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the outlet of the dryer did not exceed 6%, which indicates the adequacy of the developed model and the possibility of its implementation in industry.

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ON THE UNIQUE SOLVABILITY OF A NONLOCAL BOUNDARY VALUE PROBLEM WITH THE POINCARÉ CONDITION

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Annotation. As is known, it is customary in the literature to divide degenerate equations of mixed type into equations of the first and second kind. In the case of an equation of the second kind, in contrast to the first, the degeneracy line is simultaneously the envelope of a family of characteristics, i.e. is itself a characteristic, which causes additional difficulties in the study of boundary value problems for equations of the second kind. In this paper, in order to establish the unique solvability of one nonlocal problem with the Poincaré condition for an elliptic-hyperbolic equation of the second kind developed a new principle extremum, which helps to prove the uniqueness of resolutions assigned problem. The existence of a solution is realized by reducing the problem posed to a singular integral equation of normal type, which known by the Carleman-Vekua regularization method developed by S.G. Mikhlin and M.M. Smirnov equivalently reduces to the Fredholm integral equation of the second kind, and the solvability of the latter follows from the uniqueness of the solution delivered problem.
Keywords: Equations of the second kind, nonlocal problem, extremum principle, regularization method, generalized solution class.
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1. Introduction

Boundary value problems for degenerate equations of elliptic and equations of mixed types are in the center of attention of mathematicians and mechanics due to the presence of numerous applications in the study of problems in mechanics, physics, engineering and biology.

Starting from [1], [2], a new direction has appeared in the theory of equations of elliptic and mixed types, in which nonlocal boundary value problems (problems with a shift) and Bitsadze-Samarskii problems are considered. Further, it turned out that non-local boundary conditions arise in problems of predicting soil moisture [3], in modeling fluid filtration in porous media [4], in mathematical modeling of laser radiation processes and problems of plasma physics [5], as well as in mathematical biology [6] .

Solving various boundary value problems with the Poincaré conditions or with a conormal derivative for the Tricomi, Lavrentiev-Bitsadze and more general equations devoted to a large number of articles[7] - [13]. We note that the results of all the listed papers were obtained for equations of the first kind, and for equations of the second kind, nonlocal boundary value problems with the Poincaré condition have not been previously studied.

Therefore, the study of non-local boundary value problems with a conormal derivative for equations of mixed elliptic-hyperbolic type of the second kind seems to be very relevant and little studied. Note the works [14],[15].

In this paper, we study a nonlocal boundary value problem with the Poincaré condition for an elliptic-hyperbolic type equation of the second kind, i.e. for an equation where the line of degeneracy is a characteristic.

2. Statement of the problem

Consider the equation

$$sgny|y|^m u_{xx} + u_{yy} = 0, \qquad m \in (0;1) \tag{1}$$

Let Ω is a finite simply connected region of the plane of independent variables *x*, *y*, bounded at $y > 0$ crooked σ dot ends $A(0,0),\ B(1,0)$ and segment $AB(y=0)$, and when $y < 0$ characteristics

$$AC:\ x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0, \quad BC:\ x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$$

equations (1).

Let further $\Omega_1 = \Omega \bigcap \{ y > 0\}$, $\Omega_2 = \Omega \bigcap \{ y < 0\}$,
$J = \{(x, y): 0 < x < 1,\ y = 0\}$, $\Omega = \Omega_1 \bigcup \Omega_2 \bigcup J$, $2\beta = m/(m+2)$, and

$$\beta \in (-0,5;0). \tag{2}$$

Problem C**.** Required find function $u(x, y)$, which has the following properties:

1) $u(x,y) \in C(\overline{\Omega}) \cup C^1(\Omega_1 \cup \Omega_2 \cup \sigma \cup J)$, and the derivatives u_x and u_y can address infinity of order less than one at points $A(0,0)$ and $B(1,0)$; 2) $u(x,y) \in C^2(\Omega_1)$ is a regular solution of equation (1) in the domain Ω_1, and in the region Ω_2 is a generalized solution from the class R_2 [16]; 3) the gluing condition is satisfied on the degeneracy line

$$\lim_{y \to +0} u_y(x, y) = -\lim_{y \to -0} u_y(x, y) \tag{3}$$

4) satisfies the following boundary conditions

$$\left\{\delta(s) A_s[u] + \rho(s) u\right\}\Big|_{\sigma} = \varphi(s), \quad 0 < s < l, \tag{4}$$

$$\frac{d}{dx} u\left[\Theta_0(x)\right] + b \frac{d}{dx} u\left[\Theta_1(x)\right] = c(x), \quad (x,0) \in J, \tag{5}$$

where $l -$ the length of the whole curve σ, $s -$ are length σ, counted from the point $B(1,0)$, a $\rho(s), \delta(s), \varphi(s), c(x) -$given functions, and $b = const \neq 0$,

$$\rho(s)\delta(s) \geq 0, \quad 0 \leq s \leq l, \tag{6}$$

$$\rho(s), \delta(s), \varphi(s) \in C[0,l], \quad c(x) \in C^1[0,1] \cap C^2(0,1), \tag{7}$$

here

$$\Theta_0\left(\frac{x}{2}, -\left(\frac{m+2}{4} x\right)^{2/(m+2)}\right) \quad \text{and} \quad \Theta_1\left(\frac{x+1}{2}; -\left(\frac{m+2}{4}(1-x)\right)^{\frac{2}{m+2}}\right) \tag{8}$$

- points of intersection of the characteristics of equation (1), emerging from the points $x \in J$, with characteristics AC and BC respectively, and $A_s[u]$ determined from the formula $A_s[u] \equiv y^m \dfrac{dy}{as}\dfrac{\partial u}{\partial x} - \dfrac{dx}{as}\dfrac{\partial u}{\partial y}$.

Note that if $\delta(s) \equiv 0$, $b = 0$, then the tasks C matches the tasks T studied in [17]. Therefore, in what follows, we will assume that $\delta(s) \neq 0$.

3. Uniqueness of solutions to the problem C

To prove the uniqueness of the solution to the problem C. The following lemmas play an important role.

Lemma 1. If the function $\tau'(x)$ satisfies Hölder's condition with exponent $k > -2\beta$ at $0 < x < 1$, then the function

$$T(x) = \frac{1}{\Gamma(1-2\beta)} D_{0x}^{1-2\beta} \tau(x) \tag{9}$$

can be represented as

$$T(x) = \frac{sin 2\pi\beta}{2\pi\beta} \frac{d}{dx} \int_0^x (x-t)^{2\beta} \tau'(t)\,dt.$$

Lemma 2. Let the conditions

$$\tau(x) \in C[0,1] \cap C^{(1,k)}(0,1), k > -2\beta \tag{10}$$

and function $\tau(x)$ at the point $x = x_0$ $\left(x_0 \in (0,1)\right)$ takes on the largest positive value (LPV) and the smallest negative value (SNV). Then the function

$$E(x) = \int_0^1 \frac{(1-t)^{-2\beta} T(t)}{x-t} dt$$

at the point $x = x_0$ can be represented as

$$E(x_0) = (1-x_0)^{-2\beta} \left\{ \left[x_0^{\,2\beta-1} cos 2\beta\pi + (1-x_0)^{2\beta-1} \right] \tau(x_0) - \tau(1)(1-x)^{2\beta-1} + \right.$$

$$\left. +(1-2\beta)\left[cos 2\beta\pi \int_0^{x_0} \frac{\tau(x_0)-\tau(t)}{(x_0-t)^{2-2\beta}} d\,t - \int_{x_0}^1 \frac{\tau(t)-\tau(x_0)}{(t-x_0)^{2-2\beta}} d\,t \right] \right\}. \tag{11}$$

Lemma 3. Let conditions (2), (10) be satisfied and the function $\tau(x)$ at the point $x = x_0$ $\left(x_0 \in (0,1)\right)$ accepts refineries (SNV). Then the function $T(x)$ (see (9)) at the point $x = x_0$ can be represented as

$$T(x_0) \equiv \frac{1}{\Gamma(1-2\beta)} D_{0x}^{1-2\beta} \tau(x)\Big|_{x=x_0} =$$

$$= \frac{Sin 2\beta\pi}{\pi} \left[x_0^{\,2\beta-1}\, \tau(x_0) + (1-2\beta) \int_0^{x_0} \frac{\tau(x_0)-\tau(t)}{(x_0-t)^{2-2\beta}} d\,t \right],$$

and

$$T(x_0) < 0 \quad \left(T(x_0) > 0\right), \quad x_0 \in J\,. \tag{12}$$

Proof of Lemma 1-3 is carried out in the same way as in [22].

Lemma 1-3 implies the following.

Theorem 1.(An analogue of the extremum principle of A.V. Bitsadze). If conditions (2) are satisfied and $b < 0$, then the solution $u(x, y)$ problem C at $c(x) \equiv 0$ and $\tau(1) = 0$ own refinery and SNV in a closed area $\overline{\Omega}_1$ only reaches $\bar{\sigma}$.

Proof of Theorem 1. Indeed, due to the extremum principle for elliptic equations [5], [23], the solution $u(x,y)$ equations (1) inside the region Ω_1 cannot reach its refinery and SNV. Let us show that the solution $u(x,y)$ equation (1) does not reach its OR and SNV on the segment J. Assume the opposite, let $u(x,y)$ some point $(x_0,0)$ segment J reaches its refinery (SNV). Based on Lemma 2, if the function $\tau(x)$ at the point $(x_0,0)$ accepts the refinery (SNV), then $A(x)$ at the point $x = x_0$ can be represented in the form (11), and

$$E(x_0) > 0 \;\; \left(E(x_0) < 0\right), \;\; (x_0,0) \in J. \tag{13}$$

Now let's define the sign $\nu^-(x)$ at the point $(x_0,0) \in J$. Due to (12) and (13) at $c(x) \equiv 0$ we get

$$\nu^-(x_0) < 0 \;\; (\nu^-(x_0) > 0), \;\; (x_0,0) \in J. \tag{14}$$

But on the other hand, by virtue of the Zaremba-Giraud principle [24], [26], for the solution of equation (1), taking into account (15), we have

$$\nu^+(x_0) < 0 \;\; (\nu^+(x_0) > 0), \;\; (x_0,0) \in J. \tag{15}$$

Taking into account (4) from (14) we find

$$\nu^+(x_0) > 0 \;\; (\nu^+(x_0) < 0), \qquad (x_0,0) \in J.$$

This inequality contradicts inequality (15).

In this way, $u(x,y)$ does not reach its refinery (SNV) in the open section J.

Theorem 1 is proved.

Theorem 2. If the conditions of Theorem 1 are satisfied, and the functions $\delta(s)$ and $\rho(s)$ near points $A(0,0),\ B(1,0)$ satisfy conditions (7) and

$$\rho(0) \neq 0, \;\; \rho(l) \neq 0, \tag{16}$$

$$|\delta(s)| \le const\left[s\;(l-s)\right]^{\varepsilon_0 - \frac{m^2+2m-2}{m+2}}, \quad -1 < m < 0, \;\; \varepsilon_0 = const > 0, \tag{17}$$

then in the area D there cannot be more than one solution to the problem C.

Proof of Theorem 2. Let $\varphi(s) \equiv c(x) \equiv 0$, then, by virtue of Theorem 1, it suffices to show that the solution of the problem C cannot reach its positive maximum and negative minimum on σ.

Assume that a positive maximum (negative minimum) is reached at some point $s_0 \in \sigma$, different from the points $A(0,0)$ and $B(1,0)$. Then at this point, due to the Zaremba-Giraud principle [24], [27] $A_{s_0}\,[u] > 0 \;\left(A_{s_0}\,[u] < 0\right)$, and the boundary condition (5) takes the form

$$A_{s_0}[u] = -\frac{\rho(s_0)\delta(s_0)}{\delta^2(s_0)}u.$$

But this is impossible due to condition (7).

Therefore, at interior points σ function $u(x,y)$ does not reach its positive maximum (negative minimum).

At points $A(0,0)$ and $B(1,0)$, taking into account (2), (3), (17) we have

$$\lim_{S\to 0}\delta(s)A_s[u] = 0 \quad \text{and} \quad \lim_{S\to l}\delta(s)A_s[u] = 0 \tag{18}$$

respectively.

If a positive maximum (negative minimum) is reached at the point $A(0,0)$ or $B(1,0)$, then by virtue of (18) the boundary condition (5) takes the form

$$\rho(0)\,u(0,0)=0 \text{ or } \rho(l)\,u(1,0)=0.$$

Hence, taking into account (16), we obtain

$$u(A)=u(0,0)=\tau(0)=0,\quad u(B)=u(1,0)=\tau(1)=0. \qquad (19)$$

Means, $u(x,y)$ does not reach a positive maximum (negative minimum) at points $A(0,0)$ and $B(1,0)$. In this way, $u(x,y)$ does not reach a positive maximum (negative minimum) on the curve $\bar{\sigma}$.

Based on the extremum principle (see Theorem 1), we conclude that $u(x,y)=const$ in $\overline{\Omega}_1$. Therefore, taking into account (19), we have $u(x,y)\equiv 0$ in $\overline{\Omega}_1$. Due to the uniqueness of the solution of the Cauchy problem in the domains Ω_{2j} $(j=\overline{1,3})$ for equation (1), we obtain that $u(x,y)\equiv 0$ in $\overline{\Omega}_{2j}$ $(j=\overline{1,3})$. Hence it follows that $u(x,y)\equiv 0$ in $\overline{\Omega}$. This proves the uniqueness of the solution of the problem C.

Theorem 2. is proved.

4. Existence of a solution to the problem C

When studying the problem C an important role is played by the functional relationships between $\nu^{\pm}(x)$ and $\tau(x)$ from the elliptic and hyperbolic parts of the domain Ω, where

$$u(x,0)=\tau(x),\quad (x,0)\in\bar{J}, \qquad (20)$$

$$\lim_{y\to-0}\frac{\partial u(x,y)}{\partial y}=\nu^{-}(x),\qquad \lim_{y\to+0}\frac{\partial u(x,y)}{\partial y}=\nu^{+}(x),\quad (x,0)\in J. \qquad (21)$$

Generalized solution of the Cauchy problem with data (20), (21) for equation (1) from the class R_2 in the area of Ω_2 is given by the formula [16. 230 (27.5)], [3]:

$$u(\xi,\eta)=\int_0^{\xi}(\eta-t)^{-\beta}(\xi-t)^{-\beta}T(t)dt+\int_{\xi}^{\eta}(\eta-t)^{-\beta}(t-\xi)^{-\beta}N(t)dt, \qquad (22)$$

where

$$\xi=x-\frac{2}{m+2}(-y)^{\frac{m+2}{2}},\eta=x+\frac{2}{m+2}(-y)^{\frac{m+2}{2}},\ \gamma_2=\left[2(1-2\beta)\right]^{2\beta-1}\frac{\Gamma(2-2\beta)}{\Gamma^2(1-\beta)},$$

$$N(t)=T(t)/2\cos\pi\beta-\gamma_2\nu^{-}(t), \qquad (23)$$

$$\tau(x)=\int_0^{x}(x-t)^{-2\beta}T(t)dt, \qquad (24)$$

functions $T(x)$ and $\nu^{-}(x)$ continuous in $(0,1)$ and integrable on $[0,1]$, a $\tau(x)$ vanishes on the order of at least -2β at $x\to 0$.

Putting $\xi=0$, $\eta=x$ and $\xi=x$, $\eta=1$ respectively, in (22), taking into account (8), after some transformations we obtain

$$u\left[\Theta_0(x)\right]=\int_0^{x}(x-t)^{-\beta}t^{-\beta}N(t)dt, \qquad (25)$$

$$u\left[\Theta_1(x)\right]=\int_0^x (x-t)^{-\beta}(1-t)^{-\beta}T(t)\,dt+\int_x^1 (t-x)^{-\beta}(1-t)^{-\beta}N(t)\,dt. \qquad (26)$$

We put (25) and (26) in the boundary condition (6), by virtue of the fractional integration operators and (23) we obtain a functional relation between $T(x)$ and $\nu^-(x)$, transferred from the area Ω_2 on the J:

$$\gamma_1\left(x^{-2\beta}-2b\cos\pi\beta\cdot x^{-\beta}(1-x)^{-\beta}+b^2(1-x)^{-2\beta}\right)\nu^-(x)-\frac{x^{-2\beta}+b^2\cos 2\pi\beta(1-x)^{-2\beta}}{2\cos\pi\beta}T(x)-$$

$$-\frac{b^2\sin\pi\beta}{\pi}\int_0^1\frac{(1-t)^{-2\beta}T(t)}{x-t}dt=-\frac{x^{-\beta}}{\Gamma(1-\beta)}D_{0x}^{-\beta}c(x)+\frac{b(1-x)^{-\beta}}{\Gamma(1-\beta)}D_{x1}^{-\beta}c(x). \qquad (27)$$

The solution of the problem DK with conditions (5) and (20) for equation (1) in the region D_1 exists, is unique and can be represented in the form [16. see (10.78)]:

$$u(x,y)=\int_0^1\tau(\xi)\frac{\partial}{\partial\eta}G_2(\xi,0;x,y)d\xi+\int_0^l\frac{\varphi(s)}{\delta(s)}G_2(\xi,\eta;x,y)ds, \qquad (28)$$

where $G_2(\xi,\eta;x,y)-$ Green's function of problem DK for equation (1) [16]:

Differentiating with respect to y equation (28), then directing y to zero we get the functional relation between $\tau(x)$ and $\nu^+(x)$, transferred from the area Ω_1 on the J:

$$\nu^+(x)=\frac{k_2}{1-2\beta}\frac{d}{dx}\left[-\int_0^x(x-t)^{2\beta-1}\tau(t)dt+\int_x^1(t-x)^{2\beta-1}\tau(t)dt\right]-k_2\int_0^1\frac{\tau(t)dt}{\left(t+x-2tx\right)^{2-2\beta}}+$$

$$+\int_0^1\tau(t)\frac{\partial^2 H_2(t,0;x,0)}{\partial\eta\partial y}dt+\int_0^l\chi(s)\frac{\partial q_2(\xi(s),\eta(s);x,0)}{\partial y}ds, \qquad (29)$$

where $\chi(s)$ is a solution to the integral equation

$$\chi(s)+2\int_0^l\chi(t)\Big\{A_s\left[q_2\left(\xi(t),\eta(t);x(s),y(s)\right)\right]+$$

$$+\frac{\rho(s)}{\delta(s)}q_2\left(\xi(t),\eta(t);x(s),y(s)\right)\Big\}dt=\frac{2\varphi(s)}{\delta(s)}.$$

and $q_2(\xi,\eta,x,y)$ is the fundamental solution of equation (1) and it has the form:

$$q_2(\xi,\eta,x,y)=k_2\left(\frac{4}{m+2}\right)^{4\beta-2}\left(r_1^2\right)^{-\beta}(1-w)^{1-2\beta}F\left(1-\beta,1-\beta,2-2\beta;1-w\right)$$

where

$$\left.\begin{matrix}r^2\\ r_1^2\end{matrix}\right\}=(\xi-x)^2+\frac{4}{(m+2)^2}\left(\eta^{\frac{m+2}{2}}\mp y^{\frac{m+2}{2}}\right)^2,$$

$$w=\frac{r^2}{r_1^2},\quad \beta=\frac{m}{2(m+2)},\quad -\frac{1}{2}<\beta<0,\ k_2=\frac{1}{4\pi}\left(\frac{4}{m+2}\right)^{2-2\beta}\frac{\Gamma^2(1-\beta)}{\Gamma(2-2\beta)},$$

$F(a,b,c;z)$ is hypergeometric function of Gauss[23].

Substituting (24) into (29) and taking into account some identities of fractional differential operators, we obtain a functional relation between $T(x)$ and $\nu^+(x)$, transferred from the area Ω_1 on the J:

$$\nu^+(x)=-\frac{\pi k_2 tg\beta\pi}{1-2\beta}T(x)-\frac{k_2}{1-2\beta}\int_0^1\left(\frac{1-t}{1-x}\right)^{-2\beta}\left[\frac{1}{t-x}+\frac{1-2t}{x+t-2xt}\right]T(t)dt+$$

$$+\int_0^1 T(t)dt\int_t^1(z-t)^{-2\beta}\frac{\partial^2 H_2(z,0;x,0)}{\partial\eta\partial y}dz-\frac{2k_2}{1-2\beta}\int_0^1\left(\frac{1-t}{1-x}\right)^{-2\beta}\frac{T(t)dt}{1-2x}+$$

$$+\int_0^l\frac{\partial q_2(\xi(s),\eta(s);x,0)}{\partial y}\chi(s)ds,\quad (x,0)\in J. \quad (30)$$

Theorem 3. If conditions (2), (3), and (7) are satisfied, then in the region Ω the solution of the problem C exists.

Proof of Theorem 3. Excluding $\nu^{\pm}(x)$ from relations (27) and (30), taking into account (4) and (24), we obtain a singular integral equation of the form:

$$P_1(x)T(x)+\frac{P_2(x)}{\pi i}\int_0^1\left(\frac{1-t}{1-x}\right)^{-2\beta}\left[\frac{1}{t-x}+\frac{1-2t}{x+t-2xt}\right]T(t)dt-\int_0^1 K(x,t)T(t)dt=$$

$$=F(x),\quad 0<x<1, \quad (31)$$

where

$$P_1(x)=\frac{\pi k_2 tg\beta\pi}{1-2\beta}d_1(x)-\frac{1}{2\cos\pi\beta}d_2(x),\ P_2(x)=\frac{\pi i k_2}{1-2\beta}d_1(x)-ib^2\sin\pi\beta(1-x)^{-2\beta},$$

$$K(x,t)=d_1(x)\int_t^1(z-t)^{-2\beta}\frac{\partial^2 H_2(z,0;x,0)}{\partial\eta\partial y}dz-\frac{b^2\sin\pi\beta}{\pi}\frac{(1-2t)(1-t)^{-2\beta}}{x+t-2xt}-\frac{2k_2 d_1(x)}{(1-2\beta)(1-2x)}\left(\frac{1-t}{1-x}\right)^{-2\beta}$$

$$F(x)=d_1(x)\int_0^l\frac{\partial q_2(t,\eta;x,0)}{\partial y}\chi(s)ds-\frac{x^{-\beta}}{\Gamma(1-\beta)}D_{0x}^{-\beta}c(x)+\frac{b(1-x)^{-\beta}}{\Gamma(1-\beta)}D_{x1}^{-\beta}c(x),$$

equation (31) is an equation of normal type [23], [24].

Applying the well-known Carleman-Vekua regularization method [23], we obtain the Fredholm integral equation of the second kind, the solvability of which follows from the uniqueness of the solution of the problem C.

Theorem 3 is proved.

Conclusions

Thus, with the help developed by the authors of the article, a new principle extremum for an equation of the second kind, the uniqueness of the stated problem is proved. When studying the existence of a solution to the problem under study, with the help of functional relations, a singular integral equation was obtained of normal type, the solvability of which follows from the uniqueness of the solution of the problem. The article presents new mathematical results that are interesting for a specialist in this field. Which, can be used to compile some models of gas and hydrodynamic processes, in predicting soil moisture, in modeling fluid filtration in porous media.

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ORION Automated Fiber Management System

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Abstract: This article is devoted to the analysis of a number of existing systems for automatic control and monitoring of fiber-optic transmission systems (FOTS). The intensive development of fiber-optic communication lines, strong competition from telecom operators and the high cost of information resources transmitted over communication lines emphasize the tasks of centralized management and monitoring of telecommunication networks in order to document them, timely detect and quickly eliminate damage to FOTS. Currently, a number of systems for automatic control and monitoring of FOTS are presented on the domestic market, and new systems are being intensively developed.
Keywords: the Optical cables (OC), Software (SW), Remote test unit (RTU), Optical fiber, Fiber-optic communication

Introduction

Remote control of optical fibers is performed with an optical pulse reflectometer, which diagnoses the condition of the fiber by the backscattering of a light wave when probe pulses are introduced into the fiber. At the same time, the system allows monitoring both free and occupied fibers. In the first case, the monitoring of free spare optical fibers is performed, according to the condition of which the health of the entire fiber-optic cable is judged. In the second case, the monitoring of optical fibers is carried out, through which the traffic of digital transmission systems is transmitted. To implement this test method, the operating wavelength of the OTDR is used, which is different from the operating wavelength of the Digital Transmission System (DSP), and a number of passive optical components are introduced into the monitoring network circuit for multiplexing and separating signals from the DSP and the OTDR[1].

GN Nettest Fiber Optic Devision's Orion fiber management system is one of the most advanced systems with these capabilities. This confirms the fact that of all systems of a similar purpose currently in operation, the overwhelming majority are Orion systems. This system is installed in such telecommunication companies of the world as National Fiber Network, ADC Telecommunications and AT&T Network Systems (USA), Telsetra (Australia), Telemig (Brazil), Bezeq (Israel), SANEF and SNCF (France), FTZ (Germany) , Telia AB (Sweden), etc.

Orion offers a variety of methods to diagnose fiber cable faults and differs from similar systems by using an OTDR with the highest resolution and dynamic range of 46 dB at 1550nm and 41.5 dB at 1625nm. Special methods of detection, violations allow to test 20 optical fibers with a length of 150 km and more, in less than 12 minutes, and thanks to the Extended Range (ER) of the optical fiber monitoring mode, the Orion system allows you to detect cable violations at a distance of up to 300 km, which is not achievable with any existing OTDR.

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