo'lsa $A_1 A_2 + B_1 B_2 + C_1 C_2 = 0$

shartlar bajariladi. $M_0(x_0, y_0, z_0)$ nuqtadan

$Ax + By + Cz + D = 0$ tekislikkacha masofa

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

formuladan topiladi.

Agar koordinata boshidan tekislikka uzunligi P bo'lgan perpendikulyar o'tkazilib, u son o'qlari bilan α, β, γ burchaklar hosil qilsa, tekislik tenglamasi

$x \cos \alpha + y \cos \beta + z \cos \gamma - p = 0$ ko'rinishda bo'ladi va normal tenglama deyiladi. Bu holda

$$d = |x_0 \cos \alpha + y_0 \cos \beta + z_0 \cos \gamma - p|$$

$$A(x_0, y_0, z_0) \quad B(x_1, y_1, z_1) \quad C(x_2, y_2, z_2)$$

nuqtalardan o'tuvchi tekislik tenglamasi

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} = 0 \text{ ko'rinishda bo'ladi.}^*$$

11.1 $E(2; -1; 1)$ nuqta koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosi bo'lsa, tekislik tenglamasini yozing.

11.2 $F(3; 4; -5)$ nuqtadan o'tib $\vec{a}_1(3; 1; -1), \vec{a}_2(1; -2; 1)$ vektorlarga parallel tekislik tenglamasini tuzing

11.3 Koordinata boshidan o'tib, $5x - 3y + 2z - 3 = 0$ tekislikka parallel bo'lgan tekislik tenglamasini toping

11.4 $x - 2y + 3z - 5 = 0$ tekislikka perpendikulyar va $E(1; -1; -2), F(3; 1; 1)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing

11.5 $5x - 6y + 3z + 120 = 0$ tekislik va koordinata tekisliklari bilan chegaralangan piramida hajmini toping

11.6 $x - 2y - 2z - 12 = 0$ va $x - 2y - 2z - 6 = 0$ tekisliklar orasidagi masofani toping

11.7 $x + 2z - 6 = 0$ va $x + 2y - 4 = 0$ tekisliklar orasidagi burchakni toping.

§12. Fazoda to'g'ri chiziq.

$A(a, b, c)$ nuqtadan o'tib, $\vec{P}(m, n, p)$ vektorga parallel to'g'ri chiziq tenglamasi

$$\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p}$$

ko'rinishda bo'lib, uni kanonik tenglama deyiladi.

Kanonik tenglamani t ga tenglab to'g'ri chiziqlarning parametrik tenglamalarini olish mumkin:

$$\begin{cases} x = mt + a \\ y = nt + b \\ z = pt + c \end{cases}$$

$A(x_0, y_0, z_0)$, $B(x_1, y_1, z_1)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$$

ko'rinishda bo'ladi.

Yo'naltirilgan

vektorlari

$\vec{P}(m_1, n_1, p_1)$, $\vec{Q}(m_2, n_2, p_2)$ bo'lgan ikki to'g'ri chiziq orasidagi burchak

$$\cos \varphi = \frac{\vec{P} \cdot \vec{Q}}{|\vec{P}| \cdot |\vec{Q}|} = \frac{m_1 m_2 + n_1 n_2 + p_1 p_2}{\sqrt{m_1^2 + n_1^2 + p_1^2} \cdot \sqrt{m_2^2 + n_2^2 + p_2^2}}$$

formuladan topilib, parallellik sharti $\frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2}$,

perpendikulyarlik sharti esa

$m_1 m_2 + n_1 n_2 + p_1 p_2 = 0$ dir.

$A(a, b, c)$ va $A_1(a_1, b_1, c_1)$ nuqtalardan o'tuvchi to'g'ri chiziqlarning komplanarlik sharti

$$\begin{vmatrix} a - a_1 & b - b_1 & c - c_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix} = 0$$

bo'lib, $\vec{p}_1$ va $\vec{p}_2$ parallel bo'lmasa, bu to'g'ri chiziqlar kesishuvchiligi sharti ham bo'ladi.

Normal vektori $\vec{N}(A, B, C)$ bo'lgan tekislik va yo'naltiruvchi vektori $\vec{P}(m, n, p)$ bo'lgan to'g'ri chiziq orasidagi burchak

$$\sin \varphi = \frac{|\vec{N} \cdot \vec{P}|}{|\vec{N}| \cdot |\vec{P}|} = \frac{m \cdot A + n \cdot B + p \cdot C}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{m^2 + n^2 + p^2}}$$

formuladan topiladi.

Bunda parallellik sharti $mA + nB + pC = 0$, perpendikulyarlik sharti esa

$$\frac{m}{A} = \frac{n}{B} = \frac{p}{C} \text{ ko'rinish oladi.}$$

$$12.1. \begin{cases} x - 2y + 3z - 4 = 0 \\ 3x + 2y - 5z - 4 = 0 \end{cases} \text{ to'g'ri chiziq kanonik}$$

tenglamasini yozing.

$$12.2. \begin{cases} 2x + 3y - z - 4 = 0 \\ 3x - 5y + 2z + 1 = 0 \end{cases} \text{ to'g'ri chiziq parametrik}$$

tenglamasini yozing.

$$12.3. \frac{x+2}{3} = \frac{y-1}{-2} = \frac{z}{1} \text{ va } \begin{cases} x + y - z = 0 \\ x - y - 5z - 8 = 0 \end{cases} \text{ to'g'ri}$$

chiziqlar paralleligini ko'rsating.

$$12.4. x = 2t + 1, y = 3t - 2, z = -6t + 1 \text{ va}$$

$$\begin{cases} 2x + y - 4z + 2 = 0 \\ 4x - y - 5z + 4 = 0 \end{cases} \text{ to'g'ri chiziqlarning}$$

perpendikulyarligini isbotlang.

$$12.5. \frac{x-1}{1} = \frac{y+1}{-2} = \frac{z}{6} \text{ va}$$

$$2x + 3y + z - 1 = 0 \text{ kesishish nuqtasi topilsin.}$$

12.6. S ning qanday qiymatida
$$\begin{cases} 3x - 2y + z + 3 = 0 \\ 4x - 3y + 4z + 1 = 0 \end{cases}$$

to'g'ri chiziq va $2x - y + cz - 2 = 0$ tekislik o'zaro parallel bo'ladi?

12.7. $R(1; -1; -2)$ nuqtadan $\frac{x+3}{3} = \frac{y+2}{2} = \frac{z-8}{-2}$

to'g'ri chiziqqacha masofani toping.

12.8.
$$\begin{cases} 2x + 2y - z - 10 = 0 \\ x - y - z - 22 = 0 \end{cases}$$
 va $\frac{x+7}{3} = \frac{y-5}{-1} = \frac{z-9}{4}$

to'g'ri chiziqlar paralleligini isbotlang va ular orasidagi masofani toping.

§13. Ikkinchi tartibli sirtlar.

Fazoda

$$Ax^2 + By^2 + Cz^2 + 2Dyz + 2Exz + 2Fxy + 2Gx + 2My + 2Kz + L = 0$$

(1) tenglamani qanoatlantiruvchi nuqtalar to'plami ikkinchi tartibli sirt (ITS) deyiladi.

ITS lar koordinata o'qlarini burish, parallel ko'chirish yordamida quyidagi 15 holga keltiriladi:

1) Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, a = b = c$

bo'lsa sfera tenglamasi hosil bo'ladi.

2) Bir pallali giperboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

3) Ikki pallali giperboloid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

4) Konus: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$

5) Elliptik paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z$

$$6) \text{ Giperbolik paraboloid: } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$$

$$7) \text{ Elliptik silindr: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$8) \text{ Giperbolik silindr: } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$9) \text{ Parabolik silindr: } y^2 = 2px$$

$$10) \text{ Ikki kesishuvchi tekislik: } y^2 - k^2 x^2 = 0$$

$$11) \text{ Ikki parallel tekislik: } y^2 - k^2 = 0$$

$$12) \text{ Tekislik: } y^2 = 0$$

$$13) \text{ To'g'ri chiziq: } x^2 + y^2 = 0$$

$$14) \text{ Nuqta: } x^2 + y^2 + z^2 = 0$$

$$15) \text{ Bo'sh to'plam: } x^2 = -1$$

Fazodagi $P(x, y, z)$ nuqtaning o'rnini uning XOY tekislikka proektsiyasi $P'(x, y, 0)$ qutb koordinatalari va P nuqta applikatasi z yordamida aniqlash mumkin:

$$\varphi = \angle XOP', \quad r = OP', \quad z = P'P$$

r, φ, z kattaliklar silindrik koordinatalar deyiladi. R nuqta dekart va silindrik koordinatalari quyidagicha bog'langan: $x = r \cos \varphi, \quad y = r \sin \varphi$.

Applikata o'zgarmaydi: $P(r, \varphi, z)$. Aksincha,

$$r = \sqrt{x^2 + y^2}, \quad \operatorname{tg} \varphi = \frac{y}{x}$$

Misol. $P(2; -2; -3)$ ning silindrik koordinatalarini toping:

$$r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}, \quad \varphi = \operatorname{arctg} \frac{-2}{2} = \frac{3\pi}{4}$$

ekanligidan $P\left(2\sqrt{2}; \frac{3\pi}{4}; -3\right)$.

2) $Q(4\cos 25^\circ, -4\sin 15^\circ, 1)$ ning silindrik koordinatalarini toping.

$$r = \sqrt{16\cos^2 15^\circ + 16\sin^2 15^\circ} = 4,$$

$$\varphi = \operatorname{arctg} \frac{-4\sin 15^\circ}{4\cos 15^\circ} = -\frac{\pi}{12}. \text{ Demak, } Q\left(4; -\frac{\pi}{12}; 1\right).$$

3) $x^2 + y^2 + z^2 = 1$ tenglamani silindrik koordinatalarda yozing.

$r^2 = x^2 + y^2$ ekanligidan $r^2 + z^2 = 1$ kelib chiqadi.

Fazodagi $P(x, y, z)$ nuqta va uning OXY tekislikka proektsiyasi $P'(x, y, 0)$ berilgan bo'lsin. $OP = \rho$, $\angle ZOP = \theta$, $\angle XOP = \varphi$ kattaliklar nuqtaning sferik koordinatlari deyiladi va $P(\rho, \theta, \varphi)$ tarzida yoziladi. Dekart va sferik koordinatalar sistemalari o'zaro quyidagicha bog'langan:

$$x = \rho \cos \varphi \sin \theta, \quad y = \rho \sin \varphi \sin \theta, \quad z = \rho \cos \theta$$

. Aksincha,

$$\rho^2 = x^2 + y^2 + z^2, \quad \cos \varphi = \frac{x}{\sqrt{x^2 + y^2}}, \quad \sin \varphi = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\cos \theta = \frac{z}{\rho}, \quad \sin \theta = \frac{\sqrt{x^2 + y^2}}{\rho}.$$

Misollar: 1) $P(1; 1; 1)$ ning sferik koordinatalarini yozing.

$$\rho = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3},$$

$$\cos \varphi = \frac{1}{\sqrt{2}}; \sin \varphi = \frac{1}{\sqrt{2}} \text{ lardan } \varphi = \frac{\pi}{4}.$$

$$\cos \theta = \frac{1}{\sqrt{3}} \text{ dan } \theta = \arccos \frac{1}{\sqrt{3}}.$$

$$\text{Demak, } P\left(\sqrt{3}; \arccos \frac{1}{\sqrt{3}}; \frac{\pi}{4}\right).$$

$$2) Q(\cos 77^\circ, \sin 77^\circ, 0)$$

$$\rho = \sqrt{\cos^2 77^\circ + \sin^2 77^\circ + 0^2} = 1,$$

$$\cos \varphi = \frac{\cos 77^\circ}{1} \text{ dan } \varphi = 77^\circ;$$

$$\cos \theta = \frac{0}{1} = 0 \text{ dan } \theta = 90^\circ.$$

$$\text{Demak, } Q\left(1; \frac{\pi}{2}; \frac{77\pi}{180}\right).$$

3) $x^2 + (y-1)^2 + z^2 = 1$ tenglamani sferik koordinatalarda yozing.

$$x^2 + y^2 + z^2 - 2y + 1 = 1 \quad \text{tenglikdan}$$

$$\rho^2 - 2y = 0 \text{ yoki } \rho^2 = 2y \text{ ga egamiz.}$$

$$\rho^2 = 2 \cdot \sin \varphi \sin \theta, \quad \rho = 2 \sin \varphi \sin \theta.$$

3. Silindrik va sferik koordinatalarini toping.

Buning uchun dastlab, son o'qlarini ko'paytmalarni yo'qotadigan Eyler burchaklari deb ataluvchi φ, θ, ψ burchaklarga quyidagi tartibda

buriladi. (Ular ITCh dagiday $\operatorname{ctg} 2\alpha = \frac{A-C}{2B}$ shart yordamida aniqlanadi).

$$\begin{cases} x_1 = x \cos \varphi - y \sin \varphi \\ y_1 = x \sin \varphi + y \cos \varphi \\ z_1 = z \end{cases}$$

Bu almashtirishda $x \cdot y$ had yo'qoladi. So'ngra

$$\begin{cases} x_2 = x_1 \\ y_2 = y_1 \cos \theta - z_1 \sin \theta \\ z_2 = y_1 \sin \theta + z_1 \cos \theta \end{cases}$$

almashtirishlar $y \cdot z$ hadni yo'qotadi.

$$\begin{cases} x' = x_2 \cos \psi - z_2 \sin \psi \\ y' = y_2 \\ z' = x_2 \sin \psi + z_2 \cos \psi \end{cases}$$

almashtirishlarda $x \cdot z$ had yo'qoladi.

Natijada (1) – tenglama

$$A'x'^2 + B'y'^2 + C'z'^2 + 2ax' + 2by' + 2cz' + d = 0$$

(2)

ko'rinishga keladi. Bu tenglama parallel ko'chirish yordamida kanonik ko'rinish oladi.

13.1 $M(4; -1; -3)$ va $N(0; 3; -1)$ sferaning biror diametri uchlari bo'lsa, bu sfera tenglamasini yozing.

13.2 $x^2 + y^2 - 2z^2 = 0$ konus va $y=2$ tekislik qanday chiziq bo'yicha kesishadi?

13.3 $x^2 = yz$ tenglama qanday sirtни aniqlaydi?

13.4 Kanonik ko'rinishga keltiring:

1) $x^2 - xy - xz + yz = 0$

2) $x^2 + z^2 - 4x - 4z + 4 = 0$

3) $x^2 + 2y^2 + z^2 - 2xy - 2yz = 0$

4) $x^2 + y^2 - z^2 - 2y + 2z = 0$

- 5) $x^2 + 2y^2 + 2z^2 - 4y + 4z + 4 = 0$
 6) $4x^2 + y^2 - z^2 - 2yx - 4y + 2z + 35 = 0$
 7) $x^2 + y^2 - 6x + 6y - 4z + 18 = 0$
 8) $9x^2 - z^2 - 18x - 18y - 6z = 0$

Bobga doir misollar echish namunalari

1. $A(2; -4; 5)$ va $B(-3; 2; 7)$ nuqtalar orasidagi masofani toping.

$$|AB| = \sqrt{(-3-2)^2 + (2+4)^2 + (7-5)^2} = \sqrt{5^2 + 6^2 + 2^2} = \\ = \sqrt{25 + 36 + 4} = \sqrt{65}$$

2. $A(-1; 5; 7)$, $B(3; 3; 3)$ nuqtalar berilgan,

$$|AC| : |CB| = \frac{1}{3}$$

shartni qanoatlantiruvchi $C(x, y, z)$ nuqtani toping.

$$\lambda = \frac{1}{3} \text{ ekanligidan foydalanib,}$$

$$x = \frac{-1 + \frac{1}{3} \cdot 3}{1 + \frac{1}{3}} = 0, \quad y = \frac{5 + \frac{1}{3} \cdot 3}{1 + \frac{1}{3}} = \frac{9}{2}, \quad z = \frac{7 + \frac{1}{3} \cdot 3}{1 + \frac{1}{3}} = 6$$

ekanligini topamiz, ya'ni $C\left(0; \frac{9}{2}; 6\right)$.

3. ABC uchburchak AB tomoni EF nuqtalari yordamida teng uch qismga ajratiladi. Agar

$\overrightarrow{CA} = \vec{a}$, $\overrightarrow{CB} = \vec{e}$ bo'lsa, $\overrightarrow{CE}$ vektorni toping.

$\overrightarrow{AB} = \vec{e} - \vec{a}$ ekanligidan $\overrightarrow{AE} = \frac{1}{3}(\vec{e} - \vec{a})$. Demak,

$$\overrightarrow{CE} = \overrightarrow{CA} + \overrightarrow{AE} = \vec{a} + \frac{1}{3}(\vec{e} - \vec{a}) = \frac{1}{3}(2\vec{a} + \vec{e}).$$

4. $A(1;2;3), B(4;5;6)$ bo'lsa, $\vec{a} = \overrightarrow{AB}$ vektor koordinatalarini toping.

$x = 4 - 1 = 3; y = 5 - 2 = 3; z = 6 - 3 = 3$ ekanligidan $\vec{a}(3;3;3)$.

5. $\vec{a}(2;1;0), \vec{b}(-1;2;1), \vec{c}(0;1;1)$ bo'lsa,

$\vec{d} = 2\vec{a} - 3\vec{b} + 4\vec{c}$ ni toping.

$2\vec{a} = (4;2;0), 3\vec{b} = (-3;6;3), 4\vec{c} = (0;4;4)$ ekanligidan

$\vec{d} = (4;2;0) - (-3;6;3) + (0;4;4) = (7;0;1)$, ya'ni $\vec{d}(7;0;1)$

6. $\vec{a} = \vec{i} - 2\vec{j} - 2\vec{k}$ vektor uzunligini, yo'naltiruvchi kosinuslarini toping.

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$\cos \alpha = \frac{1}{3}; \quad \cos \beta = \frac{-2}{3}; \quad \cos \gamma = \frac{-2}{3}.$$

7. $\vec{a}(1;2;1), \vec{b}(0;4;2)$ bo'lsa, $\vec{a} \cdot \vec{e}$ va $\vec{a} \times \vec{b}$ ni toping.

$$\vec{a} \cdot \vec{b} = 1 \cdot 0 + 2 \cdot 4 + (-1) \cdot 2 = 8 - 2 = 6$$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 0 & 4 & 2 \end{vmatrix} = \vec{i} \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix}$$

$$+ \vec{k} = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = 8\vec{i} - 2\vec{j} + 4\vec{k}, \text{ ya'ni}$$

$$\vec{c}(8; -2; 4).$$

8. $\vec{a}(1; 2; -2)$, $\vec{b}(0; 6; 8)$ vektorlar orasidagi burchakni, ularga qurilgan parallelogramm yuzini toping.

$$\cos \varphi = \frac{1 \cdot 0 + 2 \cdot 6 + (-2) \cdot 8}{\sqrt{1^2 + 2^2 + (-2)^2} \cdot \sqrt{0^2 + 6^2 + 8^2}} = \frac{12 - 16}{3 \cdot 10} =$$

$$= \frac{-4}{30} = -\frac{2}{15}$$

$$\varphi = \arccos\left(-\frac{2}{15}\right)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -2 \\ 0 & 6 & 8 \end{vmatrix} = 28\vec{i} - 8\vec{j} + 6\vec{k} \quad \text{dan}$$

$$S = |\vec{a} \times \vec{b}| = \sqrt{28^2 + (-8)^2 + 6^2} = \sqrt{2^2(14^2 + 4^2 + 3^2)} =$$

$$= 2\sqrt{221} \text{ (kv.b)}$$

9. $\vec{a}(2; 3; -1)$, $\vec{b}(1; -2; 3)$, $\vec{c}(2; -1; 1)$ vektorlar berilgan. Shunday

$\vec{d}$ vektor topingki, u $\vec{a}$ va $\vec{e}$ vektorlarga perpendikulyar, $\vec{d} \cdot \vec{c} = -6$ bo'lsin.

$\vec{d}(x, y, z)$ desak, shartlardan:

$$\vec{a} \cdot \vec{d} = 2x + 3y - z = 0$$

$$\vec{b} \cdot \vec{d} = x - 2y + 3z = 0$$

$\vec{c} \cdot \vec{d} = 2x - y + z = -6$ kelib chiqadi. Hosil bo'lgan sistemani echib $x = -3, y = 3, z = 3$ ekanligini topamiz, ya'ni

$$\vec{d}(-3; 3; 3).$$

10. $|\vec{a}| = 10, |\vec{e}| = 2, \vec{a}\vec{e} = 12$ bo'lsa, $|\vec{a}\vec{e}|$ ni hisoblang.

$$\cos \varphi = \frac{12}{10 \cdot 2} = \frac{3}{5} \quad \text{ekanligidan } \varphi - \text{o'tkir}$$

$$\text{burchak va } \sin \varphi = \frac{4}{5}$$

$$|\vec{a}\vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin \varphi = 10 \cdot 2 \cdot \frac{4}{5} = 16.$$

11. Uchburchakli piramida uchlari $A(2; 2; 2), B(4; 3; 3), C(4; 5; 4)$ va $D(4; 4; 7)$ nuqtalarda bo'lsa, uning hajmini toping.

$\vec{a} = \overrightarrow{AB} = (2; 1; 1), \vec{e} = \overrightarrow{AC} = (2; 3; 2), \vec{c} = \overrightarrow{AD} = (2; 2; 5)$ vektorlar piramida yasovchilaridir.

$$(\vec{a}\vec{b})\vec{c} = \begin{vmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 2 & 5 \end{vmatrix} = 22 - 6 + (-2) = 14$$

12. $F(1;2;3)$ nuqtadan o'tuvchi va $\vec{N} = 2\vec{i} + 3\vec{j} + 2\vec{k}$ vektorga perpendikulyar tekislik tenglamasini yozing.
 $2(x-1) + 3(y-2) + 2(z-3) = 0$ ya'ni $2x + 3y + 2z - 14 = 0$

13. $E(2;-1;2)$ nuqtadan o'tib $x + y - 3z + 4 = 0$ tekislikka parallel bo'lgan tekislik tenglamasini yozing. Izlanayotgan tekislik normal vektori

$\vec{N}(1;1;-3)$ bo'lishini hisobga olib $1(x-2) + 1(y+1) - 3(z-2) = 0$ ya'ni $x + y - 3z + 5 = 0$ ekanligini topamiz.

14. $2x + 3y + 6z - 12 = 0$ tekislikni chizing.

Buning uchun tekislikni kesmalar bo'yicha tenglamasini yozamiz:

$$\frac{x}{6} + \frac{y}{4} + \frac{z}{2} = 1$$

Endi $\vec{N}$ nuqtalardan o'tuvchi tekislikni chizish mumkin

15. $A(2;3;-5)$ nuqtadan $4x - 2y + 5z - 12 = 0$ tekislikka tushirilgan perpendikulyar uzunligini toping.

Nuqtadan tekislikkacha bo'lgan eng qisqa masofa izlanayotgan perpendikulyar uzunligi bo'ladi.

Demak,

$$|p| = d = \frac{|4 \cdot 2 - 2 \cdot 3 + 5 \cdot (-5) - 12|}{\sqrt{4^2 + (-2)^2 + 5^2}} = \frac{|8 - 6 - 25 - 12|}{\sqrt{45}} = \frac{35}{\sqrt{45}}$$

16. $x - 2y + 2z - 8 = 0$ va $x + z - 6 = 0$ tekisliklar orasidagi burchakni toping.

$\vec{N}_1(1;-2;2)$, $\vec{N}_2(1;0;1)$ ekanidan:

$$\cos \varphi = \frac{1 \cdot 1 + (-2) \cdot 0 + 2 \cdot 1}{\sqrt{1^2 + (-2)^2 + 2^2} \cdot \sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}$$

ya'ni $\varphi = 45^\circ$

17. $x + 3y + 5z - 4 = 0$ va $x - y - 2z + 7 = 0$ tekisliklar kesishish chizig'i va $A(1;0;1)$ nuqtadan o'tuvchi tekislik tenglamasini toping.

Berilgan tekisliklar kesishish chizig'idan o'tuvchi tekisliklar bog'lami tenglamasi

$$x + 3y + 5z - 4 + \lambda(x - y - 2z + 7) = 0 \text{ eku}$$

$$(1 + \lambda)x + (3 - \lambda)y + (5 - 2\lambda)z + 7\lambda - 4 = 0$$

ko'rinishda bo'ladi uning $A(1;0;1)$ nuqtadan o'tishidan foydalanib $1 + 5 \cdot 1 - 4 + \lambda(1 - 2 + 7) = 0$,

$$\text{ya'ni } \lambda = -\frac{2}{6} = -\frac{1}{3} \text{ ekanligini topamiz.}$$

Demak:

$$\frac{2}{3}x + \frac{10}{3}y + \frac{17}{3}z - \frac{19}{3} = 0, \text{ yehi } 2x + 10y + 17z - 19 = 0$$

$$\text{Demak, } V_{\text{sup}} = \frac{1}{6} \cdot 14 = \frac{7}{3} \text{ (kub.b)}$$

18. Ikki tekislik

$2x - y + 3z - 1 = 0$, $5x + 4y - z - 7 = 0$ kesishishidan hosil bo'lgan to'g'ri chiziq kanonik tenglamasini yozing.

Dastlab 1 - tenglamani 4 ga ko'paytirib, 2 - tenglamaga qo'yamiz:

$$13x + 11z - 11 = 0$$

2 - tenglamani 3 ga ko'paytirib 1 - tenglikka qo'yamiz:

$$17x + 11y - 22 = 0$$

Bulardan x ni topamiz:

$$x = \frac{11(y - 2)}{-17} = \frac{11(z - 1)}{-13}, \text{ ya'ni}$$

$$\frac{x}{-11} = \frac{y - 2}{17} = \frac{z - 1}{13}$$

19. $A(1; -2; 1)$, $B(3; 1; -1)$ nuqtalardan o'tuvchi to'g'ri chiziq kanonik tenglamasini yozing.

$\vec{p} = \overrightarrow{AB} = (2; 3; 0)$ desak, $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-1}{0}$ kelib chiqadi.

20. $B(2; -1; 3)$ nuqtadan $\frac{x+1}{3} = \frac{y+2}{4} = \frac{z-1}{5}$ to'g'ri chiziqqacha bo'lgan masofani toping.

Yo'naltiruvchi vektor $\vec{p}(3; 4; 5)$ va

$A(-1; -2; 1)$ to'g'ri chiziqdagi nuqta ekanligidan:

$$|AB| = \sqrt{(2+1)^2 + (-1+2)^2 + (3-1)^2} = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$$

$\vec{AB} = (3; 1; 2)$ va $\vec{p}(3; 4; 5)$ orasidagi burchak

$$\cos \varphi = \frac{9 + 4 + 10}{\sqrt{9 + 1 + 4} \cdot \sqrt{9 + 16 + 25}} = \frac{23}{\sqrt{14} \cdot 5\sqrt{2}}$$

$$\sin \varphi = \frac{\sqrt{117 \cdot 163}}{140}$$

$$d = |AB| \sin \varphi = \sqrt{14} \cdot \frac{\sqrt{117 \cdot 163}}{140} = 0,3 \cdot \sqrt{38}$$

21. $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z-3}{3}$ to'g'ri chiziq va

$2x + y - z = 0$ tekislik orasidagi burchakni toping.

$\vec{p}(2; -1; 3)$, $\vec{N}(2; 1; -1)$ ekanligidan

$$\sin \varphi = \frac{2 \cdot 2 + 1 \cdot (-1) + (-1) \cdot 3}{\sqrt{2^2 + 1^2 + 3^2} \cdot \sqrt{2^2 + 1^2 + 1^2}} = 0, \text{ ya'ni } \varphi = 0$$

Berilgan to'g'ri chiziq va tekislik o'zaro parallel ekan.

22. $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$ sfera markazi va radiusini toping.

$$(x - 4x) + (y^2 + 2y) + z^2 - 4 = 0$$

$$(x-2)^2 - 4 + (y+1)^2 - 1 + z^2 - 4 = 0$$

$$(x-1)^2 + (y+1)^2 + z^2 = 3^2$$

Demak, sfera markazi $O(1; -1; 0)$ nuqtada, radiusi 3 ga teng.

23. $x^2 - y^2 - 4x + 8y - 2z = 0$ sirt kanonik ko'rinishga keltirilsin.

$$(x^2 - 4x) - (y^2 - 8y) = 2z$$

$$(x-2)^2 - (y-4)^2 = 2(z-6) \text{ dan}$$

$x' = x - 2$, $y' = y - 4$, $z' = z - 6$ parallel ko'chirish yordamida $x'^2 - y'^2 = 2z'$ ga ega bo'lamiz. Bu giperbolik paraboladir.

24.

- a) $x^2 + 4y^2 + 9z^2 + 12yz + 6xz + 4xy - 4x - 8y - 12z + 23 = 0$ tenglama qanday sirtni ifodalaydi?

Bu tenglamani

$$(x + 2y + 3z)^2 - 4(x + 2y + 3z) + 3 = 0 \text{ ko'rinishda}$$

yozib $x + 2y + 3z = 1$ va $x + 2y + 3z = 3$ parallel tekisliklarni olamiz.

- b). $xy + yz + xz = 1$ tenglama kanonik ko'rinishga keltirilsin.

$$\operatorname{ctg} 2\alpha = \frac{A-B}{2D} = 0, \text{ bundan } \varphi = \frac{\pi}{4}.$$

Parallel ko'chirish yordamida quyidagi ko'rinishga ega bo'lamiz:

$$\begin{cases} x = \frac{\sqrt{2}}{2}(x' - y') \\ y = \frac{\sqrt{2}}{2}(x' + y') \\ z = z' \end{cases}$$

ITS ko'rinishi: $x'^2 - y'^2 + 2\sqrt{2}x'z' = 2$

$$\operatorname{ctg} 2\theta = \frac{A-C}{2F} = \frac{1}{2\sqrt{2}}$$

2D burchakni 0° va 90° orasida, yoki 2θ to'g'ri burchakli uchburchakning o'tkir burchagi deb hisoblaymiz. 2θ burchak qarshisida yotgan katet $2\sqrt{2}$ teng bo'lsa, u holda boshqa katet 1 ga teng, gipotenuza esa 3.

$$\text{Demak, } \sin 2\theta = \frac{2\sqrt{2}}{3}, \cos 2\theta = \frac{1}{3}.$$

$$\text{Bundan, } \sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}} = \frac{1}{\sqrt{3}}, \cos \theta = \sqrt{\frac{2}{3}}$$

$$\text{va } \begin{cases} x' = x'' \sqrt{\frac{2}{3}} - z'' \frac{1}{\sqrt{3}} \\ y' = y'' \\ z' = z'' \frac{1}{\sqrt{3}} + \sqrt{\frac{2}{3}} z'' \end{cases}$$

ITS quyidagi ko'rinishda bo'ladi:

$$\begin{aligned} & \frac{2}{3} x'^{12} - \frac{2\sqrt{2}}{3} x'' z'' + \frac{1}{3} z'^{12} - y'^{12} + \\ & + 2\sqrt{2} \left(\frac{x'^{12} \sqrt{2}}{3} + \frac{2}{3} x'' z'' - \frac{1}{3} x'' z'' - \frac{\sqrt{2}}{3} z'^{12} \right) = 2 \end{aligned}$$

$$\text{Demak: } 2x'^{12} y'^{12} - z'^{12} = 2 \quad \text{yoki}$$

$$-x'^{12} + \frac{y'^{12}}{2} + \frac{z'^{12}}{2} = -1$$

3-bobga doir uy vazifalari

1. $ABCD$ piramida uchlar ko'ordinatalari berilgan. Quyidagilarni toping:

1) AB qirra uzunligini;

- 2) AB va AD qirra orasidagi burchak;
- 3) ABC tekislik tenglamasi;
- 4) AD qirra va ABC yoq orasidagi burchak;
- 5) ABC yoq yuzi;
- 6) Piramida hajmi;
- 7) AB chiziq tenglamasi;
- 8) D uchidan tushirilgan balandlik tenglamasi va uzunligi.
- 9) D uchidan tushirilgan balandlik va asosining kesishish nuqtasini toping.
- 10) AD va BC to'g'ri chiziqlar orasidagi masofani toping.

Koordinatalar sistemasida tasvirlang.

- 1) $A(4; 2; 5)$, $B(0; 7; 2)$, $C(0; 2; 7)$, $D(1; 5; 0)$.
- 2) $A(4; 4; 10)$, $B(4; 10; 2)$, $C(2; 8; 4)$, $D(9; 6; 4)$.
- 3) $A(4; 6; 5)$, $B(6; 9; 4)$, $C(2; 10; 10)$, $D(7; 5; 9)$.
- 4) $A(3; 5; 4)$, $B(8; 7; 4)$, $C(5; 10; 4)$, $D(4; 7; 8)$.
- 5) $A(10; 6; 6)$, $B(-2; 8; 2)$, $C(6; 8; 9)$, $D(7; 10; 3)$.
- 6) $A(1; 8; 2)$, $B(5; 2; 6)$, $C(5; 7; 4)$, $D(4; 10; 9)$.
- 7) $A(6; 6; 5)$, $B(4; 9; 5)$, $C(4; 6; 11)$, $D(6; 9; 3)$.
- 8) $A(7; 2; 2)$, $B(5; 7; 7)$, $C(5; 3; 1)$, $D(2; 3; 7)$.
- 9) $A(8; 6; 4)$, $B(10; 5; 5)$, $C(5; 6; 8)$, $D(8; 10; 7)$.
- 10) $A(7; 7; 3)$, $B(6; 5; 8)$, $C(3; 5; 8)$, $D(8; 4; 1)$.
- 11) $A(1; 1; 1)$, $B(5; 3; 4)$, $C(2; 0; 2)$, $D(6; 8; 10)$.
- 12) $A(0; 1; 1)$, $B(1; 0; 1)$, $C(2; 3; 0)$, $D(6; 4; 3)$.
- 13) $A(1; 2; 3)$, $B(-1; 3; 2)$, $C(7; -3; 5)$, $D(6; 10; 17)$.
- 14) $A(1; 4; 3)$, $B(6; 8; 5)$, $C(3; 1; 4)$, $D(21; 18; 33)$.
- 15) $A(7; 2; 1)$, $B(4; 3; 5)$, $C(3; 4; -2)$, $D(2; -5; -13)$.

2. Berilgan tenglamani kanonik ko'rinishga keltiring va ikkinchi tartibli sirt turini aniqlang.

- 1) $x^2 - y^2 + z^2 - 2y - 2z = 0$
- 2) $xy + yz + xz = 0$
- 3) $z^2 - xy + 4x - 4y = 12$
- 4) $x^2 + y^2 + z^2 - 4x + 6y - 1 = 0$
- 5) $x^2 - 4xy + y^2 - z^2 = 0$
- 6) $x^2 + 4y^2 + 4z^2 - 2x + 4y - 8z = 0$
- 7) $x^2 - xy + xz + yz = 0$
- 8) $z^2 - xy + y^2 = 10$
- 9) $y^2 - x^2 + z^2 - 2xy = 0$
- 10) $x^2 + y^2 - 4xz + 4yz = 0$
- 11) $x^2 - 4xy - 4xz - 4yz = 0$
- 12) $2x^2 + y^2 - 18z - 3z + y + 14 = 0$
- 13) $3x^2 - 4y^2 - 24z - x + 2y - 2 = 0$
- 14) $9x^2 + 4y^2 - z^2 - 9x + 6y - 2z - 27 = 0$
- 15) $x^2 + 3y^2 + 4z^2 + 2x - 3y + 4z - 10 = 0$

II-qism. MATEMATIK ANALIZ
4-bob. To'plam. Funktsiya. Limit va uzluksizlik.
§14. To'plam. Amallar. To'plam turlari.

Biror xususiyatiga ko'ra jamlangan predmetlar majmuasi to'plam deb qaralishi mumkin. To'plamlar $A, B, \dots, X, Y, \dots$ harflar bilan, ularni tashkil qiluvchi predmetlar — elementlari $a, b, \dots, x, y, \dots$ harflar yordamida belgilanadi. a element A to'plamga tegishliligi $a \in A$, tegishli emasligi $a \notin A$ tarzida belgilanadi.

Agar A to'plam elementlari B to'plamga ham element hisoblansa, A to'plam V to'plamning qismi deyiladi va $A \subset B$ tarzida yoziladi. $A \subset B$, $B \subset A$ bir paytda bajarilsa, $A = B$ kelib chiqadi.

A va B to'plamlar barcha elementlaridan tuzilgan to'plam ularning yig'indisi deyiladi va $A \cup B$ ko'rinishda belgilanadi. Ularning faqatgina umumiy elementlaridan tuzilgan to'plam kesishma deyilib, $A \cap B$ tarzida belgilanadi. Faqatgina A ga tegishli elementga (A ga xos elementlar) to'plami A dan B ni ayrilgani deyiladi, $A \setminus B$ tarzida belgilanadi. A va B xos elementlaridan tuzilgan to'plam ularning to'g'ri ko'paytmasi deyilib, $A \Delta B$ tarzida yoziladi. Demak, $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

$a \in A$, $b \in B$ elementlar olinib hosil qilingan barcha (a, b) ko'rinishidagi juftliklar dekart ko'paytma deyiladi va $A \times B$ kabi belgilanadi.

Birorta ham elementi bo'lmagan to'plam bo'sh to'plam deyilib, $\emptyset$ ko'rinishda yoziladi.

Qaralayotgan A , B to'plamlar biror E to'plam qismlari bo'lsa, $E \setminus A$ to'plam A ning E ga qadar to'ldiruvchisi deyiladi va $C_E A$ yoki SA ko'rinishda yoziladi.

Quyidagi ikkilanganlik printsipli deb ataluvchi tengliklar o'rinni:

$$E \setminus \bigcup_n A_n = \bigcap_n (E \setminus A_n), \quad E \setminus \bigcap_n A_n = \bigcup_n (E \setminus A_n).$$

Agar A to'plamning har bir elementiga B to'plamning bitta elementini, B ning har bir elementiga A ning bitta elementi mos qo'yilsa, A va B to'plamlar o'zaro bir qiymatli moslikda deyiladi. Bunday to'plamlar o'zaro ekvivalent deyilib, $A \sim B$ tarzida yoziladi.

To'plamlar elementlari soniga qarab solishtiriladi:

- 1) elementlari soni chekligi bo'lgan to'plam chekli to'plam deyiladi;
- 2) elementlarini sanash mumkin bo'lgan, ya'ni natural sonlar to'plamiga ekvivalent to'plamlar sanoqli to'plamlar deyiladi.
- 3) Elementlarini sanash mumkin bo'lmagan cheksiz elementli to'plamlar sanoqsiz to'plamlar (yoki S -kontinuum quvvatli), deyiladi.

14.1. Isbotlang.

- 1) $(A \cup B) \cup C = A \cup (B \cup C)$;
- 2) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$;
- 3) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$;
- 4) $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$;
- 5) $C(A \setminus B) = CA \setminus B$;
- 6) $(A \cup B) \times C = (A \times C) \cup (B \times C)$;
- 7) $A \times (B \cap C) = (A \times B) \cap (A \times C)$;
- 8)

$$(A \cup B) \setminus (A \cap B) = (A \cap CB) \cup (B \cap CA).$$

14.2. Soddashtiring: $C[C(CA \cup B) \cup (A \cup CB)]$.

14.3. Haqiqiy sonlar to'plami R va irratsional sonlar to'plami $(R \setminus Q)$ orasida o'zaro bir qiymatli moslik o'rnatilgan.

- 14.4. $[0;1]$ kesmadagi sonlarni o'nti kasrga yoyganda 9 raqami qatnashmaydigan sonlar to'plami bilan $[0;1]$ kesma orasida o'zaro bir qiymatli moslik o'rnatish.
- 14.5. Tekislikda uchlari koordinatalari ratsional bo'lgan uchburchaklar to'plami quvvatini toping.
- 14.6. Tekislikda o'zaro kesishmaydigan T harflari to'plami quvvatini toping.

§15. Funktsiya tushunchasi. Elementar funktsiyalar.

Ketma-ketliklar.

Agar X to'plamdan olingan har bir songa biror f qoidaga yoki qonunga ko'ra Y to'plamning bitta y soni mos qo'yilgan bo'lsa, u holda X to'plamda funktsiya aniqlangan deyiladi.

Bu moslik $y = f(x)$ tarzida yoziladi. X to'plam funktsiya aniqlanish sohasi, Y — o'zgarish sohasi deyiladi, x — argument, y — funktsiya deyiladi.

Aniqlanish sohasi natural sonlardan iborat funktsiya ketma-ketlik deyiladi, $y = f(n)$ o'rniga y_n yoki a_n kabi belgilanadi.

Funktsiya juft (toq) deyiladi, agar $f(-x) = f(x)$ [$f(-x) = -f(x)$] bajarilsa.

Ixtiyoriy $x \in X$ da $m \leq f(x) \leq M$ shart bajarilsa, funktsiya quyidan m soni, yuqoridan M soni bilan chegaralangan deyiladi.

Agar x ning X to'plamdagi x_1 va x_2 qiymatlari uchun $x_1 < x_2$ shartdan $f(x_1) < f(x_2)$ [$f(x_1) > f(x_2)$] kelib chiqsa, $f(x)$ bu to'plamda o'suvchi (kamayuvchi) deyiladi.

Agar shunday T son mavjud bo'lib, ixtiyoriy $x \in X$ da

$$1) x \pm T \in X, \quad 2) f(x + T) = f(x)$$

bo'lsa, bu funktsiya davriy, bu shartga bo'ysunuvchi eng kichik T soni davr deyiladi.

$y = f(x)$ funktsiyada x va y larning o'rinlarini almashtirishdan hosil bo'ladigan funktsiya teskari funktsiya deyiladi va $x = f^{-1}(y)$ tarzida belgilanadi.

Ketma - ketlik uchun chegaralangʻanlik, o'suvchilik yoki kamayuvchilikni aniqlash mumkin.

15.1. Berilgan funktsiyalar aniqlanish sohasini toping.

$$1) y = \frac{x}{1-x} \quad 2) y = \sqrt{3x - x^2}$$

$$3) y = \log(x^2 - 4) \quad 4) y = \sqrt{\sin \sqrt{x}}$$

$$5) y = \log_2 \log_3 x \quad 6) y = \sqrt{\lg \lg x}$$

$$7) y = \sqrt{\sin 2x} + \sqrt{\sin 3x}, x \in [0; 2\pi].$$

15.2. 1) $f(x+1) = x^2 - 3x + 2$ bo'lsa, $f(x)$ ni toping.

$$2) f\left(\frac{1}{x}\right) = x + \sqrt{1+x^2} \text{ bo'lsa, } f(x) \text{ ni toping}$$

($x > 0$).

15.3. Berilgan funktsiyalarga teskari funktsiyalarni toping.

$$1) y = 2x + 3$$

$$2) y = x^2, \quad 0 \leq x < +\infty$$

$$3) y = \frac{1-x}{1+x}, \quad x \neq -1$$

$$4) y = \sqrt{1-x^2}, \quad 0 \leq x \leq 1.$$

$$5) y = \begin{cases} x, & -\infty < x < 1 \\ x^2, & 1 \leq x \leq 4 \\ 2^x, & 4 < x < +\infty \end{cases}$$

15.4. Juft – toqligini tekshiring.

1) $y = 3x - x^3$;

2) $y = \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2}$;

3) $y = a^x + a^{-x}$

4) $y = \ln(x + \sqrt{1+x^2})$.

15.5. 1) $y = kx$ grafigini $k = 0, 1, -1$ hollarda chizing.

2) $y = x + b$ grafigini $b = 0, 1, -1$ bo'lganda chizing.

3) $y = ax^2$, $y = (x-a)^2$, $y = x^2 + a$ grafiglarini $a = 0$, $a = \pm 1$, $a = \pm 2$ bo'lgan hollarda chizing.

15.6. Quyidagi funktsiyalar grafiglarini chizing.

1) $y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

2) $y = \operatorname{sgn} x = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

3) $y = [x] = \{x \text{ ning butun qismi}\}$

4) $y = \{x\} = \{x \text{ ning kasr qismi}\}$

5) $y = \sin^2 x$

6) $y = \sin x \cdot \sin 3x$

7) $y = \operatorname{cthx}$

8) $y = [x] \cdot |\sin x|$

9) $y = 1 + x + e^x$

10) $y = \sin^4 x + \cos^4 x$

- 11) $y = |1 - x| + |1 + x|$.
- 12) $y = x \sin x$
- 13) $y = \ln(\sin x)$
- 14) $y = \ln \cos x$
- 15) $y = shx = \frac{e^x - e^{-x}}{2}$
- 16) $y = chx = \frac{e^x + e^{-x}}{2}$
- 17) $y = tgx$;
- 18) $y = x \cdot \operatorname{sgn}(\sin x)$
- 19) $y = \cos x \cdot \operatorname{sgn}(\sin x)$
- 20) $y = x + \sin x$
- 21) $y = |1 - x| - |1 + x|$

§16. Ketma-ketlik va funktsiya limiti.

Uzlaksizlik. Ajoyib limitlar.

Agar a nuqtaning ixtiyoriy $(a - \varepsilon, a + \varepsilon)$ atrofi ($\varepsilon > 0$) olinganda ham $\{x_n\}$ ketma-ketlikning biror hadidan boshlab, keyingi barcha hadlari shu atrofga tegishli bo'lsa, a son $\{x_n\}$ ketma-ketlikning limiti deyiladi va

$$\lim_{n \rightarrow \infty} x_n = a$$

tarzda belgilanadi.

Ketma-ketlik chekli songa intilsa yaqinlashuvchi, aks holda uzoqlashuvchi deyiladi.

Agar X to'plamning nuqtalaridan tuzilgan, a ga yaqinlashuvchi har qanday $\{x_n\}$ ketma-ketlik olinganda ham, funktsiya qiymatlaridan iborat $\{f(x_n)\}$ ketma-ketlik yagona b limitga intilsa, shu

b ga $f(x)$ ning a nuqtadagi (x ning a intilgandagi) limiti deyiladi va $\lim_{n \rightarrow \infty} x_n = b$ kabi

yoziladi. Bunda, agar $\{x_n\}$ ketma-ketlik a dan faqat katta (kichik) bo'lib a ga intilsa, o'ng (chap) limit deyiladi va

$$\lim_{n \rightarrow a+0} f(x) = F(a+0) = b \quad \left(\lim_{n \rightarrow a-0} f(x) = f(a-0) = b \right)$$

tarzida yoziladi.

Limitlar quyidagi xossalarga ega:

1) O'zgarmas son limiti o'ziga teng.

2) $\lim(u + v) = \lim u + \lim v$.

3) $\lim(u \cdot v) = \lim u \cdot \lim v$

4) $\lim \frac{u}{v} = \frac{\lim u}{\lim v}$

Agar $\lim_{x \rightarrow x_0} f(x) = f(x_0)$ o'rinli bo'lsa,

$f(x)$ $x = x_0$ nuqtada uzluksiz deyiladi. Funktsiya x_0 nuqtada uzluksiz bo'lishi uchun

$$\lim_{x \rightarrow x_0-0} f(x) = f(x_0) = \lim_{x \rightarrow x_0+0} f(x)$$

tengliklar bir paytda bajarilishi zarur va etarli.

Birorta tenglik bajarilmasa, $f(x)$ funktsiya x_0 nuqtada uzilishga ega deyiladi.

$$\lim_{x \rightarrow x_0+0} f(x) - \lim_{x \rightarrow x_0-0} f(x) = \lambda \neq 0 \text{ bo'lsa, I-}$$

tur uzilishga ega, λ esa funktsiyaning x_0 nuqtadagi sakrashi deyiladi. Boshqa turdagi uzilishlar barchasi II-tur uzilishlar deyiladi.

Limitlarni hisoblashda quyidagi ko'p uchraydigan «ajoyib limitlar»dan foydalaniladi:

$$1) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

$$2) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e.$$

$$3) \lim_{x \rightarrow 0} \frac{(1+x)^\mu - 1}{x} = \mu$$

$$4) \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$5) \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$$

16.1. Limitlar hisoblansin.

$$1) \lim_{n \rightarrow \infty} \frac{5n-1}{n+3}$$

$$2) \lim_{n \rightarrow \infty} \frac{4n^2+n}{1-n^2}$$

$$3) \lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} \quad 4) \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n-1}{n^2} \right)$$

$$5) \lim_{n \rightarrow \infty} (\sqrt{2} \cdot \sqrt[4]{2} \cdot \sqrt[8]{2} \dots \sqrt[2^n]{2}) \quad 6) \lim_{n \rightarrow \infty} \frac{\sqrt{2n^2+1}}{2n-1}$$

$$7) \lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}} \quad 8) \lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

$$9) \lim_{n \rightarrow +\infty} \frac{1-10^n}{1+10^{n+1}}$$

$$10) \lim_{n \rightarrow -\infty} \frac{3-10^n}{2+10^{n+1}}$$

$$11) \lim_{n \rightarrow \infty} \left(\frac{1+2+\dots+n}{n+2} - \frac{n}{2} \right) \quad 12) \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n} \right)^{3n}$$

$$13) \lim_{n \rightarrow \infty} \left(1 - \frac{5}{n} \right)^n \quad 14) \lim_{n \rightarrow \infty} \left(\frac{3n-2}{3n+1} \right)^{2n}$$

16.2 Limitlar hisoblansin.

- 1) $\lim_{n \rightarrow 2} \frac{x^2 - 4}{x - 2}$
- 2) $\lim_{n \rightarrow 3} \frac{x^2 - 9}{x^2 - 2x - 3}$
- 3) $\lim_{n \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 8x + 15}$
- 4) $\lim_{n \rightarrow 2} \frac{(x^2 - x - 2)^{20}}{(x^3 - 12x + 16)^{10}}$
- 5) $\lim_{n \rightarrow 1} \frac{x^m - 1}{x^n - 1}$
- 6) $\lim_{n \rightarrow 1} \left(\frac{m}{1 - x^m} - \frac{n}{1 - x^n} \right)$
- 7) $\lim_{n \rightarrow 0} \frac{x}{\sqrt{1 + 3x} - 1}$
- 8) $\lim_{n \rightarrow 0} \frac{\sqrt[3]{1 + mx} - 1}{x}$
- 9) $\lim_{n \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}$
- 10) $\lim_{n \rightarrow \infty} (\sqrt{x^2 + 3x} - x)$
- 11) $\lim_{n \rightarrow \infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 - x})$
- 12) $\lim_{n \rightarrow 1} \frac{\sqrt[n]{x} - 1}{\sqrt{x} - 1}$
- 13) $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$
- 14) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$
- 15) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{\sin^3 x}$
- 16) $\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x}$
- 17) $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}$
- 18) $\lim_{x \rightarrow a} \frac{\operatorname{tg} x - \operatorname{tg} a}{x - a}$
- 19) $\lim_{x \rightarrow 0} \frac{\sqrt{1 + \operatorname{tg} x} - \sqrt{1 + \sin x}}{x^3}$
- 20) $\lim_{x \rightarrow 0} \frac{\cos x - \sqrt[3]{\cos x}}{\sin^2 x}$
- 21) $\lim_{x \rightarrow \infty} \left(\frac{x + 2}{2x - 1} \right)^{x^2}$
- 22) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 1}{x^2 + 1} \right)^{\frac{x-1}{x+1}}$
- 23) $\lim_{x \rightarrow 0} \sqrt[3]{1 - 2x}$
- 24) $\lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \sin x} \right)^{\frac{1}{\sin^3 x}}$

Bobga doir misollar echish namunalari

1. $(A \cap B) \cup (A \cap CB) \cup (CA \cap B)$ ifodani soddalashtiring.
 $(A \cap B) \cup (A \cap CB) \cup CA \cap B = (A \cap B) \cup (a \setminus B) \cup (B \setminus A) =$
 $= (A \cap B) \cup (A \Delta B) = A \cup B.$

2. Ixtiyoriy A , B , S to'plamlar uchun $Ax(B \cup C) = (Ax B) \cup (Ax C)$ tenglik o'rinli ekanligini isbotlang.

a) $(x, y) \in Ax(B \cup C)$ bo'lsin. $x \in A$, $y \in B \cup C$, ya'ni $x \in A$, $y \in B$ yoki $x \in A$, $y \in C$. Demak, $(x, y) \in Ax B$ yoki $(x, y) \in Ax C$. Bulardan, $(x, y) \in (Ax B) \cup (Ax C)$.

b) $(x, y) \in (Ax B) \cup (Ax C)$. Demak, $(x, y) \in Ax B$ yoki $(x, y) \in Ax C$. Undan $x \in A$, $y \in B$ yoki $x \in A$, $y \in C$ kelib chiqib, $x \in A$, $y \in B \cup C$. Nihoyat, $(x, y) \in Ax(B \cup C)$.

a) va b) munosabatlar tenglik o'rinliligini bildiradi.

3. $A = (0; 1)$ va $B = (0; 1]$ to'plamlar orasida o'zaro bir qiymatli moslik o'rnatish.

A va V to'plamlarda
 $\{x_1 = \frac{1}{2}, x_2 = \frac{1}{3}, \dots, x_n = \frac{1}{n+1}, \dots\}$ nuqtalar
 to'plamini ajratamiz.

B dagi 1 ga A dagi x_1 ni, x_n ga x_{n+1} ni mos qo'yamiz. B dagi qolgan $x \in (0; 1)$ nuqtalarga A dagi mos $x \in (0; 1)$ nuqtalarni qo'yamiz.

4. Ratsional sonlar to'plami Q ning sanoqliligini isbotlang.

Har bir ratsional son qisqarmas $\frac{p}{q}$
 ($p \in \mathbb{Z}, q \in \mathbb{N}$) kasr ko'rinishda yoziladi. $|p| + q$
 yig'indi $\frac{p}{q}$ ratsional son balandligi deyiladi.

Ratsional sonlarni balandligi o'sish tartibida joylashtiramiz:

$$\frac{0}{1}, \frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{2}, \frac{2}{1}, \frac{-2}{1}, \frac{1}{3}, \frac{-1}{3}, \frac{3}{1}, \frac{-3}{1}, \dots$$

Bunda har bir ratsional son biror nomer oladi va $\mathbb{Q}^+$ ekanligi kelib chiqadi.

5. $[0; 1]$ segment nuqtalari to'plami sanoqsizligini isbotlang.

Ular sanab chiqilgan: $\alpha_1, \alpha_2, \dots, \alpha_n, \dots$ va cheksiz o'nli kasrga yoyilgan deylik:

$$\alpha_1 = 0, a_{11} a_{12} a_{13} \dots$$

$$\alpha_{21} = 0, a_{21} a_{22} a_{23} \dots$$

$$\alpha_n = 0, a_{n1} a_{n2} a_{n3} \dots$$

.....

Lekin, sanalmay qolib ketgan elementlarni topish mumkin. Masalan,

$$\alpha_0 = b_1 b_2 b_3 \dots \quad (b_1 \neq a_{11}, b_2 \neq a_{22}, \dots)$$

sanalmagan.

Demak, $[a, b], [a, b), (a, b], (a, b)$ nuqtalar to'plamlari sanoqsiz ekan.

6. a) $y = (x - 2) \sqrt{\frac{1+x}{1-x}}$ aniqlanish sohasini toping.

$\frac{1+x}{1-x} \geq 0$ o'rinli bo'lishi shart, demak,
intervallar metodidan

$$D(y) = [-1; 1] \text{ kelib chiqadi.}$$

b) $y = \lg \sin x$ aniqlanish sohasini toping.

Logarifmik funktsiya $\sin x > 0$ qiymatlarda
aniqlangan, xolos, ya'ni

$$\sin x > 0 \Rightarrow 2k\pi < x < \pi + 2k\pi, \quad k \in \mathbb{N}.$$

7. a) $f = x^2$, $g = 2^x$ bo'lsa, $f(g(x))$ - murakkab
funktsiyani tuzing.

$$f(g(x)) = (2^x)^2 = 2^{2x} = 4^x.$$

b) $f(x) = \frac{x}{\sqrt{1+x^2}}$ bo'lsa, $f_n(x) = \underbrace{f(f(\dots(f(x)\dots))}_n$

ni toping.

$$f(f(x)) = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}} = \frac{x}{\sqrt{1+x^2+x^2}} = \frac{x}{\sqrt{1+2x^2}}$$

$$f(f(f(x))) = \frac{x}{\sqrt{1+3x^2}}, \dots, f_n(x) = \frac{x}{\sqrt{1+nx^2}}.$$

8. $f(x) = \ln \frac{1-x}{1+x}$ funktsiyani juft - toqligini
tekshiring.

$$f(-x) = \ln \frac{1+x}{1-x} = \ln \left(\frac{1-x}{1+x} \right)^{-1} = -\ln \frac{1-x}{1+x} = -f(x).$$

9. Quyidagi ifodalar qiymatini toping:

$$1) \lim_{x \rightarrow \infty} \frac{2n^2 - 1}{n^2 + n} = \lim_{x \rightarrow \infty} \frac{\frac{2n^2}{n^2} - \frac{1}{n^2}}{\frac{n^2}{n^2} + \frac{n}{n^2}} = \lim_{x \rightarrow \infty} \frac{2 - \frac{1}{n^2}}{1 + \frac{1}{n}} = \frac{2}{1} = 2.$$

$$2) \lim_{x \rightarrow \infty} \left[\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} \right] = \lim_{x \rightarrow \infty} \left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{n} - \frac{1}{n+1} \right] = \lim_{x \rightarrow \infty} \left[1 - \frac{1}{n+1} \right] = 1.$$

$$3) \lim_{x \rightarrow \infty} \left(\frac{n}{n+1} \right)^n = \lim_{x \rightarrow \infty} \left(\frac{n+1-1}{n+1} \right)^n = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{n+1} \right)^n =$$

$$= \lim_{x \rightarrow \infty} \left(1 + \left(-\frac{1}{n+1} \right) \right)^{\frac{n}{-(n+1)} \cdot \frac{n}{-(n+1)}} = e^{\lim_{x \rightarrow \infty} \frac{-n}{n+1}} = e^{-1}.$$

10. Ifodalar qiymatini toping.

$$1) \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x-3)}{x-2} = \lim_{x \rightarrow 2} (x-3) = -1.$$

$$2) \lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1} =$$

$$= \lim_{x \rightarrow 1} \frac{(x-1) + (x^2-1) + \dots + (x^n-1)}{x-1} =$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)[1 + (x+1) + (x^2+x+1) + \dots + (x^{n-1} + x^{n-2} + \dots + 1)]}{(x-1)} =$$

$$= 1 + 2 + 3 + \dots + n = \frac{n+1}{2} \cdot n$$

$$3) \lim_{x \rightarrow -8} \frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}} = \lim_{x \rightarrow -8} \frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}} \cdot \frac{\sqrt{1-x}+3}{\sqrt{1-x}+3} \cdot \frac{(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{(4-2\sqrt[3]{x}+\sqrt[3]{x^2})} =$$

$$= \lim_{x \rightarrow -8} \frac{(1-x-9)(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{(8+x)(\sqrt{1-x}+3)} = - \lim_{x \rightarrow -8} \frac{(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{\sqrt{1-x}+3} =$$

$$= \frac{-12}{6} = -2.$$

$$4) \lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx} = \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \frac{nx}{\sin nx} \cdot \frac{m}{n} = \frac{m}{n}.$$

5)

$$\lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \sin x} \right)^{\frac{1}{\sin x}} = \lim_{x \rightarrow 0} \left(\frac{1 + \sin x + \operatorname{tg} x - \sin x}{1 + \sin x} \right)^{\frac{1}{\sin x}} =$$

$$= \lim_{x \rightarrow 0} \left(1 + \frac{\operatorname{tg} x - \sin x}{1 + \sin x} \right)^{\frac{1}{\sin x}} = \lim_{x \rightarrow 0} \left(1 + \frac{\operatorname{tg} x - \sin x}{1 + \sin x} \right)^{\frac{+ \sin x}{\operatorname{tg} x - \sin x} \cdot \frac{\operatorname{tg} x - \sin x}{1 + \sin x} \cdot \frac{1}{\sin x}} =$$

$$e^{\lim_{x \rightarrow 0} \frac{\sin x \left(\frac{1}{\cos x} - 1 \right)}{1 + \sin x} \cdot \frac{1}{\sin x}} = e^{\lim_{x \rightarrow 0} \frac{1 - \cos x}{\cos x (1 + \sin x)}} = e^0 = 1.$$

$$6) \lim_{x \rightarrow b} \frac{a^x - a^b}{x - b} = \lim_{x \rightarrow b} \frac{a^b (a^{x-b} - 1)}{x - b} = a^b \cdot \ln a.$$

$$7) \lim_{x \rightarrow +\infty} [\sin \ln(x+1) - \sin \ln x] =$$

$$= \lim_{x \rightarrow +\infty} 2 \sin \frac{\ln(x+1) - \ln x}{2} \cos \frac{\ln(x+1) + \ln x}{2} =$$

$$= \lim_{x \rightarrow +\infty} \frac{2 \sin \frac{1}{2} \ln \left(1 + \frac{1}{x} \right) \cdot \cos \ln \sqrt{x^2 + x}}{\frac{1}{x} \cdot x} = 0$$

8) $y = 2^{\frac{1}{x-2}}$ funktsiyani $x = 2$ da uzluksizlikka tekshiring.

$x = 2$ da funktsiya aniqlanmagan.

$$\lim_{x \rightarrow 2+0} f(x) = \lim_{x \rightarrow 2+0} 2^{\frac{1}{x-2}} = +\infty; \quad \lim_{x \rightarrow 2-0} 2^{\frac{1}{x-2}} = 0$$

II – tur uzulishga ega.

$$9) f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ 2-x, & 1 < x \leq 2 \end{cases} \text{ funktsiyani}$$

uzluksizlikka tekshiring.

$$\lim_{x \rightarrow 1-0} f(x) = \lim_{x \rightarrow 1-0} x^2 = 1; \quad \lim_{x \rightarrow 1+0} f(x) = \lim_{x \rightarrow 1+0} (2-x) = 1;$$

$$f(1) = 1^2 = 1$$

Funktsiya $x = 1$ nuqtada uzluksiz ekan.

4-bobga doir uy vazifalari

1. Funktsiya aniqlanish sohasini toping.

$$1) y = \sqrt{\frac{x^2 - 4x + 3}{x^2 - 4}} \quad 2) y = \sqrt{\frac{x^2 - 9}{x^2 - 7x + 6}}$$

$$3) y = \sqrt{\frac{x^2 - 4x - 5}{x^2 + x - 2}} \quad 4) y = \sqrt{\frac{(x-1)(x^2 - 16)}{x^2 - 10x + 16}}$$

$$5) y = \sqrt{\frac{x^2 - 16}{x^2 - 7x + 6}} \quad 6) y = \sqrt{\frac{x^2 - 36}{x^2 - 5x}}$$

$$7) y = \log_x \frac{x-1}{x+3} \quad 8) y = \ln \frac{x^2 - 4}{9 - x^2}$$

$$9) y = \lg \frac{x^2 - 4x}{x^2 + 5x - 6} \quad 10) y = \lg \sin 2x$$

$$11) y = \lg \cos 3x \quad 12) y = \sqrt{\lg 2x}$$

$$13) y = \lg(1 - 2 \cos x) \quad 14) y = \lg(\sqrt{3} - \operatorname{ctg} x)$$

$$15) y = \sqrt[4]{\lg \operatorname{tg} x}$$

2. Berilgan funktsiyaga teskari funktsiyani toping.

1) $y = 4x - 1$

2) $y = 2x + 3$

3) $y = 1 - 4x$

4) $y = 3 - 2x$

5) $y = 5x + 1$

6) $y = 2 - 5x$

7) $y = 1 - x$

8) $y = 4 + 3x$

9) $y = 4 - 5x$

10) $y = 5x - 4$

11) $y = x - 10$

12) $y = \frac{1+x}{1-x}$

13) $y = \frac{2x-1}{1+2x}$

14) $y = \frac{3x-1}{1+3x}$

15) $y = \frac{1}{4x-1}$

3. Quyidagi funktsiyalar grafigini chizing:

1) $y = \sin^2 x$

2) $y = \sin^3 x$

3) $y = \{\sin x\}$

4) $y = \{\sin x\}$

5) $y = \operatorname{sgn}(\cos x)$

6) $y = [x^2]$

7) $y = x + e^x$

8) $y = x + \sin x$

9) $y = \sin^4 x + \cos^4 x$

10) $y = |x+3| + |x-3|$

11) $y = x \cdot \cos x$

12) $y = 1 - e^{-x}$

13) $y = [x] \cdot |\cos x|$

14) $y = \cos x \operatorname{sgn}(\sin x)$

15) $y = x + \arctg x$

4. Limitlarni hisoblang

1) a) $\lim_{n \rightarrow \infty} \frac{n^4 - 3n}{1 - 2n^4}$

6) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{5x}$

6) $\lim_{x \rightarrow 0} \frac{1 - \cos 10x}{5x^2}$

2) $\lim_{n \rightarrow \infty} \left(\frac{x+3}{x-2} \right)^x$

- 2) a) $\lim_{x \rightarrow \infty} \frac{x^3 + 1}{2x^3 + 1}$; б) $\lim_{x \rightarrow 7} \frac{\sqrt{2+x} - 3}{x - 7}$;
 в) $\lim_{x \rightarrow 0} \frac{\arcsin 3x}{5x}$; г) $\lim_{x \rightarrow \infty} \left(\frac{2x-1}{2x+1} \right)^x$.
 3) a) $\lim_{x \rightarrow \infty} \frac{2x^3 + x^2 - 5}{x^3 + x - 2}$; б) $\lim_{x \rightarrow 1} \frac{x - \sqrt{x}}{x^2 - x}$;
 в) $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$; г) $\lim_{x \rightarrow \infty} \left(\frac{4x+1}{4x} \right)^{2x}$.
 4) a) $\lim_{x \rightarrow \infty} \frac{3x^4 + x^2 - 6}{2x^4 - x + 2}$; б) $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1}$;
 в) $\lim_{x \rightarrow 0} \frac{5x}{\arctg x}$; г) $\lim_{x \rightarrow 0} (1+2x)^{1/x}$.
 5) a) $\lim_{x \rightarrow \infty} \frac{2x^2 + 6x - 5}{5x^2 - x - 1}$; б) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{1-x^2}}{x^2}$;
 в) $\lim_{x \rightarrow 0} \frac{\cos x - \cos^3 x}{x^2}$; г) $\lim_{x \rightarrow \infty} x[\ln(x+1) - \ln x]$.
 6) a) $\lim_{x \rightarrow \infty} \frac{3+x+5x^4}{x^4 - 12x + 1}$; б) $\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{1-2x}}{x+x^2}$;
 в) $\lim_{x \rightarrow 0} \frac{x^2 \operatorname{ctg} x}{\sin 3x}$; г) $\lim_{x \rightarrow \infty} (2+1)[\ln(x+3) - \ln x]$.
 7) a) $\lim_{x \rightarrow \infty} \frac{x - 2x^2 + 5x^4}{2 + 3x^2 + x^4}$; б) $\lim_{x \rightarrow 0} \frac{\sqrt{1+3x^2} - 1}{x^2 + x^3}$;
 в) $\lim_{x \rightarrow 0} \frac{1 - \cos 6x}{1 - \cos 2x}$;
 г) $\lim_{x \rightarrow \infty} (x-5)[\ln(x-3) - \ln x]$.

$$8) \quad a) \lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{3x^2 + x - 5}; \quad б) \lim_{x \rightarrow 0} \frac{\sqrt{2x-1} - \sqrt{5}}{x-3};$$

$$в) \lim_{x \rightarrow 0} \frac{\operatorname{tg}^2 \frac{x}{2}}{x^2}; \quad з) \lim_{x \rightarrow 1} (7-6x)^{x/(3x-3)}.$$

$$9) \quad a) \lim_{x \rightarrow \infty} \frac{7x^4 - 2x^3 + 2}{x^4 + 3}; \quad б) \lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{2x+6}}{x^2 - 5x};$$

$$в) \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{2x \operatorname{tg} 2x}; \quad з) \lim_{x \rightarrow 2} (3x-5)^{\frac{2x}{x^2-4}}.$$

$$10) \quad a) \lim_{x \rightarrow \infty} \frac{8x^5 - 3x^2 + 9}{x^5 + 2x^2 + 5}; \quad б) \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{2x}-2};$$

$$в) \lim_{x \rightarrow 0} 5x \operatorname{ctg} 3x; \quad з) \lim_{x \rightarrow 3} (3x-8)^{\frac{2}{x-3}}.$$

$$11) \quad a) \lim_{x \rightarrow \infty} \frac{2x^2 - 4x}{5x^2 - 6}; \quad б) \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 3}{x^2 - 9};$$

$$в) \lim_{x \rightarrow 0} \frac{1 - \cos 8x}{\sin^2 x}; \quad з) \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}.$$

$$12) \quad a) \lim_{x \rightarrow \infty} \frac{1 - 2x + 3x^2}{x^2 + 1}; \quad б) \lim_{x \rightarrow -1} \frac{\sqrt{5+x} - 2}{x+1};$$

$$в) \lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 4x}}{\sin x}; \quad з) \lim_{x \rightarrow 1} (4-3x)^{\frac{1}{x-1}}.$$

$$13) \quad a) \lim_{x \rightarrow \infty} \frac{4x^2 - 4x + 5}{x^2 - 3x + 10}; \quad б) \lim_{x \rightarrow 3} \frac{\sqrt{x+6} - 3}{x^2 - 9};$$

$$в) \lim_{x \rightarrow 0} \frac{\cos x - \cos^3 x}{\sin^2 4x}; \quad з) \lim_{x \rightarrow 2} (5-2x)^{\frac{1}{2-x}}.$$

$$14) \quad a) \lim_{x \rightarrow \infty} \frac{x^3 - 4x + 1}{1 - 2x + 2x^3} \quad b) \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x+3} - 2};$$

$$b) \lim_{x \rightarrow 0} \frac{\operatorname{tg}^2 \frac{x}{2}}{\sin^2 2x}; \quad z) \lim_{x \rightarrow 0} (1 + 6x)^{\frac{2}{x}}.$$

$$15) \quad a) \lim_{x \rightarrow \infty} \frac{4x^3 - 2x}{2 + 4x + 5x^3}; \quad b) \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x^2 - 4};$$

$$b) \lim_{x \rightarrow 0} \frac{\arcsin 5x}{2x}; \quad z) \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-4} \right)^x.$$

5. Berilgan funksiylarni uzluksizlikka tekshiring, grafigini chizing.

$$1) f(x) = \begin{cases} x+4, & x < -1 \\ x^2 + 2, & -1 \leq x < 1 \\ 2x, & x \geq 1 \end{cases}$$

$$2) f(x) = \begin{cases} x+2, & x \leq -1 \\ x^2 + 1, & -1 < x \leq 1 \\ -x+3, & x > 1 \end{cases}$$

$$3) f(x) = \begin{cases} -x, & x \leq 0 \\ -(x-1)^2, & 0 < x < 2 \\ x-3, & x \geq 2 \end{cases}$$

$$4) f(x) = \begin{cases} \cos x, & x \leq 0 \\ x^2 + 1, & 0 < x < 1 \\ x, & x \geq 1 \end{cases}$$

$$5) f(x) = \begin{cases} -x & x \leq 0 \\ x^2 & , \quad 0 < x \leq 2 \\ x+1 & , \quad x > 2 \end{cases}$$

$$6) f(x) = \begin{cases} -x & x \leq 0 \\ \sin x & , \quad 0 < x \leq \pi \\ x-2 & , \quad x > \pi \end{cases}$$

$$7) f(x) = \begin{cases} -(x+1) & x \leq -1 \\ (x+1)^2 & , \quad -1 < x \leq 0 \\ x & , \quad x > 0 \end{cases}$$

$$8) f(x) = \begin{cases} -x^2 & x \leq 0 \\ \operatorname{tg} x & , \quad 0 < x \leq \frac{\pi}{4} \\ 2 & , \quad x > \frac{\pi}{4} \end{cases}$$

$$9) f(x) = \begin{cases} -2x & x \leq 0 \\ x^2 + 1 & , \quad 0 < x \leq 1 \\ 2 & , \quad x > 1 \end{cases}$$

$$10) f(x) = \begin{cases} -2x & x \leq 0 \\ \sqrt{x} & , \quad 0 < x < 4 \\ 1 & , \quad x \geq 4 \end{cases}$$

$$11) f(x) = \begin{cases} -1 & x < -2 \\ |x| & , \quad |x| \leq 2 \\ 2 & , \quad x > 2 \end{cases}$$

$$12) f(x) = \begin{cases} x^2 & x < 0 \\ x-1, & 0 < x < 1 \\ 4, & x \geq 1 \end{cases}$$

$$13) f(x) = \begin{cases} x & x < 0 \\ x^2, & |x| \leq 1 \\ x+4, & x > 1 \end{cases}$$

$$14) f(x) = \begin{cases} -x^2, & x < 0 \\ 1, & 0 \leq x < 1 \\ (x-1)^2 + 1, & x \geq 1 \end{cases}$$

$$15) f(x) = \begin{cases} -x & x \leq 0 \\ \sin x, & 0 < x \leq \pi \\ x + \pi, & x > \pi \end{cases}$$

5-bob. Hosila va differentsial.

Differentsial hisob teoremlari.

§17. Hosila. Geometrik va fizik ma'nolari.

Hosila hisoblash qoidalari.

$y = f(x)$ funktsiyaning x nuqtadagi hosilasi deb

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

limitga aytiladi va $y'(x)$, $f'(x)$, $\frac{dy}{dx}$, $\frac{df(x)}{dx}$

tarzida belgilanadi. Agar hosila chekli bo'lsa, $f'(x)$ funktsiya x nuqtada differentsiallanuvchi deyiladi.

Agar $x = x_1$ $x + \Delta x = x_2$ bo'lsa, $f'(x)$ funktsiyaning x_1 nuqtadagi hosilasi

$$\lim_{x_1 \rightarrow x_2} \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ ko'rinishda bo'ladi. Limitlardagiga}$$

o'xshash, chap va o'ng hosila tushunchasini kiritish mumkin. Hosila $y = f(x)$ funktsiyaga x_1 nuqtada o'tkazilgan urinmaning burchak koeffitsientidir:

$$y'(x_1) = \operatorname{tg} \alpha$$

Urunma to'g'ri chiziqlarning tenglamasi $y - y_1 = y'(x_1) \cdot (x - x_1)$, unga perpendikulyar $(x_1; y_1)$ nuqtadan o'tuvchi normal to'g'ri chiziq tenglamasi:

$$y - y_1 = -\frac{1}{y'(x_1)}(x - x_1) \text{ ko'rinishida bo'ladi.}$$

Moddiy nuqta $S=S(t)$ qonun bo'yicha harakat qilsa,

uning tezligi $V = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t}$, tezlanishi esa

$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t}$ formulalar yordamida topiladi.

Hosilasi mavjud bo'lgan $u(x), v(x)$ funktsiyalar uchun quyidagi qoidalar o'rinli:

1. $(C \cdot u)' = C \cdot u'$;
2. $(C_1 u \pm C_2 v)' = C_1 u' \pm C_2 v'$;
3. $(u \cdot v)' = u'v + uv'$;
4. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$;
5. $y = f(x)$ va $x = g(y)$ o'zaro teskari funktsiyalar bo'lsa, $y'(x) = \frac{1}{x'(y)}$;
6. $y = f(u), u = g(x)$, bo'lsa, u holda $y = f(g(x))$ murakkab funktsiya hosilasi $y' = f'(u) \cdot g'(x)$ formula yordamida olinadi.
7. $y = u^v$ ko'rinishdagi funktsiya daraja – ko'rsatkichli deyilib, hosilasi quyidagi Bernulli formulasidan topiladi: $y' = u^v \cdot \left[v' \cdot \ln u + \frac{u'v}{u} \right]$
8. y ga nisbatan echilmagan tenglama bilan berilgan funktsiya hosilasi, murakkab funktsiya kabi olinadi, hosil bo'lgan tenglamadan y' topiladi.

$$9. \begin{cases} x = x(t) \\ y = y(t) \end{cases} \text{ sistema yordamida parametrik}$$

berilgan funksiya hosilasi $y'_t = \frac{y'_t}{x'_t}$ formula yordamida topiladi.

Hosilalar jadvali:

- | | |
|---|---|
| 1. $(C)' = 0;$ | 2. $(x^p)' = p \cdot x^{p-1};$ |
| 3. $(\sin x)' = \cos x;$ | 4. $(\cos x)' = -\sin x;$ |
| 5. $(\operatorname{tg} x)' = \frac{1}{\cos^2 x};$ | 6. $(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x};$ |
| 7. $(a^x)' = a^x \cdot \ln a;$ | 8. $(e^x)' = e^x;$ |
| 9. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}};$ | |
| 10. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}};$ | |
| 11. $(\operatorname{arctg} x)' = \frac{1}{1+x^2};$ | |
| 12. $(\operatorname{arcctg} x)' = -\frac{1}{1+x^2};$ | |
| 13. $(\log_a x)' = \frac{1}{x \cdot \ln a};$ | 14. $(\ln x)' = \frac{1}{x};$ |
| 15. $(\operatorname{sh} x)' = \operatorname{ch} x;$ | 16. $(\operatorname{ch} x)' = \operatorname{sh} x;$ |
| 17. $(\operatorname{th} x)' = \frac{1}{\operatorname{ch}^2 x};$ | 18. $(\operatorname{cth} x)' = -\frac{1}{\operatorname{sh}^2 x};$ |

17.1. Ta'rif yordamida hosilasini toping.

$$1) y = x^3; \quad 2) y = x^{100}; \quad 3) y = \sqrt{x};$$

$$4) y = \frac{1}{x}; \quad 5) y = \frac{1}{\sqrt{x}}; \quad 6) y = \frac{1}{x^3};$$

$$7) y = \sqrt{1+x^2}; \quad 8) y = \operatorname{tg} x$$

9) $f(x) = x(x-1)(x-2)\dots(x-100)$ bo'lsa, $f'(0)$ ni hisoblang.

17.2. Hosila hisoblash qoidalarini va jadvalidan foydalanib toping.

$$1) y = \frac{x^4}{4} - \frac{x^2}{2} + 4x; \quad 2) y = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3};$$

$$3) y = 4\sqrt[3]{x} - 3\sqrt[4]{x}; \quad 4) y = x - \sin x$$

$$5) y = x^2 \cdot \operatorname{ctg} x \quad 6) y = x^2 \cdot \operatorname{ctg} x$$

$$7) y = \frac{2x}{1-x^2} \quad 8) y = \frac{\cos x}{x^2}$$

$$9) y = \sqrt{x} \cdot \cos x \quad 10) y = \frac{x^2-1}{x^2+1}$$

$$11) f(x) = \sqrt[3]{x^2} \text{ bo'lsa, } f'(-8) \text{ topilsin.}$$

$$12) f(x) = \frac{x}{2x-1} \text{ bo'lsa, } f'(0), f'(2), f'(-2)$$

topilsin.

$$13) y = \operatorname{sh}^2 x \quad 14) y = \operatorname{th} x + \operatorname{cth} x$$

$$15) y = x - \operatorname{cth} x$$

17.3. Murakkab funksiya hosilasini oling:

$$1) y = \sin 5x; \quad 2) y = (1-2x)^3;$$

$$3) y = \sqrt{1-x^2}; \quad 4) y = \sqrt{\cos 4x};$$

$$5) y = \sqrt{2x - \sin 2x}; \quad 6) y = \sin^2 x;$$

$$7) y = \sin^3 x + \cos^3 x; \quad 8) y = \sin \sqrt{x};$$

$$9) y = 3^{4x}; \quad 10) y = \ln 5x;$$

- 11) $y = e^{x^2}$; 12) $y = \ln \operatorname{tg} \frac{x}{2}$;
- 13) $y = \frac{\cos x}{2 \sin^2 x}$; 14) $y = \frac{\sin x - x \cos x}{\cos x + x \sin x}$
- 15) $y = \operatorname{tg} x - \frac{1}{3} \operatorname{tg}^3 x + \frac{1}{5} \operatorname{tg}^5 x$; 16) $y = \lg^3 x$;
- 17) $y = \frac{1}{2} \operatorname{ctg}^2 x + \ln \sin x$; 18) $y = \ln \sqrt{\frac{1 - \sin x}{1 + \sin x}}$
- 19) $y = \arccos \frac{1-x}{\sqrt{2}}$; 20) $y = \operatorname{arctg} \frac{x^2}{a}$
- 21) $y = \arcsin(\sin x)$;
- 22) $y = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a}$;
- 23) $y = \arccos(\sin x^2 - \cos x^2)$; 24) $y = \sqrt[3]{x}$;
- 25) $y = x^x$; 26) $y = x^{\operatorname{tg} x}$;
- 27) $y = x^{x^2}$; 28) $x^2 + y^2 = a^2$;
- 29) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 30) $y^2 = 2px$;
- 31) $\sqrt{x} + \sqrt{y} = \sqrt{a}$; 32) $\sqrt[3]{x^2} + \sqrt[3]{y^2} = \sqrt[3]{a^2}$;
- 33) $x = y + \operatorname{arctg} y$; 34) $e^y - e^{-x} + xy = 0$;
- 35) $y = \ln[\operatorname{ch} x]$; 36) $y = \arcsin[\operatorname{th} x]$;
- 37) $y = \sqrt{1 + sh^2 4x}$. 38) $\begin{cases} x = \sin^2 t \\ y = \cos^2 t \end{cases}$
- 39) $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$ 40) $\begin{cases} x = e^{2t} \cdot \cos^2 t \\ y = e^{2t} \cdot \sin^2 t \end{cases}$

17.4. 1) $y = (x+1)\sqrt[3]{3-x}$ funktsiyaga $A(-1;0)$, $B(2;3)$, $C(3;0)$ nuqtalarda o'tkazilgan urinma va normal tenglamalarini yozing.

2) Qanday nuqtalarda $y = 2 + x - x^2$ chiziqqa o'tkazilgan urinma

a) Ox o'qiga parallel b) $y = x$ ga parallel bo'ladf?

3) $y = \sin x$ va $y = \cos x$ chiziqlar qanday burchak ostida kesishadi?

17.5. ITCh (aylana, ellips, giperbola, parabola) larga urinma tenglamasini chiqaring.

17.6. Berilgan funktsiyalar uchun chap va o'ng hosilalarni hisoblang.

$$1) y = \sqrt[3]{x^2}; x_0 = 0 \quad 2) y = |\ln x|, x_0 = 1$$

$$3) y = \sqrt{\sin^2 x}; x_0 = 0 \quad 4) y = |x - 2|, x_0 = 2$$

17.7. 1) Jism harakat qonuni $S(t) = \frac{1}{2}t^2 + 3t + 2$

formula bilan berilgan 4s da jism qanday yo'l bosib o'tgan? Shu vaqt momentida harakat tezligi qanday?

2) $S(t) = 6t^2 - t^3$ qonun bilan harakatlanayotgan jismning eng katta tezligi qancha?

§18. Funktsiya differentsiali.

Yuqori tartibli hosila va differentsial.

Funktsiyaning $(n-1)$ tartibli hosilasidan olingan hosila n - tartibli hosila deyiladi va $y^{(n)}$ orqali belgilanadi:

$$y^{(n)} = (y^{(n-1)})'$$

$$(a^x)^{(n)} = a^x \cdot \ln^n a; (e^x)^{(n)} = e^x;$$

$$(\sin x)^{(n)} = \sin(x + \frac{n\pi}{2});$$

$$(\cos x)^{(n)} = \cos(x + \frac{n\pi}{2}); (x^m)^{(n)} =$$

$$= m(m-1) \cdots (m-n+1)x^{m-n};$$

$$(\ln x)^{(n)} = \frac{(-1)^{n-1} \cdot (n-1)!}{x^n}$$

formulalarni isbotlash mumkin.

Funktsiya orttirmasini $\Delta y = y' \cdot \Delta x + 0(\Delta x)$ ko'rinishda yozish mumkin. Δy ning bosh qismi $y' \cdot \Delta x$ funktsiya differentsiali deyiladi va dy tarzida belgilanadi.

$$dy = y' dx, \text{ chunki } dx = 1 \cdot \Delta x.$$

Hosila yordamida differentsiallashtirish qoidalari quyidagicha ko'rinish oladi:

$$d(u \pm v) = du \pm dv,$$

$$d(u \cdot v) = v du + u dv,$$

$$d\left(\frac{u}{v}\right) = \frac{v du - u dv}{v^2}$$

Yuqori tartibli differentsial $d^n y = y^{(n)} dx^n$ formuladan topiladi.

$dy \approx \Delta y$ dan $f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x$ taqribiy hisoblash formulasini keltirib chiqarish mumkin.

18.1. 2-tartibli hosilalarini toping.

$$1) y = \sin^2 x \quad 2) y = \operatorname{tg} x$$

$$3) y = \sqrt{1+x^2} \quad 4) y = x \cdot \sin x$$

18.2. n -tartibli hosilalarini toping.

$$1) y = e^{-ax} \quad 2) y = \ln x$$

$$\begin{array}{ll} 3) y = \sqrt{x} & 4) y = x^n \\ 5) y = \frac{1+x}{\sqrt{1-x}} & 6) y = x \operatorname{sh} x \quad 7) y = x \cdot \ln x \end{array}$$

18.3. Leybnits formulasidan foydalanib, funktsiyalarning 2,3 – tartibli hosilalarini yozing.

$$\begin{array}{ll} 1) y = e^x \cdot \cos x & 2) y = x \cos x \\ 3) y = x^3 \cdot e^x & 4) y = x^2 \cdot \ln x \end{array}$$

18.4. 1) $f(x) = \frac{x}{\sqrt{1+x}}$ uchun $f^{(n)}(0)$ ni

hisoblang.

2) $f(x) = x^n$ uchun

$$f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \dots + \frac{f^{(n)}(1)}{n!} = 2^n \quad \text{ekanligini}$$

ko'rsating.

18.5. Taqribiy qiymatini toping.

$$1) \sqrt{24} \quad 2) \sqrt[3]{65} \quad 3) \sin 31^\circ \quad 4) \lg 11$$

§19. Differentsial hisob asosiy teoremlari.

Teorema (Ferma). Agar $f(x)$ funktsiya $c \in (a, b)$ nuqtada o'zining eng katta (kichik) qiymatiga erishsa, bu nuqtada chekli hosilaga ega bo'lsa, $f'(c) = 0$ bo'ladi.

Teorema (Rolf). Agar $f(x)$ funktsiya $[a, b]$ segmentda uzluksiz, $f(a) = f(b)$ va (a, b) da chekli hosilaga ega bo'lsa, u holda kamida bitta $c \in (a, b)$ nuqta topiladiki, $f'(c) = 0$ bo'ladi.

Teorema (Koshi): f, g funtsiyalar $[a, b]$ da uzluksiz, (a, b) da chekli hosilalarga ega, $g'(x) \neq 0$ bo'lsa, shunday $c \in (a, b)$ topiladiki:

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)} \text{ bo'ladi.}$$

Teorema (Lagranj): $f(x)$ funktsiya Koshi teoremasi shartlarini qanoatlantirsa, u holda shunday $c \in (a, b)$ topiladiki:

$$\frac{f(b) - f(a)}{b - a} = f'(c) \text{ o'rinli.}$$

19.1. $f(x) = (x-1)(x-2)(x-3)$ funktsiya uchun Roll' teoremasi o'rinligini tekshiring.

19.2. $f(x) = 1 - \sqrt[3]{x^2}$ funktsiyaga $[-1; 1]$ oralig'ida Roll' teoremasini tatbiq etib bo'ladimi? Chizmada tushuntiring.

19.3. Qaysi nuqtada $y = x^2$ ga o'tkazilgan urinma $A(-1; 1), B(3; 9)$ nuqtalarni tutashtiruvchi vektorga parallel bo'ladi?

19.4. $f(x) = x^2$ uchun $[a, b]$ da Lagranj formulasini yozing va c ni toping.

19.5. $f(x) = \arctg x$ uchun $[0; 1]$ da Lagranj formulasini yozing va c ni toping.

19.6. $f(x) = \ln x$ uchun $[1; 2]$ da Lagranj formulasini yozing va c ni toping.

19.7. $\sin x$ va $\cos x$ uchun $[0; \frac{\pi}{2}]$ da Koshi formulasini yozing va c ni toping.

19.8. x^2 va $\sqrt{x}$ uchun $[1; 4]$ da Koshi formulasini yozing va c ni toping.

19.9. x^2 va x^3 uchun $[-1; 1]$ kesmada nega Koshi formulasi o'rinli emasligini tushuntiring.

19.10. Isbotlang:

$$1) |\sin x - \sin y| \leq |x - y|$$

$$2) \frac{a-b}{a} < \ln \frac{a}{b} < \frac{a-b}{b}, \quad 0 < b < a.$$

Bobga doir misollar echish namunalari

1. Ta'rif yordamida hosilasini oling.

$$1) y = \frac{1}{\sqrt{x}}$$

$$\begin{aligned} y' &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{-(\sqrt{x+\Delta x} - \sqrt{x})}{\sqrt{x+\Delta x} \cdot \sqrt{x} \cdot \Delta x} = \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\sqrt{x} \left(\sqrt{1 + \frac{\Delta x}{x}} - 1 \right)}{\sqrt{x+\Delta x} \cdot \sqrt{x} \cdot \frac{\Delta x}{x} \cdot x} = \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{x \cdot \sqrt{x+\Delta x}} \cdot \lim_{\Delta x \rightarrow 0} \frac{\left(1 + \frac{\Delta x}{x}\right)^{\frac{1}{2}} - 1}{\frac{\Delta x}{x}} = -\frac{1}{2} \cdot \frac{1}{x\sqrt{x}} \end{aligned}$$

$$2). \quad y = (x-1)(x-2)^2(x-3)^3 \quad \text{uchun}$$

$y'(1)$, $y'(2)$, $y'(3)$ qiymatlarini hisoblang.

$$\begin{aligned} &(x+\Delta x-1)(x+\Delta x-2)^2(x+\Delta x-3)^3 - \\ y' &= \lim_{\Delta x \rightarrow 0} \frac{-(x-1)(x-2)^2(x-3)^3}{\Delta x} \end{aligned}$$

uchun

$$y'(1) = \lim_{\Delta x \rightarrow 0} (1+\Delta x-2)^2(1+\Delta x-3)^3, \text{ ya'ni}$$

$$y'(1) = (-1)^2 \cdot (-2)^3 = -8$$

Shunga o'xshash, $y'(2) = y'(3) = 0$ topiladi.

2. Hosila hisoblash qoidasi va jadvalidan foydalanib toping:

$$1). y = \sin(\cos^2 x) \cdot \cos(\sin^2 x)$$

$$\begin{aligned} y' &= [\sin(\cos^2 x)]' \cdot \cos(\sin^2 x) + \sin(\cos^2 x) \cdot [\cos(\sin^2 x)]' = \\ &= \cos(\cos^2 x) \cdot 2 \cos x \cdot (-\sin x) \cdot \cos(\sin^2 x) + \\ &+ \sin(\cos^2 x) \cdot [-\sin(\sin^2 x) \cdot 2 \sin x \cdot \cos x] = \\ &= -\sin 2x [\cos(\cos^2 x) \cdot \cos(\sin^2 x) + \sin(\sin^2 x) \cdot \sin(\cos^2 x)] = \\ &= -\sin 2x \cdot \cos(\cos^2 x - \sin^2 x) = -\sin 2x \cdot \cos(\cos 2x); \end{aligned}$$

$$2). y = \ln(x + \sqrt{x^2 + 1});$$

$$y' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot (1 + \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x) = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}};$$

$$3). y = \sin x^{\cos x}$$

$$y' = \sin x^{\cos x} \cdot \left[-\sin x \cdot \ln \sin x + \frac{\cos x \cdot \cos x}{\sin x} \right];$$

$$4). \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1;$$

$$\frac{2x}{a^2} - \frac{2y \cdot y'}{b^2} = 0 \quad \text{dan} \quad y' = \frac{b^2 \cdot x}{a^2 \cdot y};$$

$$3. \quad 1). y = \frac{x^3}{3} \quad \text{funktsiyaga } x = -1 \text{ nuqtada}$$

o'tkazilgan urinma va normal tenglamasini yozing.

$$y(-1) = -\frac{1}{3}; \quad y' = x^2;$$

$$y'(-1) = (-1)^2 = 1 \text{ ekanligidan urinma } y + \frac{1}{3} = x + 1;$$

normal esa $y + \frac{1}{3} = -x + 1$ tenglama bilan aniqlanadi.

2). $y = x^2$ va $x = y^2$ funktsiyalar qandan burchak ostida kesishadilar?

Bu funktsiyalar kesishish nuqtasini topamiz: „ $y = y^4$ “ dan $y_1 = 0$; $y_2 = 1$. Demak, $(0;0)$, $(1;1)$ nuqtalarda kesishar ekan. Masalan, $(0;0)$ nuqtada berilgan funktsiyalar urinmalari orasidagi burchakni topamiz.

$y = x^2$ uchun $y' = 2 \cdot x$; $y'(0) = 0$; $y(0) = 0$, ya'ni $y = 0$

$y = \pm\sqrt{x}$ uchun $y' = \frac{1}{2\sqrt{x}}$; $y'(0)$ da aniqlanmagan,

lekin urinmasi $x = 0$ desak, ular 90° burchak ostida kesishishi kelib chiqadi.

$(1; 1)$ nuqtada $y - 1 = 2(x - 1)$;

$y - 1 = \pm \frac{1}{2}(x - 1)$ urinmalarga ega bo'lamiz:

$k_1 = 2$, $k_2 = -\frac{1}{2}$ uchun $\varphi = 90^\circ$; $k_1 = 2$; $k_2 = \frac{1}{2}$

uchun esa $\operatorname{tg} \varphi = \frac{2 - \frac{1}{2}}{1 + 2 \cdot \frac{1}{2}} = \frac{\frac{3}{2}}{\frac{3}{2}} = \frac{3}{2} = \frac{3}{4}$ dan $\varphi = \operatorname{arctg} \frac{3}{4}$

kelib chiqadi.

4. $y = |x|$ ning $x = 0$ nuqtada hosilasi mavjud emasligini isbotlang.

$y = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$ ekanligidan

$$y'_- = \lim_{\Delta x \rightarrow 0} \frac{|x + \Delta x| - |x|}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{|x| + |\Delta x| - |x|}{\Delta x} = -1; \quad y'_+ = 1$$

ekanligidan $y'(0)$ mavjud emas.

5. Moddiy nuqta Ox o'qi bo'ylab $x = \frac{t^3}{3} - 2t^2 + 3t$

qonun bo'yicha harakatlanayapti. Uning tezligi va tezlanishini aniqlang. Nuqta qaysi koordinatalarda yo'nalishini o'zgartiradi?

$$x' = t^2 - 4t + 3 = V(t)$$

Bundan $t_1 = 1, t_2 = 3$ da yo'nalishini o'zgartirishi kelib chiqadi.

$$a(t) = V'(t) = 2t - 4 \quad \text{dir.}$$

6. $(u \cdot v)^{(n)}$ uchun Leybnits formulasini yozing.

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$(u \cdot v)'' = u'' \cdot v + u' \cdot v' + u' \cdot v' + u \cdot v'' = u'' + 2u' \cdot v' + v''$$

$$(u \cdot v)''' = u''' + 3u'' \cdot v' + 3u' \cdot v'' + v'''$$

Demak:

$$(u \cdot v)^{(n)} = u^{(n)} + n \cdot u^{(n-1)} \cdot v' + \frac{n(n-1)}{2!} u^{(n-2)} \cdot u^{(n-2)} + \dots + v^{(n)}.$$

7. $y = \frac{1}{1-x^2}$ uchun $y^{(n)}(0)$ ni hisoblang.

$$y = \frac{1}{1-x^2} = \frac{1}{2} \left(\frac{1}{1+x} + \frac{1}{1-x} \right) = \frac{1}{2} [(1+x)^{-1} + (1-x)^{-1}]$$

ekanligidan:

$$y' = \frac{1}{2} [-1(1+x)^{-2} + (1-x)^{-2}]$$

$$y'' = \frac{1}{2} [(-1)(-2) \cdot (1+x)^{-3} - 2(1-x)^{-3}]$$

$$y''' = \frac{1}{2}[(-1)(-2)(-3)(1+x)^{-4} - 2 \cdot (-3)(1-x)^{-4}]$$

.....

$$y^{(n)} = \frac{1}{2}[(-1)^n(1+x)^{-(n+1)} \cdot n! + (-1)^{n+1} \cdot n!(1-x)^{-(n+1)}] =$$

$$= \frac{(-1)^n \cdot n!}{2}[(1+x)^{-(n+1)} - (1-x)^{-(n+1)}]$$

$$\text{Demak: } y_{(0)}^{(n)} = \begin{cases} n!, & \text{agar } n = 2m \\ 0, & \text{agar } n = 2m+1 \end{cases}$$

8. Taqribiy qiymatini toping.

$$1) \sqrt{17} = \sqrt{16+1} \approx \frac{1}{2\sqrt{16}} \cdot 1 + \sqrt{16} = \frac{1}{8} + 4 = 4\frac{1}{8} = 4,125.$$

$$2) e^{2,1} = e^{2+0,1} \approx e^2 + e^2 \cdot 0,1 = 1,1 \cdot e^2$$

9. $f(x) = x^2 - 4x + 3$ ildizlari orasida hosilaning ildizi mavjud. Sababini tushuntiring.

$$f(x) = x^2 - 4x + 3 = (x-1)(x-3)$$

ekanligidan $f(1) = f(3) = 0$. Funktsiya $[1, 3]$ da uzluksiz, chekli hosilaga ega. Demak, Roll' teoremasi shartlari bajarilayapti. Shunday $c \in (1; 3)$ nuqta mavjudki $f'(c) = 0$ bo'ladi.

Rostdan ham $f'(x) = 2x - 4 = 0$, $x = 2$ ya'ni $c = 2$ ekanligini ko'rish mumkin.

10. $f(x) = \sqrt{x}$ uchun $[1; 4]$ kesmada Lagranj formulasini yozing va c ni toping.

Funktsiya Lagranj teoremasi shartlarini qanoatlantiradi, demak,

$$\frac{f(4) - f(1)}{4 - 1} = f'(c) \text{ o'rinli.}$$

$$\frac{\sqrt{4}-\sqrt{1}}{4-1} = \frac{1}{2\sqrt{c}} \text{ ekanligidan } \sqrt{c} = \frac{3}{2} \text{ ya'ni } c = \frac{9}{4}.$$

11. $f(x) = x^3$ va $g(x) = x^2$ funksiyalar uchun Koshi formulasini yozing va c ni toping.

Koshi teoremasi shartlari bajariladi, shuning uchun

$$\frac{b^3 - a^3}{b^2 - a^2} = \frac{3c^2}{2c}.$$

$$\text{Bundan } c = \frac{2(b^3 - a^3)}{3(b^2 - a^2)} = \frac{2(b^2 + ab + a^2)}{3(b + a)}.$$

5-bobga doir uy vazifalari.

I. Berilgan funksiyaning 1) ta'rif bo'yicha hosilasi; 2)-6) jadval bo'yicha hosilasi; 7) n -tartibli hosilasi topilsin.

$$1. 1). y = x^2 - 3x \quad 2). y = 2\sqrt{4x+3} - \frac{3}{\sqrt{x^2+x}}$$

$$3). y = (e^{\cos x} + 1)^2 \quad 4). y = \ln \sin 2x$$

$$5). y = x^{\ln x} \quad 6). \operatorname{tg}(y/x) = 5x \quad 7). y = \sin 10x$$

$$2. 1). y = x^3 - 1 \quad 2). y = x^2 \sqrt{1-x^2}$$

$$3). y = \sin x / \cos^2 x \quad 4). y = \operatorname{arctg} e^{2x} \quad 5). y = x^{1/x}$$

$$6). x - y + \operatorname{arctg} y = 0 \quad 7). y = \cos 11x$$

3.

$$1). y = \frac{1}{1-x} \quad 2). y = \frac{1-x}{1+x^2} \quad 3). y = \frac{1}{\operatorname{tg}^2 2x}$$

$$4). y = \arcsin \sqrt{1-3x} \quad 5). y = (\sin x)^x$$

$$6). y \sin x = \cos(x-y) \quad 7). y = \ln 2x$$

4.

$$1). y = \frac{1}{x^2} \quad 2). y = x^2 \cos x \quad 3). y = \sin x - x \cos$$

$$4). y = x^m \ln x \quad 5). y = x^{\lg x} \quad 6). \frac{y}{x} = \operatorname{arctg} \frac{x}{y}$$

$$7). y = \frac{1}{1-x^2}$$

5.

$$1). y = \sin 3x \quad 2). y = \frac{x}{\sqrt{a^2 - x^2}} \quad 3). y = \frac{\sin^2 x}{2 + 3 \cos^2 x}$$

$$4). y = \frac{x \ln x}{x-1} \quad 5). y = (\operatorname{arctg} x)^{\ln x}$$

$$6). (e^x - 1)(e^y - 1) - 1 = 0 \quad 7). y = \frac{1}{x(1+x)}$$

$$6. \quad 1) y = \sqrt[4]{x} \quad 2) y = \frac{1}{\sqrt{x^2 + 1}} + 5\sqrt[3]{x^3 + 1}$$

$$3) y = \operatorname{tg}^3(x^2 + 1) \quad 4) y = 3^{\operatorname{arctg} x^2}$$

$$5) y = (\operatorname{arctg} x)^x \quad 6) y^2 x = e^{y/x}$$

$$7) y = \frac{x}{\sqrt{1-x}}$$

$$7.1) y = \frac{1}{\sqrt[3]{x}} \quad 2) y = \sqrt[3]{\frac{1+x^2}{1-x^2}}$$

$$3) y = \frac{1}{2} \operatorname{tg}^2 x + \ln \cos x$$

$$4) y = \operatorname{arctg} \frac{x}{1 + \sqrt{1-x^2}}$$

$$5) y = (x + x^2)^x \quad 6) x^3 + y^3 - 3axy = 0$$

$$7) y = \frac{1}{x(x+3)}$$

$$8. \quad 1) y = \log_2(1+x) \quad 2) y = 3\sqrt[3]{x^5 + 5x^4 - \frac{5}{x}}$$

$$3) y = \ln \sqrt{\frac{1-\sin x}{1+\sin x}} \quad 4) y = \arctg(tg^2 x)$$

$$5) y = (\sin x)^{\ln x} \quad 6) x - y + a \sin y = 0$$

$$7) y = \ln(1+x^2).$$

9.

$$1) y = \sqrt[5]{x+1} \quad 2) y = 5\sqrt[5]{x^2 + x + \frac{1}{x}} \quad 3) y = 2^x e^{-x}$$

$$4) y = \frac{\arcsin x}{\sqrt{1-x^2}} \quad 5) y = (\cos x)^x \quad 6) \ln y = \arctg \frac{x}{y}$$

$$7) y = \frac{x}{x-1}$$

10.

$$1) y = x^3 - x \quad 2) y = \sqrt{x^2+1} + \sqrt{x^3+1}$$

$$3) y = \frac{1}{3}tg^3 x - tg x + x \quad 4) y = \arctg \sqrt{\frac{3-x}{x-2}}$$

$$5) y = (\cos x)^{x^2} \quad 6) x - y + e^y \arctg x = 0$$

$$7) y = \sin 10x.$$

11.

$$\begin{array}{ll}
 1) y = 3 - x^2; & 2) y = \frac{\sqrt{x^2 - 1}}{x} + \arcsin \frac{1}{x}; \\
 3) y = \ln \frac{1 - \cos x}{1 + \cos x}; & 4) y = \frac{\operatorname{tg}^2 x}{2} + \ln \cos x; \\
 5) y = (1 + x)^{\frac{1}{x}}; & 6) x^2 + 2xy + y^2 - 5x = 0; \quad 7) y = e^{2x} + \frac{1}{x}.
 \end{array}$$

12.

$$\begin{array}{ll}
 1) y = \frac{1}{x}; & 2) y = \operatorname{tg} \sqrt{\frac{\sin x + \cos x}{\sin x - \cos x}}; \\
 3) y = e^{2x} \cdot \sin x & 4) y = \frac{\arccos x}{\sqrt{1 - x^2}}; \\
 5) y = (\ln x)^{\sin x} & 6) \ln(x + y) = 2xy - y^2 \quad 7) y = x \sin x
 \end{array}$$

13.

$$\begin{array}{ll}
 1) y = \frac{2}{1 + 4x} & 2) y = \ln \sqrt{\frac{1 - x}{1 + x^2}} \\
 3) y = \operatorname{tg}^2(4x + 1) & 4) y = \arccos \sqrt{1 + 4x} \\
 5) y = x^{\ln x} & 6) y + yx = \ln \frac{x}{y} \quad 7) y = \frac{1}{(x - 1)(x + 3)}
 \end{array}$$

14.

1) $y = chx$

2) $y = x\sqrt{1+x^2}$

3) $y = \lg^3 x^2$

4) $y = \frac{\ln 3 \sin x + \cos x}{3^x}$

5) $y = x^{x^x}$ 6) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ 7) $y = \sin^2 x$

15. 1) $y = \frac{1}{x^2 + 1}$ 2) $y = 4\sqrt[3]{ctg^2 x} + \sqrt[3]{ctg^8 x}$

3) $y = \sqrt{x+1} - \ln(1 + \sqrt{x+1})$

4) $y = \arccos(\sin^2 x - \cos^2 x)$

5) $y = \frac{(\ln x)^x}{x^{\ln x}}$ 6) $(x+y)^2 = x-y$

7) $y = \cos^2 x$

II. Berilgan funktsiyalar uchun $[a, v]$ oralig'ida Koshi formulasini yozing va c ni toping.

1) x^3 va $\sqrt{x}$, $[1; 4]$

2) x^3 va $3 - x^2$, $[1; \sqrt{3}]$

4) $\sin x$ va $\cos x$, $[0; \frac{\pi}{2}]$

5) tgx va $\cos x$, $[0; \frac{\pi}{4}]$

6) $1 + x^3$ va $\sqrt[3]{x}$, $[0; 4]$

7) $\ln x$ va x^2 , $[1; 3]$

8) $\cos x$ va $x + 4$, $[0; \frac{\pi}{4}]$

9) x^2 va $1 - x^3$, $[0; 2]$

10) x^3 va $1 - x^2$, $[0; 3]$

11) $x + 3$ va $\sqrt{x}$, $[1; 4]$

$$12) \operatorname{ctgx} \text{ va } x, \left[\frac{\pi}{4}; \frac{\pi}{2}\right]$$

$$13) x^4 \text{ va } \sqrt[4]{x}, [1; 4]$$

$$14) x^2 - 1 \text{ va } 1 - x, [0; 4]$$

$$15) \sin x \text{ va } \operatorname{ctgx}, \left[\frac{\pi}{6}; \frac{\pi}{3}\right]$$

6-bob. Hosila tatbiqlari.

§20 Teylor formulasi.

$f(x)$ funktsiya $x_0 \in R$ nuqtaning biror atrofida $f', f'', \dots, f^{(n)}, f^{(n+1)}$ hosilalarga ega va $f^{(n+1)}$ hosila x_0 nuqtada uzluksiz bo'lsin.

U holda

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots \\ \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + O(x-x_0)^n$$

tenglik o'rinli va bu yoyilma yagona bo'lib Teylor formulasi deyiladi.

Xususan $x = 0$ da Makloren formulasini olamiz.

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(n)}(0)}{n!}x^n + O(x^n)$$

Makloren formulasidan quyidagi yoyilmalarni olish mumkin.

I. $e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + O(x^n)$

II. $\sin x = x - \frac{x^3}{3!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + O(x^{2n})$

III. $\cos x = x - \frac{x^2}{2!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + O(x^{2n+1})$

VI.

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \dots$$

$$+ \frac{m(m-1)\dots(m-n+1)}{n!}x^n + O(x^n)$$

V. $\ln(1+x) = x - \frac{x^2}{2!} + \dots + (-1)^{n-1} \frac{x^n}{n!} + O(x^n)$

20.1 Quyidagi funktsiyalarni x ning darajalari bo'yicha yoying

- 1) $\sin^2 x$ 2) e^{2x-x^2} ni x^5 gacha
3) $\ln \cos x$ ni x^6 gacha

20.2 1) $f(x) = x^3 - 3x$ ni $(x-1)$ darajalari bo'yicha yoying.

2) $f(x) = \sin 3x$ ni $(x + \frac{\pi}{3})$ darajalari

bo'yicha yoying.

3) $f(x) = \sqrt[3]{x}$ ni $(x+1)$ darajalari

bo'yicha yoying.

20.3 Teylor formulasi yordamida taqribiy hisoblang

- 1) $\sqrt[3]{30}$ 2) $\sqrt[3]{250}$ 3) $\sqrt{e}$ 4) $\ln 1,2$ 5) $\sin 18^\circ$

20.4 Yoyilmalardan foydalanib limitni toping

1) $\lim_{x \rightarrow 0} \frac{e^x \sin x - x(1+x)}{x^3}$ 2) $\lim_{x \rightarrow 0} x^{\frac{2}{3}} (\sqrt{x+1} + \sqrt{x-1} - 2\sqrt{x})$

3) $\lim_{x \rightarrow 0} \frac{a^x - a^{-x} - 2}{x^2} \quad (a > 0)$ 4) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

5) $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x} - \operatorname{ctg} x \right)$

§21 Lopital qoidalar.

Agar $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ (yoki ∞) bo'lib,

$\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ mavjud bo'lsa, u holda

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad \text{o'rinlidir. Bu qoida } \frac{0}{0}, \frac{\infty}{\infty}$$

ko'rinishdagi aniqmasliklarni echishda Lopitalning I (II) qoidasi deyiladi.

$0 \cdot \infty, \infty - \infty, 1^\infty, 0^0$ tipidagi aniqmasliklar algebraik almashtirishlar yordamida yuqoridagi ikki holga keltiriladi.

21.1 Limitlarni hisoblang

$$1) \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \quad 2) \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} \quad 3) \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - x}{x - \sin x}$$

$$4) \lim_{x \rightarrow 0} \frac{3\operatorname{tg} 4x - 12\operatorname{tg} x}{3\sin 4x - 12\sin x} \quad 5) \lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{\operatorname{tg} x}$$

$$6) \lim_{x \rightarrow 0} \frac{\ln(\cos ax)}{\ln(\cos bx)} \quad 7) \lim_{x \rightarrow +0} x^x$$

$$8) \lim_{x \rightarrow 1} x^{1-x} \quad 9) \lim_{x \rightarrow 0} (\operatorname{ctg} x)^{\sin x}$$

$$10) \lim_{x \rightarrow 0} \left(\frac{a^x - x \ln a}{b^x - x \ln b} \right)^{\frac{1}{x^2}} \quad 11) \lim_{x \rightarrow a} \frac{a^x - x^a}{x - a}$$

$$12) \lim_{x \rightarrow 0} \frac{(a+x)^x - a^x}{x^2}$$

§ 22 Funktsiyani to'liq tekshirish.

1) Agar (a, b) intervalda $f'(x) \geq 0$ ($f'(x) \leq 0$) bo'lsa funktsiya bu oraliqda o'suvchi (kamayuvchi) bo'ladi.

2) Hosilasi nol'ga teng bo'ladigan nuqtalar kritik nuqtalar deyiladi.

Agar kritik nuqtada funktsiya hosilasi o'z ishorasini + dan - ga (- dan + ga) o'zgartirsa, bu kritik nuqta maksimum (minimum) nuqtadir.

Funktsiya kritik nuqtada ishorasini o'zgartirmasa, bu nuqtada ekstremum mavjud emas.

Bu qoida ekstremum topishning I – qoidasidir.

3) Agar (a, b) intervalda $f''(x) \geq 0$ ($f''(x)) \leq 0$ bo'lsa, funktsiya grafigi botiq (qavariq) bo'ladi.

Demak kritik nuqtada $f'(x_0) > 0$ ($f'(x_0)) < 0$

bo'lsa, $x = x_0$ nuqta minimum (maksimum) nuqtasidir.

Bu qoida ekstremum topishning II – qoidasidir.

Ikkinchi tartibli hosila ishora o'zgartiradigan nuqtalar egilish nuqtalari deyiladi.

Agar $f(x)$ funktsiya $a \in R$ nuqtaning biror atrofida aniqlanib,

$$\lim_{x \rightarrow a+0} f(x), \lim_{x \rightarrow a-0} f(x)$$

lardan biri yoki ikkalasi cheksiz bo'lsa, $x = a$ to'g'ri chiziq $f(x)$ funktsiya grafigiga vertikal asimptota deyiladi.

$$\lim_{x \rightarrow \infty} [f(x) - (kx + b)] = 0 \quad \text{bo'lsa,} \quad y = kx + b$$

to'g'ri chiziq $f(x)$ funktsiya grafigiga og'ma ($k = 0$ da gorizontal) asimptota deyiladi.

$$\text{Bunda } k = \lim_{x \rightarrow \infty} \frac{f(x)}{x}, \quad b = \lim_{x \rightarrow \infty} [f(x) - kx]$$

Funktsiya son o'qlarini kesib o'tadigan nuqtalar funktsiyaning nollari deyiladi.

Funktsiyaning to'liq tekshirish uchun navbatma – navbat quyidagi ishlar amalga oshiriladi.

- 1) Funktsiyaning aniqlanish va o'zgarish sohalarini topish;
- 2) Funktsiyaning uzluksizlikka tekshirish; uzilish nuqtalarini topish;
- 3) Funktsiya juft – toqligi, davriyligini tekshirish;
- 4) Funktsiyaning monotonlikka tekshirish;
- 5) Funktsiyaning ekstremumga tekshirish;

- 6) Funktsiyani botiqlik, qavariqlikka tekshirish;
- 7) Funktsiya grafigi asimptotalarini topish;
- 8) Funktsiya nollarini topish;
- 9) Funktsiya grafigini chizish.

Funktsiya ekstremumlarini topishni masalalar echishga ham tatbiq etish mumkin.

22.1 Quyidagi funktsiyalar grafigini chizing.

- 1) $y = 3x - x^3$
- 2) $y = \frac{x^3}{3} + x^2$
- 3) $y = \frac{x^2}{x-2}$
- 4) $y = \frac{(x-1)^2}{x^2+1}$
- 5) $y = x^3 + 6x^2 + 9x$
- 6) $y = 2x - 3\sqrt[3]{x^2}$
- 7) $y = xe^{\frac{x^2}{2}}$
- 8) $y = x - 2\ln x$
- 9) $y = 2x + \operatorname{ctg} x, x \in (0; \pi)$
- 10) $y = \sin x + \frac{1}{3}\sin 3x$
- 11) $y = \cos x \cos 3x$
- 12) $y = e^{2x-x^2}$
- 13) $y = x + e^{-x}$
- 14) $y = \frac{e^x}{1+x}$
- 15) $y = (x+2)e^{\frac{1}{x}}$
- 16) $y = x^x$
- 17) $y = x^{\frac{1}{x}}$
- 18) $y = x + 2\sqrt{-x}$
- 19) $y = \arcsin \frac{2x}{1+x^2}$
- 20) $y = \arccos \frac{1-x}{1-2x}$

22.2 1) 120 sm li simni yuzasi eng katta bo'ladigan to'g'ri to'rtburchak shaklida bukilgan. Bu to'g'ri to'rtburchak o'lchamlarini toping.

2) 10 sonini shunday ikki qo'shiluvchiga ajratingki, bu qo'shiluvchilar ko'paytmasi eng katta bo'lsin.

3) Berilgan R radiusli doiraga ichki chizilgan uchburchaklardan yuzi eng katta bo'lganini toping.

4) Trapetsiya kichik asosi va yon tomonlari 10 sm dan. Katta asosi qancha bo'lganda trapetsiya yuzi eng katta bo'ladi?

5) Shar hajmi unga ichki chizilgan eng katta hajmli tsilindr hajmidan necha marta katta?

6) R radiusli sharga ichki chizilgan, to'la sirti eng katta bo'lgan tsilindr o'lchamlarini toping.

7) $M(p,p)$ nuqtadan $y^2 = 2px$ gacha eng qisqa masofani toping.

8) $A(2,0)$ nuqtadan $x^2 + y^2 = 1$ aylanagacha bo'lgan eng qisqa va eng katta masofalarni toping.

9) Kengligi a metr daryoga perpendikulyar ravishda kengligi b metrli kanal qazildi. Maksimal uzunligi qanday bo'lgan kemalar bu kanalga o'ta oladi?

10) Yasovchisi R bo'lgan konuslar ichidan eng katta hajmlisini toping.

Bobga doir misollar echish namunalari

1. $f(x) = xe^x$ ni x darajalari bo'yicha qatorga yoying.

$$f(x) = xe^x = x\left[1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + O(x^n)\right] = x + x^2 + \frac{x^3}{2!} + \dots + \frac{x^{n+1}}{n!} + O(x^{n+1})$$

2. $f(x) = \ln \frac{1+x}{1-x}$ ni x darajalari bo'yicha qatorga

yoying:

$$f(x) = \ln \frac{1+x}{1-x} = \ln(1+x) - \ln(1-x) = x - \frac{x^2}{2} + \dots + (-1)^{n-1} \frac{x^n}{n} -$$

$$-(-x - \frac{x^2}{2} - \dots - \frac{x^n}{n}) = 2(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots)$$

3. x^4 ni $(x+1)$ darajalari bo'yicha qatorga yoying.

$$f(-1) = (-1)^4 = 1; \quad f'(-1) = -4 \quad f''(-1) = 12$$

$$f'''(-1) = -24; \quad f^{IV}(-1) = 24$$

$$f'(x) = 4x^3, \quad f''(x) = 12x^2, \quad f'''(x) = 24x, \quad f^{IV} = 24$$

Demak, Teylor formulasidan:

$$x^4 = 1 - 4(x+1) + \frac{12}{2!}(x+1)^2 - \frac{24}{3!}(x+1)^3 + \frac{24}{4!}(x+1)^4$$

4. $\ln 2$ ni hisoblang.

$$\ln \frac{1+x}{1-x} = \ln 2 \quad \text{dan}$$

$$\frac{1+x}{1-x} = 2 \Rightarrow 1+x = 2-2x \Rightarrow x = \frac{1}{3}$$

$$\ln 2 = \ln \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2 \left[\frac{1}{3} + \frac{1}{3} \left(\frac{1}{3} \right)^3 + \frac{1}{5} \left(\frac{1}{3} \right)^5 + \dots \right]$$

5. Yoyilmalardan foydalanib hisoblang.

$$\lim_{x \rightarrow 0} \frac{\cos x - e^{-\frac{x^2}{2}}}{x^4} = \lim_{x \rightarrow 0} \frac{1}{x^4} \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \left(1 - \frac{x^2}{2} + \frac{x^4}{8} \right) \right] =$$

$$= \lim_{x \rightarrow 0} \frac{1}{x^4} \left[\frac{x^4}{24} - \frac{x^4}{8} \right] = \lim_{x \rightarrow 0} \frac{-2}{24} = -\frac{1}{12}$$

$$\begin{aligned}
6. \quad \lim_{x \rightarrow 0} \frac{a^x - a^{\sin x}}{x^3} &= \lim_{x \rightarrow 0} \frac{a^x \ln a - a^{\sin x} \ln a \cos x}{3x^2} = \\
&= \lim_{x \rightarrow 0} \frac{\ln a}{3} \cdot \frac{a^x - a^{\sin x} \cos x}{x^2} = \\
&= \lim_{x \rightarrow 0} \frac{\ln a}{3} \cdot \frac{a^x \ln a - a^{\sin x} \ln a \cos^2 x + a^{\sin x} \sin x}{2x} = \\
&\quad [a^x \ln^2 a - a^{\sin x} \ln^2 a \cos^2 x + a^{\sin x} \ln a 2 \sin x \cos x + \\
&\quad \lim_{x \rightarrow 0} \frac{\ln a + a^{\sin x} \ln a \sin x \cos x + a \sin x \cos x]}{2} = \\
&= \frac{\ln a}{2 \cdot 3} (\ln^2 a - \ln^2 a + 1) = \frac{1}{6} \ln a
\end{aligned}$$

7.

$$\begin{aligned}
\lim_{x \rightarrow 0} \frac{\ln(\sin ax)}{\ln(\sin bx)} &= \lim_{x \rightarrow 0} \frac{\frac{1}{\sin ax} \cdot \cos ax \cdot a}{\frac{1}{\sin bx} \cdot \cos bx \cdot b} = \lim_{x \rightarrow 0} \frac{a \sin bx \cos ax}{b \sin ax \cos bx} = \\
&= \frac{a}{b} \lim_{x \rightarrow 0} \frac{b \cos bx \cos ax - a \sin bx \sin ax}{a \cos ax \cos bx - b \sin ax \sin bx} = 1.
\end{aligned}$$

$$8. \quad \lim_{x \rightarrow \frac{\pi}{4}} (tgx)^{tg 2x}; A = (tgx)^{tg 2x} \quad \text{bo'lsa,}$$

$$\ln A = \ln(tgx) tg 2x = \frac{\ln(tgx)}{\frac{1}{tg 2x}} \quad \text{deb,}$$

$$\begin{aligned}
\lim_{x \rightarrow \frac{\pi}{4}} \ln A &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\ln(tgx)}{\frac{1}{tg 2x}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{1}{tgx} \cdot \frac{1}{\cos^2 x}}{-tg^{-2} 2x \cdot 2 \cdot \frac{1}{\cos^2 2x}} =
\end{aligned}$$

$$= -\frac{1}{2} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{1}{\sin x \cos x}}{\frac{1}{\sin^2 2x}} =$$

$$= -\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin^2 2x}{\sin 2x} = -\lim_{x \rightarrow \frac{\pi}{4}} \sin 2x = -1; \quad \text{L'H} \rightarrow -1;$$

Demak $A \rightarrow e^{-1}$;

9. $y = \frac{x^2}{x-1}$ funktsiyani to'liq tekshiring.

1) $D(y) = (-\infty; 1) \cup (1; +\infty)$, $E(y) = (-\infty; +\infty)$

2) $x = 1$ uzilish nuqtasi bo'lib,

$$\lim_{x \rightarrow 1-0} f(x) = -\infty, \quad \lim_{x \rightarrow 1+0} f(x) = +\infty \quad \text{ekanligidan,}$$

funktsiya bu nuqtada II-tur uzilishga egaligi kelib chiqadi.

3) $y(-x) \neq \pm y(x)$ ekanligidan, funktsiya juft ham, toq ham emas.

Funktsiya davriy emas.

4,5) $y' = \frac{x(x-2)}{(x-1)^2}$ dan kritik nuqtalari

$x_1 = 0; x_2 = 2$ ekanligini topamiz.

Monotonlik oralig'ları, ekstremumlarni topish uchun quyidagi jadvalni to'ldiramiz.

x	$(-\infty; 0)$	0	$(0; 1)$	$(1; 2)$	2	$(2; +\infty)$
y'	+	0	—	—	0	+
y		0			4	

6) $y'' = \frac{2}{(x-1)^3}$ ekanligidan, egilish nuqtalari yo'q,

lekin uzilish nuqtasi bu hosila ishorasini o'zgartiradi.

x	$(-\infty; 1)$	$(1; +\infty)$
y''	$-$	$+$
y	$\cap$	$\cup$

7) $x = 1$ vertikal asimptota ekanligini bilamiz, og'ma asimptotani qidiramiz:

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^2}{x(x-1)} = 1$$

$$b = \lim_{x \rightarrow \infty} \left[\frac{x^2}{x-1} - x \right] = \lim_{x \rightarrow \infty} \frac{x}{x-1} = 1$$

demak $y = x + 1$ og'ma asimptota ekan.

8) $x = 0$ dan $y = 0$; $y = 0$ dan $x = 0$ kelib chiqadi. Demak, grafik koordinata boshida son o'qlarini kesadi, holos.

Funktsiya grafigini chizamiz

10. R radiusli sharga ichki chizilgan eng katta hajmli tsilindr o'lchamlarini toping.

Silindr asosi radiusini r , balandligini H desak, Pifagor teoremasiga ko'ra:

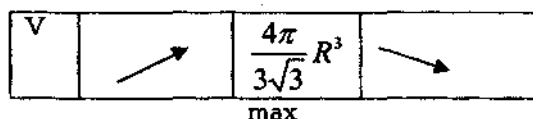
$$(2r)^2 = (2R)^2 - H^2, \text{ ya'ni } r = \frac{1}{2} \sqrt{4R^2 - H^2}$$

$$V_y(H) = \frac{\pi}{4} (4R^2 - H^2) \cdot H = \frac{\pi}{4} (4R^2 H - H^3)$$

$$V'(H) = \frac{\pi}{4} (4R^2 - 3H^2) \quad \text{da} \quad H = \frac{2}{\sqrt{3}} R$$

kritik nuqta ekanligi kelib chiqadi.

H	$(0; \frac{2}{\sqrt{3}} R)$	$\frac{2}{\sqrt{3}} R$	$(\frac{2}{\sqrt{3}} R; 2R)$
V'	$+$	0	$-$



$$V_{\max} \left(\frac{2}{\sqrt{3}} R \right) = \frac{4}{3\sqrt{3}} R^3.$$

O'lchamlari

$r = \sqrt{\frac{2}{3}} R$, $H = \frac{2}{\sqrt{3}} R$ bo'lgan silindr eng katta hajmga ega bo'ladi.

6 bobga doir uy vazifalari

I. $f(x) = e^x$ funktsiyaga Teylor formulasini qo'llab e^a qiymatni, a ning bor qiymatida taqribiy hisoblang.

- 1) $a=0.49$ 2) 0.33 3) 0.75 4) 0.63 5) 0.21
 6) 0.55 7) 0.37 8) 0.83 9) 0.13 10) 0.59 11) 0.95
 12) 0.27 13) 0.47 14) 0.18 15) 0.72

II. Berilgan $f(x)$ funktsiyaning $[a, b]$ oraliqdagi eng katta va eng kichik qiymatlarini toping.

1) $f(x) = x^3 - 12x$; $[0; 3]$ 2) $f(x) = x^5 - \frac{5}{3}x^3$; $[0; 2]$

2) $f(x) = \frac{\sqrt{3}}{2}x + \cos x$; $[0; \frac{\pi}{2}]$ 4) $f(x) = 3x^4 - 16x^2 + 2$; $[-1; 1]$

5) $f(x) = x^3 - 3x + 1$; $[0.5; 2]$ 6) $f(x) = x^4 + 4x$ $[-2; 2]$

7) $f(x) = \frac{\sqrt{3}}{2}x - \sin x$; $[0; \frac{\pi}{2}]$ 8) $f(x) = 81x - x^4$; $[-1; 4]$

9) $f(x) = 3 - 2x^2$; $[-1; 3]$ 10) $f(x) = x - \sin x$; $[-\pi; \pi]$

11) $f(x) = 2^x$; $[-1; 5]$ 12) $f(x) = x^2 - 4x + 6$; $[-1; 5]$

$$13) f(x) = |x^2 - 3x| \quad [1; 4] \qquad 14) f(x) = |1 - x^2| \quad [0; 2]$$

$$15) f(x) = \ln|(x - 4)| \quad [0; 5]$$

III. Funktsiyani to'la tekshiring, grafigini chizing.

$$1) y = \frac{4x}{4 + x^2} \qquad 2) y = \frac{x^2 - 1}{x^2 + 1}$$

$$3) y = \frac{x^2 + 1}{x^2 - 1} \qquad 4) y = \frac{x^2}{x - 1}$$

$$5) y = \frac{x^3}{x^2 + 1} \qquad 6) y = \frac{x^3}{x^2 - 1}$$

$$7) y = \frac{x^2 - 5}{x - 3} \qquad 8) y = \frac{x^4}{x^3 - 1}$$

$$9) y = \frac{4x^3}{x^3 - 1} \qquad 10) y = \frac{2 - 4x^2}{1 - x^2}$$

$$11) y = \frac{\ln x}{\sqrt{x}} \qquad 12) y = xe^{-x^2}$$

$$13) y = e^{2x-x^2} \qquad 14) y = xe^{-x^2}$$

$$15) y = x^2 - 2 \ln x$$

IV. Funktsiya ekstremumlarini topish qoidalari yordamida masalani eching.

1) Tomoni a ga teng kvadrat shaklidagi tunuka burchaklaridan teng kvadratlar qirqilib, ochiq quticha yasaldi. Qanday qilib eng katta sig'imli quti yasash mumkin?

2) Diametri d ga teng doiraviy kesimli xoda, kesimi to'g'ri to'rtburchak bo'lgan to'singa tilindi. To'g'ri to'rtburchak asosi b , balandligi h desak, to'sin eng katta mahkamlikka erishishi uchun b va h qanday o'lchamlarga ega bo'lishi kerak?

Ko'rsatma: Mahkamlik bh^2 ko'paytmaga proporsional.

- 3) R radiusli sharga eng katta sirtli tsilindr (ichki chizilgan) toping.
- 4) R radiusli doiradan eng katta hajmli konus yasash uchun qanday sektorni qirqib olish kerak?
- 5) V hajmli silindrik banka qanday o'lchamlarda eng kichik to'la sirtga ega bo'ladi?
- 6) R radiusli sharga tashqi chizilgan eng kichik hajmli konus o'lchamlarini toping.
- 7) R radiusli sferaga ichki chizilgan barcha uchburchakli muntazam prizmalar orasidan hajmi eng kattasini toping.
- 8) Asosining radiusi R, balandligi H bo'lgan konusga ichki chizilgan tsilindrlar orasidan hajmi eng kattasini toping.
- 9) O'q kesimi perimetri r ga teng bo'lgan barcha tsilindrlar orasidan hajmi eng kattasini toping.
- 10) To'g'ri prizmaning asosi teng yonli to'g'ri burchakli uchburchakdan iborat, uning katta yon yog'ining perimetri 24 sm. Prizma hajmi eng katta bo'lishi uchun, uning asosi tomonlari qanday uzunliklarga ega bo'lishi kerak?
- 11) k ning qanday qiymatida $y = x^2 + 2x$ va $y = kx + 1$ lar bilan chegaralangan figura yuzi eng kichik bo'ladi?
- 12) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning $M(x, y)$ nuqtasidan shunday urinma o'tkazingki, son o'qlari va urinma bilan chegaralangan uchburchak yuzi eng kichik bo'lsin.
- 13) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($0 < b < a$) ellipsda $B(0; -b)$ nuqtasidan o'tuvchi eng katta vatarni toping.
- 14) Yuk avtomobili ochiq kuzovi sirtining yuzi 2S bo'lgan to'g'ri burchakli paralleliped shaklida. Kuzovning hajmi eng katta, bo'yining eniga nisbati esa $\frac{5}{2}$ bo'lishi uchun uning bo'yi va eni qanday bo'lishi kerak?

15) Eni bir xil uchta taxtadan nov yasalmoqda. Nov yon devorlarining asosga og'ish burchaklari qanday bo'lganda nov ko'ndalang kesimining yuzi eng katta bo'ladi?

7-bob. Aniqmas integral

§ 23. Aniqmas integral. Jadval yordamida integrallash.

Biror (a, b) intervalda $F'(x) = f(x)$ bo'lsa, $F(x)$ funktsiya $f(x)$ ning boshlang'ich funktsiyasi deyiladi.

$F(x)$ boshlang'ich funktsiya bo'lsa, $F(x) + C$ ham boshlang'ich funktsiya bo'ladi. Ixtiyoriy boshlang'ich funktsiya $f(x)$ ning (a, b) intervaldagi aniqmas integrali deyiladi va $\int f(x)dx = F(x) + C$ tarzida yoziladi.

Aniqmas integral quyidagi xossalarga ega

$$\text{I. } d \int u dx = u dx \quad \text{II. } \int du = u + C$$

$$\text{III. } \int (Au + Bv) dx = A \int u dx + B \int v dx$$

Aniqmas integral jadvali

$$1) \int x^p dx = \frac{x^{p+1}}{p+1} + C \quad 2) \int \frac{dx}{x} = \ln|x| + C$$

$$3) \int a^x dx = \frac{a^x}{\ln a} + C \quad 4) \int e^x dx = e^x + C$$

$$5) \int \cos x dx = \sin x + C$$

$$6) \int \sin x dx = -\cos x + C$$

$$7) \int \frac{1}{\cos^2 x} = \operatorname{tg} x + C$$

$$8) \int \frac{1}{\sin^2 x} = -\operatorname{ctgx} + C$$

$$9) \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = -\arccos x + C$$

$$10) \int \frac{dx}{1+x^2} = \operatorname{arctgx} + C = -\operatorname{arcctgx} + C$$

$$11) \int \operatorname{sh} x dx = \operatorname{ch} x + C \quad 12) \int \operatorname{ch} x dx = \operatorname{sh} x + C$$

$$13) \int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C \quad 14) \int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C$$

23.1 Integrallarni toping.

$$1) \int (x^2 + 4x + \frac{1}{x}) dx \quad 2) \int (\frac{(x^2 + 1)^2 - \sqrt{x}}{x^3}) dx$$

$$3) \int (\frac{\sqrt{x} = 2\sqrt[3]{x^2} + 1}{\sqrt[4]{x}}) dx \quad 4) \int (2 - x^3)^2 dx$$

$$5) \int (\frac{a}{x} + \frac{a^2}{x^2} + \frac{a^3}{x^3}) dx \quad 6) \int (1 - \frac{1}{x^2}) \sqrt{x} \sqrt{x} dx$$

$$7) \int e^x (1 - \frac{e^{-x}}{x^2}) dx \quad 8) \int a^x (1 + \frac{a^{-x}}{\sqrt{x^3}}) dx$$

$$9) \int \frac{dx}{\sin^2 x \cos^2 x} \quad 10) \frac{3 - 2\operatorname{ctg}^2 x}{\cos^2 x} dx$$

$$11) \int \left(\sin^2 \frac{x}{2} + \cos^2 x \right) dx$$

$$12) \int (tg^2 x + \operatorname{ctg}^2 x + 2) dx \quad 13) \int \frac{x^4}{1+x^2} dx$$

§24. Bevosita va yangi o'zgaruvchi kiritib integrallash

$$x = \varphi(u), \quad dx = \varphi'(u)du \quad \text{bo'lsa,}$$

$\int f(x)dx = \int f[\varphi(u)]\varphi'(u)du$ ko'rinish olib, bunday integrallash yangi o'zgaruvchi kiritib integrallash deyiladi.

$\int f(u(x))d(u(x)) = F(u(x)) + C$ ko'rinishdan foydalanish bevosita integrallash hisoblanadi, masalan,

$$\int f(ax+b)dx = \frac{1}{a} \int f(ax+b)d(ax+b) = \frac{1}{a} F(ax+b) + C$$

24.1 Bevosita integrallang:

1) $\int \frac{2x-5}{x^2-5x+7} dx$

2) $\int \frac{2x}{x^2+1} dx$

3) $\int \lg x dx$

4) $\int \sin^2 x \cos x dx$

5) $\int \cos^3 x \sin x dx$

6) $\int \frac{\sin x dx}{1+3\cos x}$

7) $\int \frac{\sin x dx}{\cos^3 x}$

8) $\int \frac{\cos x dx}{\sin^4 x}$

9) $\int \sin x \cos x dx$

10) $\int e^{\cos x} \sin x dx$

11) $\int e^{x^2} x^2 dx$

12) $\int e^{-x^2} x dx$

13) $\int \sqrt{x^2+1} x dx$

14) $\int \sqrt[3]{x^3-8x^2} dx$

15) $\int \frac{dx}{x \ln x \ln(\ln(x))}$

16) $\int \frac{dx}{(1+x)\sqrt{x}}$

17) $\int \sin \frac{1}{x} \frac{dx}{x^2}$

24.2 Yangi o'zgaruvchi kiritib integrallang:

1) $\int \cos 3x dx$

2) $\int \sin \frac{x}{2} dx$

- 3) $\int e^{-3x} dx$ 4) $\int \frac{dx}{\cos^2 5x}$
 5) $\int \sqrt{4x-1} dx$ 6) $\int (3-2x)^4 dx$
 7) $\int \sin(a-bx) dx$

24.3. Integrallang:

- 1) $\int \frac{\sin x + \cos x}{\sqrt{\sin x - \cos x}} dx$ 2) $\int \frac{dx}{(\arcsin x)^2 \sqrt{1-x^2}}$
 3) $\int \frac{x^2+1}{x^4+1} dx$ 4) $\int \frac{dx}{\sin^2 x \sqrt{\operatorname{ctg} x}}$
 5) $\int \frac{dx}{\sin^2 x + 2 \cos^2 x}$ 6) $\int \frac{dx}{\sin x}$
 7) $\int \sin^2 x dx$ 8) $\int \cos^2 x dx$
 9) $\int \frac{dx}{\cos^4 x}$

Yangi o'zgaruvchi kiritish, bevosita integrallash yordamida quyidagilarni isbotlash mumkin

- I. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$
 II. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$
 III. $\int \frac{x dx}{a^2 \pm x^2} = \pm \frac{1}{2} \ln |a^2 \pm x^2| + C$
 IV. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
 V. $\int \frac{dx}{\sqrt{a^2 \pm x^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$

$$\text{VI. } \int \frac{x dx}{\sqrt{a^2 \pm x^2}} = \pm \sqrt{a^2 \pm x^2} + C$$

$$\text{VII. } \int \sqrt{a^2 - x^2} = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

$$\text{VIII. } \int \sqrt{x^2 \pm a^2} = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

24.4 Integrallarni toping

$$1) \int \frac{dx}{x^2 + 9}$$

$$2) \int \frac{dx}{x^2 - 25}$$

$$3) \int \frac{dx}{\sqrt{4 - x^2}}$$

$$4) \int \frac{dx}{\sqrt{x^2 + 5}}$$

$$5) \int \frac{dx}{2x^2 + 5}$$

$$6) \int \frac{dx}{4x^2 - 3}$$

$$7) \int \frac{dx}{\sqrt{5 - x^2}}$$

$$8) \int \frac{x dx}{\sqrt{3 - x^2}}$$

$$9) \int \frac{5x - 2}{x^2 + 4} dx$$

$$10) \int \frac{3x - 4}{x^2 - 4} dx$$

$$11) \int \frac{x + 1}{\sqrt{x^2 + 2}} dx$$

$$12) \int \frac{x + 2}{\sqrt{4 - x^2}} dx$$

$$13) \int \frac{dx}{x^2 + 4x + 5}$$

$$14) \int \frac{dx}{x^2 + 4x + 4}$$

$$15) \int \frac{dx}{x^2 + 4x + 3}$$

$$16) \int \frac{dx}{\sqrt{x^2 + 2x + 3}}$$

$$17) \int \frac{dx}{\sqrt{1 - 2x - x^2}}$$

$$18) \int \frac{dx}{\sqrt{4x - x^2}}$$

$$19) \int \frac{x^4 dx}{x^2 + 2}$$

$$20) \int \frac{e^x dx}{\sqrt{1 - e^{2x}}}$$

$$21) \int \frac{dx}{3 \sin^2 x - 8 \sin x \cos x + 5 \cos^2 x}$$

§25. Bo'laklab integrallash

Ko'paytmaning differentsiali $d(u,v)=u dv+v du$ formulasidan quyidagi bo'laklab integrallash formulasi kelib chiqadi:

$$\int u dv = uv - \int v du$$

Bu formula yordamida

$$\int x^k \ln^m x dx; \int x^k \sin b x dx; \int x^k \cos b x dx; \int x^k e^{ax} dx$$

va $\int e^{ax} \sin b x dx$ ko'rinishdagi integrallarni topish mumkin.

25.1 Bo'laklab integrallang

- | | |
|---|---------------------------------|
| 1) $\int x \ln x dx$ | 2) $\int x^2 \ln x dx$ |
| 3) $\int \arctg x dx$ | 4) $\int x^2 \cos x dx$ |
| 5) $\int \arcsin x dx$ | 6) $\int \frac{x dx}{\sin^2 x}$ |
| 7) $\int \frac{\ln x dx}{x^2}$ | 8) $\int \cos(\ln x) dx$ |
| 9) $\int \sin x \ln(\tg x) dx$ | 10) $\int x^3 e^{-x} dx$ |
| 11) $\int \frac{x \cos x dx}{\sin^3 x}$ | 12) $\int e^{ax} \sin b x dx$ |

§26 Ratsional algebraik funktsiyalarni integrallash

$P_n(x), Q_m(x)$ — mos darajali ko'phadlar

bo'lganda $\int \frac{P_n(x)}{Q_m(x)} dx$ ifoda quyidagicha integrallanadi:

1) $n \geq m$ bo'lsa, $\int \frac{P_n(x)}{Q_m(x)} dx$ noto'g'ri kasr

deyilib, suratni mahrajga bo'lish yordamida butun qismi ajratiladi va integrallanadi:

$$\int \frac{P_n(x)}{Q_m(x)} dx = \int \left(R(x) + \frac{P_1(x)}{Q_m(x)} \right) dx \quad \text{bunda } R - \text{butun}$$

qism, $\frac{P_1}{Q}$ - to'g'ri kasrdir.

2) $n < m$ bo'lsa, $Q_m(x)$ ko'phad algebra asosiy teoremasi natijasiga ko'ra

$$(x - \alpha_1)^{l_1} \cdot (x - \alpha_2)^{l_2} \cdots (x - \alpha_k)^{l_k} \cdot (x^2 + p_1x + q_1)^{n_1} \cdot \dots$$

$$\cdot (x^2 + p_kx + q_k)^{n_k}$$

ko'rinishda yoziladi, bunda

$$\sum_{i=1}^{k_1} l_i + 2 \sum_{j=1}^{k_2} r_j = m.$$

Kasrni quyidagicha sodda kasrlar yig'indisi sifatida yozish mumkin:

$$\begin{aligned} \frac{P_n(x)}{Q_m(x)} = & \frac{A_1}{x - \alpha_1} + \frac{A_2}{(x - \alpha_1)^2} + \dots + \frac{C_1x + D_1}{x^2 + px + q} + \\ & + \frac{C_2x + D_2}{(x^2 + px + q)^2} + \dots + \frac{C_kx + D_k}{(x^2 + px + q)^{n_k}} + \dots \end{aligned}$$

Koeffitsientlar esa aniqmas koeffitsientlar metodi yordamida topiladi.

26.1 Integrallarni toping.

1) $\int \frac{x^3}{x-2} dx$

2) $\int \frac{x^4}{x^2 + a^2} dx$

- 3) $\int \frac{x^5}{x^3 - a^3} dx$ 4) $\int \frac{x-4}{(x-2)(x-3)} dx$
- 5) $\int \frac{2x+7}{x^2+x-2} dx$ 6) $\int \frac{3x^2+2x-3}{x^3-x} dx$
- 7) $\int \frac{(x+1)^3}{x^2-x} dx$ 8) $\int \frac{x^{10}}{x^2+x-2} dx$
- 9) $\int \frac{x^3+1}{x^3-5x^2+6x} dx$ 10) $\int \frac{x^2+1}{(x+1)^2(x-1)} dx$
- 11) $\int \frac{dx}{(x+1)(x^2+1)}$ 12) $\int \frac{dx}{x^3-1}$
- 13) $\int \frac{xdx}{x^3-1}$ 14) $\int \frac{xdx}{x^4-1}$
- 15) $\int \frac{dx}{x^4+4}$ 16) $\int \frac{dx}{x^6+1}$
- 17) $\int \frac{dx}{(1+x)(1+x^2)(1+x^3)}$
- 18) $\int \frac{dx}{x^5-x^4+x^3-x^2+x-1}$

26.2 Aniqmas koiffitsientlar metodini qo'llam toping:

- 1) $\int \frac{dx}{x(x+a)}$ 2) $\int \frac{dx}{(x+a)(x+6)}$
- 3) $\int \frac{dx}{x^2-2}$ 4) $\int \frac{dx}{(x^3-3)(x^2+2)}$
- 5) $\int \frac{dx}{x^4-x^2}$ 6) $\int \frac{dx}{x^3+4x}$

§ 27 Irratsional ifodalarni integrallash

$$1^0. \int R(x, \sqrt[n]{\frac{ax+b}{cx+d}}, \dots, \sqrt[m]{\frac{ax+b}{cx+d}}) dx$$

ko'rinishdagi ifoda

EKUK $(\lambda, \dots, \mu) = m$ bo'lsa,

$$t^m = \frac{ax+b}{cx+d}$$

almashtirish yordamida ratsionallashadi.

$$2^0. \int R(x, \sqrt{a+bx+cx^2}) dx \text{ ko'rinishdagi ifoda}$$

quyidagi Eylar almashtirishlari yordamida ratsionallashadi.

a) $D = b^2 - 4ac \geq 0$ bo'lsa,

$$a+bx+cx^2 = C(x-\alpha)(x-\beta) \text{ dan } t = \sqrt{\frac{C(x-\beta)}{x-\alpha}}$$

almashtirish o'tkaziladi.

b) $D < 0$ bo'lsa, $\sqrt{a+bx+cx^2} = t \mp x\sqrt{c}$

almashtirish o'tkaziladi.

3⁰. $\int x^m (a+bx^n)^p dx$ differentsial binom quyidagi

uch holda elementar funktsiyalarda integrallanadi.

1) R - butun bo'lsa, yoyish yordamida.

2) $\frac{m+1}{n}$ butun son bo'lsa, $a+bx^n = t^s$ bunda S soni

P ning maxraji.

3) $\frac{m+1}{n} + p$ butun son bo'lsa,

$ax^{-n} + b = t^s$ yordamida ifodalanadi.

4^o. $\int R(x, \sqrt{a^2 - x^2}) dx$ ko'rinishdagi integral
 $x = a \sin t$ almashtirish yordamida,

$\int R(x, \sqrt{a^2 - x^2}) dx$ esa $x = \frac{a}{\cos t}$ almashtirishda,

$\int R(x, \sqrt{a^2 - x^2}) dx$ esa $x = a \tan t$ yordamida
 integrallanadi.

27.1 Integrallarni toping.

1) $\int \frac{dx}{1 + \sqrt{x}}$

2) $\int \frac{dx}{\sqrt[3]{x} + \sqrt{x}}$

3) $\int \frac{x^3 \sqrt{2+x}}{x + \sqrt[3]{2+x}} dx$

4) $\int \frac{dx}{(1 + \sqrt[4]{x}) \sqrt[3]{x}}$

5) $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx$

6) $\int x \sqrt{a-x} dx$

27.2 Integrallarni toping.

1) $\int \frac{dx}{x \sqrt{2x^2 + 2x + 1}}$

2) $\int \frac{dx}{x + \sqrt{x^2 + x + 1}}$

3) $\int \frac{x - \sqrt{x^2 + 3x + 2}}{x + \sqrt{x^2 + 3x + 2}} dx$

4) $\int \frac{dx}{1 + \sqrt{1 - 2x - x^2}}$

5) $\int x^2 \sqrt{x^2 - 2x + 2} dx$

27.3 Integrallarni toping.

1) $\int \frac{dx}{\sqrt[4]{1+x^4}}$

2) $\int \frac{\sqrt{x} dx}{(1 + \sqrt[3]{x})^2}$

3) $\int \frac{dx}{x^6 \sqrt{1+x^6}}$

4) $\int \frac{x^5 dx}{\sqrt{1-x^2}}$

5) $\int \sqrt[3]{3x - x^3} dx$

27.4 Integrallarni toping.

1) $\int x\sqrt{a^2 - x^2} dx$

2) $\int x^2 \sqrt{4 - x^2} dx$

3) $\int \frac{dx}{\sqrt{(4 + x^2)^3}}$

4) $\int \sqrt{3 + 2x - x^2} dx$

5) $\int \frac{x^2 dx}{(2 - x^2)^3}$

6) $\int \frac{\sqrt{x^2 - a^2}}{x} dx$

§28 Trigonometrik ifodalarni integrallash.

$\int \sin^m x \cdot \cos^n x dx$ $m, n \in \mathbb{Z}$ ko'rinishdagi

integrallar trigonometrik almashtirishlar, daraja pasaytirish, ko'paytmani yig'indiga aylantirish, yangi o'zgaruvchi kiritish yordamida topiladi.

28.1 Integrallarni toping.

1) $\int \sin^2 3x dx$

2) $\int \cos^4 x dx$

3) $\int \cos^3 x dx$

4) $\int \sin^5 x dx$

5) $\int \sin^3 x \cos^3 x dx$

6) $\int \frac{dx}{\sin 2x}$

7) $\int \frac{dx}{\cos x}$

8) $\int \lg^3 x dx$

9) $\int \operatorname{ctg}^3 x dx$

10) $\int \frac{\sin^3 x dx}{\cos^2 x}$

11) $\int \frac{\cos^3 x dx}{\sin^2 x}$

12) $\int \sin 3x \sin x dx$

13) $\int \sin(5x - \frac{\pi}{4}) \cos(x + \frac{\pi}{4}) dx$

14) $\int \frac{dx}{\sin^4 x + \cos^4 x}$

28.2. Bo'laklab integrallash formulasidan foydalanib

$\int \cos^n x dx$ uchun "Daraja pasaytirish" formulasini

keltirib chiqaring va $\int \cos^6 x dx$, $\int \frac{dx}{\cos^3 x}$ larni

hisoblang.

28.3. $\int \sin^6 x dx$ va $\int \frac{dx}{\sin^3 x}$ larni hisoblang.

Bobga doir misollar echish namunalari

1. Jadval yordamida integralni toping

$$1) \int (2 + \operatorname{tg}^2 x + 5x^4 + 2\sqrt{x} + \frac{6}{x^3} + \frac{1}{\sqrt[3]{x^2}}) dx =$$

$$= \int (1 + \frac{1}{\cos^2 x} + 5x^4 + 2x^{\frac{1}{2}} + 6x^{-3} + x^{-\frac{2}{3}}) dx =$$

$$\int 1 dx + \int \frac{dx}{\cos^2 x} + 5 \int x^4 dx + 2 \int x^{\frac{1}{2}} dx +$$

$$+ 6 \int x^{-3} dx + \int x^{-\frac{2}{3}} dx = x + \operatorname{tg} x + 5 \cdot \frac{x^5}{5} + 2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 6 \cdot \frac{x^{-2}}{-2} +$$

$$+ \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C = x + \operatorname{tg} x + x^5 + \frac{3}{2} x^{\frac{3}{2}} - \frac{1}{x^2} + 3\sqrt[3]{x} + C$$

$$2) \int \left[\frac{1}{\sqrt{1-x^2}} + \frac{2^{x+1} - 5^{x-1}}{10^x} + \frac{x^2}{1+x^2} \right] dx =$$

$$= \int \left[\frac{1}{\sqrt{1-x^2}} + 2 \left(\frac{1}{5} \right)^x - \frac{1}{5} \left(\frac{1}{2} \right)^x + \frac{1+x^2-1}{1+x^2} \right] dx =$$

$$= \int \left[\frac{1}{\sqrt{1-x^2}} + 2\left(\frac{1}{5}\right)^x - \frac{1}{5}\left(\frac{1}{2}\right)^x + \frac{1+x^2}{1+x^2} - \frac{1}{1+x^2} \right] dx =$$

$$= \arcsin x + 2\left(\frac{1}{5}\right)^x \cdot \frac{1}{\ln \frac{1}{5}} - \frac{1}{5}\left(\frac{1}{2}\right)^x \cdot \frac{1}{\ln \frac{1}{2}} + x - \arctg x + C$$

$$3) \int \left[\frac{\cos 2x}{\cos^2 x \sin^2 x} + \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2 \right] dx =$$

$$= \int \left[\frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} + \cos^2 \frac{x}{2} \right] dx =$$

$$= \int \left[\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} + 1 - \sin x \right] dx =$$

$$= -\operatorname{ctg} x - \operatorname{tg} x + x + \cos x + C$$

2. Bevosita integrallang.

$$1) \int \operatorname{ctg} x dx = \int \frac{\cos x}{\sin x} dx = \int \frac{d(\sin x)}{\sin x} = \ln |\sin x| + C$$

$$2) \int \frac{dx}{x+a} = \int \frac{d(x+a)}{x+a} = \ln |x+a| + C$$

3. Yangi o'zgaruvchilar kiritib, integrallang.

$$1) \int (x+1)^{10} dx = \int t^{10} dt = \frac{t^{11}}{11} + C = \frac{(x+1)^{11}}{11} + C$$

$$x+1=t, \quad x=t-1 \quad dx=dt$$

$$2) \int \sqrt[3]{1-6x} dx = \int \sqrt[3]{t} \left(-\frac{1}{6} dt\right) = -\frac{1}{6} \int t^{\frac{1}{3}} dt = -\frac{1}{6} \frac{t^{\frac{4}{3}}}{\frac{4}{3}} = -\frac{1}{8} \sqrt[3]{(1-6x)^4} + C$$

$$1-6x=t, \quad 6x=1-t, \quad x=-\frac{1}{6}t+\frac{1}{6}, \quad dx=-\frac{1}{6}dt$$

4. Bo'laklab integrallash

$$1) \int \ln x dx = x \ln x - \int dx = x \ln x - x + C$$

$$u = \ln x \quad dv = dx \quad du = \frac{1}{x} dx \quad v = x$$

$$2) \int x^2 \cos x dx = \int x^2 d(\sin x) = x^2 \sin x +$$

$$+ 2 \left\{ x \cos x - \int \cos x dx \right\} =$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$$3) I = \int e^{ax} \cos bxdx = \frac{1}{a} \int \cos bxd(e^{ax}) =$$

$$= \frac{1}{a} [e^{ax} \cos bx + b \int e^{ax} \sin bxdx]$$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{x^2} \int \sin bxd(e^{ax}) = \frac{1}{a} e^{ax} \cos bx +$$

$$+ \frac{b}{a^2} [e^{ax} \sin bx - b \int e^{ax} \cos bxdx] =$$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \sin bx - \frac{b^2}{a^2} I;$$

$$\text{Demak, } I = \frac{e^{ax}}{a^2 + b^2} (b \sin bx + a \cos bx);$$

$$5. \int \frac{x^4 + 3x}{x^2 + 1} dx \quad \text{ni toping.}$$

$$\int \frac{x^4 + 3x}{x^2 + 1} dx = \int x^2 dx - \int dx + \frac{3}{2} \int \frac{d(x^2 + 1)}{x^2 + 1} + \int \frac{dx}{x^2 + 1} =$$

$$= \frac{x^3}{3} - x + \frac{3}{2} \ln(x^2 + 1) + \operatorname{arctg} x + C, \quad \text{chunki}$$

$$\frac{x^4 + 3x}{x^2 + 1} = x^2 - 1 + \frac{3x + 1}{x^2 + 1}.$$

6. $\int \frac{2x^2 + x + 2}{x^3 + 2x} dx$ ni toping.

$$\frac{2x^2 + x + 2}{x(x^2 + 2)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 2} \quad \text{dan}$$

$$2x^2 + x + 2 = A(x^2 + 2) + Bx^2 + Cx \quad \text{kelib chiqadi.}$$

$$\text{Bundan } A+B=2, \quad C=1; \quad 2A=2, \quad \text{ya'ni } A=B=C=1$$

$$\begin{aligned} \int \frac{2x^2 + x + 2}{x^3 + 2x} dx &= \int \left(\frac{1}{x} + \frac{x+1}{x^2+2} \right) dx = \\ &= \ln|x| + \frac{1}{2} \ln(x^2 + 2) + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C \end{aligned}$$

7. $\int \frac{1 - \sqrt{x+1}}{1 + \sqrt[3]{x+1}} dx$ ni toping.

$$x+1 = t^6; \quad t = \sqrt[6]{x+1} \quad dx = 6t^5 dt \quad \text{almashtirishlar}$$

yordamida integral quyidagi ko'rinishga keladi.

$$\begin{aligned} \int \frac{1 - \sqrt{x+1}}{1 + \sqrt[3]{x+1}} dx &= \int \frac{1 - t^3}{1 + t^2} 6t^5 dt = \\ &= 6 \int \frac{-t^8 + t^5}{1 + t^2} dt = 6 \int (t^6 + t^4 + t^3 - t^2 - t + 1 + \frac{t-1}{t^2+1}) dt = \\ &= 6 \left[-\frac{t^7}{7} + \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} - \frac{t^2}{2} + t + \frac{1}{2} \ln(t^2 + 1) - \operatorname{arctg} t \right] + C = \\ &= 6 \left[-\frac{1}{7} \sqrt[7]{(x+1)^7} + \frac{1}{5} \sqrt[5]{(x+1)^5} + \frac{1}{4} \sqrt[4]{(x+1)^4} - \right. \end{aligned}$$

$$-\frac{1}{3}\sqrt{x+1}-\frac{1}{2}\sqrt[3]{x+1}+\sqrt[6]{x+1}+\frac{1}{2}\ln(\sqrt[3]{x+1}+1)-$$

$$-\arctg\sqrt[6]{x+1}]+C$$

8. $\int \frac{dx}{x+\sqrt{x^2+3x+2}}$ ni toping.

$x^2+3x+2=(x+1)(x+2)$ dan $(x+1)t^2=x+2$
ko'rinishda yangi o'zgaruvchi kiritamiz. Undan

$$x=\frac{2-t^2}{t^2-1}; \quad dx=\frac{-2tdt}{(t^2-1)^2}$$

Demak, berilgan integral quyidagi ko'rinishda bo'ladi:

$$\int \frac{-\frac{2t}{(t^2-1)^2} dt}{\frac{2-t^2}{t^2-1}+t(\frac{2-t^2}{t^2-1}+1)} = 2 \int \frac{t}{(t-1)(t+1)^2(t-2)} dt =$$

$$= 2 \int \left[\frac{1}{t-1} + \frac{4}{9} \frac{1}{t+1} + \frac{1}{3} \frac{1}{(t+1)^2} + \frac{5}{9} \frac{1}{(t-2)} \right] dt =$$

$$= 2 \left[\ln|1-t| + \frac{4}{9} \ln|t+1| - \frac{1}{3} \frac{1}{t+1} + \frac{5}{9} \ln|t-2| \right] + C =$$

$$= 2 \left[\ln \left| 1 - \sqrt{\frac{x+2}{x+1}} \right| + \frac{4}{9} \ln \left| 1 + \sqrt{\frac{x+2}{x+1}} \right| - \frac{1}{3} \frac{1}{\left(1 + \sqrt{\frac{x+2}{x+1}} \right)^2} \right.$$

$$\left. + \frac{5}{9} \ln \left| \sqrt{\frac{x+2}{x+1}} - 2 \right| \right] + C$$

9. $\int \sqrt{x^3+x^4} dx$ irratsionallikni yo'qoting:

$$\int \sqrt{x^3 + x^4} dx = \int x^{\frac{2}{3}} (1+x)^{\frac{1}{2}} dx \quad \text{dan} \quad \frac{m+1}{n} + p = 3$$

ya'ni $x^{-1} + 1 = t^2$
almashtirish zarurligi kelib chiqadi.

$$x = \frac{1}{t^2 - 1}; \quad dx = \frac{-2t}{(t^2 - 1)^2} dt$$

Natijada, berilgan integral quyidagi ko'rinishga keladi:

$$-2 \int \frac{t^2}{(t^2 - 1)^4} dt$$

10. $\int \sin^2 x \cdot \cos^2 x dx$ da

$$\begin{aligned} \sin^2 x \cdot \cos^2 x &= \frac{1 - \cos 2x}{2} \cdot \frac{1 + \cos 2x}{2} = \frac{1}{4} (1 - \cos^2 2x) = \\ &= \frac{1}{4} \sin^2 2x = \frac{1}{4} \cdot \frac{1 - \cos 4x}{2} \end{aligned}$$

dan

$$\begin{aligned} \int \sin^2 x \cdot \cos^2 x dx &= \\ &= \frac{1}{8} \int (1 - \cos 4x) dx = \frac{1}{8} \left[x - \frac{\sin 4x}{4} \right] + C \end{aligned}$$

11. $\int \sin^2 x \cdot \cos^3 x dx = \int \sin^2 x \cdot (1 - \sin^2 x) d(\sin x) =$

$$= \int [\sin^2 x - \sin^4 x] d(\sin x) = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

12.

$$\begin{aligned} \int \sin mx \sin nx dx &= \frac{1}{2} \int [\cos(m-n)x - \cos(m+n)x] dx = \\ &= \frac{1}{2} \left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n} \right] + C \end{aligned}$$

13. $\int \sin^n x dx$ integral uchun «daraja pasaytirish» formulasini chiqaring. Bo'laklab integrallash formulasidan

$$\begin{aligned} I_n &= -\int \sin^{n-1} x d(\cos x) = -\cos x \sin^{n-1} x + \\ &+ \int \cos x (n-1) \sin^{n-2} x \cos x dx = -\cos x \sin^{n-1} x + \\ &+ (n-1) \int (1 - \sin^2 x) \sin^{n-2} x dx = -\cos x \sin^{n-1} x + \\ &+ (n-1) \int \sin^{n-2} x dx - (n-1) I_n, \end{aligned}$$

ya'ni $n \cdot I_n = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x dx$ kelib chiqadi.

$$\text{Demak, } I_n = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x dx.$$

7-bobga doir uy vazifalari.

Aniqmas integrallarni toping.

I 1) $\int e^{\sin^2 x} \sin 2x dx$ 2) $\int \arctg \sqrt{x} dx$

3) $\int \frac{dx}{x^3 + 27}$ 4) $\int \frac{dx}{2 + \sqrt[3]{x+1}}$

5) $\int \frac{x dx}{\sqrt{x^2 + 4x - 5}}$

II 1) $\int \frac{x dx}{x^2 + 5}$ 2) $\int x \ln(x+1) dx$

3) $\int \frac{2x^2 - 1}{x^3 + 1} dx$ 4) $\int \frac{dx}{\sin x + \operatorname{tg} x}$

5) $\int \frac{dx}{(1 + \sqrt{x(1+x)})^2}$

$$\text{III} \quad 1) \int \frac{dx}{x(\ln x + 4)} \quad 2) \int x \cdot e^{-4x} dx$$

$$3) \int \frac{3x-7}{x^3+4x^2+4x+16} dx$$

$$4) \int \frac{x^2 + \sqrt{1+x}}{\sqrt[3]{1+x}} dx \quad 5) \int \frac{dx}{\sin^5 x}$$

$$\text{IV} \quad 1) \int \frac{dx}{\sin^2 x (4 \operatorname{ctg} x + 3)} \quad 2) \int \frac{x \arcsin x}{\sqrt{1-x^2}} dx$$

$$3) \int \frac{dx}{x^2 + x^2 + 2x + 2} \quad 4) \int \frac{x^2 + \sqrt{1+x}}{\sqrt[3]{1+x}} dx$$

$$5) \int \frac{dx}{\cos^6 x}$$

$$\text{V} \quad 1) \int \frac{\cos 2x}{1 + \sin 3x} dx \quad 2) \int x^2 \cdot e^{3x} dx$$

$$3) \int \frac{x dx}{x^3 + 2x + x^2 + 2} \quad 4) \int \frac{\cos x dx}{1 + \cos x}$$

$$5) \int \frac{x^2 dx}{\sqrt{1-x^6}}$$

$$\text{VI} \quad 1) \int \frac{\cos x dx}{\sqrt{\sin^2 x}} \quad 2) \int x \arcsin \frac{1}{x} dx$$

$$3) \int \frac{x+3}{x^3+x^2-2x} dx \quad 4) \frac{\sqrt[4]{x+1}}{(\sqrt{x}+4)\sqrt[4]{x^3}} dx$$

$$5) \int \frac{dx}{a+b \cos x}$$

- VII 1) $\int \frac{x + \arctg x}{1 + x^2} dx$ 2) $\int x \cdot \ln(x^2 + 1) dx$
- 3) $\int \frac{x^2 - 3}{x^4 + 5x^2 + 6} dx$
- 4) $\int \frac{\sqrt{x+5}}{1 + \sqrt[3]{x+5}} dx$ 5) $\int \frac{\sin^3 x}{\cos^5 x} dx$
- VIII 1) $\int \frac{x + \arcsin x}{\sqrt{1-x^2}} dx$ 2) $\int x \sin x \cdot \cos x dx$
- 3) $\int \frac{x^2 dx}{x^4 - 81}$ 4) $\int \frac{dx}{3 \cos x + 4 \sin x}$
- 5) $\int \frac{dx}{\sqrt[3]{x^2 + 2\sqrt{x}}}$
- IX 1) $\int \frac{\sin x dx}{\sqrt[3]{3 + 2 \cdot \cos x}}$ 2) $\int x^2 \cdot \sin 4x dx$
- 3) $\int \frac{x^2 - x + 1}{x^4 + 2x^2 - 3} dx$
- 4) $\int \frac{(\sqrt{x-1})(\sqrt[6]{x+1})}{\sqrt[3]{x^2}} dx$ 5) $\int \frac{dx}{e^{3x} - e^x}$
- X 1) $\int \frac{\sqrt[3]{4 + \ln x}}{x} dx$ 2) $\int x \ln^2 x dx$
- 3) $\int \frac{x^3 - 6}{x^4 + 6x^2 + 8} dx$ 4) $\int \frac{dx}{2 \sin x + \cos x + 2}$
- 5) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

$$\text{XI} \quad 1) \int \frac{x^2}{(1-x)^{100}} dx \quad 2) \int x \cdot \operatorname{sh} x dx$$

$$3) \int \frac{x dx}{(x+1)(x+4)(x-3)}$$

$$4) \int \frac{x+2}{x^2 \cdot \sqrt{1-x^2}} dx$$

$$5) \int \frac{\sin^2 x}{1 + \sin^2 x} dx$$

$$\text{XII} \quad 1) \int \frac{x^5}{x+1} dx \quad 2) \int x^n \cdot \ln x dx$$

$$3) \int \frac{x^2 dx}{x^4 + 5x^2 + 4} \quad 4) \int \sqrt{\frac{x}{1-x\sqrt{x}}} dx$$

$$5) \int \frac{\sin x \cos x}{\sin x + \cos x} dx$$

$$\text{XIII} \quad 1) \int \frac{2^x \cdot 3^x}{9^x - 4^x} dx \quad 2) \int x^2 \sin 2x dx$$

$$3) \int \frac{x+1}{(x^2-4)(x^2+5)} dx$$

$$4) \int \frac{x \ln(1 + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$$

$$5) \int \frac{dx}{\cos x + \cos \alpha}$$

$$\text{XIV} \quad 1) \int \frac{x^2 + 1}{x^4 + 1} dx \qquad 2) \int x^2 \arccos x dx$$

$$3) \int \frac{3x + 1}{x(x^2 + 3)} dx \qquad 4) \int \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}} dx$$

$$5) \int \frac{dx}{\sin x - \sin \alpha}$$

$$\text{XV} \quad 1) \int \frac{dx}{1 - \cos x} \qquad 2) \int \sqrt{x \cdot \ln^2 x} dx$$

$$3) \int \frac{x^2 + 5x + 4}{x^4 + 5x^2 + 4} dx$$

$$4) \int x^2 \sqrt{\frac{x}{1-x}} dx \qquad 5) \int \frac{dx}{\sqrt{\lg x}}$$

8-bob. Aniq integral va tadbirlari.
§29. Aniq integral ta'rif. N'yuton -Leybnits
formulasi.

$[a, b]$ oraliqda $f(x)$ funktsiya aniqlangan va μ uzluksiz bo'lsin. Bu oraliqni bo'laklarga bo'lamiz:
 $a = x_0 < x_1 < x_2 < \dots < x_i < x_{i+1} < \dots < x_n = b$.

$\Delta x_i = x_{i+1} - x_i$; $i = \overline{0, n-1}$ ayirmalaridan eng kattasini λ deb belgilaymiz. Har bir $[x_i, x_{i+1}]$ oraliqlardan ixtiyoriy ravishda biror $x = \xi_i$ nuqta olib

$$\sigma = \sum_{i=0}^{n-1} f(\xi_i) \cdot \Delta x_i$$

yig'indini tuzamiz. Agar bu yig'indining $\lambda \rightarrow 0$ dagi chekli limiti mavjud bo'lsa, u holda bu limit $f(x)$ ning a dan b gacha oraliqdagi aniq integrali deyiladi va quyidagicha belgilanadi:

$$\int_a^b f(x) dx = \lim_{\lambda \rightarrow 0} \sum_{k=0}^{n-1} f(\xi_k) \cdot \lambda$$

Agar $[x_i, x_{i+1}]$ oraliqdagi ξ_i o'rniga, shu oraliqdagi $f(x)$ ning aniq quyi va yuqori chegaralari m_i, M_i olinsa,

$$s = \sum_{k=0}^{n-1} m_k \Delta x_k, \quad S = \sum_{k=0}^{n-1} M_k \Delta x_k$$

Darbuning quyi va yuqori integral yig'indilari hosil bo'ladi.

Aniq integral mavjud bo'lishi uchun $\lim_{\lambda \rightarrow 0} (S - s) = 0$

o'rinni bo'lishi zarur va etarlidir.

Agar $[a, b]$ oraliqda $f(x)$ ning biror boshlang'ich funktsiyasi $F(x)$ bo'lsa, quyidagi N'yuton – Leybnits formulasi o'rinalidir:

$$\int_a^b f(x) dx = F(b) - F(a) = F(x) \Big|_a^b$$

Aniq integralda ham yangi o'zgaruvchi kiritish mumkin, lekin, chegaralari yangi o'zgaruvchi chegaralari bilan almashtiriladi, bo'laklab integrallash formulasi esa quyidagi ko'rinishda bo'ladi:

$$\int_a^b u dv = u \cdot v \Big|_a^b - \int_a^b v du$$

29.1 Integral yig'indi yordamida hisoblang.

$$\begin{array}{llll} 1) \int_0^{\frac{\pi}{2}} \sin x dx & 2) \int_{-1}^2 x^2 dx & 3) \int_a^b \frac{dx}{x^2} & 4) \int_0^{\frac{\pi}{2}} \cos x dx \\ 5) \int_0^a x dx & 6) \int_0^a e^x dx & & \end{array}$$

29.2 Hisoblang:

$$\begin{array}{lll} 1) \int_1^3 x^3 dx & 2) \int_1^2 \left(x^2 + \frac{1}{x^4} \right) dx & 3) \int_1^4 \sqrt{x} dx \\ 4) \int_0^1 \frac{dx}{\sqrt{4-x^2}} & 5) \int_a^{a\sqrt{3}} \frac{dx}{a^2+x^2} & 6) \int_0^3 e^{\frac{x}{3}} dx \\ 7) \int_0^1 \frac{dx}{\sqrt{x^2+1}} & 8) \int_0^{\frac{\pi}{4}} \sin 4x dx & 9) \int_4^9 \frac{dx}{\sqrt{x}-1} \\ 10) \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\tan^2 x}{(1+\tan x)^2} dx & 11) \int_0^4 \frac{dx}{1+\sqrt{2x+1}} & \end{array}$$

$$12) \int_0^1 \frac{x^2 dx}{\sqrt{4-x^2}} \quad 13) \int_0^1 \frac{dx}{e^x + 1} \quad 14) \int_0^{\frac{\pi}{2}} \sqrt{\frac{x}{a-x}} dx$$

$$15) \int_0^{\frac{\pi}{2}} \sin x \cos^2 x dx \quad 16) \int_0^1 \ln(x+1) dx$$

$$17) \int_0^{\sqrt{3}} x \arctg x dx \quad 18) \int_0^{2\pi} x^2 \cdot \cos x dx$$

$$19) \int_0^3 \arcsin \sqrt{\frac{x}{1+x}} dx \quad 20) I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$$

$$21) I_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$$

§30. Yuza va yoy uzunligini hisoblash.

$$y_1(x), \quad y_2(x) \quad (y_2(x) \geq y_1(x))$$

funktsiyalar va $x = a$; $x = b$ ($a < b$) to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiya yuzi

$$S = \int_a^b [y_2(x) - y_1(x)] dx \quad \text{formuladan,} \quad \text{qutb}$$

koordinatalarda

$r = r(\varphi)$, $\varphi = \alpha$, $\varphi = \beta$ ($\alpha < \beta$) lar bilan chegaralangan sektor yuzi

$$S = \frac{1}{2} \int_{\alpha}^{\beta} \sqrt{r^2(\varphi)} d\varphi$$

formula yordamida topiladi.

$y = y(x)$ chiziq $[a, b]$ oralig'idagi yoyi uzunligi

$$l = \int_a^b \sqrt{1 + y'^2(x)} dx$$

qutb koordinatalardagi $r = r(\varphi)$ chiziqlarining $[\alpha, \beta]$ oraliqdagi yoyi uzunligi esa

$$l = \int_{\alpha}^{\beta} \sqrt{r^2(\varphi) + r'^2(\varphi)} d\varphi$$

formulalar yordamida topiladi.

30.1 Quyidagi chiziqlar bilan chegaralangan soha yuzini toping.

1) $y = 4 - x^2$; $y = 0$

2) $y^2 = 2px$, $x = h$

3) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

4) $xy = 4$; $x = 1$; $x = 4$; $y = 0$

5) $y = 3 - 2x - x^2$, $y = 0$ 6) $y = \ln x$, $x = 1$; $y = 0$

7) $y = x^2$; $y^2 = 4x$ 8) $y^2 = x^3$, $y = 8$; $x = 0$

9) $4y = x^2$, $y^2 = 4x$ 10) $xy = 1$, $xy = 4$, $y = x$, $y = 4x$

11) $r^2 = a^2 \cdot \cos 2\varphi$ 12) $r = 3 + 2 \cos \varphi$

13) $r = a \cdot \cos 3\varphi$ 14) $r = a \cdot (1 + \sin^2 2\varphi)$, $r = a$

30.2 Egri chiziqlar yoy uzunligini hisoblang.

1) $x^2 + y^2 = a^2$ 2) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

3) $y = x^{\frac{3}{2}}$ ($0 \leq x \leq 4$) 4) $y^2 = 2px$ ($0 \leq x \leq x_0$)

5) $y = a \ln \frac{a^2}{a^2 - x^2}$ ($0 \leq x \leq b < a$)

$$6) y = \ln x, \quad \frac{3}{4} \leq x \leq \frac{12}{5} \quad 7) r = a\varphi, \quad 0 \leq \varphi \leq 2\pi$$

$$8) r = al^{m\varphi} (m \geq 0) \quad 0 \leq r \leq a \quad 9) r = a \sin^3 \frac{4}{3}$$

$$10) r = ath \frac{\varphi}{2} \quad (0 \leq \varphi \leq 2\pi)$$

§31. Aylanish figuralari hajmi, sirti. Momentlar va og'irlik markazi koordinatalarini topish.

Tekislikda $a \leq x \leq b, 0 \leq y \leq y(x)$ egri chiziqli trapetsiyaning Ox o'qi (Oy o'qi) atrofida aylanishidan hosil bo'lgan jism hajmi

$$V_x = \pi \cdot \int_a^b y^2 dx, \quad \left[V_y = 2\pi \int_a^b x \cdot y(x) dx \right]$$

$a \leq \alpha \leq b, y_1(x) \leq y \leq y_2(x)$ egri chiziqli trapetsiyaning Ox o'qi (Oy o'qi) atrofida aylanishidan hosil bo'lgan xalqa hajmi esa

$$V = \pi \int_a^b [y_2^2(x) - y_1^2(x)] dx \quad \left[V = 2\pi \int_a^b x \cdot [y_2(x) - y_1(x)] dx \right]$$

formula yordamida topiladi.

Aylanish jismi V_x ning sirti

$$S = 2\pi \int_a^b y \cdot \sqrt{1 + y'^2} dx \text{ formuladan topiladi.}$$

Tekislikda $0 \leq \alpha \leq \varphi \leq \beta \leq \pi, 0 \leq r \leq r(\varphi)$ silliq figuraning qutb o'qi atrofida aylanishidan hosil bo'lgan jism hajmi

$$V = \frac{2\pi}{3} \int_{\alpha}^{\beta} r^3(\varphi) \cdot \sin \varphi d\varphi \text{ formuladan topiladi.}$$

Zichligi $\rho(x)$ yuzi S bo'lgan figura massasi

$$M = \int_a^b \rho(x) dx, \text{ statistik momentlari } M_x = \int_a^b xy dy;$$

$$M_y = \int_a^b xy dx$$

og'irlik markazi koordinatalari esa

$$x = \frac{1}{S} M_y, \quad y = \frac{1}{S} M_x$$

formuladan topiladi.

31.1 1) Silindr hajmi formulasini chiqaring.

2) Konus hajmi formulasini chiqaring.

3) Shar hajmi formulasini chiqaring.

4) $y^2 = 2px$ va $x = h$ bilan chegaralangan sohaning Ox o'qi atrofida aylanishidan hosil bo'lgan jism hajmini toping.

5) $y^2 = (x+4)^3$, $x = 0$ chiziqlar bilan chegaralangan sohaning

Oy o'qi atrofida aylanishidan hosil bo'lgan jism hajmini toping.

6) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $y = \pm \epsilon$ chiziqlar bilan chegaralangan sohaning Ox o'qi atrofida aylanishidan hosil bo'lgan jism hajmini toping.

31.2 Quyidagi chiziqlar aylanishidan hosil bo'lgan jism sirtini toping.

1) $x^2 + y^2 = R^2$ ning Ox o'qi atrofida.

2) $4x^2 + y^2 = 4$ ning Oy o'qi atrofida .

3) $x^2 + (y - a)^2 = a^2$ ning Ox o'qi atrofida .

31.3 Qutb o'qi atrofida aylanishdan hosil bo'lgan jism sirtini toping .

1) $r = a(1 + \cos \varphi)$

2) $r^2 = a^2 \cos 2\varphi$.

31.4 Quyidagi chiziqlar bilan chegaralangan sohalar inertsia momentlari, og'irlik markazi koordinatalarini toping .

1) $bx + ay = ab$, $x = 0$, $y = 0$

2) $y = x^2$, $x = 2$; $y = 0$

3) $x^2 + y^2 = a^2$; $y \geq 0$.

§32 Xosmas integrallar

Agar $\lim_{A \rightarrow \infty} \int_a^A f(x) dx$ mavjud va chekli bo'lsa, uni

$[a; +\infty)$ oraliqdagi xosmas integral deyiladi va

$\int_a^{+\infty} f(x) dx$ tarzida yoziladi.

Shunga o'hshash $\int_a^{+\infty} f(x) dx$, $\int_{-\infty}^{+\infty} f(x) dx$ larni ham

kiritiladi.

$[a, b]$ oraliqning S nuqtadan boshqa nuqtalarida uzluksiz, S nuqtada II-tur uzilishga ega funktsiya $f(x)$ dan $[a, b]$ da olingan xosmas integral deb

$\lim_{v \rightarrow 0} \int_a^{c-v} f(x) dx + \lim_{v \rightarrow 0} \int_{c+v}^b f(x) dx$ yigindiga aytiladi.

Xosmas integrallar chekli chiqsa yaqinlashuvchi, aks xolda uzoqlashuvchi deyiladi.

32.1. Yaqinlashishini tekshiring.

$$1) \int_0^{+\infty} \frac{dx}{\sqrt{1+x^3}}$$

$$2) \int_2^{+\infty} \frac{dx}{\sqrt[3]{x^3-1}}$$

$$3) \int_1^{+\infty} \frac{1}{xe^x} dx$$

$$4) \int_0^2 \frac{dx}{\ln x}$$

$$5) \int_0^{\frac{\pi}{2}} \frac{dx}{\sin^p x \cdot \cos^q x}$$

$$6) \int_0^1 \frac{x^n dx}{\sqrt{1-x^4}}$$

32.2. Xisoblang.

$$1) \int_0^1 \ln x dx$$

$$2) \int_a^{+\infty} \frac{dx}{x^2}$$

$$3) \int_0^1 \frac{dx}{(2-x)\sqrt{1-x}}$$

$$4) \int_{-1}^1 \frac{dx}{\sqrt{1-x^2}}$$

$$5) \int_{-\infty}^{+\infty} \frac{dx}{(x^2+x+1)^2}$$

$$6) \int_0^{+\infty} e^{-ax} \cdot \cos x dx$$

$$7) \int_b^{\frac{\pi}{2}} \ln(\sin x) dx$$

32.3. 1) $y = \frac{1}{1+x^2}$ va uning asimtotasi orasidagi yuzani hisoblang.

2) $x > 0$ da $y = e^{-x}$ chiziq Ox o'qi atrofida aylanishidan hosil bulgan jism hajmini toping.

Integrallarni taqribiy hisoblash

$$\int_a^b f(x) dx \text{ aniq integralni hisoblashda } f(x)$$

funktsiyaning boshlang'ich funktsiyasi $F(x)$ ma'lum bo'lganda, N'yuton-Leybnits formulasi yordamida oson hisoblanadi. $F(x)$ - boshlang'ich funktsiyani topish mumkin bo'lmasa, aniq integralni taqribiy hisoblashga to'g'ri keladi, buning uchun $f(x)$ funktsiya soddaroq funktsiyalar bilan almashtiriladi.

2. To'g'ri to'rtburchaklar formulasi.

$[a; b]$ kesmani teng n ta bo'lakka

$$x_k = a + k \cdot \frac{b-a}{n} \quad (k = 0, 1, \dots, n) \text{ nuqtalar yordamida}$$

bo'lamiz va quyidagi

$$\int_a^b f(x) dx \approx \frac{b-a}{n} \cdot \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

taqribiy tenglikka ega bo'lamiz. Uni aniq integralni to'g'ri to'rtburchaklar formulasi yordamida hisoblash formulasi deyiladi.

2. Trapetsiyalar formulasi.

$[a; b]$ kesmani teng n ta bo'lakka bo'lganda

$$S_k = \{(x, y) \in R : x_{k-1} \leq x \leq x_k, 0 \leq y \leq f(x)\} \text{ yuzani}$$

taqriban

$$x = x_{k-1}, x = x_k, y = 0,$$

$y = f(x_{k-1}) + [f(x_k) - f(x_{k-1})]$ to'g'ri chiziqlar chegaralangan trapetsiya bilan almashtirish quyidagi

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

trapetsiyalar formulasi deb ataluvchi taqribiy tenglikni olib keladi.

3. Simpson (parabolik) formulasi:

Agar S_k yuza $x = x_{k-1}$, $x = x_k$, $y = 0$, va uchi

$$f\left(\frac{x_{k-1} + x_k}{2}\right) \text{ nuqtada, } (x_{k-1}, f(x_{k-1})), (x_k, f(x_k))$$

nuqatalardan o'tuvchi parabola bilan chegaralangan egri chiziqli trapetsiya bilan almashtirilsa, parabolik formula deb ataluvchi quyidagi Simpson formulasi hosil bo'ladi.

$$\int_a^b f(x) dx = \frac{b-a}{6n} \left[(y_0 + y_n) + 2(y_1 + \dots + y_{n-1}) + 4\left(y_{\frac{1}{2}} + \dots + y_{n-\frac{1}{2}}\right) \right]$$

Bu holda oraliq $2n$ qismga bo'linadi.

1. Trapetsiyalar formulasiga ko'ra $\ln 2 = \int_1^2 \frac{dx}{x}$ hisoblansin.
2. To'g'ri to'rtburchaklar formulasiga ko'ra $\int_0^{2\pi} x \sin x dx$ hisoblansin va aniq qiymati bilan solishtirilsin ($n = 12$).
3. Simpson formulasiga ko'ra $\pi = 6 \int_0^1 \frac{dx}{\sqrt{1-x^2}}$ taqribiy qiymati hisoblansin.
4. $n = 10$ da Katalon doimiysi deb ataluvchi $G = \int_0^1 \frac{\arctg x}{x} dx$ hisoblansin.
5. $\int_0^1 e^{x^2} dx$ ni 0,001 gacha aniqlikda hisoblang.
6. Yarim o'qlari $a = 10$, $b = 6$ bo'lgan ellips uzunligini taqriban hisoblang.

7. $\int_0^{+\infty} e^{-x^2} dx$ ni 0,001 gacha aniqlikda hisoblang.

Integrallash sohasini teng 20 qismga bo'lib, Simpson formulasi yordamida hisoblang.

1) $\int_{-2}^8 \sqrt{x^3 + 16} dx$

2) $\int_{-3}^7 \sqrt{x^3 + 32} dx$

3) $\int_{-2}^8 \sqrt{x^3 + 8} dx$

4) $\int_0^{10} \sqrt{x^3 + 4} dx$

5) $\int_{-1}^9 \sqrt{x^3 + 11} dx$

6) $\int_1^{11} \sqrt{x^3 + 12} dx$

7) $\int_{-2}^8 \sqrt{x^3 + 14} dx$

8) $\int_{-4}^6 \sqrt{x^3 + 1} dx$

9) $\int_{-1}^9 \sqrt{x^3 + 5} dx$

10) $\int_{-7}^3 \sqrt{x^3 + 11} dx$

11) $\int_{-12}^{-2} \sqrt{x^3 + 13} dx$

12) $\int_{-1}^9 \sqrt{x^3 + 15} dx$

13) $\int_{-4}^6 \sqrt{x^3 + 9} dx$

14) $\int_{-2}^8 \sqrt{x^3 + 22} dx$

15) $\int_{-1}^9 \sqrt{x^3 + 7} dx$

Bob bo'yicha misollar echish namunalari

1. Integral yig'indi yordamida integralni hisoblang:

$$\int_a^b x^p dx$$

$[a, b]$ oraliqni $q_n = \sqrt[n]{\frac{b}{a}}$ yordamida

$a, aq, aq^2, \dots, aq^n = b$ bo'laklarga bo'lamiz. Har bir bo'lakdagi eng kichik sonni ξ_k sifatida olamiz:

$(p \neq -1)$

$$\begin{aligned}\sigma_n &= \sum_{k=0}^{n-1} (a \cdot q^k)^p \cdot [aq^{k+1} - aq^k] = a^{p+1} \cdot (q-1) \cdot \sum_{k=0}^{n-1} (q^{p+1})^k \\ &= a^{p+1} \cdot (q-1) \cdot \frac{\left(\frac{b}{a}\right)^{p+1} - 1}{q^{p+1} - 1} = (b^{p+1} - a^{p+1}) \cdot \frac{q-1}{q^{p+1} - 1}\end{aligned}$$

Demak,

$$\int_a^b x^p dx = \lim_{n \rightarrow \infty} \sigma_n = (b^{p+1} - a^{p+1}) \lim_{q \rightarrow 1} \frac{q-1}{q^{p+1} - 1} = \frac{b^{p+1} - a^{p+1}}{p+1}$$

$p = -1$ bo'lgan holda

$$\sigma_n = n(q_n - 1) = n\left(\sqrt[n]{\frac{b}{a}} - 1\right) \quad \text{dan}$$

$$\int_a^b \frac{dx}{x} = \lim_{n \rightarrow \infty} \sigma_n = \lim_{n \rightarrow \infty} n\left(\sqrt[n]{\frac{b}{a}} - 1\right) = \ln b - \ln a$$

kelib chiqadi.

2. N'yuton - Leybnits formulasi yordamida hisoblang.

1)

$$\int_{-1}^8 \sqrt[3]{x} dx = \left. \frac{3}{4} x^{\frac{4}{3}} \right|_{-1}^8 = \frac{3}{4} [\sqrt[3]{8^4} - \sqrt[3]{(-1)^4}] = \frac{3}{4} [16 - 1] = \frac{45}{4} = 11,25$$

2)

$$\int_0^2 |1-x| dx = \int_0^1 (1-x) dx + \int_1^2 (x-1) dx = \left(x - \frac{x^2}{2}\right) \Big|_0^1 + \left(\frac{x^2}{2} - x\right) \Big|_1^2 =$$

$$= 1 - \frac{1}{2} + \frac{2^2}{2} - 2 - \left(\frac{1}{2} - 1\right) = 1$$

$$3) \int_b^a x^2 \cdot \sqrt{a^2 - x^2} dx$$

$x = a \sin t$ almashtirish o'tkazamiz. $x = 0$ da $t = 0$;

$x = a$ da esa $t = \frac{\pi}{2}$ ekanligini topamiz.

$dx = a \cos t dt$ ni hisobga olib:

$$\int_0^{\frac{\pi}{2}} a^2 \cdot \sin^2 t \cdot \sqrt{a^2 - a^2 \sin^2 t} \cdot a \cos t dt =$$

$$= a^4 \cdot \int_0^{\frac{\pi}{2}} \sin^2 t \cdot \cos^2 t dt = \frac{a^4}{4} \int_0^{\frac{\pi}{2}} \sin^2 2t dt =$$

$$\frac{a^4}{8} \int_0^{\frac{\pi}{2}} [1 - \cos 4t] dt = \frac{a^4}{8} \left[t - \frac{\sin 4t}{4} \right] \Big|_0^{\frac{\pi}{2}} = \frac{\pi a^4}{16}$$

$$4) \int_0^1 \arccos x dx = x \cdot \arccos x \Big|_0^1 + \int_0^1 \frac{x}{\sqrt{1-x^2}} dx =$$

$$= -\frac{1}{2} \int_0^1 \frac{d(1-x^2)}{\sqrt{1-x^2}} = -1 \sqrt{1-x^2} \Big|_0^1 = 1$$

$$u = \arccos x$$

$$dv = dx$$

$$du = -\frac{1}{\sqrt{1-x^2}} dx$$

$$v = x$$

3. $y = 2x - x^2$ va $x + y = 0$ chiziqlar bilan chegaralangan soha yuzini toping.

Dastlab bu chiziqlar kesishish nuqtalarini topamiz:

$$2x - x^2 = -x \Rightarrow 3x - x^2 = 0 \quad x(3-x) = 0$$

$$x_1 = 0; \quad x_2 = 3$$

$$y = -x^2 + 2x = -[x^2 - 2x] = -[(x-1)^2 - 1] = -(x-1)^2.$$

$[0;3]$ oraliqda $2x - x^2 \geq -x$ ekanligidan

$$S = \int_0^3 [2x - x^2 + x] dx = \int_0^3 [3x - x^2] dx = \left(3 \cdot \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_0^3 = \frac{27}{2} - \frac{27}{3}$$

$$= 13,5 - 9 = 4,5 \text{ (kv.b)}$$

4. $r = a \cdot \sin 3\varphi$ (uch yaproqli gul) bilan chegaralangan soxa yuzini toping.

$\left[0; \frac{\pi}{6}\right]$ oraliqda uch yaproqli gulning $\frac{1}{6}$ qismi joylashadi. Demak,

$$\begin{aligned} S &= \frac{1}{2} \cdot 6 \int_0^{\frac{\pi}{6}} a^2 \cdot \sin^3 3\varphi d\varphi = 3a^2 \int_0^{\frac{\pi}{6}} \frac{1 - \cos 6\varphi}{2} d\varphi = \\ &= \frac{3a^2}{2} \left(\varphi - \frac{\sin 6\varphi}{6} \right) \Big|_0^{\frac{\pi}{6}} = \frac{\pi a}{4} = \frac{3a^2}{2} \text{ (kv.b)} \end{aligned}$$

5. $y = \ln \cos x \quad \left(0 \leq x \leq a \left(\frac{\pi}{2}\right)\right)$ chiziq yoyi uzunligini toping.

$$y' = \frac{1}{\cos x} \cdot (-\sin x) = -\operatorname{tg} x \text{ ekanligidan}$$

$$l = \int_0^a \sqrt{1 + tg^2 x} dx = \int_0^a \sqrt{\frac{1}{\cos^2 x}} dx = \int_0^a \frac{dx}{\cos x} = \ln \left| \operatorname{tg} \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| \Big|_0^a$$

$$= \ln \left| \operatorname{tg} \left(\frac{a}{2} + \frac{\pi}{4} \right) \right| + \ln \left| \operatorname{tg} \left(0 + \frac{\pi}{4} \right) \right| = \ln \left| \operatorname{tg} \left(\frac{a}{2} + \frac{\pi}{4} \right) \right|$$

6. $r = a(1 + \cos \varphi)$ (kardoida) yoyi uzunligini toping.
 $[0, \pi]$ da bu chiziq yoyi yarmi joylashganligidan

$$l = 2 \int_0^{\pi} \sqrt{a^2 (1 + \cos \varphi)^2 + a^2 \cdot \sin^2 \varphi} d\varphi =$$

$$= 2a \int_0^{\pi} \sqrt{1 + 2 \cos \varphi + \cos^2 \varphi + \sin^2 \varphi} d\varphi =$$

$$2\sqrt{2} \cdot a \int_0^{\pi} \sqrt{2 \cos^2 \frac{\varphi}{2}} d\varphi = 4a \int_0^{\pi} \cos \frac{\varphi}{2} d\varphi = 4a \cdot \frac{\sin \frac{\varphi}{2}}{\frac{1}{2}} \Big|_0^{\pi} = 8a$$

7. Kesik konus hajmi formulasini keltirib chiqaring.

Kesik konus asoslari radiuslari r, R , balandligi H bo'lsin. Uni $[0; H]$ kesmada $A(0; r)$ va $B(H; R)$ nuqtalardan o'tuvchi to'g'ri chiziq $x = 0; x = H; y = 0$ lar bilan chegaralangan trapetsiyani Ox o'qi atrofida aylantirib hosil qilish mumkin.

$$AB: \quad y = r + \frac{R-r}{H} \cdot x$$

$$V_x = \pi \int_0^H \left[r + \frac{R-r}{H} \cdot x \right]^2 dx = \frac{\pi H}{3(R-r)} \left[r + \frac{R-r}{H} \cdot x \right]^3 \Big|_0^H =$$

$$\begin{aligned} &\approx \frac{H}{3(R-r)}(R^3 - r^3) = \\ &\approx \frac{\pi H}{3}[R^2 + rR + r^2] = \frac{H}{3}[S + \sqrt{S \cdot s} + s]; \end{aligned}$$

8. $x^2 + (y-b)^2 = a^2$ ($b \geq a$) ning Ox o'qi atrofida hosil bo'lgan xalqa sirtini toping.

$y = b \pm \sqrt{a^2 - x^2}$ bo'lgani uchun $[-a, a]$ da

$$S_1 = 4\pi \int_b^a \left(b + \sqrt{a^2 - x^2} \right) \cdot \sqrt{1 + \frac{x^2}{a^2 - x^2}} dx = 4\pi a \left(\frac{b\pi}{2} + a \right),$$

$$S_2 = 4\pi a \left(\frac{b\pi}{2} - a \right) \text{ lardan}$$

$S = 4\pi^2 \cdot ab$ kelib chiqadi.

9. $\int_1^{+\infty} \frac{1}{x^p} dx$ yaqinlashishini tekshiring.

$$\begin{aligned} \int_1^{+\infty} \frac{1}{x^p} dx &= \lim_{A \rightarrow +\infty} \int_1^A \frac{1}{x^p} dx = \lim_{A \rightarrow +\infty} \frac{x^{-p+1}}{-p+1} \Big|_1^A = \\ &= \lim_{A \rightarrow +\infty} \left[\frac{A^{1-p}}{1-p} - \frac{1}{1-p} \right] = \begin{cases} \frac{1}{p-1}; & p < 1 \\ +\infty, & p > 1 \end{cases} \end{aligned}$$

$p = 1$ holda alohida tekshiramiz:

$$\int_1^{+\infty} \frac{dx}{x} = \lim_{a \rightarrow \infty} \ln|x| = \lim_{A \rightarrow \infty} \ln|x| \Big|_1^A = +\infty.$$

Demak, $r > 1$ ta yaqinlashuvchi; $r \leq 1$ da uzoqlashuvchi.

10. $\int_a^b \frac{dx}{(b-x)^p}$ yaqinlashishga tekshirilsin

$$\begin{aligned} a \int \frac{dx}{(b-x)^p} &= \lim_{v \rightarrow 0} \int_a^{b-v} \frac{dx}{(b-x)^p} = \lim_{v \rightarrow 0} \left[-\frac{(b-x)^{-p+1}}{-p+1} \right] \Big|_a^{b-v} = \\ &= \lim_{v \rightarrow 0} \left[\frac{v^{-p+1}}{1-p} - \frac{(b-a)^{-p+1}}{1-p} \right] = \begin{cases} \frac{(b-a)^{1-p}}{p-1} & \text{agar } p < 1 \\ \infty; & \text{agar } p > 1. \end{cases} \end{aligned}$$

$r=1$ da

$$\begin{aligned} \int_n^b \frac{dx}{b-x} &= -\lim_{v \rightarrow 0} \int_a^{b-v} \frac{d(b-x)}{b-x} = -\lim_{v \rightarrow 0} \ln |b-x| \Big|_a^{b-v} = \\ &= -\lim_{v \rightarrow 0} [\ln |v| - \ln |b-a|] = +\infty \end{aligned}$$

Demak, bu xosmas integral $p < 1$ da yaqinlashuvchi, $p \geq 1$ da uzoqlashuvchi ekan.

11. Xisoblang.

$$\begin{aligned} \int_1^{\infty} \frac{dx}{1+x^2} &= \lim_{A \rightarrow \infty} \int_1^A \frac{dx}{1+x^2} = \lim_{A \rightarrow \infty} \arctg x \Big|_1^A = \\ &= \lim_{A \rightarrow \infty} [\arctg A - \arctg 1] = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}. \end{aligned}$$

12. Son uklari va $x > 0$ da $y = e^{-2x}$ chizik bilan chegaralangan soha yuzini toping.

$$\begin{aligned} S &= \int_0^{+\infty} e^{-2x} dx = \lim_{A \rightarrow \infty} \int_0^A e^{-2x} dx = \lim_{A \rightarrow \infty} \left[-\frac{e^{-2x}}{2} \right]_0^A = 0 + \frac{1}{2} = \frac{1}{2} \\ &\quad \text{(kv.b)} \end{aligned}$$

8-bobga doir uy vazifalari.

Quyidagi chiziqlar bilan chegaralangan figura yuzini hisoblang:

1) $y = 3x^2 + 1$ va $y = 3x + 7$

2) $y = x$; $y = x + \sin^2 x$ ($0 \leq x \leq \pi$)

3) $r = 3(1 - \cos \varphi)$

4) $r = \frac{p}{1 + \varepsilon \cos \varphi}$ ($0 < \Sigma < 1$)

5) $y = x$; $y = x + 2$; $y = 3$; $y = 4$

Yoy uzunligini hisoblang.

6) $y = \sqrt{(x-2)^3}$ $A(2;0)$ dan $B(6;8)$ gacha

7) $x = \frac{1}{4}y^2 - \frac{1}{2}ly$ ($1 \leq y \leq e$)

8) $y^2 = \frac{x^3}{2a-x}$ ($0 \leq x \leq \frac{5}{3}a$)

9) $\sqrt[3]{x^2} + \sqrt[3]{y^2} = \sqrt[3]{a^2}$

10) $y = ach \frac{x}{a}$, $A(0;a)$ dan $B(b,h)$ gacha

Quyidagi chiziqlar bilan chegaralangan figura aylanishidan hosil bo'lgan jism hajmini toping

11) $xy = 4$, $x = 1$, $x = 4$, $y = 0$ Ox o'qi atrofida;

12) $y^2 = (x+4)^3$ va $x = 0$ OY o'qi atrofida;

13) $y = x^2$ va $y = \sqrt{x}$ Ox o'qi atrofida;

14) $y = \frac{2}{1+x^2}$ va $y = \sqrt{x^2}$ Oy o'qi atrofida;

**Xosmas integralni hisoblang yoki
uzoqlashishini hisoblang.**

$$1) \int_0^{+\infty} x e^{-x^2} dx$$

$$2) \int_{-\infty}^{-3} \frac{x dx}{(x^2 + 1)^2}$$

$$3) \int_{-1}^{+\infty} \frac{dx}{x^2 + x + 1}$$

$$4) \int_0^1 \frac{x^2 dx}{\sqrt{1-x^3}}$$

$$5) \int_1^2 \frac{dx}{(x-1)^2}$$

$$6) \int_{-3}^2 \frac{dx}{(x+3)^2}$$

$$7) \int_2^{+\infty} \frac{dx}{x \ln x}$$

$$8) \int_0^3 \frac{dx}{(x-2)^2}$$

$$9) \int_0^1 \frac{dx}{\sqrt[3]{(x-3)^2}}$$

$$10) \int_{-\infty}^{+\infty} \frac{dx}{x^2 + 4x + 1}$$

$$11) \int_1^{+\infty} \frac{\arctg x}{x^2} dx$$

$$12) \int_1^{+\infty} \frac{dx}{x^2 + 2x}$$

$$13) \int_2^{+\infty} \frac{dx}{x \sqrt{x^2 - 1}}$$

$$14) \int_1^{+\infty} \frac{e^{-x}}{x} dx$$

$$15) \int_0^{+\infty} e^{-x^2} dx$$

9-bob. Ko'p o'zgaruvchili funktsiyalar. Qatorlar.

§33. Ikki o'zgaruvchili funktsiyalar. Limit.

Uzluksizlik.

Ikki x, y o'zgaruvchilarning x, y juftligiga biror qonun yordamida Z ning yagona qiymati mos qo'yilgan bo'lsa, bu qonun ikki x, y o'zgaruvchili funktsiya deyiladi va $Z = f(x, y)$ ko'rinishida yoziladi.

$$\lim_{\substack{x \rightarrow x_0 \\ y \rightarrow y_0}} f(x, y) = A \quad \text{bo'lsa, } A \text{ soni } f(x, y)$$

funktsiyaning $P(x, y)$ nuqta $P_0(x_0, y_0)$ nuqtaga intilgandagi limiti deyiladi.

$f(x, y)$ funktsiya $P_0(x_0, y_0)$ nuqtada uzluksiz deyiladi, agar

$$\lim_{P \rightarrow P_0} f(P) = f(P_0) \quad \text{ tenglik o'rinli bulsa.}$$

33.1. Aniqlanish sohaslarini toping va tasvirlang.

$$1) z = x + \sqrt{y} \quad 2) z = \sqrt{1-x^2} + \sqrt{y^2-1}$$

$$3) z = \sqrt{\alpha^2 \cdot b^2 - bx^2 - ay^2}$$

$$4) z = \frac{1}{\sqrt{x^2 + y^2 - 1}} \quad 5) z = \sqrt{\frac{x^2 + y^2 - x}{2x - x^2 - y^2}}$$

$$6) z = \arccos \frac{x}{x+y}$$

$$7) z = \sqrt{\sin(x^2 + y^2)} \quad 8) z = \ln xy$$

$$9) z = \ln(-x - y)$$

33.2 Takroriy limitlarni hisoblang:

$$1) \lim_{x \rightarrow \infty} \left\{ \lim_{y \rightarrow \infty} \frac{x^2 + y^2}{x^2 + 4} \right\} \quad 2) \lim_{x \rightarrow \infty} \left\{ \lim_{y \rightarrow +0} \frac{x^y}{1 + x^y} \right\}$$

$$3) \lim_{x \rightarrow 0} \left\{ \lim_{y \rightarrow \infty} \frac{1}{xy} \cdot \operatorname{tg} \frac{xy}{1+xy} \right\} \quad 4) \lim_{x \rightarrow 1} \left\{ \lim_{y \rightarrow 0} \log_x (x+y) \right\}$$

33.3. Karrali limitlarni hisoblang.

$$1) \lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \frac{x+y}{x^2 - xy + y^2} \quad 2) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow a}} \frac{\sin xy}{x}$$

$$2) \quad 3) \lim_{\substack{x \rightarrow +\infty \\ y \rightarrow +\infty}} \left(\frac{xy}{x^2 + y^2} \right)^{x^2}$$

$$4) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^2 + y^2)^{x^2 \cdot y^2} \quad 5) \lim_{\substack{x \rightarrow 1 \\ y \rightarrow 0}} \frac{\ln(x + e^y)}{\sqrt{x^2 + y^2}}$$

33.4 Quyidagi funktsiyalar uzilish nuqtalarini toping.

$$1) z = \frac{1}{\sqrt{x^2 + y^2}} \quad 2) z = \frac{xy}{x+y}$$

$$3) z = \frac{1}{x \cdot y \cdot z}$$

$$4) z = \ln \frac{1}{\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}}$$

§34. Xususiy hosilalar. To'la diferentsial.

$z = f(x, y)$ funktsiyada y ni o'zgarmas son deb faraz qilib, x bo'yicha olingan hosila, (yoki aksincha) xususiy hosila deyiladi va mos

ravishda $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ yoki z'_x , z'_y tarzida belgilanadi.

Ulardan olingan xususiy xosilalar ikkinchi tartibli xususiy xosilalar deyiladi va

$$\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y \partial x} \quad \text{yoki} \quad z''_{xx}, z''_{yy},$$

$$z''_{xy}, z''_{yx}$$

ko'rinishda belgilanadi. SHunga o'xshash yuqori tartibli xususiy hosilalarni ham kiritiladi.

Agar funktsiya orttirmasini

$$\Delta z = \frac{\partial z}{\partial x} \Delta x + \frac{\partial z}{\partial y} \Delta y + o(\sqrt{\Delta x^2 + \Delta y^2})$$

ko'rinishda yozish mumkin bo'lsa, funktsiya diferentsiallanuvchi deyiladi.

Orttirmaning chiziqli bosh qismi funktsiya to'la diferentsiali deyiladi:

$$dz = \frac{\partial z}{\partial x} \Delta x + \frac{\partial z}{\partial y} \Delta y.$$

$z = x$ va $dx = \Delta x$, $z = y$ va $dy = \Delta y$ ekanligidan

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy.$$

34.1. Funktsiyalar xususiy hosilalarini toping.

1. $z = x^3 + 3x^2y - y^2$
2. $y = \ln(x^2 + y^2)$
3. $z = \frac{y}{x}$
4. $z = \operatorname{arctg} \frac{y}{x}$
5. $z = \frac{x \cdot y}{x - y}$
6. $z = x \cdot e^{-yx}$
7. $z = \operatorname{arctg} \frac{x + y}{1 - xy}$
8. $z = \arcsin \frac{x}{\sqrt{x^2 + y^2}}$
9. $z = x^y$

34.2. Quyidagi funktsiyalar uchun $z''_{xy} = z''_{yx}$ tenglikni tekshiring:

$$1. z = x^2 - 2xy - 3y^2 \quad 2. z = x^{y^2}$$

$$3. z = \arccos \sqrt{\frac{x}{y}}.$$

34.3 To'la diferentsialini toping;

$$1. z = x^m \cdot y^n \quad 2. z = e^{xy} \quad 3. z = \frac{x}{y}$$

$$4. z = \sqrt{x^2 + y^2} \quad 5. z = \ln \sqrt{x^2 + y^2}$$

$$6. z = x \cdot \ln y$$

34.4. Ko'rsatilgan xususiy hosilalarni toping.

$$1. z = x \ln(xy) \text{ bo'lsa, } z = \frac{\partial^2 z}{\partial x^2 \partial y}.$$

$$2. z = e^{xy} \text{ bo'lsa, } \frac{\partial^2 z}{\partial x \partial y}$$

$$3. z = \frac{x+y}{x-y} \text{ bo'lsa, } \frac{\partial^{m+n} z}{\partial x^m \partial y^n}.$$

$$4. z = xy \cdot e^{x+y} \text{ bo'lsa, } \frac{\partial^{p+q} z}{\partial x^p \partial y^q}.$$

§35. Ikki o'zgaruvchili funktsiya ekstremumlari.

$z = f(x, y)$ funktsiya $z'_x = 0$, $z'_y = 0$ yoki $df=0$ shartlar bajariladigan kritik nuqtalardagina ekstremumga erishishi mumkin.

$P(x_0, y_0)$ kritik nuqta uchun $A = f''_{xx}(x_0, y_0)$,
 $B = f''_{xy}(x_0, y_0)$, $C = f''_{yy}(x_0, y_0)$ belgilashlar kiritamiz;
 $P(x_0, y_0)$ nuqta

1. Minimum nuqta, agar $AC - B^2 > 0$, $A > 0$ ($C > 0$) bo'lsa,
2. Maksimum nuqta, agar $AC - B^2 > 0$, $A < 0$ ($C < 0$) bo'lsa,
3. Ekstremum mavjud emas, agar $AC - B^2 < 0$ bo'lsa.

Agar $AC - B^2 = 0$ bo'lsa, bu nuqtada ekstremum bo'lishi ham, bo'lmasligi ham mumkin.

$z = f(x, y)$ funktsiyaning $\varphi(x, y) = 0$ shart ostidagi ekstremumni topish uchun yordamchi

$$L(x, y, \lambda) = f(x, y) + \lambda \varphi(x, y)$$

Lagranj funktsiyasining ekstremumini topish kifoya.

35.1. Funktsiyalar ekstremumlarini toping.

- 1) $z = x^2 - xy + y^2 + 9x - 6y + 20$
- 2) $z = x^2 + (y-1)^2$
- 3) $z = x^3 + y^3 - 3xy$
- 4) $z = y\sqrt{x} - y^2 - x + 6y$
- 5) $z = e^{x^2-y} \cdot (5 - 2x + y)$

35.2. Funktsiyalar shartli ekstremumlarini toping.

1. $z = \frac{1}{x} + \frac{1}{y}$, $x + y = 2$
2. $z = x + y$, $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{2}$

$$3. z = x \cdot y, \quad x^2 + y^2 = 2$$

$$4. z = \frac{x}{a} + \frac{y}{b}, \quad x^2 + y^2 = 1$$

$$5. z = x^2 + 12xy + 2y^2, \quad 4x^2 + y^2 = 25$$

35.3. $z = x^2 - 9xy + 10$ funksiyaning

$D = \{x \geq 0, y \geq 0, x + y \leq 2\}$ sohadagi eng katta va eng kichik qiymatini toping.

§36 Ikki karrali integral yordamida yuzani hisoblash.

1⁰. Agar (S) soha

$a \leq x \leq b, y_1(x) \leq y \leq y_2(x)$ tengsizlik bilan aniqlangan bo'lsa, shu soha yuzasi

$$S = \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \sum \sum \Delta x \Delta y = \iint_{(S)} dx dy = \int_a^b dx \int_{y_1(x)}^{y_2(x)} dy \quad \text{formula}$$

yordamida hisoblanadi.

2⁰. Agar (S) soha

$h \leq y \leq l, x_1(y) \leq x \leq x_2(y)$ tengsizlik bilan aniqlangan bo'lsa, shu soha yuzasi

$$S = \iint_{(S)} dx dy = \int_h^l dy \int_{x_1(y)}^{x_2(y)} dx$$

3⁰. Agar (S) soha qutb koordinatasida

$\varphi_1 \leq \varphi \leq \varphi_2, r_1(\varphi) \leq r \leq r_2(\varphi)$ tengsizlik bilan aniqlangan bo'lsa, shu soha yuzasi

$$S = \iint_{(S)} r dr d\varphi = \int_{\varphi_1}^{\varphi_2} d\varphi \int_{r_1(\varphi)}^{r_2(\varphi)} r dr.$$

Agar S soha $x = x(u; v)$ va $y = y(u; v)$ almashtirish yordamida sodda sohaga o'tkazilishi mumkin bo'lsa, u holda yuza $\iint J |du dv|$ yordamida aniqlanadi, bu erda

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$

36.1 Ikki karrali integral ko'rinishida yozing va chiziqlar bilan chegaralangan yuzani hisoblang.

1. $xy = 4$, $y = x$, $x = 4$
2. $y = x^2$, $4y = x^2$, $y = 4$
3. $y = x^2$, $4y = x^2$, $x = \pm 2$
4. $xy = \frac{a^2}{2}$, $xy = 2a^2$, $y = \frac{x}{2}$, $y = 2x$

Egri chiziqli integrallar. Grin formulasi. Yuzalarni hisoblash.

10. Egri chiziqli integralning aniqlanishi. Silliqliq AB yoyda $P(x, y, z)$ uzluksiz funktsiya aniqlangan bo'lsin. AB yoyini $A(x_0; y_0; z_0)$ $M_1(x_1; y_1; z_1)$... $M_{n-1}(x_{n-1}; y_{n-1}; z_{n-1})$ $B(x_n; y_n; z_n)$ nuqtalar yordamida qismlarga bo'lamiz, bunda $x_i - x_{i-1} = \Delta x_i$.

$$\lim_{\Delta x_i \rightarrow 0} \sum_{i=1}^n P(x_i, y_i, z_i) \Delta x_i \text{ integral yig'indi limiti}$$

AB yoy bo'yicha olingan egri chiziqli integral deb ataladi va $\int_{AB} P(x, y, z) dx$ tarzida yoziladi.

$$\int_{AB} Q(x, y, z) dy + \int_{AB} R(x, y, z) dz,$$

$\int_{AB} (Pdx + Qdy + Rdz)$ egri chiziqli integrallar ham yuqoridagiga o'xshash aniqlanadi.

Quyidagi ko'rinishdagi egri chiziqli integrallar ham uchrab turadi:

$$\int_{AB} P(x, y, z) ds = \lim_{\Delta s_i \rightarrow 0} \sum_{i=1}^n P(x_i, y_i, z_i) \Delta s_i, \quad \text{bu erda}$$

$$\Delta s_i = M_{i-1} M_i.$$

2^o. Agar AB yoy $x = f(t)$, $y = \varphi(t)$, $z = \psi(t)$ tenglamalar bilan aniqlansa, t parametr esa $M(t)$ nuqta AB yoy bo'yicha bir yo'nalish bo'yicha harakatlanganda monoton o'zgarsa, u holda

$$\int_{AB} P(x, y, z) dx = \int_{t_1}^{t_2} P[f(t), \varphi(t), \psi(t)] f'(t) dt.$$

Yopiq L kontur bo'yicha olingan ikkinchi tur egri chiziqli integralni va shu kontur bilan chegaralangan D soha bo'yicha olingan ikki karrali integralni bog'lovchi formula Grin formulasi deb

$$\text{ataladi: } \oint_L Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

Bu formuladagi $P(x, y)$, $Q(x, y)$ funktsiyalar va ularning birinchi tartibli hususiy hosilalari D sohada va L konturda uzluksiz bo'lishi kerak. Egri chiziqli integralda L kontur bo'yicha integrallash musbat yo'nalishda olinadi. Ikkinchi tur egri chiziqli integral orqali oddiy bo'lakli — silliq kontur bilan chegaralangan S yuzani hisoblash mumkin:

$$S = \oint_L x dy = - \oint_L y dx = \frac{1}{2} \oint_L x dy - y dx$$

Misollar:

1. Grin formulasi bo'yicha
 $J = \oint_L (2(x^2 + y^2)dx + (x + y)^2 dy)$ integralni

hisoblansin, bu erda $L - ABC$ uchburchak konturi:
 $A(1;1), B(2;2), C(1;3)$.

Ecish: AB, BC, CA to'g'ri chiziqlar tenglamasini

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$ tenglama yordamida topamiz: AB
da $y = x$.

BC da $y = -x + 4$, AC da $x = 1$. ABC
uchburchak konturi bilan chegaralangan D soha
 $x = 1$, $x = 2$, $y = x$, $y = 4 - x$ to'g'ri chiziqlar
orasiadir. Bu ma'lumotlar ikki karrali integralni
hisoblash uchun kerak. Endi

$P = 2(x^2 + y^2)$, $Q = (x + y)^2$ larni topib, Grin
formulasiga qo'yamiz va

$$\begin{aligned} J &= \iint_D (2x + 2y - 4y) dx dy = 2 \int_1^2 dx \int_x^{4-x} (x - y) dy = \\ &= - \int_1^2 (x - y) \Big|_x^{4-x} dx = - \int_1^2 (2x - 4)^2 dx = -\frac{4}{3} \end{aligned}$$

2. $\oint_L (e^y + 2x \cos y) dx + (e^y - x^2 \sin y) dy$ integralni L

kontur bilan chegaralangan D soha bo'yicha olingan
ikki karrali integralga keltiring.

Echish: Misolni echish uchun Grin formulasidan
foydalamiz. Berilgan $P(x, y) = e^y + 2x \cos y$,

$$Q(x, y) = e^{xy} - x^2 \sin y \text{ uchun } \frac{\partial Q}{\partial x} = ye^{xy} - 2x \sin y.$$

$$\frac{\partial P}{\partial y} = xe^{xy} - 2x \sin y.$$

$$\oint_L Pdx + Qdy = \iint_D [(ye^{xy} - 2x \sin y) - (x \cdot e^{xy} - 2x \sin y)] dxdy = \iint_D (y - x) e^{xy} dxdy \text{ natijani}$$

hosil qilamiz.

$$36.2. A(2;2), B(2;0) \text{ nuqtalar berilgan. } \int_{(C)} (x+y) dx$$

integral 1) OA to'g'ri chiziq bo'yicha; 2) $y = \frac{x^2}{2}$

parabolaning OA yoyi bo'yicha; 3) OBA sinik chiziq bo'yicha hisoblansin.

$$36.3. 1). \oint_L (1-x^2) dx + x(1+y^2) dy \text{ integral } L \text{ kontur}$$

$x^2 + y^2 = R^2$ aylana bo'yicha a) Grin formulasi orqali va b) bevosita hisoblansin.

$$2). \oint_L (xy + x + y) dx + (xy + x - y) dy \text{ integral a) Grin}$$

bo'yichava b) bevosita hisoblansin.

$$1\text{-hol. } L \text{ kontur } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ellips;}$$

$$2\text{-hol. } L \text{ kontur } x^2 + y^2 = ax \text{ aylana.}$$

§37 Sonli qatorlar va ularning yaqinlashish alomatlari.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots + a_n + \dots$$
 qator
yaqinlashuvchi deyiladi,
agar $S_n = a_1 + a_2 + \dots + a_n$ qisman yigindilarining

limiti $\lim_{n \rightarrow \infty} S_n = S$ mavjud va chekli son bo'lsa, aks holda kator uzoqlashuvchi deyiladi.

Qator yaqinlashuvchi bulishi uchun $\lim_{n \rightarrow \infty} a_n = 0$ bo'lishi zarur.

Yaqinlashuvchilikning etarli shartlarini quyidagi alomatlar bera oladi.

I. 1^o. $\sum_{n=1}^{\infty} a_n$ (1) va $\sum_{n=1}^{\infty} b_n$ (2) qatorlar uchun biror $n \geq n_0$ nomerdan boshlab $0 \leq a_n \leq b_n$ shart bajarilsa, (2) - qator yaqinlashishidan (1) - qator yaqinlashishi, (1) - qator uzoqlashishidan (2) - qatorning uzoqlashishi kelib chiqadi.

2^o. Agar $a_n = O\left(\frac{1}{n^p}\right)$, bo'lsa, $\sum_{n=1}^{\infty} a_n$ qator $p > 1$ da yaqinlashuvchi, $p \leq 1$ da uzoqlashuvchi bo'ladi.

Musbat hadli $\sum a_n$ qator berilgan bo'lsin.

II. 1^o. Dalamber alomati.

$$D = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = q$$
 bo'lib, $q < 1$ bo'lsa,

yaqinlashuvchi, $q > 1$ bo'lsa uzoqlashuvchidir.

2^o. Koshi alomati.

$K = \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = q$ bo'lib, $q < 1$ bo'lsa, qator

yaqinlashuvchi, $q > 1$ bo'lsa uzoqlashuvchidir.

30. Koshining integral alomati. Agar $f(x)$ funksiya $x \geq 1$ da manfiymas o'smaydigan uzluksiz funksiya bo'lsa,

$$\sum_{n=1}^{\infty} f(n) \quad \text{va} \quad \int_1^{+\infty} f(x) dx$$

bir paytda yaqinlashadilar yoki uzoqlashadilar.

III. Ishorasi almashinuvchi qator $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$

yaqinlashuvchi bo'ladi,

agar $a_1 > a_2 > \dots$ va $\lim_{n \rightarrow \infty} a_n = 0$ bajarilsa.

Agar $\sum_{n=1}^{\infty} |b_n|$ qator yaqinlashsa, $\sum_{n=1}^{\infty} b_n$ qator

absolyut yaqinlashuvchi deyiladi.

Agar $\sum |b_n|$ uzoqlashsa va $\sum b_n$

yaqinlashsa, u holda $\sum b_n$ shartli yaqinlashuvchi deyiladi.

37.1. Yaqinlashuvchiligini isbotlang va yig'indilarni toping.

$$1) \left(\frac{1}{2} + \frac{1}{3} \right) + \left(\frac{1}{2^2} + \frac{1}{3^2} \right) + \dots + \left(\frac{1}{2^n} + \frac{1}{3^n} \right) + \dots$$

$$2) \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} + \dots$$

$$3) \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{n(n+1)} + \dots$$

37.2 Alomatlar yordamida yaqinlashishini tekshiring.

$$\begin{array}{lll}
 1) \sum_{n=1}^{\infty} \frac{(n!)^2}{2n} & 2) \sum_{n=1}^{\infty} \frac{2^n \cdot n}{n^n} & 3) \sum_{n=1}^{\infty} \frac{1}{1+n^2} \\
 4) \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} & 5) \sum_{n=3}^{\infty} \frac{1}{n^2-1} & 6) \sum_{n=1}^{\infty} \frac{n}{1+n^2} \\
 7) \sum_{n=2}^{\infty} \frac{1}{\ln n} & 8) \sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}} &
 \end{array}$$

36.3 Absolyut va shartli yaqinlashishni tekshiring.

$$\begin{array}{ll}
 1) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p} & 2) \sum_{n=1}^{\infty} \frac{(-1)^n}{x+n} \\
 3) \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} & 4) \sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}
 \end{array}$$

§38 Funktsional va darajali qatorlar.

1⁰. X to'plamda aniqlangan $f_1(x), f_2(x), \dots, f_n(x), \dots$ funktsiyalar ketma-ketligi uchun

$$a) \lim_{n \rightarrow \infty} f_n(x) = f(x), \quad b)$$

$$\lim_{n \rightarrow \infty} \sup_{x \in X} |f_n(x) - f(x)| = 0$$

shartlar bajarilsa, bu ketma-ketlik X to'plamda $f(x)$ funktsiyaga tekis yaqinlashadi deyiladi va $f_n \Rightarrow f$ tarzda yoziladi.

$$2^0. \sum_{n=1}^{\infty} u_n(x) = u_1(x) + u_2(x) + \dots + u_n(x) + \dots$$

funktsional qator yaqinlashadigan X nuqtalar to'plami bu qatorning yaqinlashish sohasi deyiladi.

$S(x) = \lim_{n \rightarrow \infty} S_n(x)$ funktsiya funktsional qator yigindisi,

$R_n(x) = S(x) - S_n(x)$ esa qator qoldig'i deyiladi.

$\sum_{n=1}^{\infty} u_n(x)$ qator $[a, b]$ kesmada tekis

yaqinlashuvchi deyiladi, agar ixtiyoriy $\varepsilon > 0$ uchun shunday N nomer topilsaki, $n > N$ va ixtiyoriy $x \in [a, b]$ larda $|R_n(x)| < \varepsilon$ tengsizlik o'rinli bo'lsa,

30. **Veyershtass alomati:** Agar shunday $\sum_{n=1}^{\infty} c_n$

yaqinlashuvi sonli qator mavjud bo'lib, $|U_n(x)| \leq C_n$,

$x \in [a, b]$, $n \in \mathbb{N}$ shart bajarilsa, $\sum_{n=1}^{\infty} u_n(x)$

funktionalqator $[a, b]$ kesmada absolyut va tekis yaqinlashuvchi bo'ladi.

40.

$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$ darajali

qator uchun

$R = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right|$ soni yaqinlashish radiusi deyiladi:

Qator $|x| < R$ da yaqinlashuvchi, $|x| > R$ da esa uzoqlashuvchi bo'ladi.

$(-R, R)$ yaqinlashish intervali ichida qator absolyut va tekis yaqinlashuvchi buladi.

50. Berilgan sohada tekis yaqinlashuvchi funktional, darajali qatorlarni hadma-had differentsiallash va integrallash mumkin.

38.1. Tekis yaqinlashishini tekshiring.

a) $f_n(x) = x^n$, $0 \leq x \leq 1$

b) $f_n(x) = x^n - x^{2n}$, $0 \leq x \leq 1$

$$v) f_n(x) = \frac{1}{x+n}, \quad 0 \leq x < +\infty$$

$$g) f_n(x) = n\left(\sqrt{x + \frac{1}{n}} - \sqrt{x}\right), \quad 0 < x < +\infty$$

38.2. Tekis yaqinlashishini tekshiring.

$$a) \sum_{n=1}^{\infty} \frac{(-1)^n}{x+2^n}, \quad -2 < x < +\infty$$

$$b) \sum_{n=1}^{\infty} \frac{x}{1+n^4 x^2}, \quad 0 \leq x < +\infty$$

$$v) \sum_{n=1}^{\infty} \frac{nx}{1+n^5 x^2}, \quad |x| < +\infty$$

$$g) \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}, \quad |x| < +\infty.$$

38.3. Yaqinlashish intervalini toping.

$$a) \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} x^n \quad b) \sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n} (x+1)^n$$

$$v) \sum_{n=1}^{\infty} \frac{x^{n^2}}{2^n} \quad g) \sum_{n=1}^{\infty} \left(\frac{x}{\sin n}\right)^n$$

38.4. Qator yig'indisini toping.

$$a) x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$$

$$b) \frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \dots$$

$$v) 1 + \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \dots$$

$$g) x + 2x^2 + 3x^3 + \dots$$

$$d) 1 \cdot 2x - 2 \cdot 3x^2 + 3 \cdot 4x^3 - \dots$$

Bob bo'yicha misollar echish namunalari

1. $Z = \sqrt{(x^2 + y^2 - 1)(4 - x^2 - y^2)}$
funktsiyaning aniqlanish sohasini toping.

$(x^2 + y^2 - 1)(4 - x^2 - y^2) \geq 0$ shartidan
quyidagilarga egamiz:

$$\begin{cases} x^2 + y^2 - 1 \geq 0 \\ 4 - x^2 - y^2 \geq 0 \end{cases} \quad \text{yoki}$$

$$\begin{cases} x^2 + y^2 - 1 \leq 0 \\ 4 - x^2 - y^2 \leq 0 \end{cases}$$

Bulardan $\begin{cases} x^2 + y^2 \geq 1 \\ x^2 + y^2 \leq 4 \end{cases}$ yoki

$$\begin{cases} x^2 + y^2 \leq 1 \\ x^2 + y^2 \geq 4 \end{cases} \quad \text{kelib chiqadi.}$$

Ikkinchi sistema echimga ega bo'la olmaydi.
Demak, berilgan funksiya $1 \leq x^2 + y^2 \leq 4$ xalqada aniqlangan ekan.

$$2. \lim_{\substack{x \rightarrow \infty \\ y \rightarrow a}} \left(1 + \frac{1}{x}\right)^{\frac{x^2}{x+y}} =$$

$$\lim_{\substack{y \rightarrow a \\ x \rightarrow \infty}} \left(1 + \frac{1}{x}\right)^{x \cdot \frac{1}{1 + \frac{y}{x}}} = e^{\lim_{\substack{x \rightarrow \infty \\ y \rightarrow a}} \frac{1}{1 + \frac{y}{x}}} = e^{\lim_{\substack{x \rightarrow \infty \\ y \rightarrow a}} \frac{1}{1 + \frac{y}{x}}} = e^1 = e.$$

3. $z = \ln(1 - x^2 - y^2)$ uzilish nuqtasini toping.

Funktsiya $1 - x^2 - y^2 > 0$ da uzluksiz, $x^2 + y^2 = 1$ da esa, ya'ni birlik aylana har bir nuqtasida uzilishga ega.

4. Birinchi va ikkinchi tartibli xususiy hosilalarni toping.

$$z = y^4 + x^4 - 4x^2y^2;$$

$$z'_x = 4x^3 - 8x \cdot y^2; \quad z'_y = 4y^3 - 8x^2y;$$

$$z''_{xx} = 12x^2 - 8y^2; \quad z''_{yy} = 12y^2 - 8x^2.$$

$$z''_{xy} = -16xy = z''_{yx};$$

5. $z = (x^2 + y^2) \cdot e^{x+y}$ uchun $\frac{\partial^{m+n}z}{\partial x^m \partial y^n}$ ni toping.

$$z'_x = e^{x+y} [x^2 + y^2 + 2x]$$

$$z''_{xx} = e^{x+y} [x^2 + y^2 + 2x + 2x + 2] = e^{x+y} [x^2 + y^2 + 4x + 2]$$

$$z'''_{xy} = e^{x+y} [x^2 + y^2 + 4x + 2 + 2x + 4] = e^{x+y} [x^2 + y^2 + 6x + 6]$$

$$\frac{\partial^{(4)}z}{\partial x^4} = e^{x+y} [x^2 + y^2 + 8x + 12]$$

Yuqoridagilardan :

$$\frac{\partial^m z}{\partial x^m} = [m(m-1) + 2mx + x^2 + y^2] e^{x+y}$$

$$\frac{\partial^{m+1}z}{\partial x^m \partial y} = [2y + m(m-1) + 2mx + x^2 + y^2] e^{x+y}$$

$$\frac{\partial^{m+n}z}{\partial x^m \partial y^n} = [n(n-1) + m(m-1) + 2(mx + ny) + x^2 + y^2] e^{x+y}$$

6. $z = \ln \sqrt{x^2 + y^2}$ ning to'liq differentsialini toping.

$$z'_x = \frac{1}{\sqrt{x^2 + y^2}} \cdot \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{x^2 + y^2} \text{ va}$$

$$z'_y = \frac{y}{x^2 + y^2}$$

ekanligidan $dz = \frac{x}{x^2 + y^2} dx + \frac{y}{x^2 + y^2} dy$.

7. $z = e^{\frac{x}{2}}(x + y^2)$ ekstremumini toping.

$$z'_x = e^{\frac{x}{2}}\left(\frac{x}{2} + \frac{y^2}{2} + 1\right) = 0; \quad z'_y = 2ye^{\frac{x}{2}} = 0 \quad \text{dan } x = -2;$$

$$y = 0$$

kelib chiqadi. $P(-2, 0)$ kritik nuqta.

$$z''_{xx} = \frac{1}{2} e^{\frac{x}{2}} \left[\frac{x}{2} + \frac{y^2}{2} + 2 \right]; \quad z''_{xy} = ye^{\frac{x}{2}};$$

$$z''_{yy} = e^{\frac{x}{2}}(2 + y) \quad \text{lardan} \quad A = \frac{1}{2e}; \quad B = 0; \quad C = \frac{2}{e}$$

ekanligi kelib chiqadi.

$$AB - C^2 = \frac{1}{e^2} > 0; \quad A = \frac{1}{2e} > 0 \quad \text{bo'lganligi uchun}$$

funktsiya $P(-2, 0)$ nuqtada minimum qiymatiga erishadi:

$$z_{\min}(-2, 0) = -\frac{2}{e};$$

8. $z = x^2 + y^2$ parabolaning $\frac{x}{a} + \frac{y}{b} = 1$ shartdagi ekstremumlarini toping.

Lagranj funktsiyasi

$$L(x, y, \lambda) = x^2 + y^2 + \lambda\left(\frac{x}{a} + \frac{y}{b} - 1\right) \quad \text{ko'rinishida}$$

bo'ladi.

$$L'_x = 2x + \frac{\lambda}{a} = 0, \quad L'_y = 2y + \frac{\lambda}{b} = 0 \quad \text{va}$$

$$\frac{x}{a} + \frac{y}{b} - 1 = 0$$

$$\text{shartlaridan } \lambda = -\frac{2a^2 \cdot b^2}{a^2 + b^2}; \quad x = \frac{ab^2}{a^2 + b^2};$$

$$y = \frac{ba^2}{a^2 + b^2} \quad \text{kelib chiqadi.}$$

$$P\left(\frac{ab^2}{a^2 + b^2}; \frac{a^2b}{a^2 + b^2}\right) \text{ kritik nuqta ekan.}$$

$$L''_{xx} = 2; \quad L''_{xy} = 0; \quad L''_{yy} = 2 \quad \text{lardan} \quad A = 2, \quad B = 0,$$

$$C = 2 \quad \text{ekanligi,} \quad AC - B^2 = 4 > 0, \quad A = 2 > 0$$

lardan esa

$$z_{\min} = \left(\frac{ab^2}{a^2 + b^2}\right)^2 + \left(\frac{a^2b}{a^2 + b^2}\right)^2 = \frac{a^2b^4 + a^4b^2}{(a^2 + b^2)^2} = \frac{a^2b^2}{a^2 + b^2}$$

9. $z = x^2 - xy + y^2 + 1$ funktsiyaning

$$D = \{|x| + |y| \leq 1\}$$
 sohadagi eng katta va eng kichik qiymatlarini toping.

$$z'_x = 2x - y = 0, \quad z'_y = -x + 2y \quad \text{dan kritik nuqta}$$

$$P(0;0) \text{ ekanligi kelib chiqadi.}$$

$$A = z''_{xx} = 2, \quad B = z''_{xy} = -1, \quad C = z''_{yy} = 2 \quad \text{va}$$

$$D = AC - B^2 = 2 \cdot 2 - (-1)^2 = 3 \neq 0 \text{ ekanligidan}$$

$$z_{\min}(0;0) = 1 \text{ kelib chiqadi.}$$

Bundan tashqari, D soha chegaralari, masalan, $x + y = 1$ da $z = 3x^2 - 3x + 2$ ko'rinish oladi va $z \leq 2$ bo'ladi.

$z(1;0) = z(0;1) = z(-1;0) = z(0;-1) = 2$ ekanligidan

esa $z_{e.kat}(1;0) = 2$, $z_{e.kich}(0;0) = 1$ kelib chiqadi.

10. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \frac{(-1)^{n-1}}{2^{n-1}} \dots$ qator

yaqinlashishini isbotlang va yig'indisining toping.

$$\begin{aligned} S &= (1 + \frac{1}{4} + \frac{1}{16} + \dots) - (\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \dots) = \\ &= \frac{1}{1 - \frac{1}{4}} - \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3} \end{aligned}$$

Berilgan qator yig'indisi chekli son bo'lganligi uchun, ta'rifga

ko'ra yaqinlashuvchidir.

11. 1) $\sum_{n=1}^{\infty} \frac{1000^n}{n!}$ yaqinlashishga tekshirilsin:

Dalamber alomatiga ko'ra;

$$D = \lim_{n \rightarrow \infty} \frac{\frac{1000^{n+1}}{(n+1)!}}{\frac{1000^n}{n!}} = \lim_{n \rightarrow \infty} \frac{1000}{n+1} = 0 < 1$$

Qator yaqinlashuvchi.

2) $\sum_{n=2}^{\infty} \left(\frac{n-1}{n+1}\right)^{n(n-1)}$ yaqinlashishga tekshirilsin.

Koshi alomatiga ko'ra

$$\begin{aligned} K &= \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n-1}{n+1}\right)^{n(n-1)}} = \lim_{n \rightarrow \infty} \left(1 - \frac{2}{n+1}\right)^{\frac{n+1}{2} \cdot \left(-\frac{2}{n+1}\right)^{n(n-1)}} = \\ &= e^{-2 \lim_{n \rightarrow \infty} \frac{n-1}{n+1}} = e^{-2} = \frac{1}{e^2} < 1 \end{aligned}$$

Qator yaqinlashuvchi.

3) $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln^p n}$ yaqinlashishga tekshirilsin.

Koshining integral alomatiga ko'ra $p \neq 1$ da:

$$\begin{aligned} \int_2^{+\infty} \frac{dx}{x \ln^p x} &= \lim_{A \rightarrow +\infty} \int_2^A \frac{d(\ln x)}{\ln^p x} = \lim_{A \rightarrow +\infty} \frac{\ln x}{-p+1} \Big|_2^A = \\ &= \lim_{A \rightarrow +\infty} \left[\frac{\ln^{1-p} A}{1-p} - \frac{\ln^{1-p} 2}{1-p} \right] \\ &= \begin{cases} p > 1 & \text{da yaqinlashuvchi} \\ p < 1 & \text{da uzoqlashuvchi} \end{cases} \\ &\quad p=1 \end{aligned}$$

$$\text{da } \int_2^{+\infty} \frac{dx}{x \ln x} = \lim_{A \rightarrow +\infty} \int_2^A d(\ln(\ln x)) = \lim_{A \rightarrow +\infty} \ln(\ln x) \Big|_2^A = +\infty$$

Demak berilgan qator ham $p \leq 1$ da uzoqlashuvchi,
 $p \geq 1$ da yaqinlashuvchidir.

12. $\sum_{n=1}^{\infty} \frac{n}{n+1} \left(\frac{x}{2x+1} \right)^n$ absolyut yaqinlashishga tekshirilsin.

Koshi alomatiga ko'ra:

$$K = \lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{n}{n+1} \cdot \left(\frac{x}{2x+1} \right)^n \right|} = \left| \frac{x}{2x+1} \right|$$

$\left| \frac{x}{2x+1} \right| < 1$ da, ya'ni $x < -1$ va $x > -\frac{1}{3}$ bo'lganda

berilgan qator absolyut yaqinlashuvchi.

13. Tekis yaqinlashishga tekshiring.

$$1. f_n(x) = \sqrt{x^2 + \frac{1}{n^2}}; \quad -\infty < x < +\infty$$

$$a) \lim_{n \rightarrow \infty} f_n(x) = \lim_{n \rightarrow \infty} \sqrt{x^2 + \frac{1}{n^2}} = \sqrt{x^2} = |x|$$

b)

$$\lim_{n \rightarrow \infty} \sup_{x \in \mathbb{R}} \left| \sqrt{x^2 + \frac{1}{n^2}} - |x| \right| = \lim_{n \rightarrow \infty} \frac{1}{n^2} \sup_{x \in \mathbb{R}} \frac{1}{\sqrt{x^2 + \frac{1}{n^2}} + |x|} = \lim_{n \rightarrow \infty} \frac{n}{n^2} = 0$$

$$f_n(x) \Rightarrow |x|$$

2). $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot x^n}{n}$ qator $[0,1]$ kesmada tekis yaqinlashishini ko'rsating. n ning qanday qiymatlarida ixtiyoriy x uchun $|R_n(x)| < 0,1$ bo'ladi?

Ishora almashinuvchi qatorlarda har bir hadlari o'zidan keyingi hadlar yig'indisidan katta bo'ladi, ya'ni

$$|R_n(x)| < \frac{x^{n+1}}{n+1} < \frac{1}{n+1} \leq 0,1$$

Demak, $n+1 \geq 10$ yoki $n \geq 9$ dan boshlab qoldik 0,1 dan kichik bo'ladi. Bu natija qator tekis yaqinlashishini ta'minlaydi.

$$3). \sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}, \quad -\infty < x < +\infty \quad \text{qatorni tekis}$$

yaqinlashishga tekshiring.

$$\frac{1}{x^2 + n^2} \leq \frac{1}{n^2} \quad \text{ekanligidan Veyershtrass}$$

alomatiga ko'ra

$\sum \frac{1}{n^2 + x^2}$ tekis yaqinlashuvchi, chunki $\sum \frac{1}{n^2}$ — yaqinlashuvchidir.

14. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$ yaqinlashish intervalini ko'rsating.

$$R = \lim_{n \rightarrow \infty} \left| \frac{(n!)^2}{(2n)!} \cdot \frac{[2(n+1)]!}{[(n+1)!]^2} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2n+1)(2n+2)}{(n+1)^2} \right| = 4,$$

ya'ni yaqinlashish intervali $(-4, 4)$ dir. $x = \pm 4$ da qator uzoqlashuvchi, masalan, $x = -4$ bo'lsa, uzoqlashuvchi qator.

$$\sum \frac{(-1)^n (n!)^2 \cdot 2^{2n}}{2^n \cdot n!} = \sum (-1)^n n! 2^n$$

kelib chiqadi.

15. $\sum_{n=1}^{\infty} nx^n$ qator yig'indisini toping.

$$R = \lim_{n \rightarrow \infty} \frac{n}{n+1} = 1 \text{ ekanligidan } (-1, 1) \text{ intervalda}$$

qator yig'indisi chekliligi kelib chikadi.

$$S(x) = x \cdot \sum_{n=1}^{\infty} n \cdot x^{n-1}$$

$$S_1(x) = \sum_{n=1}^{\infty} nx^{n-1} \text{ qatorni hadma-had}$$

integrallaymiz:

$$\int S_1(x) dx = \int \sum_{n=1}^{\infty} nx^{n-1} dx = \sum_{n=1}^{\infty} \int nx^{n-1} dx = \sum_{n=1}^{\infty} x^n = \frac{x}{1-x};$$

$$\text{Demak, } S_1(x) = \left[\frac{x}{1-x} \right]' = \frac{1}{(1-x)^2} \text{ va}$$

$$S(x) = \frac{x}{(1-x)^2}$$

9-bobga doir uy vazifalari

I. $z = f(x, y)$ funktsiyaning yopiq D sohadagi eng katta va eng kichik qiymatlarini toping.

1) $z = x^2 + y^2 - 9xy + 27$; $D = \{0 \leq x \leq 3, 0 \leq y \leq 3\}$.

2) $z = 3 - 2x^2 - xy - y^2$; $D = \{x \leq 1; y \geq 0; y \leq x\}$.

3) $z = x^2 + 2y^2 + 1$; $D = \{x \geq 0, y \geq 0, x + y \leq 3\}$.

4)

$z = x^2 + 3y^2 + x - y$; $D = \{x \geq 1, y \geq -1; x + y \leq 1\}$.

5) $z = x^2 + 2xy + 2y^2$; $D = \{|x| \leq 1; 0 \leq y \leq 2\}$.

6) $z = 5x^2 - 3xy + y^2 + 4$; $D = \{x \geq -1, y \geq -1, x + y \leq 1\}$.

7) $z = 10 + 2y - x^2$; $D = \{0 \leq y \leq 4 - x^2\}$.

8) $z = x^2 + 2xy - y^2 + 4x$; $D = \{x \leq 0; y \leq 0, x + y + 2 \geq 0\}$.

9) $z = x^2 + xy - 2$; $D = \{4x^2 - 4 \leq y \leq 0\}$.

10) $z = x^2 + xy$; $D = \{|x| \leq 1, 0 \leq y \leq 3\}$.

11) $z = x^2 + y^2 - 12x + 16y$; $D = \{x^2 + y^2 \leq 25\}$.

12) $z = x^2 - xy + y^2$; $D = \{|x| + |y| \leq 1\}$.

13) $z = x^2 - 4xy + 4y^2$; $D = \{x \geq 0; y \geq 0; x + y \leq 2\}$.

14) $z = x^2 + 4xy - 4y^2$; $D = \{x \geq -1; y \geq -1; x + y \leq 1\}$.

15) $z = x^2 + 4xy$; $D = \{x \leq 2, y \leq 2; y \geq x\}$.

II. $\sum_{n=1}^{\infty} a_n$ sonli qatorni yaqinlashishini tekshiring:

1) $a_n = \frac{n+3}{n^3-2}$

2) $a_n = \frac{e^{-\sqrt{n}}}{\sqrt{n}}$

3) $a_n = \frac{1}{(2n+1)^2-1}$

4) $a_n = \frac{3^n}{(2n)!}$

$$5) a_n = \frac{n^3}{e^n}$$

$$6) a_n = \frac{1}{(n+1)[\ln(n+1)]^2}$$

$$7) a_n = \frac{2n+1}{\sqrt{n \cdot 2^n}}$$

$$8) a_n = \frac{n^2}{(3n)!}$$

$$9) a_n = \frac{1}{(n+1)\ln(n+1)}$$

$$10) a_n = \frac{n^{n+1}}{(n+1)!}$$

$$11) a_n = \frac{(n!)}{3^{n^2}}$$

$$12) a_n = \frac{1}{(3n+1)^2 - 2}$$

$$13) a_n = \frac{n-1}{n^4 + 1}$$

$$14) a_n = \frac{n^2}{e^n}$$

$$15) a_n = \frac{1}{n \ln n \ln(\ln n)}$$

III. $\sum_{n=1}^{\infty} a_n \cdot x^n$ darajali qator yaqinlashish intervalini toping.

$$1) a_n = \frac{\sqrt[n]{(n+1)^n}}{n!}$$

$$2) a_n = \frac{2^n}{n(n+1)}$$

$$3) a_n = \frac{(2n)!}{n^n}$$

$$4) a_n = \frac{3^n \cdot n!}{(n+1)^n}$$

$$5) a_n = \frac{n}{3^n \cdot (n+1)}$$

$$6) a_n = \frac{5^n}{\sqrt[n]{n}}$$

$$7) a_n = \left(1 + \frac{1}{n}\right)^n$$

$$8) a_n = \frac{n+1}{3^n \cdot (n+2)}$$

$$9) a_n = \frac{3^n}{\sqrt{2^n(3n-1)}}$$

$$10) a_n = \frac{n+2}{n(n+1)}$$

$$11) a_n = \left(1 + \frac{1}{n}\right)^{n^2}$$

$$12) a_n = \frac{3^n + (-1)^n}{n}$$

$$13) a_n = \frac{(2n)!!}{(2n+1)!!}$$

$$14) a_n = \frac{(-1)^n}{n!} \left(\frac{n}{e}\right)^n$$

$$15) a_n = \frac{n!}{3^{n^2}}$$

IV. Hadma-had differentsiallash, integrallash yordamida qator yig'indisini toping.

$$1) x - \frac{x^7}{7} + \frac{x^{13}}{13} - \frac{x^{19}}{19} + \dots$$

$$2) x^2 + 2x^3 + 3x^4 + \dots$$

$$3) x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$4) x^2 + \frac{x^3}{2} + \frac{x^4}{3} + \dots$$

$$5) x - 2x^2 + 3x^3 - \dots$$

$$6) \frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \dots$$

$$7) \frac{x^2}{1 \cdot 2} - \frac{x^3}{2 \cdot 3} + \frac{x^4}{3 \cdot 4} - \dots$$

$$8) x - 4x^2 + 9x^3 - 16x^4 + \dots$$

$$9) 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$10) 1 \cdot 2x - 2 \cdot 3 \cdot x^2 + 3 \cdot 4 \cdot x^3 - \dots$$

$$11) 1 - \frac{x^4}{4} + \frac{x^8}{8} + \dots$$

$$12) \frac{x^2}{1 \cdot 2} - \frac{x^3}{2 \cdot 3} + \frac{x^4}{3 \cdot 4} - \dots$$

$$13) x + 4x^2 + 9x^3 + 16x^4 + \dots$$

$$14) x + \frac{x^3}{3} + \frac{x^5}{5} + \dots$$

$$15) x + 3x^3 + 5x^5 + \dots$$

III-qism. Differentsial tenglamalar.

Noma'lum funksiya hosila yoki differentsial belgisi ostida qatnashgan tenglamalar differentsial tenglamalar deyiladi. Hosilaning eng yuqori tartibi differentsial tenglama tartibi deyiladi. n -tartibli differentsial tenglama

$$F(x, y, y', y'', \dots, y^{(n)}) = 0$$

tenglama bilan berilishi mumkin.

Bu tenglamani ayniyatga aylantiruvchi $y = \varphi(x)$ funksiya differentsial tenglama echimi deyiladi. Tarkibida n ta o'zgarmas qatnashuvchi $\Phi(x, y, c_1, c_2, \dots, c_n) = 0$ funktsiyalar oilasi differentsial tenglamani qanoatlantirsa, umumiy echim deyiladi. O'zgarmaslarning ma'lum bir qiymatida xususiy echimlar yuzaga keladi. Ma'lum shartlarda echimni topish Koshi masalasi deyiladi.

10-bob. Birinchi tartibli differentsial tenglamalar.

§39. Birinchi tartibli sodda differentsial tenglamalar.

Birinchi tartibli differentsial tenglamalar $F(x, y, \frac{dy}{dx}) = 0$ ko'rinishga ega. Bu tenglamani ko'p

hollarda $\frac{dy}{dx}$ ga nisbatan uchib $\frac{dy}{dx} = f(x, y)$ ko'rinishga keltiriladi.

$\frac{dy}{dx} = f(x)$ ko'rinishdagi tenglamani $dy = f(x)dx$ ko'rinishda yozib, tomonlarni integrallasak $y = \int f(x)dx + c$ umumiy echim kelib chiqadi.

Shunga o'xshash $\frac{dy}{dx} = g(y)$ tenglama umumiy echimi $dx = \frac{dy}{g(y)}$ dan $x(y) = \int \frac{dy}{g(y)} + c$ yoki $\int \frac{dy}{g(y)} = x + c$ ko'rinishda bo'ladi.

39.1. Quyidagi umumiy echimlarga mos differentsial tenglamalarni tuzing.

- | | |
|---------------------|----------------------|
| 1) $y = e^{Cx}$, | 2) $y = (x - e)^3$ |
| 3) $y = Cx^3$ | 4) $y = \sin(x + C)$ |
| 5) $y^2 + Cx = x^3$ | 6) $y = C(x - C)^2$ |
| 7) $Cy = \sin Cx$ | |

39.2. Quyidagi differentsial tenglamalar echilsin.

- | | | |
|----------------|------------------|-------------------|
| 1) $y' = 3x^2$ | 2) $y' = \cos x$ | 3) $y' = 3e^{3x}$ |
| 4) $y' = y$ | 5) $y' = \sin y$ | 6) $y' = e^y$ |

39.3 Koshi masalasi echimini toping.

$$y' = \sin x, \quad y(0) = 1$$

§40. O'zgaruvchilari ajraladigan differentsial tenglamalar.

$$y' = f(x) \cdot g(y) \quad \text{yoki}$$

$$M(x) \cdot N(y)dx + P(x) \cdot Q(y)dy = 0 \quad \text{ko'rinishda}$$

yoziladigan differentsial tenglamalar o'zgaruvchilari ajraladigan differentsial tenglamalar deyiladi. Bunday tenglamalarni echish uchun ikkala tomonni shunday ifodalarga bo'lish (ko'paytirish) kerakki, natijada tenglamaning bir tomonida faqat y ga, ikkinchi tomonida faqat x ga bog'liq ifodalar hosil bo'lsin.

$$\frac{dy}{g(y)} = f(x)dx \quad \text{yoki} \quad \frac{Q(y)}{N(y)} dy = -\frac{M(x)}{P(x)} dx$$

So'ngra ikkala tomonni integrallab umumiy echim hosil qilinadi.

Ikkala tomon x, y qatnashgan ifodalarga bo'linganda, bu ifodalarni nolga aylantiradigan xususiy echimlar yo'qolishi mumkin.

$y' = f(ax + by + c)$ ko'rinishdagi differentsial tenglamalar

$z = ax + by + c$ yangi o'zgaruvchi kiritish yordamida o'zgaruvchilari ajraladigan differentsial tenglamalarga keltiriladi.

40.1. Quyidagi differentsial tenglamalarni eching.

$$1. xy' - y = 0 \qquad 2. xy' + y = 0$$

$$3. yy' + x = 0 \qquad 4. y' = y$$

$$5. x^2 y' + y = 0 \qquad 6.$$

$$x + xy + y'(y + xy) = 0$$

$$7. \sqrt{y^2 + 1} dx = xy dy \qquad 8. 2x^2 yy' + y^2 = 0$$

$$9. y' - xy^2 = 2xy \qquad 10. y' = e^{x+y}$$

40.2. Berilgan boshlang'ich shartni qanoatlantiruvchi xususiy echimlarni toping.

$$1. 2y' \sqrt{x} = y; \quad y(4) = 1.$$

$$2. y' = (2y + 1) \operatorname{ctg} x; \qquad y\left(\frac{\pi}{4}\right) = \frac{1}{2}.$$

$$3. x^2 y' + y^2 = 0; \quad y(-1) = 1.$$

$$4. y' = 2\sqrt{y} \ln x; \qquad y(e) = 1.$$

$$5. (x^2 - 1)y' + 2xy^2 = 0; \quad y(0) = 1.$$

$$6. xy' + y = y^2, \quad y(1) = \frac{1}{2}.$$

40.3. Yangi o'zgaruvchi kiritib o'zgaruvchilari ajraladigan differentsial tenglamaga keltiring va eching.

$$1. y' = \sqrt{4x + 2y - 1}.$$

$$2. y' = \cos(y - 1);$$

$$3. (x + 2y)y' = 1; \quad y(0) = -1.$$

$$4. (2x - y + 1)y' = 1;$$

§41. Bir jinsli differentsial tenglamalar.

$$M(x, y)dx + N(x, y)dy = 0 \quad \text{tenglamada}$$

$M(\lambda x, \lambda y)$, $N(\lambda x, \lambda y)$ almashtirishlarda tenglama ko'rinishi o'zgarmasa, bunday tenglama bir jinsli deyiladi. Bunday tenglamalar

$$\frac{dy}{dx} = \varphi\left(\frac{y}{x}\right) \text{ ko'rinishga keladi va } \frac{y}{x} = u \text{ yoki}$$

$y = ux$ yangi o'zgaruvchi kiritish yordamida o'zgaruvchilari ajraladigan differentsial tenglamaga keltiriladi.

$$y' = f\left(\frac{a_1x + b_1y + c_1}{ax + by + c}\right) \text{ ko'rinishdagi differentsial}$$

tenglamalar koorinatalar boshini $a_1x + b_1y + c_1 = 0$ va $ax + by + c = 0$ to'g'ri chiziqlar kesishish nuqtasiga parallel ko'chirish yordamida bir jinsliga keltiriladi. Agar bu to'g'ri chiziqlar kesishmasa, $a_1x + b_1y = k(ax + by)$ bajarilib, $z = ax + by$ almashtirish yordamida o'zgaruvchilari ajraladigan differentsial tenglamaga keladi.

Ba'zi tenglamalarda $y = z^m$ almashtirish yordamida bir jinsliga keltirib olinadi. Buning uchun m soni differentsial tenglama bir jinsli bo'ladigan qilib tanlab olinadi. Bunday m soni mavjud bo'lmasa, bu usul bilan tenglamani bir jinsliga keltirib bo'lmaydi.

41.1. Bir jinsli ekanligini tekshiring va eching.

1. $yy' = 2y - x$

2. $x^2 + y^2 - 2xyy' = 0$

3. $\frac{dy}{dx} = \frac{y}{x} - \frac{x}{y}$

4. $xy' + 2\sqrt{xy} = y$

5. $(x - y)dx + (x + y)dy = 0$

6.

$(y^2 - 2xy)dx + x^2dy = 0$

7. $xy' = y - x \cdot e^{\frac{y}{x}}$

8. $xy' = y \cos \ln \frac{y}{x}$

41.2. Parallel ko'chirish yordamida bir jinsliga keltiring va eching.

$$1. (2x + y + 1)dx - (4x + 2y - 3)dy = 0$$

$$2. (x + 4y)y' = 2x + 3y - 5$$

$$3. (y + 2)dx = (2x + y - 4)dy$$

$$4. y' = 2 \left(\frac{x+2}{x+y-1} \right)^2$$

$$5. (y'+1) \ln \frac{y+x}{x+3} = \frac{y+x}{x+3}$$

41.3. Yangi o'zgaruvchi kiritib bir jinsliga keltiring va eching.

$$1. 2x^2 y' = y^3 + xy \quad 2.$$

$$2x dy + (x^2 y^4 + 1) y dx = 0$$

$$3. y dx + x(2xy + 1) dy = 0 \quad 4. 2y' + x = 4\sqrt{y}$$

$$5. y' = y^2 - \frac{2}{x^2} \quad 6.$$

$$2y + (x^2 \cdot y + 1)xy' = 0$$

§42. Birinchi tartibli chiziqli tenglamalar.

Noma'lum funktsiya va uning hosilalari birinchi darajada qatnashgan differentsial tenglamalar chiziqli deyiladi. Birinchi tartibli chiziqli tenglama $y' + P(x)y = Q(x)$ ko'rinishda bo'ladi.

Bunday tenglamani $y = u \cdot v$ almashtirish yordamida ikkita o'zgaruvchilari ajraladigan differentsial tenglamaga keltirish mumkin. O'zgarmas sonni variatsiyalash deb ataluvchi ikkinchi usulda bunday tenglamani echish uchun dastlab $y' + P(x)y = 0$ tenglama umumiy echimi olinadi, undagi o'zgarmas S soni $S(x)$ funktsiya bilan o'zgartiriladi, berilgan tenglamaga qo'yiladi va $S(x)$ funktsiya topiladi.

Bu xususiy echim va bir jinsli tenglama umumiy echimi yig'indisi berilgan tenglama echimi hisoblanadi.

$y' + P(x)y = Q(x) \cdot y^n \quad (n \neq 1)$ ko'rinishdagi tenglama Bernulli tenglamasi deyiladi. Bu tenglamaning ikkala tomoni y^n ga bo'linib

$$\frac{1}{y^{n-1}} = z \quad \text{almashtirish o'tkazilsa, chiziqli}$$

tenglamaga ega bo'lamiz.

$y' + P(x)y + Q(x) \cdot y^2 = R(x)$ ko'rinishdagi tenglama Rikkati tenglamasi deyiladi. Bunday tenglamaning biror xususiy $y_0(x)$ echimi ma'lum bo'lsagina, $y = y_0(x) + z$ almashtirish yordamida Bernulli tenglamasiga keltirish mumkin.

42.1. Chiziqli tenglamalarni eching.

$$1. y' - \frac{3y}{x} = x$$

$$2. (2x+1)y' = 4x+2y$$

$$3. y' + y \operatorname{tg} x = \frac{1}{\cos x}$$

$$4. (xy + e^x)dx - xdy = 0$$

$$5. 2x(x^2 + y)dx = dy$$

$$6. x^2 \cdot y' + (x+1)y = 3x^2 \cdot e^{-x}$$

42.2. Izlanayotgan funktsiya va bog'liqsiz o'zgaruvchi «rollarini» almashtiring, hosil bo'lgan tenglamani eching.

$$1. y = (2x + y^3) \cdot y' \quad 2.$$

$$(x + y^2)dy = ydx$$

$$3. (2 \cdot e^y - x)y' = 1 \quad 4.$$

$$(\sin^2 y + x \operatorname{ctg} y) \cdot y' = 1$$

42.3. Bernulli tenglamalarini eching:

$$1. x^2 y' - xy = y^2$$

$$2. y' - xy = -y^3 \cdot e^{-x^2}$$

$$3. y'x + y = -xy^2$$

$$4. xydy = (y^2 + x)dx$$

$$5. xy' + 2y + x^5 y^3 \cdot e^x = 0$$

$$6. y'x^3 \sin y = xy' - 2y.$$

42.4. Xususiy echimi berilgan Rikkati tenglamalarini eching.

$$1. x^2 y' + xy + x^2 y^2 = 4 \quad y_0 = \frac{2}{x}$$

$$2. 3y' + y^2 + \frac{2}{x^3} = 0 \quad y_0 = \frac{1}{x}$$

$$3. xy' - (2x+1)y + y^2 = -x^2, \quad y_0 = x$$

$$4. y' + 2ye^x - y^2 = e^{2x} + e^x, \quad y_0 = e^x$$

§43. To'la differentsial tenglamalar

$P(x, y)dx + Q(x, y)dy = 0$ tenglama chap tomoni biror $F(x, y)$ funktsiyaning to'la differentsiali bo'lsa, bu tenglama to'la differentsial tenglama deyiladi.

Masalan, $xdy + ydx = 0$,

$\frac{xdy + ydx}{x^2} = 0$ tenglamalar chap tomoni mos

ravishda $F(x, y) = x \cdot y$, $F(x, y) = \frac{y}{x}$ funktsiyalar to'la differentsiali bo'lib, umumiy echimlari $x \cdot y = C$;

$\frac{y}{x} = C$ ko'rinishda bo'ladi.

$P(x, y)dx + Q(x, y)dy = 0$ tenglama chap tomoni to'la differentsial bo'lishi uchun $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ shart bajarilishi zarur. Bu shart bajarilsa, $dF = F'_x dx + F'_y dy = Pdx + Qdy = 0$ dan $F'_x = P$, $F'_y = Q$ kelib chiqadi.

$F = \int P(x, y)dx + \varphi(y)$ desak, (o'zgarmas son o'rniga $\varphi(y)$ olamiz.)

$$F'_y = \left(\int P(x, y)dx \right)'_y + \varphi'_y(y) = Q(x, y) \quad \text{dan} \quad \varphi(y),$$

ya'ni $F(x, y)$ topiladi.

Agar $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$ bo'lsa, ba'zi hollarda shunday

$\mu Pdx + \mu Qdy = 0$ tenglama to'la differentsial tenglama bo'ladi. Bu ko'paytuvchi integrallovchi ko'paytuvchi deyilib, quyidagi hollarda oson topiladi:

$$1) \frac{P'_y - Q'_x}{Q} = \phi(x) \text{ bo'lsa, } \ln \mu = \int \phi(x) dx.$$

$$2) \frac{Q'_x - P'_y}{P} = \phi_1(y) \text{ bo'lsa, } \ln \mu = \int \phi_1(y) dy.$$

Dastlabki paragraflardan differentsial tenglamalarning har biri to'la yoki to'la differentsial tenglamaga biror integrallovchi ko'paytuvchi yordamida keltiriluvchi tenglamalardir. Masalan, $y' + a(x)y = b(x)$ chiziqli tenglama uchun integrallovchi ko'paytuvchi

$$\mu(x) = e^{\int a(x) dx} \text{ ko'rinishda bo'ladi.}$$

43.1. To'la differentsialga keltirib eching.

$$1. x^2 dy + xy dy = dx$$

$$2. y^2 x dy - y^3 dx = x^2 dy$$

$$3. y dx + (x - y^3) dy = 0$$

$$4. y dx - (x - y^3) dy = 0$$

$$5. x^2 y^2 + 1 + x^3 y y' = 0$$

$$6. x dy - y dx = x^2 dx$$

$$7. xy' + y = \frac{2x}{\cos y}$$

$$8. y(y \cdot e^{-\frac{x}{2}} + 1) = x \cdot y'$$

43.2. To'la differentsial tenglama ekanligini tekshiring va eching.

$$1. (4 - \frac{y^2}{x^2}) dx + \frac{2y}{x} dy = 0$$

$$2. 3x^2 e^y dx + (x^3 \cdot e^y - 1) dy = 0$$

$$3. e^{-y} dx + (1 - x e^{-y}) dy = 0$$

$$4. 2x \cos^2 y dx + (2y - x^2 \sin 2y) dy = 0$$

$$5. (3x^2 y - 4xy^2) dx + (x^3 - 4x^2 y + 12y^3) dy = 0$$

$$6. (x \cos 2y + 1) dx - x^2 \sin 2y dy = 0$$

$$7. 3x^2(1 + \ln y)dx = (2y - \frac{x^3}{y})dy$$

$$8. 2x(1 + \sqrt{x^2 - y})dx - \sqrt{x^2 - y}dy = 0.$$

43.3. Integrallovchi ko'paytuvchini toping va
eching.

$$1. (x^2 - y)dx + xdy = 0 \quad 2. \quad "$$

$$2xtdydx + (x^2 - 2\sin y)dy = 0$$

$$3. (e^{2x} - y^2)dx + ydy = 0 \quad 4.$$

$$(\sin x + e^y)dx + \cos xdy = 0$$

$$5. (1 + 3x^2 \cdot \sin y)dx - xctgydy = 0 \quad 6.$$

$$x(\ln y + 2nx - 1)dy = 2ydx$$

$$7. (x^2 - y)dx + x(y + 1)dy = 0 \quad 8.$$

$$y^2(ydx - 2xdy) = x^3(xdy - 2ydx)$$

§44. Hosilaga nisbatan echilmagan 1- tartibli differentsial tenglamalar. Lagranj va Klero tenglamalari.

Hosilaga nisbatan echilmagan $F(x, y, y') = 0$
tenglama asosan $y' = \frac{dy}{dx} = p$ parametr kiritish usuli
bilan echiladi. Tenglamani $y = f(x, p)$ ko'rinishda
yozib, ikkala tomondan to'liq differentsial olamiz.

$dy = pdx$ ekanligidan
 $M(x, p)dx + N(x, p)dp = 0$ ko'rinishdagi tenglamani
hosil qilamiz. Bu tenglama echimi $x = \varphi(p)$ bo'lsa,
berilgan tenglama echimi $y = f(\varphi(p), p)$ bo'ladi.

Differentsial tenglama $x = f(y, y')$ ko'rinishga
kelsa ham, shu usulda umumiy echimdan tashqari
maxsus echimlarni $F(x, y, p) = 0$, $F'_p(x, y, p) = 0$
tenglamalarda p ni yo'qotib topish mumkin.

$y = xf(y') + \varphi(y')$ tenglama Lagranj tenglamasi deyilib, $y' = p$ almashtirishdan quyidagi

$$p = f(p) + [xf'(p) + \varphi'(p)] \frac{dp}{dx}$$

x ga nisbatan chiziqli tenglamani hosil qilamiz.

$y = px + \varphi(p)$ tenglama Klero nomi bilan yuritilib, Lagranj tenglamasi xususiy holidir. Bunday tenglamalar maxsus echimga ham egadir.

44.1. Tenglamalar barcha echimlarini toping.

- 1) $y'^2 - y^2 = 0$
- 2) $8y'^3 = 27y$
- 3) $(y'+1)^3 = 27(x+y)^2$
- 4) $y^3(y'^2+1) = 1$
- 5) $y'^2 - 4y^3 = 0$
- 6) $xy'^2 = y$

44.2. y' ga nisbatan echib, so'ngra umumiy echimlarini toping.

- 1) $xy'(xy'+y) = 2y^2$
- 2) $xy'^2 - 2yy'+x = 0$
- 3) $xy'^2 = y(2y'-1)$
- 4) $y'^2 + x = 2y$
- 5) $y'^2 - 2xy' = 8x^2$
- 6) $(xy'+3y)^2 = 7x$

44.3. Yangi parametr kiritib eching.

- 1) $x = y'^3 + y'$
- 2) $x(y'^2 - 1) = 2y'$
- 3) $y'^4 - y'^2 = y^2$
- 4) $y'^2 - y'^3 = y^2$
- 5) $5y + y'^2 = x(x + y')$
- 6) $y = 2xy' + y^1 y'^3$

44.4. Lagranj va Klero tenglamalarini eching.

- 1) $y = xy'^2 + y'^2$
- 2) $y = 2xy' + \frac{1}{y'^2}$
- 3) $2y = \frac{xy'^2}{y'+2}$
- 4) $y = xy' - y'^2$
- 5) $y = xy' - a\sqrt{1+y'^2}$
- 6) $y = xy' + \frac{1}{2y'^2}$

Bobga doir misollar echish namunalari

1. Echimi $x^2 + cy^2 = 2y$ bo'lgan differentsial tenglamani tuzing.

Ikkala tomondan hosila olamiz:

$$2x + 2c \cdot y \cdot y' = 2y'$$

Bundan, $c = \frac{y'-x}{yy'}$. Berilgan tenglamaga qo'yib

$$x^2 + \frac{y'-x}{yy'} \cdot y^2 = 2y \text{ ni hosil qilamiz.}$$

Soddalashtirib $x^2 y' - xy = yy'$ tenglamani hosil qilamiz.

2. $y = Cx^3$ funktsiya $3y - xy' = 0$ differentsial tenglama echimi ekanligini tekshiring va $R(1;1)$ nuqtadan o'tuvchi xususiy echimini toping.

$y' = 3Cx^2$ ni differentsial tenglamaga qo'ysak,

$3Cx^3 - x \cdot 3Cx^2 = 0$ ayniyat hosil bo'ladi. Demak, $y = Cx^3$ umumiy echim, $x = y = 1$ ekanligidan $C = 1$, ya'ni $y = x^3$ funktsiya $R(1;1)$ nuqtadan o'tuvchi xususiy echimdir.

3. $\frac{dy}{dx} = \frac{1}{1+x^2}$, $x \in R$ tenglama umumiy echimi,

$y(1) = \pi$ shartga bo'ysinuvchi xususiy echimini toping.

$dy = \frac{dx}{1+x^2}$ dan $y = \arctg x + c$ umumiy echim.

$x = 1$ da $y = \pi$ ekanligidan $\pi = \arctg 1 + c$, ya'ni

$$c = \frac{3\pi}{4}.$$

Koshi masalasi echimi $y = \arctg x + \frac{3\pi}{4}$ dir.

4. $xydx + (x+1)dy = 0$ tenglamani eching.

$(x+1)dy = -xydx$ ko'rinishda yozib olib, ikkala tomonni $y \cdot (x+1)$ ga bo'lamiz. Bunda tenglamani

qanoatlantiruvchi $y=0$, $x=-1$ echimlar borligini yodda tutamiz.

Tenglama $\frac{dy}{y} = -\frac{x}{x+1} dx$ ko'rinishga keladi.

Ikkala tomonni integrallaymiz:

$$\int \frac{dy}{y} = - \int \frac{x}{x+1} dx$$

$\ln|y| = -x + \ln|x+1| + \ln C$ ya'ni $y = C \cdot (x+1) \cdot e^{-x}$ umumiy echimdir.

5. $y' \cdot \operatorname{ctgx} + y = 2$ tenglamaning $y(\frac{\pi}{3}) = 0$ shartni qanoatlantiruvchi echimini toping.

$\frac{dy}{dx} \cdot \operatorname{ctgx} = 2 - y$ ko'rinishda yozib, tomonlarni

$\frac{dy}{2-y} = \operatorname{tg} x dx$ ko'rinishga keltiramiz. Ikkala tomonni

integrallab $-\ln|2-y| = -\ln|\cos x| - \ln C$ yoki $-2+y = C \cdot \cos x$

Demak, $y = 2 + C \cdot \cos x$ umumiy echimdir.

Endi boshlang'ich shartni qanoatlantiruvchi echimni topamiz. $y(\frac{\pi}{3}) = 0$ dan $0 = 2 + C \cdot \cos \frac{\pi}{3}$, ya'ni

$$0 = 2 + C \cdot \frac{1}{2} \text{ dan } C = -4.$$

Izlanayotgan echim $y = 2 - 4 \cos x$ bo'ladi.

6. $y' = y + 2x - 3$ tenglamani o'zgaruvchilari ajraladigan differentsial tenglamaga keltiring va eching.

$z = y + 2x - 3$ ko'rinishda yangi o'zgaruvchi kiritamiz.

$$y = z - 2x + 3 \text{ dan } y' = z' - 2$$

$z' - 2 = z$ ko'rinishdagi tenglamaga ega bo'lamiz.

$$\frac{dz}{z+2} = dx \text{ dan}$$

$$\ln|z+2| = x + \ln C \quad \text{yoki } z+2 = C \cdot e^x$$

Eski o'zgaruvchilarga qaytib $y = C \cdot e^x - 2x + 1$ ekanligini topamiz.

7. $(x + 2y)dx - xdy = 0$ tenglamani eching.

$\lambda \neq 0$ uchun $(\lambda x + 2\lambda y)dx - \lambda xdy = 0$ tenglama berilgan tenglamaning aynan o'zi, demak, tenglama bir jinsli $y = u \cdot x$ almashtirish o'tkazamiz.

$y' = u'x + u$, $dy = xdu + udx$
 ekanligidan $(x + 2 \cdot ux)dx - x \cdot (xdu + udx) = 0$
 $x \cdot [(1 + 2u)dx - (xdu + udx)] = 0$ da $x = 0$
 xususiy echim bo'ladi. Qavs ichini ixchamlab

$$(1 + u)dx = xdu$$

$$\int \frac{du}{1+u} = \int \frac{dx}{x}$$

$$\ln|1+u| = \ln x + \ln C$$

$$1+u = C \cdot x$$

$$\frac{y}{x} = Cx - 1 \quad \text{dan} \quad y = x \cdot (Cx - 1) \quad \text{umumiy}$$

echimdir.

8. $(2x - 4y + 6)dx + (x + y - 3)dy = 0$ tenglamani bir jinsliga keltiring va eching.

$2x - 4y + 6 = 0$ va $x + y - 3 = 0$ to'g'ri chiziqlar kesishish nuqtasi $R(1;2)$ dir. Demak, $X = x - 1$,
 $Y = y - 2$ almashtirishlar o'tkazamiz.

$$[2(X+1) - 4(Y+2) + 6]dx + [(X+1) + (Y+2) - 3]dy = 0$$

$$(2X - 4Y)dX + (X + Y)dY = 0$$

hosil bo'lgan tenglama bir jinslidir.

$$(2x - 4 \cdot u \cdot X)dX + (X + u \cdot X)[u dX + X du] = 0$$

$X = 0$, ya'ni $x - 1 = 0$ xususiy echim bo'lishi mumkin.

$$(2 - 4u)dX + (1 + u)(u dX + x du) = 0$$

$$\frac{1+u}{(u-1)(u-2)} = -\frac{dX}{X}$$

$$-\int \frac{2}{u-1} du + \int \frac{3}{u-2} du = -\int \frac{dX}{X}$$

$$-2\ln|u-1| + 3\ln|u-2| = -\ln X + \ln C$$

$$3\ln|u-2| + \ln X = 2\ln|u-1| + \ln C$$

$$(u-2)^3 \cdot X = C(u-1)^2$$

$$u = \frac{Y}{X} = \frac{y-2}{x-1} \text{ ekanligini hisobga olsak,}$$

$$\left(\frac{y-2}{x-1} - 2\right)^3 \cdot (x-1) = C\left(\frac{y-2}{x-1} - 1\right)^2$$

$$(y+2x)^3 = C(y-x-1)^2 \text{ umumiy echimdir.}$$

9. $x^3(y'-x) = y^2$ tenglamani bir jinsliga keltiring.

$$y = z^m, \quad y' = mz^{m-1} \cdot z' \text{ almashtirish o'tkazamiz.}$$

$$x^3(mz^{m-1} \cdot z' - x) = z^{2m}$$

Bu tenglama bir jinsli bo'lishi uchun

$$3 + m - 1 = 4 = 2m \quad \text{tengliklar bajarilishi, } m = 2$$

bo'lishi zarur.

Unda tenglama $x^3(2 \cdot z \cdot z' - x) = z^4$ ko'rinishdagi bir jinsli differentsial tenglamaga aylanadi.

10. $y' - \frac{2}{x} \cdot y = 2x^3$ chiziqli differentsial tenglamani eching.

$$y' - \frac{2}{x} \cdot y = 0 \quad \text{tenglamani umumiy echimi}$$

$$y = Cx^2.$$

$y = C(x) \cdot x^2$ deb olamiz va berilgan tenglamaga qo'yamiz:

$$C'(x) \cdot x^2 + 2x \cdot C(x) - \frac{2}{x} \cdot C(x)x^2 = 2x^3$$

$$\text{Soddalashtirib } C'(x) = 2x, \quad \text{ya'ni } C(x) = x^2$$

ekanligini topamiz. Xususiy echim $y = x^4$ ekan.

Berilgan tenglamaning umumiy echimi $y = Cx^2 + x^4$ ko'rinishda bo'ladi.

11. $y' - \frac{1}{x}y = \frac{x}{y^2}$ Bernulli tenglamasini eching.

$$n = -2 \quad \text{ekanligidan} \quad z = \frac{1}{y^{-2-1}} = y^3$$

$$\text{almashtirish o'tkazamiz. } y = z^{\frac{1}{3}}; \quad y' = \frac{1}{3} z^{-\frac{2}{3}} \cdot z'$$

$$\text{ekanligidan} \quad \frac{1}{3} z^{-\frac{2}{3}} \cdot z' - \frac{1}{x} \cdot z^{\frac{1}{3}} = \frac{x}{z^{\frac{2}{3}}};$$

Tomonlarni $3z^{\frac{2}{3}}$ ga ko'paytirsak:

$$z' - \frac{3}{x}z = 3x \quad \text{chiziqli tenglama hosil bo'ladi.}$$

$$z' - \frac{3}{x}z = 0 \quad \text{ning umumiy echimi} \quad z = C \cdot x^3.$$

$$z = C(x) \cdot x^3 \quad \text{deb tenglamaga qo'yamiz: } C' = \frac{3}{x^2}$$

dan $C(x) = -\frac{3}{x}$. Xususiy echim $z = -3x^2$ ko'rinishda,

umumiy echim esa $z = C \cdot x^3 - 3x^2$ dir. Eski o'zgaruvchiga qaytib $y^3 = Cx^3 - 3x^2$

Bernulli tenglamasi echimi ekanligini topamiz.

12. $y' - 2xy + y^2 = 5 - x^2$ Rikkati tenglamasining xususiy echimi $y_1 = x + 2$ ma'lum bo'lsa, umumiy echimini toping.

$$y = x + 2 + z, \quad y' = 1 + z' \quad \text{almashtirish bajaramiz:}$$

$$1 + z' - 2x(x + 2 + z) + (x + 2 + z)^2 = 5 - x^2.$$

Soddalashtirib

$$z' + 4z = -z^2 \quad \text{Bernulli tenglamasiga ega bo'lamiz.}$$

$$\frac{1}{z^{2+1}} = t; \quad z = \frac{1}{t}; \quad z' = -\frac{1}{t^2} \cdot t' \text{ almashtirish}$$

yordamida $t'-4t=1$ chiziqli tenglamaga kelamiz.

$$t'-4t=0 \text{ tenglama echimi } t=C \cdot e^{4x},$$

$$t'-4t=1 \text{ tenglama echimi esa } t=\frac{4Ce^{4x}-1}{4}$$

ekanligini topish mumkin.

Mos Bernulli tenglamasini echimi

$$z = \frac{4}{4Ce^{4x}-1} \text{ ko'rinishda, Rikkati tenglamasi umumiy}$$

$$\text{echimi esa } y = x + 2 + \frac{4}{4Ce^{4x}-1} \text{ ko'rinishda bo'ladi.}$$

13. $y \cos x dx + \sin x dy = \cos 2x dy$ tenglamani to'la differentsialga keltirib eching.

Tenglama chap tomonini $d(y \sin x)$,

O'ng tomonini $d\left(\frac{\sin 2x}{2}\right)$ deyish mumkin.

$$\text{Bundan } d\left(y \sin x - \frac{\sin 2x}{2}\right) = 0.$$

Umumiy echim $y \sin x - \sin x \cos x = C$ yoki

$$y = \cos x + \frac{C}{\sin x} \text{ ko'rinishda bo'ladi.}$$

14. $\frac{y}{x} dx + (y^3 + \ln x) dy = 0$ to'la differentsial tenglama ekanligini tekshiring va eching.

$$P = \frac{y}{x}; \quad Q = y^3 + \ln x \text{ uchun } P'_y = Q'_x = \frac{1}{x}.$$

Demak, berilgan tenglama to'la differentsial tenglamadir.

$$F'_x = \frac{y}{x}, \quad F'_y = y^3 + \ln x.$$

$$F(x, y) = \int \frac{y}{x} dx + \varphi(y) = y \ln x + \varphi(y).$$

$F'_y(x, y) = \ln x + \phi'_y(y) = y^3 + \ln x$ tenglikdan
 $\phi'_y(y) = y^3$, ya'ni $\phi(y) = \frac{y^4}{4}$. Demak, umumiy echim

quyidagi $y \ln x + \frac{y^4}{4} = C$ ko'rinishda bo'ladi.

15. $(x \cdot \sin y + y)dx + (x^2 \cdot \cos y + x \ln x)dy = 0$
 tenglama uchun integrallovchi ko'paytuvchi toping va
 eching.

$$P'_y = x \cos y + 1; Q'_x = 2x \cos y + \ln x + 1.$$

$$\frac{P'_y - Q'_x}{Q} = \frac{-x \cos y - \ln x}{x^2 \cos y + x \ln x} = -\frac{1}{x}.$$

$$\ln \mu = \int \left(-\frac{1}{x}\right) dx = -\ln x \text{ dan } \mu(x) = \frac{1}{x}.$$

Demak, $(\sin y + \frac{y}{x})dx + (x \cos y + \ln x)dy = 0$
 tenglama to'la differentsial bo'ladi. Haqiqatan, hosil
 bo'lgan tenglama uchun $P'_y = \cos y + \frac{1}{x} = Q'_x$

$$F'_x = \sin y + \frac{y}{x}; F'_y = x \cos y + \ln x.$$

$$F(x, y) = \int (\sin y + \frac{y}{x}) dx = x \sin y + y \ln x + \phi(y)$$

$$F'_y(x, y) = x \cos y + \ln x + \phi'_y(y) = x \cos y + \ln x$$

$$\text{Bundan. } \phi'_y(y) = 0, \phi = C.$$

Demak, umumiy echim $x \sin y + y \ln x = C_1$
 ko'rinishda bo'ladi.

16. $yy'^2 + x = 1$ tenglamani eching.

$$y' \text{ ga nisbatan echaniz: } y' = \pm \sqrt{\frac{x-1}{y}}$$

O'zgaruvchilari ajraladigan differentsial
 tenglama hosil bo'ldi. Uning umumiy echimi

$$y^{\frac{4}{3}} + (x-1)^{\frac{4}{3}} = \frac{4}{3}C \text{ ko'rinishda bo'ladi.}$$

17. $y'^2 + xy = y^2 + xy'$ tenglamani y' ga nisbatan eching, so'ngra umumiy echimini toping.

$y'^2 - xy' + xy - y^2 = 0$ tenglama y' ga nisbatan kvadrat tenglamadir.

$$(y')_{1,2} = \frac{x \pm \sqrt{x^2 - 4(xy - y^2)}}{2} = \frac{x \pm (x - 2xy)}{2} \text{ dan}$$

a) $y' = x - y$ (chiziqli)

b) $y' = y$ (o'zgaruvchilari ajraladigan)

Bu tenglamalar mos ravishda

$y = C \cdot e^{-x} + x - 1$, $y = Ce^x$ umumiy echimlarga egadir.

18. $y = y'^2 + 2y'^3$ tenglamani parametr kiritib eching.

$y' = p$ belgilash kiritsak, $y = p^2 + 2p^3$

dan $p = 2p \cdot p' + 6p^2 \cdot p'$ Tomonlarni $p \neq 0$ ga qisqartirsak $1 = 2p' + 6pp'$ yoki $x' = 2 + 6p$. Bunda

$x' = \frac{dx}{dp}$. Demak, $x = 2p + 3p^2 + C$, $y = p^2 + 2p^3$.

$p = 0$ bo'lgan holda $y' = 0$, ya'ni $y = C$ lardan $y = 0$ maxsus echim bo'ladi.

19. $y = xy' - y'^2$ Klero tenglamasini eching.

$y' = p$ belgilash kiritib,

$p = x \cdot p' + p - 2pp'$ ga egamiz. Undan

$p'(x - 2p) = 0$ kelib chiqadi. Agar $p' = 0$ bo'lsa

$y' = C$ yoki $y = Cx + C_1$, $x - 2p = 0$ dan $y' = \frac{x}{2}$

yoki $4y = x^2 + 4c$ ga ega bo'lamiz.

10-bobga doir uy vazifalari.

I. Bir jinsli yoki unga keltiriluvchi differentsial tenglama umumiy echimlarini toping.

$$1) (x^2 - y^2)y' = 2xy$$

$$2) xy' = y \ln \frac{y}{x}$$

$$3) xy' + xe^x - y = 0$$

$$4) xy' - y = \sqrt{x^2 + y^2}$$

$$5) xy' + y = \sqrt{x^2 + y^2}$$

$$6) xy' + y = x$$

$$7) y - xy' = 2(x + yy')$$

$$8) xy'(\ln y - \ln x) = y$$

$$9) y'\sqrt{x} = \sqrt{y-x} + \sqrt{x}$$

$$10) x^2 y' = y(x + y)$$

$$11) (x + y)^2 \cdot y' = xy$$

$$12) xyy' = x^2 - y^2$$

$$13) (2x - y)y' = y - x - 1$$

$$14) 2y' + x = 4\sqrt{y}$$

$$15) (x - y + 1)y' = y - x + 3$$

II. Birinchi tartibli, chiziqli yoki chiziqqligiga keltiriluvchi differentsial tenglama umumiy echimini toping.

$$1) y' + y = xy^3$$

$$2) yy' + y^2 \operatorname{ctgx} = \cos x$$

$$3) (1 + x^2)y' - 2xy = (1 + x^2)^2$$

$$4) x^2 \cdot y' - 2xy + y^3 = 0$$

$$5) xy' + y = \frac{1}{y}$$

$$6) xy' + y = (x + 1)y^2$$

$$7) (1 - x^2)y' - xy = 2x^2$$

$$8) 3y^2 y' + y^3 = x - 1$$

$$9) (2x + 1)y' + y = \frac{x}{y}$$

$$10) y' + y - y^2 = 1 - x - x^2, y_0 = x + 1$$

$$11) y' + xyy^2 = -2x - 3, y_0 = x + 2$$

$$12) y' + 4xy - 4y^2 = 12x - 5; y_0 = x - 3$$

$$13) y' - 3x^2y + xy^2 = 1 - 2x^3; \quad y_0 = x$$

$$14) y' - xy + 4y^2 = 5x^2 - 1; \quad y_0 = -x$$

$$15) xy' - y + y^2 = x^2; \quad y_0 = x$$

III. Quyidagi to'la yoki to'laga keltiriluvchi differentsial tenglamalarni eching.

$$1) 2xydx + (x^2 - y^2)dy = 0$$

$$2) (2 - 9xy^2)xdx + (4y^2 - 6x^3)ydy = 0$$

$$3) e^{-y}dx - (2y + xe^{-y})dy = 0$$

$$4) \frac{x}{y}dx + (y^3 + \ln x)dy = 0$$

$$5) (1 + y^2 \sin 2x)dx - 2y \cos^2 x dy = 0$$

$$6) 3x^2(1 + \ln y)dx = (2y - \frac{x^3}{y})dy$$

$$7) (1 - x^2)dy - xydx = 0$$

$$8) 3dy + ydx = 0$$

$$9) (2x + 1)dy + ydx = 0$$

$$10) \cos x dy = (y + 1) \sin x dx$$

$$11) (3 + 2xy)dx - x^2 dy = 0$$

$$12) (x + 1 - y)dx = x dy$$

$$13) (2y - x)dy - ydx = 0$$

$$14) dy + y \operatorname{ctg} x dx = 0$$

$$15) xdy - ydx = 0$$

IV. Hosilaga nisbatan echilmagan quyidagi differentsial tenglamani eching.

$$1) y = y'^2 + y'^3 \quad 2)$$

$$y = 2y'x + \frac{x^2}{2} + y'^2 \quad 3) y = y'x + \frac{1}{x}$$

$$4) y = xy' + y' + y'^2$$

$$5) y'^2 + 4xy' - y^2 - 2x^2y = x^4 - 4x^2 \quad 6) y = xy' + y'^2$$

$$7) y = 4\sqrt{y'} - xy' \quad 8)$$

$$y = 2xy' - 4y'^3$$

$$9) y = xy' - (2 + y')$$

$$10) y'^3 = 3(xy' - y) \quad 11) y = xy'^2 - 2y'^3$$

$$12) 2xy' - y = \ln y' \quad 13) xy' - y = \ln y'$$

$$14) xy'(y' + 2) = y$$

$$15) 2y'^2 (y - xy') = 1$$

11-Bob.

Yuqori tartibli differentsial tenglama va sistemalar.

§45. Tartibi pasayadigan yuqori tartibli differentsial tenglamalar

$y^{(n)} = f(x)$ ko'rinishdagi differentsial tenglama ketma-ket n -marta integrallash yordamida umumiy echimi topiladi. Har bir integrallashda bittadan o'zgarmas qo'shiladi, natijada, umumiy echimda n ta o'zgarmas qatnashadi.

$F(x, y^{(k)}, y^{(k+1)}, \dots, y^{(n)}) = 0$ ko'rinishidagi, noma'lum funktsiyaning o'zi qatnashmaydigan differentsial tenglamalar $y^{(k)} = z$ yangi o'zgaruvchi kiritish yordamida tartibi pasayadi.

Erkin o'zgaruvchi x qatnashmagan $F(x, y', y'', \dots, y^{(n)}) = 0$ ko'rinishdagi differentsial tenglamalar $y' = p(y)$, $y'' = pp'$ almashtirishlar yordamida tartibi pasayadi.

Agar tenglama funktsiya va uning hosilalariga nisbatan bir jinsli bo'lsa $(y, y', y'', \dots, y^{(n)})$ lar $(ky, ky', ky'', \dots, ky^{(n)})$ lar bilan almashtirganda tenglama o'zgarmasa), $y' = yz$ yangi o'zgaruvchi kiritish yordamida tartibi pasayishi mumkin.

Agar tenglama tomonlari to'la differentsiallar bo'lsa, integrallash yordamida tartibi pasayadi.

45.1. Tenglamalarni eching.

$$1) y'' = 4 \cos 2x$$

$$2) y'' = \frac{1}{\cos^2 x}$$

$$3) y'' = \frac{1}{1+x^2}$$

$$4) x^3 \cdot y'' + x^2 \cdot y' = 1$$

$$5) yy'' + y'^2 = 0$$

$$6) y'' + y' \operatorname{tg} x = \sin 2x$$

$$7) y'' + 2y \cdot y'^2 = 0$$

$$8) y'' x \ln x = y'$$

$$8) y'' x \ln x = y'$$

$$10) 2yy'' = y'^2$$

$$11) 2yy'' = 1 + y'^2$$

$$12) y'' \operatorname{tg} x = y' + 1$$

$$13) xy'' - y' = e^x \cdot x^2$$

45.2. Tenglama tomonlarini to'la hosilaga keltirib eching.

$$1) yy''' + 3y'y'' = 0 \quad 2)$$

$$yy'' = y'(y'+1) \quad 3) yy'' + y'^2 = 1 \quad 4)$$

$$y'' = xy' + y + 1 \quad 5) xy'' + y' = 2yy' \quad 6)$$

$$xy'' - y' = x^2 \cdot yy'$$

45.3 Bir jinsliligidan foydalanib eching:

$$1) yy'' = y'^2 + 15y^2 \cdot \sqrt{x} \quad 2)$$

$$(x^2 + 1)(y'^2 - yy'') = xy'y'$$

$$3) xyy'' + xy'^2 = 2yy' \quad 4) x^2 yy'' = (y - xy')^2$$

$$5) x^2 yy'' + y'^2 = 0 \quad 6) xyy'' = y'(y + y')$$

$$7) x^2 (y'^2 - 2yy'') = y^2$$

§46. O'zgarmas koeffitsientli, chiziqli, bir jinsli differentsial tenglamalar

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0 \quad (1) \text{ tenglamada}$$

$$y = e^{kx} \quad \text{almashtirish} \quad \text{yordamida}$$

$k^n + a_1 k^{n-1} + \dots + a_n = 0 \quad (2)$ xarakteristik tenglamaga ega bo'lamiz.

1) Agar (2) tenglama o'zaro tengmas $k_1, k_2, \dots, k_n$ — haqiqiy ildizlarga ega bo'lsa, $e^{k_1 x}, e^{k_2 x}, \dots, e^{k_n x}$ funksiyalar (1) ning xususiy, $y_0 = C_1 e^{k_1 x} + C_2 e^{k_2 x} + \dots + C_n e^{k_n x}$ esa umumiy echim bo'ladi.

2) Agar (2) tenglama $k_1 = k_2 = \dots = k_m, k_{m+1}, \dots, k_n$ — haqiqiy ildizlarga ega bo'lsa, ya'ni k_1 — m karali ildiz bo'lsa, u holda dastlabki m ta ildizga mos xususiy eichmlar $e^{k_1 x}, x e^{k_1 x}, \dots, x^{m-1} e^{k_1 x}$, ularga mos umumiy echim esa

$y_0 = (C_1 + C_2x + C_3x^2 + \dots + C_nx^{n-1})e^{kx}$ ko'rinishda bo'ladi.

3) Har bir qo'shma kompleks $\alpha \pm \beta i$ ildizlarga $(C_1 \cos \beta x + C_2 \sin \beta x)e^{\alpha x}$ echim, agar bu ildizlar m -karrali bo'lsalar,

$$y_0 = [(C_1 + C_2x + \dots + C_mx^{m-1})\cos \beta x + (C_1 + C_2x + \dots + C_mx^{m-1})\sin \beta x]$$

echim mos keladi.

46.1. Tenglamalar umumiy echimlarini toping.

1. $y'' - 4y' = 0$

2. $y'' - 4y' + 4y = 0$

3. $y'' - 4y' + 13y = 0$

4. $y'' - 4y' = 0$

5. $y'' + 4y = 0$

6. $y'' + 4y' = 0$

7. $y'' + 3y' - 4y = 0$

8. $y'' + 2ay' + a^2y = 0$

9. $y''' - 5y'' + 8y' - 4y = 0$

10. $y''' - 3y'' + 4y = 0$

11. $y''' + 3ay'' + 3a^2y' + a^3y = 0$

12. $y'''' + 4y = 0$

13. $4y'''' - 3y''' - y = 0$

14. $y'''' - 3y''' - 4y = 0$

15. $y'''' + 8y'' + 16y = 0$

§47. O'zgarmas koeffitsientli, chiziqli, bir jinsli bo'lmagan differentsial tenglamalar.

$$y^{(n)} + a_1y^{(n-1)} + \dots + a_ny = f(x) \quad (1) \text{ va}$$

$$y^{(n)} + a_1y^{(n-1)} + \dots + a_ny = 0 \quad (2) \text{ tenglamalarni}$$

qaraymiz.

Agar y_1 (1) tenglama xususiy echimi, y_0 esa (2) tenglama umumiy echimi bo'lsa, (1) tenglama umumiy echimi $y = y_0 + y_1$ ko'rinishda bo'ladi.

(1) tenglamaning xususiy echimi ikki xil usulda topilishi mumkin:

1. Aniqmas koeffitsientlar metodi.

(1) - tenglama xususiy echimi, bu metod yordamida quyidagi hollarda topiladi:

1) $f(x) = P_n(x)e^{mx}$ — ko'phad

2) $f(x) = e^{mx}(a \cos nx + b \sin nx)$

3) Funktsiya yuqoridagilarning yig'indisi yoki ko'paytmasi.

Bu hollarda y_1 — xususiy echim ham noma'lum koefitsientli $f(x)$ funktsiya ko'rinishida izlanadi.

Agar 1) holda $k = m$, 2) holda $k = m \pm ni$ xarakteristik tenglamaning r — karrali ildizlari bo'lsa, izlanayotgan noma'lum koefitsientli funktsiya $x' \cdot f(x)$ ko'rinishda bo'ladi.

Ko'p hollarda $f(x)$ tarkibida sinus va kosinus qatnashganda Eylearning

$$\cos \beta x = \frac{1}{2}(e^{i\beta x} + e^{-i\beta x}), \quad \sin \beta x = \frac{1}{2i}(e^{i\beta x} - e^{-i\beta x})$$

formulalari yordamida yuqoridagi hollarga keltiriladi.

II. Lagranjning o'zgarishni variatsiyalash usuli.

Agar $y_0 = C_1 y_1 + C_2 y_2 + \dots + C_n y_n$ bir jinsli (2) tenglama umumiy echimi bo'lsa, (1) — tenglama umumiy echimi $y_0 = C_1(x) y_1 + C_2(x) y_2 + \dots + C_n(x) y_n$

ko'rinishda izlanadi. Noma'lum $C_i x$ funktsiyalar

$$C_1^1 y_1 + \dots + C_n^1 y_n = 0$$

$$C_1^1 y_1 + \dots + C_n^1 y_n^1 = 0$$

.....

$$C_1^1 y_1^{(n-2)} + \dots + C_n^1 y_n^{(n-2)} = 0$$

$$C_1^1 y_1^{(n-1)} + \dots + C_n^1 y_n^{(n-1)} = f(x)$$

sistemadan topiladi.

47.1. Tenglamalarni eching.

1) $y'' - 2y' + y = e^{2x}$ 2) $y'' - 4y = 8x^3$

3) $y'' + 3y' + 2y = \sin 2x + 2 \cos 2x$

4) $y'' + y = x + 2e^x$ 5) $y'' + 3y' = 9x$

$$6) y''+4y'+5y = 5x^2 - 32x + 5$$

$$7) y''-3y'+2y = e^x \quad 8) y''-2y = x \cdot e^{-x}$$

$$9) y''-2y' = x^2 - x \quad 10) y''+5y'+6y = e^{-x} + e^{-2x}$$

$$11) y'''+y'' = 6x + e^{-x} \quad 12) y^{IV} - 81y = 27e^{-3x}$$

$$13) y'''+8y = e^{-2x} \quad 14) y^{IV} - 3y''+4y = 3\sin x$$

$$15) y'''-3y''+3y'-y = e^x.$$

47.2. O'zgarasni variatsiyalash yordamida eching:

$$1) y''+4y = \frac{1}{\sin 2x} \quad 2) y''-4y'-5y = \frac{e^{2x}}{\cos x}$$

$$3) y''-2y'+y = x^{-2} \cdot e^x \quad 4) y''+y = \operatorname{tg} x$$

$$5) y''+y' = \frac{1}{1+e^x} \quad 6) y''+4y'+4 = \frac{e^{-2x}}{x^3}$$

$$7) y''+4y'+4y = e^{-2x} \cdot \ln x \quad 8) y''+y = \frac{1}{\cos^3 x}$$

$$9) y''-2y'+y = \frac{e^x}{\sqrt{4-x^2}} \quad 10) y''+4y = \frac{1}{\sin^2 x}$$

§48. O'zgaras koeffitsientli, chiziqli differentsial tenglamalar sistemalari.

Noma'lumlarni ketma-ket yo'qotish yordamida murakkab bo'lmagan sistemalarni echish mumkin.

48.1. Bir jinsli sistemani eching:

$$1) \begin{cases} \dot{x} = x - y \\ \dot{y} = y - 4x \end{cases} \quad 2) \begin{cases} \dot{x} + x - 8y = 0 \\ \dot{y} - x - y = 0 \end{cases}$$

$$3) \begin{cases} \dot{x} = x + y \\ \dot{y} = 3y - 2x \end{cases} \quad 4) \begin{cases} \dot{x} = x + z - y \\ \dot{y} = x + y - z \\ \dot{z} = 2x - y \end{cases}$$

$$5) \begin{cases} \dot{x} = x - 2y - z \\ \dot{y} = y - x + z \\ \dot{z} = x - z \end{cases}$$

48.2. Bir jinsli bo'lmagan sistemani eching:

$$1) \begin{cases} \dot{x} = y + 2e^t \\ \dot{y} = x + t^2 \end{cases} \quad 2) \begin{cases} \dot{x} = y - 5 \cos t \\ \dot{y} = 2x + y \end{cases}$$

$$3) \begin{cases} \dot{x} = 3x + 2y + 4e^{5t} \\ \dot{y} = x + 2y \end{cases} \quad 4) \begin{cases} \dot{x} = 2x - 4y + 4e^{-2t} \\ \dot{y} = 2x - 2y \end{cases}$$

$$5) \begin{cases} \dot{x} = 4x + y - e^{2t} \\ \dot{y} = y - 2x \end{cases} \quad 6) \begin{cases} \dot{x} = 2y - x + 1 \\ \dot{y} = 3y - 2x \end{cases}$$

Bobga doir misollar echish namunalari

1. $y''' = \frac{6}{x^3}$ tenglamaning $x = 1$ da
 $y = 2$, $y' = 1$; $y'' = 1$ shartlarga bo'ysunuvchi
 echimini toping.

Ketma – ket integrallab quyidagilarni topamiz:

$$y'' = -\frac{3}{x^2} + C, \quad y' = \frac{x}{3} + C_1x + C_1, \quad y = 3 \ln x + C \frac{x^2}{2} + C_1x + C_2$$

$x = 1$ da o'zgaraslarni topish uchun quyidagi sistemaga ega bo'lamiz:

$$\begin{cases} 1 = -3 + C \\ 1 = 3 + C + C_1 \\ 2 = \frac{C}{2} + C_1 + C_2 \end{cases}$$

Bundan esa $C = 4$; $C_1 = -6$; $C_2 = 6$. Xususiylar
echim $y = 3 \ln x + 2x^2 - 6x + 6$

2. $x^2 \cdot y'' = y'^2$ tenglamani eching.

$y' = z$, $y'' = z'$ almashtirish yordamida
 $x^2 \cdot z' = z^2$ o'zgaruvchilari ajraladigan differentsial
tenglamaga ega bo'lamiz. Uning echimi quyidagi
ko'rinishda bo'ladi: $z \neq 0$, $\frac{1}{z} = \frac{1}{x} - C$, яъни $\frac{1}{y'} = \frac{1}{x} - C$.

Bundan $y' = \frac{x}{1 - Cx}$, $Cy = C^2x + \ln|1 - Cx| = C_1$

umumiy echimni topamiz. Agar $z = 0$ bo'lsa,
 $y' = 0$, $y = C$.

3. $2yy' - 1 = y'^2$ tenglamani eching.

$y' = p$, $y'' = pp'$ almashtirishlardan

$$2ypp' - 1 = p^2, \quad \frac{2pdp}{p^2 + 1} = \frac{dy}{y} \quad \text{va} \quad \ln|p^2 + 1| = \ln y + \ln C.$$

Bundan $p^2 + 1 = C \cdot y$, yoki $y' = \pm \sqrt{Cy - 1}$.

Bundan, $4(Cy - 1) = C^2(x + C_2)$ umumiy echimni
olamiz.

4. $y' \cdot y''' = 2y'^2$ tenglama tomonlarini to'la hosilalar
ko'rinishida keltirib eching.

Tomonlarni $y' \cdot y''$ ga bo'lib, $\frac{y'''}{y''} = 2 \frac{y''}{y'}$ yoki $(\ln y'')' = (2 \ln y')'$ dan $y''' = C \cdot y'^2$ ga ega bo'lamiz. Bu tenglamani ham $\frac{y''}{y'} = Cy'$ yoki $(\ln y')' = (C \cdot y)'$ ko'rinishda yozish mumkin. Demak, $\ln y' = Cy + \ln C_1$ yoki $y' = C_1 e^{Cy}$ lardan $-\frac{1}{C} e^{-Cy} = C_1 x + C_2$ yoki $y = -\frac{1}{C} \ln |CC_2 - CC_1 x|$ kelib chiqadi.

5. Bir jinsliligidan foydalanib tarkibini pasaytiring va eching: $xyy'' - xy'^2 = yy'$

$y' = y \cdot z$; $y'' = y'z + y \cdot z' = yz^2 + yz'$
 almashtirishlar o'tkazamiz:
 $xy(yz^2 + yz') - xy^2 \cdot z^2 = y \cdot yz$. $y^2 \neq 0$ deb tomonlarni qisqartirsak, $xz^2 + xz' - xz^2 = z$ yoki $xz' = z$ tenglama hosil bo'ladi. Bundan $z = Cx$ yoki $y' = C \cdot yx$. Bu tenglama echimi esa $y = C_1 \cdot e^{\frac{1}{2}Cx^2}$ ko'rinishda bo'ladi.

6. $y'' - 4y' + 3y = 0$ tenglama umumiy echimini toping.

Xarakteristik tenglama $k^2 - 4k + 3 = 0$ bo'lib, $k_1 = 1$, $k_2 = 3$ dir. Demak, $y_0 = C_1 e^x + C_2 e^{3x}$

45.2. $y''' - 3y'' + 3y' - y = 0$ tenglama umumiy echimini toping.

$k^3 - 3k^2 + 3k - 1 = 0$ tenglama $(k-1)^3 = 0$ ga ekvivalent tenglamadir. Demak, $k_{1,2,3} = 1$ va

$y_0 = (C_1 + C_2 x + C_3 x^2) e^x$.

7. $y'' - 2y' + 5 = 0$ tenglamaning umumiy echimini toping.

$k^2 - 2k + 5 = 0$ xarakteristik tenglama $k_{1,2} = 1 \pm 2i$ ildizlarga ega. Umumiy echim esa $(C_1 \cos 2x + C_2 \sin x)e^x$ ko'rinishda bo'ladi.

8. $y'' + 8y'' + 16 = 0$ tenglamaning umumiy echimini toping.

Xarakteristik tenglama $k^4 + 8k^2 + 16 = (k^2 + 4)^2 = 0$ ko'rinishda bo'lib, $k_{1,2} = 2i$, $k_{3,4} = -2i$ ildizlardir.

$y_0 = [(C_1 + C_2 x) \cos 2x + (C_3 + C_4 x) \sin 2x]e^{0x}$ bo'ladi.

9. Berilgan differentsial tenglama xarakteristik tenglamasi

$k_1 = 2$; $k_2 = 3$; $k_{3,4} = 4$; $k_{5,6} = -1 \pm 5i$; $k_{7,8,9,10} = 2 \pm 7i$

ildizlarga ega. Umumiy echim ko'rinishini yozing.

Ildizlar barcha xususiy hollarni o'z ichiga oladi.

Umumiy echim esa

$y_0 = C_1 e^{2x} + C_2 e^{3x} + (C_3 + C_4 x) e^{4x} + (C_5 \cos 5x + C_6 \sin 5x) e^{-x} +$
 $+ [(C_7 + C_8 x) \cos 7x + (C_9 + C_{10} x) \sin 7x] e^{2x}$

10. $y'''' - 3y'' + 3y' - y = e^x + x$ tenglamani aniqmas koeffitsientlar metodi bilan yozing.

Bir jinsli tenglama xarakteristik tenglamasi ildizlari

$k_{1,2,3} = 1$ ekanligidan $y_0 = (C_1 + C_2 x + Cx^2)e^x$.

a) $y'''' - 3y'' + 3y' - y = e^x$ xususiy echim $y_1 = Ax^3 e^x$ ko'rinishda izlanadi.

$y_1' = A[3x^2 e^x + x^3 e^x] = A(3x^2 + x^3)e^x$.

$y_1'' = A[6x + 3x^2 + 3x^2 + x^3]e^x = A[6x + 6x^2 + x^3]e^x$.

$y_1''' = A[6 + 12x + 3x^2 + 6x + 6x^2 + x^3]e^x = A[6 + 18x + 9x^2 + x^3]e^x$.

Topilganlarni o'rniga qo'yib:

$A[6 + 18x + 9x^2 + x^3 - 18x - 18x^2 - 3x^3 + 9x^2 + 3x^3 - 3x^2 - x^3]e^x = e^x$.

ya'ni $A[6 - 3x^2] = 1$ dan $A = \frac{1}{6}$ ba $y_1 = \frac{x^3}{6} \cdot e^x$ bo'ladi.

b) $y''' - 3y'' + 3y' - y = x$ xususiy echimi $y_2 = Ax + B$ tarzida izlanadi. $y_2' = A$; $y_2'' = 0$. Bundan: $3A - Ax - B = x$, ya'ni $A = -1$; $B = -3$ ba $y_2 = -x - 3$.

Umumiy echim esa $y = (C_1 + C_2x + Cx^2 + \frac{x^3}{6})e^x - x - 3$ ko'rinishda bo'ladi.

11. $y'' - 3y' + 2y = \frac{e^{3x}}{1 + e^{2x}}$ tenglamani o'zgarmasni variatsiyalash yordamida eching.

$k^2 - 3k + 2 = 0$ echimlari $k_1 = 1$, $k_2 = 2$ ekanligidan

tenglama xususiy echimlari e^x va e^{2x} dir. Bundan

$y_0 = C_1e^x + C_2e^{2x}$ va

$$\begin{cases} C_1' \cdot e^x + C_2' e^{2x} = 0 \\ C_1' e^x + 2C_2' e^{2x} = \frac{e^{3x}}{1 + e^{2x}} \end{cases}$$

sistemaga ega bo'lamiz. $C_1' = -C_2' e^x$ ni ikkinchi

tenglamaga qo'yib $C_2' e^{2x} = \frac{e^{3x}}{1 + e^{2x}}$, ya'ni

$C_2' = \frac{e^x}{1 + e^{2x}}$ ga ega bo'lamiz. Bundan

$C_2 = \arctg e^x$.

$C_1' = -\frac{e^{2x} + 1 - 1}{1 + e^{2x}}$ dan $C_1' = -1 + \frac{1}{1 + e^{2x}}$ va

$C_1 = -\ln \sqrt{1 + e^{2x}}$.

Demak, umumiy echim

$y = C_1e^x + C_2e^{2x} - \ln \sqrt{1 + e^{2x}} \cdot e^x + e^{2x} \arctg e^x$.

$$12. \begin{cases} \dot{x} = 2x + y \\ \dot{y} = 3x + 4y \end{cases} \quad \text{sistemani eching, bunda}$$

$$\dot{x} = \frac{dx}{dt}, \quad \dot{y} = \frac{dy}{dt}.$$

Birinchi tenglamadan $y = \dot{x} - 2x$ ekanligidan, uni ikkinchi tenglamaga qo'yib $\ddot{x} - 2\dot{x} = 3x + 4(\dot{x} - 2x)$ yoki $\ddot{x} - 6\dot{x} + 5x = 0$ tenglamaga ega bo'lamiz. Xarakteristik tenglama ildizlari $k_1 = 1, k_2 = 5$ ekanligidan

$x = C_1 e^t + C_2 e^{5t}, \quad \dot{x} = C_1 e^t + 5C_2 e^{5t}$ bo'lganligi uchun

$y = C_1 e^t + 5C_2 e^{5t} - 2C_1 e^t - 2C_2 e^{5t} = -C_1 e^t + 3C_2 e^{5t}$ kelib chiqadi.

Demak, $x = C_1 e^t + C_2 e^{5t}$

$$y = -C_1 e^t + 3C_2 e^{5t}.$$

$$13. \begin{cases} \dot{x} = x - y + 8t \\ \dot{y} = 5x - y \end{cases} \quad \text{bir jinsli bo'lmagan sistemani eching.}$$

Ikkinchi tenglamadan $x = \frac{y}{5} + \frac{y}{5}, \quad \dot{x} = \frac{\dot{y}}{5} + \frac{\dot{y}}{5}$ larni topib birinchi tenglamaga qo'yamiz.

$$\frac{\dot{y}}{5} + \frac{\dot{y}}{5} = \frac{y}{5} + \frac{y}{5} - y + 8t$$

$\dot{y} + 4y = 40t$ tenglama hosil bo'ladi.

$$k^2 + 4 = 0 \quad \text{dan} \quad k_{1,2} = \pm 2i \quad \text{ya'ni}$$

$$y_0 = C_1 \cos 2t + C_2 \sin 2t \quad y_1 = At + B \quad \text{ko'rinishda izlanadi.}$$

$$4At + 4B = 40t \quad \text{dan} \quad A = 10; B = 0.$$

Demak,

$$y_1 = C_1 \cos 2t + C_2 \sin 2t + 10t,$$

$$x = \frac{1}{5}(-2C_1 \sin 2t + 2C_2 \cos 2t + C_1 \cos 2t + C_2 \sin 2t + 10t)$$

11-bobga doir uy vazifalari

I. Tartibini pasaytiring va yeching.

$$1) 2x' y'' = y'^2 - 2 \quad 2) y'^2 + 4yy'' = 0$$

$$3) yy'' + 3 = y'^2 \quad 4) y''' = 4y''^2$$

$$5) y''' = 2(y' - 5) \operatorname{ctgx} \quad 6) y''' = xy'' = 6y'$$

$$7) y'' + y'^2 = 7e^{-y} \quad 8) y''^2 = y'^2 + 8$$

$$9) y'' - xy''' + y''^2 = 0 \quad 10) y^4 - y^3 \cdot y'' = 10$$

$$11) y''(2y' + x) = 11 \quad 12) (1 - x^2)y'' + xy' = 12$$

$$13) (y' + 13y)y'' = y'^2 \quad 14) y''^3 \cdot y'^2 = 4y'^3$$

$$15) xy'' = y' + x(y'^2 + x^2)$$

II. Bir jinsli bo'lmagan tenglamalarni yeching.

$$1) y'' + 4y' - 12y = 8 \sin 2x$$

$$2) y'' - 6y' + 9y = x^2 - x + 3$$

$$3) y'' + 4y' = e^{-2x}$$

$$4) y'' - 2y' + 5y = xe^{2x}$$

$$5) y'' + 5y' + 6y = \cos 2x$$

$$6) y'' - 5y' + 6y = (12x - 7)e^{-x}$$

$$7) y'' - 4y' + 13y = 26x + 5$$

- 8) $y'' - 2y' + y = 16e^x$
- 9) $y'' - 4y' = 6x^2 + 1$
- 10) $y'' + 6y' + 9y = 10e^{-3x}$
- 11) $y'' + 4y' = e^x + x$
- 12) $y'' - 3y = x^2 + 5$
- 13) $y'' + y' + y = e^x$
- 14) $y'' + 2y' + 4y = e^{2x}$
- 15) $y'' - 4y = e^{2x}$

III. Sistemani yeching:

$$1) \begin{cases} \dot{x} = 4x + 6y \\ \dot{y} = 4x + 2y \end{cases}$$

$$2) \begin{cases} \dot{x} = -5x - 4y \\ \dot{y} = -2x - 3y \end{cases}$$

$$3) \begin{cases} \dot{x} = 3x + y \\ \dot{y} = 8x + y \end{cases}$$

$$4) \begin{cases} \dot{x} = 6x + 3y \\ \dot{y} = -8x - 5y \end{cases}$$

$$5) \begin{cases} \dot{x} = -x + 5y \\ \dot{y} = x + 3y \end{cases}$$

$$6) \begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = 2x + 5y \end{cases}$$

$$7) \begin{cases} \dot{x} = -4x - 6y \\ \dot{y} = -4x - 2y \end{cases}$$

$$8) \begin{cases} \dot{x} = 5x - 8y \\ \dot{y} = -3x - 3y \end{cases}$$

$$9) \begin{cases} \dot{x} = -x - 5y \\ \dot{y} = -7x - 3y \end{cases}$$

$$10) \begin{cases} \dot{x} = -7x + 5y \\ \dot{y} = 4x - 8y \end{cases}$$

$$11) \begin{cases} \dot{x} = 2x - y \\ \dot{y} = x + 2e^t \end{cases} \quad 12) \begin{cases} \dot{x} = 2x - 4y \\ \dot{y} = x - 3y + 3e^t \end{cases}$$

$$13) \begin{cases} \dot{x} = x + 2y \\ \dot{y} = x - 5 \sin t \end{cases} \quad 14) \begin{cases} \dot{x} = 2x - y \\ \dot{y} = y - 2x + 18t \end{cases}$$

$$15) \begin{cases} \dot{x} = 5x - 3y + 2e^{3t} \\ \dot{y} = x + y + 5e^{-t} \end{cases}$$