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Algebra va geometriya kafedrasi

# SONLAR NAZARIYASI ASOSLARIDAN MASALA VA MASHQLAR

**«Algebra va sonlar nazariyasi» fanidan amaliy mashg’ulotlar o’tkazish uchun uslubiy tavsiyalar**

«**5 460100 MATEMATIKA**»
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(Uslubiy qo‘llanma)

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Ushbu uslubiy qo‘llanma « Algebra va sonlar nazariyasi » fani bo‘yicha «5460100 – matematika» ta'lim yo‘nalishi bakalavr talabalari va «5A460100 – Matematik mantiq, Algebra va sonlar nazariyasi» mutaxassisligi magistrantlari uchun mo‘ljallangan bo‘lib, unda shu fanning namunaviy o‘quv dasturidan kelib chiqib, butun sonlar xalqasida bo'linish nazariyasi va taqqoslamar nazariyasining usullariga oid qisqacha nazariy ma'lumotlar, bu usullarning taqbiqiga oid namunaviy misollar yechimlari, mustaqil ish topshiriqlari va boshqa tarqatma materiallar keltirilgan. Uslubiy qo'llanma talabalarga shu fanni yanada chuqurroq o‘zlashtirishga yaqindan yordam beradi degan umiddamiz.

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# I-BOB
# BUTUN SONLAR XALQASIDA BO'LINISH NAZARIYASI

Tayanch iboralar**:** *bo'linma; bo'luvchi; qoldiqli bo'lish haqidagi teorema; to'liqmas bo'linma; qoldiq; umumiy bo'luvchi; eng katta umumiy bo'luvchi; juft-juft tub sonlar; umumiy karrali; eng kichik umumiy bo'linuvchi; Yevklid algoritmi; murakkab son; Eratosfen g'alviri; arifmetikaning asosiy teoremasi; kanonik yoyilma; chekli uzluksiz kasrlar; aniq bo'linmalar; munosib kasrlar; butun qism; kasr qism; antye funksiya; Eyler funksiyasi; Myobius funksiyasi.*

## 1-§. Butun sonlarning bo'linishi

Agar shunday $q$ butun son mavjud bo'lib, $a = bq$ tenglik o'rinli bo'lsa, $a$ butun son $b$ butun songa ($b \neq 0$) bo'linadi yoki $b$ son $a$ sonni bo'ladi deyiladi. Bu yerda $q$ bo'linma, $b$ bo'luvchi, $a$ bo'linuvchi deb ataladi. $a$ sonning $b$ songa bo'linishini $b|a$ shaklda belgilanadi, agar $a$ son $b$ songa bo'linmasa, uni $b \nmid a$ bilan belgilaymiz.

***Bo'linish xossalari*:**

*a*) bo'linish refleksiv, ya'ni $a|a$;

*b*) bo'linish tranzitiv, ya'ni agar $b|a$ va $c|b$ bo'lsa, u holda $c|a$;

*c*) $c|a$ dan ixtiyoriy butun $b$ son uchun $c|ab$ o'rinli;

*d*) $c|a$ va $c|b$ dan ixtiyoriy butun $x$ va $y$ sonlar uchun $c|ax+by$ o'rinli (masalan, $c|a\pm b$). Bu xossa ikkidan ko'p sonlar uchun ham o'rinli;

*e*) $b|a$ va $a|b$ bo'lsa, $a = \pm b$;

*f*) $b|a$, $a > 0$, $b > 0$ dan $b \leq a$ kelib chiqadi.

*Qoldiqli bo'lish haqidagi teorema:* $a$ – butun son, $b$ – butun musbat son bo'lsin. $a$ son hamma vaqt $b$ songa bo'linmaydi, lekin hamma vaqt *a son b songa qoldiqli bo'linadi*, ya'ni shunday yagona butun $q$ va $r$ sonlar topiladiki, ular uchun

$$a = bq + r,\ 0 \leq r < b$$

tenglik o'rinli bo'ladi, bu yerda $q$ - *to'liqmas bo'linma*, $r$ - soni $a$ ni $b$ ga bo'lgandagi *qoldiq* deyiladi.

1-m i s o l. $a$ sonni 13 ga bo'lganda to'liqmas bo'linma 17 ga teng bo'lsa, $a$ ning eng katta qiymatini toping.

*Yechish.* Masala shartiga ko'ra, $a = 13\cdot17+r$, $0 \leq r < 13$. Demak, $r = 12$ bo'lganda $a$ eng katta qiymatga erishadi, ya'ni $13\cdot17 + 12 = 233$. ■

2-m i s o l. Bo'linuvchi 371, to'liqmas bo'linma 14 ga teng bo'lsa, bo'luvchi va unga mos qoldiqlarni toping.

*Yechish.* Masala shartiga ko'ra, $371 = b\cdot14 + r$, $0 \leq r < b$ , bundan $14b < 371$, $b \leq 26$. Boshqa tomondan $15b > 371$, bundan $b > 24$. Demak, $b$=25; 26 va $r$ = 21; 7 bo'ladi. ■

3-m i s o l. $a$ sonni $b$ songa bo'lganda bo'linma $q$ va nolmas qoldiq $r$ ga teng. $a$ ni qanday natural $n$ songa ko'paytirganda bo'linma $n$ marta ortadi?

*Yechish.* $an = bqn + rn$ dan $rn < b$ va $n < \frac{b}{r}$. ■

4-m i s o l. Uchta ketma-ket natural sonlardan bittasi 3 ga bo'linishini isbotlang.

*Yechish.* Natural sonni $3k$, $3k + 1$, $3k + 2$ sonlarning bittasi shaklida ifodalash mumkin. Agar $n = 3k$ bo'lsa, u holda $3|n$; agar $n = 3k + 1$ bo'lsa, u holda $3|n + 2$; agar $n = 3k + 2$ bo'lsa, u holda $3|n+1$. ■

5-m i s o l. Agar besh xonali son 41 ga bo'linsa, shu sonni tashkil qilgan raqamlarni aylanma almashtirish yordamida hosil bo'lgan har qanday sonning 41 ga bo'linishini isbotlang.

*Yechish.* Besh xonali son $N=10^4a+10^3b+10^2s+10d+e$ bo'lsin va u 41 ga bo'linsin. Raqamlarni aylanma almashtirishdan (chapga bir raqamga) quyidagi sonni hosil qilamiz:

$$N_1=10^4 b + 10^3 c + 10^2 d + 10\, e + a =$$
$$10(10^4 a + 10^3 b + 10^2 c + 10\, d + e) - 10^5 a + a = 10N - 99999\, a.$$

$41|$ N va $41|99999$ dan $41|$ $N_1$ kelib chiqadi. ■

6-m i s o l. $2^{2^n} + 1\ (n = 2,3,...)$ ko'rinishdagi barcha sonlar 7 raqam bilan tugashini isbotlang.

*Yechish.* $2^{2^2} + 1 = 17$. Agar $2^{2^n} + 1 = 10q + 7$, bo'lsa, u holda

$$2^{2^{n+1}} + 1 = \left(2^{2^n}\right)^2 + 1 = (10q + 6)^2 + 1 = (10Q + 6) + 1 = 10Q + 7.$$ ■

7-m i s o l. $7\cdot 11\cdot 13=1001$ ni bilgan holda 7, 11, 13 ga umumiy bo'linish alomatini keltirib chiqaring. Bu alomatni 368312 ga qo'llang.

*Yechish.* $N = 1000q + r = 1001q + r - q$ dan $N$ son 7, 11 va 13 ga bo'linishi uchun shu sondan uning 1000 ga bo'linganida hosil bo'lgan qoldiqdan ayirmasi 7, 11 yoki 13 ga bo'linishi zarur va yetarligi kelib chiqadi, ya'ni $7 \cdot 11 \cdot 13/(1 - q)$. Agar $N = 368312$ bo'lsa, yuqorida keltirilgan ayirma $368 - 312 = 56$.

56 faqat 7 ga bo'linganligi sababli 368312 7 ga bo'linadi, lekin 11 va 13 ga bo'linmaydi. ■

8-m i s o l. To'rtta ketma-ket joylashgan butun sonlar ko'paytmasiga bir qo'shilganda to'liq kvadrat hosil bo'lishini isbotlang.

*Yechish.* $n - 1$, $n$, $n + 1$, $n + 2$ – to'rtta ketma-ket keladigan butun sonlar bo'lsin. U holda

$$(n-1)n(n+1)(n+2)+1 = (n-1)(n+2)(n^2+n)+1 = (n^2+n-1)^2.$$ ■

9-m i s o l. $11^{10} - 1$ sonni 100 ga bo'linishini isbotlang.

*Yechish.* Nyuton binomini qo'llaymiz:

$$(1+10)^{10} = 1 + C_{10}^1 \cdot 10 + C_{10}^2 \cdot 10^2 + C_{10}^3 \cdot 10^3 + ... + 10^{10}.$$

Bundan

$$(1+10)^{10} - 1 = 10 \cdot 10 + C_{10}^2 \cdot 10^2 + C_{10}^3 \cdot 10^3 + .... + 10^{10}$$

har bir qo'shiluvchi 100 ga bo'linadi. ■

10-m i s o l. Har bir butun $n$ uchun $n^5 - n$ son 5 ga bo'linishi isbotlang.

*Yechish.* $n^5 - n = n(n^2-1)(n^2+1)$. Butun sonni 5 ga bo'lganda qoldiqlar 0, 1, 2, 3, 4 bo'ladi va bundan butun son $5k$, $5k+1$, $5k+2$, $5k+3$, $5k+4$, ko'rinishdan biriga teng bo'lishi kelib chiqadi.

Agar $n = 5k$ bo'lsa, $n$ son 5 ga bo'linadi; agar $n = 5k + 2$ yoki $n = 5k + 3$ bo'lsa, $(n^2 + 1)$ son 5 ga bo'linadi; agar $n = 5k + 1$ yoki $n = 5k + 4$ bo'lsa, $(n^2-1)$ 5 ga bo'linadi. ■

11-m i s o l. Raqamlar yig'indisi bir xil bo'lgan ikki son ayirmasi 9 ga bo'linishini isbotlang.

*Yechish.* $N_1 = \overline{a_n...a_1a_0}$ va $N_1 = \overline{b_m...b_1b_0}$ bo'lsin.

$$N_1 = 9Q_1 + \sum_{i=0}^{n} a_i \text{ va } N_2 = 9Q_2 + \sum_{j=0}^{m} b_j \text{ dan shartga ko'ra,}$$

$$\sum a_i = \sum b_j \text{, demak, } N_1 - N_2 = 9(Q_1 - Q_2). \ ■$$

12-m i s o l. Ketma-ket kelgan to'rtta raqam birin-ketin yozilgan bo'lib, dastlabki ikkita raqam o'rni almashtirilgandan so'ng to'la kvadrat bo'lgan to'rt xonali son hosil qilingan. Shu sonni toping.

*Yechish.* Masala shartiga ko'ra,

$$N^2 = 1000(x + 1) + 100x + 10(x + 2) + (x + 3) = 11(101x + 93).$$

Bundan $N = 11 \cdot k$ va $N$ to'la kvadrat bo'lganligidan $11k^2 = 101x + 93$, ya'ni $k^2 = \frac{101x+93}{11} = 9x + 8 + \frac{2x+5}{11}$. Bu yerdan $x = 3$ kelib chiqadi. Demak, $N = 11(101 \cdot 3 + 93) = 4356 = 66^2$. ■

## M A S H Q L A R

**1**. Agar bo'linuvchi va bo'linma berilgan bo'lsa, bo'luvchi va qoldiqni toping: *a*) 25 va 3; *b*) – 30 va – 4.

**2***. Isbotlang:

*a*) toq natural sonning kvadratini 8 ga bo'lganda 1 qoldiq qoladi;

*b*) ketma-ket ikki natural son kvadratlari yig'indisini 4 ga bo'lganda 1 qoldiq qoladi.

**3***. 15 soni har qanday natural darajaga ko'tarilib, 7 ga bo'linsa 1 qoldiq qolishini isbotlang.

**4***. Agar $mn + pq$ $m - p$ ga bo'linsa, u holda $mq + np$ ham $m - p$ ga bo'linishini ko'rsating, bu yerda $m, n, p, q \in \mathbf{Z}$.

**5***. $a, b, c, d, n$ – butun sonlar. $ad - bc$, $a - b$ sonlar $n$ ga bo'linadi va $b, n$ sonlar birdan farqli natural bo'luvchilarga ega emas. $c - d$ ni $n$ ga bo'linishini isbotlang.

**6.** Ixtiyoriy butun $n$ son uchun isbotlang:

*a*) $n^3 - n$ son 3 ga bo'linadi; *b*) $n^7 - n$ son 7 ga bo'linadi;

*c**) $n^5 - n$ son 30 ga bo'linadi.

**7***. Olti raqamli son 5 bilan tugaydi, agar bu sonni chap tomonga birinchi o'ringa o'tkazsak, u holda berilgan sondan 4 marta katta son hosil bo'ladi. Shu sonni toping.

**8***. $n(n+1)(2n+1)$ ($n\in \boldsymbol{N}$) sonni 6 ga bo'linishini isbotlang.

**9***. Kasr sonning surati ikki toq sonning kvadatlari ayirmasi, maxraji esa shu sonlar kvadratlari yig'indisiga teng. Shu kasr surat va maxrajini ikkiga qisqartirish mumkin, 4 ga esa qisqarmasligini ko'rsating.

**10***. To'la kvadrat bo'lgan to'rt xonali sonning minglar va o'nlar xonasidagi raqamlari bir xil, yuzlar xonasidagi raqam birlik raqamdan 1 ga katta. Shu sonni toping.

**11*.** Ketma-ket joylashgan beshta butun sonlar kvadratlarining yig'indisi to'la kvadrat bo'lmasligini isbotlang.

**12***. Agar biror sonni 9 ga bo'lganda qoldiq 2, 3, 5, 6, 8 sonlardan birortasi bo'lsa, shu son to'la kvadrat bo'la olmasligini ko'rsating.

**13**. $S_n = 7 + 77 + 777 + ... + \underbrace{77...7}_{n-ta}$ ketma-ketlikning $n$ ta hadlari yig'indisini toping.

**14***. 16 sonning raqamlari o'rtasiga 15 soni yozilgan, 1156 son o'rtasiga yana 15 yozilgan va hokazo. Shu sonlar to'la kvadrat bo'lishini ko'rsating.

**15***. Har qanday natural $m$ va $n$ lar uchun $mn(m^4 - n^4)$ sonni 30 ga bo'linishini isbotlang.

**16***. Hech qanday butun $x$ uchun $3x^2+2$ son to'la kvadrat bo'laolmasligini ko'rsating.

**17***. $3^n$ $(n\in N)$ ta bir xil raqamlardan tuzilgan natural sonni $3^n$ ga bo'linishini isbotlang.

## 2-§. Eng katta umumiy bo'luvchi va eng kichik umumiy bo'linuvchi

*a, b, …, l* sonlarni bo'luvchi butun son shu sonlarni *umumiy bo'luvchisi* deyiladi.

Shu bo'luvchilarning eng kattasi *eng katta umumiy bo'luvchi* (EKUB) deyiladi va $d = (a, b,\dots, l)$ bilan belgilanadi.

Agar $(a, b,\dots,l) = 1$ bo'lsa, *a, b, …, l* sonlar o'zaro *tub sonlar* deyiladi. Agar *a, b,…, l* sonlarning har biri qolganlari bilan o'zaro tub bo'lsa, bu sonlar *juft-juft bilan o'zaro tub sonlar* deyiladi.

Yevklid algoritmini qo'llab, sonlarni EKUB ini topish mumkin, bu usul quyidagicha: agar $a$ va $b$ natural sonlar va $a > b$ bo'lsa, u holda

$$a = bq_1+r_1,\ 0 < r_1 < b,$$
$$b = r_1 q_1 + r_2,\ 0 < r_2 < r_1,$$
$$r_1 = r_2 q_3 + r_3,\ 0 < r_3 < r_2,$$
$$\dots\dots\dots\dots\dots\dots$$
$$r_{n-2} = r_{n-1}q_n + r_n,\ 0 < r_n < r_{n-1},$$
$$r_{n-1} = r_n q_{n+1},\ r_{n+1}=0.$$

Noldan farqli oxirgi $r_n$ qoldiq $a$ va $b$ sonlarni EKUB ini beradi.

Har qanday *a, b,…, l* sonlarga bo'linadigan son berilgan sonlarni *umumiy karralisi* deyiladi. Umumiy karralilarning eng kichigi *eng kichik umumiy bo'linuvchi* (EKUK) deyiladi va $m = [a, b,\ldots,l]$ bilan belgilanadi.

*a* va *b* sonlarni umumiy karralisi

$$M = \frac{ab}{d}t, \quad t \in \mathbf{Z}, \qquad d = (a,b)$$

tenglik yordamida topiladi. Agar $t = 1$ bo'lsa, bu tenglikdan $a$ va $b$ sonlarning EKUK i kelib chiqadi, ya'ni

$$m = \frac{ab}{d}, \quad \text{yoki} \quad [a,b] = \frac{ab}{(a,b)}.$$

Juft-juft o'zaro tub sonlarning EKUK i shu sonlar ko'paytmasiga teng.

Agar $a = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$ *ва* $b = p_1^{\beta_1} p_2^{\beta_2} \ldots p_k^{\beta_k}$, *бу ерда* $p_1, p_2, \ldots, p_k$ – turli tub sonlar, $\alpha_i$, $\beta_j$ – butun musbat sonlar bo'lsin. U holda

$$(a,b) = p_1^{\min(\alpha_1,\beta_1)} p_2^{\min(\alpha_2,\beta_2)} \ldots p_k^{\min(\alpha_k,\beta_k)},$$

$$[a,b] = p_1^{\max(\alpha_1,\beta_1)} p_2^{\max(\alpha_2,\beta_2)} \ldots p_k^{\max(\alpha_k,\beta_k)}.$$

quyidagi rekurrent formulalar yordamida bir nechta sonlarni EKUK va EKUB ini topish mumkin:

$$(a_1, a_2, \ldots, a_{n-1}, a_n) = ((a_1, a_2, \ldots, a_{n-1}), a_n),$$

$$[a_1, a_2, \ldots, a_{n-1}, a_n] = [[a_1, a_2, \ldots, a_{n-1}], a_n]$$

Demak bu formulalardan bir nechta sonlarni EKUB va EKUK ini topish ikkita sonni EKUB va EKUK ini topish masalasiga keltiriladi.

1-m i s o l. (1734, 822) va [1734, 822] ni toping.

*Yechish.* Bu sonlar uchun Yevklid algoritmini topamiz:

$$1734 = 822 \cdot 2 + 90;$$
$$822 = 90 \cdot 9 + 12;$$
$$90 = 12 \cdot 7 + 6;$$
$$12 = 6 \cdot 2.$$

Demak, (1734, 822) = 6.

$[1734,\ 822] = \frac{1734 \cdot 822}{6} = 237558.$ ■

2-m i s o l. Ikkita ketma-ket juft sonlarning EKUB i 2 ga, toq sonlarning EKUB i esa 1 ga tengligini isbotlang.

*Yechish.*

$(2n, 2n + 2) = 2(n, n + 1) = 2$

$2n + 3 = (2n + 1)\cdot 1 + 2$

$2n + 1 = 2\cdot n + 1$

$2 = 1\cdot 2$, bundan $(2n + 1 , 2n + 3) = 1$. ■

3-m i s o l. $(a, b) = 1$ dan $(a + b, a - b)$ 1 yoki 2 ga tengligi kelib chiqishini isbotlang.

*Yechish.* $(a + b, a - b) = d$ bo'lsin, u holda $d|2a$ va $d|2b$. $(2a, 2b) =$
$= 2(a,b) = 2$ bo'lganligi sababli $d|2$.

Demak, $d = 1$ yoki 2. ■

4-m i s o l. Agar $u_1v_2 - u\ v_1 = 1$ bo'lsa, $(a,b) = (u_1\ a+v_1b,\ u_2a+v_2b)$ ni isbotlang.

*Yechish.* $(a, b) = d$ va $(u_1a + v_1b, u_2a + v_2b) = d_1$ bo'lsin. $d_1|(u_1a + v_1b)$, $d_1|(u_2a + v_2b)$ va $u_1v_2 - u_2v_1 = 1$ dan $d_1 \mid a$ , $d_1 \mid b$, kelib chiqadi, demak, $d_1 \mid d$. $d \mid a$, $d \mid b$ dan $d_1 \mid d$ kelib chiqadi. Demak, $d = d_1$. ■

5-m i s o l. $3 = (51, 21)$ ni $51x + 21y$ shaklda ifodalang.

*Yechish.* $51 = 21\cdot 2 + 9$, $21 = 9\cdot 2 + 3$. Bundan
$3 = 21 - 2\cdot 9 = 21 - 2(51 - 21\cdot 2) = 21\cdot 5 - 51\cdot 2$. ■

6-m i s o l. $ab$ va $m = [a, b]$ sonlarni EKUB ini toping.

*Yechish.* $(ab, m) = (dm, m) = m(d, 1) = m$, bu yerda $d = (a,b)$. ■

7-m i s o l. Uchta ketma-ket natural sonlarning EKUB va EKUK ini toping.

*Yechish.* $(n, n + 1, n + 2) = ((n, n + 1), n + 2) = (1, n + 2) = 1$.

$$[n,n+1,n+2]=[[n,n+1],n+2]=[n(n+1),n+2]=$$
$$=\frac{n(n+1)(n+2)}{(n(n+1),n+2)}=\frac{n(n+1)(n+2)}{(n,n+2)},$$

$(n,n+2)$ $n$ ning juft-toqligiga qarab 2 yoki 1 bo'ladi.

Demak, agar $n$ toq bo'lsa, $[n, n + 1, n + 2] = n(n + 1)(n + 2)$, va agar $n$ juft bo'lsa, $[n, n + 1, n + 2] = \dfrac{n(n+1)(n+2)}{2}$ . ■

8-m i s o l. Ikkita sonning EKUB i shu sonlar ayirmasidan katta bo'lishi mumkinmi?

*Yechish.* $a > b$ va $(a, b) = d$ bo'lsin. Bundan $a = dx$, $b = dy$ va
$x - y > 0$ bo'ladi. Agar $d > a - b = d(x - y)$ bo'lsa, $1 > x - y$ va
$0 < x - y < 1$ ni hosil qilamiz. Bu tengsizlik o'rinli emas, chunki $x$ va $y$ – butun sonlar. Demak, $(a, b) \le a - b$ $(a > b)$ bo'ladi. ■

9-m i s o l. $\begin{cases} x+y=150 \\ (x,y)=30 \end{cases}$ sistemani natural yechimlarini toping.

*Yechish.* $(x, y) = 30$ quyidagi sistemaga teng kuchli.

$$\begin{cases} x = 30u \\ y = 30v \\ (u,v)=1. \end{cases}$$

Bundan berilgan sistemaning birinchi tenglamasi $u + v = 5$ ko'rinishga keladi va $u = 1,2,3,4$ qiymatlar qabul qiladi. Demak, $x = 30, 60, 90, 120$ ga teng bo'lishi mumkin. $y = 150 - x$ dan $y = 120, 90, 60, 30$. ■

10-m i s o l. Agar $(a, b) = 24$, $[a,b] = 2496$ bo'lsa, $a$ va $b$ larni toping.

*Yechish.* $(a, b) = 24$ dan $a = 24x$, $b = 24y$ va $(x,y) = 1$ kelib chiqadi. $x < y$ bo'lsin. $[a,b] = \dfrac{ab}{(a,b)}$ dan

$$2496 = \frac{24x \cdot 24y}{24} \text{ yoki } xy = 104 = 2^3 \cdot 13.$$

$(x, y) = 1$ dan $xy = 1{\cdot}104$ yoki $xy = 8{\cdot}13$ bo'lishi mumkin. Bu yerdan $x = 1$ va $y = 104$ bo'lganda $a = 24{\cdot}1 = 24$, $b = 24{\cdot}104 = 2496$; $x = 8$ va $y = 13$ bo'lganda $a = 24{\cdot}8 = 192$, $b = 24{\cdot}13 = 312$. ■

## M A S H Q L A R

**18.** Yevklid algoritmi yordamida sonlarning EKUB va EKUK ini toping:
*a*) 546 va 231; *b*) 1001 va 6253; *c*) 2737, 9163 va 9639;
*d*) 420, 126 va 525; *e*) 529, 1541 va 1817.

**19.** Sonlarni tub ko'paytuvchilarga ajratib sonlarning EKUB ini toping:
*a*) 360 va 504; *b*) 220 va 6600; *c*) 187 va 533;
*d*) 420, 126 va 525; *e*) 529, 1541 va 1817.

**20*.** Agar $a = cq+r$, $b = cq_1+r_1$ bo'lib, $a, b, q, q_1$ $r, r_1$ – butun nomanfiy sonlar; $c$ – butun musbat son bo'lsa,

$$(a, b, c) = (c, r, r_1)$$

tenglikni isbotlang. Bu tenglikdan $(a,b,c)$ ni topish qoidasini keltirib chiqaring va shu qoidani $n$ ta son uchun umumlashtiring.

**21.** 20-masaladan foydalanib quyidagi sonlarni EKUB ini toping:
*a*) 299, 391 va 667; *b*) 588, 2058 va 2849;
*c*) 31605, 13524 , 12915 va 11067.

**22.** $[a,b] = \dfrac{ab}{(a,b)}$ formuladan foydalanib quyidagi sonlarning EKUK ini toping:
*a*) 252 va 468; *b*) 279 va 372; *c*) 178 va 381;
*d*) 299 va 234; *e*) 493 va 221.

**23*.** Agar $(a,b)=1$ bo'lsa, quyidagilarni toping:
*a*) $((a,b), [a,b])$; *b*) $(a+b, ab)$; *c*) $(a+b, [a,b])$.

**24*.** Ikki son yig'indisi 667, EKUK i va EKUB i nisbatlari 120 ga teng bo'lsa, shu sonlarni toping.

**25*.** Ikki sonni har birini ularning EKUB iga bo'lganda hosil bo'lgan bo'linmalar yig'indisi 18 ga teng. Sonlarning EKUK i 975 ga teng bo'lsa, shu sonlarni toping.

**26*.** $a = 899$, $b = 493$ berilgan. $d = (a,b)$ ni toping va shunday $x$ va $y$ larni aniqlangki, $d = ax + by$ ko'rinishda ifodalash mumkin bo'lsin.

**27.** 26-masalani quyidagi juftliklar uchun bajaring:
*a*) $a = 1445$, $b = 629$; *b*) $a = 903$, $b = 731$; *c*) $a = 1786$, $b = 705$.

**28*.** Sistemalarni natural yechimlarini toping:

$$a)\ \begin{cases}(x,y)=45\\ \dfrac{x}{y}=\dfrac{11}{7}\end{cases};\ b)\ \begin{cases}xy=8400\\ (x,y)=20\end{cases};$$

$$c)\ \begin{cases}\dfrac{x}{y}=\dfrac{5}{9}\\ (x,y)=28\end{cases};\ d)\ \begin{cases}xy=20\\ [x,y]=10.\end{cases}$$

**29*.** Agar *a, b, c* – toq sonlar bo'lsa,

$(a,b,c)=\left(\dfrac{a+b}{2},\dfrac{a+c}{2},\dfrac{b+c}{2}\right)$ ni isbotlang.

**30*.** Isbotlang:

$a)\ [a,b,c]=\dfrac{abc(a,b,c)}{(a,b)(a,c)(b,c)};$

$b)\ (a,b)\,(a,c)\,(b,c)\,[a,b]\,[a,c]\,[b,c]=a^2b^2c^2.$

**31*.** $N=10a+b$ $(0\le b\le 9)$ natural son $m=10q+1$ songa bo'linishi uchun faqat va faqat $a-bq$ $m$ ga bo'linishi kifoya ekanligini isbotlang.

**32*.** Toping:

*a*) $(n,2n+1)$; *b*) $(10n+9,\ n+1)$; *c*) $(3n+1,\ 10n+3)$.

**33*.** $N=10a+b$ $(0\le b\le 9)$ natural son $m=10q+9$ ga bo'linishi uchun faqat va faqat $a+b(q+1)$ ni $m$ ga bo'linishi kifoya ekanligini isbotlang.

**34.** Ixtiyoriy natural $a$ va $b$ lar uchun:

$$(a,b)=(5a+3b,\ 13a+8b)$$

tenglik o'rinli ekanligini isbotlang.

**35.** Agar $(a,b)=1$ bo'lsa, $\dfrac{1}{a}+\dfrac{1}{a+b}$ – qisqarmas kasr ekanligini isbotlang.

## 3-§. Tub va murakkab sonlar

Agar natural son *tub son* deyiladi, u ikkita turli natural bo'luvchiga (bir va o'zi) ega bo'lsa va *murakkab son* deyiladi, agar uning bo'luvchilar soni ikkitadan ko'p bo'lsa.

Bir son na tub, na murakkab songa tegishli emas. Tub sonlar (va ularning natural darajalari) o'zaro tubdir. Murakkab sonning birdan farqli natural bo'luvchisi $\sqrt{a}$ dan katta emas. Bu shartdan foydalanib *a* sonning tub bo'luvchilarini faqat $\sqrt{a}$ dan katta bo'lmagan tub sonlar orasidan izlash kerakligi kelib chiqadi.

*a* sondan katta bo'lmagan tub sonlarni jadvalini tuzish uchun *Eratosfen g'alviri* deb ataluvchi usul mavjud. Bu usul bo'yicha sonlar qatorida bi-rinchi topilgan $p_1$ tub songa karrali bo'lgan sonlarni o'chirish, so'ng ikkinchi $r_2$ tub sonni topib, unga karrali sonlarni o'chirish va hokazo. Bu prosessni $\sqrt{a}$ dan katta bo'lmagan tub songacha davom ettirib, 1 dan $a$ gacha sonlar qatorida o'chirilmay qolgan sonlar $a$ dan katta tub sonlarni beradi.

Birdan katta har qanday butun $a$ sonni $p_1, p_2, \ldots, p_n$ tub sonlar ko'paytmasi shaklida ko'paytuvchilar yozilishi tartibi aniqligida yagona ra-vishda yozish mumkin (*arifmetikaning asosiy teoremasi*):

$$a = p_1 p_2 \ldots p_n .$$

Ba'zi ko'paytuvchilar takrorlanib kelishi mumkin, shuning uchun ularni karralilarini mos ravishda $\alpha_1, \alpha_2, \ldots, \alpha_n$ lar bilan belgilab, $a$ sonning *kanonik yoyilmasi*ni hosil qilamiz, ya'ni:

$$a = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_n^{\alpha_n} .$$

Bundan $a$ sonning har qanday bo'luvchisi

$$d = p_1^{\beta_1} p_1^{\beta_2} \ldots p_n^{\beta_n}$$

ko'rinishga ega bo'lishi kelib chiqadi, bu yerda $0 \le \beta_1 \le \alpha_1$, $0 \le \beta_2 \le \alpha_2$, …., $0 \le \beta_n \le \alpha_n$.

1-m i s o l. Tub sonlar ayirmasi shaklida yoziladigan barcha toq sonlarni toping.

*Yechish.* Tub sonlardan bittasi albatta juft bo'lishi kerak, shuning uchun $N = p - 2$, bu yerda $p$ – juft bo'lmagan tub son. ■

2-m i s o l. $N = 3m+2$ ($m$=1,2…,) sonning kvadratini natural son kvadrati va tub son yig'indisi shaklida yozish mumkin emasligini isbotlang.

*Yechish.* Agar $N^2 = n^2 + p$ bo'lsa, u holda $p = (N-n)(N+n)$, bundan $N - n = 1$, $N + n = p$ va demak, $2N = 1 + p$ yoki $p = 2N - 1 = 6m + 3$, bu esa mumkin emas. ■

3-m i s o l. 127 tub yoki murakkab son ekanligini aniqlang.

*Yechish.* $\sqrt{127}$ dan oshmaydigan 2, 3, 5, 7, 11 tub sonlar 127 ini bo'luvchilari emas, demak, bu son tub sondir. ■

4-m i s o l. 2320 va 2350 sonlari orasida joylashgan barcha tub sonlarni toping.

*Yechish.* Yechimni soddalashtirish maqsadida 2321 dan 2349 gacha bo'lgan sonlar qatorida juft, 0 va 5 bilan tugallanadigan sonlarni yozmaslik mumkin, chunki bu sonlar tub emas. Demak: 2321, 2323, 2327, 2329, 2331, 2333, 2337, 2339, 2341, 2343, 2347, 2349.

Bu sonlar qatoridan 3 ga bo'linadiganlarni o'chiramiz (3 ga bo'linish alomatidan foydalanamiz). Bu sonlar:

2331, 2337, 2343, 2349

Qolgan sonlar:

2321, 2323, 2327, 2329, 2333, 2339, 2341, 2347.

Bu qatorda 5 ga karrali son bo'lmaganligi sababli 7 ga karrali sonlarni o'chiramiz. Bu quyidagicha amalga oshiriladi. Qatordagi birinchi sonni 7 ga bo'lamiz:

$$2321 = 7 \cdot 331 + 4.$$

Qoldiq 4 dan (7 gacha 3 yechishmaydi) 7 ga karrali son natural sonlar qatoridagi 2321 dan keyingi uchinchi sonligi kelib chiqadi, ya'ni 2324 va shu 2324 dan keyingi barcha 7 chi sonlar bo'ladi. Ya'ni: 2331, 2338, 2345. 11 ga karrali son 2321. Bundan keyin keladigan 11 ga karrali sonlar 2332, 2343 sonlar o'chirilgan. 13 ga karrali sonlarni topamiz: qolgan sonlardan birinchi son 2323 ni 13 ga bo'lamiz:

$$2323=13\cdot 178+9.$$

Demak, 13 ga karrali son natural sonlar qatorida 2323 dan to'rtta keyin kelgan (9+4=13) bo'ladi, ya'ni 2327. Bu sonni o'chiramiz. 13 ga bo'linadigan keyingi son 2340, bu son o'chirilgan. $\sqrt{2350}<49$ bo'lganligi sababli bu prosessni to 47 tub songacha davom ettirish kerak. 2329 – 17 ga karrali, 2323 – 23 ga karrali sonlar. Qolgan 2333, 2339, 2341, 2347 sonlar tub sonlar bo'ladi. ■

5-m i s o l. $2^{18}+3^{18}$ yig'indini tub ko'paytuvchilarga ajrating.

*Yechish.*

$$2^{18}+3^{18}=\left(2^2+3^2\right)\left(2^4-2^2 3^2+3^4\right)\left(2^{12}-2^6\cdot 3^6+3^{12}\right)=$$
$$=13\cdot 61\left(2^{12}-2^6\cdot 3^6+3^{12}\right)=13\cdot 61\cdot 488881=13\cdot 61\cdot 37\cdot 73\cdot 181.$$ ■

6-m i s o l. 3, 5 va 7 sonlar yagona uch egizak sonlar (ya'ni ayirmasi 2 ga teng bo'lgan arifmetik progressiya tashkil etuvchi 3 ta tub son) tashkil etishini isbotlang.

*Yechish.* $p, p+2$ va $p+4$ ( $p>3$ ) sonlarni ko'ramiz. $p=3q+1$ ($q$ = 2,4,…,) bo'lsa, $p+2$ – son murakkab son bo'ladi (3 ga bo'linadi). Agar $p=q+2$ $(q=1,2,...)$ bo'lsa, $p+4$ murakkab son bo'ladi. ■

7-m i s o l. $2^n-1$ va $2^n+1$ ($n>2$) sonlar bir vaqtda tub sonlar bo'laolmasligini isbotlang.

*Yechish.* $2^n=3q+1$ yoki $2^n=3q+2$ ko'rinishga ega. Birinchi holda $2^n-1=3q$ – murakkab son, chunki $n>2$ da $q>1$ bo'lishi kerak. Ikkinchi holda $2^n+1=3q+3$ – yana murakkab son. ■

8-m i s o l. $3n+2$ ($n$ = 1, 2,…) ko'rinishdagi eng katta tub son mavjud emasligini isbotlang.

*Yechish.* $N=3\cdot 5\cdot 7\cdots p+2$ ko'rinishdagi sonni qaraymiz, bu yerda $p=3n+2$ ko'rinishdagi son ($N$ soni ham shu ko'rinishdagi son). $N$ ning kanonik yoyilmasida $p$ dan katta tub sonlar mavjud va bular orasida $3n+2$ ko'rinishdagi tub son mavjud. $3n+1$ ko'rinishdagi tub sonlar ko'paytmasi yana shu shaklga ega bo'lganligi sababli u $N$ ga teng bo'laolmaydi. Demak, $p$ qanday bo'lishidan qat'iy nazar $3n+2$ ko'rinishdagi $p$ dan katta tub son mavjud. ■

9-m i s o l. $n>2$ sondan katta bo'lmagan barcha tub sonlar ko'paytmasi $n$ dan katta bo'lishini isbotlang.

*Yechish.* $p$ – tub son $p\le n$ shartni qanoatlantiruvchi eng katta tub son bo'lsin. $N=2\cdot 3\cdot 5\cdots(p\text{-}1)$ sonning kanonik yoyilmasi faqat $n$ dan katta tub sonlardan iborat.

Demak, $N>n$ va bundan $N=2\cdot 3\cdot 5\cdots p>n$. ■

10-m i s o l. $p$ va $8p^2+1$ – tub sonlar bo'lsa, $8p^2+2p+1$ tub son bo'lishini isbotlang.

*Yechish.* $p$ va $8p^2 + 1$ – tub sonlar bo'lganligi sababli $p = 3$ bo'lishi kerak, chunki $p = 3k + 1$ yoki $3k + 2$ bo'lganda $8p^2 + 1$ tub bo'lmaydi. Demak, $8p^2 + 2p + 1 = 79$ – bu tub son. ■

## M A S H Q L A R

**36.** Sonlar orasida joylashgan tub sonlarni toping:
*a*) 200 va 220; *b*) 2540 va 2570; *c*) 1200 va 1250.

**37*.** $n > 1$ natural sonlar uchun $n^4 + 4$ va $n^4 + n^2 + 1$ murakkab sonlar bo'lishini isbotlang.

**38*.** Qanday tub $p$ son uchun $4p^2 + 1$ va $6p^2 + 1$ tub sonlar bo'ladi.

**39*.** Qanday tub $p$ son uchun $p + 10$ va $p + 14$ tub sonlar bo'ladi.

**40*.** Agar $a > 3$, natural $m$ va $n$ sonlarni 3 ga bo'lganda mos ravishda 1 va 2 ga teng qoldiqga ega bo'lsa, $a$, $a + m$, $a + n$ sonlar bir vaqtda tub bo'laolmasligini ko'rsating.

**41*.** $n$ va $n!$ ($n > 2$) sonlar orasida hech bo'lmaganda bitta tub son borligini isbotlang.

**42*.** Barcha $2p + 1$ ko'rinishdagi butun sonlar ichida bitta son to'la kub bo'lishini isbotlang, bu yerda $p$ – tub son.

**43*.** Agar tub sonlarni 5 tub sondan boshlab nomerlab chiqilsa, u holda har bir tub son o'zini uchlangan nomeridan katta bo'lishini isbotlang.

**44*.** Agar $p > 5$ tub son bo'lsa, uning kvadratini 30 ga bo'lganda qoldiq 1 yoki 19 bo'lishini ko'rsating.

**45*.** $p$ va $q$ – 3 dan katta tub sonlar bo'lsa, $p^2 - q^2$ son 24 ga karrali bo'lishini ko'rsating.

**46*.** Sonlar bir vaqtda tub son bo'laolmasligin isbotlang: *a*) $p + 5$ va $p + 10$; *b*) $p$, $p + 2$ va $p + 5$.

**47*.** Agar toq $p$ sonni ikki son kvadratlari ayirmasi shaklida yagona ravishda ifodalash mumkin bo'lsa, u tub, aks holda murakkab bo'lishini isbotlang.

**48*.** 47 masala yechimidan foydalanib toq sonlarni ko'paytuvchilarga ajratish usulini keltirib chiqaring.
*a*) 6643; *b*) 1769; *s*) 3551; *d*) 6497 sonlarni ko'pay-tuvchilarga ajrating.

**49*.** Agar $N$ son ikki sonlar kvadratlari yig'indisi shaklida ikki xil ifodalansa, ya'ni $N = a^2 + b^2 = c^2 + d^2$, u holda $N$ murakkab son bo'lishini isbotlang.

**50*.** $235^2 + 972^2$ sonni ko'paytuvchilarga ajrating.

**51*.** $3^{10} + 3^5 + 1$ sonni ko'paytuvchilarga ajrating.

**52*.** Agar $1+2^k$ tub son bo'lsa, $k = 0$ yoki $k = 2^n$ ($n = 0, 1, 2, \ldots$) bo'lishini isbotlang.

**53*.** O'zaro tub $a,b$ sonlar uchun $a^\alpha + b^\beta$ tub son bo'lsa, $(\alpha, \beta) = 1$ yoki $(\alpha, \beta) = 2^k$ o'rinli bo'lishini ko'rsating.

**54.** Agar $2^n-1$ tub son bo'lsa, $n$ – tub son ekanligini ko'rsating.

## 4-§. Chekli uzluksiz kasrlar

Agar $\frac{a}{b}$ – qisqarmas kasr (to'g'ri yoki noto'g'ri) bo'lsa, bu kasrni Yevklid algoritmi yordamida quyidagi ko'rinishda tasvirlash mumkin:

$$\frac{a}{b}=q_0+\cfrac{1}{q_1+\cfrac{1}{q_2+\cfrac{1}{q_3+\cfrac{}{\ldots\ldots\ldots\ldots+\cfrac{1}{q_n}}}}},$$

bu yerda $q_0$ – butun nomanfiy son; $q_1, q_2, \ldots, q_n$ – butun musbat sonlar.

Bu tenglikning o'ng tomonida yozilgan kasr *chekli uzluksiz kasr* yoki *zanjirli kasr* deyiladi.

Bu kasrlarni qisqacha

$$\frac{a}{b}=(q_0, q_1, q_2, \ldots, q_n)$$

ko'rinishda yozish mumkin.

Yevklid algoritmidagi $q_1, q_2, \ldots, q_n$ lar uzluksiz zanjirning *maxrajlari*; $q_0, q_1, q_2, \ldots, q_{n-1}$ – *to'liqmas bo'linmalar*; $q_0, q_1, q_2, \ldots, q_n$ lar esa *aniq bo'linmalar* deyiladi.

$$\delta_0=\frac{P_0}{Q_0}=\frac{q_0}{1}, \qquad \delta_1=\frac{P_1}{Q_1}=q_0+\frac{1}{q_1}; \quad \delta_2=\frac{P_2}{Q_2}=q_0+\cfrac{1}{q_1+\cfrac{1}{q_2}}, \ldots\ldots.$$

$$\ldots\ldots, \delta_n=\frac{P_n}{Q_n}=q_0+\cfrac{1}{q_1+\cfrac{1}{q_2 \cfrac{}{\ldots\ldots\ldots\ldots\ldots\ldots+\cfrac{1}{q_n}}}}$$

lar *munosib kasrlar* deyiladi va $\frac{p_n}{Q_n}=\frac{a}{b}$.

Munosib kasrlar va $\frac{a}{b}$ kasr orasida quyidagi munosibatlar o'rinli:

$$\frac{P_0}{Q_0} < \frac{P_2}{Q_2} < \frac{P_4}{Q_4} < .... < \frac{a}{b} < .... < \frac{P_5}{Q_5} < \frac{P_3}{Q_3} < \frac{P_1}{Q_1}.$$

Bu tengsizliklardan berilgan $\frac{a}{b}$ kasr ikkita qo'shni munosib kasrlar orasida joylashganligi va tartib oshgani sari bu qo'shni kasrlar intervali kichrayib borishi ko'rinyapti. Shuning uchun ham bunday kasrlar «munosib kasrlar» deyiladi.

Ketma-ket uchta munosib kasrlar suratlari va maxrajlari $k = 2$ dan boshlab quyidagi bog'lanish o'rinli:

$$\frac{P_k}{Q_k} = \frac{P_{k-1}q_k + P_{k-2}}{Q_{k-1}q_k + Q_{k-2}}$$

Agar shartli ravishda $P_{-1}$=1, $Q_{-1}$= 0, $Q_0$ = 1 qabul qilsak, u holda barcha munosib kasrlarni quyidagi sxema yordamida topish mumkin:

| $k$ | | 0 | 1 | 2 | … | k | … | n |
|---|---|---|---|---|---|---|---|---|
| $q_k$ | | $q_0$ | $q_1$ | $q_2$ | … | $q_k$ | … | $q_n$ |
| $P_k$ | 1 | $P_0 = q_0$ | $P_1 = q_0q_1 + 1$ | $P_2 = P_1q_2 + P_0$ | … | $P_k = P_{k-1}q_k + P_{k-2}$ | … | $P_n$ |
| $Q_k$ | 0 | $Q_0 = 1$ | $Q_1 = q_1$ | $Q_2 = Q_1q_2 + Q_0$ | … | $Q_k = P_{k-1}q_k + Q_{k-2}$ | … | $Q_n$ |

Ikkita qo'shni munosib kaslar ayirmasini

$$\frac{P_{k+1}}{Q_{k+1}} - \frac{P_k}{Q_k} = \frac{(-1)^k}{Q_kQ_{k+1}}$$

formula yordamida topish mumkin.

$\frac{a}{b}$ kasrni $\frac{P_k}{Q_k}$ munosib kasr bilan almashtirganda hosil bo'lgan xatoni

$$\left|\frac{a}{b} - \frac{P_k}{Q_k}\right| \le \frac{1}{Q_kQ_{k+1}}$$

tengsizlik bilan baholanadi.

1-m i s o l. $\frac{245}{83}$ sonni shunday munosib kasr bilan almashtiringki, uning xatosi 0, 001 dan katta bo'lmasin.

*Yechish.* Sonni uzluksiz kasrga yoyamiz:

$$\frac{245}{83} = (2,1,19,1,3).$$

Demak kasrlarni topamiz:

| $k$ | | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $q_k$ | | 2 | 1 | 19 | 1 | 3 |
| $P_k$ | 1 | 2 | 3 | 59 | 62 | 245 |
| $Q_k$ | 0 | 1 | 1 | 20 | 21 | 83 |

$$\left|\frac{245}{83}-\frac{59}{20}\right|<\frac{1}{20\cdot 21}>\frac{1}{1000}.$$

$\delta_2$ shartni qanoatlantirmaydi.

$\delta_3=\dfrac{62}{21}$ ni keltiramiz: $\left|\dfrac{245}{83}-\dfrac{62}{21}\right|<\dfrac{1}{21\cdot 83}<\dfrac{1}{1000}$. Demak, masala yechimi $\delta_3=\dfrac{62}{21}$.

■

2-m i s o l. $\dfrac{a}{b}=(2,1,1,3,1,2)$ uzluksiz kasrga mos kasrni toping.

*Yechish.* Munosib kasrlarni topamiz:

| $k$ | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| $q_k$ | | 2 | 1 | 1 | 3 | 1 | 2 |
| $P_k$ | 1 | 2 | 3 | 5 | 18 | 23 | 64 |
| $Q_k$ | 0 | 1 | 1 | 2 | 7 | 9 | 25 |

Bu jadvaldan $\dfrac{a}{b}=\dfrac{64}{25}$.

Bu masalani yechimini quyidagicha topish mumkin:

$$\frac{a}{b}=2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2}}}}}=2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{2}{3}}}}=$$

$$=2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{3}{11}}}=2+\cfrac{1}{1+\cfrac{11}{14}}=2+\frac{14}{25}=\frac{64}{25}.$$

Bu usuldan zanjirdagi sonlar miqdori oz bo'lganda foydalanish mumkin. ■

3-m i s o l. $\dfrac{3587}{2743}$ kasrni kasrga yoyish yordamida qisqartiring.

*Yechish.* Sonni uzluksiz kasrga yoyamiz: $\dfrac{3587}{2743}=(1,3,4)$.

Demak, $\dfrac{3587}{2743}=1+\cfrac{1}{3+\cfrac{1}{4}}=1+\dfrac{4}{13}=\dfrac{17}{13}$. ■

4-m i s o l. $a$ va $b$ – o'zaro tub musbat sonlar. $\frac{a}{b}$ ni uzluksiz kasrga yoygandagi oxiridan ikkinchi munosib kasr $\frac{P_{n-1}}{Q_{n-1}}$ bo'lsin. $ax + by = 1$ Diofant tenglamasini xususiy yechimi

$$x_0 = (-1)^{n-1} Q_{n-1}; \quad y_0 = (-1)^n P_{n-1}, \quad яъни \quad ax_0 + by_0 = 1$$

ko'rinishda bo'lishini isbotlang.

*Yechish.* $\frac{a}{b}$ ni uzluksiz kasr ko'rinishda tasvirlaymiz:

$$\frac{a}{b} = (q_0, q_1, ..., q_n)$$

Ikkita munosib kasrlar orasidagi formuladan

$\frac{P_n}{Q_n} - \frac{P_{n-1}}{Q_{n-1}} = \frac{(-1)^{n-1}}{Q_n Q_{n-1}}$, lekin $\frac{P_n}{Q_n} = \frac{a}{b}$, shuning uchun $\frac{a}{b} - \frac{P_{n-1}}{Q_{n-1}} = \frac{(-1)^{n-1}}{bQ_{n-1}}$, bundan $aQ_{n-1} - bP_{n-1} = (-1)^{n-1}$, yoki $a(-1)^{n-1} Q_{n-1} + b(-1)^n P_{n-1} = 1$.

Bu tenglikni $ax_0 + by_0 = 1$ tenglik bilan solishtirsak

$x_0 = (-1)^{n-1} Q_{n-1}, \quad y_0 = (-1)^n P_{n-1}$ ni hosil qilamiz. ■

5-m i s o l. $ax + by = c$ diofant tenglamasi yechimlarini toping.

*Yechish.* 4-misoldan

$$x_0 = (-1)^{n-1} Q_{n-1} c, \quad y_0 = (-1)^n P_{n-1} c$$

kelib chiqadi.

Agar tenglamada $b$ koeffisiyentning ishorasi manfiy bo'lsa, u holda $y_0$ formulasida $(-1)^{n-1}$ ni olish kerak. Bu $x_0$ va $y_0$ qiymatlarini $x = x_0 - bt$ , $y = y_0 + at$ ga qo'yib berilgan tenglamani umumiy yechimini hosil qilamiz: $ax + by = c$. ■

6-m i s o l. Uzluksiz kasrlar yordamida $38x + 117y = 209$ tenglama umumiy yechimini toping.

*Yechish.* $\frac{38}{117}$ ni uzlksiz kasrga yoyamiz: $\frac{38}{117} = (0,3,12,1,2)$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $k$ | | | 0 | 1 | 2 | 3 | 4 |
| $Q_k$ | | | 0 | 3 | 12 | 1 | 2 |
| $P_k$ | 0 | 1 | 0 | 1 | 12 | 13 | 38 |
| $Q_k$ | 1 | 0 | 1 | 3 | 37 | 40 | 117 |

kasrlarni topamiz.

Bundan: $P_{n-1} = 13$, $Q_{n-1} = 40$, $n = 4$.

5-misoldagi formulalardan

$$x_0 = (-1)^3 \cdot 40 \cdot 209 = -8360,$$
$$y_0 = (-1)^4 \cdot 13 \cdot 209 = 2717$$

ni topamiz. Demak, tenglamani umumiy yechimi:

$$x = -8360 - 117\,t,$$
$$y = 2717 + 38\,t.$$

*Tekshirish:* 38 (- 8360) + 117 · 2717 = - 317680 + + 317889 = 209. ■

7-m i s o l. Uzluksiz kasrlar yordamida 119 $x$ – 68 $y$ = 34 tenglamani umumiy yechimimni toping.

*Yechish.* $\frac{119}{68}$ ni uzluksiz kasrga yoyamiz: $\frac{119}{68} = (1,1,3)$. Munosib kasrlarni topamiz:

| $k$ | | | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $q_k$ | | | 1 | 1 | 3 |
| $P_k$ | 0 | 1 | 1 | 2 | 7 |
| $Q_k$ | 1 | 0 | 1 | 1 | 4 |

Bundan: $P_{n-1} = 2$, $Q_{n-1} = 1$, $n = 2$ ni aniqlaymiz.

(119, 68) = 17 va $c$ = 34 son 17 ga bo'linadi. Berilgan tenglamani 17 ga bo'lib, $7x - 4y = 2$ ni hosil qilamiz.

Tenglamaning xususiy yechimi:

$$x_0 = (-1)^1 \cdot 1 \cdot 2 = -2,\ y_0 = (-1)^1 \cdot 2 \cdot 2 = -4.$$

Umumiy yechim esa: $\left.\begin{matrix} x = -2 + 4t \\ y = -4 + 7t \end{matrix}\right\}.$

*Tekshirish*: 7 (-2) – 4 (-4) = - 14 + 16 = 2. ■

## M A S H Q L A R

**55.** Kasrlarni uzluksiz kasrlarga yoying:

$a)$ 2,71828; $b)$ $\frac{103993}{33102}$; $c)$ $\frac{99}{170}$; $d)$ $\frac{355}{113}$.

**56.** Kasrlarni uzluksiz kasrlarga yoying:

$a)$ $\frac{247}{74}$; $b)$ $\frac{77}{187}$; $c)$ $\frac{333}{100}$; $d)$ $\frac{103993}{33102}$.

**57.** Uzluksiz kasrlarga yoyilmasidan foydalanib kasrlarni qiqartiring:

$a)$ $\frac{3953}{871}$; $b)$ $\frac{6059}{1241}$; $c)$ $\frac{6821}{2147}$; $d)$ $\frac{10027}{32671}$; $e)$ $\frac{3653}{3107}$

**58.** Berilgan kasrni uzluksiz kasrga yoying va uni $\frac{P_4}{Q_4}$ kasr bilan almashtiring. Almashtirish xatosini toping va xatosi ko'rsatilgan holda taqribiy almashtirishga mos tengligini yozing:

$$a)\ \frac{29}{37};\qquad b)\ \frac{648}{385};\qquad c)\ \frac{571}{359}.$$

**59.** Ko'rsatilgan chekli uzluksiz kasrlarga mos oddiy qisqarmaydigan kasrlarni toping:

$$a)\ \frac{a}{b}=(2,3,1,4);\quad b)\ \frac{a}{b}=(1,1,2,3,4);\quad c)\ \frac{a}{b}=(1,3,2,4,3,1,1,1,5);$$

$$d)\ \frac{a}{b}=(2,1,1,2,1,6,2,5);\quad e)\ \frac{a}{b}=(0,1,2,3,4,5);$$

$$f)\ \frac{a}{b}=(-2,3,1,5,4,2);\quad g)\ \frac{a}{b}=(0,13,2,2,2,1,1,7).$$

**60.** Tenglamani yeching:

$$a)\ (x,2,3,4)=\frac{73}{30};\qquad b)\ (2,1,2,x)=\frac{19}{7}.$$

**61.** Diofant tenglamalarini yeching:

*a*) 41x + 114y = 5; *b*) 19x – 15y = 1;
*c*) 23x – 17y = 11; *d*) 53x – 47y = 11;
*e*) 35x – 18y = 3; *f*) 85x – 71y = 5;
*g*) 41x – 11y = 7.

## 5-§. Sonli funksiyalar

**1.** Sonning butun qismi

$x$ sonning *butun qismi*, ya'ni $[x]$ qo'sh tengsizlik bilan $[x]\le x\le[x]+1$ yoki $x-1<[x]\le x$; yoki $x=[x]+\alpha,\ 0\le\alpha\le1$ tenglik bilan aniqlanadi va *ant'ye funksiya* deyiladi.

Agar $x_1$ va $x_2$ sonlardan birortasi butun bo'lsa,

$$[x_1+x_2]=[x_1]+[x_2]$$

o'rinli bo'ladi.

$\left[\frac{x}{m}\right]=\left[\frac{[x]}{m}\right]$ o'rinli bo'ladi.

$m!$ ko'paytmaning kanonik yoyilmasiga $p$ tub son

$$\left[\frac{m}{p}\right]+\left[\frac{m}{p^2}\right]+...+\left[\frac{m}{p^S}\right]$$

darajada keladi, bu yerda $S$ son $p^S\le m<p^{S+1}$ tengsizlikdan aniqlanadi.

1-m i s o l. $3-2\cos\frac{90\pi}{181}$ sonning butun qismini toping.

*Yechish.* $a\in\mathbf{Z}$ va $x$ kasr son uchun $[a-x]=a+[-x]$ formula o'rinli. Bu formulani qo'llab

$$\left[3-2\cos\frac{90\pi}{181}\right]=3+\left[-2\cos\frac{90\pi}{181}\right]=3+(-1)=2$$

ni hosil qilamiz. ■

2-m i s o l. $\left[\frac{x+y}{n}\right]$ ni $\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]$ yoki $\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]+1$ ga tengligini isbotlang.

*Yechish.* $\frac{x+y}{n}=\left[\frac{x}{n}\right]+\alpha+\left[\frac{y}{n}\right]+\beta$

bo'lib, bu yerda $0\le\alpha<1,\ 0\le\beta<1$. Demak,

$$\left[\frac{x+y}{n}\right]=\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]+[\alpha+\beta].$$

$0\le\alpha+\beta<2$ bo'lganligi sababli $[\alpha+\beta]$ 0 yoki 1 ga teng bo'ladi. ■

$n$ dan katta bo'lmagan va $p_1, p_2,..., p_k$ tub sonlar bilan o'zaro tub bo'lgan sonlar sonini quyidagi formula bilan hisoblash mumkin:

$$B(n;p_1,p_2,...,p_k)=[n]-\left[\frac{n}{p_1}\right]-....-\left[\frac{n}{p_k}\right]+\left[\frac{n}{p_1p_2}\right]+....+$$

$$+\left[\frac{n}{p_{k-1}p_k}\right]-\left[\frac{n}{p_1p_2p_3}\right]-....-\left[\frac{n}{p_{k-2}p_{k-1}p_k}\right]+....+(-1)^k\left[\frac{n}{p_1p_2....p_k}\right].$$

3-m i s o l. 180 dan katta bo'lsagan va 5, 7, 11 larga bo'linmaydigan sonlar sonini toping.

*Yechish.* $n = 180$ va $p_1 = 5, p_2 = 7, p_3 = 11$ lar uchun

$$B(180;5,7,11)=[180]-\left[\frac{180}{5}\right]-\left[\frac{180}{7}\right]-\left[\frac{180}{11}\right]+\left[\frac{180}{5\cdot7}\right]+\left[\frac{180}{5\cdot11}\right]+$$

$$+\left[\frac{180}{7\cdot11}\right]-\left[\frac{180}{5\cdot7\cdot11}\right]=180-36-25-16+5+3+2-0=113.$$ ■

4-m i s o l. 2002! son nechta 0 bilan tugaydi.

*Yechish.* Misol yechimi 2002! Ning kanoniy yoyilmasiga 5 nechanchi daraja bilan kirishini aniqlash masalasiga keltiriladi:

$$\left[\frac{2002}{5}\right]+\left[\frac{2002}{25}\right]+\left[\frac{2002}{125}\right]+\left[\frac{2002}{625}\right]+\left[\frac{2002}{3125}\right]=$$

$$=400+80+16+3+0=499.$$

Demak, 2002! son 499 ta 0 bilan tugaydi. ■

5-m i s o l. $(2m)!!$ ning kanonik yoyilmasiga $p$ tub son nechanchi darajada kirishini aniqlang.

*Yechish.* $(2m)!! = m!\,2^m$ bo'lganligi sababli $p = 2$ ga teng bo'lsa,

$$m+\sum_{i=1}^{k}\left[\frac{m}{2^i}\right],\ \ 2^k \le m < 2^{k+1}\ .$$

$p > 2$ bo'lsa,

$$\sum_{i=1}^{s}\left[\frac{m}{p^i}\right],\quad p^s \le m < p^{s+1}$$

ga teng bo'ladi. ■

2. H a q i q i y   s o n n i n g   k a s r   q i s m i

Haqiqiy $x$ sonning kasr qismi $\{x\}$ quyidagi formula bilan aniqlanadi: $\{x\} = x - [x]$.

6-m i s o l. {-4,35} ni toping.

*Yechish.* {-4,35} = –4,35 – (–5) = 0,65. ■

3. N a t u r a l   s o n n i n g   b o' l u v ch i l a r
s o n i   v a   u l a r   y i g' i n d i s i

Ixtiyoriy natural $a$ son uchun $\tau(a)$ va $S(a)$ funksiyalar mos ravishda $a$ sonning natural bo'luvchilari soni va ularni yig'indisini ifodalaydi. Bu funksiyalar uchun quyidagi formulalar o'rinli:

$$\tau(a)=(\alpha_1+1)(\alpha_2+1).....(\alpha_n+1)=\prod_{i=1}^{n}(\alpha_i+1)$$

$$S(a)=\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_2+1}-1}{p_2-1}....\frac{p_n^{\alpha_n+1}-1}{p_n-1}=\prod_{i=1}^{n}\frac{p_i^{\alpha_i+1}-1}{p_i-1},$$

bu yerda $a=p_1^{\alpha_1}p_2^{\alpha_2}...p_n^{\alpha_n}=\prod_{i=1}^{n}p_i^{\alpha_i}$ – $a$ sonning kanonik yoyilmasi.

Bu funksiyalar multiplikativ, ya'ni agar $(a,b) = 1$ lar uchun

$$\tau(ab) = \tau(a)\,\tau(b) \text{ va } S(ab) = S(a)S(b)$$

o'rinli.

7-m i s o l. 2002 sonni bo'luvchilar soni va ularni yig'indisini toping.

*Yechish.* 2002 = 2 · 7 · 11 · 13, bundan

$$\tau(2002)=(1+1)(1+1)(1+1)(1+1)=16,$$

$$S(2002)=\frac{2^{1+1}-1}{2-1}\cdot\frac{7^{1+1}-1}{7-1}\cdot\frac{11^{1+1}-1}{11-1}\cdot\frac{13^{1+1}-1}{13-1}=3\cdot8\cdot12\cdot14=4032\,.$$ ■

8-m i s o l. 2002 sonni barcha bo'luvchilarini toping.

*Yechish.* 2002=2·7·11·13 – kanonik yoyilmasidan foydalanamiz: (1+2)(1+7)(1+11)(1+13)=1+2+7+11+13+14+22+26+77+91+143+154+182+286+1001+2001 – 2002 ning barcha bo'luvchilari yig'indisi va demak har bir qo'shiluvchi izlanayotgan bo'linmalarni beradi. ■

9-m i s o l. Natural $a$ sonning barcha natural bo'luvchilarining ko'paytmasi funksiyasi δ($a$) bo'lsa,

$$\delta(a)=\sqrt{a^{\tau(a)}}$$

tenglik to'g'riligini isbotlang.

*Yechish.* $d_1, d_2, \ldots, d_{\tau(a)}$ –$a$ sonning barcha natural bo'luvchilari bo'lsin. U holda $\delta(a)=\prod_{i=1}^{\tau(a)} d_i = d_1 d_2 \ldots d_{\tau(a)}$. $\frac{a}{d_1}, \frac{a}{d_2}, \ldots, \frac{a}{d_{\tau(a)}}$ – sonlar $a$ ning bo'luvchilaridir, bundan $\delta(a)=\prod_{i=1}^{\tau(a)} \frac{a}{d_i} = a^{\tau(a)} \prod_{i=1}^{\tau(a)} \frac{1}{d_i}.$

δ($a$) uchun hosil bo'lgan tengliklarni ko'paytirib $\delta^2(a)=a^{\tau(a)}$ ni hosil qilamiz, bundan $\delta(a)=\sqrt{a^{\tau(a)}}$. ■

10-m i s o l. 2002 sonining barcha natural bo'luvchilari ko'paytmasini toping.

*Yechish.* $\delta(2002)=\sqrt{2002^{16}}=2002^8$. ■

11-m i s o l. Barcha natural bo'luvchilari ko'paytmasi 5832 ga teng bo'lgan natural sonni toping.

*Yechish.* $\sqrt{a^{\tau(a)}}=5832=2^3\cdot 3^6$, bundan $a=2^x\cdot 3^y$ va

$$\begin{cases} x(1+x)(1+y)=6 \\ y(1+x)(1+y)=12. \end{cases}$$

Bu sistemaning yechimi: $x$ = 1, $y$ = 2. Demak, $a$ = 18. ■

12-m i s o l. 3 va 4 ga bo'linadigan va 14 ta bo'luvchiga ega bo'lgan sonni toping.

*Yechish.* Misol shartiga ko'ra, τ(a) = 14 = 2·7 == (1+1)(6+1),

demak, $a=p_1^{\alpha_1}p_2^{\alpha_2}$, ya'ni $a=2^{\alpha_1}\cdot 3^{a_2}$, bu yerda $\alpha_1\geq 2$, $\alpha_2\geq 1$. Demak, $a = 2^6\cdot 3 = 192$. ■

## 3. Berilgan musbat sondan katta bo'lmagan tub sonlar soni

π ($x$) barcha natural $x$ lar uchun aniqlangan bo'lib, natural sonlar qatorida $x$ dan katta bo'lmagan tub sonlar sonni bildiradi. π ($x$) ni qiymatini tub sonlar jadvali yordamida aniqlanadi yoki yetarlicha katta $x$ lar uchun taqribiy hisoblash mumkin:

$$\pi(x)\approx\frac{x}{\ell n x} \quad \text{va} \quad \pi(x)\approx\int_2^x \frac{du}{\ell n u}.$$

13-m i s o l. $\pi(x)=\frac{x}{\ell n\, x}$ formula yordamida π (1000) ni qiymatini toping va natijaning nisbiy xatosini hisoblang.

*Resheniye.* $\pi(1000)\approx\frac{1000}{3\ell n 10}\approx\frac{1000}{6{,}9078}\approx 145.$

Tub sonlar jadvalidan π(1000)=168, demak nisbiy xato

$$\frac{\Delta\pi(1000)}{\pi(1000)}=\frac{168-145}{168}\approx 14\% .\ ■$$

## 4. E y l ye r f u n k s i ya s i

*φ (a) – Eyler funksiyasi a* sonning barcha natural qiymatlarida aniqlangan bo'lib, *a* dan katta bo'lmagan va u bilan o'zaro tub bo'lgan sonlar sonini bildiradi. φ(1) = 1 deb qabul qilingan. Eyler funksiyasi:

$$\varphi(a)=a\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)....\left(1-\frac{1}{p_n}\right)=\left(p_1^{\alpha_1}-p_1^{\alpha_1-1}\right)\left(p_2^{\alpha_2}-p_2^{\alpha_2-1}\right)....\left(p_n^{\alpha_n}-p_n^{\alpha_n-1}\right)$$

formula yordamida hisoblanadi, bu yerda $a=p_1^{\alpha_1}\,p_2^{\alpha_2}...p_n^{\alpha_n}$ – sonning kanonik yoy-ilmasi.

Xususan, $\varphi(p^{\alpha})=p^{\alpha}-p^{\alpha-1}$, $\qquad\varphi(p)=p-1$.

Eyler funksiyasi multiplikativ, ya'ni o'zaro tub $a,b,\dots,\ell$ sonlar uchun $\varphi(ab...\ell)=\varphi(a)\varphi(b)...\varphi(\ell)$ shart bajariladi.

14-m i s o l. φ (1956) ni hisoblang.

*Yechish.* 1956=$2^2$·3·163 bo'lganligi sababli

$$\varphi(1956)=\left(2^2-2\right)(3-1)(163-1)=2\cdot 2\cdot 162=648.\ ■$$

15-m i s o l. φ (12 · 5 · 1956) ni hisoblang.

*Yechish.* O'zaro tub ko'paytuvchilarni aniqlash uchun ko'paytmani kanonik yoyilmasini topamiz:

$12\cdot 5\cdot 1956=2^2\cdot 3\cdot 5\cdot 2^2\cdot 3\cdot 163=2^4\cdot 3^2\cdot 5\cdot 163$. Bundan

$$\varphi(12\cdot 5\cdot 1956)=\varphi\left(2^4\cdot 3^2\cdot 5\cdot 163\right)=\left(2^4-2^3\right)\left(3^2-3\right)(5-1)(163-1)=$$
$$=8\cdot 6\cdot 4\cdot 162=31104.\ ■$$

16-m i s o l. $\varphi\left(3^x\cdot 5^y\right)=600$ tenglamani yeching.

*Yechish.* 600 = $2^3$ · 3 · $5^2$ dan $\varphi\left(3^x\cdot 5^y\right)=2^3\cdot 3\cdot 5^2$. Boshqa tomondan

$$\varphi\left(3^x5^y\right)=\left(3^x-3^{x-1}\right)\left(5^y-5^{y-1}\right).$$

Demak, $3^{x-1}\cdot 2\cdot 5^{y-1}\cdot 4=2^3\cdot 3\cdot 5^2$ yoki $3^{x-1}5^{y-1}=3\cdot 5^2$ va $x=2$, $y=3$. ■

17-m i s o l. *a* = 72 uchun *Gauss formulasini* to'g'riligini ko'rsating: $\sum_{d/a}\varphi(d)=a$.

*Yechish.* Gauss formulasi $\sum_{d/a}\varphi(d)=a$ da *a* = 72 deb olamiz: *a* = 72 = $2^3$ · $3^2$ .

72 ning barcha bo'luvchilari:

(1 + 2 + $2^2$ + $2^3$)(1 + 3 + $3^2$).

$$\sum_{d/72}\varphi(d)=\left[\varphi(1)+\varphi(2)+\varphi\left(2^2\right)+\varphi\left(2^3\right)\right]\left[\varphi(1)+\varphi(3)+\varphi\left(3^2\right)\right]=$$
$$=(1+1+2+4)\,(1+2+6)=8\cdot 9=72=a.\ ■$$

18-m i s o l. $\varphi(x) = p - 1$ tenglamani yeching.

*Yechish.* $x = r^{\alpha}\cdot y$ deb olamiz, bu yerda $(y, r) = 1$.
$r^{\alpha-1}\varphi(y) = 1$, bundan $\alpha = 1$ va $\varphi(y) = 1$. Demak, $r = 2$ da tenglama yagona $x = 2$ (chunki bu holda $y = 1$); $r > 2$ da tenglama ikkita: $x = r$; $2r$ yechimga ega. ■

19-m i s o l. Eyler funksiyasining xossalaridan foydalanib tub sonlar soni cheksiz ko'pligini isbotlang.

*Yechish.* $r_1, r_2,\ldots,r_k$ – barcha tub sonlar bo'lsin, u holda

$a = r_1 \cdot r_2 \ldots r_k$ son uchun $\varphi(a) = (r_1 - 1)(r_2 - 1)\ldots(r_k - 1)$ bo'ladi. Boshqa tomondan $\varphi(a)=1$, chunki ixtiyoriy birdan farqli va $a$ dan katta bo'lmagan son oddiy bo'luvchiga ega va bu bo'luvchi $r_i$ lardan birortasiga teng, shu sababli bu son $a$ bilan o'zaro tub bo'la olmaydi. Demak, $(r_1-1)(r_2-1)\ldots(r_k-1)=1$, lekin bu tenglik $k = 2$ dan boshlab o'rinli emas, (2-1)(3-1) > 1 hosil qilingan qarama-qarshilik tub sonlar soni cheksizligini bildiradi.

## 5. Myobius funksiyasi

Barcha natural sonlar uchun aniqlangan

$$\mu(a)=\begin{cases}1, & \text{агар} \quad a=1\\ (-1)^k, & \text{агар} \quad a=p_1p_2\ldots p_k, \quad p_i\neq p_j \quad i\neq j\\ 0, & \text{агар} \quad a \text{ туб сон квадратига} \quad \text{булинса}\end{cases}$$

ko'rinishdagi funksiyaga *Myobius funksiyasi* deb ataladi.

Bu funksiya multiplikativdir, ya'ni agar $(a,b)=1$ bo'lsa, $\mu(a,b)=\mu(a)\mu(b)$.

Agar $\theta(a)$ – ixtiyoriy multiplikativ funksiya bo'lsa, u holda

$$\sum_{d/a}\mu(d)\,\theta(d)=\begin{cases}1, & \text{агар} \quad a=1\\ \prod_{p/a}(1-\theta(p)), & \text{агар} \quad a\neq 1.\end{cases}$$

Agar bu formulada $\theta(a)\equiv 1$ va $\theta(a)=\dfrac{1}{a}$ deb olsak quyidagshi formulalarni hosil qilamiz:

$$\sum_{d|a}\mu(d)=\begin{cases}1, & \text{агар} \quad a=1\\ 0, & \text{агар} \quad a\neq 1\end{cases}$$

$$\sum_{d|a}\frac{\mu(d)}{d}=\begin{cases}1, & \text{агар} \quad a=1\\ \prod_{p/a}\left(1-\dfrac{1}{p}\right), & \text{агар} \quad a>1.\end{cases}$$

Agar butun $a$ lar uchun $f(a)$ – funksiya birqiymatli bo'lib,

$$F(a)=\sum_{d|a} f(d) \quad (d>0)$$

o'rinli bo'lsa, u holda

$$f(a)=\sum_{d|a}\mu(d)F\left(\frac{a}{d}\right)$$

tenglik o'rinlidir (*Myobiusning teskarilash formulasi*).

20-m i s o l. μ (2002) ni hisoblang.

*Yechish.* 2002 = 2·7·11·13 dan μ (2002) = $(-1)^4$ = 1 kelib chiqadi.■

21-m i s o l.

$$\sum_{d|a}\mu(d)=0 \quad a=18$$ uchun to'g'riligini isbotlang.

*Yechish.* 18 ning bo'luvchilari: 1, 2, 3, 6, 9, 18. Bundan

$$\sum_{d|18}\mu(d)=\mu(1)+\mu(2)+\mu(3)+\mu(6)+\mu(9)+\mu(18)=$$
$$=1+(-1)+(-1)+1+0+0=0.$$

22-m i s o l. $\sum_{d|a}\frac{\mu(d)}{d}=\prod_{p|a}\left(1-\frac{1}{p}\right)$ formula to'g'riligini $a=12$ uchun tekshiring.

*Yechish.* 12 ning bo'luvchilari: 1, 2, 3, 4, 6, 12. Bundan

$$\sum_{d|a}\frac{\mu(d)}{d}=\frac{\mu(1)}{1}+\frac{\mu(2)}{2}+\frac{\mu(3)}{3}+\frac{\mu(4)}{4}+\frac{\mu(6)}{6}+\frac{\mu(12)}{12}=$$
$$=1-\frac{1}{2}-\frac{1}{3}+0+\frac{1}{6}+0=\frac{1}{3}.$$

$$\prod_{p|12}\left(1-\frac{1}{p}\right)=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)=\frac{1}{2}\cdot\frac{2}{3}=\frac{2}{6}=\frac{1}{3}.$$ ■

## M A S H Q L A R

**62.** Toping:

$a)\ [-2{,}7];\quad b)\ [2+\sqrt[3]{987}];\quad c)\ \left[\frac{7-\sqrt{21}}{2}\right];\quad d)\ \left[\frac{10}{3+\sqrt{3}}\right];$

$e)\ \left[1{,}(3)+2tg\frac{\pi}{4}\right];\quad f)\ \left[3+\sin\frac{13\pi}{7}\right];\quad g)\ [2-\lg 2512];$

$h)\ [2-\lg abcd];\quad i)\ [\sqrt{30}+\sqrt[3]{10}];\quad j)\ [1-\ln 50].$

**63*.** Barcha haqiqiy $x$ va $y$ lar uchun $[x+y]\ge[x]+[y]$ to'g'riligini isbotlang.

**64*.** $[ax]=m$ tenglamani yechimini toping, bu yerda $a\ne 0$, $x\in\mathbf{R}$.

**65*.** $m$ ning qanday butun musbat qiymati uchun
$[12{,}4\cdot m]=86$ tenglik o'rinli bo'ladi.

**66*.** Agar $p > 2$ tub son bo'lsa, $\left[\frac{p}{4}\right]$ ning qiymati $\frac{p-1}{4}$ yoki $\frac{p-3}{4}$ ga tengligini isbotlang.

**67*.** $a$ sonni $m$ ga bo'lganda qoldiq $r$ bo'lsa, $\left[\frac{a}{m}\right] = \frac{a-r}{m}$ tenglikni isbotlang.

**68*.** Agar $m$ toq son bo'lsa, $\left[\frac{m}{2}\right] = \frac{m-1}{2}$ ni isbotlang.

**69***. Tenglamani yeching:

$a)\ [x^2] = 2;\qquad b)\ [3x^2 - x] = x + 1;$

$c)\ [x] = \frac{3}{4}x;\quad d)\ [x^2] = x.$

**70*.** $10^6$ va $10^7$ sonlar orasida 786 ga karrali bo'lgan nechta natural son bor?

**71*.** 1000 kichik natural sonlardan nechtasi 5 va 7 ga bo'linadi?

**72*.** 100 dan katta bo'lmagan natural sonlardan nechtasi 36 bilan o'aro tub?

**73.** 1000! ning kanonik yoyilmasida 11 nechanchi darajada keladi?

**74.** 1964! soni nechta nol bilan tugaydi?

**75.** 2311 dan oshmaydiganva 5, 7, 13, 17 larga bo'linmaydigan butun musbat sonlar soni nechta?

**76.** Nayti kolichestvo selыx polojitelnыx chisel, ne prevosxodyaщix 110 i vzaimno prostыx s chislom 36.

**77.** 12317 dan katta bo'lmagan va 1575 bilan o'zaro tub bo'lgan butun musbat sonlar sonini toping.

**78.** 1000 dan katta bo'lmagan va 363 bilan o'zaro tub bo'lgan butun musbat sonlar sonini toping.

**79.** $r^n$ ! ning kanonik yoyilmasiga $p$ tub son nechanchi darajada keladi?

**80.** Sonlarni kanonik yoyilmasini toping:

*a)* 10! ; *b*) 15! ; *c)* 20! ; *d*) 25! ; *e*) 30! .

**81.** $\frac{20!}{10!\ \ 10!}$ ni kanonik yoyilmaini toping.

**82*.** $\alpha$ ning shunday eng katta qiymatini topingki, bunda

$$N = \frac{101 \cdot 102 ... 1000}{7^{\alpha}} -$$ butun son bo'lsin.

**83*.** $(2m+1)!!$ ning kanonik yoyilmasida $p$ tub son nechanchi darajada bo'lishini aniqlang.

**84*.** $a \le x \le b, \quad 0 \le y \le f(x),$ egri chiziqli trapesiyada butun koordinatali nuqtalar soni nechta? Bu yerda $a$ va $b$ – natural sonlar; $f(x)$ – berilgan kesmada uzluksiz va nomanfiy funksiya.

**85.** $x^2 + y^2 = 6{,}5^2$ doirada nechta butun koordinatali nuqta bor?

**86*.** Agar $(a, 4) = 1$ bo'lsa,

$\left[\frac{a}{4}\right] + \left[\frac{2a}{4}\right] + \left[\frac{3a}{4}\right] = \frac{3(a-1)}{2}$ tenglik to'g'riligini isbtolang.

**87*.** Agar $(a, m) = 1$, $m \geq 2$, $a \geq 2$ bo'lsa,

$$\left[\frac{a}{m}\right]+\left[\frac{2a}{m}\right]+\ldots+\left[\frac{(m-1)a}{m}\right]=\frac{(m-1)(a-1)}{2}$$

tenglik to'g'riligini isbotlang.

**88*.** $x$ ning qanday qiymatlarida $[x]-2\left[\frac{x}{2}\right]=1$ tenglik o'rinli.

**89*.** $\left[\frac{x}{m}\right]=\left[\frac{x}{m-1}\right]$ tenglamani yeching, bu yerda $m = 2, 3, 4...$

**90**. Qanday shartlar bajarilganda $[ax^2 + bx + c] = d$ tenglama yechimga ega bo'ladi, bu yerda $a \neq 0$, $d \in \mathbf{Z}$.

**91**. Toping: $a)\ \{2{,}6\}$; $b)\ \left\{\frac{8}{3}\right\}$; $c)\ \{7\}$; $d)\ \left\{-2\frac{1}{2}\right\}$.

**92**. Berilgan sonlarni natural bo'luvchilari va ular yig'indisini toping:
*a*) 375 ; *b*) 720 ; *c*) 957 ; *d*) 988 ; *e*) 990 ; *f*) 1200.

**93**. Berilgan sonlarning barcha bo'luvchilarini toping:

*a*) 360 ; *b*) 375.

**94*.** $S(m) = 2m - 1$ sharti qanoatlantiruvchi natural $m$ sonlar cheksiz ko'pligini isbotlang.

**95*.** Agar $(m, n) > 1$ bo'lsa, $\tau(m\,n)$ yoki $\tau(m)\,\tau(n)$ lardan qaysisi katta, $S(mn)$ va $S(m)\,S(n)$ larchi?

**96**. Agar $m = 1968$ bo'lsa, $\tau(m)$, $S(m)$, $\delta(m)$ larni toping.

**97*.** O'zining natural bo'luvchilari ko'paytmasiga teng bo'lgan barcha natural sonlar to'plami barcha tub sonlar to'plami bilan ustma-ust tushishini isbotlang.

**98*.** $a$ natural sonning barcha natural bo'luvchilarining $n$-darajasi ($n \in \mathbf{Z}$) yig'indisi $S_n(a)$ formulasini keltirib chiqaring.

**99**. Toping: *a*) $S_2(12)$; *b*) $S_2(18)$; *c*) $S_2(16)$.

**100**. 28, 496, 8128 sonlar mukammal, ya'ni o'zining bo'luvchilari yig'indisining yarmiga tengligini isbotlang.

**101*.** *Yevklid teoremasi*ni isbotlang: $2^{\alpha}(2^{\alpha+1} - 1)$ ko'rinishdagi juft natural sonlar mukammal sonlardir, bu yerda $2^{\alpha+1} - 1$ – tub son.

**102*.** *Eyler teoremasi*ni isbotlang: $2^{\alpha}(2^{\alpha+1} - 1)$ ko'rinishdagi natural sonlar, yagona mukammal juft sonlardir, bu yerda $2^{\alpha+1} - 1$ – tub son.

**103*.** *Ferma masalasi*: $2^{\alpha}\cdot r_1\, r_2$ ko'rinishdagi shunday eng kichik son topingki, uning barcha bo'luvchilari yig'indisi o'zidan uch marta katta bo'lsin, bu yerda $r_1$ va $r_2$ – tub sonlar.

**104*.** Shunday son topingki, uning ikkita tub bo'luvchisi bo'lib, barcha bo'luvchilarning soni 6 ta yig'indisi 28 ga teng bo'lsin.

**105*.** Natural son ikkita tub bo'luvchiga ega. Shu son kvadratining barcha bo'luvchilari soni 15 ta bo'lsa, uning kubi nechta bo'luvchiga ega?

**106***. Natural son ikkita tub bo'luvchiga ega. Shu son kvadratining barcha bo'luvchilari soni 81 ta bo'lsa, uning kubi nechta bo'luvchiga ega?

**107***. Isbotlang:

$$N = \frac{d_1 + d_2 + ... + d_{n-1} + d_n}{\frac{1}{d_1} + \frac{1}{d_2} + ... + \frac{1}{d_{n-1}} + \frac{1}{d_n}},$$

bu yerda $d_1$, $d_2$,..., $d_n$ – $N$ sonning barcha bo'luvchilari.

**108.*** Agar $N = a^{\alpha}\ b^{\beta}\ \dots\ m^{\mu}$ *(a, b,., m*$\in$ ***Z****)* bo'lsa, shu sonni ikkita son ko'paytmasi shaklida necha xilda yozish mumkin?

**109.*** $N = 2^{\alpha}\,5^{\beta}\,7^{\Upsilon}$ son berilgan**.** Agar $5N$ $N$ dan kichik 8 ta bo'luvchiga, $8N$ – $N$ dan katta .

**110.** $N = 2^x\ 3^{-y}\ 5^z$ son berilgan. Agar $N$ ni 2 ga bo'lsak, yangi sonning bo'luvchilari $N$ ning bo'luvchilaridan 30 ta kam; agar $N$ ni 3 ga bo'lsak, yangi sonning bo'luvchilari $N$ ning bo'luvchilaridan 35 ta kam; agar $N$ ni 5 ga bo'lsak, yangi sonning bo'luvchilaridan 42 ta kam. Shu sonni toping.

**111.** Agar biror son to'la kvadrat bo'lishi uchun faqat va faqat uning bo'luvchilari soni toq bo'lishini isbotlang.

**112.** Quyidagilarni aniq qiymatini hisoblang:

*a)* π (4); *b*) π (7); *c*) π (10); *d*) π (12); *e*) π (25);

*f)* π (50); *g*) π (200); *h*) π (500).

**113.** $\pi(x) \approx \frac{x}{\ell nx}$ formula yordamida quyidagilarni taqribiy qiymatini va natijaning nisbiy xatosini toping:

*a*) π (50), b) π (100); *c*) π (500).

**114*.** Chebыshev tengsizligi yordamida

$$\frac{\pi(x)}{x} \to 0 \quad (x \to +\infty)$$

ni isbotlang.

**115*.** Ixtiyoriy p tub son uchun $\frac{\pi(p-1)}{p-1} < \frac{\pi(p)}{p}$ o'rinli, lekin $m$ - murakkab son bo'lsa, $\frac{\pi(m)}{m} < \frac{\pi(m-1)}{m-1}$ o'rinligini ko'rsating.

**116.** Toping:

$a)\ \varphi(375);\quad b)\ \varphi(720);\quad c)\ \varphi(988);\quad d)\ \varphi(1200);$

$e)\ \varphi(1500);\quad f)\ \varphi(4320)$

**117.** Ko'paytma qiymatini topmasdan ko'paytuvchilarning Eyler funksiyasini qiymatini toping:

*a*) φ (5 · 7 · 13) ; *b*) φ (12 · 17); *c*) φ (11 · 14 · 15·);

*d*) φ (990 · 1890).

**118.** 1 dan 120 gacha sonlar intervalida 30 bilan o'zaro tub bo'lmagan sonlar nechta?

**119***. Agar $a = 3^{\alpha} 5^{\beta} 7^{\Upsilon}$ va $\varphi(a) = 3600$ bo'lsa, $a$ ni toping.

**120***. Agar $a = pq$, $p - q = 2$ va $\varphi(a) = 120$ bo'lsa, $a$ ni toping. Bu yerda $p$ va $q$ – har xil tub sonlar har xil tub sonlar.

**121***. Agar $a = p^2 q^2$ va $\varphi(a) = 11424$ bo'lsa, $a$ ni toping. $p$ va $q$ – har xil tub sonlar.

**122***. Agar $a = p_1^{\alpha_1} p_2^{\alpha_2} ... p_n^{\alpha_n}$ ($\alpha_1>1$, $\alpha_2>1,\dots$, $\alpha_n > 1$) va $\varphi(a) = 462000$ bo'lsa, $a$ ni toping.

**123***. $m$ dan kichik va u son bilan o'zaro tub sonlar yig'indisi $S = \frac{1}{2} m \cdot \varphi(m)$ formula yordamida hisoblashini isbotlang.

**124**. $S = \frac{1}{2} a \cdot \varphi(a)$ formulani quyidagi sonlar uchun qo'llang: *a*) 12; *b*) 18; *c*) 375.

**125***. Isbotlang:

$a)\ \varphi(2^{\alpha}) = 2^{\alpha-1};\ b)\ \varphi(p^{\alpha}) = p^{\alpha-1}\varphi(p);\quad c)\ \varphi(a^{\alpha}) = a^{\alpha-1}\varphi(a),\ a \in \boldsymbol{N}.$

**126**. φ (2*a*) ni φ (*a*) yoki $2\varphi$ (*a*) ga tengligini isbotlang. Shu sonlar o'rinli bo'ladigan shartlarni toping.

**127***. Isbotlang: a) φ (4*n* + 2) = φ (2*n* + 1);

$$b)\ \varphi(4n) = \begin{cases} 2\varphi(n), & agar \quad (n,2) = 1 \\ 2\varphi(2n), & agar \quad (n,2) = 2 \end{cases}$$

**128**. Tenglamalarni yeching: *a*) $\varphi(5^x) = 100$; *b*) $\varphi(7^x) = 294$; *c*) $\varphi(7^x) = 705894$; *d*) $\varphi(r^x) = r^{x-1}$, $x \in N$.

**129**. Berilgan *b* maxrajli nechta to'g'ri qisqarmas musbat kasrlar mavjud?

**130**. 129 masala yordamida maxrajlari quyidagilar bo'lgan qisg'armas musbat kasrlar sonini toping:

*a*) 10; *b*) 16; *c*) 36; d) 72.

**131**. $\frac{a}{b}$ musbat, to'g'ri qisqarmas kasr bo'lsin. Agar $b = 2$ dan $b = n$ gacha qiymatlar qabul qilsa, bunday kasrlar nechta?

**132**. 131 masala shartida *b*: a) 2 dan 5 gacha; b) 2 dan 10 gacha; c) 2 dan 15gacha qiymatlar qabul qilsa, kasrlar sonini toping.

**133***. 300 dan kichik natural sonlar ichida 20 bilan teng umumiy bo'luvchiga ega bo'lgan sonlar nechta?

**134**. 1665 dan kichik natural sonlar ichida u bilan 37 ga teng umumiy bo'luvchiga ega bo'lgan sonlar nechta?

**135**. 1476 dan kichik natural sonlar ichida u bilan 41 ga teng umumiy bo'luvchiga ega bo'lgan sonlar nechta?

**136***. $a \geq 3$ lar uchun φ (*a*) ning qiymati doimo juft son bo'lishini isbotlang.

**137***. Agar φ (*x*) = *a* tenglama *x* = *m* ildizga ega bo'lsa,

$x = 2m$ ham tenglama ildizi bo'lishini isbotlang, bu yerda $(m,2) = 1$.

**138***. $(m,n) > 1$ bo'lsa, $\varphi(m\,n)$ yoki $\varphi(m)\,\varphi(n)$ larni solishtiring?

**139***. $\varphi(mn) = \varphi(m)\varphi(n)\cdot\dfrac{d}{\varphi(d)}$ tenglikni isbotlang, bu yerda $(m,n) > 1$.

**140***. $\varphi(m\,n) = \varphi(\delta)\,\varphi(\mu)$ tenglikni isbotlang, bu yerda $\delta = (m,n)$, $\mu = [m,n]$.

**141**. $\varphi(1) + \varphi(p) + \varphi(p^2) + \ldots + \varphi(p^\alpha)$, $\alpha \in \mathbf{N}$ ni hisoblang.

**142**. $\varphi\left(\dfrac{a}{d_1}\right) + \varphi\left(\dfrac{a}{d_2}\right) + \ldots + \varphi\left(\dfrac{a}{d_k}\right)$ ni hisoblang, bu yerda $d_i$ –$a$ ning barcha bo'luvchilari?

**143**. Quyidagi sonlar uchun $\sum\limits_{d/a} \varphi(d) = a$ to'g'riligini tekshiring: a) 80; b) 360; c) 375; d) 957; e) 2800.

**144**. Tenglamalarni yeching: a) $\varphi(x) = 2^\alpha$; b) $\varphi(p^x) = 6 \cdot p^{x-2}$.

**145**. Tenglamalarni yeching: a) $\varphi(x) = 14$; b) $\varphi(x) = 8$;

c) $\varphi(x) = 12$.

**146***. Tenglamani yeching: $\varphi(2x) = \varphi(3x)$.

**147**. $\varphi(5x) = \varphi(7x)$ tenglama butun sonlar to'plamida yechimga ega emasligini isbotlang.

**148**. Tenglamalarni yeching: *a*) $\varphi(x) = \varphi(p\,x)$;

*b*) $\varphi(p\,x) = p\varphi(x)$;

*c*) $\varphi(p_1 x) = \varphi(p_2 x)$ ($p_1$, $p_2$ – turli tub sonlar).

**149***. Tenglamani yeching:

$$a)\ \varphi(x) = \frac{x}{2};\quad b)\ \varphi(x) = \frac{x}{3};\quad c)\ \varphi(x) = \frac{x}{4}.$$

**150**. $\varphi(p^x) = a$ tenglamani tekshiring.

**151.** $a = 1, 2, \ldots, 100$ sonlar uchun $\mu(a)$ funksiyaning jadvalini tuzing.

**152.** $a = 24$ uchun $\sum\limits_{1/a} \mu(d) = 0$ formula to'g'riligini tekshiring.

**153**. $a = 18$ uchun $\sum\limits_{d/a} \dfrac{\mu(d)}{d} = \prod\limits_{p/a}\left(1 - \dfrac{1}{p}\right)$ formula to'g'riligini isbotlang.

II-BOB

# BUTUN SONLAR XALQASIDA TAQQOSLAMALAR NAZARIYASI

Kalit so'zlar va ifodalar: taqqoslanuvchi sonlar*; taqqoslamaning ma'nosi haqidagi teorema; sonlar sinfi; berilgan modul bo'yicha chegirma; berilgan modul bo'yicha chegirmalarning to'la sinfi, berilgan modul bo'yicha chegirmalarning keltirilgan sinfi, Eyler teoremasi, Ferma teoremasi; berilgan modul bo'yicha chegirmalarning additiv gruppasi; berilgan modul bo'yicha chegirmalarning xalqasi; modul bilan o'zaro tub chegirmalar sinfi; modul bilan o'zaro tub chegirmalarningn multiplikativ gruppasi; absolyut psevdotub son; Bir noma'lumli n-darajali taqqoslama; taqqoslamaning yechimi; teng kuchli taqqoslamalar; birinchi darajali taqqoslamalar; birinchi darajali bir xil noma'lumli taqqoslamalar sistemasi; birinchi darajali bir xil noma'lumli taqqoslamalar sistemasining yechimlari.*

## §1. Taqqoslama tushunchasi va uning xossalari

$a$ va $b$ *butun sonlarni* butun musbat $m$ soniga bo'lganda bir xil qoldiq qoladigan, ya'ni

$$a = mq_1 + r \qquad \text{va} \qquad b = mq_2 + r,$$

bo'lsa, $a$ va $b$ *sonlar* teng qoldiqdli yoki $m$ modul bo'yicha o'zaro taqqoslanadigan sonlar deyiladi va quyidagicha yoziladi:

$$a \equiv b \ (mod\ m)$$

"*a son b bilan m modul bo'yicha taqqoslanadi*" deb o'qiladi.

Agar $a \equiv b \ (mod\ m)$ bo'lsa, u holda $a - b$ ayirma $m$ ga qoldiqsiz bo'linadi, va aksincha, agar $a$ va $b$ sonlarning ayirmasi $m$ ga bo'linsa, u holda $a \equiv b \ (mod\ m)$ o'rinli bo'ladi (*taqqoslamaning ma'nosi haqidagi teorema*).

Har qanday butun son $m$ modul bo'yicha o'zining qoldig'i bilan taqqoslanadi, ya'ni, agar $a = mq + r$ bo'lsa, u holda $a \equiv r \ (mod\ m)$ bo'ladi.

Xususiy holda, agar $r = 0$ bo'lsa, u holda $a \equiv 0 \ (mod\ m)$ bo'ladi; bu taqqoslama $m \mid a$ ekanligini, ya'ni $m$ soni $a$ ning bo'luvchisi ekanligini bildiradi, aksincha ham o'rinli, agar $m \mid a$ bo'lsa, u holda $a \equiv 0 \ (mod\ m)$ deb yoziladi.

Taqqoslamalarning asosiy xossalari
(tengliklarning xossalariga o'xshash)

1. Agar $a \equiv cs \ (mod\ m)$ va $b \equiv c \ (mod\ m)$ bo'lsa, u holda $a \equiv b \ (mod\ m)$ bo'ladi.
2. Agar $a \equiv b \ (mod\ m)$ va $c \equiv d \ (mod\ m)$ bo'lsa, u holda $a \pm c \equiv b \pm d \ (mod\ m)$ bo'ladi.
3. Agar $a + b \equiv c \ (mod\ m)$ bo'lsa, u holda $a \equiv c - b \ (mod\ m)$ *bo'ladi*.
4. Agar $a \equiv b \ (mod\ m)$ bo'lsa, u holda $a \pm mk \equiv b \ (mod\ m)$, yoki $a \equiv b \pm mk \ (mod\ m)$ bo'ladi.

5. Agar $a \equiv b \ (mod\ m)$ va $c \equiv d \ (mod\ m)$ bo'lsa, u holda $ac \equiv bd \ (mod\ m)$ *bo'ladi*.
6. Agar $a \equiv b \ (mod\ m)$ bo'lsa, u holda $a^n \equiv b^n \ (mod\ m)\ (n \in \mathbf{N})$ *bo'ladi*.
7. Agar $a \equiv b \ (mod\ m)$ bo'lsa, u holda ixtioriy $k$ butun son uchun $ak \equiv bk \ (mod\ m)$ *bo'ladi,.*
8. Agar $ak \equiv bk \ (mod\ m)$ va $(k,m) = 1$ bo'lsa, u holda $a \equiv b \ (mod\ m)$ *bo'ladi*.
9. Agar $f(x) = a_0 x^n + a_1 x^{n-1} + ... + a_n \ (a_i \in \mathbf{Z})$ va $x \equiv x_1 \ (mod\ m)$ bo'lsa, u holda $f(x) \equiv f(x_1) \ (mod\ m)$ *bo'ladi*.

Taqqoslamalarninng maxsus xossalari

1. Agar $a \equiv b \ (mod\ m)$ bo'lsa, u holda $k \in \mathbf{N}$ *uchun* $ak \equiv bk \ (mod\ mk)$ bo'ladi.
2. Agar $a \equiv b \ (mod\ m)$ va $a = a_1 d, \quad b = b_1 d, \quad m = m_1 d$ bo'lsa, u holda $a_1 \equiv b_1 \ (mod\ m_1)$ bo'ladi.
3. Agar $a \equiv b \ (mod\ m_1), a \equiv b \ (mod\ m_2), ..., a\ (\ b \ (mod\ m_k)$ bo'lsa, u holda $a \equiv b \ (mod\ M)$ bo'ladi, bu yerda $M = [m_1, m_2,..., m_k]$.
4. Agar taqqoslama $m$ modul bo'yicha o'rinli bo'lsa, u holda bu taqqoslama $m$ ning ixtiyoriy bo'luvchisi bo'lgan $d$ modul bo'yicha ham o'rinli bo'ladi.
5. Agar taqqoslamaning bir tomoni biror songa bo'linsa, u holda uning ikkinchi tomoni va moduli ham shu songa bo'linadi.

**1-Misol**. Quyidagi shartlarni taqqoslamalar yordamida yozing:

a) 219 va 128 sonlarni 7 ga bo'lganda bir xil qoldiq qoladi;
b) (-352) sonini 31 ga bo'linganida qoldiq 20 ga teng bo'ladi ;
c) 487 - 7 ayirma 12 ga bo'linadi; d) 20 – soni 389 ni 41 ga bo'lgandagi qoldiqdan iborat;
e) $N$ soni juft; f) $N$ soni toq; g) $N$ sonining ko'rinishi $4k + 1$ dan iborat;
h) $N$ sonining ko'rinishi $10k + 3$ dan iborat; i) $N$ sonining ko'rinishi $8k - 3$ dan iborat.

*Yechilishi.* Taqqoslamaning ma'nosi haqidagi teoremaga asosan:

a) $219 \equiv 128 \ (mod\ 7)$; b) $-352 \equiv 20 \ (mod\ 31)$; c) $487 \equiv 7 \ (mod\ 12)$; d) $389 \equiv 20 \ (mod\ 41)$;

e) $N \equiv 0 \ (mod\ 2)$; f) $N \equiv 1$ yoki $-1 \ (mod\ 2)$; g) $N \equiv 1 \ (mod\ 4)$; h) $N \equiv 3 \ (mod\ 10)$; i) $N \equiv -3 \ (mod\ 8)$. ■

**2-Misol**. Quyidagi shartni qanoatlantiradigan $m$ ning qiymatlarini toping:

$$20 \equiv 8 \ (mod\ m).$$

*Yechilishi*. $m$ ning qiymatlari (taqqoslamaning ma'nosi haqidagi teoremaga asosan) $20 - 8 = 12$ ning bo'luvchilaridan iborat, ya'ni: 1; 2; 3; 4; 6; 12. ■

**3-Misol**. $2^{5n} - 1$ ning 31 ga bo'linishini isbotlang $(n \in \mathbf{N})$.

*Yechilishi*. $2^5 - 1 = 31$ bo'lganligi uchun $2^5 \equiv 1 \ (mod\ 31)$. Bu taqqoslamaning ikkala tomonini (6-xossaga asosan) $n$ darajaga ko'tarib, $2^{5n} \equiv 1 \ (mod\ 31)$ ni hosil qilamiz, bu esa $31 \mid (2^{5n} - 1)$ ni anglatadi. ■

**4-Misol**. $2^{100}$ sonining oxirgi ikkita raqamini toping.

*Yechilishi*. Berilgan sonning oxirgi ikki raqami bu sonni 100 ga bo'lganda hosil bo'ladigan qoldiqdan iborat. Demak, quyidagi taqqoslamani qanoatlantiradigan $x$ sonini topish talab qilinadi:

$$2^{100} \equiv x \ (mod\ 100).$$

Ikkining kichik darajalaridan boshlab, 100 ga bo'lganda hosil bo'ladigan qoldiqlarni ketma-ket ajratamiz:

$$2^{100} = (2^{10})^{10} = (1024)^{10};\ (1024)^{10} \equiv (24)^{10}\ (mod\ 100).$$

$(24)^{10} = (576)^5 \equiv 76^5 \equiv (76)^4 \cdot 76 = (5776)^2 \cdot 76 \equiv (76)^2 \cdot 76 = 5776 \cdot 76 \equiv 76^2 \equiv 5776 \equiv 76$ $(mod\ 100)$.

Shunday qilib, $2^{100}$ sonining oxirgi ikki raqamir 7 va 6 dan iborat. ■

**5-Misol**. Agar p – tub son bo'lsa, u holda $C_{p-1}^{k} \equiv (-1)^k\ (mod\ p)$ taqqoslamani isbotlang.

*Yechilishi*. Ma'lumki, ixtiyoriy $p$ va $k$ sonlar uchun $C_{p-1}^{k} + C_{p-1}^{k-1} = C_p^k$ formula o'rinli, $C_p^k$ - butun sondan iborat bo'lib, $p$ ga bo'linadi, chunki $k < p$, $p$ esa tub sondan iborat, shuning uchun u maxrajning birorta ham ko'paytuvchisi bilan qisqarib ketmaydi. Shunday qilib, $C_p^k \equiv 0\ (mod\ p)$. U holda $C_{p-1}^{k} \equiv (-1)\ C_{p-1}^{k-1}\ (mod\ p)$.

Bu rekurrent munosabatni ketma-ket qo'llab, yuqori ko'rsatkichni 1 gacha kamaytiramiz:

$$C_{p-1}^{k} \equiv (-1)\ C_{p-1}^{k-1} \equiv (-1)^2\ C_{p-1}^{k-2} \equiv (-1)^3\ C_{p-1}^{k-3} \equiv \dots \equiv (-1)^{k-1}(p-1) \equiv (-1)^k\ (mod\ p).\ ■$$

**6-Misol**. Agar $a$ va $b$ – ixtiyoriy butun sonlar, $p$ – tub son bo'lsa, quyidagi taqqoslamani isbotlang

$$(a+b)^p \equiv a^p + b^p\ (mod\ p).$$

*Yechilishi*. Binomni yoyish formulasidan:

$$(a+b)^p = a^p + C_p^1 a^{p-1} b + C_p^2 a^{p-2} b^2 + \dots + C_p^{p-1} a b^{p-1} + b^p.$$

O'ng tomonda ikkinchi qo'shiluvchidan boshlab, p-*1* –nchi qo'shiluvchigacha barcha qo'shiluvchilar $p$ ga bo'linadi, chunki

$$C_p^k = \frac{p(p-1)\dots(p-(k-1)}{1 \cdot 2 \cdot \dots \cdot k},\ \text{bu yerda}\ k < p.$$

Demak, $C_p^i \equiv 0 (mod\ p)$, $i = 1, 2, \dots, (p-1)$.

Bu yerdan $(a+b)^p \equiv a^p + b^p\ (mod\ p)$ kelib chiqadi. ■

## MAShQLAR

**1.** Qanday modul bo'yicha barcha butun sonlar o'zaro taqqoslanadi?

**2.** Quyidagi taqqoslamalardan qaysilari to'g'ri:

a) $1 \equiv -5\ (mod\ 6)$; b) $546 \equiv 0\ (mod\ 13)$; c) $1956 \equiv 5\ (mod\ 12)$;

d) $2^3 \equiv 1\ (mod\ 4)$; e) $3m \equiv -1\ (mod\ m)$?

**3*.** Berilgan modul bo'yicha har qanday butun son o'zining qoldig'i bilan taqqoslanishini isbot qiling.

**4.** Quyidagi taqqoslamalarni qanoatlantiradigan $x$ ning barcha qiymatlarini toping:

a) $x \equiv 0\ (mod\ 3)$;  b) $x \equiv 1\ (mod\ 2)$.

**5.** Quyidagi taqqoslamalarni qanoatlantiradigan $m$ ning barcha qiymatlarini toping: $3r + 1 \equiv r + 1\ (mod\ m)$.

**6.** Agar $x = 13$ soni $x \equiv 5\ (mod\ m)$ taqqoslamani qanoatlantirsa, modulning mumkin bo'lgan qiymatlarini toping.

**7*.** Agar $n$ – toq son bo'lsa, u holda $n^2 - 1 \equiv 0\ (mod\ 8)$ taqqoslama o'rinli ekanligini ko'rsating.

**8*.** Agar $100a + 10b + c \equiv 0\ (mod\ 21)$ bo'lsa, u holda $a - 2b + 4c \equiv 0\ (mod\ 21)$ taqqoslamaning o'rinli ekanligini ko'rsating.

**9.** Agar $3^n \equiv -1\ (mod\ 10)$ bo'lsa, u holda $3^{n+4} \equiv -1\ (mod\ 10)$ $(n \in \mathbf{N})$ taqqoslamaning o'rinli ekanligini ko'rsating,.

**10*.** $2^{11 \cdot 31} \equiv 2\ (mod\ 11 \cdot 31)$ taqqoslamaning to'g'riligini ko'rsating.

**11*.** Agar $x = 3n + 1$, $n = 0, 1, 2, ....$ bo'lsa, u holda $1 + 3^x + 9^x$ ning 13 ga bo'linishini ko'rsating.

**12.** $N = 11 \cdot 18 \cdot 2322 \cdot 13 \cdot 19$ soni 7 modul bo'yicha absolyut qiymati bo'yicha eng kichik qanday son bilan taqqoslanadi?

**13.** $3^{14} \equiv -1\ (mod\ 29)$ ni tekshiring.

**14.** $1532^5 - 1$ ni 9 ga bo'lganda hosil bo'ladigan qoldiqni toping.

**15*.** Agar $a \equiv b\ (mod\ p^n)$ bo'lsa, u holda $a^p \equiv b^p\ (mod\ p^{n+1})$ ni isbotlang.

**16** Agar $ax \equiv bx\ (mod\ m)$ bo'lsa, u holda $a \equiv b\left(mod\dfrac{m}{(x,,m)}\right)$ ni isbotlang.

**17*.** Agar $\overline{a_4a_3a_2a_1a_0} \equiv 0\ (mod\ 33)$ bo'lsa, u holda $a_4 + \overline{a_3a_2} + \overline{a_1a_0} \equiv 0\ (mod\ 33)$ ni isbotlang. $a_{i+1} = 0$ da $\overline{a_{i+1}a_i} = a_i$ deb oling.

**18*.** Berilgan sonning oxirgi ikkita raqamini toping: a) $9^{9^9}$ ; b) $7^{9^9}$ .

**19*.** $r^{r+2} + (r+2)^r \equiv 0\ (mod\ 2r+2)$ taqqoslamani isbot qiling, bu yerda $r > 2$.

**20*.** Quyidagi sonlarni

$$-\frac{p-1}{2}, -\frac{p-3}{2}, ...,\ -1, 0, 1, ...,\ \frac{p-3}{2}, \frac{p-1}{2}$$

r > 2 modul bo'yicha o'zaro taqqoslanmasligini ko'rsating.

**21*.** $2^{3^n} \equiv -1\ (mod\ 3^{n+1})$, $n \in \mathbf{N}$ taqqoslamani isbotlang.

**22*.** $N = 3^{2^{4n+1}} + 2$ va $M = 2^{3^{4n+1}} + 3$ $(n \in \mathbf{N})$ ko'rinishdagi sonlarning murakkab sonlardan iboratligini isbot qiling.

**23*.** Agar

$$\left.\begin{aligned} ac &\equiv bd \\ a &\equiv b \end{aligned}\right\}(mod\ m)$$

taqqoslamalar berilgan va $(a, m) = 1$ bo'lsa, u holda birinchi taqqoslamani ikkinchi taqqoslamaga hadma-had bo'lish natijasida $c \equiv d\ (mod\ m)$ ni hosil qilinishini ko'rsating.

**24.** $a^{100} \equiv 2$ (*mod* 73) va $a^{101} \equiv 69$ (*mod* 73) ekanligi ma'lum. *a* ni 73 ga bo'linganida hosil bo'ladigan qoldiqni toping.

**25*.** $\frac{11a+2b}{19} \in \boldsymbol{Z}$ ifoda berilgan. $\frac{18a+5b}{19} \in \boldsymbol{Z}$ ni isbotlang.

**26.** $2^x + 7^y = 19^z$ va $2^x + 5^y = 19^z$ tenglamalar natural sonlarda yechimga ega emasligini ko'rsating.

**27.** $p > 2$ (*p* – tub son) bo'lganda

$$1^{2k+1} + 2^{2k+1} + 3^{2k+1} + ... + (p-1)^{2k+1} \equiv 0 \ (mod\ p)$$

taqqoslamani to'g'riligini ko'rsating.

## § 2. Chegirmalar sinflari. Eyler va Ferma teoremalari

*m* natural songa bo'linganida bir xil *r* qoldiq qoladigan barcha butun sonlar to'plami *m* modul bo'yicha sonlar sinfini tashkil qiladi. Bu sinfning har bir soni umumiy holda $mk+r$, $k\in\boldsymbol{Z}$ ko'rinishda yoziladi. Barcha sinflar soni *m* ga teng.

Sinfning ixtiyoriy soni *m* modul bo'yicha *chegirma* deyiladi (shu sinfning boshqa sonlariga nisbatan).

Har bir sinfdan ixtiyoriy ravishda bittadan olingan sonlar to'plami berilgan *m* modul bo'yicha *chegirmalarning to'la sinfi* deyiladi.

Odatda chegirmalarning to'la sinfi sifatida berilgan *m* bo'yicha eng kichik manfiy bo'lmagan chegirmalar, ya'ni 0, 1, 2,..., *m* - 1 sistema olinadi.

Ba'zan berilgan *m* modul bo'yicha chegirmalardan absolyut qiymati bo'yicha eng kichik musbat bo'lmagan chegirmalarning to'la sistemasi ham qaraladi: -(*m*-1), -(*m*-2),..., -2, -1, 0. *m* modul bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning to'la sinfi ham ishlatiladi. Masalan, $m = 7$ bo'lganda bu sistema -3, -2, -1, 0, 1, 2, 3 chegirmalardan iborat bo'ladi; $m = 8$ bo'lganda esa -3, -2, -1, 0, 1, 2, 3, 4 yoki –4, -3, -2, -1, 0, 1, 2, 3 chegirmalardan tashkil topadi.

Chegirmalarning to'la sistemasidan olingan va *m* modul bilan o'zaro tub bo'lgan chegirmalar *m* modul bo'yicha chegirmalarning keltirilgan sistemasi deyiladi. Kelitirilgan sistemada chegirmalar soni $\varphi(m)$- Eyler funksiyasining qiymatiga teng.

Chegirmalarning to'la sistemasidagi kabi keltirilgan sistemaning ham uch turi ishlatiladi: *eng kichik musbat chegirmalarning keltirilgan sistemasi, absolyut qiymati bo'yicha eng kichik manfiy chegirmalarning keltirilgan sistemasi va absolyut qiymati bo'yicha eng kichik chegirmalarning keltirilgan sistemasi.*

$x_1, x_2,..., x_s$ butun sonlar sistemasi $s = m$ va $i \neq j$ da $x_i \equiv x_j\ (mod\ m)$ bo'lganda va faqat shu holda *m* modul bo'yicha chegirmalarning to'la sistemasidan iborat bo'ladi. $(a, m) = 1$ bo'lganda $ax + b$ chiziqli formaning qiymatlari *m* modul bo'yicha chegirmalarning to'la sistemasidan iborat bo'lishi uchun *x* qabul qiladigan qiymatlar ham chegirmalarning to'la sistemasidan iborat bo'lishi zarur va yetarlidir.

$x_1, x_2,..., x_s$ butun sonlar sistemasi $s = \varphi(m)$ va $i \neq j$, $(x_i, m) = 1$ da $x_i \equiv x_j\ (mod\ m)$ bo'lganda va faqat shu holda *m* modul bo'yicha chegirmalarning keltirilgan sistemasidan iborat bo'ladi. $(a, m) = 1$ bo'lganda *ax* chiziqli formaning qiymatlari *m* modul bo'yicha chegirmalarning keltirilgan sistemasidan iborat bo'lishi uchun *x* qabul qiladigan qiymatlar ham chegirmalarning keltirilgan sistemasidan iborat bo'lishi zarur va yetarlidir.

$m > 1$ va $(a, m) = 1$ bo'lganda quyidagi taqqoslama o'rinli:

$$a^{\varphi(m)} \equiv 1 \ (mod\ m),$$

bu yerda $\varphi(m)$ –Eyler funksiyasi (*Eyler teoremasi*).

$p$ tub son va $(a, p) = 1$ bo'lganda quyidagi taqqoslama o'rinli:

$$a^{p-1} \equiv 1 \ (mod\ p)$$ (*Ferma teoremasi*).

*a* butun sonni o'zida saqlaydigan *m* bo'yicha chegirmalar sinfini *a mod m* bilan belgilaymiz. Demak,

$$a\ mod\ m = a + m\mathbf{Z} = \{a + km \mid k \in \mathbf{Z}\}.$$

**Z/*m*Z** bilan *m* modul bo'yicha barcha chegirmalar sinflari to'plamini belgilaymiz:

$$\mathbf{Z}/m\mathbf{Z} = \{0\ mod\ m,\ 1\ mod\ m, ...,\ (m-1)\ mod\ m\}.$$

Bu to'plamda qo'shish va ko'paytirish amallarini quyidagi tengliklar orqali kiritiladi:

$$a\ mod\ m + b\ mod\ m = (a + b)\ mod\ m,$$

$$(a\ mod\ m) \cdot (b\ mod\ m) = ab\ mod\ m.$$

($\mathbf{Z}/m\mathbf{Z}$, +) – abel gruppasidan, hamda **Z** gruppaning *m***Z** qism gruppa bo'yicha faktor gruppasidan iborat bo'lib, *m modul bo'yicha chegirmalar sinfining additiv gruppasi deyiladi.*

*(Z/mZ, +, ·)* – birlik elementli kommutativ xalqadan iborat bo'lib, *m modul bo'yicha chegirmalar sinfinig xalqasi deyiladi.*

Agar $(a, m) = 1$ bo'lsa, *a mod m* sinf *m modul bilan o'zaro tub bo'lgan chegirmalar sinfi deyiladi.*

*m modul bilan o'zaro tub bo'lgan chegirmalar sinflari to'plami* ko'paytirishga *nisbatan* abel gruppasi tashkil etadi va u *m modul bilan o'zaro tub bo'lgan chegirmalar sinflarining multiplikativ gruppasi deyiladi.*

Agar $ab \equiv 1 \ (mod\ m)$ bo'lsa, *a* chegirma *b* chegirmaga *m* modul bo'yicha teskari deyiladi.

**1-Misol**. 10 modul bo'yicha chegirmalar to'la sistemasining uchta turini yozing.

*Yechilishi.* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 – 10 modul bo'yicha eng kichik manfiy bo'lmagan chegirmalarning to'la sistemasi.

-9, -8, -7, -6, -5, -4, -3, -2, -1, 0 – 10 modul bo'yicha absolyut qiymati jihatidan eng kichik manfiy chegirmalarning to'la sistemasi.

-4, -3, -2, -1, 0, 1, 2, 3, 4, 5 yoki –5, -4, -3, -2, -1, 0, 1, 2, 3, 4 – 10 modul bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasi. ■

**2-Misol**. 10 modul bo'yicha chegirmalarning keltirilgan sistemasining uchta turini yozing.

*Yechilishi.* 1, 3, 7, 9 – 10 modul bo'yicha eng kichik manfiy bo'lmagan chegirmalarning keltirilgan sistemasi.

-9, -7, -3, -1 – 10 modul bo'yicha absolyut qiymati jihatidan eng kichik manfiy chegirmalarning keltirilgan sistemasi.

-3, -1, 1, 3 chegirmalar 10 modul bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning keltirilgan sistemasi. ■

**3-Misol**. 20, -4, 22, 18, -1 sonlar qanday modul bo'yicha chegirmalarning to'la sistemasini tashkil etadi?

*Yechilishi.* 5 modul bo'yicha berilgan sonlar mos ravishda 0, 1, 2, 3, 4 sonlar bilan taqqoslanadi, shuning uchun izlanayotgan modul 5 ga teng. ■

**4-Misol**. $3^1, 3^2, 3^3, 3^4, 3^5, 3^6$ sonlar sistemasi 7 modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil eitishini ko'rsating.

*Yechilishi.* Berilgan sonlardan eng kichik musbat chegirmalarni tuzamiz:

3, 2, 6, 4, 5, 1, chunki $3^2 \equiv 2$ (*mod* 7), $3^3 \equiv 6$ (*mod* 7), $3^4 \equiv 4$ (*mod*7), $3^5 \equiv 5$ (*mod* 7), $3^6 \equiv 1$ (*mod* 7). ■

**5-Misol**. $383^{175}$ ni 45 ga bo'lganda hosil bo'ladigan qoldiqni toping.

*Yechilishi.* $383 \equiv 23$ (*mod* 45) bo'lganligi uchun $383^{175} \equiv 23^{175}$ (*mod* 45). Endi $\varphi(45) = 24$ va $(23, 45) = 1$ dan Eyler teoremasiga ko'ra:

$23^{24} \equiv 1$ (*mod* 45) ni hosil qilamiz. Demak,

$23^{175} = 23^{24\bullet 7+7} = (23^{24})^7 \cdot 23^7 \equiv 1^7 \cdot 23^7$ (*mod* 45).

Shu taxlitda davom etib, $23^7 = (23^2)^3 \cdot 23 \equiv 34^3 \cdot 23 = 34^2 \cdot 34 \cdot 23 \equiv 1156 \cdot 782 \equiv 31 \cdot 17 = 527 \equiv$ 32 (*mod* 45) ni hosil qilamiz.

Shunday qilib, $383^{175} \equiv 32$ (*mod* 45). Izlanayotgan qoldiq 32 dan iborat. ■

**6-Misol**. $x$ ning har qanday butun qiymatida $x^7 \equiv x$ (*mod* 42) taqqoslamani to'g'riligini ko'rsating.

*Yechilishi.* Ferma teoremasiga ko'ra, $x^7 \equiv x$ (*mod* 7). Endi $x^7 \equiv x$ (*mod* 2 va 3) ekanligini isbot qilamiz, buning uchun 2 va 3 modullar bo'yicha chegirmalarning to'la sistemasini, y'ani 0, 1, 2 sonlarni sinash yetarli.

**7-Misol**. Butun sonning 100-darajasini 125 ga bo'lganda hosil bo'ladigan qoldiqni toping.

*Yechilishi.* Agar $(a, 5) = 1$ bo'lsa, u holda Eyler teoremasiga ko'ra:

$a^{\varphi(125)} = a^{100} \equiv 1$ (*mod* 125).

Agarda $(a, 5) = 5$ bo'lsa, u holda $a^{100} \equiv 0$ (*mod* 125).

Demak, agar $(a, 5) = 1$ bo'lsa, u holda izlanayotgan qoldiq 1 ga teng. Agarda $(a, 5) = 5$ bo'lsa, u holda $a^{125}$ soni 125 ga bo'linadi. ■

**8-Misol**. $2^{\varphi(m)-1}$ ni toq $m$ soniga bo'linganida hosil bo'ladigan qoldiqni toping.

*Yechilishi.* $2^{\varphi(m)-1} \equiv r$ *(mod m)*, $0 \le r < m$ bo'lsin. U holda $2^{\varphi(m)} \equiv 2r \equiv 1$ *(mod m)* yoki $r = \frac{1+mq}{2}$, bu yerda $q \in \mathbf{Z}$. $0 \le r < m$ shartni $q = 1$ da yagona $\frac{1+mq}{2}$ qiymat qanoatlantiradi, bu yerdan $r = \frac{1+m}{2}$ ni hosil qilamiz. ■

**9-Misol**. 341 soni uchun $2^{341} \equiv 2$ (*mod* 341) taqqoslamaning o'rinli ekanligini ko'rsating.

*Yechilishi.* 341 – murakkab son, $341 = 11 \cdot 31$. $2^5 \equiv 1$ (*mod* 31) va $2^{10} \equiv 1$ (*mod* 31) taqqoslamalar o'rinli ekanligini osongina tekshirish mumkin.

Ferma teoremasiga asosan $2^{10} \equiv 1$ (*mod* 11). 11 va 13 sonlar o'zaro tub bo'lganligi uchun bu yerdan $2^{10} \equiv 1$ (*mod* 11·31) kelib chiqadi, ya'ni $2^{10} \equiv 1$ (*mod* 341). Demak, $2^{340} \equiv 1$ (*mod* 341) va $2^{341} \equiv 2$ (*mod* 341) taqqoslamalar o'rinli. ■

**10-Misol**. Agar har bir butun $a$ soni uchun $a^n \equiv a$ *(mod n)* taqqoslama o'rinli bo'lsa, $n$ murakkab soni *absolyut psevdotub son* deyiladi. 561 ning *absolyut psevdotub son* ekanligini ko'rsating.

*Yechilishi.* Berilgan sonni tub ko'paytuvchilarga ajratamiz 561 = 3·11·17. Ferma teoremasiga asosan 561 bilan o'zaro tub bo'lgan har bir butun $a$ soni uchun $a^2 \equiv 1$ (*mod* 3), $a^{10} \equiv 11$ (*mod* 11), $a^{16} \equiv 17$ (*mod* 17) taqqoslamalar o'rinli bo'ladi. 3, 11, 17 tub sonlardan iborat bo'lganligi uchun va [2, 10, 16] = 80 bo'lganligidan bu taqqoslamalardan quyidagi taqqoslamalar kelib chiqadi: $a^{80} \equiv 1$ (*mod* 561), $a^{560} \equiv 1$ (*mod* 561). Demak, 561 *absolyut psevdotub sondan iborat* . ■

## MAShQLAR

**28.** 10 modul bo'yicha chegirmalaraning hamma sinflarini taqqoslamalar ko'rinishida yozing.

**29.** 10 modul bo'yicha chegirmalaraning hamma sinflarini $x = 10q + r$, $0 \le r < 10$ ko'rinishda yozing.

**30.** Quyidagi modullar bo'yicha ko'rsatilgan chegirmalarning sinflarini yozing: a) 10 modul bilan o'zaro tub bo'lgan; b) 10 modul bilan EKUBi 2 ga teng bo'lgan; s) 10 modul bilan EKUBi 5 ga teng bo'lgan; d) 10 modul bilan EKUBi 10 ga teng bo'lgan.

**31.** Berilgan modullar bo'yicha chegirmalarning to'la va keltirilgan sistemalarining barcha turlarini yozing: a) $m = 9$; b) $m = 8$; c) $p = 7$; d) $m = 12$.

**32*.** 25, -20, 16, 46, -21, 18, 37, -17 sonlarning 8 modul bo'yicha chegirmalarning to'la sistemasini tashkil qilishini ko'rsating.

**33.** 32, -9, 15, 42, -18, 30, 6 sonlarni $p$ =7 modul bo'yicha chegirmalarning to'la sistemasini tashkil etishini ko'rsating.

**34.** 21, 2, -18, 28, -19, 40, -22, -2, 15 sonlarning 9 modul bo'yicha chegirmalarning to'la sistemasini tashkil etishini ko'rsating.

**35.** 24, 18, -19, 37, 28, -23, -32, 5, 41, -35, -33 sonlarning 11 modul bo'yicha chegirmalarning to'la sistemasini tashkil etishini ko'rsating.

**36.** 19, 23, 25, -19 sonlarning 12 modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil etishini ko'rsating.

**37.** 11, -1, 17, -19 sonlarning 8 modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil etishini ko'rsating.

**38*.** 24, 14, 25, 37, -8, -19, -40 sonlarning 6 modul bo'yicha eng kichik manfiy bo'lmagan, absloyut qiymati bo'yicha eng kichik musbat bo'lmagan va absolyut qiymati jihatidan eng kichik chegirmalarini toping. Bu sonlar berilgan modul bo'yicha nechta har xil sinflarga tegishli bo'ladi? Qaysi sonlar berilgan modul bo'yicha bir sinfga tegishli bo'ladi?

**39.** Oldingi masalaning shartini 8 modul bo'yicha 17, -14, 19, -49, -22, 21, -29 sonlarga qo'llang.

**40.** 100 sonining quyidagi modullar bo'yicha eng kichik manfiy bo'lmagan, absloyut qiymati bo'yicha eng kichik musbat bo'lmagan va absolyut qiymati jihatidan eng kichik chegirmalarini toping: 5, 7, 11, 25, 120, 200.

**41.** Oldingi masalani 50 soni va 3, 8, 12, 25, 70, 100 modullarga nisbatan yeching.

**42*.** Har qanday ketma-ket keladigan $m$ ta butun sonlar $m$ modul bo'yicha chegirmalarning to'la sistemasini tashkil eitishini ko'rsating.

**43*.** 10 modul bo'yicha $3x–1$ ko'rinishdagi chegirmalarning to'la sistemalaridan birortasini ko'rsating.

**44.** 4 modul bo'yicha $5x$ ko'rinishdagi chegirmalarning to'la sistemalaridan birortasini ko'rsating.

**45.** $5, 5^2, 5^3, 5^4, 5^5, 5^6$ sonlar sistemasi 7 modul bo'yicha keltirilgan sistemani tashkil etishini ko'rsating.

**46.** m modul bo'yicha chegirmalarning additiv gruppasi ***Z/mZ*** ning siklik gruppa ekanligini ko'rsating.

**47.** (***Z/mZ***, +) gruppaning $m$ modul bo'yicha chegirmalar sinfi xalqasining additiv gruppasidan iborat ekanligini ko'rsating.

**48.** Agar $a$ va $b$ o'zaro teskari bo'lsa, u holda $a \bmod m \cdot b \bmod m = 1 \bmod m$ ni ko'rsating, ya'ni $a \bmod m$ va $b \bmod m$ chegirmalar sinflari ham $m$ modul bo'yicha o'zaro teskari bo'ladi.

**49.** $m$ modul bo'yicha chegirmalar sinfi xalqasining teskarilanuvchi elementlari gruppasi $m$ modul bo'yicha o'zaro tub bo'lgan chegirmalar sinfining multiplikativ gruppasi bilan ustma-ust tushishini ko'rsating.

**50.** $m$ – tub sondan iborat bo'lganda $m$ modul bo'yicha chegirmalar sinfining xalqasi maydon tashkil qilishini ko'rsating.

**51*.** Kanonik yoyilmasi 2 va 5 ni o'zida saqlamaydigan natural sonning 12-darajasining birlik raqami 1 ga tengligini isbotlang.

**52*.** Eyler teoremasini quyidagi hollarda tekshiring: a) $a = 5, m = 24$; b) $a = 2, m = 33$; c) $a = 3, m = 18$; d) a = 3, m = 24.

**53*.** Eyler va Ferma teoremalaridan foydalanib, quyidagi modullar bo'yicha taqqoslamalr tuzing va har bir taqqoslamani qanoatlantiradigan $a$ ning qiymatlarini va chegirmalar sinfini yozing: a) 6; b) 5; c) 8; d) 7.

**54*.** a) Agar $(a, 7) = 1$ bo'lsa, u holda $a^{12} – 1$ ning 7 ga bo'linishini; b) Agar $(a, 65) = (b, 65) = 1$ bo'lsa, u holda $a^{12} – b^{12}$ ning 65 ga bo'linishini ko'rsating.

**55.** $a \equiv 0 \pmod p$ bo'lganda $a^{p-1} + p - 1$ sonning murakkab ekanligini ko'rsating.

**56*.** $\sum\limits_{i=1}^{p-1} i^{k(p-1)} + 1 \equiv 0 \pmod p$ taqqoslamani to'g'ri ekanligini isbot qiling.

**57.** $\left(\sum\limits_{i=1}^{n} a_i\right)^p \equiv \sum\limits_{i=1}^{n} a_i^{\,p} \pmod p$ taqqoslamani to'g'ri ekanligini isbot qiling.

**58.** $a^{n(p-1)+1} \equiv a \ (mod\ p)$ taqqoslamani to'g'ri ekanligini isbot qiling..

**59.** Bo'linishning qoldiqlarini toping: a) $109^{345}$ ni 14 ga; b) $439^{291}$ ni 60 ga; s) $293^{275}$ ni 48 ga.

**60.** Bo'linishning qoldiqlarini toping: : a) $3^{80} + 7^{80}$ ni 11 ga; b) $3^{100} + 5^{100}$ ni 7 ga; s) $2^{100} + 3^{100}$ ni 5 ga; d) $5^{70} + 7^{50}$ ni 12 ga.

**61.** $243^{402}$ sonining oxirgi uchta raqamini toping.

**62*.** $(a, m) = 1$ bo'lganda $a^x \equiv 1 \ (mod\ m)$ taqqoslamani qanoatlantiradigan $x$ ning eng kichik natural qiymati $\varphi(m)$ sonning bo'luvchisidan iborat ekanligini isbotlang.

**63*.** $a^{561} \equiv a \ (mod\ 1)$ ni isbotlang.

**64*.** $x^{(p-1)m} + x^{(p-1)n} \equiv 0 \ (mod\ p)$ taqqoslamani $x$ ning $p > 2$ ga karrali qiymatlari qanoatlantirishini ko'rsating.

**65*.** 2, 3 va 5 ga bo'linmaydigan $m$ natural soni 11...1 ko'rinishdagi $\varphi(m)$-xonali sonning bo'luvchisi ekanligini ko'rsating.

**66.** a) Agar $(a, 561) = 1$ bo'lsa, u holda $a^{560} \equiv 1 \ (mod\ 561)$ ni isbotlang; b) $2^{1093 \bullet 1092} \equiv 1 \ (mod\ 1093^2)$ ni isbotlang.

**67*.** Agar $a^r \equiv \pm 1 (mod\ p)$ bo'lsa, u holda $a^r \equiv \pm 1 (mod\ p^2)$ ni isbotlang ($r$ – tub son).

**68*.** Agar $p$ va $q$ – o'zaro teng bo'lmagan tub sonlar bo'lsa, u holda $p^{q-1} + q^{p-1} \equiv 1 (mod\ pq)$ ni isbotlang.

**69*.** $x$ ning qanday butun qiymatida $x^{13} \equiv x \ (mod\ 2730)$ taqqoslama to'g'ri bo'ladi?

**70*.** Agar $\sum_{i=1}^{n} a_i \equiv 0 \ (mod\ 30)$ bo'lsa, u holda $\sum_{i=1}^{n} a_i^5 \equiv 0 \ (mod\ 30)$ ni ko'rsating.

**71*.** Agar $m > 1$- toq son bo'lsa, u holda $2^{\varphi(m)-1}$ soni $m$ ga bo'linganida $m - \left[\frac{m}{2}\right]$ qoldiq qolishini ko'rsating.

**72*.** Agar $(a, 10) = 1$ bo'lsa, u holda $a^{100n+1} \equiv a \ (mod\ 1000)$, $n \in \mathbf{N}$ ni ko'rsating.

**73*.** $2^{19 \bullet 73-1} \equiv 1 \ (mod\ 19 \cdot 73)$ taqqoslamani to'g'ri ekanligini ko'rsating.

**74*.** Agar $p_1$ va $p_2$ – har xil tub sonlar bo'lsa, $p_1^{P_2-1} + p_2^{P_1-1} \equiv 1 \ (mod\ p_1p_2)$ taqqoslamani to'g'ri ekanligini ko'rsating.

**75*.** Agar $2r + 1$ $(r \neq 3)$ – tub son bo'lsa, u holda $4r + 1 \equiv 0 \ (mod\ 3)$ ni ko'rsating.

**76*.** Agar $(a, m) = 1$ va $\alpha_1 \equiv \alpha_2 \ (mod\ \varphi(m))$ bo'lsa, u holda $a^{\alpha_1} \equiv a^{\alpha_2} (mod\ m)$ ni isbotlang.

**77*.** $a^{6m} + a^{6n} \equiv 0 \ (mod\ 7)$, $m, n \in \mathbf{N}$ taqqoslama faqat $a$ soni 7 ga karrali bo'lganda o'rinli bo'lishini ko'rsating.

**78*.** Agar $(n, 6) = 1$ bo'lsa, u holda $n^2 \equiv 1 \ (mod\ 24)$.

**79*.** Quyidagi shartdan $p$ tub sonni toping:

$$5^{P^2} + 1 \equiv 0 \ (mod\ p^2).$$

**80*.** Agar uchta ketma-ket keladigan butun sonlardan o'rtadagisi biror butun sonning kubidan iborat bo'lsa, bu sonlarning ko'paytmasi 504 ga bo'linishini ko'rsating.

**81*.** Agar $r > 3$, $r$ va $2r+1$ lar tub sonlar bo'lsa, u holda $4r+1$ – murakkab son ekanligini ko'rsating.

## § 3. Bir noma'lumli algebraik taqqoslamalar.

## Birinchi darajali taqqoslamalar.

*n-darajali bir noma'lumli taqqoslama deb quyidagi ko'rinishdagi taqqoslamaga aytiladi*:

$$a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n \equiv 0 \ (mod\ m),$$

bu yerda $a_0 \equiv 0 \ (mod\ m)$, $a_i \in \mathbf{Z}$, $i = \overline{0, n}$, $n$ – manfiy bo'lmagan butun son.

*Taqqoslamani yechish – uni qanoatlantiradigan x ning barcha qiymatlarini topish demakdir.*

Agar berilgan taqqoslamani biror $x = \alpha$ qiymat qanoatlantirsa, u holda bu taqqoslamani $\alpha$ bilan $m$ modjul bo'yicha taqqoslanaidgan barcha sonlar ham qanoatlantiradi: $x \equiv \alpha \ (mod\ m)$, yoki, $x = mk + \alpha$, ya'ni, $m$ modul bo'yicha $\alpha$ tegishli bo'lgan chegirmalar sinfining barcha chegirmalari qanoatlantiradi. Har bir sinf bitta yechimni tashkil etadi. Demak, *taqqoslamani yechish* – uni qanoatlantiradigan chegirmalarning barcha sinflarini topishdan iborat.

Har bir sinfdan bittadan olingan chegirmalar to'la sistemani tashkil etganligi uchun taqqoaslamani qanoatlantiradigan sonlar sinfini topish chegirmalarning to'la sistemasidan ularga mos keladigan chegirmalarni topishdan iborat ekan. Odatda $\alpha$ sifatida berilgan modul bo'yicha manfiy bo'lmagan eng kichik yoki absolyut qiymati jihatidan eng kichik chegirmalar olinadi. Shunday qilib, to'la sistemaning nechta chegirmasi berilgan taqqoslamani qanoatlantirsa, taqqoslama shuncha yechimga ega bo'ladi.

Agar bir xil $x$ noma'lumli va bir xil modulli ikkita taqqoslamani $x$ noma'lumninng bir xil qiymatlari qanoatlantirsa, bunday taqqoslamalar teng kuchli deyiladi.

Berilgan taqqoslamaga teng kuchli taqqoslamalar quyidagi almashtirishlar natijasida hosil bo'ladi:

a) berilgan taqqoslamaning ikkala tomoniga ham bir xil sonni qo'shish natijasida;

b) berilgan taqqoslamaning ixtiyoriy bir qismiga modulga karrali bo'lgan sonni qo'shish natijasida;

c) berilgan taqqoslamaning ikkala tomonini modul bilan o'zaro tub bo'lgan songa ko'paytirish (bo'lish) natijasida;

d) taqqoslamaning ikkala tomonini va modulini bir xil songa bo'lish natijasida.

**1-Misol**. Quyidagi taqqoslamalarni yeching:

a) $x^3 - 2x + 6 \equiv 0 \ (mod\ 11)$;

b) $x^4 + 2x^3 + 6 \equiv 0\ (mod\ 8)$;

c) $x^4 - x^3 - x^2 + 5x - 2 \equiv 0\ (mod\ 6)$.

*Yechilishi.* a) 11 modul bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasidan iborat

-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5

sonlarni bevosita taqqoslamaga qo'yib tekshirish natijasida 5 soni taqqoslamani qanoatlantirishini hosil qilamiz. Yechimni $x \equiv 5\ (mod\ 11)$ ko'rinishda yozamiz.

b) 8 modul bo'yicha chegirmalarning to'la sistemasi -3, -2, -1, 0, 1, 2, 3, 4 da birorta ham chegirma taqqoslamani qanoatlantirmaydi, shuning uchun berilgan taqqoslama yechimga ega emas.

c) 6 modul bo'yicha chegirmalarning to'la sistemasi -2, -1, 0, 1, 2, 3 da faqat ikkita son taqqoslamani qanoatlantiradi: -1 va 2. Berilgan taqqoslama ikkita yechimga ega: $x \equiv -1\ (mod\ 6)$ va $x \equiv 2\ (mod\ 6)$.

Modulning bo'luvchisi bo'yicha olingan taqqoslama berilgan modul bo'yicha taqqoslamaning natijasidan iborat bo'ladi. ■

**2-Misol**. $x^2 - 5x + 6 \equiv 0\ (mod\ 9)$ taqqoslamani yeching.

*Yechilishi.* Modulning bo'luvchisi bo'yicha olingan taqqoslamani qaraymiz: $x^2 - 5x + 6 \equiv 0\ (mod\ 3)$, bu yerdan $x^2 + x \equiv 0\ (mod\ 3)$ yoki $x\ (x + 1) \equiv 0\ (mod\ 3)$, ko'paytuvchilarning har birini alohida yechib $x \equiv 0, 2\ (mod\ 3)$ ni hosil qilamiz. Yechimlarni chegirmalar sinfi orqali $x = 3q$; $3q + 2$ shaklda yozamiz.

Endi $x = 3q$ ni berilgan taqqoslamaga qo'yamiz:

$9q^2 - 15q + 6 \equiv 0\ (mod\ 9)$, bu yerdan $3q \equiv 3\ (mod\ 9)$, ya'ni. $q \equiv 1\ (mod\ 3)$ yoki $q = 1 + 3t$. Bu yerdan $x = 3 + 9t$ yoki $x \equiv 3\ (mod\ 9)$ yechimni hosil qilamiz.

$x = 3q + 2$ da berilgan taqqoslama quyidagi ko'rinishda bo'ladi:

$9q^2 + 12q + 4 - 15q - 10 + 6 \equiv 0\ (mod\ 9)$. Bu taqqoslamani soddalashtirishlardan so'ng $3q \equiv 0\ (mod\ 9)$ yoki $q \equiv 0\ (mod\ 3)$ ni hosil qilamiz. $q = 3t$ bo'lganda berildgan taqqoslamaning ikkinchi yechimi $x = 9t + 2$ yoki $x \equiv 2\ (mod\ 9)$ ni hosil qilamiz.

Shunday qilib, berilgan taqqoslama ikkita yechimga ega ekan: $x \equiv 2; 3\ (mod\ 9)$. ■

**3-Misol**. Teng kuchli taqqoslamaga o'tish bilan quyidagi taqqoslamani yeching: $13x \equiv 5\ (mod\ 47)$.

*Yechilishi.* Taqqoslamaning o'ng tomoniga 47 ni qo'shamiz:

$13x \equiv 52\ (mod\ 47)$. Endi taqqoslamaning ikkala tomonini 13 ga qisqartirib, uning yechimini hosil qilamiz: $x \equiv 4\ (mod\ 47)$. ■

*Birinchi darajali taqqoslamaning* umumiy ko'rinishi quyidagicha yoziladi:

$$ax \equiv b\ (mod\ m).$$

Bu taqqoslamani yechishda quyidagi hollar bo'lishi mumkin:

a) Agar $(a, m) = 1$ bo'lsa, u holda taqqoslama faqat yagona yechimga ega.

b) Agar $(a, m) = d > 1$ bo'lib, $b$ ozod had $d$ ga bo'linmasa, u holda taqqoslama yechimga ega emas.

s) Agar $(a, m) = d > 1$ bo'lib, $b$ ozod had $d$ ga bo'linsa, u holda taqqoslama $d$ ta yechimga ega bo'ladi va bu yechimlar quyidagi formulalar bilan topiladi:

$$x_k \equiv \alpha + \frac{(k-1)m}{d} \ (mod\ m), \ k=1, 2, ...., d$$

bu yerda $\alpha$ - quyidagi taqqoslamaning yechimidan iborat:

$$\frac{a}{d}x \equiv \frac{b}{d}(mod\ \frac{m}{d}).$$

$ax \equiv b \ (mod\ m)$ taqqoslamani yechish usullarini faqat $(a, m) = 1$ bo'lganda qarab chiqamiz, uchinchi holda taqqoslama $d$ ga qisqartirilgandan so'ng birinchi holga keltiriladi.

Birinchi darajali taqqoslamalarni yechishda quyidagi uchta usul qo'llaniladi:

a) yechim $m$ modul bo'yicha eng kichik manfiy bo'lmagan yoki absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasidagi sonlarni bevosita sinash usuli bilan topiladi.

b) *Eyler usuli*. Yechim quyidagi formula bilan topiladi:

$$x \equiv ba^{\varphi(m)-1} \ (mod\ m),$$

bu yerda $\varphi(m)$ –Eyler funksiyasi;

s) chekli uzluksiz kasrlar yordamida quyidagi formula bilan yechim topiladi:

$$x \equiv (-1)^n\, b\, P_{n-1} \ (mod\ m),$$

bu yerda $P_{n-1}$ – $\frac{m}{a}$ kasrni uzluksiz kasrga yoyganda hosil bo'ladigan oxirgisidan bitta oldingi munosib kasrning suratidan iborat.

Ba'zi hollarda taqqoslamalarning xossalariga asoslangan almashtirishlar orqali berilgan taqqoslama oson yechiladi (3-misolga qarang).

**4-Misol** . Quyidagi taqqoslamani Eyler usuli bilan yeching:

$$9x \equiv 8 \ (mod\ 34).$$

*Yechilishi.* (9, 34) = 1 bo'lganligi uchun berilgan taqqoslama yagona yechimga ega bo'ladi. $\varphi(34) = 16$ ni hisoblab quyidagilarga ega bo'lamiz:

$$x \equiv 8\cdot 9^{15} \equiv 8\cdot 3^{30} \equiv 8\cdot 3^{14} \equiv 8\cdot(2187)^2 \equiv 8\cdot 11^2 \equiv 16 \ (mod\ 34). \ \blacksquare$$

Misol 5. Taqqoslamani uzluksiz kasrlar orqali yeching:

$$285x \equiv 177 \ (mod\ 924).$$

*Yechilishi.* (285, 924) = 3 va 177 = 59·3 bo'lganligi uchun berilgan taqqoslama uchta yechimga ega.

Taqqoslamaning ikkala tomonini va modulini 3 ga bo'lamiz:

$$95x \equiv 59 \ (mod\ 308).$$

$\frac{308}{95}$ kasrni uzluksiz kasrga yoyamiz: $\frac{308}{95}$ = (3, 4, 7, 1, 2). Munosib kasrlar jadvalini tuzamiz:

| $q_i$ | | 3 | 4 | 7 | 1 | 2 |
|---|---|---|---|---|---|---|
| $P_i$ | 1 | 3 | 13 | 94 | 107 | 308 |

Shunday qilib, $P_{n-1} = P_4 = 107$, demak,

$$x \equiv (-1)^4 \cdot 107 \cdot 59 \ (mod\ 308),$$

Bu yerdan natija taqqoslamaning yechimi $x \equiv 153\ (mod\ 308)$ ni hosil qilamiz.
Berilgan taqqoslamaning yechimlari quyidagicha tasvirlanadi:

$$x \equiv 153;\ 461;\ 769\ (mod\ 924).$$ ■

Birinchi darajali taqqoslamalarni birinchi darajali ikki noma'lumli aniqmas tenglamalarni (diofant tenglamalari) yechishga tatbig'ini qarab chiqamiz.
*Quyidagi aniqmas tenglama*

$$ax + by = c; \quad a, b, c \in \mathbf{Z}$$

ni yechish talab qilinsin. Agar $(a, b) = 1$ bo'lsa, u holda berilgan tenglama butun yechimlarga ega bo'lib, uning umumiy yechimi quyidagicha ifodalanadi:

$$x = x_1 + bt,$$
$$y = y_1 - at$$

yoki $b$ manfiy bo'lganda quyidgicha ifodalash qulay:

$$x = x_1 - bt,$$
$$y = y_1 + at.$$

Bu formulalarda $x_1$ va $y_1$ lar $x$ va $y$ larning tenglamani qanoatlantiradigan qandaydir qiymatlaridan iborat va $t \in \mathbf{Z}$.

Agar $(a, b) = d > 1$ va $c$ soni $d$ ga bo'linmasa, u holda $ax + by = c$ tenglama butun sondagi yechimlarga ega emas.

Birinchi darajali aniqmas tenglamalar nazariyasidan noma'lumlarni xususiy yechimlarini topishning bir necha usullari mavjud.

Taqqoslamalar yordamida bu xususiy yechim quyidagicha topiladi: $ax + by = c$ dan taqqoslamaning ma'nosi haqidagi teoremaga ko'ra $ax \equiv c\ (mod\ b)$ bir noma'lumli taqqoslamani hosil qilamiz, bu yerda $b$ o'z ishorasi bilan olinadi, taqqoslamani qanoatlantiradigan $x$ ning qiymati $x_1$ sifatida olinadi, $y_1$ ning qiymati esa bevosita berilgan tenglamaga $x_1$ ni qo'yib topiladi.

Misol 6. Quyidagi tenglamani butun sonlarda yechimlarini toping:

$$39x - 22y = 10.$$

*Yechilishi.* Tenglamadan quyidagi taqqoslama kelib chiqadi:

$$39x \equiv 10\ (mod\ 22).$$

Bu taqqoslamadagi koeffisiyentlarni 22 modul bo'yicha eng kichik musbat chegirmalariga keltirsak, $17x \equiv 10\ (mod\ 22)$ ni hosil qilamiz, bu yerdan $x_1 = 20$ ni hosil qilamiz. Bu qiymatni berilgan tenglamaga qo'yib, $y_1 = 35$ ni topamiz. Demak, berilgan tenglamaning umumiy yechimi quyidagicha bo'ladi:

$$\begin{cases} x = 20 + 22t, \\ y = 35 + 39t. \end{cases}$$ ■

**7-Misol**. Tug'ilgan kunning 12 ga ko'paytmasi va oyning 31 ga ko'paytmalarining yig'indisi 299 ekanligi ma'lum bo'lsa, tug'ilgan kunni toping.

*Yechilishi.* $x$ – sana, $y$ – oyning raqami bo'lsin. U holda quyidagi tenglamani hosil qilamiz

$$12x + 31y = 299.$$

Bu yerdan $12x \equiv 299\ (mod\ 31)$ yoki $12x \equiv 20\ (mod\ 31)$ taqqoslama kelib chiqadi. Oxirgi taqqoslamani yechib, $x_1 = 12$ ni hosil qilamiz. Topilgan qiymatni berilgan tenglamaga quyib, $y_1 = 5$ ni hosil qilamiz. Demak, tug'ilgan kun 12 - may ekan. ■

## MAShQLAR

**82.** Manfiy bo'lmagan eng kichik chegirmalarni bevosita sinash usuli bilan quyidagi taqqoslamalarni yeching:

a) $5x^2 - 15x + 22 \equiv 0 \ (mod\ 3)$; b) $x^2 + 2x + 2 \equiv 0 \ (mod\ 5)$; c) $3x \equiv 1 \ (mod\ 5)$;
d) $8x \equiv 3 \ (mod\ 14)$; e) $x^3 - 2 \equiv 0 \ (mod\ 5)$; f) $x^2 - 2x + 1 \equiv 0 \ (mod\ 4)$;
g) $27x^2 - 13x + 11 \equiv 0 \ (mod\ 5)$.

**83.** Taqqoslamalarning xossalari yordamida dastlab soddalashtirib, so'ngra absolyut qiymati jihatidan eng kichik chegirmalarni bevosita sinash usuli bilan quyidagi taqqoslamalarni yeching:

a) $12x \equiv 1 \ (mod\ 7)$; b) $8x \equiv 1 \ (mod\ 5)$; c) $3x \equiv 13 \ (mod\ 11)$; d) $6x \equiv 3 \ (mod\ 7)$;
e) $6x + 5 \equiv 1 \ (mod\ 7)$; f) $90x^{20} + 46x^2 - 52x + 46 \equiv 0 \ (mod\ 15)$.

**84.** Quyidagi taqqoslamalarni yechimga ega emasligini ko'rsating:

a) $2x - 3 \equiv 0 \ (mod\ 6)$; b) $x^2 - 2x + 3 \equiv 0 \ (mod\ 4)$; c) $x^3 + x + 4 \equiv 0 \ (mod\ 5)$;
d) $x^4 + 2 \equiv 0 \ (mod\ 5)$; e) $x^5 - 2x^3 + 13x - 1 \equiv 0 \ (mod\ 4)$.

**85.** Noma'lumning ixtiyoriy butun qiymatlari quyidagi taqqoslamalarni qanoatlantirishini ko'rsating:

a) $x^2 - x + 6 \equiv 0 \ (mod\ 2)$; b) $x(x^2 - 1) \equiv 0 \ (mod\ 6)$;
c) $x^4 + 2x^3 - x^2 - 2x \equiv 0 \ (mod\ 4)$; d) $x^r - x \equiv 0 \ (mod\ r)$.

**86.** Taqqoslamaning xossalaridan foydaldanib, almashtirishlar orqali quyidagi taqqoslamalarni yeching:

a) $2x \equiv 7 \ (mod\ 15)$; b) $5x \equiv 2 \ (mod\ 8)$; c) $7x \equiv 2 \ (mod\ 13)$;
d) $3x \equiv 23 \ (mod\ 37)$; ye) $27x \equiv 14 \ (mod\ 25)$; f) $13x \equiv 10 \ (mod\ 11)$;
g) $5x \equiv 3 \ (mod\ 11)$; h) $7x \equiv 5 \ (mod\ 24)$.

**87.** $x$ ning qanday butun qiymatlarida $5x^2 + x + 4$ kvadrat uchhad 10 ga bo'linadi?

**88.** $x^2 - 4x + 3 \equiv 0 \ (mod\ 6)$ taqqoslamani $x^2 - 4x + 3 \equiv 0 \ (mod\ 2)$ zaruriy shartdan foydalanib yeching.

**89.** $x^{\varphi(30)} \equiv 1 \ (mod\ 30)$ taqqoslamani yeching.

**90.** $x^{\varphi(m)} \equiv 1 \ (mod\ m)$ taqqoslama nechta yechimga ega?

**91*.** Agar $(n, m) = 1$ bo'lsa, u holda $n$-darajali $x^n + a_1x^{n-1} + ... + a_n \equiv 0 \ (mod\ m)$ taqqoslamani yangi $y$ o'zgaruvchini kiritish bilan $(n$-$1)$ darajali hadi qatnashmaydigan $y^n + b_2\, y^{n-2} + ... + b_n \equiv 0 \ (mod\ m)$ taqqoslamaga keltirish mumkinligini ko'rsating.

**92*.** Oldingi masaladan foydalanib, $x^3 + 5x^2 + 6x - 8 \equiv 0 \ (mod\ 13)$ taqqoslamani uch hadli $y^3 + ry + q \equiv 0 \ (mod\ 13)$ taqqoslamaga keltiring.

**93.** Eyler usuli bilan quyidagi taqqoslamalarni yeching:

a) $5x \equiv 7 \ (mod\ 10)$; b) $3x \equiv 8 \ (mod\ 13)$; c) $7x \equiv 5 \ (mod\ 17)$;
d) $13x \equiv 3 \ (mod\ 19)$; e) $27x \equiv 7 \ (mod\ 58)$.

**94.** Uzluksiz kasrlar usuli bilan quyidagi taqqoslamalarni yeching:

a) $7x \equiv 4$ (*mod* 19); b) $143x \equiv 41$ (*mod* 221); c) $13x \equiv 178$ (mod 153);
d) $67x \equiv 64$ (*mod* 183); y) $89x \equiv 86$ (*mod* 241); f) $213x \equiv 137$ (*mod* 516);
g) $111x \equiv 81$ (*mod* 447); h) $186x \equiv 374$ (*mod* 422); i) $129x \equiv 321$ (*mod* 471).

**95.** Qulay usul bilan quyidagi taqqoslamalarni yeching:

a) $12x \equiv 9$ (*mod* 18);
b) $20x \equiv 10$ (*mod* 25);
c) $-50x \equiv 67$ (*mod* 177);
d) $-73x \equiv 60$ (*mod* 311);
e) $-53x \equiv 84$ (*mod* 219);
f) $90x + 18 \equiv 0$ (*mod* 138);
g) $78x \equiv 42$ (*mod* 51).

Javoblarni berilgan taqqoslamaga qo'yish bilan tekshirib ko'ring.

**96*.** Birinchi darajali 21 modul bo'yicha quyidagi taqqoslamalarni tuzing: a) faqat yagona yechimga ega bo'lgan; b) 3 va 7 ta yechimga ega bo'lgan; c) 2, 10, 15 ta yechimga ega bo'lgan.

**97.** Tug'ilgan kunning 12 ga ko'paytmasi va oyning 31 ga ko'paytmalarining yig'indisi 198 bo'lsa, tug'ilgan kunni toping.

**98*.** 523 sonning chap tomonidan shunday uch xonali sonni yozingki, hosil bo'lgan olti xonali son 7, 8 va 9 ga bo'linsin.

**99.** 629 sonning o'ng tomonidan shunday uch xonali sonni yozingki, hosil bo'lgan olti xonali son 5, 8 va 11 ga bo'linsin.

**100.** 723 sonning o'ng tomonidan shunday ikki xonali sonni yozingki, hosil bo'lgan 5 xonali sonni 31 ga bo'lganda 7 qoldiq qolsin.

**101.** Quyidagi tenglamalarni butun sonlarda yeching:

a) $3x + 4y = 13$;
b) $8x - 13y = 63$;
c) $43x + 37y = 21$;
d) $45x - 37y = 25$;
e) $81x - 48y = 33$;
f) $26x + 3y = 13$;
g) $53x + 17y = 25$;
h) $47x - 105y = 4$;
i) $18x - 33y = 112$;
j) $11x + 16y = 156$;
k) $12x - 37y = -3$;
l) $23x + 15y = 19$.

**102.** Don tashish uchun 60 kg va 80 kg lik qoplar bor. 440 kg donni tashish uchun shu xaltalardan nechtadan kerak bo'ladi?

**103.** 14900 so'mga 300 so'mlik va 500 so'mlik chiptalardan nechtadan sotib olsa bo'ladi?

**104.** $ax + by = c$ to'g'ri chiziqning abssissalari $a_1$ va $a_2$ ga teng bo'lgan kesmasida nechta butun son mavjudligini toping:

a) $8x - 13y + 6 = 0$; $a_1 = -100$; $a_2=150$;
b) $7x + 29y = 584$; $a_1 = -20$, $a_2 = 160$;
c) $90x - 74y = 50$; $a_1 = -100$, $a_2 = 200$.

**105*.** $a$ va $b$ ning qanday natural qiymatlarida $ax - by = 31$ tenglama $x = 5$, $y = 9$ yechimga ega?

**106*.** Uchlari $A(x_1, y_1)$, $V(x_2, y_2)$ butun nuqtalarda bo'lgan kesmaning ichki butun nuqtalari soni $d - 1$ ga tengligini ko'rsating, bu reda $d = (y_1 - y_2, x_1 - x_2)$.

**107.** Uchlari $A(2, 1)$, $B(20, 7)$, $C(8, 15)$ nuqtalarda bo'lgan uchburchak ning tomonlari nechta butun nuqtalardan o'tadi?

**108.** $x$ qanday butun qiymatlarida quyidagi funksiyalar butun qiymatlarni qabul qiladi:

a) $f(x) = \frac{9x-1}{7}$; b) $F(x) = \frac{7x-1}{15}$?

**109*.** Fabrikaga 18 ta vagonda 500 tonna paxta olib kelishdi. Vagonlarda 15, 20 va 30 tonnadan paxta bo'lgan. 15, 20 va 30 tonnalik vagonlardan nechtadan bo'lgan?

## § 4. Birinchi darajali taqqoslamalar sistemalari

Bir noma'lumli har xil modulii birinchi darajali taqqoslamalr sistemasining umumiy ko'rinishi quyidagidan iborat:

$$\left.\begin{array}{l} a_1x \equiv b_1(mod\ m_1) \\ a_2x \equiv b_2(mod\ m_2) \\ \dots\dots\dots\dots \\ a_nx \equiv b_n(mod\ m_n) \end{array}\right\} \quad (1)$$

Bu sistema yechimini topishning umumiy usuli quyidagicha: dastlab sistemaning birinchi taqqoslamasining $x \equiv \alpha\ (mod\ m_1)$ yechimi topiladi, bu yerda $\alpha$ - $m_1$modul bo'yicha manfiy bo'lmagan eng kichik yoki absolyut qiymati jihatidan eng kichik chegirmadan iborat, bu yechimni sonlar sinfi shaklida yozib olinadi:

$$x = m_1t + \alpha. \quad (2)$$

(Agar birinchi taqqoslama yechimga ega bo'lmasa, berilgan sistema ham yechimga ega bo'lmaydi).

So'ngra $x$ ning (2) dagi qiymati sistemaning ikkinchi taqqoslamasiga qo'yilib,

$$a_2(m_1t + \alpha) \equiv b_2(mod\ m_2) \quad (3)$$

taqqoslama hosil qilinadi . (3) taqqoslamadan $t$ ning sonlar sinfi shaklidagi

$$t = m_2t_1 + \beta$$

ko'rinishi topilib, u (2) tenglikka qo'yiladi va $x$ ning yangi qiymati hisoblanadi. (Agar (3) taqqoslama yechimga ega bo'lmasa, berilgan sistema ham yechimga ega bo'lmaydi).

Natijada $x$ ning sonlar sinfi shaklida yozilgan va berilgan sistemaning dastlabki ikkita taqqoslamasini qanoatlantiradigan qiymati hosil bo'ladi. $x$ ning topilgan qiymati uchinchi taqoslamaga qo'yilib, hosil bo'lgan taqqoslama $t_1$ ga nisbatan yechiladi va $t_1$ ning sonlar sinfi shaklida yozilgan qiymati $x$ ning ifodasiga qo'yladi, so'ngra $x$ ning bu qiymati to'rtinchi taqqoslamag qo'yiladi va shu taxlitda sistemaning oxirgi taqqoslamasigacha yechiladi. *x ning oxirgi qiymati berilgan sistemaning yechimidan iborat bo'ladi.*

Berilgan sistemani yechishda dastavval har bir taqqoslamani alohida yechib, sistema quyidagi ko'rinishga keltirib olinadi:

$$\left.\begin{array}{l} x \equiv \alpha_1(mod\ m_1) \\ x \equiv \alpha_2(mod\ m_2) \\ \dots\dots\dots\dots \\ x \equiv \alpha_n(mod\ m_n) \end{array}\right\} \quad (4)$$

So'ngra yuqoridagi usul qo'llaniladi.

Agar (1) sistemaning $a_i x \equiv b_i \ (mod\ m_i)\ (i = \overline{1, n})$ taqqoslamalari uchun $(a_i, m_i) = d_i$ va $d_i | b_i$ bo'lsa, u holda har bir $i$-nchi taqqoslamaning hadlarini va modulini $d_i$ ga qisqartirib, (1) sistemaga teng kuchli bo'lgan quyidagi sistema hosil qilinadi:

$$\left.\begin{array}{l} \frac{a_1}{d_1}x \equiv \frac{b_1}{d_1}\left(mod\ \frac{m_1}{d_1}\right) \\ \frac{a_2}{d_2}x \equiv \frac{b_2}{d_2}\left(mod\ \frac{m_2}{d_2}\right) \\ \dots\dots\dots\dots\dots\dots\dots \\ \frac{a_n}{d_n}x \equiv \frac{b_n}{d_n}\left(mod\ \frac{m_n}{d_n}\right) \end{array}\right\}. \qquad (5)$$

Bu sistemaning taqqoslamalirini $x$ ga nisbatan yechib, (5) sistemaning yechimini quyidagi sistemaning yechimiga keltirish mumkin:

$$\left.\begin{array}{l} x \equiv \alpha_1\left(mod\ \frac{m_1}{d_1}\right) \\ x \equiv \alpha_2\left(mod\ \frac{m_2}{d_2}\right) \\ \dots\dots\dots\dots\dots\dots \\ x \equiv \alpha_n\left(mod\ \frac{m_n}{d_n}\right) \end{array}\right\} \qquad (6)$$

Agar (4) sistemada $m_1, m_2, \dots, m_n$ modullar juft-jufti bilan o'zaro tub bo'lsa, $i \neq j$ da $(m_i, m_j) = 1$ bo'lsa, u holda uning yechimini quyidagi formula bilan ham topish mumkin

$$x_0 = \frac{M}{m_1} y_1 \alpha_1 + \frac{M}{m_2} y_2 \alpha_2 + \dots + \frac{M}{m_n} y_n \alpha_n, \qquad (7)$$

bu yerda $M = [m_1, m_2, \dots, m_n]$ va $y_1, y_2, \dots, y_n$ lar

$$\frac{M}{m_i} y_i = 1(mod\ m_i),\ i = \overline{1, n}$$

taqqoslamalarning yechimlaridan iborat. Sistemaning yechimi

$x \equiv x_0\ (mod\ M)$ taqqoslamadan iborat bo'ladi.

Agar $\frac{m_1}{d_1}, \frac{m_2}{d_2}, \dots, \frac{m_n}{d_n}$ muodullar juft-jufti bilang o'zaro tub bo'lsa, Bu usul bilan (6) sistemani ham yechish mumkin.

Misol 1. Quyidagi taqqoslamalr sistemasini yeching:

$$\left.\begin{array}{l} x \equiv 13(mod\ 16) \\ x \equiv 3(mod\ 10) \\ x \equiv 9(mod\ 14) \end{array}\right\}$$

*Yechilishi.* Birinchi taqqoslamadan:

$$x = 16t + 13.$$

ni hosil qilamiz. $x$ ning bu qiymatini ikkinchi taqqoslamag qo'yamiz:

$$16t + 13 \equiv 3\ (mod\ 10), \quad \text{yoki} \quad 16t + 10 \equiv 0\ (mod\ 10),$$

Bu yerdan $8t \equiv 0\ (mod\ 5)$, yoki $16t \equiv 0\ (mod\ 5)$ ni hosil qilamiz.
Demak, $t = 5t_1$.
$t = 5t_1$ ni $x = 16t + 13$ ifodaga qo'yamiz:

$$x = 16\cdot 5t_1 + 13 = 80t_1 + 13.$$

$x$ ning topilgan qiymatini uchinchi taqqoslamag qo'yamiz:

$80t_1 + 13 \equiv 9\ (mod\ 14)$, yoki $80t_1 \equiv -4\ (mod\ 14)$, bu yerdan
$80t_1 \equiv 10\ (mod\ 14)$, yoki $40t_1 \equiv 5\ (mod\ 7)$, yoki
$8t_1 \equiv 1\ (mod\ 7)$, bu yerdan $t_1 \equiv 1\ (mod\ 7)$, ya'ni, $t_1 = 7t_2 + 1$.

$t_1 = 7t_2 + 1$ ni $x = 80t_1 + 13$ ifodaga qo'yib,

$$x = 80\ (70t_2 + 1) + 13 = 560t_2 + 93$$

ni hosil qilamiz. Shunday qilib, $x \equiv 93$ (mod 560). ■

*Tekshirish*: 93 – 13 ayirma 16 ga bo'linadi; 93 – 13 ayirma 10 ga bo'linadi; 93 – 9 ayirma 14 ga bo'linadi.

*Eslatma.* $16t \equiv 0\ (mod\ 10)$ taqqoslamani yechishda biz $8t \equiv 0\ (mod\ 5)$ taqqoslamani hosil qildik, uning yechimi $t \equiv 0\ (mod\ 5)$, yoki $t = 5t_1$ berilgan taqqoslamaning $x = 80t_1 + 13$ yechimiga olib keldi. Ammo $16t \equiv 0\ (mod\ 10)$ taqqoslamaning ikkinchi $t \equiv 5\ (mod\ 10)$, yoki $t = 10t_1 + 5$ yechimi ham mavjud (chunki, $d = (16, 10) = 2$). Bu yechimni $x = 16t + 13$ ifodaga qo'yib, $x = 16(10t_1 + 5) + 13 = 160t_1 + 93$ yechimni hosil qilamiz. Lekin $93 \equiv 13\ (mod\ 80)$ bo'lganligi uchun, ya'ni 93 va 13 sonlari 80 modul bo'yicha bir sinfga tegishli bo'lganligi uchun $x$ ning bu qiymatiga mos bo'lgan yechim qaralmaydi.

Bu eslatmadan (1-misol) agar sistemaning biror taqqoslamasi yoki $t_1$ ga nisbatan biror taqqoslama $m$ modul bo'yicha $d$ ta yechimga ega bo'lsa, u holda sistemani yechimini topish uchun $d$ ta yechimga ega bo'lgan taqqoslama yechimini unga teng kuchli bo'lgan $m/d$ modul bo'yicha taqqoslama yechimi bilan almashtirish yetarlidir.

Misol 2. Taqqoslamalar sistemasini yeching:

$$\left.\begin{aligned} 7x &\equiv 3(mod\ 11) \\ 15x &\equiv 5(mod\ 35) \\ 3x &\equiv 2(mod\ 5) \end{aligned}\right\}$$

*Yechilishi.* Sistemaning har bir taqqoslamasini alohida yechib, bu sistemaga teng kuchli bo'lgan quyidagit sistemani hosil qilamiz:

$$\left.\begin{aligned} x &\equiv 2(mod\ 11) \\ x &\equiv 5(mod\ 7) \\ x &\equiv 4(mod\ 5) \end{aligned}\right\}$$

Bu sistemaning modullari juf-jufti bilan o'zaro tub sonlardan iborat bo'lganligi uchun uning yechimini (7) formula bilan topish mumkin.

$M = [11, 7, 5] = 385,$

$$\frac{M}{m_1} = 35, \quad \frac{M}{m_2} = 55, \quad \frac{M}{m_3} = 77\,.$$

sonlarni topib, quyidagi taqqoslamalarni tuzamiz:

$$35u_1 \equiv 1\ (mod\ 11),\ 55u_2 \equiv 1\ (mod\ 7),\ 77u_3 \equiv 1\ (mod\ 5),$$

bu yerdan $u_1 = 6$, $u_2 = -1$, $u_3 = 3$ larni hosil qilamiz.
Endi (7) formuladan quyidagini hosil qilamiz:

$$x_0 = 35 \cdot 6 \cdot 2 + 55 \cdot (-1) \cdot 5 + 77 \cdot 3 \cdot 4 = 1069 \equiv 299 \ (mod\ 385).$$

Shunday qilib, $x \equiv 299$ (*mod* 385). ■

Misol 3. Taqqoslamalar sistemasini yeching:

$$\left.\begin{array}{l} 5x \equiv 7(mod\ 9) \\ 4x \equiv 3(mod\ 7) \\ 3x \equiv 8(mod\ 12) \end{array}\right\}$$

*Yechilishi*. Berilgan sistemaning uchinchi taqqoslamasida (3, 12) = 3, ammo 8 soni 3 ga bo'linmaydi, shuning uchun bu taqqoslama ham berilgan sistema ham yechimga ega emas.

Misol 4. Taqqoslamalar sistemasini yeching:

$$\left.\begin{array}{l} x \equiv 2(mod\ 3) \\ x \equiv 3(mod\ 2) \\ x \equiv -1(mod\ 6) \end{array}\right\}$$

*Yechilishi*. Sistemaning dastlabki ikkita taqqoslamasi $x \equiv -1$ (*mod* 3) va $x \equiv -1$ (*mod* 2) taqqoslamalarga teng kuchli, shuning uchun ularni uchinchi taqqoslamaning natijasi bo'lganligi uchun tashlab yuborilsa bo'ladi. Shunday qilib, sistema uchinchi taqqoslamasining yechimi sistemaning ham yechimi bo'ladi, ya'ni. $x \equiv -1 \equiv 5$ (*mod* 6). ■

Misol 5. 2, 3, 4, 5, 6 va 7 sonlariga bo'linganida mos ravishda 1, 2, 3, 4, 5 va 0 qoldiq hosil bo'ladigan sonni toping.

*Yechilishi*. Masala yuidagi taqqoslamalr sistemasiga keltiriladi:

$$\left.\begin{array}{l} x \equiv 1(mod\ 2) \\ x \equiv 2(mod\ 3) \\ x \equiv 3(mod\ 4) \\ x \equiv 4(mod\ 5) \\ x \equiv 5(mod\ 6) \\ x \equiv 0(mod\ 7) \end{array}\right\}$$

$x \equiv 1$ (*mod* 2) yoki $x \equiv 3$ (*mod* 2) taqqoslama $x \equiv 3$ (*mod* 4) taqqoslamaning natijasi sifatida tashlab yuborilishi mumkin. Xuddi shunday $x \equiv 2$ (*mod* 3) taqqoslama ham olinmaydi.

Shunday qilib, quyidagi sistemani hosil qilamiz:

$$\left.\begin{array}{l} x \equiv 3(mod\ 4) \\ x \equiv 4(mod\ 5) \\ x \equiv 5(mod\ 6) \\ x \equiv 0(mod\ 7) \end{array}\right\}$$

Bu sistemani yechib, $x \equiv 119$ (*mod* 420) ni hosil qilamaiz. ■

Misol 6. Quyidagi taqqoslama yechimga ega bo'ladigan $a$ ning qiymatlarini toping:

$$\left.\begin{aligned} x &\equiv 5 \pmod{18} \\ x &\equiv 8 \pmod{21} \\ x &\equiv a \pmod{35} \end{aligned}\right\}$$

*Yechilishi.* Birinchi taqqoslamadan

$$x = 18t + 5$$

ni hosil qilamiz. $x$ ning bu qiymatini ikkinchi taqoslamaga qo'yib, $t$ ning qiymatini topamiz:

$18t + 5 \equiv 8$ (*mod* 21), yoki $18t \equiv 3$ (*mod* 21), yoki $6t \equiv 1$ (*mod* 7), $t \equiv 6$ (*mod* 7). $t \equiv -1$ (*mod* 7) ni olish qulayroq, bu yerdan $t = 7t_1 - 1$. Bu qiymatni $x$ ning ifodasiga qo'yib,

$$x = 16\ (7t_1 - 1) = 5 = 126t_1 - 13.$$

$x$ ning hosil qilingan qiymatini sistemaning uchinchi taqqoslamaga qo'yamiz:

$$126t_1 - 13 \equiv a\ (mod\ 35),\ \text{t.ye. } 21t_1 \equiv a = 13\ (mod\ 35).$$

(21, 35) = 7 bo'lganligi uchun oxirgi taqqoslama yechimga ega bo'lishi uchun $a + 13 \equiv 0$ (*mod* 7) taqqoslama yechimga ega bo'lishi kerak, bu yerdan $a \equiv 1$ (*mod* 7). *Shunday qilib, berilgan sistema* a *≡ 1 (*mod *7) bo'lganda yechimga ega.* ■

Misol 7. O'nlik sanoq sistemasida berilgan $4x87u6$ soni 56 ga bo'linadi. Shu sonni toping.

**Yechilishi.** Masala shartidan quyidagi taqqoslamalarni tuzamiz:

$$\left.\begin{aligned} 4x87y6 &\equiv 0 \pmod{8} \\ 4x87y6 &\equiv 0 \pmod{7} \end{aligned}\right\}$$

Birinchi taqqoslamadan $7y6$ ning 8 ga bo'linishi va 8 ga bo'linish alomatiga asosan $y = 3$ va $y = 7$ qiymatlarni hosil qilamiz.

Bu qiymatlarni ikkinchi taqqoslamag qo'yib,:

4x8736 ≡ 0 (*mod* 7),

4x8776 ≡ 0 (*mod* 7)

taqqoslamalarni hosil qilamiz. Bu taqqoslamalrni quyidagi ko'rinishda tasvirlab olamiz

$400000 + 10000x + 8736 \equiv 0$ (*mod* 7), $\qquad$ $4x \equiv 1$ (*mod* 7),

yoki

$400000 + 10000x + 8776 \equiv 0$ (*mod* 7), $\qquad$ $4x \equiv 3$ (*mod* 7).

Birinchi taqqoslama $x \equiv 2$ (*mod* 7), yoki $x = 7t+2$ yechimga ega. Bu yerdan $t=0$ da $x_1=2$ va $t=1$ da $x_2=9$ ni hosil qilamiz. $t$ ning boshqa qiymatlariga mo keluvchi $x$ ning qiymatlari yaramaydi.

Ikkinchi taqqoslama $x \equiv 6$ (*mod* 7) yoki $x = 7t + 6$ yechimga ega. Bundan yagona qiymat $x_3 = 6$ ni hosil qilamiz. $x$ ning hosil qilingan qiymatlarini berilgan sonning ifodasiga qo'yib, 428736, 498736, 468776 sonlarni hosil qilamiz. ■

Misol 8. Quydagi taqqoslamani o'zaro tub modullar bo'yicha taqqoslamalar sistemasiga keltirib yeching:

$$x^3 + 2x + 3 \equiv 0\ (mod\ 15)\ .$$

*Yechilishi.* Berilgan taqqoslama quyidagi sistemaga teng kuchli:

$$\left.\begin{array}{l} x^3+2x+3\equiv 0(mod\ 5) \\ x^3+2x+3\equiv 0(mod\ 3). \end{array}\right\}$$

Bu sistemaningn ikkinchi taqqoslamasi $(x-1)x(x+1)\equiv 0$ $(mod\ 3)$ taqqoslamaga teng kuchli va u $x$ ning barcha butun qiymatlari uchun o'rinli. Demak berilgan taqqoslama quyidagi taqqoslamaga teng kuchli bo'ladi

$$x^3+2x+3\equiv 0\ (mod\ 5),$$

bu yerdan $x\equiv 2;\ 4\ (mod\ 5)$ ni hosil qilamiz.

Berilgan 15 modul bo'yicha quyidagi yechimlarni hosil qilamiz:

$x\equiv 2;\ 7;\ 12;\ 4;\ 9;\ 14\ (mod\ 15)$. ■

Misol 9. Quyidagi chiziq qaysi butun nuqtalardan o'tadi:

$$15u=2x^3-5x^2+4x+11,\ \text{bu yerda}\ -2<x<8\ ?$$

*Yechilishi.* Chiziq tenglamasidan $2x^3-5x^2+4x+11\equiv 0\ (mod\ 15)$ taqqoslamaga ega bo'lamiz. Bu taqqoslama esa quyidagi sistemaga teng kuchli

$$2x^3-5x^2+4x+11\equiv 0\ (mod\ 5)$$
$$2x^3-5x^2+4x+11\equiv 0\ (mod\ 3).$$

Birinchi taqqoslama $x\equiv 2;\ 4\ (mod\ 5)$ yechimlarga, ikkinchisi esa $x\equiv 1;\ 2\ (mod\ 3)$ yechimlarga ega.

Endi

$$\begin{cases} x\equiv 2\ (mod\ 5) \\ x\equiv 1\ (mod\ 3) \end{cases} \begin{cases} x\equiv 2\ (mod\ 5) \\ x\equiv 2\ (mod\ 3) \end{cases} \begin{cases} x\equiv 4\ (mod\ 5) \\ x\equiv 1\ (mod\ 3) \end{cases} \begin{cases} x\equiv 4\ (mod\ 5). \\ x\equiv 2\ (mod\ 3), \end{cases}$$

taqqoslamalarni yechib, $x\equiv 7;\ 2;\ 4;\ 14\ (mod\ 15)$ yechimlarni topamiz.

Shartda ko'rsatilgan oraliqqa $x$ ning quyidagi qiymatlari tushadi: $x=7;\ 2;\ 4;\ -1$. $u$ ning mos qiymatlari chiziqningn berilgan tenglamasidang topiladi. ■

Misol 10. Taqqoslamalar sistemasini yeching:

$$\begin{cases} 9u\equiv 15 \\ 7x-3u\equiv 1 \end{cases} (mod\ 12),$$

*Yechilishi.* Birinchi taqqoslamaning ikkala tomonini va modulini 3 ga qisqartirib, $3u\equiv 5\ (mod\ 4)$, yoki $3u\equiv 9\ (mod\ 4)$, yoki $u\equiv 3\ (mod\ 4)$ ni hosil qilamiz.

12 modul bo'yicha $u\equiv 3;\ 7;\ 11\ (mod\ 12)$ yechimlar kelib chiqadi.

Bu yerdan quyidagi uchta sistemani hosil qilamiz:

$$\begin{cases} 7x\equiv 1+3u \\ u\equiv 3 \end{cases} (mod\ 12),\quad \begin{cases} 7x\equiv 1+3u \\ u\equiv 7 \end{cases} (mod\ 12),\quad \begin{cases} 7x\equiv 1+3u \\ u\equiv 11 \end{cases} (mod\ 12)$$

Bu sistemalarni soddalashtirib,

$$\begin{cases} x\equiv 10 \\ u\equiv 3 \end{cases} (mod\ 12),\quad \begin{cases} x\equiv 10 \\ u\equiv 7 \end{cases} (mod\ 12),\quad \begin{cases} x\equiv 10 \\ u\equiv 11. \end{cases} (mod\ 12)$$

yechimlarni hosil qilamiz. ■

Misol 11. Taqqoslamalar sistemasin yeching:

$$\begin{cases} x+2u\equiv 3 \\ \end{cases}$$

$(mod\ 5)$.

$4x + u \equiv 2$

*Yechilishi.* Ikkinchi taqqoslamani 2 ga ko'paytirib, hosil bo'lgan taqqoslamadan birinchi taqqoslamani hadma-had ayiramiz: $7x \equiv 1\ (mod\ 5)$, bu yerdan $x \equiv 3\ (mod\ 5)$ ni hosil qilamiz. Birinchi taqqoslamani ikkala tomonini 4 ga ko'paytirib, hosil qilingan taqqoslamadan ikkinchisini ayiramiz: $u \equiv 0\ (mod\ 5)$. Tekshirish:

$$\begin{cases} x \equiv 3 \\ \qquad (mod\ 5) \\ u \equiv 2 \end{cases}$$

sistema berilgan sistemaning yechimidan iborat ekanligini ko'rsatadi. ■

## MAShQLAR

**110.** Quyidagi taqqoslamalar sistemalarini yeching:

a) $\left.\begin{array}{r} x \equiv 4\ (mod\ 5) \\ x \equiv 1\ (mod\ 12) \\ x \equiv 7\ (mod\ 14) \end{array}\right\}$; b) $\left.\begin{array}{r} x \equiv 1\ (mod\ 25) \\ x \equiv 2\ (mod\ 4) \\ x \equiv 3\ (mod\ 7) \\ x \equiv 4\ (mod\ 9) \end{array}\right\}$;

c) $\left.\begin{array}{r} 2x \equiv 7\ (mod\ 13) \\ 5x \equiv 8\ (mod\ 17) \\ 3x \equiv 7\ (mod\ 31) \\ 14x \equiv 35\ (mod\ 19) \end{array}\right\}$; d) $\left.\begin{array}{r} 4x \equiv 7\ (mod\ 13) \\ x \equiv 2\ (mod\ 17) \\ 5x \equiv 3\ (mod\ 9) \\ 8x \equiv 4\ (mod\ 14) \end{array}\right\}$;

e) $\left.\begin{array}{r} 3x \equiv 7\ (mod\ 10) \\ 2x \equiv 5\ (mod\ 15) \\ 7x \equiv 5\ (mod\ 12) \end{array}\right\}$; f) $\left.\begin{array}{r} 4x \equiv 1\ (mod\ 9) \\ 5x \equiv 3\ (mod\ 7) \\ 4x \equiv 5\ (mod\ 12) \end{array}\right\}$;

g) $\left.\begin{array}{r} 5x \equiv 1\ (mod\ 12) \\ 5x \equiv 2\ (mod\ 8) \\ 7x \equiv 3\ (mod\ 11) \end{array}\right\}$; h) $\left.\begin{array}{r} 3x \equiv 1\ (mod\ 10) \\ 4x \equiv 3\ (mod\ 5) \\ 2x \equiv 7\ (mod\ 9) \end{array}\right\}$;

i) $\left.\begin{array}{r} 3x \equiv 5\ (mod\ 7) \\ 2x \equiv 3\ (mod\ 5) \\ 3x \equiv 3\ (mod\ 9) \end{array}\right\}$; j) $\left.\begin{array}{r} 5x \equiv 200\ (mod\ 251) \\ 11x \equiv 192\ (mod\ 401) \\ 3x \equiv -15\ (mod\ 907) \end{array}\right\}$.

**111.** 2, 3, 4 ga bo'linganida 1 qoldiq qoladigan va 5 ga qoldiqsiz bo'linadigan barcha natural sonlarni toping.

**112.** 4, 5, 7 ga bo'linganida mos ravishda 3, 4, 5 qoldiq qoladigan 200 va 500 sonlari orasidagi barcha butun sonlarni toping.

**113*.** Abssissalar o'qiga perpendikulyar bo'lgan bitta chiziqda yotadigan $4x - 7u = 9$, $2x + 9u = 15$ va $5x - 13u = 12$ to'g'ri chiziqlarning butun nuqtalarini toping.

**114.** Quyidagi taqqoslamalar sistemalarini yeching:

a) $\left.\begin{array}{l} x \equiv a \ (mod\ 6) \\ x \equiv 1 \ (mod\ 8) \end{array}\right\}$; b) $\left.\begin{array}{l} x \equiv 2 \ (mod\ 6) \\ x \equiv \mathrm{a} \ (mod\ 8) \end{array}\right\}$;

c) $\left.\begin{array}{l} x \equiv 5 \ (mod\ 18) \\ x \equiv 8 \ (mod\ 21) \\ x \equiv a \ (mod\ 35) \end{array}\right\}$; d) $\left.\begin{array}{l} x \equiv a \ (mod\ 7) \\ x \equiv b \ (mod\ 5) \\ x \equiv c \ (mod\ 3) \end{array}\right\}$.

**115.** Quyidagi sistemalar yechimga ega bo'ladigan $a$ ning barcha qiymatlarini toping:

a) $\left.\begin{array}{l} x \equiv a \ (mod\ 6) \\ x \equiv 1 \ (mod\ 10) \\ x \equiv 2 \ (mod\ 21) \\ x \equiv 3 \ (mod\ 11) \end{array}\right\}$; b) $\left.\begin{array}{l} x \equiv 3 \ (mod\ 11) \\ x \equiv 11 (mod\ 20) \\ x \equiv 1 \ (mod\ 15) \\ x \equiv a \ (mod\ 18) \end{array}\right\}$; c) $\left.\begin{array}{l} 2x \equiv a \ (mod\ 4) \\ 3x \equiv 4 \ (mod\ 10) \end{array}\right\}$.

**116*.** O'nlik sanoq sistemasida $xuz138$ ko'rinishda yozilgan $N$ soni 7 ga bo'linadi, $138xuz$ soni esa 13 ga bo'linganida 6 qoldiq qoladi va $x1u3z8$ sonini 11ga bo'linganida 5 qoldiq qoladi. $N$ sonini toping.

**117*.** $13xu45z$ sonini 792 ga bo'linishini bilgan holda $x, u, z$ larni toping.

**118*.** Shunday uch xonali sonlarni topingki, ularni har birining o'ng tomoniga shu sondan keyin keladigan sonni yozsak aniq kvadrat hosil bo'lsin.

**119*.** Noma'lum sonni 7 ga bo'lsak, 3 qoldiq hosil bo'ladi, shu noma'lumning kvadratini $7^2$ ga bo'lsak 44 qoldiq hosil bo'ladi; uning kubini $7^3$ ga bo'lsak 111 qoldiq hosil bo'ladi. Noma'lum sonni toping.

**120.** Quydagi taqqoslamalarni o'zaro tub modullar bo'yicha taqqoslamalar sistemasiga keltirib yeching:

a) $13x \equiv 32 \ (mod\ 28)$; b) $245x \equiv 405 \ (mod\ 475)$; c) $78x \equiv 49 \ (mod\ 77)$;
d) $56x \equiv 81 \ (mod\ 45)$; ye) $x^2 \equiv -1 \ (mod\ 20)$; g) $x^2 \equiv -1 \ (mod\ 85)$.

**121.** Quyidagi chiziq qaysi butun nuqtalardan o'tadi:

$$14y = 3x^3 - 4x^2 + 11x + 4, \text{ bu yerda } -7 < x < 7?$$

**122.** Quyidagi taqqoslamalar sistemalarini yeching:

a) $\left\{\begin{array}{l} x + 3u \equiv 5 \\ 4x \equiv 5 \end{array}\right. (mod\ 7)$; b) $\left\{\begin{array}{l} x \equiv 2 \\ x - 2u \equiv 1 \end{array}\right. (mod\ 4)$;

c) $\left\{\begin{array}{l} 9u \equiv 15 \\ 3x - 7u \equiv 1 \end{array}\right. (mod\ 12)$; d) $\left\{\begin{array}{l} 3x - 5u \equiv 1 \\ 9u \equiv 15 \end{array}\right. (mod\ 12)$;

e) $\left\{\begin{array}{l} x + 2u \equiv 0 \\ 3x + 2u \equiv 2 \end{array}\right. (mod\ 5)$; f) $\left\{\begin{array}{l} 3x + 4u \equiv 29 \\ 2x - 9u \equiv -84 \end{array}\right. (mod\ 143)$;

g) $\left\{\begin{array}{l} x + 2u \equiv 4 \\ \end{array}\right. (mod\ 5)$; h) $\left\{\begin{array}{l} 4x - u \equiv 2 \\ \end{array}\right. (mod\ 6)$.

$3x + u \equiv 2$ $\qquad\qquad\qquad\qquad$ $2x + 2u \equiv 0$

**123.** Quyidagi tenglamalar sistemasini butun sonlarda yeching:

a) $\left.\begin{array}{r} x + 2y + 5z = 1 \\ 3x + y + 5z = 3 \end{array}\right\}$; $\qquad$ b) $\left.\begin{array}{r} x - y - 3z = 1 \\ x + y - 2z = 1 \end{array}\right\}$.

**124.**

$\dfrac{3x - u + 1}{7}$ va $\dfrac{2x + 3u - 1}{7}$ ifodalar butun son bo'ladigan.

$x$ va $u$ ninng butun qiymatlarini toping,

## JAVOBLAR va KO'RSATMALAR
## I-BOB
## BUTUN SONLAR XALQASIDA BO'LINISH NAZARIYASI

### 1-§

**1.** *a*) $q = 7; 8$ va $r = 2; 6$; *b*) $q = 8; 9$ va $r = 2; 6$.

**2.** *a*) *Yechish:* $(2n+1)^2 = 4n(n+1) + 1$, bu yerda $n(n + 1)$ 2 ga bo'linadi;

*b*) *Yechish:* $n^2 + (n + 1)^2 = 2n(n + 1) + 1$, bu yerda $n(n + 1)$ 2 ga bo'linadi.

**3.** *Ko'rsatma.* $15 = 7\cdot 2 + 1$. Agar $15^n = 7q+1$, u holda $15^{n+1} = 5^n \cdot 15 = 7Q + 1$.

**4.** *Yechish.* Masala sharti bo'yicha, $\frac{mn+pq}{m-p} = t -$ butun son.

$$\frac{mq+np}{m-p} - t = \frac{mq+np}{m-p} - \frac{mn+pq}{m-p} = \frac{q(m-p)-n(m-p)}{m-p} = q-n.$$

Bundan $\frac{mq+np}{m-p} = q - n + t -$ butun son. Demak, $mq + np$ $m - p$ ga bo'linadi.

**5.** *Yechish.* Masala sharti bo'yicha, $ad - bc = nt$ *va* $a - b = nt_1$. Ikkinchi tenglikni $d$ ga ko'paytirib, birinchisidan ayiramiz:
$b(c\text{-}d) = n(dt_1 - t)$. Bundan $b$ va $n$ ga qo'yilgan shartlarlarga asosan $s–d$ ni $n$ ga bo'linishi kelib chiqadi.

**6.** *c) Yechish.* $m^5 - m = (m - 1)m(m + 1)(m^2 + 1) = (m - 1)m(m + 1).[(m^2 - 4) + 5] = (m - 2)(m - 1)m(m + 1)(m + 2) + 5(m - 1)(m + 1)$. Qo'shiluvchilarning har biri 30 ga bo'linadi, chunki $k$ ta ketma-ket sonlar ko'paytmasi $k!$ ga bo'linadi (bu $C_n^k = \frac{n(n-1)(n-2)...(n-k+1)}{1.2.3....k}$ - butun son bo'lishidan kelib chiqadi). Bundan yig'indi ham 30 ga bo'linadi, demak $m^5 - m$ 30 ga bo'linadi.

**7.** *Yechish.* $10x + 5$ – izlanayotgan sonl bo'lsin. 5 raqamni chap tomondan birinchi o'ringa qo'yib $5 \cdot 10^5 + x$ hosil qilamiz . Berilgan shartlarga ko'ra $5 \cdot 10^5 + x = 4(10x + 5)$ tenglamaga kelamiz. Bundan
$x = 112820$ kelib chiqadi.

**8.** *Yechish.* Masala shartini quyidagicha yozib olamiz: $n(n + 1)(2n + 1) = n(n + 1)[(n - 1) + (n + 2)] = (n-1)n(n + 1) + n(n + 1)(n + 2)$. Har bir qo'shiluvchi 6 ga bo'linishidan (6 masala yechimidan) yig'indini 6 ga bo'linishi kelib chiqadi.

**9.** *Yechish.* $\frac{(2m+1)^2-(2n+1)^2}{(2m+1)^2+(2n+1)^2} = \frac{4(m+n+1)(m-n)}{2[2(m^2+n^2+m+n)]}$ hosil bo'lgan kasrni faqat 2 ga qisqartirish mumkin.

**10.** *Yechish.* $N^2 = 1000x + 100(y+1) + 10x + y = 101(10x+y) + 100$.

Bundan $10x + y = \frac{(N+10)(N-10)}{101}$, $N = 91$, $N^2 = 8181$.

**11.** *Yechish.* $(n-2)^2+(n-1)^2+n^2+(n+1)^2+(n+2)^2=5(n^2+2)$ to'la kvadrat bo'lishi uchun $n^2+25$ ga karrali bo'lishi kerak yoki $n^2$ ning oxirgi raqami 8 yoki 3 bo'lishi kerak, bu mumkin emas.

**12.** *Yechish.* Har qanday butun sonni quyidagilardan birortasi shaklida yozish mumkin: $9k$, $9k\pm 1$, $9k\pm 2$, $9k\pm 3$, $9k\pm 4$. Bu sonlar kvadratlari:
$(9k)^2=9(9k^2)$; $(9k\pm 1)^2=9(9k^2\pm 2k)+1$; $(9k\pm 2)^2=9(9k^2\pm 4k)+4$;
$(9k\pm 3)^2=9(9k^2\pm 6k+1)$; $(9k\pm 4)^2=9(9k^2\pm 8k+1)+7$. Natijada butun son kvadrati 9 ga bo'lganda qoldiq faqat 0, 1, 4, 7 bo'lishi mumkinligi kelib chiqadi.

**13.** *Yechish.*

$$S_n=7(1+11+111+\ldots+\underbrace{111\ldots1}_{n\ ta\ raqam})=7\Big(\frac{10-1}{9}+\frac{10^2-1}{9}+\frac{10^3-1}{9}+\ldots+\frac{10^n-1}{9}=$$

$$=\frac{7}{81}(10^{n+1}-9n-10).$$

**14.** *Yechish.*

$$\underbrace{111\ldots1}_{n\ ta\ raqam}\underbrace{555\ldots56}_{n\ ta\ raqam}=\frac{10^{n+1}-1}{9}\cdot 10^{n+1}+5\cdot 10\cdot\frac{10^n-1}{9}+6=$$

$$\left(\frac{10^{n+1}+2}{3}\right)^2=\left(\frac{10^{n+1}-1}{3}+1\right)^2=\left(\underbrace{333\ldots3}_{n\ ta\ raqam}+1\right)^2.$$

**15**. *Yechish.* $m\,n\,(m^4-n^4)=n\,(m^5-m)-m\,(n^5-n)$ 30 ga karrali (1 misolga ko'ra).

**16**. *Yechish.* $y^2=3x^2+2$ tenglama butun sonlarda yechimga ega emas. Haqiqatdan ham, $u$ ni $y=3n$ yoki $y=3n\pm 1$ shakllardan birortasi ko'rinishida ifodalash mumkin va bundan $y^2$ ni 3 ga bo'lganda qoldiq faqat 0 yoki 1 bo'ladi masala shartiga ko'ra qoldiq 2 bo'lishi kerak.

**17**. *Yechish.* Matematik induksiya usulini qo'llaymiz: $\overline{aaa}$ son 3 ga bo'linadi, chunki $a+a+a=3a;$ Agar $\overline{\underbrace{aa\ldots a}_{3^m}}$ son $3^n$ ga bo'linsa, u holda

$$\overline{\underbrace{aa\ldots a}_{3^{n+1}}}=\overline{\underbrace{aa\ldots a}_{3^n}\underbrace{aa\ldots a}_{3^n}\underbrace{aa\ldots a}_{3^n}}=\overline{\underbrace{aa\ldots a}_{3^n}}\cdot\left(10^{3^n}\right)^2+\overline{\underbrace{aa\ldots a}_{3^n}}\cdot 10^{3^n}+\overline{\underbrace{aa\ldots a}_{3^n}}=$$

$$=aa\ldots a\cdot 100\ldots0100\ldots01$$

$3^{n+1}$ ga bo'linadi.

## 2-§

**18**. *a*) 21, *b*) 13; *c*) 119; *d*) 3; *e*) 23.

**19**. *a*) 2520; *b*) 138600; *c*) 99671; *d*) 881200.

**20**. *Yechish.* $(a, b, c)=d$ bo'lsin, u holda $a=cq+r$, $b=cq_1+r_1$ dan $d|r$ va $d|r_1$ kelib chiqadi. $d=(c, r, r_1)$ ni isbotlaymiz. $(c, r, r_1)=D$ bo'lsin. $a=cq+r$ va $b=cq_1+r_1$ tengsizliklardan $D\,|\,a$, $D\,|\,b$ va shart bo'yicha $D|c$. Bundan $D=(a, b, c)$

va demak, $D = d$. $n$ ta son uchun $(a_1, a_2,\dots,a_n) = (a_n, r_1,r_2,.,r_{n-1})$ ni olamiz, bu yerda $r_1, r_2,\dots,r_{n-1} - a_1, a_2,\dots, a_{n-1}$ sonlarni $a_n$ ga bo'lgandagi qoldiq.

**21**. *a*) 23; *b*) 7; *c*) 21.

**22**. *a*) 3776; *b*) 1116; *c*) 67818; *d*) 5382; *e*) 6409.

**23**. *a*) *Yechish.* $(d,m) = (d, [dx, dy]) = d(1, [x, y]) = d$. Agar $d = (a_1, a_2,\dots,a_n)$ va $m = [a_1, a_2,\dots,a_n]$ deb olsak, natija o'zgarmaydi;

*b*) *Yechish.* Agar $p - a + b$ va $a \cdot b$ sonlarning umumiy bo'luvchichi bo'lsa, u holda $a$ yoki $b$ sonlardan birortasi $p$ ga bo'linishi kerak. $a + b$ ni $p$ ga bo'linishidan $p$ son $a$ va $b$ larni umumiy bo'luvchisi ekanligi kelib chiqadi. Bu masala shartiga zid, chunki $(a, b) =1$;

*c*) *Yechish.* $(a, b) = d$ va $a = dx$, $b = dy$ bo'lsin, bunda $(x, y) = 1$. Bu holda $(a + b, m ) = (d (x + y), dxy) = d(x + y, xy) = d$. Demak,
$(a+b, [a,b]) = (a, b)$.

**24**. *Yechish.* $x$ va $y$ – izlanayotgan sonlar bo'lsin va $(x, y) = d$, bundan $x = dm$ va $y = dn$ va $(m, n) = 1$. Shartga ko'ra, $x + y = d(m + n) =$
$= 667 = 23\cdot 29$. Shart bo'yicha $\frac{[x,y]}{(x,y)} = 120$, bundan $[x,y]=120\cdot(x, y)=120\, d$, boshqa tomondan $[x,y] = \frac{xy}{d} \Rightarrow \frac{xy}{d} = 120d$, *yoki* $xy = 120d^2$. Bulardan

$$\begin{cases} x + y = 23\cdot 29 \\ xy = 120d^2 \end{cases}$$

sistemani hosil qilamiz. $d\,(m + n) = 23 \cdot 29$ dan $d = 23$ va $d = 29$ ($d = 1$ yoki $d = 23 \cdot 29$ - o'rinli bo'lmaydi) bo'lishi mumkin. $d = 23$ bo'lganda, $x = 552$, $y = 115$. $d = 29$ da $x = 435$, $y = 232$.

**25**. *Yechish.* $x$ va $y$ – noma'lum sonlar va $(x,y) = d$ bo'lsin. U holda $\frac{x}{d} = m, \frac{y}{d} = n$, bunda $(m,n) = 1$. Shart bo'yicha $m + n = 18$,

$$[x, y] = \frac{xy}{d} = \frac{dm\cdot dn}{d} = mnd = 975 = 3\cdot 5^2\cdot 13.$$

Bundan $\begin{cases} m + n = 18 \\ mnd = 3\cdot 5^2 \cdot 13 \end{cases}$ ni hosil qilamiz va uning yechimi $m = 5, n = 13, d = 15$ bo'ladi. Demak, $x = 75$, $y = 195$.

**26.** *Yechish*. Shart bo'yicha, $a = 899$, $b = 493$. Yevklid algoritmiga ko'ra: $a = b\cdot 1 + 406$, $b = 406\cdot 1 + 87$, $406 = 87\cdot 4 + 58$, $87 = 58\cdot 1 + 29$, $58 = 29\cdot 2$ bo'ladi. Oxiridan ikkinchi tenglikdan boshlab: $29 = 87– 58 =87 – (406–87\cdot 4) = 87\cdot 5–406 = (b - 406)\cdot 5–406 = 5b - 406\cdot 6 = 5b - (a - b)6 = a(-b) + b\cdot 11$ ni olamiz. $29 = 899\, x + 493\, y$ bilan solishtirsak, $x = - 6$, $y = 11$ kelib chiqadi.

**27**. *a*) $17 = a\,(-10) + b \cdot 23 = ax + by$;

*b*) $43 = a \cdot (-4) + b \cdot 5 = ax + by$;

*c*) $47 = a \cdot 2 + b\,(-5) = ax + by$.

**28**. *a*) *Yechish.* $x = 45u$ va $y = 45v$, bu yerda $(u, v) = 1$, $\frac{u}{v} = \frac{11}{7}$ dan $u = 11$ va $v = 7$, demak $x = 495$ va $y = 315$; *b*) *Yechish.* $x = 20u$ va $y = 20v$, bu yerda $(u,v) = 1$, $uv = 21$ dan $u = 1; 3; 7; 21$ va $x = 20; 60; 140; 420$. $y = \frac{8400}{x}$ bo'lganligi sababli $y = 420, 140, 60, 20$. *d*) $x = 140$, $y = 252$.

**29**. *Yechish.* $(a, b, c) = d$ bo'lsin, u holda $a = md$, $b = nd$, $c = kd$

$\frac{a+b}{2} = \frac{m+n}{2}d$; $\frac{a+c}{2} = \frac{m+k}{2}d$; $\frac{b+c}{2} = \frac{n+k}{2}d$. Bundan $d$ son

$\frac{a+b}{2}$, $\frac{a+c}{2}$, $\frac{b+c}{2}$ sonlarning umumiy bo'luvchisi bo'lishini ko'rsatadi.

Faraz qilamiz, $\left(\frac{a+b}{2}, \frac{a+c}{2}, \frac{b+c}{2}\right) = D$ bundan $d|D$,

$\frac{a+b}{2} = m_1 D, \frac{a+c}{2} = n_1 D; \frac{b+c}{2} = k_1 D.$ Birinchit va ikkinchi tengliklar yig'indisidan uchinchi tenglikni ayirib, $a = (m_1 + n_1 - k_1)D$ ni hosil qilamiz. Shu usulda $b = (m_1 - n_1 + k_1)D$, $c = (-m_1 + n_1 + k_1)D$ larni hosil qilamiz. Bu tengliklardan $a, b, c$ larni $D$ ga bo'linishi kelib chiqadi va demak, $D|d$. Natijada

$$D = d, \quad \text{яъни} \quad (a,b,c) = \left(\frac{a+b}{2}, \quad \frac{a+c}{2}, \quad \frac{b+c}{2}\right).$$

**30**. *a*) *Yechish.* $(a, b, c) = d$ bo'lsin, u holda $(a, b) = md$, $(a, c) = nd$, $(b,c) = kd$, bu yerda $(m, n, k) = 1$. Bu tenglikdan *adm* va *dn* ga bo'linishi kelib chiqadi, demak, $a = dmn\alpha$. Xuddi shunday: $b = dmk\beta$; $c = dnk\gamma$. Bu yerda $(\alpha, \beta, \gamma) = 1$.

$$[a,b,c] = dmnk\alpha\beta\gamma = \frac{\alpha^4 m^2 n^2 k^2 \alpha\beta\gamma}{d^2 mnk} = \frac{(bmn\alpha)(dmk\beta)(dnk\gamma)d}{dm \cdot dn \cdot dk} = \frac{abc(a,b,c)}{(a,b)(a,c)(b,c)};$$

*b*) *Ko'rsatma*: $[a,b] = \frac{ab}{(a,b)}$ dan foydalaning.

**31**. *Yechish.* $qN = 100q + bq = 100q + a - a + bq = am - (a - bq)$, bundan tasdiq to'g'riligi kelib chiqadi, chunki $(q, m) = 1$.

**32**. *a*) 1;

*b*) *Yechish.* $(10n + 9, n + 1) = d$ va $10n + 9 = dx$, $n + 1 = dy$ bo'lsin. U holda $10(dy - 1) + 9 = dx$ yoki $10dy - 1 = dx$ va natijada $d = 1$.

*c*) *Yechish.* Agar $(3n + 1, 10n + 3) = d$ bo'lsa, u holda $\begin{cases} 3n+1 = dx \\ 10n+3 = dy \end{cases}$ *yoki* $\begin{cases} 30n+10 = 10dx \\ 30n+9 = 3dy \end{cases}$, bundan $1 = d(10x - 3y)$ va $d = 1$. Masalani Yevklid algoritmi yordamida ham yechish mumkin.

**33**. *Yechish.* $(q + 1)N = 10a(q + 1) + b(q + 1) = am + [a + b(q + 1)]$, bu yerda $(q + 1, 10q + 9) = 1$ (32 masalaga qarang).

**34**. *Yechish.* $(a, b) = d$ bo'lsin, u holda $a = md$, $b = nd$, $(m, n) = 1$. $5a + 3b = (5m + 3n)d$, $13a + 8b = (13m + 8n)d$ tengliklardan $5a + 3b$ va $13a + 8b$ larning umumiy bo'luvchisi $d$ bo'ladi. $(5a + 13b, 13a + 8b) = D$ bo'lsin, u holda $D|d$, $5a + 3b = m_1D$, $13a + 8b = n_1D$.

Bundan $a = (8m_1 - 3n_1)D$, $b = (5n_1 - 13m_1) = D$ va $D$ $a$ va $b$ larning bo'luvchisi, demak, $D|d$. Natijada $d = D$.

**35**. *Yechish.* $\frac{1}{a} + \frac{1}{a+b} = \frac{2a+b}{a(a+b)}$. $(a,b) =1$ dan $(a, 2a + b) = 1$ kelib chiqadi. $(2a + b, a + b) = 1$ ni ko'rsatamiz. $(2a + b, a + b) = d > 1$ bo'lsin, u holda $2a + b = dm$, $a + b = dn$, $(m, n) = 1$ va demak, $a = d(m - n)$,
$b = d(2n - m)$, ya'ni $d/a$, $d/b$ masala shartiga ziddir.

## 3-§

**36**. *a*) 211;

*b*) 2543, 2549, 2551, 2557;

*c*) 1201, 1213, 1217, 1223, 1229, 1231, 1237, 1249.

**37.** *Yechish.*

$$n^4 + 4 = n^4 + 4n^2 + 4 - 4n^2 = (n^2 + 2)^2 - 4n^2 = (n^2 + 2n + 2)(n^2 - 2n + 2).$$

**38**. *Yechish.* Barcha natural sonlarni $5n$, $5n \pm 1$, $5n \pm 2$ ko'rinishda yozish mumkin. $5n$ ko'rinishdagi son tub son bo'ladi, agar $n = 1$ bo'lsa va bu holda $p = 5$, $4p^2 + 1 = 101$, $6p^2 + 1 = 151$. Bu $p$ ning qiymati masala shartini qanoatlantiradi. Boshqa bunday sonlar mavjud emasligini ko'rsatamiz. Agar $p = 5n\pm1$ bo'lsa, $4p^2 + 1 = 5(20n^2 \pm 8n + 1)$ – murakkab son; agar $p = 5n + 2$ bo'lsa, $6p^2 + 1 = 5(30n^2 \pm 24n + 1)$ – murakkab son.

**39**. *Yechish.* Barcha natural sonlarni $6k$, $6k \pm 1$, $6k \pm 2$, $6k \pm 3$ ko'rinishda yozish mumkin. 2 va 3 dan tashqari $6k \pm 1$ ko'rinishdagi sonlar tub bo'lishi mumkin (teskarisi hamma vaqt o'rinli emas, ya'ni har qanday $6k \pm 1$ ko'rinishdagi sonlar tub son bo'lmasligi ham mumkin). Agar $p = 6k - 1$ bo'lsa, u holda $p + 10 = 6k - 1 + 10 = 3(2k + 3)$ – murakkab son; agar $p = 6k + 1$ bo'lsa, u holda $p + 14 = 6k + 1 + 14 = 3(2k + 5)$ – murakkab son. Shunday qilib, bir vaqtda $p + 10$ va $p + 14$ sonlar tub bo'ladigan 3 dan katta $p$ tub son mavjud emasligi ko'rsatdik.

Agar $p = 2$ bo'lsa, $p + 10$ va $p + 14$ – murakkab sonlar bo'ladi. Agar $p = 3$ bo'lsa, $p + 10$ va $p + 14$ – tub sonlar bo'ladi. Demak, bitta $p = 3$ son masala shartini qanoatlantiradi.

**40**. *Yechish.* Shart bo'yicha, $a > 3$, $m = 3t + 1$, $n = 3t_1 + 2$. 2 va 3 dan farqli tub sonlarni $p = 6k \pm 1$ ko'rinishda ifodalash mumkin (39 masalaga qarang). Agar $a = p = 6k + 1$, u holda $a + n = 6k + 1 + 3t + 2 =$
$= 3(2k + t + 1)$ – murakkab son; agar $a = p = 6k–1$, to $a + m = 6k - 1 + 3t + + 1 = 3(2k + t)$ murakkab son.

**41**. *Yechish.* $p - n!$ ning tub bo'luvchisi. $p \le n! - 1$ bo'lganligi sababli $p < n!$ Boshqa tomondan $n!$ $p$ ga bo'linmaydi, bundan $n < p$. Shunday qilib, $n < p < n!$ (bu isbotdan tub sonlar soni cheksiz ko'pligi kelib chiqadi).

**42**. *Yechish.* Shart bo'yicha, $2p + 1$ – to'la kub, ya'ni

$2p + 1 = (2x + 1)^3 = 8x^3 + 12x^2 + 6x + 1 = 2x(4x^2 + 6x + 3) + 1$, bundan $p = x(4x^2 + 6x + 3)$. $p$ – tub sonligidan $x = 1$ va $p = 13$, shuning uchun
$2p + 1 = 27 = 3^3$ – yagona son.

**43**. *Yechish.* Oldin natural sonlar qatorida 5 dan boshlab uchta ketma-ket kelgan toq sonlar barchasi tub bo'laolmasligini ko'rsatamiz. Faraz qilamiz, har bir tub sonlar jufti oralarida bitta murakkab son joylashgan (egzak sonlar). Tub sonlarni bunday joylashishi yetarlicha ziya bo'ladi. Bu holda tub sonlar $6n – 1$ va $6n + 1$ shaklida tasvirlash mumkin va ularning nomerlari $2n – 1$ va $2n$ bo'ladi. Haqiqatdan ham, $n = 1, 2, 3, 4, 5, 6, \dots$ deb
$6n – 1 = 5, 11, 17, 23, 29, 35, \dots$(bu sonlar nomerlari $2n – 1 = 1, 3, 5, 7, 9, 11, \dots$) va $6n + 1 = 7, 13, 19, 25, 31, 37, \dots$ (bu sonlar nomerlari $2n = 2, 4, 6, 8, 10, 12, \dots$). Bundan ko'rinyaptiki, har bir son o'zining nomeri uchlanganidan katta: $6n – 1 > 3(2n - 1)$ va $(6n + 1) > 3 \cdot 2n$.

**44**. *Ko'rsatma.* Natural sonlar qatoridagi sonlarni $30k$, $30k \pm \pm 1$, $30k \pm 2$, …, $30k \pm 15$ shaklida tasvirlaymiz. Bu sonlardan $p = 30k \pm 1$; $30k \pm 7$; $30k \pm 11$, $30k \pm 13$ lar tub sonlar bo'lishi mumkin.

**45**. *Yechish.* Agar $p – 1$ va $p + 1$ sonlar orasiga 3 dan katta $p$ son joylashtirilsa, $(p - 1)p(p + 1)$ ko'paytma 3 ga bo'linadi. $p>3$ bo'lganligi sababli $(p - 1)(p + 1)$ ko'paytma 3 ga bo'linishi kerak. Boshqa tomondan
$(p - 1)(p + 1)8$ ga bo'linadi, chunki agar $p – 12$ ga bo'linsa, $p + 1$ – hech bo'lmasa 4 ga bo'linishi kerak.

$p^2 – q^2 = (p - 1)(p + 1) – (q - 1)(q + 1)$, bu yerda $(p - 1)(p + 1)$ va $(q - 1)(q + 1)$ lar har biri 3 ga va 8 ga bo'linadi, demak, $p^2 – q^2$ 24 ga bo'linadi.

**46**. *a*) *Yechish.* Agar $p = 2$ bo'lsa, u holda $p + 10$ – murakkab son. Agar $p = 2q + 1$ ($q = 1,2,\dots,$) bo'lsa, u holda $p + 5$ – murakkab son.

**47**. *Yechish.* $p = 2k + 1$ ko'rinishdagi toq son. $p = mn$ ($m > n$) ko'rinishda ko'paytuvchilarga ajralsin. U holda shunday $x$ va $y$ sonlar topiladiki, bular uchun quyidagi sistema o'rinli:

$$\begin{cases} x + y = m \\ x - y = n \end{cases}, \quad bundan \quad x = \frac{m+n}{2},\ y = \frac{m-n}{2}.$$

Demak, murakkab $p$ uchun:

$$p = mn = (x + y)(x - y) = x^2 - y^2 = \left(\frac{m+n}{2}\right)^2 - \left(\frac{m-n}{2}\right)^2.$$

Agar $p$ tub bo'lsa, uni $p = (2k + 1)\cdot 1$ yagonashaklda yozish mumkin. Bu holda $m = 2k + 1 = p$, $n = 1$, demak,

$$p = \left(\frac{m+n}{2}\right)^2 - \left(\frac{m-n}{2}\right)^2 = \left(\frac{p+1}{2}\right)^2 - \left(\frac{p-1}{2}\right)^2.$$

Shunday qilib, $p=\left(\frac{p+1}{2}\right)^2-\left(\frac{p-1}{2}\right)^2$ ko'rinishda tasvirlanish yagona bo'lsa, $p$ – tub; agar $p=\left(\frac{m+n}{2}\right)^2-\left(\frac{m-n}{2}\right)^2$ ko'rinishda tasvirlangan bo'lsa, $p$ – murakkab son.

**48**. 47-masala shartidan toq sonlarni ($x$ + $y$) ($x$ - $y$) ko'rinishdagi ko'paytuvchilarga ajratishning quyidagi usuli kelib chiqadi: $p = x^2 - y^2$ tenglikdan $p + y^2 = x^2$, ya'ni $x$ ni topish uchun $p$ ga shunday $y$ $\left(y \le \frac{p-1}{2}\right)$ natural son kvadratini qo'yish kerakki natijada $p + y^2$ yig'indi kvadratdan ($x^2$) iborat bo'lsin. Shu usulda $y$ va $x$ ni topib

$p\ (x + y)\ (x - y) = m\ n.$

*a*) Kvadratlar jadvalidan foydalanib 6643 soniga yaqin bo'lgan son

$6724 = 82^2$ olamiz. $6724 – 6643 = 81 = 9^2$. Demak, $6643 = 82^2 - 9^2 = (82 + 9)\ (82 - 9) = 91 \cdot 73 = 7 \cdot 13 \cdot 73$;

*b*) $1769 = 61 \cdot 29$; *c*) $3551 = 67 \cdot 53$; *d*) $6497=89 \cdot 73$.

**49**. *Yechish.* $N = a^2 + b^2 = c^2 + d^2$ va $a$ va $b$, $c$ va $d$ – sonlarning juft toqligi har xil bo'lsin. $a$ va $c$, $b$ va $d$ – larning juft toqligi bir xil deb olamiz. $(a - c)\ (a + c) = (d - b)\ (d + b)$ tenglikdan

$$\frac{a-c}{d-b}+\frac{d+b}{a+c}=\frac{u}{v}$$

kelib chiqadi. Bunda birinchi kasrni $t$ ga va ikkinchi kasrni $s$ ga qisqartirilgan deb olsak, ya'ni $a – c = tu$, $d + b = su$, $ak = sv$, $d – b = tv$. U holda

$a=\frac{tu+sv}{2}$, $b=\frac{su-tv}{2}$ bo'ladi. Natijada

$$N=a^2+b^2=\frac{1}{4}\left[(tu+sv)^2+(sn-tv)^2\right]=\frac{1}{4}\left(u^2+v^2\right)\left(t^2+s^2\right).$$

**50**. *Yechish.* $972^2 + 235^2 = 1000009 = 1000^2 + 3^2$ dan 1000009 son ikki usulda ikki son kvadratlari yig'indisi ko'rinishda yozilishi kelib chiqadi, demak, bu son murakkab va $293 \cdot 3413$ ga teng.

**51**. *Yechish.* Quyidagi yoyilmani ko'ramiz:

$$a^{10}+a^5+1=\frac{a^{15}-1}{a^5-1}=\frac{\left(a^3-1\right)\left(a^{12}+a^9+a^6+a^3+1\right)}{(a-1)\left(a^4+a^3+a^2+a+1\right)}=$$

$$=\left(a^2+a+1\right)\left(a^8-a^7+a^5-a^4+a^3-a+1\right);$$

Bu yerda $a^{12} + a^9 + a^6 + a^2 + 1$ ko'phad $a^4 + a^3 + a^2 + a + 1$ ko'phadga ko'phadlarni bo'lish qoidasiga asosan bo'lingan. Natijada $3^{10} + 3^5 + 1 = (3^2 + 3 + 1)(3^8 - 3^7 + 3^5 - 3^4 + 3^3 - 3 + 1) = 13 \cdot 4561$.

**52**. *Yechish.* Agar $k$ – toq bo'lsa, $1 + 2^k$ son $1 + 2 = 3$ ga karrali. Agar $k$ – juft bo'lsa, u $k = 2^n$ ga yoki $k = 2^n m$ ($m \ge 1$ va toq son), yoki $k = 0$. Lekin

$1+2^k=1+2^{2^n m}=\left(1+2^{2^n}\right)^m$ $1+2^{2^n}$ ga karrali (agar $k$ = 0 bo'lsa, 2 ga karrali). Demak, barcha $k = 2^n$ dan farqli $k$ lar uchun $1+2^{2^n}$ son murakkab son bo'ladi.

**53**. *Yechish.* $(\alpha, \beta) = 1$, $(\alpha, \beta) = 2^n$ shartlarni qanoatlantiruvchi barcha $\alpha$ va $\beta$ lar uchun $a^\alpha + b^\beta$ murakkab son ekanligini ko'rsatamiz. Haqiqatdan ham. $(\alpha, \beta) = 1$ – bo'lib, toq bo'lsa, u holda $\alpha = dm$, $\beta = dk$, $(m, k) = 1$ va $a^\alpha + b^\beta = (a^m)^d + (b^k)^d$ $a^m + b^k$ ga karrali. Agar $(\alpha, \beta) = 2^n d$ juft son bo'lib, $d > 1$ – toq bo'lsa, u holda $\alpha = 2^n d m$, $\beta = 2^n d k$ bundan

$$a^\alpha + b^\beta = \left(a^{2^n \cdot m}\right)^\alpha + \left(b^{2^n k}\right)^\alpha \text{ son } a^{2^n m} + b^{2^n k} \text{ ga karrali.}$$

Demak, $(\alpha, \beta) = 1$ va $(\alpha, \beta) = 2^n$ shartlarni qanoatlantiruvchi $\alpha$ va $\beta$ lardan tashqari barcha hollarda $a^\alpha + b^\beta$ son murakkab bo'ladi. Teskari tasdiq noto'g'ri, masalan $2^4 + 3^2 = 25$ – murakkab son.

**54**. *Yechish.* $n$ – murakkab son bo'lsin, $n = ab$ ($a > 1$, $b > 1$), u holda $2^n - 1 = 2^{ab} - 1 = (2^a)^b - 1$ – murakkab son. Teskari tasdiq noto'g'ri: $2^p - 1$ hamma vaqt tub emas, masalan $2^{11} - 1 = 23 \cdot 89$; $2^{23} - 1 = 47 \cdot 178421$.

## 4-§

**55.** $a)\ \dfrac{271828}{10^5} = (2,1,2,1,1,4,1,1,6,10,1,1,2)$;

$b)\ \dfrac{103993}{33102} = (3,7,15,1,292)$;

$c)\ \dfrac{99}{170} = (0,1,1,2,1,1,6,2)$;

$d)\ \dfrac{355}{113} = (3;7,16)$.

**56.** $a)\ \dfrac{247}{74} = (3,2,1,2,2)$, munosib kasrlari: $\dfrac{3}{1}, \dfrac{4}{3}, \dfrac{11}{3}, \dfrac{15}{4}, \dfrac{131}{35}; \dfrac{277}{74}$;

$b)\ \dfrac{77}{187} = (0,2,2,3)$, munosib kasrlari: $\dfrac{0}{1}, \dfrac{1}{2}, \dfrac{2}{5}, \dfrac{7}{12}$;

$c)\ \dfrac{333}{100} = (3,3,33)$, munosib kasrlari: $\dfrac{3}{1}; \dfrac{10}{3}; \dfrac{333}{100}$;

$d)\ \dfrac{103993}{33102} = (3,7,15,1,292)$, munosib kasrlari:

$$\frac{3}{1}; \frac{22}{7}; \frac{333}{106}; \frac{355}{113}; \frac{103993}{33102}.$$

**57**. $a)$ $\dfrac{29}{37}$ sonni uzluksiz kasrga yoyamiz: $\dfrac{29}{7} = (0,1,3,1,1,1,2)$. Sxema yordamida

| $k$ | | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| $q_k$ | | 0 | 1 | 3 | 1 | 1 | 1 | 2 |
| $P_k$ | 1 | 0 | 1 | 3 | 4 | 7 | 11 | 29 |
| $Q_k$ | 0 | 1 | 1 | 4 | 5 | 9 | 14 | 37 |

kasrlarni topamiz

$$\frac{P_4}{Q_4}=\frac{7}{9} \qquad \frac{1}{Q_4Q_5}=\frac{1}{9\cdot 14}=\frac{1}{126}\approx 0{,}008<0{,}01;$$

$$\frac{7}{9}\approx 0{,}78 \qquad ortig'i \quad bilan$$

Shunday qilib, $\frac{29}{37}\approx\frac{7}{9}(+0{,}01)=0{,}78.$ $\frac{P_4}{Q_4}<\frac{a}{b}$ bo'lganligidan xatoni + ishora bilan olinadi. 7 ni 9 ga bo'lganda bo'linma ortig'i bilan olinishi sababi $\frac{7}{9}$ jami bilan yaqinlashishi bo'lganligidir. O'nli yaqinlashish $\frac{29}{37}\approx 0{,}78$ da xato ko'rsatilmaganligi sababi bu xatoni maxsus hisoblashdadir, ya'ni bu + 0, 008 xato va 7 ni 9 ga bo'lganda yaxlitlash xatolar yig'indisi.

$$b)\frac{648}{385}\approx\frac{69}{41}(+0{,}0003)\approx 1{,}6830;\ c)\frac{571}{359}\approx\frac{35}{22}(-0{,}0005)\approx 1{,}5909.$$

**59**. $a)\frac{43}{19};\quad b)\frac{73}{43};\quad c)\frac{2633}{1810};\quad d)\frac{1421}{552};$
$e)\frac{157}{225};\quad f)-1\frac{159}{215};\quad g)\frac{893}{11953}.$

**60**. a) $x = 2$; b) $x = 2$.

**61**. a) $x = -125 - 114t$, $y = 45 + 41t$;

b) $x = 4 + 15t$, $y = 5 + 19t$;

c) $x = 33 + 17t$, $y = 44 + 23t$;

d) $x = 88 + 47t$, $y = 99 + 53t$;

e) $x = -3 + 18t$, $y = 6 + 35t$;

f) $x = -25 + 71t$, $y = -30 + 85t$;

g) $x = -28 + 11t$, $y = -105 + 41t$, $t \in \mathbf{Z}$.

## 5-§

**62**. *a*) – 3; *b*) 11; *c*) 1; *d*) 2; *e*) 3; *f*) 2; *g*) – 2; *h*) – 2; agar $abcd > 1000,\ \ va\ \ -1, agar\ \ \overline{abcd}=1000;$ i) 7; j) – 3.

**63.** *Yechish.*

$x=[x]+\theta_1$ ва $y=[y]+\theta_2$ $bo'l\sin, bu\ yerda\ \ 0\le\theta_1<1, 0\le\theta_2<1,$

u holda $x+y=[x]+[y]+(\theta_1+\theta_2).\ Agar\ 0\le\theta_1+\theta_2<1,\quad [x+y]=[x]+[y];$

agar

$1 \le \theta_1 + \theta_2 < 2$ bo'lsa $[x+y] > [x] + [y]$ bo'ladi. Natijalarni birlashtirsak, $[x + y] \ge [x] + [y]$ ni hosil qilamiz.

**64**. *Yechish.* $[x]$ ni ta'rifiga ko'ra, masala shartiga asosan

$ax = m + \theta$, bu yerda $0 \le \theta < 1$ va $a \ne 0$, bu tenglikdan $x = \dfrac{m+\theta}{a}$ ni hosil qilamiz.

**65**. *Yechish.* 12,4 $m$ = 86 + θ, bu yerda 0 ≤ θ < 1. Tenglikni 5 ga ko'paytiramiz: $62m = 430 + 5\theta$, bundan $m = \dfrac{430+5\theta}{62} = 6 + \dfrac{58+5\theta}{62}$. $0 \le \theta < 1$ dan $0 \le 5\theta < 5$ va $m$ butun musbat son bo'lishi uchun $\dfrac{58+5\theta}{62} = t$ butun bo'lishi lozim. $t = 1$ deb olsak, $\theta = \dfrac{4}{5}$ ва $m = 7$ ni hosil qilamiz.

**66.** *Yechish.* $\left[\dfrac{p}{4}\right]_{p=4n+1} = n = \dfrac{p-1}{4};\quad \left[\dfrac{p}{4}\right]_{p=4n+3} = n = \dfrac{p-3}{4}.$

**67.** *Yechish.*

$a = mq + 1,\quad 0 \le r < m,\quad$ *ёки* $\quad \dfrac{a}{m} = q + \dfrac{r}{m},\ 0 \le \dfrac{r}{m} < 1,$ bu yerdan $q = \left[\dfrac{a}{m}\right]$ *u* $\left[\dfrac{a}{m}\right] = \dfrac{a-r}{m}.$

**68.** $\left[\dfrac{m}{2}\right]_{m=2k+1} = \left[k + \dfrac{1}{2}\right] = k = \dfrac{m-1}{2}.$

**69.** a) *Yechish.* $2 \le x^2 < 3$ yoki $\sqrt{2} \le |x| < \sqrt{3}$, *bundan* $-\sqrt{3} < x \le -\sqrt{2}$ va $\sqrt{2} \le x < \sqrt{3}$ ni olamiz;

b) *Yechish.* $x + 1$ ning qiymatlari va bundan $x$ ning qiymatlari ham butun bo'lishi zarur. Bu qiymatlarda $3x^2 - x$ ham butun bo'ladi va berilgan tenglama $3x^2 - x = x + 1$ teng kuchli bo'ladi, bundan $x = 1$ ni olamiz.

c) *Yechish.* Berligan tenglamani $0 \le x < 4$ qiymatlar qanoatlantiradi, bu qiymatlarda $\dfrac{3}{4}x$ butun qiymatlarni qabul qiladi, ya'ni $x = 0;\ 1\dfrac{1}{3};\quad 2\dfrac{2}{3};$

*d*) $x$=0; 1.

**70.** $\left[\dfrac{10^7}{786}\right] - \left[\dfrac{10^6}{786}\right] = 11450.$

**71.** *Yechish.* $999 - \left[\dfrac{999}{5}\right] - \left[\dfrac{999}{7}\right] + \left[\dfrac{\left[\dfrac{999}{5}\right]}{7}\right] = 686.$

**72.** *Yechish.* $100 - \left[\dfrac{100}{2}\right] - \left[\dfrac{100}{3}\right] + \left[\dfrac{100}{6}\right] = 33.$

**73.** 98.
**74.** 488.
**75.** $B$ (2311; 5, 7, 13, 17) = 1378;
**76.** $B$ (110; 2,3) = 37.
**77.** $B$ (12317; 3,5,7) = 5634.
**78.** 393.
**79.** $\left[\frac{p^n}{p}\right]+\left[\frac{p^n}{p^2}\right]+...+\left[\frac{p^n}{p^{n-1}}\right]+\left[\frac{p^n}{p^n}\right]=p^{n-1}+...+p+1=\frac{p^n-1}{p-1}$
**80.**
$a)10!=2^8\cdot 3^4\cdot 5^2\cdot 7;$

$b)15!=2^{11}\cdot 3^6\cdot 5^3\cdot 7^2\cdot 11\cdot 13;$

$c)20!=2^{18}\cdot 3^8\cdot 5^4\cdot 7^2\cdot 11\cdot 13\cdot 17\cdot 19;$

$d)25!=2^{22}\cdot 3^{10}\cdot 5^6\cdot 7^3\cdot 11^2\cdot 13\cdot 17\cdot 19\cdot 23;$

$e)30!=2^{26}\cdot 3^{14}\cdot 5^7\cdot 7^4\cdot 11^2\cdot 13^2\cdot 17\cdot 19\cdot 23\cdot 29.$

**81.** $\frac{20!}{10!10!}=2^2\cdot 11\cdot 13\cdot 17\cdot 19.$

**82**. *Yechish.* $N=\frac{1000!}{100!\,7^{\alpha}}$, bu yerda α

$$\left[\frac{1000}{7}\right]+\left[\frac{1000}{49}\right]+\left[\frac{1000}{301}\right]=\left[\frac{100}{7}\right]+\left[\frac{100}{49}\right]+\alpha$$

shartni qanoatlantiradi, bundan α = 148 kelib chiqadi.

**83**. *Yechish.* $(2m+1)!!=\frac{(2m+1)!}{(2m)!!}=\frac{(2m+1)!}{m!2^m}$. Agar $p > 2$ bo’lsa, u holda

$$\sum_{i=1}^{k}\left[\frac{2m+1}{p^i}\right]-\sum_{i=1}^{k}\left[\frac{m}{p^i}\right], \qquad \textit{бу ерда} \qquad p^k\le 2m+1<p^{k+1}.$$

**84**. *Yechish.* Ixtiyoriy butun $x = k$ $(a \le k \le b)$ abssissa uchun $[f(x)]+1$ butun ordinatali va berilgan trapesiyaning ichida va chegarasida joylashadi. Demak, nuqtalar soni $\sum_{k=a}^{b}([f(k)]+1)$ ga teng.

**85.** 126.

**86.** *Yechish.* Shartga asosan, $a = 4q + 1$ yoki $a = 4q + 3$ ga teng. Birinchi holda $\left[\frac{a}{4}\right]+\left[\frac{2a}{4}\right]+\left[\frac{3a}{4}\right]=q+2q+3q=6q=\frac{3(a-1)}{2}$. Ikkinchi hol ham xuddi shunday tekshiriladi.

**87.** *Yechish.* $a = mq + r$ bo’lsin, bu yerda $0 \le r < m$ va $(r, m) = 1$. $(r, m) = 1$ shart barcha $m \le 2$ lar uchun bajarilishidan $r = 1$ kelib chiqadi. Demak,

$$\sum_{i=1}^{m-1}\left[i\left(q+\frac{1}{m}\right)\right]=q\sum_{i=1}^{m-1}i=\frac{a-1}{m}\cdot\frac{m(m-1)}{2}=\frac{(a-1)(m-1)}{2}.$$

**88.** *Yechish.* $x=[x]+\alpha,\quad 0\le\alpha<1$ bo'lganligidan

$[x]+\left[x+\frac{1}{2}\right]=2[x]+\left[\alpha+\frac{1}{2}\right]$, bundan $\frac{1}{2}\le\alpha+\frac{1}{2}<1\frac{1}{2}$ va $\left[\alpha+\frac{1}{2}\right]$ 0 ga yoki 1 ga teng. $2x=2[x]+2\alpha$ va $[2x]=2[x]+[2\alpha]$ bo'lganligidan

$$[x]+\left[x+\frac{1}{2}\right]=[2x]-[2\alpha]+\left[\alpha+\frac{1}{2}\right].$$

Bu yerda $\left[\alpha+\frac{1}{2}\right]=[2\alpha]=0,$ yoki $\left[\alpha+\frac{1}{2}\right]=[2\alpha]=1.$

**89**. *Yechish.* Tenglamaning har bir qismini $y$ deb belgilab, $y\le\frac{x}{m}<\frac{x}{m-1}<y+1$ *ни оламиз*, бундан $my\le x<(m-1)\,(y+1)$. Bu tengsizlikni qanoatlantiruvchi $x$ lar mavjud bo'lishi uchun

$my<(m\text{-}1)\,(y+1)$ yoki $y<m-1$ shartlar bajarilishi zarur va yetarli. Bundan quyilagi natija kelib chiqadi: $my\le x<(m-1)\,(y+1)$, bu yerda $y-y<m-1$ shartni qanoatlantiruvchi butun son.

**90**. *Yechish.* $ax^2+bx+c$ funksiya va shu bilan birgalikda $[ax^2+bx+c]$ funksiya $a>0$ da quyidan, $a<0$ da esa yuqoridan chegaralangan. Ikala holda ham $[ax^2+bx+c]$ funksiya chegarasi $\left[-\frac{b^2-4ac}{4a}\right]$. songa teng. Shu sababli $a>0$ da berilgan tenglama yechimga ega bo'ladi, agar $\left[-\frac{b^2-4ac}{4a}\right]\le d;$ shart bajarilsa va faqat shartda, agar $a<0$ bo'lsa, bu shart quyidagicha: $\left[-\frac{b^2-4ac}{4a}\right]\ge 1.$

**91**. $a)\,0{,}6;\quad b)\,\frac{2}{3};\quad c)\,0;\quad d)\,\frac{1}{2}.$

**92**. a) 624; b) 2418; 30; c) 1440; 8; d) 1960; 12; e) 2808; 24; f) 3844;30.

**93**. a) 1, 2, 4, 8, 3, 6, 12, 24, 9, 18, 36, 72, 5, 10, 20, 40, 15, 30, 60, 120, 45, 90, 180, 360;

b) 1, 5, 25, 125, 3, 15, 75, 375.

**94**. *Yechish.* $S\,(2^{\alpha})=2^{\alpha+1}-1=2\cdot 2^{\alpha}-1$, demak, $m=2^{\alpha}$, $\alpha\in\mathbf{N}$.

**95**. *Yechish.* Agar $p$ son $m$ yoki $n$ ning kanonik yoyilmalarining birortasiga $\alpha$ ko'rsatkich kirsa, u holda $\tau\,(mn)$, va $\tau\,(m)\cdot\tau\,(n)$ da $\alpha+1$ ko'paytma mavjud. Agar $m$ va $n$ ning kanonik yoyilmalarida mos ravishda $\rho^{\alpha}$ va $\rho^{\beta}$ Lar bo'lsa, u holda $mn$ ning kanonik yoyilmasida $\rho^{\alpha+\beta}$ mavjud va $\tau\,(mn)$ dagi $\alpha+\beta+1$ ko'paytmaga $\tau\,(m)\cdot\tau\,(n)$ da qatnashuvchi $(\alpha+\beta)\,(\beta+1)>\alpha+\beta+1$ ko'paytma mos keladi. Demak, agar $(m, n)>1$, u holda $\tau\,(m)\,\tau\,(n)>\tau(mn)$. Agar $r$ $m$ yoki $n$ ning kanonik yoyilmasiga qatnashsa, yuqorida qayd qilinganidek, $S\,(x)$ ni hisoblash mumkin.

Ikkinchi holda S ($mn$) ga kiruvchi $\frac{p^{\alpha+\beta+1}}{p^{-1}}$, ko'paytmaga S ($m$) S ($n$) ga kiruvchi $\frac{p^{\alpha+1}-1}{p-1}\cdot\frac{p^{\beta+1}-1}{p-1}=\frac{p^{\alpha+\beta+2}-p^{\alpha+1}}{(p-1)^2}+\frac{p^{\beta+1}+1}{(p-1)^2}$, ko'paytma mos keladi.

$$\frac{p^{\alpha+\beta+2}-p^{\beta+1}+1}{p-1}-\left(p^{\alpha+\beta-1}-1\right)=\frac{p\left(p^{\alpha}-1\right)\left(p^{\beta}-1\right)}{p-1},$$

tenglikni o'rinliligini osongina ko'rsatish mumkin, bundan

$$\frac{p^{\alpha+1}-1}{p-1}\cdot\frac{p^{\beta+1}-1}{p-1}>\frac{p^{\alpha+\beta-1}-1}{p-1}.$$

Demak, agar ($m, n$) > 1 bo'lsa, S ($m$) S ($n$) > S ($mn$) bo'ladi.

**96**. $\tau$ ($m$) = 20, S ($m$) = 5208, $\delta$ ($m$) = $1968^{10}$.

**97**. *Yechish.* ¡zining barcha bo'luvchilari ko'paytmasiga teng bo'lgan $m$ natural son $m=\sqrt{m^{\tau(m)}}$, tenglama yordamida aniqlanadi, ya'ni $\tau$ ($m$) = 2. bundan masala yechimi kelib chiqadi.

**98**. $S_n(a)=\prod_{i=1}^{k}\frac{p_i^{n(\alpha_i+1)}-1}{p_i^n-1}$. *Ko'rsatma.* Matematik induksiya usulidan foydalaning.

**99**. a) $S_2$ (12) = 120; b) $S_2$ (18) = 455; c) $S_2$ (16) = 341.

**101**. *Yechish.* $2^{\alpha}\left(2^{\alpha+1}-1\right)=m \quad u \qquad 2^{\alpha+1}-1=p,$ bo'lsin, u holda

$S(m)=S\left(2^{\alpha}\cdot p\right)=\left(2^{\alpha+1}-1\right)(p+1)=\left(2^{\alpha+1}-1\right)2^{\alpha+1}=2m.$

**102**. *Yechish.* 101-masalaga asosan, har qanday mukammal son $2^{\alpha}\left(2^{\alpha+1}-1\right)$, ko'rinishda bo'lishini isbotshan kerak, bu urda $2^{\alpha+1}-1$ – tub son. $m = 2^{\alpha}q$ bo'lsin, ($q$, 2) = 1 va $S(m) = 2m$, ya'ni ($2^{\alpha+1}$-1) $S$ ($q$) = $2^{\alpha+1}\cdot q$, bundan $S(q)= 2^{\alpha+1}\cdot k$ va $q$ = ($2^{\alpha+1}$ - 1)$k$, $k\in$**N**. $k$ va ($2^{\alpha+1}$ - 1)$k$ sonlar $q$ ning bo'luvchilari bo'lib ular yig'indisi $k\cdot 2^{\alpha+1} = S(q)$ ga teng, bundan $q$ boshqa natural bo'luvchilarga ega emas. Demak $q$ = ($2^{\alpha+1}$-1)$k$ – tub son, bundan $k$ = 1 va $2^{\alpha+1}$ – 1 – tub sondir.

**103.** *Yechish.* S ($m$) = 3$m$ tenglama $m = 2^{\alpha}\cdot p_1p_2$ uchun ($2^{\alpha+1}$ - 1)(1 + $p_1$)(1 + $p_2$) = 3 · $2^{\alpha}p_1p_2$ ko'rinishga ega. Agar $\alpha$ = 0 bo'lsa (1 + $p_1$) (1 + $p_2$) = 3$p_1p_2$ yoki 1 + $p_1$ + $p_1$, bundan $p_1$ va $p_2$ juft son bo'lishi kerak, bu esa o'rinli emas, chunki 1 + $p_1$ va 1 + $p_2$ juft sonlar. Demak $\alpha\neq 0$. Agar $\alpha$ = 1 (1 + $p_1$) (1 + $p_2$) = 2$p_1p_2$ yoki 1 + $p_1$ + $p_2$ = $p_1p_2$, ya'ni 1 + $p_1$ = $p_2$ ($p_1$ - 1); $p_1$ – 1 = 2$n$ bo'lganligidan $n$ + 1 = $p_2 n$, bundan $n$ = 1 va $p_2$ = 2, bu esa o'rinli emas. Demak $\alpha\neq$ 1. Agar $\alpha$ = 2 bo'lsa, 7 (1 + $p_1$) (1 + $p_2$) = 12 $p_1p_2$ yoki 7 + 7 ($p_1$ + $p_2$) = 5$p_1p_2$, bundan $p_1$ = 7 (yoki $p_2$ = 7) va $p_2$ = 2 (yoki $p_1$ = 2), bunday bo'lishi mumkin emas. Demak $\alpha\neq$ 1. $\alpha$ = 3 bo'lganda 5(1 + $p_1$)(1 + $p_2$) = $\gamma$ $p_1p_2$ yoki 5 + 5 ($p_1$ + $p_2$) = 3 $p_1p_2$, bundan $p_1$ = 5 va $p_2$ = 3. Shunday qilib, masala shartini qanoatlantiruvchi eng kichik natural son $m$ =$2^3$ · 3 · 5 = 120 bo'ladi.

**104**. *Yechish.* Shart bo'yicha, $m = p_1^{\alpha_1} p_2^{\alpha_2}$ va $(1+p_1)(1+p_2) = 6$, bo'lganligidan $\alpha_1=1$, $\alpha_2 = 2$ va $m = p_1 p_2^2$. Bundan tashqari, S(m)=28, ya'ni $(1+p_1)(p_2^2+p_2+1)=28$, bundan $1+p_1=4$ va $p_2^2+p_2+1=7$, ya'ni $p_1=3, p_2=2$. Demak, m = $3 \cdot 2^2 = 12$.

**105.** *Yechish.* Masala sharti bo'yicha $m = p_1^{\alpha_1} p_2^{\alpha_2}$, $m^2 = p_1^{2\alpha_1} p_2^{2\alpha_2}$ $u\,(2\alpha_1+1)(2\alpha_2+1)=15$, bundan $2\alpha_1+1=3$ va $2\alpha_2+1=5$, ya'ni $\alpha_1=1$, $\alpha_2=2$. Demak, $\tau(m^2) = (3\alpha_1+1)(3\alpha_2+1) = 4 \cdot 7 = 28$.

**106**. *Yechish.* Shart bo'yicha $(1 + 2\alpha_1)(1 + 2\alpha_2) = 81$, ikki hol o'rinli bo'lishi mumkin: $(1 + 2\alpha_1)(1 + 2\alpha_2) = 3 \cdot 27$ va $(1 + 2\alpha_1)(1+2\alpha_2) = 9 \cdot 9$, ya'ni $\alpha_1 = 1$, $\alpha_2=13$ va $\alpha_1=\alpha_2=4$, bulardan $\tau(m^3) = 160$ yoki $\tau(m^3) = 169$.

**107**. *Ko'rsatma.* $d_1 = \frac{N}{d_n}$, $d_2 = \frac{N}{d_{n-1}}$,....dan foydalaning.

**108**. *Yechish.* $N$ ning barcha bo'luvchilarini o'sish tartibida yozamiz: $1, d_1, d_2, ..., \frac{N}{d_2}, \frac{N}{d_1}, \frac{N}{1}$, bular $(\alpha + 1)(\beta + 1)\dots(\mu + 1)$. Bularni juftliklarga bo'linsa, $1 \cdot \frac{N}{1}$, $d_1 \cdot \frac{N}{d_1}$, $d_2 \cdot \frac{N}{d_2}$,... barcha turli bo'linmalarni hosil qilamiz, ular soni $N$ – to'la kvadrat bo'lganda $\frac{(\alpha+1)(\beta+1)...(\mu+1)}{2}$, ga teng. Bu natijalarni birlashtirib, turli yoyilmalar soni $\left[\frac{1+(\alpha+1)(\beta+1)...(\mu+1)}{2}\right]$. ga tengligini olamiz.

**109**. *Ko'rsatma.* Masala yechimi $\begin{cases}(\alpha+1)(\gamma+1)=8\\(\alpha+1)(\beta+1)=12\\(\beta+1)(\gamma+1)=6\end{cases}$ sistemaga keladi. Bundan $N = 1400$.

**110**. $N = 2 \cdot 3 \cdot 5^4$.

**111.** *Yechish.* $m = p_1^{\alpha_1} p_2^{\alpha_2} ... p_k^{\alpha_k}$. bo'lsin, u holda $\tau(m) = (1+\alpha_1)(1+\alpha_2)\dots(1+\alpha_k)$. Agar $\tau(m) \equiv 1 (\mathrm{mod}\ 2)$ bo'lsa, u holda $1 + \alpha_i \equiv 1 (\mathrm{mod}\ 2)$ bo'ladi, bundan $\alpha_i \equiv 0 (\mathrm{mod}\ 2)$, bu esa $m$ – butun soni kvadrati bo'lishini ko'rsatadi. Teskaridan, agar $m$ – butun son kvadrati bo'lsa, u holda $\alpha_i \equiv 0 (\mathrm{mod}\ 2)$ va bundan $\tau(m) \equiv 1 (\mathrm{mod}\ 2)$ ni hosil qilamiz.

**112**. a) 2; b) 4; c) 4; d) 5; e) 9; f) 15; g) 46; h) 95.

**113**. a) ≈13; 13%; b) ≈ 22; 12%; c) ≈ 80; 16%.

**114**. *Yechish.* $\pi(p) < p$ tengsizlikdagi

$-p < -\pi(p)$, $p\pi(p) - p < (p-1)\pi(p)$ va $\frac{\pi(p)-1}{p-1} < \frac{\pi(p)}{p}$ ni hosil qilamiz.

$\pi(p)-1 = \pi(p-1)$ dan $\frac{\pi(p-1)}{p-1} < \frac{\pi(p)}{p}$ kelib chiqadi. $\pi(m-1) = \pi(m)$ tengsizlikdan $\frac{\pi(m)}{m} < \frac{\pi(m-1)}{m-1}$ ni olamiz.

**116**. a) 200; b) 192; c) 432; d) 320; e) 400; f) 1152.

**117**. a) 288; b) 24; c) 480; d) 388800.

**118**. 88.

**119**. *Yechish.* Masala sharti bo'yicha, $a = 3^{\alpha}5^{\beta}7^{\Upsilon}$. Bu soning Eyler funksiyasi $\varphi(a) = 3^{\alpha-1}\cdot 2\cdot 5^{\beta-1}\ 4\cdot 7^{\Upsilon-1}\cdot 6 = 2^4\ 3^{\alpha-1}\ 5^{\beta-1}\ 7^{\gamma-1}$ . shart bo'yicha $\varphi(a) = 3600 = =2^4\cdot 3^2\cdot 5^2$, demak $2^4\cdot 3^{\alpha}\cdot 5^{\beta-1}\cdot 7^{\gamma-1} = 2^4\cdot 3^2\cdot 5^2$, bundan $\alpha = 2$, $\beta = 3$, $\gamma = 1$ va $a = 3^2\cdot 5^3\cdot 7 = 7875$.

**120**. *Yechish.* Shart bo'yicha: $\varphi(a) = \varphi(pq) = (p-1)(q-1) = 120$ va $p - q = 2$. Natijada $\begin{cases}(p-1)(q-1)=120\\ p-q=2\end{cases}$; sistemani hosil qilamiz. Uning yechimi $p = 13$, $q = 11$. Demak, $a = p\,q = 143$.

**121**. *Yechish.* Shart bo'yicha $\varphi(a)=\varphi(p^2q^2)=p(p-1)q(q-1)$ va $\varphi(a)=11424=2^5\cdot 3\cdot 7\cdot 17$. Demak, $p(p-1)q(q-1)=2^5\cdot 3\cdot 7\cdot 17$, yoki $p(p-1)q(q-1)=17\cdot 16\cdot 7\cdot 6$, bundan $p=17, q=7$ va $a=17^2\cdot 7^2=14161$.

**122**. *Yechish.* Shart bo'yicha

$\varphi(a)=p_1^{\alpha_1-1}(p_1-1)p_2^{\alpha_2-1}(p_2-1)...p_n^{\alpha_n-1}(p_n-1)$ va $\varphi(a)=462000=2^4\cdot 3\cdot 5^3\cdot 7\cdot 11$. Demak,

$$p_1^{\alpha_1-1}(p_1-1)p_2^{\alpha_2-1}(p_2-1)...p_n^{\alpha_n-1}(p_n-1)=2^4\cdot 3\cdot 5^3\cdot 7\cdot 11.$$

O'ng tomondagi ko'paytuvchilarni chap tomondagi kabi ko'rinishda ko'paytuvchilarni o'rnini almashtiramiz: $p_1^{\alpha_1-1}(p_1-1)p_2^{\alpha_2-1}(p_2-1)...p_n^{\alpha_n-1}(p_n-1)=(11\cdot 10)(7\cdot 6)(5^2\cdot 4)$, bundan $p_1=11$ va $\alpha_1=2$; $p_2=7$ va $\alpha_2=2$; $p_3=5$ va $\alpha_3=3$; $a=11^2\cdot 7^2\cdot 5^3=741125$.

**123**. *Yechish*. $(a, m)=1$ shart bajarilganda $(a, m-a) = 1$ ni bajarilishini ko'rsatamiz. Teskarisini faraz qilamiz, ya'ni $(a, m - a) = d > 1$, u holda $a = dk$, $m – a = dt$, bundan $m = d(t + k)$ va $(a, m) = d > 1$, bu esa $(a, m)= 1$ shartga ziddir.

$m$ dan kichik va u bilan tub bo'lgan sonlarni tartib bilan yozamiz: $1, a_1, a_2, \dots, m – a_2, m – a_1, m – 1$; bu qatorda $\varphi(m)$ son bor. Har qanday $a_i$ songa $m – a_i$ son mos keladi; ular yig'indisi

$a_i + (m - a_i) = m$, bu juftliklar soni $\frac{1}{2}\varphi(m)$ va demak $S = \frac{1}{2}m\varphi(m)$.

**124**. a) 24; b) 54; c) 37500.

**125**. a) *Yechish*. $a = p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$. bo'lsin. U holda

$$\varphi(a^\alpha) = a^\alpha\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\dots\left(1-\frac{1}{p_k}\right) = a^{\alpha-1}\left[a\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\dots\left(1-\frac{1}{p_k}\right)\right] =$$

$$= a^{\alpha-1}\varphi(a).$$

**126**. Birinchi hol $(a, 2) = 1$ bo'lganda o'rinli, ikkinchi hol esa $(a, 2) = 2$ bo'lganda o'rinli bo'ladi.

**127**. a) *Yechish*. $\varphi(4n + 2) = \varphi(2)\varphi(2n + 1) = \varphi(2n + 1)$;

b) *Yechish*. Agar $(n, 2)=1$ bo'lsa, u holda $\varphi(4n) = \varphi(4)\varphi(n) = 2\varphi(n)$. Agar $n = 2^\alpha \cdot k$ bo'lib, $(k, 2) = 1$ bo'lsa, u holda $\varphi(4n) = \varphi(2^{\alpha+2} \cdot k) = 2^{\alpha+1} \cdot \varphi(k) = 2 \cdot \varphi(2^{\alpha+1} \cdot k) = 2\varphi(2n)$.

**128**. a) $x = 3$; b) $x = 3$; c) tenglama $p > 2$ da yechimga ega emas. $p = 2$ da ixtiyoriy natural sonlar uchun o'rinli.

**129**. $\varphi(b)$.

**130**. a) 4 kasr: $\frac{1}{10}, \frac{3}{10}, \frac{7}{10}. \frac{9}{10}$. c) 12 kasr.

**131**. $\varphi(2) + \varphi(3) + \dots + \varphi(n)$.

**132**. a) 9; b) 31; c) 71.

**133**. *Yechish*. Shart bo'yicha, $(300, x) = 20$ va barcha $x$ lar 300 dan kichik, 20 ga qisqartirgandan so'ng $(15, y) = 1$, bu yerda barcha $y$ Lar 15 dan kichik va 15 bilan o'zaro tub; ular soni $\varphi(15) = 8$. Bu $y = 1, 2, 4, 7, 8, 11, 13, 14$ sonlar va bundan $x = 20, 40, 80, 140, 160, 220, 260, 280$.

**134**. $\varphi(45) = 24$.

**135**. $\varphi(36) = 12$.

**136**. *Ko'rsatma*. $\varphi(a)$ ning juftligi 123 masala yordamidan kelib chiqadi.

**137**. *Yechish*. Agar $(m, 2) = 1$ bo'lsa, u holda $\varphi(m) = \varphi(2m)$.

**138**. *Yechish*. $m$ va $n$ larning tub bo'luvchisi $p$ uchun $\varphi(mn)$ sonda $1-\frac{1}{p}$, ko'paytuvchi bor, a $\varphi(m)\varphi(n)$ – sonida esa $\left(1-\frac{1}{p}\right)^2$. ko'paytuvchi bor. $1-\frac{1}{p} < 1$, bo'lganligi sababli $\varphi(m)\varphi(n) < \varphi(mn)$. Xususiy holda $\varphi^2(m) \le \varphi(m^2)$, tenglik $m = 1$ bo'lganda bajariladi.

**139**. *Ko'rsatma.* $q_1, q_2, \ldots, q_t$ – $m$ ning kanonik yoyilmasidagi tub sonlar; $p_1, p_2, \ldots, p_k$ –$m$ va $n$ ning kanonik yoyilmasidagi tub sonlar va $\ell_1, \ell_2, \ldots, \ell_S$ – faqat $n$ ning kanonik yoyilmasidagi tub sonlar bo'lsin. U holda

$$\varphi(mn) = mn\prod_{i=1}^{t}\left(1-\frac{1}{q_i}\right)\cdot\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\cdot\prod_{i=1}^{S}\left(1-\frac{1}{\ell_i}\right) = \left[m\prod_{i=1}^{t}\left(1-\frac{1}{q_i}\right)\cdot\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\right]\times$$

$$\times\left[n\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\prod_{i=1}^{S}\left(1-\frac{1}{\ell_i}\right)\right]\cdot\frac{d}{d\cdot\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)} = \varphi(m)\varphi(n)\frac{d}{\varphi(d)}.$$

1-*eslatma*. Shu usulda $m$ va $n$ sonlarning umumiy bo'luvchisi $A$ uchun $\varphi(mn) = \varphi(m)\varphi(n)\frac{A}{\varphi(A)}$ munosabatni keltirib chiqarish mumkin.

2-*eslatma.* Chiqarilgan formula yordamida 138 masala yechimi juda osonlik bilan topiladi: $\varphi(m)\varphi(n) \le \varphi(mn)$, chunki $\frac{d}{\varphi(\alpha)} \ge 1$. Tenglik o'rinli bo'lishi uchun $d = 1$ zarur va yetarlidir.

**140**. *Yechish.* $\varphi(mn) = \varphi(\delta\mu) = \varphi(\delta)\varphi(\mu)\cdot\frac{\delta}{\gamma^{\tau}} = \delta\varphi(\mu).$

**141**. $p^{\alpha}$.

**142**. *m.*

**144**. a) *Yechish*. Gaussa formulasidan $x = 2^y 3^z 5^u$ ($y \ge 0$, $z = 0$; 1 va $U = 0$; 1) kelib chiqadi. $x = 2^y$; $2^y \cdot 3$; $2^y \cdot 5$; $2^y \cdot 15$; 3; 5; 15 imkoniyatlarni tekshirishi $x = 2^{\alpha+1}$; $2^{\alpha}\cdot 3$; $2^{\alpha-1}\cdot 5$; $2^{\alpha-2}\cdot 15$ ($\alpha \ge 2$); 15 ($\alpha = 3$)larni beradi;

b) $p \ne 3$ da yechim yo'q. $r = 3$ da tenglamalarni ixtiyoriy butun $x \ge 2$ qiymatlar qanoatlantiradi.

**145**. a) yechimi yo'q;

II-BOB

## BUTUN SONLAR XALQASIDA TAQQOSLAMALAR NAZARIYASI

### § 1.

**1**. 1 modul bo'yicha. **2**. a); v); s); **3**. *Yechilishi.* Agar $a = mq + r$ bo'lsa, u holda $a - r = mq$ va $a \equiv r(mod\ m)$. **4**. a) $x = 3q$; v) $x = 1+2q$. **5**. $2r$ ning bo'luvchilari. **6**. 8 ning bo'luvchilari. **7**. *Yechilishi.* Agar $n$ toq son bo'lsa, u holda $n$-1 va $n$+1- ketma-ket keladigan juft sonlar. Agar ulardan biri 2 ga karrali bo'lsa, u holda ikkinchisi kaimda 4 ga karrali, shuning uchun: $(n-1)(n+1) = n^2 - 1 \equiv 0\ (mod\ 8)$, yoki $n^2 \equiv 1\ (mod\ 8)$. **8**. *Yechilishi.* Berilgan taqqoslamani ikkala tomonini 4 ga ko'paytiramiz: $400a + 40b + 4c \equiv 0\ (mod\ 21)$, lekin $400a \equiv a\ (mod\ 21)$, $40b \equiv -2\ (mod\ 21)$, $4c \equiv 4\ (mod\ 21)$. Oxirgi uchta taqoslamani hadma-had qo'shamiz: $400a + 40b + 4c \equiv a - 2b + 4c\ (mod\ 21)$. Bu yerdan $a - 2b + 4c \equiv 0(mod\ 21)$ ni hosil qilamiz. **10.** *Yechilishi.* $11\cdot 31 - 1 = 340 = 5\cdot 68$ va $2^5 \equiv -1(mod\ 11)$ bo'lganligi uchun oxirgi taqqoslamani ikkala tomonini 68-nchi darajaga ko'tarib: $(2^5\ )^{68} = 2^{340} = 2^{11\bullet 31\text{-}1} \equiv 1(mod\ 11)$ ni hsil qilamiz. So'ngra $2^5 \equiv 1(mod\ 31)$ dan $(2^5\ )^{68} = 2^{11\bullet 31\text{-}1} \equiv 1(mod\ 31)$ ni hosil qilamiz. Bu yerdan $2^{11\bullet 31\text{-}1} \equiv 1(mod\ 11\cdot 31)$. Bu taqqoslamani ikkala tomonini 2 ga ko'paytirib izlanayotgan taqqoslamani hosil qilamiz. **11.** *Yechilishi.* $x = 3n + 1$ bo'lganda bo'linuvchi $1+3\cdot 27^n + 9\cdot 729^n$ ga teng bo'ladi. Berilgan modul bo'yicha $27^n \equiv 729^n \equiv 1$, shuning uchun $1 + 3\cdot 27^n + 9\cdot 729^n \equiv 1 + 3 + 9 = 13$. **14.** 4. **15.** *Yechilishi.* Shartga asosan $a = b + p^n q$ $(q = 0, \pm 1, \pm 2, ...)$. Bu tenglikningn ikkala tomonini $r$ – nchi darajaga ko'tarib, $a^p = b^p + p^{n+1}\cdot\alpha$ $(\alpha = 0, \pm 1, \pm 2, ...)$ ni hosil qilamiz. **17. K**o'rsatma. $a_4\cdot 10^4 + a_3a_2\cdot 10^2 + a_1a_0 \equiv 0(mod\ 33)$ taqqoslamaning chap tomonidan modulga karrali bo'lgan $999a_4 + 99a_3a_2$ sonni ayirish kerak. **18.** *Yechilishi*. a) $9^{10} \equiv 1(mod\ 100)$ bo'lganligi uchun $9^{10q+r} \equiv 9^r\ (mod\ 100)$. $9^9 \equiv 9(mod\ 10)$ bo'lganligi uchun $9^{9^9} \equiv 9^9 \equiv 89\ (\text{mod } 100)$. Izlanayotgan raqamlar 8 va 9; b) $7^4 = 2401 \equiv 1(mod\ 100)$ bo'lganligi uchun $7^{100} \equiv 1(mod\ 100)$, bu yerdan $7^{9^{9^9}} \equiv 7^{100q+89} \equiv 7^{89}\ (mod\ 100)$. $7^{88} \equiv 1(mod\ 100)$ bo'lganligi uchun $7^{89} \equiv 7\ (mod\ 100)$. Izlanayotgan raqamlar 0 va 7. **19.** *Yechilishi*. $r \equiv r + 2 \equiv 1\ (mod\ 2)$ taqqoslamadan $r^{r+2} + (r+2)^r \equiv 0\ (mod\ 2)$ kelib chiqadi, $r \equiv -1\ (mod\ r+1)$, $r + 2 \equiv 1\ (mod\ r+1)$ dan esa $r^{r+2} + +(r+2)^r \equiv 0\ (mod\ r+1)$ kelib chiqadi. **20.** *Ko'rsatma.* Noldan tashqari $\pm\frac{p-x}{2}(x = 1, 2, ..., p-2)$ ko'rinishdagi sonlar berilgan.

$$\pm\frac{p-x}{2} \equiv 0(mod\ \ p), \quad \frac{p-x_1}{2} \equiv \pm\frac{p-x_2}{2} \equiv 0(mod\ \ p),$$

Taqqoslamalar mos ravishda noto'g'ri $x \equiv r(mod\ \ r)$, $x_1 \equiv \pm x_2\ (mod\ \ r)$ taqqoslamalarga olib keladi. **21.** *Yechilishi.* **M**atematik induksiya metodini qo'llaymiz. $n = 1$ da taqqoslama to'g'ri. $n$ uchun taqqoslama to'g'ri bo'lsin. Uni $n + 1$ da ham o'rinli ekanligini ko'rsatamiz. Haqiqatdan ham, $2^{3^{n+1}} + 1 = \left(2^{3^n}\right)^3 + 1 = \left(2^{3^n} + 1\right)\left(2^{2\cdot 3^n} - 2^3 + 1\right) \equiv 0\left(mod\ \ 3^{n+2}\right)$, induksiya faraziga asosan $2^{3^n} + 1 \equiv 0\ (mod\ 3^{n+1})$ va $2^{2\bullet 3} - 2^3 + 1) \equiv 0\ (mod\ 3)$ , chunki $2 \equiv -1(mod\ 3)$. **22.** a) *Yechilishi.* $4 \equiv -1(mod\ 5)$ bo'lganligi uchun $2^{4n+1} = 2\cdot 4^{2n} \equiv 2(mod\ 5)$. Shunday qilib, $2^{4n+1} = 2 + 5k$, bu yerda $k \in \boldsymbol{N}$, va $N = 3^{2+5k} + 2 = 9\cdot 243^k + 2 \equiv 0\ (mod\ 11)$, chunki $243 \equiv 1(mod\ 11)$. Shunday qilib, $11|\ N$ va $N > 11$. Demak, $N$ – murakkab son. b) *Yechilishi.* $9 \equiv -1(mod\ 10)$ bo'lganligi uchun $3^{4n+1} = 3\cdot 9^{2n} \equiv 3(mod\ 10)$. Demak, $3^{4n+1} = 3 + 10k$, bu yerda $k \in \boldsymbol{N}$, va $M = 2^{3+10k} + 3 = 8\cdot 32^{2k} + 3 \equiv 0\ (mod\ 11)$, chunki $32 \equiv -1(mod\ 11)$. $M > 11$ va $11|\ M$ dan $M$ ning murakkab son ekanligi kelib chiqadi. **23.** *Yechilishi.* Dastavval agar $(a, m) = k$ bo'lsa, u holda $(b, m) = k$ ekanligini ko'rsatamiz. $a \equiv b\ (mod\ m)$ taqqoslamadan $a = mt + b$, yoki $b = a - mt$, bu yerdan ko'rinib turibdiki, agar $(a, m) = k$ bo'lsa, u holda $k \mid b$. Shunday qilib, agar $(a, m) = 1$ bo'lsa, u holda $(b, m) = 1$. Masala shartidagi ikkinchi taqqoslamani $s$ ga ko'paytirib: $ac \equiv bc\ (mod\ m)$ taqqoslamani hosil qilamiz. U holda $bc \equiv bd\ (mod\ m)$, bu yerdan $(b, m) = 1$ ni hisobga olib $c \equiv d\ (mod\ m)$ ni hosil qilamiz. **24.** *Yechilishi.* Shartga asosan, $a^{100} \equiv 2\ (mod\ 73)$; bu taqqoslamaning ikkala tomonini $a$ ga ko'paytirib, $a^{101} \equiv 2a\ (mod\ 73)$ ni hosil qilamiz; ammo, shartga ko'ra, $a^{101} \equiv 69\ (mod\ 73)$. Bu taqqoslamalardan $2a \equiv 69\ (mod$

73) kelib chiqadi. Bu taqqoslamaninng o'ng tomoniga 73 ni qo'shamiz: $2a \equiv 142(mod\ 73)$. (2, 73) = 1 bo'lganligi uchuntaqqoslamaning ikkala tomonini 2 ga qisqartirib, $a \equiv 71(mod\ 73)$ ni hosil qilamiz. Bu yerdan qoldiq 71 ekanligi kelib chiqadi. **25.** *Yechilishi.* Masala shartidan $11a + 2b \equiv 0\ (mod\ 19)$ taqqoslamani hosil qilamiz. Bu taqqoslamaning ikkala tomonini 12 ga ko'paytirib: $132a + 24b \equiv 0\ (mod\ 19)$ ni hosil qilamiz, bu yerdan esa $18a + 5b \equiv 0\ (mod\ 19)$.

**27.** $$1, 2, 3, \ldots, \frac{p-1}{2}, \frac{p+2}{2}, \ldots, p-2, p-1$$

sonlardan $\frac{p-1}{2}$ ta taqqoslamalrni hosil qilamiz: $1 \equiv -(r-1)\ (mod\ r)$, $2 \equiv -(r-2)\ (mod\ r)$,

$\ldots, \frac{p-1}{2} \equiv -\frac{p+1}{2}\ (mod\ p)$.

Bu taqqoslamalarning har birini *(2k + 1)*- nchi darajaga ko'tarib, so'ngra ularni qo'shib talab qilingan taqqoslamani hosil qilamiz.

## §2.

**28.** $x \equiv 0; 1; 2; \ldots; 9\ (mod\ 10)$. **30.** a) $x \equiv 1; 3; 7; 9\ (mod\ 10)$; b) $x \equiv 2; 4; 6; 8\ (mod\ 10)$; s) $x \equiv 5\ (mod\ 10)$; d) $x \equiv 0\ (mod\ 10)$. **31.** a) $m = 9$ da chegirmalarning to'la sistemalari: 0, 1, 2, 3, 4, 5, 6, 7, 8; -8, -7, -6, -5, -4, -3, -2, -1, 0; -4, -3, -2, -1, 0, 1, 2, 3, 4. Chegirmalarning keltirilgan sistemalari: 1, 2, 4, 5, 7, 8; -8, -7, -5, -4, -2, -1; -4, -2, -1, 1, 2, 4. b) $m = 8$ da chegirmalarning to'la sistemalari: 0, 1, 2, 3, 4, 5, 6, 7; -7, -6, -5, -4, -3, -2, -1, 0; -3, -2, -1, 0, 1, 2, 3, 4 yoki -4, -3, -2, -1, 0, 1, 2, 3. chegirmalarning keltirilgan sistemalari: 1, 3, 5, 7; -7, -5, -3, -1; -3, -1, 1, 3.

**32.** *Yechilishi*. Berilgan sinf sonlarining umumiy ko'rinishidan quyidagilarni topamiz: $25 = 8\cdot3 + 1$; $-20 = 8(-3) + 4$; $16 = 8\cdot2 + 0$; $46 = 8\cdot5 + 6$; $-21 = 8(-3) + 3$; $18 = 8\cdot2 + 2$; $37 = 8\cdot4 + 5$; $-17 = 8(-3) + 7$. Hosil qilingan qoldiqlarning hammasi har xil va ular manfiy bo'lmagan eng kichik chegirmalarning to'la sistemasini tashkil qiladi: 0, 1, 2, 4, 5, 6, 7, demak, berilgan sonlar ham chegirmalarning to'la sistemasini tashkil qiladi. **38.** *Yechilishi*. Har bir sonni 6 ga bo'lib: 0, 2, 1, 1, 4, 5, 2 qoldiqlarni hosil qilamiz. Topilgan manfiy bo'lmagan chegirmalardan (noldan tashqari) 6 ni ayirib, 0, -4, -5, -5, -2, -1, -4 –absolyut qiymati jihatidan eng kichik musbat bo'lmagan chegirmalarni hosil qilamiz. Absolyut qiymati jihatidan eng kichik chegirmalar 0, 2, 1, 1, -2, -1, 2 lardan iborat. **40.** manfiy bo'lmagan eng kichik chegirmalar: 0, 2, 1, 0, 100, 100; absolyut qiymati jihatidan eng kichik musbat bo'lmagan chegirmalar: 0, -5, -10, 0, -20, -100; absolyut qiymati jihatidan eng kichik chegirmalar: 0, 2, 1, 0, -20, 100 ili –100. **42.** *Ko'rsatma*. Berilgan sonni $a + x$ ($x = 0, 1, 2, \ldots, m-1$), bu yerda $a$ – ixtiyoriy butun son, ko'rinishda yozib olib, chiziqli formaning chegirmalari haqidagi teoremani qo'llanilsin. **43.** *Ko'rsatma.* 10 modul bo'yicha qoldiqlarning to'las sistemasini beradigan $x$ ningn qiymatlaridan foydalanish kerak. **51.** *Yechilishi.* Shartga asosan $x^4 \equiv 1(mod\ 10)$, bu yerdan $x^{12} \equiv 1\ (mod\ 10)$. **52.** *Yechilishi*. a) (5, 24) = 1 va $\varphi(24) = 8$ bo'lganligidan $5^8 \equiv 1\ (mod\ 24)$ kelib chiqadi. Haqiqatdan ham, $5^8 = (5^2)^4 = 25^4 \equiv 1^4$

= 1 (*mod* 24); s) *Yechilishi* (3, 18) = 3 > 1 bo'lganligi uchun Eyler teoremasi o'rinli emas. Haqiqatdan ham, $\varphi(18) = 6$ va $3^6 = 3^4 \cdot 3^2 = 81 \cdot 9 \equiv 9 \cdot 9 = 81 \equiv 9$ (*mod* 18). **53.** *Yechilishi*. a) $\varphi(6) = 2$ bo'lganligi uchun $a^2 \equiv 1$ (*mod* 6). Bu taqqoslamani modul bilan o'zaro tub bo'lgan $a = 1$ va $a = 5$, yoki $6k + 1$ va $6k + 5$ sonlar sinflari qanoatlantiradi. **54.** *Yechilishi*. b) $a^{12} \equiv b^{12} \equiv 1$ (*mod* 13) va $a^4 \equiv b^4 \equiv 1$ (*mod* 5) bo'lganligi uchun $a^{12} \equiv b^{12} \equiv 1$ (*mod* 5); demak, $a^{12} \equiv b^{12} \equiv 1$ (*mod* 65) yoki $a^{12} - b^{12}$ ayirma 65 ga bo'linadi. **56.** *Ko'rsatma*. $i = 1, p - 1$ da $i^{k(p-1)} \equiv 1$ *(mod p)* taqqoslamalarni hadma-had qo'shish kerak. **57.** *Ko'rsatma*. $a^r \equiv a$ *(mod r)* taqqoslamadan foydalanish kerak. **59.** a) 1; b) 19; c) 29. **60.** a) 2; b) 6; c) 2; d) 2. **61.** 049. **62.** *Yechilishi*. $\varphi(m) = R_m(a) \cdot q + r$ bo'lsin, bu yerda $q \geq 0$ va $0 \leq r \leq R_m(a) - 1$. $a^{P_m(a)} \equiv 1$ *(mod m)* dan $a^{\varphi(m)} \equiv a^r \equiv 1$ *(mod m)* kelib chiqadi, bu yerdan esa $r = 0$. **63.** *Ko'rsatma*. Oldingi masaladan foydalanish kerak. **64.** *Ko'rsatma*. Agar $(x, r) = 1$ bo'lsa, u holda $x^{(p-1)m} + x^{(p-1)n} \equiv 2$ *(mod p)*. **65.** *Ko'rsatma*. $(m, 10) = 1$ bo'lganligi uchun $10^{\varphi(m)} \equiv 1$ *(mod m)* yoki $10^{\varphi(m)} - 1 = 99...9 \equiv 0$ *(mod m)*. $(9, m) = 1$ bo'lganligi uchun hosil qilingan taqqoslamaning ikkala tomonini 9 ga bo'lish mumkin. **66.** b) *Ko'rsatma*. 1093 – tub son. **67.** *Yechilishi*. $a^{r-1} - 1 = (a-1)(a^{r-1} + a^{r-2} + ... + a + 1) \equiv 0$ *(mod r)* bo'lsin. $a^r \equiv a$ *(mod r)* bo'lganligi uchun $a^r - 1 \equiv a - 1$ *(mod r)*. Shunday qilib, agar $a^r - 1 \equiv 0$ *(mod r)* bo'lsa, u holda $a - 1 \equiv 0$ *(mod r)*. Oxirgi taqqoslamadan quyidagilarni hosil qilamiz: $a^{r-1} \equiv 1$ *(mod r)*, $a^{r-2} \equiv 1$ *(mod r)*, ... , $a \equiv 1$ *(mod r)*, $1 \equiv 1$ *(mod r)*. Bu taqqoslamalarni hadma-had qo'shib: $a^{r-1} + a^{r-2} + ... + a + 1 \equiv r \equiv 0$ *(mod r)* ni hosil qilamiz, demak, $a^r - 1 \equiv 0$ *(mod $r^2$)*. Shunga o'xshash agar $a^r + 1 \equiv 0$ *(mod r)* bo'lsa, u holda $a^r + 1 \equiv 0$ (*mod* $r^2$) ni hosil qilamiz. **68.** *Yechilishi*. Ferma teoremasiga asosan $r^{q-1} - 1 \equiv 0$ *(mod q)*, bu yerdan $r^{q-1} - 1 = qt_1$. Shunga o'xshash, $q^{p-1} - 1 \equiv 0$ *(mod p)*, bu yerdan esa $q^{p-1} - 1 = pt_2$. Hosil qilingan tengliklarni ko'paytirib, izlanayotgan taqqoslamani hosil qilamiz. **69.** *Yechilishi*. $2730 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 13$. $x^{13} \equiv x$ (*mod* 13) ga egamiz. $x^{13} \equiv x$ (*mod* 2, 3, 5 va 7) taqqoslamalarning to'g'riligi 58 masalaning taqqoslamasidan kelib chiqadi.

**70.** *Yechilishi*. $a_i^5 \equiv a_i$ (*mod* 2, 3, 5) bo'lganligi uchun $a_i^5 \equiv a_i$ (*mod* 30), (58 masalaga qarang). Shunday qilib, $\sum_{i=1}^{n} a_i^5 \equiv \sum_{i=1}^{n} a_i \equiv 0 \pmod{30}$. **71.** *Yechilishi*. $m - \left[\frac{m}{2}\right] = m - \frac{m-1}{2} = \frac{m+1}{2}$ tenglik to'g'ri. $2^{\varphi(m)-1} \equiv r \pmod{m}$. yoki $2^{\varphi(m)} - 1 \equiv 2r - 1 \pmod{m}$. Ammo Eyler teoremasiga ko'ra $2^{\varphi(m)-1} \equiv r$ *(mod m)*. Demak, $2r - 1 \equiv 0$ (*mod m*) bu yerdan esa $2r - 1 = mt, r = \frac{mt+1}{2}, r = \frac{m+1}{2}$. **72.** *Yechilishi*. Shartga asosan, $(a, 10) = 1$, bu yerdan $(a, 5) = 1$ va $(a, 2) = 1$. $1000 = 125 \cdot 8$ ni hisobga olib, 125 va 8 modullar bo'yicha taqqoslamalarni qaraymiz. 7 masalaning yechilishidan $a^{100} \equiv 1$ (*mod* 125) ni hosil qilamiz. Ikkinchi tomondan Eyler teoremasiga ko'ra $a^4 \equiv 1$ (*mod* 8); bu taqqoslamani 25 –nchi darajaga ko'tarib, $^{100} \equiv 1$ (*mod* 8) ni hosil qilamiz. Bu yerdan $a^{100} \equiv 1$ (*mod* 1000) kelib chiqadi. Oxirgi taqqoslamani *n*-darajaga ko'tarib, so'ngra uning ikkala tomonini *a* ga ko'paytirib, $a^{100n+1} \equiv a$ (*mod* 1000) ni hosil qilamiz. **73.** *Yechilishi*. $19 \cdot 73 - 1 = 1386 = 18 \cdot 77$

bo'lganligi uchun Ferma teoremasiga asosan $2^{18} \equiv 1$ (*mod* 19). U holda $2^{18\bullet 77} = 2^{19\bullet 73 - 1} \equiv 1$ (*mod* 19). $2^9 = 512 \equiv 1$ (*mod* 73), to $2^{9\bullet 154} = 2^{19\bullet 73-1} \equiv 1$ (*mod* 73) bo'lganligi uchun bu yerdan $2^{19\bullet 73 - 1} \equiv 1$ (*mod* 19·73) ni hosil qilamiz. **74.** *Ko'rsatma.* $p_1^{p_2-1} - 1 = p_2 \cdot q_1$ va $p_1^{p_1-1} - 1 = p_1 \cdot q_2$ tengliklarni hadma-hado' ko'paytirish kerak, bu yerda $q_1, q_2 \in \mathbf{Z}$. **75.** *Yechilishi.* $(2r + 1, 3) = 1$ bo'lsa, u holda $(2r + 1)^2 \equiv 1$ (*mod* 3), bu yerdan $4r + 1 \equiv 0$ (*mod* 3). **76.** *Yechilishi.* Shartga asosan $a^{\varphi(m)} \equiv 1(mod\ m)$ va $\alpha_1 = \alpha_2 + \varphi(m) \cdot q$. Demak, $a^{\alpha_2 + \varphi(m) \cdot q} \equiv a^{\alpha_2}(mod\ m)$ yoki $a^{\alpha_1} \equiv a^{\alpha_2}(mod\ m)$. **77.** *Yechilishi.* Agar $a$ soni 7 ga karrali bo'lmasa, u holda $(a, 7) = 1$, bu yerdan $a^6 \equiv 1$ (*mod* 7), bu taqqoslamadan $a^{6m} \equiv 1$ (*mod* 7) va $a^{6n} \equiv 1$ (*mod* 7) taqqoslamalarni hosil qilamiz. Oxirgi taqqoslamalarni qo'shib: $a^{6m} + a^{6n} \equiv 2$ (*mod* 7). Bu yerdan talab qilingan shart kelib chiqadi. **78.** *Yechilishi.* Agar $(n, 6) = 1$ bo'lsa, u holda $(n, 2) = 1$. Demak, $n$ – toq son va $(n - 1)(n + 1)$ ifoda ikkita ketma-ket joylashgan juyaye sonning ko'paytmasi sifatida 8 ga bo'linadi, ya'ni, $n^2 - 1 \equiv 0$ (*mod* 8), yoki $n^2 \equiv 1$ (*mod* 8). Ikkinchi tomondan $(n, 6) = 1$ dan $(n, 3) = 1$ ham kelib chiqadi. Shuning uchun $n^2 \equiv 1$ (*mod* 3). Hosil qilingan taqqoslamalardan $n^2 \equiv 1$ (*mod* 24) kelib chiqadi. **79.** *Yechilishi.* $r \neq 5$ ekanligi ko'rinib turibdi. Bu yerdan $5^{r-1} \equiv 1$ (*mod r*), $5^{p^2-1} \equiv 1$ (*mod r*) va $5^{p^2} + 1 \equiv 6$ (*mod* 6). Shartga asosan $6 \equiv 0$ (*mod r*) bo'lsa, u holda $r$ ninng qiymatini 2 va 3 sonlardan izlash kerak. Tekshirishdan $r = 3$ ni topamiz. **80.** *Yechilishi.* $(x^3 - 1)x^3(x^3 + 1) \equiv 0$ (*mod* 504), yoki $x^2(x^7 - x) \equiv 0$ (*mod* 7·8·9) taqfqoslamalarni isbotlash kerak. ixtiyoriy $x \in \mathbf{Z}$ da $x^7 - x \equiv 0$ (*mod* 7) bo'lganligidan $(x^3 - 1)x^3(x^3 + 1) \equiv 0$ (*mod* 7) kelib chiqadi. Shu bilan bir vaqtda $x$ ning juft qiymatlari uchun ham toq qiymatlari uchun ham $(x^3 - 1)x^3(x^3 + 1) \equiv 0$ (*mod* 8) o'rinli, $\varphi(9) = 6$ bo'lganligidan $x^3(x^6 - 1) \equiv 0$ (*mod* 9) kelib chiqadi. Bu yerdan: $(x^3 - 1)x^3(x^3 + 1) \equiv 0$ (*mod* 504). **81.** *Yechilishi.* Shartga asosan $r$ va $2r + 1$ – lar tub sonlar, shuning uchun $(2r + 1)^2 \equiv 1$(*mod* 3), $r^2 \equiv 1$(*mod* 3). Ikkinchi taqqoslamani 4 ga ko'paytirib, birinchisidan ayiramiz, $4r + 1 \equiv -3 \equiv 0$ (*mod* 3), ya'ni. $4r + 1$ – murakkab son (3 ga bo'linadi). **82.** a) $x_1 \equiv 1$(*mod* 3), $x_2 \equiv 2$ (*mod* 3); b) $x_1 \equiv 1$(*mod* 5), $x_2 \equiv 2$ (*mod* 5); s) $x \equiv 2$ (*mod* 5); d) yechimlari yo'q; ye) $x \equiv 3$ (*mod* 5); f) $x_1 \equiv 1$(*mod* 4), $x_2 \equiv 3$ (*mod* 4); g) $x_1 \equiv 1$(*mod* 5), $x_2 \equiv 3$ (*mod* 5). **83.** a) $x \equiv 3$ (*mod* 7); b) $x \equiv 2$ (*mod* 5); c) $x \equiv -3$ (*mod* 11); d) $x \equiv -3$ (*mod* 7); e) $x \equiv -1$ (*mod* 7); f) $x \equiv -4$ (*mod* 15). **86.** a) $x \equiv 11$ (*mod* 15); b) $x \equiv 2$ (*mod* 8); c) $x \equiv 4$ (*mod* 13); d) $x \equiv 20$ (*mod* 37); e) $x \equiv 7$ (*mod* 25); f) $x \equiv 5$ (*mod* 11); g) $x \equiv 5$ (*mod* 11); h) $x \equiv 11$ (*mod* 24). **91.** *Yechilishi.* $x = u + \alpha$ almashtirishni kiritib, $(u + \alpha)^n + a_1(y + \alpha)^{n-1} + ... + a_n \equiv 0(mod\ m)$ ni hosil qilamiz. Bu yerdan qavslarni ochib chiqib, qaytadan gruppalashlardan so'ng $u^n + (n\alpha + a_1)\, y^{n-1} + ... + (\alpha^n + a_1\alpha^{n-1} + ... + a_n) \equiv 0(mod\ m)$ ni hosil qilamiz. $\alpha$ ni shunday tanlaymizki, $n\alpha + a_1 \equiv 0\ (mod\ m)$ o'rinli bo'lsin. Natijada $u^{n-1}$ ni o'zida saqlaydigan had yo'qoladi: $u^n + b_1y^{n-2} + ... + b_n \equiv 0\ (mod\ m)$. **92.** *Yechilishi.* $n\alpha + a_1 \equiv 0\ (mod\ m)$ taqqoslamani tuzamiz. Shartdan $3\alpha + 5 \equiv 0$ (*mod* 13) kelib chiqadi, uning yechimi: $\alpha \equiv 7$ (*mod* 13), demak, $x = u + 7$ almashtirish olamiz. Bu almashtirishni berilgn taqqoslamaga qo'yib, $(u + 7)^3 + 5(y + 7)^2 + 6(u + 7) - 8 = u^3 + 26y^2 + 223u + 622 \equiv u^3 + 2u - 2 \equiv 0$ (*mod* 13) ni hosil qilamiz. **94.** a) $(5, 10) = 5$ va 7 soni 5 ga bo'linmagshanligi uchun bekrilgan taqqoslama yechimga ega emas. b) $x \equiv 7(mod\ 13)$; c) $x \equiv 8\ (mod\ 17)$; d) $x \equiv 9\ (mod\ 19)$; e) $x \equiv 11\ (mod\ 58)$. **94.** a) $x \equiv 6(mod\ 19)$; b) yechimi yo'q; c) $x \equiv 49\ (mod\ 153)$; d) $x \equiv 3\ (mod\ 183)$; e) $x \equiv 47\ (mod\ 241)$. f) yechimi yo'q; g) $x \equiv 41, 190, 339\ (mod\ 447)$; h) $x \equiv 61, 248\ (mod\ 422)$; i) $x \equiv 39, 196, 353\ (mod\ 471)$. **95.** a) yechimi yo'q; b) $x \equiv 3, 8, 13, 18, 23\ (mod\ 25)$; c) $x \equiv 73\ (mod\ 177)$; d) $x \equiv 29\ (mod\ 311)$; e) $x \equiv 48\ (mod\ 219)$; f) $x \equiv 9, 32, 55, 78, 101, 124\ (mod\ 138)$; g) $x \equiv 11, 28, 45\ (mod\ 51)$. **96.** *Yechilishi*. a) $ax \equiv b\ (mod\ 21)$, bu yerda $(a, 21) = 1$, $b \in \mathbf{Z}$; b) $ax \equiv b\ (mod\ 21)$ taqqoslama yechimga ega bo'lishi uchun, masalan, 3 ta yechimga ega bo'lishi uchun $(a, 21) = 3$ va $b$ soni 3 ga bo'linishi zarur va yetarlidir; s) bunday taqqoslamani tuzish mumkin emas. **97.** 1 iyun. **98.** *Yechilishi*. Qo'shib yoziladigan sonni $x$ bilan beliglaymiz, u holda $523 \cdot 10^3 + x \equiv 0\ (mod\ 7 \cdot 8 \cdot 9)$, bu yerdan $x \equiv -523000 \equiv -352 \equiv 152\ (mod\ 504)$, yoki $x = 504t + 152$. $x$ ning qiymati $t = 0$ va $t = 1$ da uch xonali bo'ladi. Bu yerdan $x_1 = 152$, $x_2 = 656$. **99.** $x \equiv 200\ (mod\ 440)$, ya'ni $x \equiv 200; 640$. **100.** $x \equiv 30\ (mod\ 31)$, ya'ni $x = 30, 61, 42$. **101.** a) $x = 3 + 4t$, $y = 1 - 3t$; b) $x = 3 + 13t$, $y = -3 + 8t$; c) $x = 22 - 37t$, $y = -25 + 43t$; d) $x = 17 + 37t$, $y = 20 + 45t$; e) $x = 1 + 16t$, $y = 1 + 27t$; f) yechimga ega emas g) $x = 4 + 17t$, $y = -11 - 53t$; h) $x = 47 + 105t$, $y = 21 + 47t$; i) yechimi yo'q; j) $x = 4 + 16t$, $y = 7 - 11t$; k) $x = 9 + 37t$, $e = 3 + 12t$; l) $x = -7 + 15t$, $y = 12 - 23t$. **102.** $x = 2 - 4t$, $y = 4 + 3t$. $t = 0$ va $t = -1$ da talab qilingan hosil bo'ladi. **103.** $x = 3 - 5t$, $y = 28 + 3t$. **104.** a) $x = -4 + 13t$, $-100 < -4 + 13t < 150$, $-7 \le t \le 11$; 19 ta nuqta; b) 7 ta nuqta; s) 8 ta nuqta. **105.** *Yechilishi*. $5a - 9b = 31$ yoki $5a \equiv 31\ (mod\ 9)$, bu shartni qanoatlantiradigan $a$ ning eng kichik natural qiymati $a = 8$ dan iborat. $b$ ning qiymatini tenglamadan topamiz: $b = 1$. **106.** *Ko'rsatma*. AV to'g'ri chiziqning burchak koeffisiyenti $\frac{y_1 - y_2}{x_1 - x_2}$ qisqarmaydigan kasrdan iborat ekanligidan kelib chiqadi, $x_1 = x_2$ bo'lgan hol ko'rinib turibdi. **107.** Oldingi masalaga asoasan (uchburchakning uchlarini ham hisobga olganda) izlanayotgapn butun nuqtalar soni $(18, 6) + (12, 8) + (6, 14) + 3 = 12$ ga teng. **108.** a) $9x \equiv 1\ (mod\ 7)$, bu yerdan $x = 4 + 7t$; b) $x = 13 + 15t$. **109.** *Yechilishi*. Shartga asosan $15x + 20u + 30z = 500$ va $x + y + z = 18$, bu yerdan $y + 3z = 46$, $y \equiv 1\ (mod\ 3)$ yoki $u = 1 + 3t$. Demak, $3z = 45 - 3t$ va $z = 15 - t$. Bu yerdan $x + 16 + 2t = 18$ yoki $x = 2 - 2t$. Faqat $t = 0$ da natural yechimni hosil qilamiz. Demak, $x = 2$, $y = 1$, $z = 15$.

## § 4.

**110.** a) $x \equiv 49\ (mod\ 420)$; b) $x \equiv 4126\ (mod\ 6300)$; c) $x \equiv 85056\ (mod\ 130169)$; d) $x \equiv 9573\ (mod\ 13923)$; e) yechimi yo'q; f) yechimi yo'q; g) yechimi yo'q; h) $x \equiv 17\ (mod\ 90)$; i) $x \equiv 4\ (mod\ 105)$; j) $x \equiv 7777777\ (mod\ 91290457)$. **111.** $x \equiv 25\ (mod\ 60)$. **112.** 299 va 439. **113.** *Ko'rsatma.* $4x \equiv 9\ (mod\ 7)$; $2x \equiv 15\ (mod\ 9)$; $5x \equiv 12\ (mod\ 13)$ taqqoslamalar sistemasini yechish kerak, bu yerdan $x \equiv 291\ (mod\ 819)$. Nuqtalarning ordinatalari to'g'ri chiziqlarning berilgan tenglamalaridan kelib chiqadi. **114.** a) $x \equiv 4a - 3\ (mod\ 24)$, bu yerda $a \equiv 1\ (mod\ 2)$; b) $x \equiv 8 - 3a\ (mod\ 24)$, bu yerda $a \equiv 0\ (mod\ 2)$; c) $x \equiv 36a - 175\ (mod\ 630)$, bu yerda $a \equiv 1\ (mod\ 7)$; d) $x \equiv 15a + 21b - 35c\ (mod\ 105)$. **115.** a) $a \equiv 5\ (mod\ 6)$; b) $a \equiv 1\ (mod\ 6)$; c) $a \equiv 0\ (mod\ 4)$.

**116.** *Yechilishi*. Shartdan quyidagi sistemani hosil qilamiz: $xyz138 \equiv 0$ ($mod$ 7), $138xyz \equiv 6$ ($mod$ 13), $x1y3z8 \equiv 5$ ($mod$ 11). Birinchi taqqoslamani $10^3xyz + 138 \equiv 0$ ($mod$ 7) ko'rinishda yozib olamiz. U holda $3xyz \equiv 1$ ($mod$ 7), bu yerdan $xyz \equiv 5$ ($mod$ 7). Ikkinchi taqqoslama bilan ham xuddi shunday amallarni bajaramiz: $138000 + xyz \equiv 6$ ($mod$ 113), bu yerdan $xyz \equiv 1$ ($mod$ 13). Endi $xyz \equiv 1$ ($mod$ 13), $xyz \equiv 5$ ($mod$ 7) taqqoslamalar sistemasini $xyz$ ga nisbatan yechib, $xyz \equiv 40$ ($mod$ 91), yoki $xyz = 91t + 40$ ni hosil qilamiz. $t = 1, 2, 3, \dots, 10$ da $xyz = 131, 222, 313, \dots, 950$ larni topamiz. Uchinchi taqqoslamani $x \cdot 10^5 + 10^4 + u \cdot 10^3 + 3 \cdot 10^2 + z \cdot 10 + 8 \equiv 5$ ($mod$ 11) shaklda tasvirlab olamiz, soddalashtirishlardan so'ng $x + y + z \equiv 7$ ($mod$ 11)ni hosil qilamiz, ya'ni $x + y + z = 11t + 7$. $0 < x + y + z < 27$ tengsizlikni e'tiborga olib, $x + y + z = 7$ va $x + y + z = 18$ larni hosil qilamiz. U holda 131, 222, 313, ... , 950 sonlar ketma-ketligidan shartni qanoatlantiradigan 313138 va 495138 sonlarni topamiz. **117.** *Yechilishi*. Shartga asosan $13xy45z \equiv 0$ ($mod$ 792), ammo $792 = 8 \cdot 9 \cdot 11$, shuning uchun quyidagi sistemaga ega bo'lamiz:

$$\left.\begin{aligned} 13xy45z &\equiv 0 \pmod 8 \\ 13xy45z &\equiv 0 \pmod 9 \\ 13xy45z &\equiv 0 \pmod{11} \end{aligned}\right\}.$$

Birinchi taqqoslamadan va 8 ga bo'linish alomatidan $450 + z \equiv 0$ ($mod$ 8) ni hosil qilamiz, bu yerdan $z \equiv 6$ ($mod$ 8). $z = 6$ ni ikkinchi va uchinchi taqqoslamalarga qo'yib, sistemani hosil qilamiz:

$$\left.\begin{aligned} 13xy456 &\equiv 0 \pmod 9 \\ 13xy456 &\equiv 0 \pmod{11} \end{aligned}\right\}$$

Bu sistemaning birinchi taqqoslamasidan 9 ga bo'linish alomatiga asosan $x + y + 19 \equiv 0$ ($mod$ 9), yoki $x + y \equiv 0$ ($mod$ 9) kelib chiqadi. Ikkinchi taqqoslamani $1300000 + x \cdot 10^4 + u \cdot 10^3 + 456 \equiv 0$ ($mod$ 11) ko'rinishda tasvirlab olamiz, soddalashtirishlardan so'ng $x - u \equiv 8$ ($mod$ 11). U holdaa

$$\left.\begin{aligned} x + u &= 9t_1 + 8 \\ x - y &= 11t_2 + 8 \end{aligned}\right\}$$

Bu yerdan $x = 8$ va $u = 0$ ekanligi kelib chiqadi. Shunday qilib, izlangan son 1380456 dan iborat.

**118.** *Yechilishi*. Izlanayotgan sonii $x$ bilan belgilaymiz. U hol-da$x \cdot 1000 + (x+1) = 1001x + 1 = N^2$, yoki $(N+1)(N-1) = 7 \cdot 11 \cdot 13x$, bu yerdan

$$x = \frac{(N+1)(N-1)}{7 \cdot 11 \cdot 13}.$$

Bu tenglikdan $N$ va $x$ aniqlash uchun quyidagi taqqoslamalar sistemalarini hosil qilamiz:

1) $$\left.\begin{aligned} N+1 &\equiv 0 \pmod 7 \\ N-1 &\equiv 0 \pmod{143} \end{aligned}\right\}$$

Odatdagi usul bilan yechib $N=573$, $N^2=328329$, $x_1=328$ larni topamiz.

2) $$\left.\begin{aligned} N+1 &\equiv 0 \pmod{143} \\ N-1 &\equiv 0 \pmod 7 \end{aligned}\right\}.$$

Bu yerdan $N$=428, $N^2$=183184, $x_2$=183.

3) $\left.\begin{array}{l} N+1\equiv 0 \ (mod\ 11) \\ N-1\equiv 0 \ (mod\ 91) \end{array}\right\}$.

Bu yerdan $N$=274, $N^2$=075076, ammo $x$=075 ikki xonali son bo'lganligi uchun yechim emas.

4) $\left.\begin{array}{l} N+1\equiv 0 \ (mod\ 91) \\ N-1\equiv 0 \ (mod\ 11) \end{array}\right\}$

Sistemadan $N$=727, $N^2$=528529, $x_3$=528 larni hosil qilamiz.

5) $\left.\begin{array}{l} N+1\equiv 0 \ (mod\ 13) \\ N-1\equiv 0 \ (mod\ 77) \end{array}\right\}$

$N$=155, $N^2$=024025, ammo $x$=025 ikki xonali son bo'lganligi uchun yechim emas.

6) $\left.\begin{array}{l} N+1\equiv 0 \ (mod\ 77) \\ N-1\equiv 0 \ (mod\ 13) \end{array}\right\}$

Bu yerdan $N$=846, $N^2$=715716, $x_4$=715.

**119**. Shartdan quyidagi sistemani hosil qilamiz: $x\equiv 3 \ (mod\ 7)$, $x^2\equiv 44 \ (mod\ 7^2)$, $x^3\equiv 111 \ (mod\ 7^3)$. Birinchi taqqoslamadan $x=7t+3$ ni topamiz. $x$ ning bu qiymatini ikkinchi taqqoslamaga qo'yib, uni $t$ ga nisbatan yechamiz: $(7t+3)^2=44 \ (mod\ 7^2)$, soddalashtirishlardan so'ng, $42t \equiv 35 \ (mod\ 7^2)$. Bu taqqoslamani 7 ga qisqartirib, $6t\equiv 5(mod\ 7)$ ni hosil qilamiz, bu yerdan $t\equiv 2 \ (mod\ 7)$, ya'ni $t=7t_1+2$ bo'ladi. $t$ ning topilgan qiymatini $x=7t+3$ tenglikka qo'yib, $x=7(7t_1+2)+3=49t_1+17$ ni hosil qilamiz. $x$ ninng oxirgi topilgan qiymatini uchinchi taqqoslamaga qo'yib $(49\ t_1+17)^3 \equiv 111(mod\ 7^3)$ ni hosil qilamiz. Soddalashtirishlardan so'ng $t_1\equiv 0 \ (mod\ 7)$ ni topamiz, bu yerdan $t_1=7t_2+0$. $t_1$ bu qiymatini $x=49t_1+17$ tenglikka qo'yib, nihoyat $x\equiv(mod\ 7^3)$ ni topamiz. **120.** a) $x\equiv 24 \ (mod\ 28)$; b) $x\equiv 54 \ (mod\ 95)$; c) $x\equiv 39 \ (mod\ 77)$; d) $x\equiv -9 \ (mod\ 45)$; e) yechim yo'q; f)$x\equiv\pm 13;\ \pm 47 \ (mod\ 85)$. **121.** (-6,-61), (-1,-1), (1,1), (6,41). **122.** a) $x\equiv y\equiv 3 \ (mod\ 7)$; b) yechim yo'q; s) yechim yo'q; d) $x_1\equiv 0(mod\ 12)$, $y_1\equiv 7(mod\ 12)$ $x_2\equiv 4 \ (mod\ 12)$, $y_2\equiv 7 \ (mod\ 12)$; $x_3\equiv 8 \ (mod\ 12)$, $y_3\equiv 7 \ (mod\ 12)$. **123.** a) $x=k+5n$, $y=2k-2+10n$, $z=1-k-5n$, bu yerda $k$=0; 1; 2; 3; 4 va $n\in\mathbb{Z}$; b) Berilgan shartdan $x-u\equiv 1 \ (mod\ 3)$, $x+y\equiv 1(mod\ 2)$ sistemaga kelib chiqadi. Birinchi taqqoslamadan $y\equiv x - 1+3i \ (mod\ 6)$, bu yerda $i$=0; 1 kelib chiqadi. Bu qiymatni ikkinchi taqqoslamag qo'yib, $2x\equiv 2-3i \ (mod\ 2)$ ni hosil qilamiz, bu yerdan $3i\equiv 0 \ (mod\ 2)$ va $i$=0 kelib chiqadi. Demak, $x\equiv k(mod\ 6)$, $y\equiv k-1 \ (mod\ 6)$, $k$=0; 1; 2; 3; 4; 5, yoki $x=k+6n$, $y=k-1+6m$, bu yerda $m, n \in \mathbb{Z}$. Bu qiymatlarni berilgan sistemaning tenglamalariga qo'yib, $z=2n-2m=k-1+3n+3m$ ni hosil qilamiz, bu yerdan $n=1-k-5m$. Shunday qilib, berilgan tenglamalar sistemasining yechimlari $z=6-5k-30m$, $y=k-1+6m$, $x=2-2k-12m$ lardan iborat, bu yerda $k$=0; 1; 2; 3; 4; 5 va $m \in \mathbb{Z}$. **124.** *Ko'rsatma*. Masalani quyidagi taqqoslamalar sistemasini bilan yechish mumkin:

$$\left\{\begin{array}{l} 3x-u+1\equiv 0 \ (mod\ 7) \\ 2x+3y-1\equiv 0 \ (mod\ 7), \end{array}\right. \quad \text{bu yerdan} \quad \left\{\begin{array}{l} x\equiv 3 \ (mod\ 7) \\ u\equiv 5 \ (mod\ 7). \end{array}\right.$$

## MUNDARIJA

### I-bob. BUTUN SONLAR XALQASIDA BO'LINISH NAZARIYASI



### II-bob. BUTUN SONLAR XALQASIDA TAQQOSLAMALAR NAZARIYASI